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Noise Analysis in Operational Amplifier Circuits

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Noise Analysis in Operational Amplifier Circuits

ABSTRACT

This application report uses standard circuit theory and noise models to calculate noise in op amp circuits. Example analysis of the inverting, noninverting, and differential-amplifier circuits shows how calculations are performed. Characteristics of noise sources are presented to help the designer make informed decisions when designing for noise.

Introduction

"Statistical fluctuation of electric charge exists in all conductors, producing random variation of potential between the ends of the conductor. The electric charges in a conductor are found to be in a state of thermal agitation, in thermodynamic equilibrium with the heat motion of the atoms of the conductor. The manifestation of the phenomenon is a fluctuation of potential difference between the terminals of the conductor" – J.B. Johnson[1]

"The term spontaneous fluctuations, although, perhaps, theoretically the most appropriate, is not commonly used in practice; usually it is simply called noise" – Aldert van der Ziel[2]

Early investigators of noise likened spontaneous fluctuations of current and voltage in electric circuits to Brownian motion. In 1928 Johnson[1] showed that electrical noise was a significant problem for electrical engineers designing sensitive amplifiers. The limit to the sensitivity of an electrical circuit is set by the point at which the signal-to-noise ratio drops below acceptable limits.

Notational Conventions

In the calculations throughout this report, lower case letters *e* and *i* indicate independent voltage and current noise sources; upper case letters *E* and *I* indicate combinations or amplified versions of the independent sources.

Spectral Density

A spectral density is a noise voltage or noise current per root hertz, i.e. $V/\sqrt{Hz}$ or $A/\sqrt{Hz}$. Spectral densities are commonly used to specify noise parameters. The characteristic equations that identify noise sources are always integrated over frequency, indicating that spectral density is the natural form for expressing noise sources. To avoid confusion in the following analyses, spectral densities are identified, when used, by stating them as volts or amps per root hertz.

Types of Noise

In electrical circuits there are 5 common noise sources:

- Shot noise
- Thermal noise
- Flicker noise
- Burst noise
- Avalanche noise

In op amp circuits, burst noise and avalanche noise are normally not problems, or they can be eliminated if present. They are mentioned here for completeness, but are not considered in the noise analysis.

Shot Noise

Shot noise is always associated with current flow. Shot noise results whenever charges cross a potential barrier, like a pn junction. Crossing the potential barrier is a purely random event. Thus the instantaneous current, i, is composed of a large number of random, independent current pulses with an average value, i_D. Shot noise is generally specified in terms of its mean-square variation about the average value. This is

written as $\overline{i_n^2}$, where :

$$\overline{i_n^2} = \overline{\left(i - i_D\right)^2} = \int 2qi_D df \tag{1}$$

Where q is the electron charge (1.62×10^{-19} C) and df is differential frequency.

Shot noise is spectrally flat or has a uniform power density, meaning that when plotted versus frequency, it has a constant value. Shot noise is independent of temperature.

The term qi_D is a current power density having units A^2/Hz.

Thermal Noise

Thermal noise is caused by the thermal agitation of charge carriers (electrons or holes) in a conductor. This noise is present in all passive resistive elements.

Like shot noise, thermal noise is spectrally flat or has a uniform power density, but thermal noise is independent of current flow.

Thermal noise in a conductor can be modeled as voltage or current. When modeled as a voltage it is placed in series with an otherwise noiseless resistor. When modeled as a current it is placed in parallel with an otherwise noiseless resistor. The average mean-square value of the voltage noise source or current noise source is calculated by:

$$\overline{e^2} = \int 4kTRdf \text{ or } \overline{i^2} = \int \left(4kT \,/\, R\right)df \tag{2}$$

Where k is Boltzmann's constant (1.38×10^{-23} j/K), T is absolute temperature in Kelvin (K), R is the resistance of the conductor in ohms (Ω) and df is differential frequency.

The terms *4kTR* and *4kT/R* are voltage and current power densities having units of V^2/Hz and A^2/Hz.

Flicker Noise

Flicker noise is also called *1/f* noise. It is present in all active devices and has various origins. Flicker noise is always associated with a dc current, and its average mean-square value is of the form:

$$\overline{e^2} = \int \left(K_e^2 / f \right) df \text{ or } \overline{i^2} = \int \left(K_i^2 / f \right) df \quad (3)$$

Where K_e and K_i are the appropriate device constants (in volts or amps), *f* is frequency, and *df* is differential frequency.

Flicker noise is also found in carbon composition resistors where it is often referred to as excess noise because it appears in addition to the thermal noise. Other types of resistors also exhibit flicker noise to varying degrees, with wire wound showing the least. Since flicker noise is proportional to the dc current in the device, if the current is kept low enough, thermal noise will predominate and the type of resistor used will not change the noise in the circuit.

The terms K_e^2/f and K_i^2/f are voltage and current power densities having units of V^2/Hz and A^2/Hz.

Burst Noise

Burst noise, also called popcorn noise, appears to be related to imperfections in semiconductor material and heavy ion implants. Burst noise makes a popping sound at rates below 100 Hz when played through a speaker. Low burst noise is achieved by using clean device processing.

Avalanche Noise

Avalanche noise is created when a pn junction is operated in the reverse breakdown mode. Under the influence of a strong reverse electric field within the junction's depletion region, electrons have enough kinetic energy that, when they collide with the atoms of the crystal lattice, additional electron-hole pairs are formed. These collisions are purely random and produce random current pulses similar to shot noise, but much more intense.

Noise Characteristics

Since noise sources have amplitudes that vary randomly with time, they can only be specified by a probability density function. Thermal noise and shot noise have Gaussian probability density functions. The other forms of noise noted do not. If δ is the standard deviation of the Gaussian distribution, then the instantaneous value lies between the average value of the signal and $\pm\delta$ 68% of the time. By definition, δ^2 (variance) is the average mean-square variation about the average value. This means that in noise signals having Gaussian distributions of amplitude, the average mean-square variation about the average value, $\overline{i^2}$ or $\overline{e^2}$, is the variance δ^2, and the rms value is the standard deviation δ.

Theoretically the noise amplitude can have values approaching infinity. However, the probability falls off rapidly as amplitude increases. An effective limit is $\pm 3\delta$, since the noise amplitude is within these limits 99.7% of the time. Figure 1 shows graphically how the probability of the amplitude relates to the rms value.

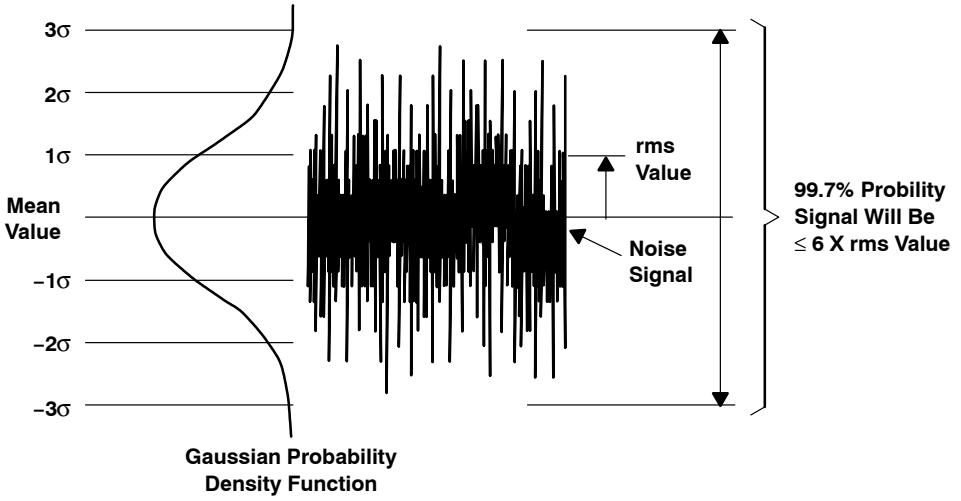


Figure 1. Gaussian Distribution of Noise Amplitude

Since the rms value of a noise source is equal to δ, to assure that a signal is within peak-to-peak limits 99.7% of the time, multiply the rms value by 6(+3δ–(–3δ)): *Erms* $\times$ *6* = *Epp*. For more or less assurance, use values between 4(95.4%) and 6.8(99.94%).

Adding Noise Sources

With multiple noise sources in a circuit, the signals must be combined properly to obtain the overall noise signal.

Consider the example of two resistors, R_1 and R_2, connected in series. Each resistor has a noise generator associated with it as shown in Figure 2 where

$$\overline{e_1^2} = \int 4kTR_1 df \quad \text{and} \quad \overline{e_2^2} = \int 4kTR_2 df .$$

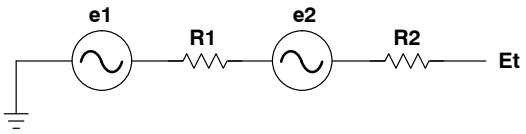


Figure 2. R1 and R2 Noise Model

To calculate the average mean square voltage, $\overline{E_t^2}$, across the two resistors, let $E_t(t) = e_1(t) + e_2(t)$ be the instantaneous values. Then

$$\overline{E_t(t)^2} = \overline{\left[e_1(t) + e_2(t)\right]^2} = \overline{e_1(t)^2} + \overline{e_2(t)^2} + \overline{2e_1(t)e_2(t)} \tag{4}$$

Since the noise voltages, $e_1(t)$ and $e_2(t)$, arise from separate resistors, they are independent, and the average of their product is zero:

$$\overline{2e_1(t)e_2(t)} = 0 \tag{5}$$

This results in

$$\overline{E_t^2} = \overline{e_1^2} + \overline{e_2^2}. \tag{6}$$

Therefore, as long as the noise sources arise from separate mechanisms and are independent, which is usually the case, the average mean square value of a sum of separate independent noise sources is the sum of the individual average mean square values. Thus in our example $\overline{E_t^2} = \int 4kT(R_1 + R_2)df$, which is what would be expected. This is derived using voltage sources, but also is true for current sources. The same result can be shown to be true when considering two independent sine wave sources.

Noise Spectra

A pure sine wave has power at only one frequency. Noise power, on the other hand, is spread over the frequency spectrum. Voltage noise power density, $\overline{e^2}/Hz$, and current noise power density, $\overline{i^2}/Hz$ are often used in noise calculations. To calculate the mean-square value, the power density is integrated over the frequency of operation. This application report deals with noise that is constant over frequency, and noise that is proportional to 1/f.

Spectrally flat noise is referred to as white noise. When plotted vs frequency, white noise is a horizontal line of constant value.

Flicker noise is 1/f noise and is stated in equation form as:

$$\overline{e^2} = \int \left(K_e^2/f\right) df \text{ or } \overline{i^2} = \int \left(K_i^2/f\right) df$$

See equation (3). When plotted vs frequency on log-log scales, 1/f noise is a line with constant slope. If the power density V^2/Hz is plotted, the slope is –1 decade per decade. If the square root of the power density, $V_{rms}/\sqrt{Hz}$, is plotted, the slope is –0.5 decade per decade.

Figure 3 shows the spectra of 1/f and white noise per root hertz.

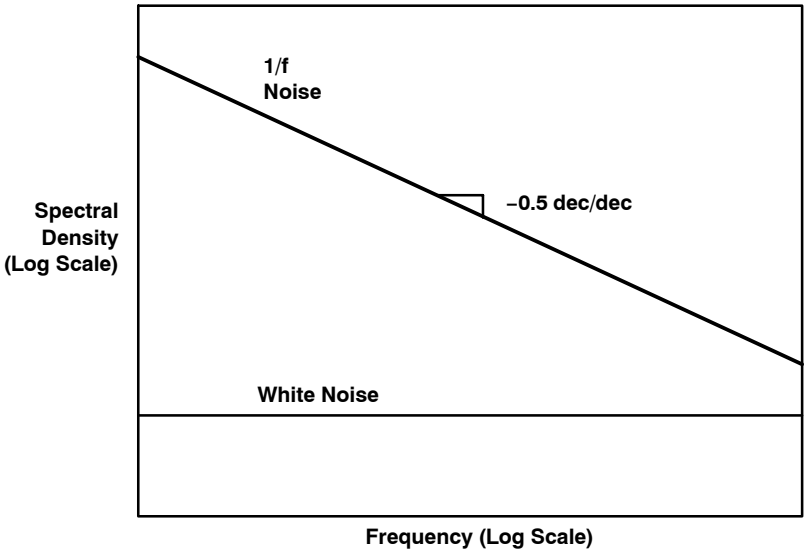


Figure 3. 1/f and White Noise Spectra

Integrating Noise

To determine the noise or current voltage over a given frequency band, the beginning and ending frequencies are used as the *f* integration limits and the integral evaluated. The following analysis uses voltages; the same is true for currents.

Given a white or constant voltage noise versus frequency source then:

$$\overline{e^2} = \int_{f_L}^{f_H} C\,df = C\left(f_H - f_L\right) \tag{7}$$

where $\overline{e^2}$ is the average mean-square voltage, C is the spectral power density per hertz (constant), f_L is the lowest frequency, and f_H is the highest frequency.

Given a 1/f voltage noise versus frequency source then:

$$\overline{e^2} = \int_{f_L}^{f_H} \frac{K^2}{f}\,df = K^2\,ln\,\frac{f_H}{f_L} \tag{8}$$

where $\overline{e^2}$ is the average mean-square voltage, K is the appropriate device constant in volts, f_L is the lowest frequency, and f_H is the highest frequency.

The input noise of an op amp contains both 1/f noise and white noise. The point in the frequency spectrum where 1/f noise and white noise are equal is referred to as the noise corner frequency, f_{nc}. Using the same notation as in the equations above, this means that $K^2/fnc = C$. It is useful to find f_{nc} because the total average mean-square noise can be calculated by adding equations (7) and (8) and substituting Cf_{nc} for K^2:

$$\overline{E^2} = C\left(f_{nc}\,ln\,\frac{f_H}{f_L} + f_H - f_L\right) \tag{9}$$

Where C is the square of the white noise voltage specification for the op amp.

Figure 4 shows the equivalent input noise voltage vs frequency graph for the TLV2772 as normally displayed in the data sheet.

f_{nc} can be determined visually from the graph of equivalent input noise per root hertz vs. frequency graph that is included in most op amp data sheets. Since at f_{nc} the white noise and 1/f noise are equal, f_{nc} is the frequency at which the noise is $\sqrt{2}$ x white noise specification. This would be about *17 nV/$\sqrt{Hz}$* for the TLV2772, which is at 1000 Hz as shown in Figure 4.

Another way to find f_{nc}, is to determine K^2 by finding the equivalent input noise voltage per root hertz at the lowest possible frequency in the 1/f noise region, square this value, subtract the white noise voltage squared, and multiply by the frequency. Then divide K^2 by the white noise specification squared. The answer is f_{nc}.

For example, the TLV2772 has a typical noise voltage of *130 nV/$\sqrt{Hz}$* at 10 Hz.

The typical white noise specification for the TLV2772 is 12 *nV/$\sqrt{Hz}$*

$$K^2 = \left[\left(130\ nV/\sqrt{Hz}\right)^2 - \left(12\ nV/\sqrt{Hz}\right)^2\right] \times (10\ Hz) = 167560\ (nV)^2$$

Therefore, f_{nc} = (167560(nV)^2)/(144(nV)^2/Hz) = 1163 Hz

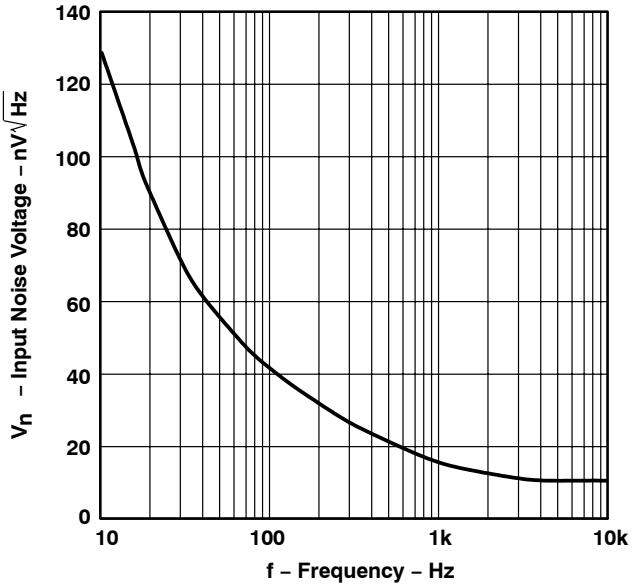


Figure 4. Equivalent Input Noise Voltage vs Frequency for TLV2772 as Normally Presented

Figure 5 was constructed by interpreting the equivalent input noise voltage versus frequency graph for the TLV2772 and plotting the values on log-log scales. The –0.5 dec/dec straight line nature of 1/f noise when plotted on log-log scales can be seen.

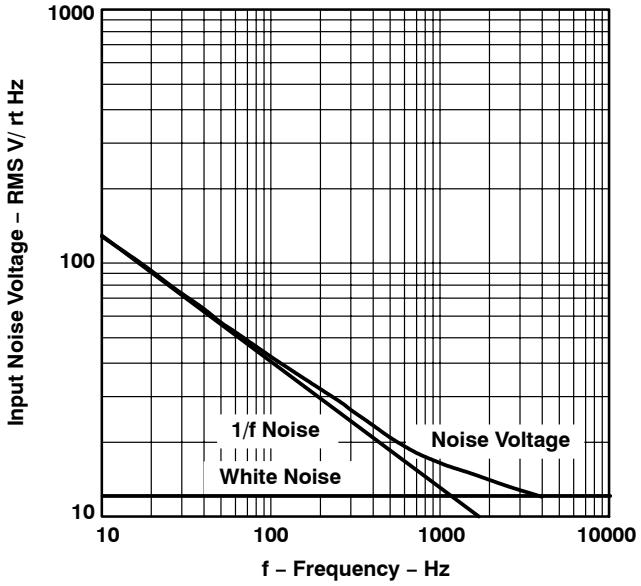


Figure 5. Equivalent Input Noise Voltage vs Frequency for TLV2772 on Log-Log Scale

Equivalent Noise Bandwidth

Equations (7), (8), and (9) are only true if the bandwidth If the op amp circuit is brick-wall. In reality there is always a certain amount of out-of-band energy transferred. The equivalent noise bandwidth (ENB) is used to account for the extra noise so that brick-wall frequency limits can be used in equations (7), (8), and (9). Figure 6 shows the idea for a first order low-pass filter.

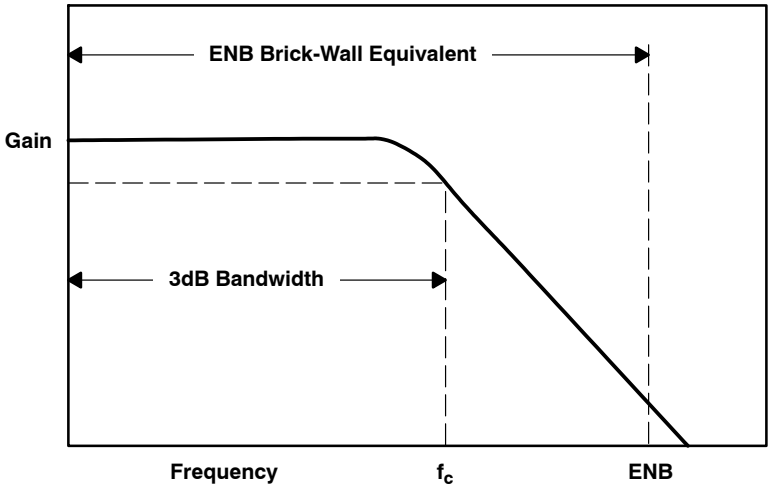


Figure 6. ENB Brick-Wall Equivalent

Figure 7 shows an example of a simple RC filter used to filter a voltage noise source, e_{in}.

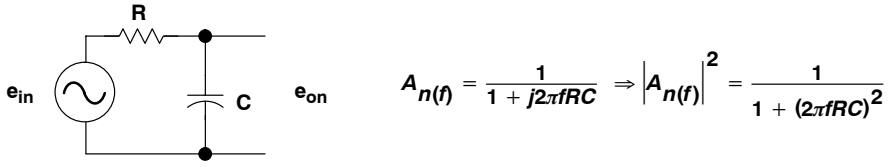


Figure 7. RC Filter

$A_{n(f)}$ is the frequency-dependant gain of the circuit, and e_{on} is calculated:

$$e_{on} = \sqrt{\int_0^\infty |A_{n(f)}|^2 \, e_{in}{}^2 \, df} \tag{10}$$

Assuming e_{in} is a white noise source (specified as a spectral density in V/$\sqrt{\text{Hz}}$), using radian measure for frequency, and substituting for $A_{n(f)}$, the equation can be solved as follows:

(11)

$$e_{on} = e_{in} \sqrt{\int_0^\infty \frac{1}{1 + (2\pi fRC)^2} df} = e_{in} \sqrt{\frac{1}{2\pi RC} \Big|_0^\infty \tan^{-1} 2\pi fRC} = e_{in} \sqrt{\frac{1}{2\pi RC} \frac{\Pi}{2}}$$

So that the ENB = 1.57 x 3dB bandwidth in this first-order system. This result holds for any first-order low-pass function. For higher order filters the ENB approaches the normal cutoff frequency, f_c, of the filter. Table 1 shows the ENB for different order low-pass filters.

Table 1. ENB vs Filter Order for Low-Pass Filters

FILTER ORDER	ENB
1	1.57 x f_c
2	1.11 x f_c
3	1.05 x f_c
4	1.025 x f_c

Resistor Noise Model

To reiterate, noise in a resistor can be modeled as a voltage source in series, or as a current source in parallel, with an otherwise noiseless resistor as shown in Figure 8. These models are equivalent and can be interchanged as required to ease analysis. This is explored in Appendix A.

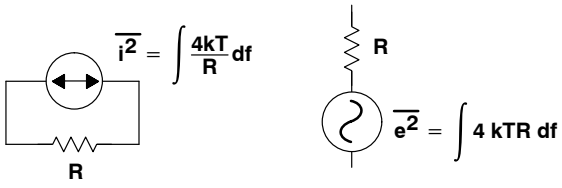


Figure 8. Resistor Noise Models

Op Amp Circuit Noise Model

Op amp manufacturers measure the noise characteristics for a large sampling of a device. This information is compiled and used to determine the typical noise performance of the device. The noise specifications published by Texas Instruments in their data sheets refer the measured noise to the input of the op amp. The part of the internally generated noise that can properly be represented by a voltage source is placed in series with the positive input to an otherwise noiseless op amp. The part of the internally generated noise that can properly be represented by current sources is placed between each input and ground in an otherwise noiseless op amp. Figure 9 shows the resulting noise model for a typical op amp.

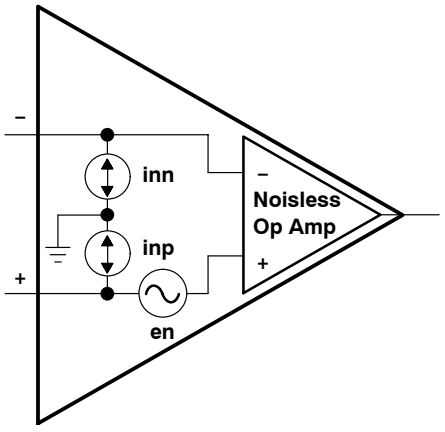


Figure 9. Op Amp Noise Model

Inverting and Noninverting Op Amp Circuit Noise Calculations

To perform a noise analysis, the foregoing noise models are added to the circuit schematic and the input signal sources are shorted to ground. When this is done to either an inverting or a noninverting op amp circuit, the same circuit results, as shown in Figure 10. This circuit is used for the following noise analysis.

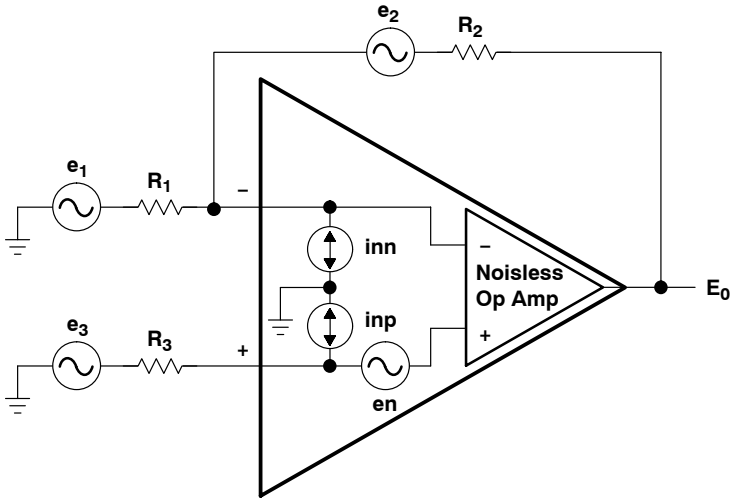
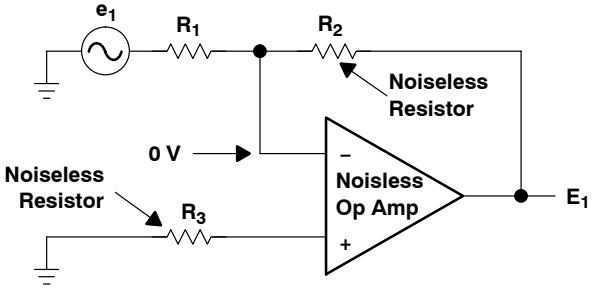


Figure 10. Inverting and Noninverting Noise Analysis Circuit

At first, this analysis may appear somewhat daunting, but it can be deciphered piece by piece. Using the principles of superposition, each of the noise sources is isolated, and everything else is assumed to be noiseless. Then the results can be added according to the rules for adding independent noise sources. An ideal op amp is assumed for the noiseless op amp. In the end it will all seem simple, if a little tedious the first time through.

Figures 11 through 13 show the analysis.

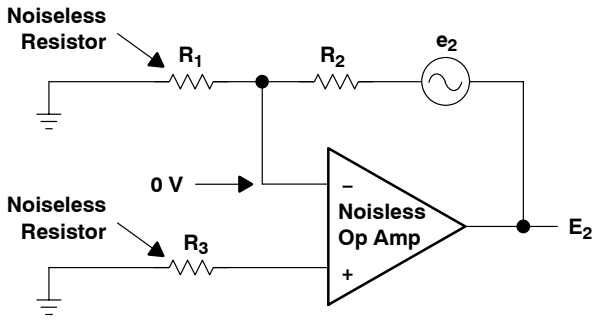


$$\overline{E_1} = \overline{e_1}\,\frac{R_2}{R_1}$$

$$\overline{E_1}^2 = \overline{e_1}^2\left(\frac{R_2}{R_1}\right)^2$$

$$\overline{e_1}^2 = \int 4kTR_1\,df$$

Figure 11. E1

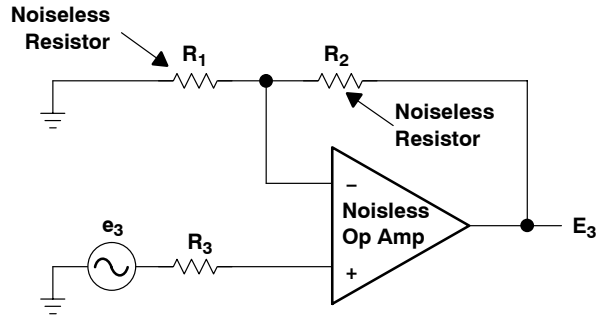


$$\overline{E_2} = \overline{e_2}$$

$$\overline{E_2}^2 = \overline{e_2}^2$$

$$\overline{e_2}^2 = \int 4kTR_2\,df$$

Figure 12. E2



$$\overline{E_3} = \overline{e_3}\left(\frac{R_1 + R_2}{R_1}\right)$$

$$\overline{E_2}^2 = \overline{e_2}^2\left(\frac{R_1 + R_2}{R_1}\right)^2$$

$$\overline{e_3}^2 = \int 4kTR_3\,df$$

Figure 13. E3

Combining to arrive at the solution for the circuit's output rms noise voltage, E_{Rrms}, due to the thermal noise of the resistors in the circuit:

(12)

$$E_{Rrms} = \sqrt{\overline{E_1}^2 + \overline{E_2}^2 + \overline{E_3}^2}$$

$$E_{Rrms} = \sqrt{\int \left[4kTR_1\left(\frac{R_2}{R_1}\right)^2 + 4kTR_2 + 4kTR_3\left(\frac{R_1 + R_2}{R_1}\right)^2\right] df}$$

$$E_{Rrms} = \sqrt{\int \left[4kTR_2\left(\frac{R_1 + R_2}{R_1}\right) + 4kTR_3\left(\frac{R_1 + R_2}{R_1}\right)^2\right] df}$$

If it is desired to know the resistor noise referenced to the input, E_{iRrms}, the output noise is divided by the noise gain, A_n, of the circuit:

$$A_n = \left(\frac{R_1 + R_2}{R_1}\right) \tag{13}$$

(14)

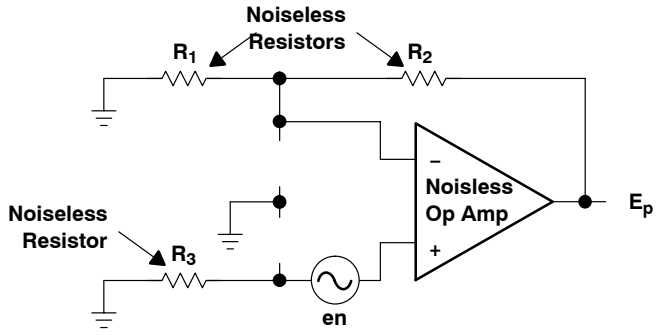
$$E_{iRrms}^{2} = \left(\frac{E_{Rrms}}{A_n}\right)^2 = \int \frac{\left[4kTR_2\left(\frac{R_1+R_2}{R_1}\right) + 4kTR_3\left(\frac{R_1+R_2}{R_1}\right)^2\right]df}{\left(\frac{R_1+R_2}{R1}\right)^2} = \int 4kT\left[\left(\frac{R_1R_2}{R_1+R_2}\right) + R_3\right]df$$

Normally R_3 is chosen to be equal to the parallel combination of R_1 and R_2 to minimize offset voltages due to input bias current. If this is done, the equation simplifies to:

(15)

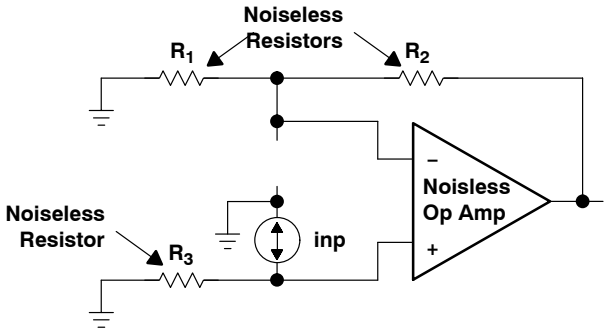
$$E_{iRrms} = \sqrt{\int 8kTR_3\,df} \qquad When\ R3 = \left(\frac{R_1R_2}{R_1+R_2}\right)$$

Now consider the noise sources associated with the op amp itself. The analysis proceeds as before as shown in Figures 14 through 16.



$$\overline{E_n^2} = \int\left[(en)\left(\frac{R_1+R_2}{R_1}\right)\right]^2 df$$

Figure 14. Ep



$$\overline{E_{np}^2} = \int\left[(i_{np})(R_3)\left(\frac{R_1+R_2}{R_1}\right)\right]^2 df$$

Figure 15. E_{np}

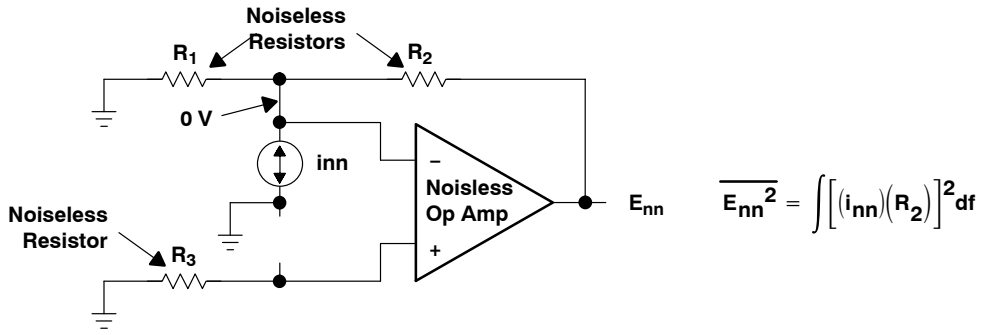


Figure 16. E_{nn}

Combining to arrive at the solution for the circuit's output rms noise voltage, E_{oarms}, due to the input referred op amp noise in the circuit:

(16)

$$E_{oarms} = \sqrt{\overline{E_n}^2 + \overline{E_{np}}^2 + \overline{E_{nn}}^2}$$

$$E_{oarms} = \sqrt{\int \left[\left(i_{nn}(R_2) \right)^2 + \left((i_{np}) R_3 \left(\frac{R_1 + R_2}{R_1} \right) \right)^2 + \left(e_n \left(\frac{R_1 + R_2}{R_1} \right) \right)^2 \right] df}$$

Now combining the resistor noise and the op amp noise to get the total output rms noise voltage, E_{Trms}.

(17)

$$E_{Trms} = \sqrt{\int \left[4kTR_2 \left(\frac{R_1 + R_2}{R_1} \right) + 4kTR_3 \left(\frac{R_1 + R_2}{R_1} \right)^2 + \left((i_{nn}) R_2 \right)^2 + \left((i_{np}) R_3 \left(\frac{R_1 + R_2}{R_1} \right) \right)^2 + \left(e_n \left(\frac{R_1 + R_2}{R_1} \right) \right)^2 \right] df}$$

The only work left is to evaluate the integral. Most of the terms are constants that can be brought straight out of the integral. The resistors and their associated noise are constant over frequency so that the first two terms are constants. The last three terms contain the input referred noise of the op amp. The voltage and current input referred noise of op amps contains flicker noise, shot noise, and thermal noise. This means that they must be evaluated as a combination of white and 1/f noise. Using equation (9) and Table 1, the output noise is:

$$E_{Trms} = \sqrt{ENB \left(4kTR_2 A + 4kTR_3 A^2 \right) + i_w^2 \left(R_2^2 + R_3^2 A^2 \right) \left(f_{inc} \ln \frac{f_H}{f_L} + ENB \right) + e_w^2 A^2 \left(f_{enc} \ln \frac{f_H}{f_L} + ENB \right)}$$

Where $A = (R_1 + R_2)/R_1$, i_w is the white current noise specification (spectral density in $A/\sqrt{Hz}$), f_{inc} is the current noise corner frequency, e_w is the white voltage noise specification (spectral density in $V/\sqrt{Hz}$, and f_{enc} is the voltage noise corner frequency. ENB is determined by the frequency characteristics of the circuit. f_H/f_L is set equal to ENB.

In CMOS input op amps, noise currents are normally so low that the input noise voltage dominates and the i_w terms are not factored into the noise computation. Also, since bias current is very low, there is no need to use R_3 for bias current compensation, and it, too, is removed from the circuit and the calculations. With these simplifications the formula above reduces to:

$$E_{Trms} = \sqrt{ENB\ 4kTR_2 A + e_w^2 A^2 \left(f_{enc} \ln \frac{f_H}{f_L} + ENB \right)}$$ CMOS input op amps

Differential Op Amp Circuit Noise Calculations

A noise analysis for a differential amplifier can be done in the same manner as the previous example. Figure 17 shows the circuit used for the analysis.

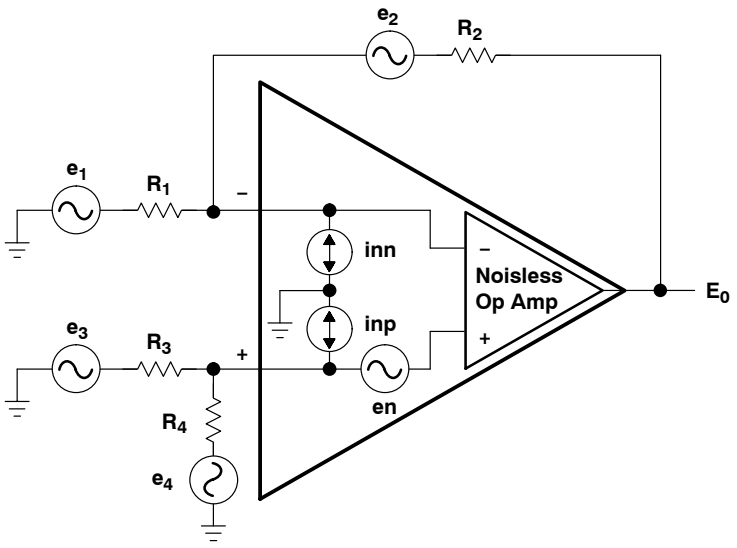
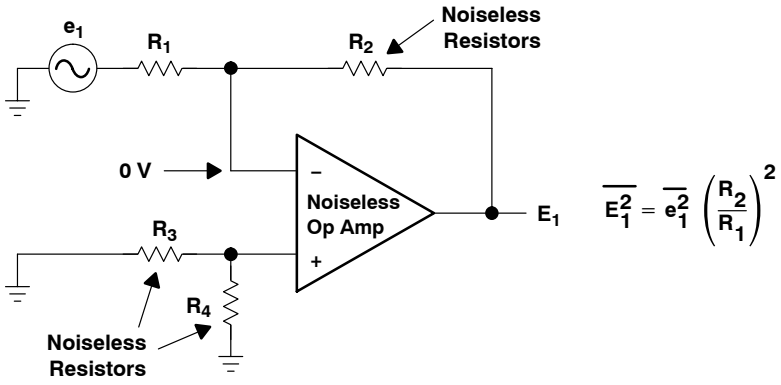


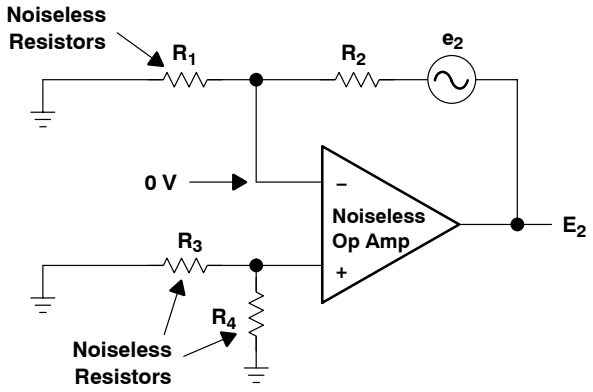
Figure 17. Differenital Op Amp Circuit Noise Model

Figures 18 through 21 show the analysis.



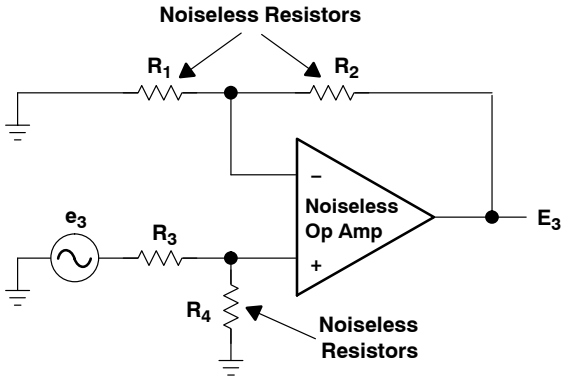
$$\overline{E_1^2} = \overline{e_1^2} \left(\frac{R_2}{R_1} \right)^2$$

Figure 18. e1



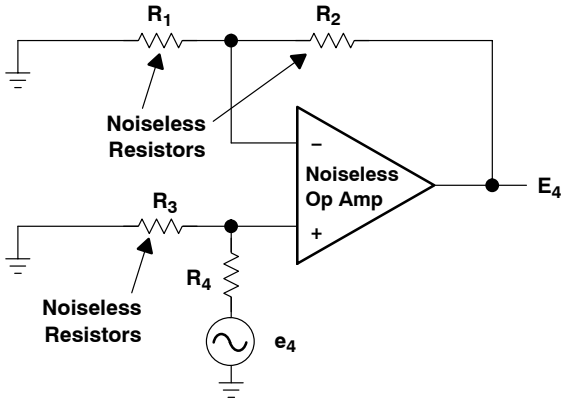
$$\overline{E_2^2} = \overline{e_2^2}$$

Figure 19. e2



$$\overline{E_3^2} = \overline{e_3^2}\left[\left(\frac{R_4}{R_3 + R_4}\right)\left(\frac{R_1 + R_2}{R_1}\right)\right]^2$$

Figure 20. e3



$$\overline{E_4^2} = \overline{e_4^2}\left[\left(\frac{R_3}{R_3 + R_4}\right)\left(\frac{R_1 + R_2}{R_1}\right)\right]^2$$

Figure 21. e4

Combining to arrive at the solution for the circuit's output rms noise voltage, E_{Rrms}, due to the thermal noise in the resistors in the circuit:

(18)

$$E_{Rrms} = \sqrt{E1^2 + E2^2 + E3^2 + E4^2}$$

$$E_{Rrms} = \sqrt{\int \left[\left(4kTR1\left(\frac{R2}{R1}\right)^2 \right) + (4kTR2) + \left(4kTR3\left(\frac{R4}{R3+R4}\right)^2 \left(\frac{R1+R2}{R1}\right)^2 \right) + \left(4kTR4\left(\frac{R3}{R3+R4}\right)^2 \left(\frac{R1+R2}{R1}\right)^2 \right) \right] df}$$

$$E_{Rrms} = \sqrt{\int 4kT \left[\frac{R2^2}{R1} + R2 + \left(R3\left(\frac{R4}{R3+R4}\right)^2 \left(\frac{R1+R2}{R1}\right)^2 \right) + \left(R4\left(\frac{R3}{R3+R4}\right)^2 \left(\frac{R1+R2}{R1}\right)^2 \right) \right] df}$$

Normally $R_1 = R_3$ and $R_2 = R_4$. Making this substitution reduces the above equation to:

$$E_{Rrms} = \sqrt{\int 8kTR2\left(1 + \frac{R2}{R1}\right) df} \quad \textit{If } R1 = R3 \textit{ and } R2 = R4 \tag{19}$$

Now consider the noise sources associated with the op amp itself. The analysis proceeds as before as shown in Figures 22 through 24.

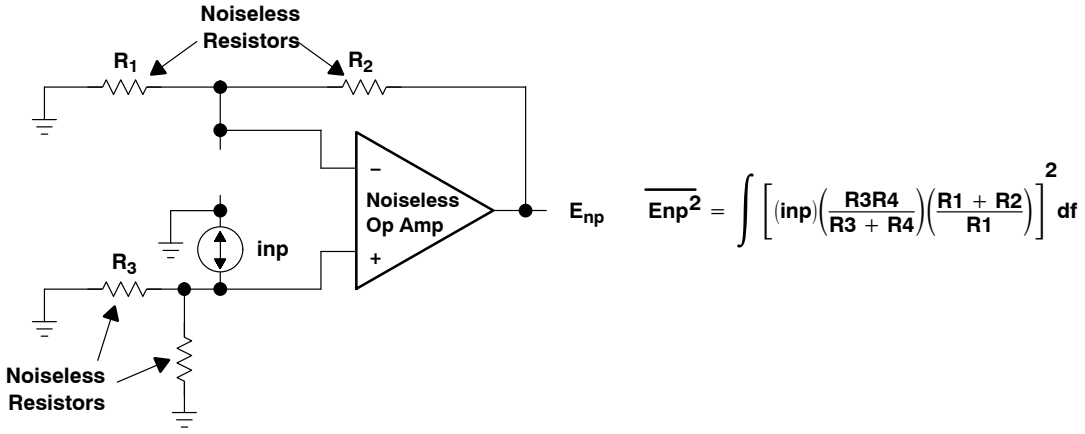


Figure 22. inp

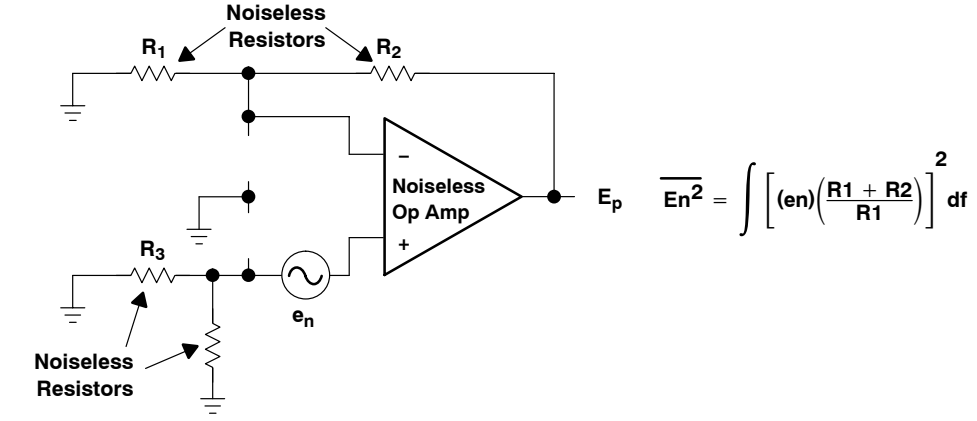


Figure 23. en

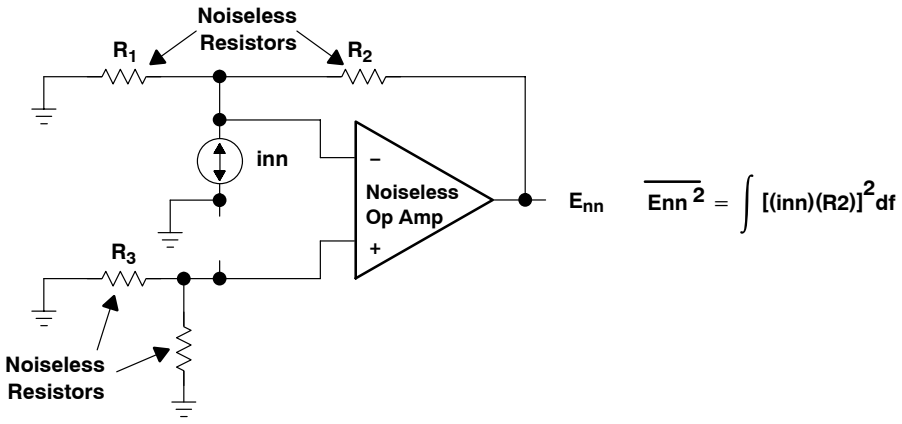


Figure 24. inn

Combining to arrive at the solution for the circuit's output rms noise voltage, E_{oarms}, due to the input referred op amp noise in the circuit:

(20)

$$E_{oarms} = \sqrt{\overline{En}^2 + \overline{Enp}^2 + \overline{Enn}^2}$$

$$E_{oarms} = \sqrt{\int\left[((inn)\,R2)^2 + \left((inp)\left(\frac{R3R4}{R3 + R4}\right)\left(\frac{R1 + R2}{R1}\right)\right)^2 + \left(en\left(\frac{R1 + R2}{R1}\right)\right)^2\right]df}$$

Normally $R_1 = R_3$, $R_2 = R_4$, and $inn = inp = in$. Making this substitution reduces the above equation to:

(21)

$$E_{oarms} = \sqrt{\int\left[(2inR2)^2 + \left(en\left(\frac{R1 + R2}{R1}\right)\right)^2\right]df}$$

$$R1 = R3,\ R2 = R4 \text{ and } inn = inp = in$$

Now combine the resistor noise and the op amp noise to get the total output rms noise voltage, E_{Trms}.

(22)

$$E_{Trms} = \sqrt{\int \left[((inn)\,R2)^2 + \left((inp)\left(\frac{R3R4}{R3+R4}\right)\left(\frac{R1+R2}{R1}\right)\right)^2 + \left(en\left(\frac{R1+R2}{R1}\right)\right)^2 + \left(4kTR1\left(\frac{R2}{R1}\right)^2\right) + (4kTR2) + \left(4kTR3\left(\frac{R4}{R3+R4}\right)^2\left(\frac{R1+R2}{R1}\right)^2\right) + \left(4kTR4\left(\frac{R3}{R3+R4}\right)^2\left(\frac{R1+R2}{R1}\right)^2\right)\right] df}$$

$$E_{Trms} = \sqrt{\int \left[((inn)\,R2)^2 + \left((inp)\left(\frac{R3R4}{R3+R4}\right)\left(\frac{R1+R2}{R1}\right)\right)^2 + \left(en\left(\frac{R1+R2}{R1}\right)\right)^2 + 4kT\left[\frac{R2^2}{R1} + R2 + \left(R3\left(\frac{R4}{R3+R4}\right)^2\left(\frac{R1+R2}{R1}\right)^2\right) + \left(R4\left(\frac{R3}{R3+R4}\right)^2\left(\frac{R1+R2}{R1}\right)^2\right)\right] df}$$

Substituting $R_1 = R_3$, $R_2 = R_4$, and $inn = inp = in$:

(23)

$$E_{Trms} = \sqrt{\int \left[2\,(inR2)^2 + \left(en\left(\frac{R1+R2}{R1}\right)\right)^2 + 8kTR2\left(\frac{R1+R2}{R1}\right)\right] df}$$

$$R1 = R3,\ R2 = R4 \text{ and } inn = inp = in$$

Evaluating the integral using these simplifications results in:

(24)

$$E_{Trms} = \sqrt{ENB\ 8kTR_2A + 2\left(i_w^2 R_2^2\right)\left(f_{inc}\,ln\,\frac{f_H}{f_L} + ENB\right) + e_w^2 A^2\left(f_{enc}\,ln\,\frac{f_H}{f_L} + ENB\right)}$$

$$R1 = R3,\ R2 = R4 \text{ and } inn = inp = in$$

Where $A = (R_1 + R_2)/R_1$, i_w is the white current noise specification (spectral density in A/$\sqrt{\text{Hz}}$), f_{inc} is the current noise corner frequency, e_w is the white voltage noise specification (spectral density in V/$\sqrt{\text{Hz}}$), and f_{enc} is the voltage noise corner frequency. ENB is determined by the frequency characteristics of the circuit. f_H/f_L is set equal to ENB.

Summary

The techniques presented here can be used to perform a noise analysis on any circuit. Superposition was chosen for illustrative purposes, but the same solutions can be derived by using other circuit analysis techniques.

Noise is a purely random signal; the instantaneous value and/or phase of the waveform cannot be predicted at any time. The only information available for circuit calculations is the average mean-square value of the signal. With multiple noise sources in a circuit, the total root-mean-square (rms) noise signal that results is the square root of the sum of the average mean-square values of the individual sources.

$$E_{Totalrms} = \sqrt{e_{1rms}^2 + e_{2rms}^2 + ...e_{nrms}^2}$$

Because noise adds by the square, when there is an order of magnitude or more difference in value, the lower value can be ignored with very little error. For example:

$$\sqrt{1^2 + 10^2} = 10.05$$

If the 1 is ignored, the error is 0.5%. With modern computational resources, evaluation of all the terms is trivial, but it is important to understand the principles so that time will be spent reducing the 10 before working on the 1.

Noise is normally specified as a spectral density in rms volts or amps per root Hertz, $V/\sqrt{Hz}$ or $A/\sqrt{Hz}$. To calculate the amplitude of the expected noise signal, the spectral density is integrated over the equivalent noise bandwidth (ENB) of the circuit.

Very often the peak-to-peak value of the noise is of interest. Once the total rms noise signal is calculated, the expected peak-to-peak value can be calculated. The instantaneous value will be equal to or less than 6 times the rms value 99.7% of the time.

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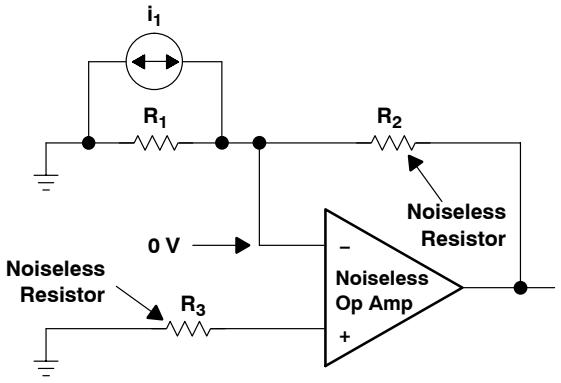
5. Sergio Franco. *Design with Operational Amplifiers and Analog Integrated Circuits*. McGraw-Hill, Inc., 1988.
6. David E. Johnson, Johnny R. Johnson, and John L. Hilburn. *Electric Circuit Analysis.* Prentice-Hall, Inc., 1989.

Appendix A Using Current Sources for Resistor Noise Analysis

Figures A1 through A3 show analysis of the resistor noise in the inverting/noninverting op amp noise analysis circuit using current sources in parallel with the noiseless resistors.

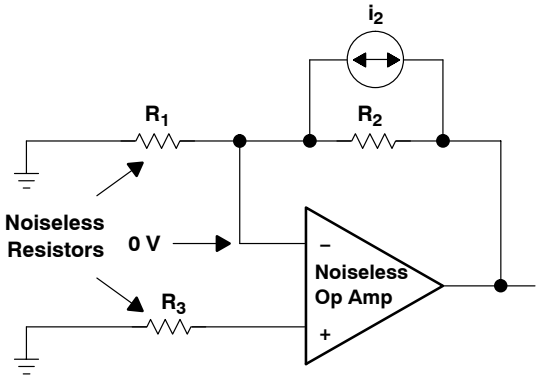


$$\overline{E_1} = \overline{i_1} R_1 \frac{R_2}{R_1} = \overline{i_1} R_2$$

$$\overline{E_1}^2 = \left(\overline{i_1} R_2\right)^2$$

$$\overline{i_1}^2 = \int \frac{4kT}{R_1}\, df$$

Figure A–1. E1

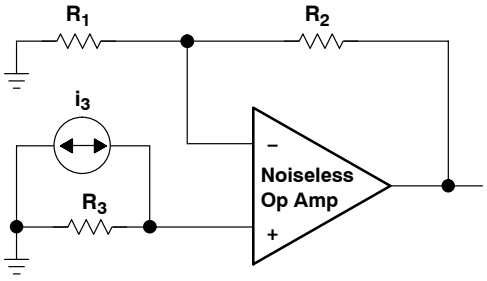


$$\overline{E_2} = \overline{i_2} R_2$$

$$\overline{E_2}^2 = \left(\overline{i_2} R_2\right)^2$$

$$\overline{i_2}^2 = \int \frac{4kT}{R_2}\, df$$

Figure A–2. E2



$$\overline{E_3} = \overline{i_3} R_3 \left(\frac{R_1 + R_2}{R_1}\right)$$

$$\overline{E_3}^2 = \left[\overline{i_3} R_3 \left(\frac{R_1 + R_2}{R_1}\right)\right]^2$$

$$\overline{i_3}^2 = \int \frac{4kT}{R_3}\, df$$

Figure A–3. E3

Combining the independent noise signals:

(A–1)

$$E_{Rrms} = \sqrt{\overline{E_1}^2 + \overline{E_2}^2 + \overline{E_3}^2}$$

$$E_{Rrms} = \sqrt{\int \left[\frac{4kT}{R_1} R_2^2 + \frac{4kT}{R_2} R_2^2 + \frac{4kT}{R_3} R_3^2 \left(\frac{R_1 + R_2}{R_1} \right)^2 \right] df}$$

$$E_{Rrms} = \sqrt{\int \left[4kTR_2 \left(\frac{R_1 + R_2}{R_1} \right) + 4kTR_3 \left(\frac{R_1 + R_2}{R_1} \right)^2 \right] df}$$

The resulting equation is the same as equation (12) presented earlier.

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