TECHNISCHE UNIVERSITÄT CHEMNITZ

Fakultät für Mathematik

Chemnitz FEM-Symposium 2007

Programme

Collection of abstracts

List of participants

Chemnitz, September 24 - 26, 2007

Scientific topics:

The symposium is devoted to all aspects of finite elements and wavelet methods in partial differential equations.

The topics include (but are not limited to)

- adaptive methods,
- parallel implementation,
- high order methods.

This year we particularly encourage talks on

- Computational Fluid Dynamics
- Contact problems and large deformations
- Mortar methods / Discontinuous Galerkin methods
- Solvers and preconditioners

Invited Speakers:

Lutz Tobiska (Magdeburg)

Ralf Kornhuber (Berlin)

Peter Hansbo (Goeteborg)

Arnold Reusken (Aachen)

Scientific Committee:

Th. Apel (München), G. Haase (Graz), B. Heinrich (Chemnitz), M. Jung (Dresden), U. Langer (Linz), A. Meyer (Chemnitz), O. Steinbach (Graz)

WWW: http://www.tu-chemnitz.de/mathematik/fem-symposium/

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Chemnitz, September 24 - 26, 2007

Programme for Monday, September 24, 2007

9:00 A. Meyer
Welcome

Computational Fluid Dynamics
Chairman: A. Meyer
Room: "Schlossblick"

9:05 L. Tobiska 50
Stabilization by local projection. Convection-diffusion and incompressible flow problems

9:55 A. Baran 7
A high-order non-conforming finite element family

10:20 M. Wohlmuth 52
Numerical Analysis of Stokes Equations with Improved LBB Dependency

10:45 *Tea and coffee break*

	CFD stabilisation *Chairman*: L. Tobiska *Room*: "Schlossblick"	New developments *Chairman*: R. Griesse *Room*: "von Taube"
11:10	G. Lube .... 31 Stabilized FEM for incompressible flows: Equal-order vs. inf-sup stable approximation	B. Haasdonk .... 18 Reduced Basis Method for Evolution Schemes with Nonlinear Explicit Operators
11:35	G. Matthies .... 33 On streamline-diffusion methods for inf-sup stable discretisations of the generalised Oseen Problem	T.J. Dijkema .... 10 Solving elliptic PDEs in high space dimensions
12:00	H.-G. Roos .... 39 Stabilized FEM and related difference schemes	L. Nannen .... 34 Hardy space infinite element method
12:25	M. Roland .... 38 Numerical Simulation of a Calcium Carbonate Precipitation	M.V. Popov .... 35 Piecewise Parabolic Method on Local stencil

12:50 *Lunch break*

Mortar, Nitsche and Discontinuous Galerkin methods
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Room: “Schlossblick”

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14:50	R. Ernst Decoupling of Nonlinearities in Acoustic-Structure Interaction	13
15:15	M. Juntunen An Unconditionally Stable Mixed Discontinuous Galerkin Method	25
15:40	M. Feistauer On some aspects of the DGFEM for convection-diffusion problems	14
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	Mortar, Nitsche and DG *Chairman*: M. Feistauer *Room*: “Schlossblick”		**Adaptive methods** *Chairman*: T. Apel *Room*: “von Taube”	
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16:55	S. Franz A new Approach to Recovery of Discontinuous Galerkin	15	S. Zaglmayr Equilibrated residual-based error estimators for Poisson and Maxwell’s equations	53
17:20	M. Vlasak Higher order time discretizations for scalar nonlinear convection-diffusion equation	51	M. Grajewski A fast and accurate method for grid deformation	17
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	Mortar, Nitsche and DG *Chairman*: P. Hansbo *Room*: “Schlossblick”		**Adaptive methods** *Chairman*: M. Fröhner *Room*: “von Taube”	
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18:20	M. Huber Simulation of Diffraction in periodic Media with a coupled Finite Element and Plane Wave Approach	21	O. Sander Dune: The Distributed and Unified Numerics Environment	41
18:45	B. Jung Nitsche mortaring for parabolic initial-boundary value problems	24	J.M. Fried Parallel Segmentation of Large Images via Level Sets	16
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10:45	M. Stiemer ... 48 Finite element simulation of electromagnetic metal forming
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	CFD stabilisation *Chairman*: H.-G. Roos *Room*: "Schlossblick"	**FEM and BEM** *Chairman*: O. Steinbach *Room*: "von Taube"
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Fluid-structure interaction
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Room: "Schlossblick"

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Solver
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Overlapping Additive Schwarz preconditioners for degenerate problems

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Mixed finite elements for linear elasticity

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Room: "Schlossblick"

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On domain-robust preconditioners for the Stokes equations

11:10 A. Linke 30
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On discrete projection methods for rotating incompressible flow with Coriolis force

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Preconditioning of Elliptic Boundary Value Problems

12:25 A. Meyer
Closing

A high-order non-conforming finite element family

Agnes Baran[1] Gisbert Stoyan[2]

We describe a triangular non-conforming finite element pair for the two-dimensional Stokes problem [6]. It is a generalization of the low order cases described by Crouzeix and Raviart [3], by Fortin and Soulie [4] and by Crouzeix and Falk [2]. The velocity and the pressure are approximated trianglewise by polynomials of order k and k-1, respectively, and the element is defined for all k≥1. For even k the finite element pair can be obtained by adding trianglewise a non-conforming bubble function of order k to the local basis of the velocity space of the conforming $\mathbb{P}_k/\mathbb{P}_{k-1}$ element, and we give a general formula for these bubble functions. In the case of odd k we describe a set of degrees of freedom for the velocity part of the element. For even k using a modification of the macroelement technique of Stenberg [5] we prove that the finite element pair is inf-sup stable [1].

References:

[1] Á. Baran, G. Stoyan, Gauss-Legendre elements: a stable, higher order non-conforming finite element family, *Computing* **79**, 1–21 (2007).
[2] M. Crouzeix, R. S. Falk, Nonconforming finite elements for the Stokes problem, *Mathematics of Computation* **186**, 437–456 (1989).
[3] M. Crouzeix, P. A. Raviart, Conforming and non conforming finite element methods for solving the stationary Stokes equations, *RAIRO Analyse Numérique* **7**, 33–76 (1973)
[4] M. Fortin, M. Soulie, A non-conforming piecewise quadratic finite element on triangles, *International Journal for Numerical Methods in Engineering* **19**, 505–520 (1983).
[5] R. Stenberg, Analysis of mixed finite element methods for the Stokes problem: a unified approach, *Mathematics of Computation* **165**, 9–23 (1984).
[6] G. Stoyan, Á. Baran, Crouzeix-Velte decompositions for higher-order finite elements, *Computers and Mathematics with Applications* **51**, 967–986 (2006).

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Overlapping Additive Schwarz preconditioners for degenerate problems

Sven Beuchler[1]

In this paper, we consider some degenerated boundary value problems on the unit square. These problems are discretized by piecewise linear finite elements on a triangular mesh of isosceles right-angled triangles. The system of linear algebraic equations is solved by a preconditioned gradient method using a domain decomposition preconditioner with overlap. We prove that the condition number of the preconditioned system is bounded by a constant which independent of the discretization parameter. Moreover, the preconditioning operation requires $\mathcal{O}(N)$ operations, where N is the number of unknowns. Several numerical experiments show the preformance of the proposed method.

This a joint work with S. Nepomnyaschikh (Novosibirsk).

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BEM-based FEM for Helmholtz and Maxwell equations on arbitrary polyhedral meshes

Dylan Copeland[1]

We present new finite element methods for the Helmholtz and Maxwell equations on arbitrary three-dimensional polyhedral meshes, with boundary elements on the surfaces of the polyhedral elements. The methods are based on domain decomposition techniques, treating the polyhedral elements as subdomains. On a triangular mesh of the skeleton, we use the lowest order polynomial spaces and obtain sparse, symmetric linear systems despite the use of boundary elements. Moreover, piecewise constant coefficients are permissible. The resulting approximation on the skeleton mesh can be extended throughout the domain via representation formulas.

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Solving elliptic PDEs in high space dimensions

Tammo Jan Dijkema[1] Rob Stevenson[2]

We consider the problem of finding $u \in H_0^1(0,1)^d$ such that $a(u,v) = f(v)$ for all $v \in H_0^1(0,1)^d$, where a is an elliptic bilinear form. Specifically, we try to do this for large space dimensions d. When the unknown solution u is approximated using standard isotropic approximation with piecewise polynomials of a fixed degree, we run into the so-called 'curse of dimensionality': the convergence rate is inversely proportional to d.

Using that $(0,1)^n$ is a tensor-product domain, the curse of dimensionality can be circumvented using a sparse tensor product approximation. However, this can only be expected to work when special regularity conditions are met. Already for the Poisson equation with constant (non-zero) right-hand side, this is not the case.

We use an adaptive wavelet approximation method, which reaches a convergence rate as that of the best N-term approximation, in linear complexity. For this, we use orthogonal tensor product wavelets based on the multiwavelets of Donovan et al.

Numerical results will be shown for experiments in high dimensions, illustrating the convergence rate.

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On domain-robust preconditioners for the Stokes equations

Manfred Dobrowolski[1]

Many, if not all preconditioners for the Stokes equations depend on the LBB constant $L(\Omega)$ which behaves like $O(a^{-1})$ on domains Ω with aspect ratio a. This fact leads to a poor convergence behavior of the preconditioned method on elongerated domains such as channels which are typical for fluid flow problems. This drawback can be completely removed by using a second, very rough grid on which the small eigenvectors of the corresponding Schur complement eigenvalue problem can be efficiently represented.

In the second part we present a class of BPX-type preconditioners for the Stokes equations which are domain-robust and efficient but also completely parallel. Numerical results are given which demonstrate the effectivity of the presented methods.

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On discontinuous Galerkin methods for convection-dominated elliptic problems

Herbert Egger[1] Joachim Schöberl[2]

Convection dominated elliptic problems cannot be solved on a reasonable discretization level by standard conforming Galerkin FEM. Moreover, stabilization by upwinding techniques like SUPG is known to introduce significant artificial diffusion, which has certain drawbacks, e.g., the widening of boundary layers.

For this reason, discontinuous Galerkin (DG) methods, which are well-known for their good performance for the limiting hyperbolic problem, have been applied also to elliptic problems. A major disadvantage of DG methods is that the discretization of the elliptic operator leads to increased stencils, i.e., the resulting system matrices are much less sparse then the ones coming from conforming methods.

We propose a new method for convection-diffusion problems, which is composed of a mixed method for the elliptic operator and a discontinuous Galerkin discretization of the convective part. This method has the stability and conservation property of DG methods while leading to much sparser systems. Moreover, we outline the convergence analysis and present numerical tests.

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Decoupling of Nonlinearities in Acoustic-Structure Interaction

Roland Ernst[1] Bernd Flemisch[2] Barbara Wohlmuth[3]

The wide field of fluid-structure interactions contains many applications for elasto-acoustic coupling problems, especially nonlinear ones. Linear elasticity is restricted to the case of small deformations. Nonlinear acoustic effects including dissipation and generation of higher harmonics play an important role in ultrasonics. The choice of a velocity potential or pressure based formulation of the acoustic part and quite often a displacement one of the elastic part results in Dirichlet-Neumann type interface conditions.
In this presentation, decoupling strategies and algorithms for the nonlinear elasto-acoustic system are discussed. The nonlinear aspect is investigated in both, the elastic and acoustic part, and compared with each other. A mortar finite element discretization in combination with a generalized Newmark scheme leads to a system on which a subspace iteration for solving the nonlinear equation and a second iteration for the elasto-acoustic connection is applied. The corresponding solvability results for this kind of coupling problems are analyzed. Two illustrative numerical schemes using different solvers for the subspace iteration, a Newton-like and a fixed point iteration method are given. In particular a better performance, i.e. smaller convergence rates, is obtained for a multiplivative Schwarz method. Computational calculations show the superior behavior to an additive Schwarz method. An additional improvement of the convergence rate is achieved with the introduction of a small region where both subdomains overlap.

References:

[1] B. Flemisch, M. Kaltenbacher, and B. I. Wohlmuth. Elasto-acoustic and acoustic-acoustic coupling on non-matching grids. *Internat. J. Numer. Methods Engrg.*, 67(13):1791–1810, 2006.
[2] R. Ernst, B. Flemisch and B. I. Wohlmuth. Decoupling of Nonlinearities in Acoustic-Structure Interaction. In preparation, 2007.

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On some aspects of the DGFEM for convection-diffusion problems

Miloslav Feistauer[1] V. Dolejsi[2] V. Kucera[3]

In this paper we shall be concerned first with optimal error estimates for the numerical solution of nonlinear convection-diffusion problems by the discontinuous Galerkin finite element method (DGFEM). The main emphasis will be paid to the analysis of $L^\infty(L^2)$-optimal error estimate using general nonconforming meshes.

In the second part, some applications of the DGFEM to the simulation of compressible flow will be presented. Our goal is to develop sufficiently accurate, efficient and robust numerical schemes allowing the solution of compressible flow for a wide range of Mach numbers.

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A new Approach to Recovery of Discontinuous Galerkin

Sebastian Franz[1] Lutz Tobiska[2] Helena Zarin[3]

A new recovery operator $P : Q_n^{disc}(\mathcal{T}) \to Q_{n+1}^{disc}(\mathcal{M})$ for discontinuous Galerkin is derived. It is based on the idea of projecting the local polynomial solution on a given mesh $\mathcal{T}$ into a higher order polynomial space on a macro mesh $\mathcal{M}$. Therefore, we define local degrees of freedom using Legendre polynomials and provide global degrees of freedom on the macro mesh.
We prove consistency

$$Pu = Pu^I, \forall u \in L_1(M)$$

with u^I as the local L_2-projection, stability results in several norms and optimal anisotropic error estimates.
In case of bilinear elements and the singularly perturbed problem

$$\begin{aligned} -\varepsilon\Delta u + \mathbf{b}\cdot\nabla u + cu &= f, \quad \text{in } \Omega = (0,1)\times(0,1) \\ u &= 0, \quad \text{on } \Gamma = \partial\Omega \end{aligned}$$

we use a supercloseness result by H.-G. Roos and H. Zarin to prove superconvergence of the postprocessed numerical solution.

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Parallel Segmentation of Large Images via Level Sets

J. Michael Fried[1]

In several applications like remote sensing or medical imaging, a fast and accurate segmentation of large two or three dimensional images into a a priori unknown number of classes (i.e. segments) is wanted. The aim is to find homogeneous regions Ω^i and their boundaries Γ inside a given, possibly noisy two or three dimensional image $g : \Omega \to [0, 1]$ and a piecewise constant approximation u to g which is constant inside of the segments Ω^i but may jump across the boundary Γ. We present a parallel adaptive finite element algorithm for segmentation of large images with automatic adjustment of the maximal number of possible classes, which scales close to optimal. It is based on a level set formulation of the Minimal Partition approach proposed by Chan and Vese and suitable also for two and three dimensional multichannel data.

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A fast and accurate method for grid deformation

Matthias Grajewski[1] Stefan Turek[2]

In the context of hardware-oriented numerics, grid deformation, e.g. the relocation of grid points preserving the topology of the mesh, is a viable way for grid adaption and offers an alternative to the widespread h-adaptivity. We present a fast and accurate grid deformation method which -unlike many common approaches to grid deformation- works independently of the underlying problem and thus acts as black-box tool. It requires the solution of one global Poisson problem and one ODE for each grid point only. We show the convergence of our method. Moreover, exploiting the given grid hierarchy, this grid deformation method is of almost optimal complexity with respect to the number of vertices in the mesh.

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Reduced Basis Method for Evolution Schemes with Nonlinear Explicit Operators

Bernard Haasdonk[1] Mario Ohlberger[2] Gianluigi Rozza[3]

During the last decades, reduced basis (RB) methods have been developed to a wide methodology for model reduction of problems that are governed by parametrized partial differential equations [1]. In particular equations of elliptic and parabolic type for linear, low polynomial or monotonic nonlinearities have been treated successfully by RB methods using finite element schemes. Due to the characteristic offline-online decomposition, the reduced models often become suitable for a multi-query or real-time setting, where simulation results, such as field-variables or output estimates, can be approximated reliably and rapidly for varying parameters.

In the current study, we address a certain class of time-dependent nonlinear evolution schemes. We extend the linear scheme [2] with a general nonlinear explicit space-discretization operator. We extend the RB-methodology to these cases by applying the *empirical interpolation* method [3] to localized discretization operators. The main technical ingredients are: (i) generation of a *collateral reduced basis* modelling the nonlinearity under parameter variations in the offline-phase and (ii) an online simulation scheme based on localized evaluations of the evolution operator.

Numerical experiments on a parametrized convection-diffusion problem, discretized with a finite volume scheme, demonstrate the applicability of the model reduction technique. We obtain a parametrized reduced model, which enables parameter variation with fast simulation response. We quantify the computational gain with respect to the non-reduced model and investigate the error convergence.

[1] A.T. Patera and G. Rozza. *Reduced Basis Approximation and A Posteriori Error Estimation for Parametrized Partial Differential Equations.* Version 1.0, Copyright MIT 2006-2007, to appear in (tentative rubric) MIT Pappalardo Graduate Monographs in Mechanical Engineering.

[2] B. Haasdonk and M. Ohlberger. *Reduced Basis Method for Finite Volume Approximations of Parametrized Evolution Equations.* Preprint 12/2006, Institute of Mathematics, University of Freiburg, 2006, submitted.

[3] M. Barrault, Y. Maday, N.C. Nguyen, and A.T. Patera. *An 'Empirical Interpolation' Method: Application to Efficient Reduced-Basis Discretization of Partial Differential Equations.* C.R. Acad. Sci. Paris Series I, 339:667-672, 2004.

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Stabilized mortar methods for interface problems

Peter Hansbo[1]

Usually, mortaring of meshes across interfaces is performed by use of Lagrange multiplier techniques in order to enforce continuity of the solution. The multiplier space can not be chosen independently of the trace meshes adjacent to the interface if stability of the method is to be maintained. In this talk I will present three different approaches to avoiding this problem by introducing artificial but consistent stabilization. These methods have different characteristics: one requires a mesh to define the multiplier space, one allows for greater freedom in choosing multipliers (e.g., as global polynomials), and one formally does without multipliers altogether (Nitsche's method). Finally, I will give an outlook on different uses of Nitsche's method in computational mechanics.

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Sparse Second Moment Analysis for Potentials on Stochastic Domains

Helmut Harbrecht[1]

This talk is concerned with the numerical solution of Dirichlet problems in domains $D \in \mathbb{R}^d$ with random boundary perturbations. Assuming normal perturbations with small amplitude and known mean field and two-point correlation function, we derive, using a second order shape calculus, deterministic equations for the mean field and the two-point correlation function of the random solution for the Dirichlet problem in the stochastic domain.

The two-point correlation of the random solution satisfies a boundary value problem on the tensor product domain $D \times D$. It can be approximated in sparse tensor product spaces. This yields densely populated system matrices, independently of using the finite element method in $D \times D$ or the boundary element method on $\partial D \times \partial D$.

We present and analyze algorithms to approximate the random solution's two-point correlation function in essentially $\mathcal{O}(N)$ work and memory, where N denotes the number of unknowns required for consistent discretization of the domain (in case of finite element methods) or its boundary (in case of boundary element methods). Here "essentially" means up to powers of $\log N$.

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Simulation of Diffraction in periodic Media with a coupled Finite Element and Plane Wave Approach

Martin Huber[1] Joachim Schöberl[2]

Electromagnetic waves propagating towards a grating are diffracted and transmitted into certain spatial directions. While the calculation of these directions, which depend only on the period of the structure, is easy to perform, the corresponding intensities, depending strongly on the shape, are much more complicated to compute.

Modeling such a grating with FEM, we have to solve Maxwell's equations. Due to the periodicity of the system, we are able to use the theorem of Bloch-Floquet, and the computational domain can be reduced to a single unit cell by formulating quasi periodic boundary conditions. In order to compare simulation results with optical experiments, it is useful to express the far field by plane waves and exponential decaying functions. A critical point is to couple these plane waves with the polynomial basis functions of the FEM domain. The innovation of our approach is to perform this coupling by Nitsche's method.

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Mortar discretization for dynamical thermomechanical contact problems with friction

Stefan Hüeber[1] Barbara Wohlmuth[2]

The numerical simulation of dynamical nonlinear contact problems with friction plays an important role for a wide range of technical appplications. For such problems, nonconforming domain decomposition techniques such as mortar methods provide a powerful and flexible tool. In this talk we present the formulation and the discretization of such type of problems with the mortar approach. To solve the arising nonlinear equations we use primal-dual active set strategies which can be interpreted as a semi-smooth Newton method. We focus on the treatment of Coulomb friction in the three-dimensional case and investigate a full Newton method which shows a superior convergence behavior in contrast to the widely used fixed point approach. To avoid the spurious oscillations in the Lagrange multiplier modeling the contact stresses over time, we use a modified mass matrix. This matrix results from a different integration formula on the elements near the possible contact zone. Furthermore, we extend this approach to the case where thermomechanical effects are involved. We especially consider the formulation, the discretization and algorithmic aspects of dynamical thermomechanical effects such as frictional heating and thermal softening at the contact interface. In addition we focus on the Robin-type interface condition for the heat and its treatment by the mortar method.To solve the arising nonlinear conditions at the contact interface we extend semi-smooth Newton methods for the purely mechanical problem to the thermo-dynamical case.In the last part we consider the extension of the primal-dual active set method to the case of large deformation contact problems. Numerical examples both in the two-dimensional and the three-dimensional setting illustrate the flexibility, robustness and efficiency of the proposed algorithms.

References:

[1] C. Hager, S. Hüeber, and B.I. Wohlmuth. A stable energy conserving approach for frictional contact problems based on quadrature formulas. *Internat. J. Numer. Methods Engrg.*, to appear.
[2] S. Hüeber and G. Stadler and B.I. Wohlmuth, A primal-dual active set algorithm for three-dimensional contact problems with Coulomb friction, *SIAM J. Sci. Comput.*, to appear.
[3] S. Hüeber, and B.I. Wohlmuth. Mortar discretization for dynamical thermomechanical contact problems with friction. In preparation.

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Adaptive High Order Nodal Finite Elements on Simplices For Electrocardiological Models

Nicolas Jeannequin[1] Jonathan Whiteley[2] David Gavaghan[3]

We present an implementation of a pseudo-spectral scheme on unstructured simplices to solve the bidomain equations. They describe the macroscale electrical activity of cardiac tissue and is coupled to a cellular model, such as the Luo-Rudy '91 model (C.H. Luo and Y. Rudy. A model of the ventricular cardiac action potential-depolarisation, repolarisation and their interaction. Circulation Research, 68:1501-1526, 1991). The interaction of many different spatial and temporal scales makes this model difficult to solve accurately.

We describe an explicit method for the construction of nodes that are suitable for polynomial interpolation on simplices. These nodes are used in our implementation of the pseudo-spectral scheme. They show a performance on par with Fekete nodes up to 12th order. However, their very simple construction makes them much more desirable.

The bidomain model is solved using an adaptive method in both time and space. All known approaches to time and space adaptivity applied to this problem have concentrated on the use of low order methods. Our spatial discretisation is a high order h-adaptive continuous Galerkin method which is coupled to an adaptive linearly implicit Rosenbrock time integration scheme. The spatial mesh size is determined by the gradient of the transmembrane and extracellular potentials. We also show high order an h-adaptive discontinuous Galerkin method which is coupled to an adaptive linearly implicit Rosenbrock time integration scheme. Both these approaches are compared in terms of computational time and accuracy.

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Nitsche mortaring for parabolic initial-boundary value problems

Beate Jung[1] Bernd Heinrich[2]

In this talk we present a method for the numerical solution of parabolic initial-boundary value problems in two-dimensional domains. The Nitsche-finite-element method (as a mortar method) is applied for the discretization in space, i.e. non-matching meshes are used. For the discretization in time, the backward Euler method is employed.
The rate of convergence in some H^1-like norm and in the L_2-norm is proved for the semi-discrete as well as for the fully discrete problem. In order to improve the accuracy of the method in presence of singularities arising in case of non-convex domains, meshes with local grading are employed for the Nitsche-finite-element method.

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An Unconditionally Stable Mixed Discontinuous Galerkin Method

Mika Juntunen[1] Rolf Stenberg[2]

In discontinuous Galerkin method continuity can be forced in various ways. One quite popular way to force continuity in discontinuous approach is the Nitsche method. The method is based on handling of non-homogenous Dirichlet boundary conditions by J.A. Nitsche (1970). However, for non-mixed problems the method is elliptic (or stable) only for large enough values of stability parameter. Lower limit of the stability parameter depends on the degrees of freedom, e.g. on polynomial order of the elements. Even though the lower limit can be computed analytically, it may cause problems in for example p-refinement (i.e. refinement by increasing the polynomial order of the elements). Therefore it is interesting to notice that we can give up the stability parameter if we pose the problem in a mixed form.

In this work we show that for mixed problem the Nitsche method is stable for all values of the stability parameter, hence we can give up the parameter entirely. We also propose a residual based a posteriori error estimate for the method. The proof the a posteriori estimate is based on the Helmholtz decomposition, in which the gradient is decomposed into divergence free and rotation free parts. The Helmholtz decomposition technique is also applicable in other similar proofs. The advantage of the Helmholtz decomposition in the discontinuous framework is that we can still use the conventional interpolants, for example Clément's interpolants, since the members of the decomposition are continuous and we only need the interpolants for the decomposition.

Our model problem is the mixed form of the Poisson equation, in other words, the gradient of the solution is also a variable. Solving mixed equations naturally increases size of the problem. However, we can reduce the increase of the computational work conciderably with local condensation. We show that it is possible to solve the gradient part of the mixed solution already from the local, elementwise, matrix equation. Thus the size of the ultimate matrix equation is the same as usual, only the assembly is more toilsome since every element requires solving a small matrix equation.

In addition to the analytical results we show numerical examples supporting the results.

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On the definition of the SUPG parameter

Petr Knobloch[1]

The SUPG method is one of the most popular finite element approaches for the numerical solution of convection dominated problems. However, the quality of the approximate solution strongly depends on the definition of a stabilization parameter. The aim of this talk is to propose a new definition of this parameter and to demonstrate that it significantly reduces spurious oscillations in the approximate solution. As a model problem we shall consider a two-dimensional steady scalar convection-diffusion equation.

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Adaptive Finite Element Algorithms of Optimal Complexity

Yaroslav Kondratyuk[1]

Aiming for an optimal algorithm for solving numerically partial differential equations (PDEs), we are naturally led to the so-called adaptive method. Simply speaking, the task of an adaptive method is to adapt the approximation to the unknown solution of a differential equation during the solution process, using only a posteriori information, and to produce eventually the (quasi-) best possible approximation with optimal computational costs. Nowadays Adaptive Finite Element algorithms are being used to solve efficiently PDEs arising in science and engineering.

In this talk we present a detailed design of Adaptive Finite Element Algorithms and analysis of their convergence, convergence rates and computational complexity.

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First steps towards efficient and reliable simulation of the human gait.

Ralf Kornhuber[1] Peter Deuflhard Rolf Krause Oliver Sander

A precise prediction of loads and forces within human joints would support surgical decisions and thus increase the overall success of surgery procedures. However, in vivo measurements are hardly possible and credible numerical simulation is demanding task. In this talk, we consider 3D finite element models of the human knee and particularly concentrate on efficient and reliable multigrid solvers for the arising two body contact problems, heterogeneous domain decomposition methods for the coupling/decoupling of bones and ligaments and contact-stabilized time discretizations.

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Nonsmooth Decomposition Methods for Frictional Contact Problems in Elasticity

Rolf Krause[1]

In this talk, we give a framework for the construction of non-linear and non-smooth decomposition methods for the efficient and robust solution of frictional contact problems in space and time. Taking the locality of the friction law and the non-penetration constraints into account, multiscale methods offer the possibility to use different models on different scales concurrently for the modeling as well as the simulation process. These can be used to "separate" interface processes and the processes within the material. From the point of view of the arising discrete large scale problems, however, domain decomposition seem to be a natural approach for the construction of non-linear solution methods. We present an adaptive non-linear parallel solution method for frictional contact problems, which uses ideas from multiscale as well as domain decomposition methods. Our general approach allows also for the treatment of coupled problems as occur in thermoelasticity.

In case of complicated gepometries and materials as occur, moreover topics as the information transfer between non-matching meshes at curvilinear boundaries as well the treatment of non-linear material laws have to be considered. We discuss some new ideas for mortar-based transfer operators at the contact interface and consider also shortly the case of non-linear material laws and large deformations.

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Non-Nested Multi-Grid Solvers for Mixed Divergence-free Scott-Vogelius Discretizations

Alexander Linke[1] Gunar Matthies[2] Lutz Tobiska[3]

In [1] a general framework for analyzing the convergence of multi-level methods for mixed finite element problems has been established. Important applications for this framework are nearly all practically relevant conforming and non-conforming mixed finite element discretizations of the Stokes problem. When applying this framework to discretizations of the Navier-Stokes equation, additional difficulties have to be taken into account, e.g. the grad-div stabilization for stabilizing weak mass conservation.

In the talk, we present basic results for a recently proposed conforming stabilized discretization of the full Navier-Stokes problem [2]. The discretization is based on the Scott-Vogelius element, and uses symmetric stabilization operators for stabilizing dominant convection. Here, discrete approximations to the Navier-Stokes problem are pointwise divergence-free. Problems induced by the cumbersome grad-div stabilization are therefore circumvented. For this discretization, we apply the general multi-grid framework [1] and analyze a generalized Stokes problem including the symmetric stabilization operator for dominant convection. Since the Scott-Vogelius element has only been proven to be LBB-stable on meshes, which are derived from a macroelement triangulation, the multigrid hierarchy is non-nested.

References:

[1] V. John, P. Knobloch, G. Matthies L. Tobiska. Non-nested multi-level solvers for finite element discretisations of mixed problems. *Computing*. 68:313–341, 2002.
[2] E. Burman, A. Linke. Stabilized finite element schemes for incompressible flow using Scott-Vogelius elements. *submitted to Applied Numerical Mathematics*. 2007.

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Stabilized FEM for incompressible flows: Equal-order vs. inf-sup stable approximation

Gert Lube[1] G. Rapin[2]

A recent approach to the variational multiscale approach to incompressible flows consists in stabilization of the standard Galerkin finite element method using the local projection approach [1]. In the present paper, we take advantage of a general framework for the a-priori analysis of such methods given in [2] for linearized Navier-Stokes problems of Oseen type.

The case of equal-order approximation for velocity and pressure can be found in [1], [2]. The case of inf-sup stable finite element pairs is considered in [3]. Here, we give a critical comparison for a h/p-version of both variants. Numerical examples for incompressible Navier-Stokes flows support the discussion.

References:

[1] M. Braack, E. Burman, "Local projection stabilization for the Oseen problem and its interpretation as a variational multiscale method", SIAM J. Numer. Anal. 43, 6 (2006), 2544-2566.
[2] G. Matthies, P. Skrzypacz, L. Tobiska "A unified convergence analysis for local projection stabilizations applied to the Oseen problem", to appear in M^2AS 2007.
[3] G. Rapin, G. Lube "Local projection stabilizations for inf-sup stable finite elements applied to the Oseen problem", NAM-Preprint. University of Göttingen 2007.

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Mixed fem-bem coupling for non-linear transmission problems with Signorini contact

Matthias Maischak[1]

Here we generalize the approach in [4] and discuss an interface problem consisting of a non-linear partial differential equation in $\Omega \subset \mathbb{R}^n$ (bounded, Lipschitz, $n \geq 2$) and the Laplace equation in the unbounded exterior domain $\Omega_c := \mathbb{R}^n \backslash \bar{\Omega}$ fulfilling some radiation condition, which are coupled by transmission conditions and Signorini conditions imposed on the interface. The interior pde is discretized by a mixed formulation, whereas the exterior part of the interface problem is rewritten using a Neumann to Dirichlet mapping (NtD) given in terms of boundary integral operators.

We treat the general numerical approximation of the resulting variational inequality and discuss the non-trivial discretization of the NtD mapping. Assuming some abstract approximation properties and a discrete inf-sup condition we prove existence and uniqueness and show an a-priori estimate, which generalizes the results in [4]. Choosing Raviart-Thomas elements and piecewise constants in Ω and hat functions on $\partial\Omega$ the discrete inf-sup condition is satisfied [1]. We present a solver based on a modified Uzawa algorithm, reducing the solution procedure of the non-linear saddle point problem with an inequality constraint to the repeated solution of a standard non-linear saddle point problem and the solution of a variational inequality based on an elliptic operator. Finally, we present a residual based a-posteriori error estimator compatible with the Signorini condition and a corresponding adaptive scheme, see [5].

Some numerical experiments are shown which illustrate the convergence behavior of the uniform h-version with triangles and rectangles and the adaptive scheme as well as the bounded iteration numbers of the modified Uzawa algorithm, underlining the theoretical results.

REFERENCES

[1] I. Babuška, G. Gatica "On the mixed finite element method with Lagrange multipliers", Numer. Methods Partial Differential Equations, Vol. 19, pp. 192–210, (2003).

[2] M. Barrientos, G. Gatica, M. Maischak "A-posteriori error estimates for linear exterior problems via mixed-FEM and DtN mappings", Mathematical Modelling and Numerical Analysis, Vol. 36, pp. 241–272 (2002).

[3] G. Gatica, M. Maischak "A posteriori error estimates for the mixed finite element method with Lagrange multipliers", Numerical Methods for Partial Differential Equations, Vol. 21, pp. 421–450, (2005).

[4] G. Gatica, M. Maischak, E.P. Stephan "Mixed FEM and BEM coupling for a transmission problem with Signorini contact", NMPDE (submitted)

[5] M. Maischak "A posteriori error estimates for a mixed fem-bem coupling method with Signorini contact", (submitted).

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On streamline-diffusion methods for inf-sup stable discretisations of the generalised Oseen Problem

Gunar Matthies[1] Gert Lube[2]

We consider stabilised finite element methods for the generalised Oseen problem with homogeneous Dirichlet boundary conditions. The unique solvability based on a modified stability condition and an error estimate are proved for inf-sup stable discretisations of velocity and pressure. We will show that it is possible to separate the stabilisation terms of streamline-diffusion type (SUPG) and the pressure stabilisation terms (PSPG). This extends a recent result in [1] on quasi-uniform meshes and continuous pressure approximations to general shape-regular meshes and to discontinuous pressure interpolation. Numerical examples which show the influence of the different stabilisation parameters will be presented.

References:

[1] T. Gelhard, G. Lube, M. A. Olshanskii, and J.-H. Starcke Stabilized finite element schemes with LBB-stable elements for incompressible flows, J. Comput. Appl. Math., 177 (2005), pp. 243–267.

[2] G. Matthies and G. Lube, On streamline-diffusion methods of inf-sup stable discretisations of the generalised Oseen problem, Preprint 2007-02, Institut für Numerische und Angewandte Mathematik, Georg-August-Universiät Göttingen, 2007.

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Hardy space infinite element method

Lothar Nannen[1] Thorsten Hohage[2]

We present a new infinite element method for solving Helmholtz-type scattering and resonance problems. Physical solutions are characterised by their Laplace transform in radial direction, which belong to the Hardy space of the negative complex half plane. After another transformation a symmetric variational formulation in the Hardy space of the complex unit disc is derived. Using a Galerkin method with respect to the monomial basis of this Hardy space we obtain super-algebraic convergence. Uniqueness and equivalence to usual characterisations of physical solutions can be proven. In contrast to usual infinite element methods the Hardy space method depends linearly on the square of the wave number. Therefore it can be used for computing resonances in unbounded domains.

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Piecewise Parabolic Method on Local stencil

Mikhail V. Popov[1] Sergey D. Ustyugov[2]

A new numerical method for solving of fluid dynamics and magnetic hydrodynamics equations is suggested. This method is a modification of popular Piecewise Parabolic Method (PPM) (P. Collela, P. Woodward, J. Comp. Phys.,**54**, 1984, 174.). PPM approved itself in different problems of computational physics, it has third-order approximation error for space variables and second-order for time variable. One of its disadvantages is the use of the interpolation procedure on four-point stencil for computation of quantities' values on the boundaries of the cells of difference grid to construct piecewise parabolas. This procedure leads to dissipation of quantities and to smoothing shock's fronts and contact discontinuities in Riemann problem for discontinuity decay. In our paper we suggested a way to avoid the interpolation procedure with the help of information from previous time step - we used the property of Riemann invariants to conserve when moving along the characteristic curves of hyperbolic system of equations. For the first time a similar idea was applied to construct a difference scheme with linear approximation of quantities inside cells long before PPM was invented (B. van Leer, J. Comp. Phys., **14**, 1974, 361.). We suggest the name of modified method as Piecewise Parabolic Method on Local stencil (PPML).

PPML was compared with PPM by numerical solution of Cauchy problem for linear advection equation and inviscid Burgers' equation. The comparison was carried out on the basis of values of errors in different norms and showed that PPML is more accurate. A numerical scheme based on PPML was developed for fluid dynamics and MHD problems solution. Much attention was paid to a monotonicity preserving problem. A question about maintaining of nondivergence constraint on magnetic field was also considered in MHD.

We made a number of standard tests: Sod, Lax, Shu etc problems for 1D hydrodynamics; 2D Riemann problem with contact discontinuities, the double-Mach reflection problem in 2D hydrodynamics; Woodward and Colella's interacting-blast-wave problem, 2.5D shock tube problem, Rotor problem, Orszag-Tang vortex problem etc in MHD. The method was successfully applied to study 3D isothermal MHD turbulence, which is rather difficult problem because of strong shock-waves and regions of high rarefaction.

PPML approved itself as accurate, low-dissipative and convenient to use in adaptive grids.

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Adaptive Techniques for Dynamic Contact Problems

Andreas Rademacher[1] Heribert Blum[2] Andreas Schröder[3]

The necessity to approximate dynamic contact problems arises in many engineering processes. Because of the local effects in the contact zone adaptive techniques are promising for improving the discretisations of these kind of problems. In this talk finite difference methods in time and finite element methods in space are considered to obtain numerical approximations of the solution. Refinement indicators and strategies based on this discretisation are discussed. The talk concludes with numerical results for the presented algorithms.

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Finite Element Techniques for Two-Phase Incompressible Flows

Arnold Reusken[1]

We consider a domain $\Omega \subset \mathbb{R}^3$ which contains two different immiscible incompressible newtonian phases (fluid-fluid or fluid-gas). The time-dependent domains which contain the phases are denoted by $\Omega_1 = \Omega_1(t)$ and $\Omega_2 = \Omega_2(t)$. The interface between the two phases ($\partial\Omega_1 \cap \partial\Omega_2$) is denoted by $\Gamma = \Gamma(t)$. To model the forces at the interface we make the standard assumption that the surface tension balances the jump of the normal stress on the interface, i.e., we have a free boundary condition

$$[\boldsymbol{\sigma}\mathbf{n}]_\Gamma = \tau\kappa\mathbf{n} \ ,$$

with $\mathbf{n} = \mathbf{n}_\Gamma$ the unit normal at the interface, τ the surface tension coefficient (material parameter), κ the curvature of Γ and $\boldsymbol{\sigma}$ the stress tensor. We assume continuity of the velocity across the interface. In combination with the conservation laws of mass and momentum this yields the following standard model:

$$\begin{cases} \rho_i\big(\frac{\partial \mathbf{u}}{\partial t} + (\mathbf{u}\cdot\nabla)\mathbf{u}\big) = -\nabla p + \rho_i \mathbf{g} + \operatorname{div}(\mu_i \mathbf{D}(\mathbf{u})) & \text{in } \ \Omega_i \\ \operatorname{div}\mathbf{u} = 0 & \text{in } \ \Omega_i \end{cases} \quad \text{for } i = 1,2$$
$$[\boldsymbol{\sigma}\mathbf{n}]_\Gamma = \tau\kappa\mathbf{n}, \quad [\mathbf{u}\cdot\mathbf{n}]_\Gamma = 0 \ .$$

The vector $\mathbf{g}$ is a known external force (gravity). In addition we need initial conditions for $\mathbf{u}(x,0)$ and boundary conditions at $\partial\Omega$. For simplicity we assume homogeneous Dirichlet boundary conditions.

In this talk we present an overview of a solver that has been developed and implemented in our group. Important characteristics of the method are the following. For capturing the interface between the two phases the level set method is applied. The spatial discretization is based on a stable hierarchy of consistent tetrahedral grids. For discretization of velocity, pressure and the level set function we use conforming finite elements. For the pressure variable an extended linear finite element space (XFEM) is used which allows an accurate approximation of the pressure discontinuity across the interface. For the treatment of the surface tension force a special Laplace-Beltrami method has been developed. The time discretization is based on a variant of the fractional step θ-scheme. For solving the linearized discrete problems we use inexact Uzawa techniques and Krylov subspace methods combined with multigrid preconditioners. We apply a variant of the Fast Marching method for the reparametrization of the level set function.

We will discuss certain aspects of our solver in more detail. Results of numerical experiments for a three dimensional instationary two-phase fluid-fluid flow problem are presented.

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Numerical Simulation of a Calcium Carbonate Precipitation

Michael Roland[1] Volker John

The precipitation of barium sulphate is modeled by a population balance system describing

- an incompressible flow field (Navier–Stokes equations),
- an isothermal chemical reaction (nonlinear convection–diffusion–reaction equation),
- a population balance equation for the particle size distribution (linear transport equation).

The solution of each individual equation possesses its difficulties:

- in applications, the flow is often turbulent,
- it is important that one obtains oscillation–free solutions for the concentrations in the chemical reaction,
- the particle size distribution depends not only on time and space but also on one or even more properties of the particles (interior coordinates); that means, one has to solve an equation in $\mathbb{R}^d$, $d \geq 4$ in applications.

It turns out that the simulation of population balance systems is rather challenging.

We will present first steps in the simulation of the precipitation of barium sulphate. The flow and the chemical reaction will be considered in a two–dimensional domain. The particle size distribution possesses one interior coordinate (the diameter of the particles). For the particles, nucleation and growth are modeled. Topics included into the talk are the discretization of the individual equations and numerical simulations for different physical and chemical parameters.

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Stabilized FEM and related difference schemes

Hans-Görg Roos[1]

Recently developed stabilization methods for convection-diffusion problems as the continuous interior penalty method or local projection stabilization generate (in 1D) five-point difference schemes. It is extremely useful to compare the schemes generated by different methods.

First we review well known properties of four- and five-point difference schemes and its stability properties. Moreover, we discuss in detail the crucial question to determine the stabilization parameters in such a way that the error of the method behaves nice even if layers exist.

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Interaction of a channel flow with a vibrating airfoil

Martin Ruzicka[1] M. Feistauer[2] J. Horacek[3]

The subject of the contribution is the numerical simulation of the interaction of two-dimensional incompressible viscous flow in a channel and a vibrating airfoil. A solid airfoil with two degrees of freedom, which can rotate around the elastic axis and oscillate in the vertical direction, is considered. The numerical simulation consists of the finite element solution of the Navier-Stokes equations coupled with the system of ordinary differential equations describing the airfoil motion. High Reynolds numbers considered $(10^5 - 10^6)$ require the application of a suitable stabilization of the finite element discretization. The time dependent computational domain and a moving grid are taken into account with the aid of the Arbitrary Lagrangian-Eulerian (ALE) formulation of the Navier-Stokes equations. A special attention is paid to the time discretization and the solution of the nonlinear discrete problem on each time level is performed. As a result a sufficiently accurate and robust method is developed, which is applied to the case of flow induced airfoil vibrations with large amplitudes after loosing the aeroelastic stability. The computational results are compared with known aerodynamical data and with results of aeroelastic calculations obtained by NASTRAN code for a linear approximation.

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Dune: The Distributed and Unified Numerics Environment

Oliver Sander [1]

Dune is a framework for grid-based numerical computations. Its main feature is the introduction of an abstract interface which separates grid implementations from the algorithms that use them. Applications are written for the interface instead of for a specific grid implementation. Hence grid implementations can be changed at any moment in algorithm development with minimal effort. Several such implementations are available. Some of them are grids specifically written for Dune, others encapsulate existing well-known finite element codes such als Alberta and UG. Due to the use of modern programming techniques the extra abstraction layer comes at very little additional cost.

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The role of Nitsche type boundary conditions for edge stabilization and local projection method

Friedhelm Schieweck[1]

We investigate the edge stabilization method and the local projection method for higher order finite elements applied to a convection diffusion equation with a small diffusion parameter ε. Performing numerical experiments, it turns out that strongly imposed Dirichlet boundary conditions lead to relatively bad numerical solutions. However, if the Dirichlet boundary conditions are imposed on the inflow part of the boundary in a weak sense and additionally on the whole boundary in an ε-weighted weak sense due to Nitsche then one obtains reasonable numerical results. In many cases, this holds even in the limit case where the parameter of the edge stabilization is zero, i.e., where the standard Galerkin discretization is applied.

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Anisotropic mesh adaption based on a posteriori estimates and optimisation of node positions

Rene Schneider[1] P. Jimack[2]

Efficient numerical approximation of solution features like boundary or interior layers by means of the finite element method requires the use of layer adapted meshes. Anisotropic meshes, like for example Shishkin meshes, allow the most efficient approximation of these highly anisotropic solution features. However, application of this approach relies on *a priori* analysis on the thickness, position and stretching direction of the layers. If it is impossible to obtain this information *a priori*, as it is often the case for problems with interior layers of unknown position for example, automatic mesh adaption based on *a posteriori* error estimates or error indicators is essential in order to obtain efficient numerical approximations.

Historically the majority of work on automatic mesh adaption used locally uniform refinement, splitting each element into smaller elements of similar shape. This procedure is clearly not suitable to produce anisotropically refined meshes. The resulting meshes are over-refined in at least one spatial direction, rendering the approach far less efficient than that of the anisotropic meshes based on *a priori* analysis.

Automatic anisotropic mesh adaption is an area of active research. Here we present an update regarding a new approach to this problem, based upon using not only an *a posteriori* error estimate to guide the mesh refinement, but its sensitivities with respect to the positions of the nodes in the mesh as well. Once this sensitivity information is available, techniques from mathematical optimisation can be used to minimise the estimated error by moving the positions of the nodes in the mesh appropriately.

The discrete adjoint technique is utilised to evaluate the sensitivities of an error estimate, reducing the cost of this evaluation to solving one additional equation system. This approach is crucial to make gradient based optimisation techniques, such as BFGS-type schemes, applicable. For more details, see [1].

References:

[1] R. Schneider and P.K. Jimack, Toward anisotropic mesh adaption based upon sensitivity of a posteriori estimates, School of Computing Research Report Series 2005.03, University of Leeds, 2005.
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Mixed finite elements for linear elasticity

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In this talk, we derive a mixed variational formulation for the equations of linear elasticity

$$\begin{aligned} -\mathrm{div}(\sigma) &= f \\ A\sigma &= \varepsilon(u) = \frac{1}{2}(\nabla u + \nabla u^T). \end{aligned}$$

Here u denotes the displacement field, and σ the stress tensor.

We choose $u \in H(\mathrm{curl})$, which implies the following continuity condition for the stresses

$$\mathrm{div}\sigma \in H(\mathrm{curl})^* = H^{-1}(\mathrm{div}).$$

We refer to this space as $H(\mathrm{div}\,\mathrm{div})$. We see that the variational problem is well posed.

We discretise the problem using Nédélec finite elements for the displacements. For σ, we propose a new family of symmetric, tensor-valued finite elements. The normal-normal component $\sigma_{nn} = n^T\sigma n$ is continuos across interfaces, therefore we see that they are conforming for the space $H(\mathrm{div}\,\mathrm{div})$. We present stable elements of arbitrary order.

These elements are suitable for the discretization of beams or shells, as they do not suffer from shear locking. We see that the optimal order error estimates for the solution still hold true in the case of thin structures. Also, there occurs no volume locking, when the material gets nearly incompressible, i.e. the Poisson ratio ν approaches 1/2.

The mixed formulation corresponds to a saddle point problem. In order to obtain a positive definite matrix, we hybridize the stress space. This means leave the stress-space noncontinuous, and introduce Lagrange parameters which guarantee the required normal-normal continuity. We use these additional variables to find a preconditioner that works for nearly incompressible materials.

We give numerical results, including the cases of nearly incompressible materials and thin structures.

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Stabilisation methods of local projection type for convection-diffusion-reaction problems

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Stabilisation methods by local projection are commonly used for solving convection dominated problems, especially for chemically reactive flows. We present a unified analysis of projection methods for scalar problems. In contrast to the known two-level local projection methods our approach allows to construct new finite element schemes employing only one mesh. The main advantage of local projection methods is their symmetric nature. Compared to standard residual methods additional couplings in stabilising terms can be avoided for diffusion-convection-reaction systems. In reaction problems with nonlinear kinetics the interactions between species can cause troubles. In general our method leads to the reduction of computing effort in order to resolve iteratively nonlinear reaction terms. We study numerically mixed boundary layer problems as well as systems occuring in reactor chemistry.

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On discrete projection methods for rotating incompressible flow with Coriolis force

Andriy Sokolov[1] Stefan Turek[2] Maxim A. Olshanskii[3]

In many physical and industrial processes there is a necessity of numerical simulations of models with moving boundaries in a 3-dimensional space. As proposed in the literature, approaches like fictitious boundary or Arbitrary Lagrangian Eulerian methods require large amount of CPU time to simulate even a 2D benchmark models. Moreover, they could provide a source of additional errors in velocity and pressure fields. At the same time, there is a large class of rotating models, where the use of the above methods can be avoided by some modifications of Navier-Stokes equations and/or transformations of the coordinate system. In our talk we present a numerical analysis and algorithmic details for treating the system of Stokes and Navier-Stokes equations with Coriolis force term. Using a Discrete Projection Method with modified Pressure Schur Complement operator, we examine the influence of the Coriolis force on every step of the algorithm: modification of the momentum equation and building of a new block diagonal preconditioner, construction of a new Pressure Schur Complement operator by inserting the off-diagonal parts, which are due to the rotating terms and an explicit formation of its inverse, and finally, correction of pressure and velocity to satisfy the incompressibility constraints. Detailed numerical studies of the improvements in the convergence rate, its dependence on the magnitude of the time step and angular velocity are provided. We also consider the addition of the convection term into the Schur Complement, discuss its influence on the iterative behaviour of the solver and propose a generalized coriolis-convection oriented Pressure Schur Complement. Finally, we provide numerical simulations for polygonal geometries and check the efficiency of our solver by comparing simulation results with those obtained by the fictitious boundary method for moving boundary parts. The used solver is based on the 3-dimensional PP3D CFD-code from the Featflow package (www.featflow.de).

References:

[1] S. Turek. On discrete projection methods for the incompressible Navier-Stokes equations: an algorithmical approach, Comput. Methods Appl. Mech. Engrg., Vol. 143, page 271-288, 1997.
[2] M. A. Olshanskii. An iterative Solver for the Oseen Problem and Numerical Solution of Incompressible Navier-Stokes Equations, Numer. Linear Algebra Appl., Vol. 6, page 353-378, 1999
[3] S. Turek. Efficient Solvers for Incompressible Flow Problems: An Algorithmic and Computational Approach, LNCSE 6, Springer, 1999.
[4] S. Turek, D. C. Wan and L. P. Rivkind. The Fictitious Boundary Method for the implicit treatment of Dirichlet boundary conditions with applications to incompressible flow simulations, In E. Bänsch editor, Challenges in Scientific Computing CISC 2002, LNCSE, page 37-68, Berlin, 2002. Springer, Berlin.

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Preconditioning of Elliptic Boundary Value Problems

Olaf Steinbach[1]

The discretization of second order elliptic boundary value problems by either finite or boundary element methods leads to large linear systems of algebraic equations. Since the spectral condition number of the stiffness matrix depends in general on the discretization parameter h, preconditioning seems to be mandatory.

Here we present a general concept of preconditioning which is based on the use of operators of the opposite order. This involves different approaches, e.g. the use of different boundary integral operators, the use of appropriate multilevel operators, and the use of certain Newton potentials in finite element methods.

The numerical analysis is based on an approximate discretization of inverse operators by using a mixed scheme. Hence, a suitable stability condition is required.

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Finite element simulation of electromagnetic metal forming

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Electromagnetic metal forming is a high speed forming process in which strain rates of over 1000 s^{-1} are achieved. The deformation of the work piece is driven by the Lorentz force, a material body force, which results from the interaction of eddy currents induced by a pulsed magnetic field with the triggering magnetic field itself. In this talk, a mathematical model of the coupled process is presented that is based on a rate-dependent elasto-viscoplastic material model in a dynamic large deformation context on the mechanical side and on the quasistatic approximation to Maxwell's equations on the electromagnetic side. Then, a finite element discretization is derived both for the mechanical and for the electromagnetic subsystem. While standard finite element methods suffice for the mechanical subsystem, the electromagnetic equations are discretized with the help of edge elements. The electromagnetic field equations are transformed into an arbitrary Lagrangian Eulerian (ALE) formulation to guarantee a precise data transfer between the two finite element meshes involved. The coupled system is finally solved by an implicit staggered scheme. After simulation results have been presented, methods of a posteriori error control for the particular subsystems as well as for the coupled system are discussed.

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On coupling of incompressible fluid and structure models in an aeroelastic application

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In this paper the numerical approximation of a two dimensional aeroelastic problem is addressed. The mutual interaction of fluids and structure can be met in many different situations, cf. [1]. The main objectives of the engineering problems is to determine the critical velocity for loose of the system stability. In many cases simplifications of the aeroelastic model are used, e.g. linearized models, etc.

Here, the fully coupled formulation of incompressible viscous fluid flow over a structure is used. For the flow model we use the incompressible system of Navier-Stokes equations with large values of the Reynolds number $10^4 - 10^6$. The Navier-Stokes equations are spatially discretized by the FE method and stabilized with a modification of the Galerkin Least Squares (GLS) method; cf. [2].

The motion of the computational domain is treated with the aid of Arbitrary Lagrangian Eulerian(ALE) method. The GLS stabilizing terms are modified in a consistent way with the weak formulation of the ALE method.

The structure model is considered as a solid body with two/three degrees of freedom (bending, torsion and torsion of the control section). The motion is described with the aid of a system of nonlinear differential equations. The construction of the ALE mapping is based on the solution of an elastic problem.

The method is applied onto several benchmark problems, where several technical parameters are compared with reference values. The nonlinear behaviour of the coupled system is shown for the nearby critical velocity.

References:

[1] Dowel E. H., "A Modern Course in Aeroelasticity", Kluwer Academic Publishers, Dodrecht, 1995.

[2] Gelhard T., Lube G. and Olshanskii M.A., "Stabilized finite element schemes with LBB-stable elements for incompressible flows". Journal of Computational and Applied Mathematics, 177:243-267, 2005.

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Stabilization by local projection. Convection-diffusion and incompressible flow problems

Lutz Tobiska[1]

The discretisation of the Oseen problem by finite element methods may suffer in general from two shortcomings. First, the discrete inf-sup (Babuška–Brezzi) condition can be violated. Second, spurious oscillations occur due to the dominating convection. One way to overcome both difficulties is the use of local projection techniques. Studying the local projection method in an abstract setting, we show that the fulfilment of a local inf-sup condition between approximation and projection spaces allows to construct an interpolation with additional orthogonality properties. Based on this special interpolation, optimal a-priori error estimates are shown with error constants independent of the Reynolds number. Applying the general theory, we extend the results of Braack and Burman for the standard two-level version of the local projection stabilisation to discretisations of arbitrary order on simplices, quadrilaterals, and hexahedra. Moreover, our general theory allows to derive a novel class of local projection stabilisation by enrichment of the approximation spaces. This class of stabilised schemes uses approximation and projection spaces defined on the same mesh and leads to much more compact stencils than in the two-level approach. Finally, on simplices, the spectral equivalence of the stabilising terms of the local projection method and the subgrid modeling introduced by Guermond is shown. This clarifies the relation of the local projection stabilisation to the variational multiscale approach.

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Higher order time discretizations for scalar nonlinear convection-diffusion equation

Miloslav Vlasak[1] Vit Dolejsi[2]

We deal with a numerical solution of a scalar non-stationary nonlinear convection-diffusion equation, which we use as a model simplified problem for the solution of the system of the compressible Navier-Stokes equations. For the space discretization we use the discontinuous Galerkin finite element method (DGFEM). We employ the so-called NIPG (nonsymmetric interior penalty Galerkin) approach. After discretization by DGFEM we obtain system of the ordinary differential equations (ODEs), which is often discretized by the explicit Runge-Kutta methods since these schemes have a high order of accuracy and they are simple for implementation. Their drawback is a strong restriction to the length of the time step. In order to avoid this disadvantage it is suitable to use an implicit time discretization, but a fully implicit scheme leads to a necessity to solve a nonlinear system of algebraic equations at each time step which is rather expensive and complicated. Therefore, we develop a higher order unconditionally stable (or with a large stability domain) time discretization technique which do not require a solution of nonlinear problem at each time step. We present several approaches and derive some a priori error estimates in the discrete analogous of the $L^\infty(0,T;L^2(\Omega))$-norm and the $L^2(0,T;H^1(\Omega))$-seminorm.

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Numerical Analysis of Stokes Equations with Improved LBB Dependency

Matthias Wohlmuth[1] Manfred Dobrowolski[2]

We provide a priori bounds with improved domain dependency both for the solution of Stokes Equations and for the numerical error of an approximation by conforming Finite Element Methods. The domain dependency mainly appears in terms of the inverse of the LBB constant $\frac{1}{L}$. It is known from several previous works, that L decreases with the aspect ratio of the domain. In the first part we explain the LBB dependency of common a priori bounds on Du and p. Then, we improve most of these estimates by avoiding a global inf-sup condition and by the assumption of *locally-balanced flow*.
In fact, most common Finite Element Methods do not show a dependency of the aspect ratio at all. Hence, in a last step we explain how these methods, which we characterize as *LBB-friendly*, reduce the LBB dependency by automatically subtracting an appropriate base flow. For certain domains, if this modification is carried out manually, at least the condition of locally balanced flow can be roughly achieved. But if the same procedure is accomplished automatically by the choice of an appropriate method, locally-balanced flow can even be assumed for the dual problem and hence improve the L^2-pressure error estimate. Finally, we present some numerical computations which demonstrate the sharpness of the theoretical results.

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Equilibrated residual-based error estimators for Poisson and Maxwell's equations

Sabine Zaglmayr[1] Dietrich Braess[2] Joachim Schöberl[3]

Equilibrated a-posteriori error estimators for Poisson equations rely on the following principle: Any flux σ which fulfils the equilibrium condition, $div\,\sigma = f$, can be used to obtain an upper bound for the discretization error without generic constants. We present a patch-wise construction of a flux correction $\sigma^{\Delta} = \sigma + grad(u_h)$ that is also applicable for high-order FE-schemes and for Maxwell's equations.

The flux correction has to satisfy $div(\sigma^{\Delta}) = f + \Delta u_h$ in the distributional sense. The existence of a solution of the local vertex-patch problems is guaranteed by the exactness of discrete distributional de Rham sequences. The appropriate finite element spaces are a broken Raviart-Thomas space for the flux correction and an element-wise polynomial space extended by polynomials living only on the skeleton for the residual.

The extension to curl-curl problems follows an analogue principle. Here, the divergence-free localization of the residual to local patch-problems plays a crucial role.

We illustrate the advantages of our method by numerical experiments with h- and p-refinement strategies.

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