On the ARMA Approximation for Fading Channels Described by the Clarke Model with Applications to Kalman-based Receivers

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***Abstract*—We consider a terrestrial wireless channel, whose statistical model under flat-fading conditions is due to Clarke. A lot of papers in the literature deal with receivers for this scenario, aiming at estimating and tracking the time-varying channel, possibly with the aid of known (pilot) symbols. A common approach to derive receivers of reasonable complexity is to resort to a Kalman filter which is based on an approximation of the actual fading process as autoregressive moving-average (ARMA) of a given order. The aim of this paper is to show that the approximation of the actual fading process, usually exploited in the literature, is far from effective. Thus, we present a novel technique, based on an off-line minimization of the mean square error of the channel estimate, which ensures a considerable gain in terms of bit-error rate for Kalman-based receivers without increasing the receiver complexity. Moreover, we also propose a novel approximation, to be employed in Kalman smoothers proposed for iterative detection schemes, which allows further performance improvements without a significant increase of the computational complexity.**

***Index Terms*—Fading channels, time-varying channels, parameter estimation, autoregressive moving average processes, Wiener filtering, Kalman filtering.**

I. Introduction

STATISTICAL models to describe terrestrial wireless channels are necessary to design effective receivers. Among all models, the Clarke model [1] is widely accepted in the literature for frequency-flat correlated Rayleigh fading channels [2].

Typically, when no channel state information (CSI) is available at the receiver, the time-varying channel amplitude and phase must be estimated and tracked. This is often carried out by taking advantage of known (pilot) symbols, periodically inserted into the coded data stream [3]. Moreover, when iterative joint detection and decoding is adopted at the receiver, channel estimates are iteratively updated using the soft information coming from the decoder [4]–[7].

An approximation often employed in the literature to design receivers for correlated fading channels consists of modeling the fading process as autoregressive (AR) or, more generally, as autoregressive moving average (ARMA). This allows the application of a low-complexity Kalman filter [8] to track the channel variations (e.g., see [4], [7], [9]–[11]). Moreover, since the complexity of a Kalman filter increases with the order of the ARMA process, a tradeoff between complexity and accuracy is necessary—the larger the order of the model, the better the approximation of the actual fading statistics but also the larger the tracking complexity. Several papers published so far deal with the approximation of the fading process with a finite order Markov model, as for instance an ARMA process, (e.g., see [12]–[14]). In most of them the authors conclude that a first order approximation can capture most of the dynamics of the actual fading process.

However, the choice of the parameters of an AR process is based upon trying to best match the actual fading process and its approximation. This is done by solving a Yule-Walker linear system with respect to the coefficients of an AR model [15]. We propose a different approach whose aim is to find the coefficients of an ARMA model which minimize the mean square error (MSE) of the channel estimate under the hypothesis of perfect knowledge of the transmitted symbols. It turns out that the optimized coefficients depend in this case on the signal-to-noise ratio (SNR). We will show that, following the proposed method, not only the MSE of the channel estimate is reduced, but also the bit-error rate (BER) performance of Kalman-based receivers can be dramatically improved. This performance improvement comes at practically no cost, since the complexity of the receiver is unchanged, the only modification being the different parameters of the ARMA model.

The remainder of this paper is organized as follows. In Section II, we define the system model and formulate the problem. The proposed method to obtain the coefficients of the approximating ARMA process is described in Section III. Mathematical details are shown for the case of first- and second-order AR models. In Section IV, the receiver proposed in [4], [7], based on a Kalman smoother [8], is briefly summarized and an improvement is proposed. Numerical results, comparing the performance of classical receivers based on Kalman smoothing with that of the proposed one, are shown in Section V by approximating the fading process using the proposed optimization method. Finally, some concluding remarks are drawn in Section VI.

II. System Model and Problem Formulation

We consider the transmission of a sequence of complex modulation symbols $\mathbf{c} = (c_0, c_1, \ldots, c_{K-1})$ over a flat correlated Rayleigh fading channel. We assume that the sequence $\mathbf{c}$ is a codeword, possibly interleaved, of a channel code $\mathcal{C}$ constructed over a modulation constellation $\mathcal{X} \subset \mathbb{C}$. We include possible pilot symbols (known to the receiver) as a part of the code $\mathcal{C}$. Symbols $\{c_k\}$ are linearly modulated. Assuming Nyquist transmitted pulses, matched filtering, and channel variations slow enough so that no intersymbol interference arises, the discrete-time baseband complex equivalent of the

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received signal is given by

$$r_k = f_k c_k + w_k, \quad k = 0, \ldots, K-1 \tag{1}$$

where $\{w_k\}$ is the additive white Gaussian noise (AWGN) and $\{f_k\}$ is the fading process. The vector of noise samples $\mathbf{w} = (w_0, w_1, \ldots, w_{K-1})$ has independent and identically distributed (i.i.d.) complex circularly symmetric components, with $w_k \sim \mathcal{N}_{\mathbb{C}}(0, 2\sigma^2)$.[1] The fading process $\{f_k\}$ is a sequence of zero-mean complex Gaussian random variables with autocorrelation sequence modeled according to isotropic scattering[2] [1], i.e., given by

$$R_f(m) = E\{f_{k+m} f_k^*\} = J_0(2\pi f_D T m) \tag{2}$$

where $J_0(\cdot)$ is the zero-th order Bessel function, T is the signaling rate, and f_D is the Doppler spread, assumed known to the receiver. This latter assumption is common in the literature (e.g. see [7], [16]). Moreover, f_D can be estimated using, for example, the algorithm in [17]. The vector $\mathbf{f} = (f_0, f_1, \ldots, f_{K-1})$, unknown to both transmitter and receiver, is statistically independent of $\mathbf{c}$ and $\mathbf{w}$.

Receivers for this scenario have been widely studied in the literature and many different approaches have been pursued (e.g., see [4], [7], [9]–[11], [18]–[21]). In this paper, we are interested in receivers based on a Kalman filter. In particular, we will consider the receiver, proposed in [4], [7] and briefly reviewed in Section IV, based on a Kalman smoother and iteratively refining the channel estimate by using the extrinsic information coming from a soft-input soft-output (SISO) decoder. A common characteristic of receivers based on a Kalman filter is that they model the fading process as ARMA(N, M), i.e., $\{f_k\}$ is approximated with a new process $\{g_k\}$ that can be generated as

$$g_k = \sum_{n=1}^{N} \rho_n g_{k-n} + \sum_{m=0}^{M} \gamma_m v_{k-m} \tag{3}$$

where $\{v_k\}$ is a complex circularly symmetric white Gaussian process with unitary variance.[3] Therefore, in the considered scenario it is necessary to find a proper approximation for the considered fading process, that is, for any given N and M, to find the coefficients $(\rho_1, \ldots, \rho_N)$ and $(\gamma_0, \ldots, \gamma_M)$ for which the best approximation, in some sense, of the actual fading process results. In the case of an AR(N) model, the common approach in the literature consists of obtaining the coefficients $(\rho_1, \ldots, \rho_N)$ by solving the Yule-Walker linear system $R_g(m) = R_f(m)$, for $m = 1, 2, \ldots, N$ [15], whereas γ_0 can be expressed as a function of coefficients $(\rho_1, \ldots, \rho_N)$ from the constraint $R_g(0) = R_f(0)$. This is exactly the same approach followed in [4], [7] and many other papers (e.g. [11], [14]). In the next section, we propose an alternative method to derive the ARMA coefficients.

[1]A complex circularly symmetric Gaussian random variable v with mean μ and covariance Σ is denoted by $v \sim \mathcal{N}_{\mathbb{C}}(\mu, \Sigma)$. We denote the complex circularly symmetric Gaussian probability density function (pdf) with mean μ, covariance Σ and argument x by $g_{\mathbb{C}}(x; \mu, \Sigma)$.

[2]Although in this paper we always assumed the Clarke model because it is widely used in the literature, we wish to point out that any arbitrary autocorrelation function can be used in (2).

[3]When $M = 0$, the process is said to be autoregressive of order N [AR(N)].

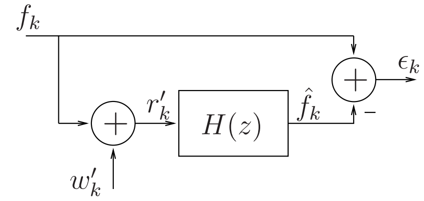

Fig. 1. Pictorial representation of the proposed method.

III. ARMA Approximation of the Actual Fading Channel

Let assume that we know the transmitted symbols. We are in fact interested in coded systems with iterative joint detection and decoding, where this information on code symbols can be derived from the decoder and becomes more and more reliable through the iterations. Hence, we can remove the modulation from the observation obtaining the sequence $\{r'_k\}$, with

$$r'_k = \frac{r_k}{c_k} = f_k + w'_k \tag{4}$$

having defined $w'_k = w_k / c_k$. With reference to Fig. 1, which represents a pictorial description of the proposed method, let us design the minimum MSE linear estimator of a *generic* ARMA(N, M) process $\{g_k\}$ from the observation $\{g_k + w'_k\}$. The impulse response of this filter will be denoted to as $\{h_k\}$ and its transfer function, in terms of Z-transform, as $H(z)$. Obviously, $\{h_k\}$ will be a function of the parameters of the ARMA process. Then, we employ this filter to estimate f_k from the observation $\{r'_k\}$. We will choose the parameters of the ARMA model such that the mean square value of the error $\epsilon_k = f_k - \hat{f}_k$ is minimized.

The rationale for such an approach is very simple. The described procedure is exactly what we want to do with a Kalman filter. In fact, this Kalman filter works on the true observation but is designed with an ARMA model in mind and our aim is the minimization of the MSE of the estimate produced by the Kalman filter. For simplicity, and also because this case will be considered in the numerical results, we assume a phase shift keying (PSK) modulation with $|c_k| = 1$. Hence, sequences $\{w_k\}$ and $\{w'_k\}$ are statistically equivalent. Since in the numerical results we will consider a Kalman smoother, as a filter $H(z)$ we may consider the non-causal minimum MSE estimator that, in this case, can be expressed in closed form [15]:

$$H(z) = \frac{S_g(z)}{S_g(z) + 2\sigma^2} \tag{5}$$

where $S_g(z)$ is the Z-transform of the autocorrelation function $R_g(m)$ which depends on the parameters of the ARMA model. Since the estimate $\hat{f}_k$ can be expressed as

$$\hat{f}_k = \sum_{n=-\infty}^{\infty} h_n r'_{k-n} = \sum_{n=-\infty}^{\infty} h_n (f_{k-n} + w'_{k-n}) \tag{6}$$

the MSE to be minimized as a function of the parameters of the ARMA model is given by

$$E\{|\epsilon_k|^2\} = E\left\{\left|f_k - \sum_{n=-\infty}^{\infty} h_n(f_{k-n} + w'_{k-n})\right|^2\right\}$$
$$= 1 + 2\sigma^2\sum_n |h_n|^2 + \sum_m\sum_n h_n h_m^* R_f(m-n)$$
$$-2\mathrm{Re}\left[\sum_n h_n^* R_f(n)\right]. \quad (7)$$

The MSE (7) cannot be evaluated in closed form due to the presence of the Bessel function in the expression of the autocorrelation $R_f(n)$. Hence, it must be numerically evaluated by truncating the summations in (7). This can be done since both h_n and $R_f(n)$ go to zero for $n \to \pm\infty$. We now detail the proposed method in the case of AR(1) and AR(2) models.

AR(1) approximation

We consider a first order autoregressive process $\{g_k\}$ which is defined by the following recursive equation

$$g_k = \rho g_{k-1} + \sqrt{1-\rho^2}v_k \quad (8)$$

where the real parameter ρ is such that $|\rho| < 1$ to ensure wide-sense stationarity. The Z-transform of the autocorrelation function of such a process becomes

$$S_g(z) = \frac{1-\rho^2}{(1-\rho z^{-1})(1-\rho z)}. \quad (9)$$

After substituting (9) in (5), the expression of the non-causal minimum MSE estimator $H(z)$ becomes

$$H(z) = \frac{1-\rho^2}{1-\rho^2+2\sigma^2(1-\rho z^{-1})(1-\rho z)} \quad (10)$$

which has two real poles $p_1 < 1$ and $p_2 > 1$, with the property that $p_1 = 1/p_2$. It follows that the corresponding impulse response is

$$h_n = Ap_1^{|n|} \quad (11)$$

where

$$p_1 = \frac{1}{2}\left(\rho^{-1} + \rho + \frac{\rho^{-1}-\rho}{2\sigma^2}\right)$$
$$-\frac{1}{2}\sqrt{\left(\rho^{-1}+\rho+\frac{\rho^{-1}-\rho}{2\sigma^2}\right)^2 - 4} \quad (12)$$
$$A = \frac{1}{2\sigma^2}\frac{\rho^{-1}-\rho}{p_1^{-1}-p_1}.$$

After substituting (11) in (7), the remaining step is to search for $\rho \in (-1, 1)$ such that the MSE $E\{|\epsilon_k|^2\}$ is minimized. ◇

AR(2) approximation

We now consider the case of the AR(2) model. Two parameters have to be optimized in this case, since an AR(2) process is defined by this recursive equation

$$g_k = \rho_1 g_{k-1} + \rho_2 g_{k-2} + \gamma_0 v_k \quad (13)$$

where γ_0 can be expressed as a function of ρ_1 and ρ_2, exploiting the constraint $E\{|g_k|^2\} = 1$, and the search for ρ_1 and ρ_2 must be performed with the constraint that the polynomial $z^2 - \rho_1 z - \rho_2 = 0$ must have both roots inside the unit circle to ensure the wide-sense stationarity of $\{g_k\}$. In this case, the filter $H(z)$, computed from (5), has four poles: p_1 and p_2 inside the unit circle and $p_3 = 1/p_1, p_4 = 1/p_2$ outside. Assuming for simplicity distinct poles, two scenarios are possible: i) real poles, leading to the following impulse response

$$h_n = A_1 p_1^{|n|} + A_2 p_2^{|n|} \quad (14)$$

with A_1 and A_2 real coefficients, and ii) complex conjugate poles, for which (14) holds with A_1 and A_2 complex conjugate coefficients. The expression of the constants appearing in (14) as a function of ρ_1 and ρ_2 is omitted. Again, after substituting (14) in (7) the remaining step is to search for ρ_1 and ρ_2 such that the MSE $E\{|\epsilon_k|^2\}$ is minimized. ◇

The parameters of the ARMA model computed using the proposed method will depend on the Doppler spread and the SNR. However, they can be computed off-line and stored in a look-up table. Alternatively, simple approximations of the coefficients as a function of the Doppler spread may be introduced by fitting the numerically computed data. For example, it turns out that, for the AR(1) coefficient ρ, the approximation $\log(1-\rho) = m\log(f_D T) + q$ (where the real coefficients m and q depend on the SNR, but are independent of the Doppler spread) is excellent on a wide range of Doppler values, for suitably selected m and q. Therefore, in practice the computationally intensive evaluation of the optimal coefficients can be carried out off-line, and the optimal parameters m and q derived for a set of values of the SNR. At the receiver, these coefficients are used to evaluate ρ for any Doppler spread of interest, by calculating

$$\rho = 1 - e^{m(SNR)\log(f_D T)+q(SNR)} \quad (15)$$

where we pointed out the dependency of m and q from the SNR. The above function can be evaluated easily by means of two look-up tables. We choose to express the coefficient ρ as a function of the Doppler spread, instead of the SNR, since the dependence on the first is much more pronounced than on the latter. Therefore, even if m and q are evaluated and stored only for a few values of the SNR, the approximation will be very good.

Similarly, in the AR(2) case it turns out that a very good fitting is obtained by means of the following functions

$$\rho_1 = 2 - e^{m_1(SNR)\log(f_D T)+q_1(SNR)}$$
$$\rho_2 = -1 + e^{m_2(SNR)\log(f_D T)+q_2(SNR)}$$

where again m_1, m_2, q_1, q_2 have to be evaluated off-line for a set of SNR values of interest.

IV. Iterative Kalman Smoother

We now briefly review the receiver proposed in [4], [7] for the considered scenario, based on a Kalman smoother and on an iterative refinement of the channel estimate using the soft information coming from the decoder. This Kalman smoother will be designed for a fading channel modeled as ARMA(N, M) and whose parameters can be chosen according to the classical or the proposed optimization method.

Omitting for simplicity of notation the explicit reference to the current iteration, let us denote by $P(c_k)$ the extrinsic soft-output produced by the SISO decoder in the iterative receiver. Using these probabilities, the pdf $p(r_k|f_k)$ can be computed as

$$\begin{aligned} p(r_k|f_k) &= \sum_{c_k} P(c_k) p(r_k|f_k, c_k) \\ &= \sum_{c_k} P(c_k) g_{\mathbb{C}}(r_k; c_k f_k, 2\sigma^2) \end{aligned} \tag{16}$$

where the assumption of independence between data symbols and fading coefficients has been used. From (16), it turns out that $p(r_k|f_k)$ is Gaussian only when the k-th code symbol is perfectly known, e.g., it is a pilot symbol or the decoder has taken a decision on it with high reliability, while otherwise it is a linear combination of Gaussian pdfs. In [4], [7] the following approximation is proposed

$$p(r_k|f_k) \simeq g_{\mathbb{C}}\left(r_k; f_k \alpha_k, 2\sigma^2\right) \tag{17}$$

where $\alpha_k = \sum_{c_k} c_k P(c_k)$ is the first-order moment of the code symbol c_k. As a consequence, in order to estimate and track the channel, we may assume the following observation model

$$r_k = f_k \alpha_k + n_k \tag{18}$$

where the noise samples $\{n_k\}$ are i.i.d., with $n_k \sim \mathcal{N}_{\mathbb{C}}(0, 2\sigma^2)$. Hence, the modulation is removed and modeling $\{f_k\}$ as ARMA(N, M), a Kalman smoother [4], [7], [8] can be adopted to obtain a Bayesian channel estimator. From this channel estimate, the probabilities of symbols $\{c_k\}$ are then updated and passed to the decoder [4], [7]. The approximation (17) will be denoted to as *approximation* σ_0.

We propose a different approximation based on the minimum Kullback-Leibler distance [22]:

$$p(r_k|f_k) \simeq g_{\mathbb{C}}(r_k; E\{r_k|f_k\}, \mathrm{var}\{r_k|f_k\}) \,. \tag{19}$$

It is straightforward to show that

$$E\{r_k|f_k\} = f_k \alpha_k \tag{20}$$

$$\mathrm{var}\{r_k|f_k\} = 2\sigma^2 + |f_k|^2 \left(\beta_k - |\alpha_k|^2\right) \tag{21}$$

where β_k is the second-order moment of the code symbol c_k, i.e., $\beta_k \triangleq \sum_{c_k} |c_k|^2 P(c_k)$. On the other hand, the pdf $p(r_k|f_k)$ in (19) as a function of f_k is not Gaussian since $|f_k|^2$ appears in the expression of $\mathrm{var}\{r_k|f_k\}$. In order to derive a receiver based on a Kalman filter, the dependence on $|f_k|^2$ in (21) has to be neglected in some way. We propose the substitution of each fading sample f_k in (21) with its estimate $\hat{f}_k$ made by the Kalman filter at the previous iteration, i.e.,

$$p(r_k|f_k) \simeq g_{\mathbb{C}}\left(r_k; f_k \alpha_k, 2\sigma^2 + |\hat{f}_k|^2 \left(\beta_k - |\alpha_k|^2\right)\right) \,. \tag{22}$$

Since no estimate is available at the first iteration, $\hat{f}_k = 0$ is assumed. This approximation will be denoted to as *approximation* σ_{KL}. The approximation σ_0 in [4], [7] can be considered as a particular case of the proposed approximation based on the minimum Kullback-Leibler distance by using, at every iteration, $\hat{f}_k = 0$ instead of the estimate produced by the Kalman smoother.

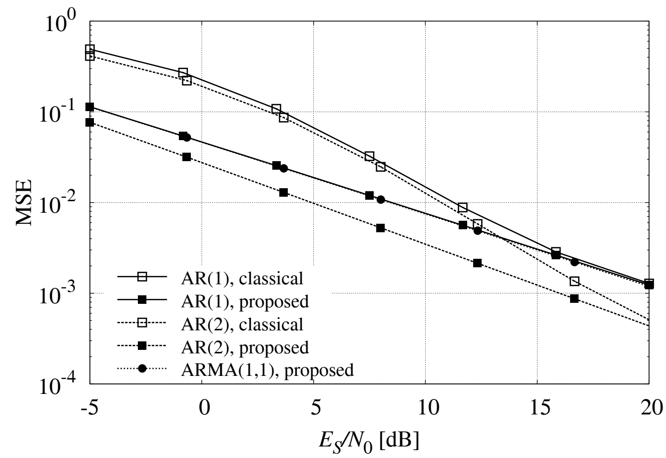

Fig. 2. MSE of the fading estimate, for different optimization methods for the parameters of the ARMA model. The considered normalized Doppler bandwidth is $f_D T = 10^{-2}$.

It is worth noticing that, with respect to [4], [7], the proposed method entails only a minor increase in complexity. In particular, it requires to store the estimates $\{\hat{f}_k\}$ which will be used at the next iteration, and to evaluate the second-order moments $\{\beta_k\}$. Moreover, two real additions and two magnitude computations are required to evaluate (21).

V. Numerical Results

In order to illustrate the benefits that can be obtained with a proper optimization of the parameters of the employed ARMA model, in Fig. 2 the MSE $E\{|\epsilon_k|^2\}$ as a function of E_S/N_0, E_S being the received signal energy per modulation symbol and N_0 the one-sided noise power spectral density, is shown. Both the classical (based on the Yule-Walker linear system for AR processes) and the proposed optimization methods have been considered. A correlated Rayleigh fading process $\{f_k\}$ with Clarke statistics has been considered, with normalized Doppler bandwidth $f_D T = 10^{-2}$. The channel realizations were generated according to the method proposed in [23]. It can be observed that AR(1) and ARMA(1,1) models lead to approximately the same MSE. This is confirmed by the BER curves also. On the other hand, when the proposed optimization method is employed, the AR(2) approximation exhibits a quite large gain, in terms of MSE, with respect to the AR(1) model. Hence, it may be worth employing Kalman filters designed on the AR(2) approximation in practical receivers, despite the increased complexity. Finally, it is important to point out that the gain of the proposed method with respect to classical method, for both AR(1) and AR(2), can be very large, especially for low to medium SNR values. For a large enough SNR, the MSE of the classical method asymptotically approaches that of the proposed one. As pointed out in Section III, the infinite summations in (7) must be truncated. Since, for example in the AR(1) case, the summations in (7) behave as geometric series with base p_1, an effective way to carry out those summations in practice is to truncate them to the first $\left\lceil \frac{\log \epsilon}{\log p_1} \right\rceil$ terms, where ϵ is a design parameter. In the numerical simulations of this paper we chose $\epsilon = 10^{-9}$. The same criterion can be used in the AR(2) case, where the

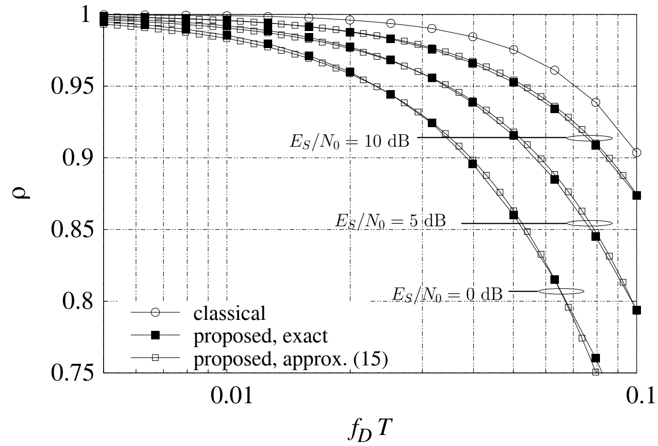

Fig. 3. Coefficient ρ in the AR(1) scenario, along with its approximation (15).

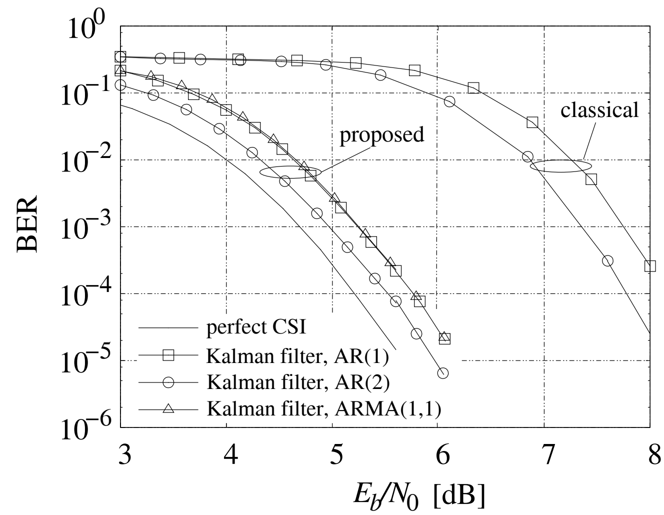

Fig. 4. Performance for an LDPC-coded BPSK modulation. The considered normalized Doppler bandwidth is $f_D T = 10^{-2}$.

largest (in magnitude) between p_1 and p_2 of eq. (14) has to be used to determine the truncation.

Another interesting but also intuitive conclusion derived from different simulation results, not reported in this paper, is that the gain of the AR(2) model with respect to the AR(1), decreases for increasing values of the equivalent Doppler bandwidth. In other words, for very large values of the Doppler bandwidth, the effective advantage of using a larger order AR approximation disappears. This of course holds also for the classical approach, and it is well in line with the results in [12]–[14].

In order to show the accuracy of the approximation (15), in Fig. 3 the optimized coefficient ρ in the AR(1) scenario is shown along with its approximation, as a function of the normalized Doppler spread and for three values of the SNR. For comparison, the coefficient obtained with the classical approach (that does not depend on the SNR) is also shown. As it can be seen, when suitable values of m and q are used in (15), the resulting approximation for ρ is excellent. As an example, for $E_S/N_0 = 5$ dB, $m = 1.34$ and $q = 1.51$.

The improvement in terms of MSE produces an improvement also in the BER performance. The sequence $\mathbf{c}$ is now assumed a codeword, possibly interleaved, of a channel code $\mathcal{C}$ constructed over a PSK constellation. For every considered scenario, simulation results for the case of known fading samples is carried out and the relevant curves (labeled "perfect CSI") added to every figure for comparison purposes. The BER performance will be shown versus E_b/N_0, where E_b denotes the received signal energy per information bit.

Fig. 4 refers to a binary PSK (BPSK) transmitted over a Rayleigh fading channel with $f_D T = 10^{-2}$. A (3,6)-regular rate-1/2 low-density parity-check (LDPC) code is employed, with codewords of 4000 bits. At the receiver, iterative decoding is carried out allowing at most 200 iterations. In order to ensure convergence of the iterative detection and decoding algorithm, 1 pilot symbol (known at the receiver) every 19 code symbols is placed in the transmitted codeword. This corresponds to a decrease in the effective transmission rate of a factor 19/20, resulting in an increase in the required signal-to-noise ratio of 0.223 dB which has been introduced artificially in the curves labeled "perfect CSI" for the sake of comparison. Hence, the gap between the "perfect CSI" curve and the others is uniquely due to the need for fading estimation/compensation, and not to the rate decrease due to pilot symbols. The receiver described in Sect. IV based on the approximation σ_0 is considered, for several channel models: AR(1), AR(2), and ARMA(1,1). There is a significant performance gain deriving from the proposed optimization method, reaching about 2.5 dB for both AR(1) and AR(2) models. As already pointed out, the ARMA(1,1) model gives no gain with respect to the AR(1) approximation, hence its use can be avoided since the computational complexity of receivers based on this model is larger than that of receivers based on the AR(1) model. For a very similar scenario, in [7] simulation results are reported showing that a simple low-pass filtering approach can outperform the first-order Kalman filter of a fraction of dB. Hence, the authors conclude that this simple approach is favorable. However, this conclusion does not hold when a proper optimization of the parameters of the employed ARMA model is adopted—a gain of about 2.5 dB is obtained in this case.

In Fig. 5, an LDPC-coded 8-PSK modulation over a fading channel with $f_D T = 5 \cdot 10^{-3}$ is considered. The irregular rate-2/3 LDPC code proposed for the DVB-S2 standard, whose codeword size is 64800 bits, is employed. At the receiver, iterative decoding is carried out allowing at most 40 iterations. As in the previous considered scenario, 1 pilot symbol every 19 code symbols is placed in the transmitted codeword to ensure convergence. All curves shown in the figure are obtained by using the proposed optimization method to choose the parameters of the AR models. The aim of this figure is to show the advantage of the proposed approximation σ_{KL} with respect to the classical approximation σ_0. This gain is about 1.5 dB in the case of a Kalman-based receiver for the AR(1) model, while the gain for the AR(2) model is negligible, since with both approximations the perfect CSI performance is reached thanks to the proposed method for optimizing the parameters of the AR model.

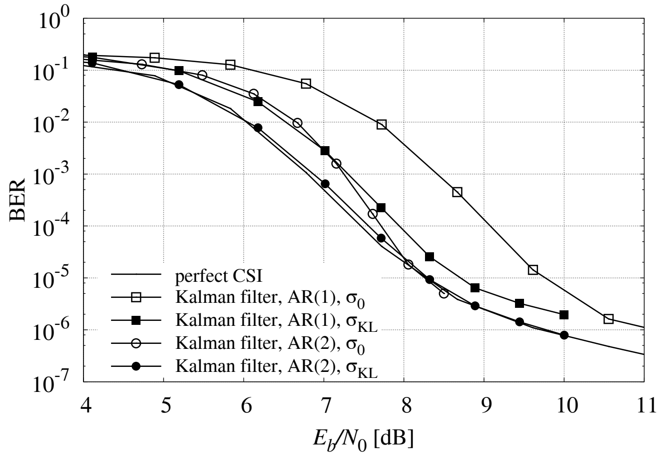

Fig. 5. Performance for an LDPC-coded 8-PSK modulation. The considered normalized Doppler bandwidth is $f_D T = 5 \cdot 10^{-3}$.

VI. Conclusions

In this paper, a new method to approximate a Rayleigh fading process with Clarke dynamics with an ARMA model has been proposed, showing that the classical method is highly suboptimal in terms of mean square error. We have shown that with the proposed optimization method not only the mean square estimation error is reduced, but also the bit-error rate of classical receivers based on a Kalman smoother can be dramatically reduced. It is worth pointing out that this gain comes at no further computational cost, since the optimization may be done once during the receiver design and the receiver itself does not change, since only the model parameters plugged in the receiver change.

Moreover, a novel approximation to be employed in detection algorithms based on a Kalman smoother, suitable for receivers based on iterative detection and decoding, has been proposed. This approximation offers a better performance than similar receivers presented in the literature with a similar computational complexity.

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