

# A Comparison of Two Fitting Functions for Saccadic Pulse Component Mathematical Modelling

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**Abstract.** An accepted model for the saccade signal of ocular motor neurons comprises two components in the form of a pulse and a step. In this contribution, an assessment of two fitting functions for the saccadic pulse component is made, in order to obtain a reduced set of descriptors that could be used for the early diagnosis of ataxia. Results show that both models have achieved to describe the waveform of the saccadic pulse signal, revealing higher performance of Gauss series over the gamma function.

## 1 Introduction

Ocular movements are affected by inherited spinocerebellar ataxias [3, 1, 2, 4], specially the behavior of saccadic system is modified, patients show increased latencies to respond to visual stimulation with slower saccades among other impairments of this system [5].

An accepted model for saccades generation involves two components, oculomotor plant is driven by a pulse-step to produce a saccade [6]. Several works have been using independent component analysis to isolates pulse and step components in saccadic and vergence ocular movements [7, 9, 8]. Based in these results in a precedent work we have applied independent component analysis to noisy electro-oculographic records of patients, of ataxia SCA2, characterized by severely deformed saccades and the pulse and step components were obtained [10]. In order to evaluate the pulse component several of its parameters has been used, as duration, amplitude, the time to reach a determined percent of the final value (not including the latent period) [12, 9, 11] among others. An important limitation common in the estimation of these variables is the need to use thresholds to identify onset and offset of the pulse.

In the present work an evaluation of two fitting functions for the pulse component is made in order to obtain a reduced set of descriptors to be used for classification purposes of ataxia patients and presymptomatics with respect to healthy subjects.

## 2 Materials and Methods

All the experiments and material were handled by the medical staff of the Centre for the Research and Rehabilitation of Hereditary Ataxias (CIRAH) at Holguín (Cuba). A two-channel electronystagmograph (Otoscreen, Jaeger-Toennies, Hochberg, Germany) was used to record saccadic ocular movements. Subjects were seated on chair with special fixation accessory to avoid head movements, and asked to follow a divergence stimuli conformed by a white circular target in a blank screen. The target appeared suddenly at each side of the screen at random time slots between 1s and 3 s, the distance from subject to screen was adjusted to obtain an angular distance of 30 degrees.

A group of 19 records of patients of ataxia was collected and the 29 records of presymptomatic individuals with genetic evidence of the disease but without detected symptoms and a third group of 23 records of healthy subjects. Saccades were identified using an automated algorithm based in a velocity threshold of 10 degrees per second, after this step a manual process of visual inspection was made to eliminate saccades considered anticipatory (latencies lower than 100 ms) or with artifacts like blinkings, excessive noise, muscle or extra ocular movements in the close time before onset, or in the next fixation. In the next step the mean of amplitude, duration and latency were calculated and saccades with deviations higher than 20% were excluded, in similar way as it is described in [10].

### 2.1 Obtaining Pulse and Step Components

Independent component analysis (ICA) is a well known method for estimation of underlying components in mixtures of non gaussian and statistically independent variables. The aim of ICA is to find the linear matrix  $W$  which mixes the  $x$  independent components to produce a set of  $y$  observed signals, as it is shown in the equation 1:

$$y = W \times x. \quad (1)$$

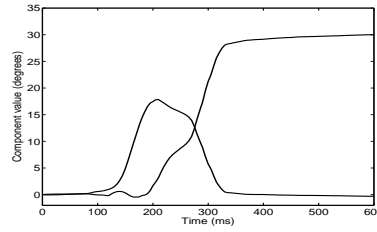
To apply ICA as observations are considered an ensemble of saccades after the process of identification and exclusion of non valid saccades, where each row is a vector containing a saccade:

$$S = \begin{pmatrix} s_{11} & s_{12} & \dots & s_{1n} \\ \vdots & \vdots & \dots & \vdots \\ s_{m1} & s_{m2} & \dots & s_{mn} \end{pmatrix} \quad (2)$$

An Infomax ICA [13] algorithm implemented in *matlab* [14] was used accordingly to the procedure described in [10]. Figure 1 shows the pulse and step obtained for a patient of ataxia.

### 2.2 Pulse Component Fitting

The fitting of step component using a sigmoid function was treated in a precedent work [10], four coefficients were estimated and used in conjunction with the value and latency



**Fig. 1.** Pulse and step components.

of the maximum of the pulse component for classification purposes. The best results of experimental tests with several functions to fit the pulse components were achieved with the gamma function, previously used to model the velocity behavior of saccadic movements [15], and gaussian series with different numbers of terms. The equation of the gamma function is the following:

$$f(x) = a \left[ \frac{x}{b} \right]^{c-1} e^{-\frac{x}{b}} \quad (3)$$

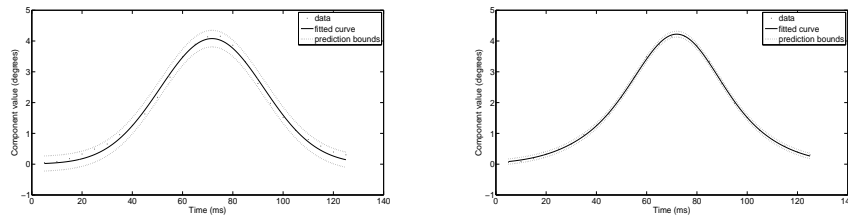
Where  $a$ ,  $b$ ,  $c$  and  $d$  are the coefficients to be adjusted.

While the gaussian series is expressed by:

$$f(x) = \sum_{i=1}^n a_i e^{-\left(\frac{x-b_i}{c_i}\right)^2} \quad (4)$$

Where the coefficients for each term are  $a_i$ ,  $b_i$ ,  $c_i$

The function fitting procedure was made by means of the *fit* function implemented in *matlab*. Prior to the fitting process the active pulse segment was identified, in order to consider only this part of the signal. The onset and offset points were marked examining the first derivative of the signal at left (onset point) and right (offset point) sides of the central maximum value, until a value of 0 was found. Figure 2 shows a segmented pulse fitted by gamma function (left) and using a two terms Gauss series (right).



**Fig. 2.** Pulse component fitted using a gamma (left) and Gauss1 (right) function.

### 3 Results

The fitting was applied to all the records, using gamma function and Gauss series from one up to five terms. A visual inspection of the graphics for every record and fitting function was done to rank the results, when no differences were seen among several

**Table 1.** Ranking of fitting functions.

Fitting functions	1	2	3	4	5	6
Gamma	0.97%	28.64%	49.51%	11.65%	1.94%	0.49%
Gauss1	3.88%	38.83%	32.04%	19.9%	5.34%	0%
Gauss2	62.14%	28.16%	6.8%	2.43 %	0%	0%
Gauss3	80.01%	7.28%	2.43%	0%	0%	0%
Gauss4	82.04%	1.46%	0%	0%	0%	0%
Gauss5	73.79%	1.46%	0.49%	0%	0%	0%

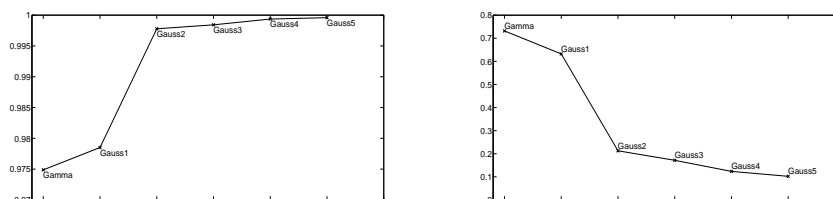
functions, same rank was assigned to them for this record. This is summarized in Table 1.

Results from Table 1 reveal gamma function worse performance when compared with the Gauss series. Best results are achieved by Gauss2 and higher. Table 2 shows an analysis of the success in fitting of each function, gamma function is below Gauss1 and Gauss2, from Gauss3 this indicator deteriorates. An overall evaluation of both tables points to Gauss2 as the best, with a very low percent of failed fittings and ranked first or second about 80% of the cases, while gamma is in general sense worst than Gauss series.

**Table 2.** Failures of fitting functions.

Fitting functions	Number of failed fittings	Percent of failed fittings
Gamma	14	6.8%
Gauss1	0	0%
Gauss2	1	0.49%
Gauss3	21	10.19%
Gauss4	34	16.5%
Gauss5	50	24.27%

Gauss series of higher order has the inconvenience of the increased number of coefficients (three per term), otherwise gamma and Gauss1 with only three coefficients. An analysis of the correlation coefficient of Pearson (Figure 3 left) and root medium square error (Figure 3 right) as metrics for the goodness of the fit accounts for gamma as the worst function, while the Gauss series has sustained improvements with the increment in the number of terms.

**Fig. 3.** Pearson's correlation coefficients (left) and RMSE (right) results of fitting functions.

### 3.1 Analysis of Fitting Coefficients

The fitting procedure for gamma functions failed for 5 records of patients of ataxia (14 well fitted), six presymptomatics (23 well fitted) and 3 healthy subjects (20 well fitted),

Gauss1 showed no failures and Gauss2 failed for one patient (not coincident with those ones failed by gamma). Given the limited quantity of available records it was decided to compare gamma and Gauss1 fitting functions. A random selection of the resultant records of presymptomatics and healthy subjects was made to obtain fourteen records in each class of subjects for this preliminary analysis of the relationships among fitting coefficients and classes of subjects.

Figure 4 represents the coefficients of gamma function versus the category of the subjects, it can be seen overlapping among categories for the three variables. Otherwise, in Figure 5 is possible to observe better defined clusters in the categories of healthy subjects and patients of ataxia for coefficients  $b$  and  $c$ , while presymptomatics are located between these categories, probably depending on their progression in the disease. It must be noticed the bigger separation between means and similar dispersion observed for coefficient  $b$ , this is confirmed by the values of the mean and standard deviation of each coefficient (Table 3).



Fig. 4. Coefficients  $a$  (left),  $b$  (center) and  $c$  (right) of gamma fitting function *vs* category.

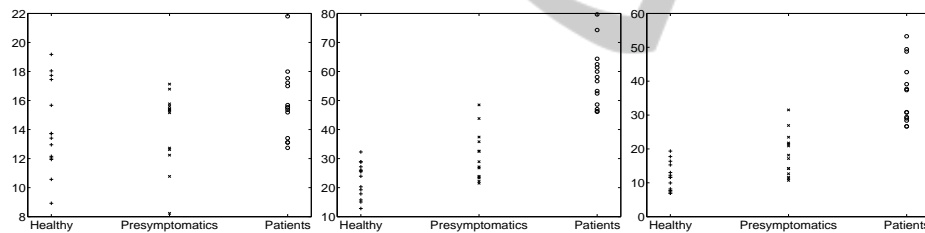


Fig. 5. Coefficients  $a$  (left),  $b$  (center) and  $c$  (right) of Gauss1 fitting function *vs* category.

Table 3. Means and standard deviations of coefficients of Gauss1 fitting function.

Category	Coefficients					
	a		b		c	
	mean	std	mean	std	mean	std
Healthy	14,1	3,1	22,8	6,0	11,8	4,2
Presymptomatics	14,2	2,5	30,6	8,3	18,3	6,3
Patient	15,8	2,4	57,9	10,2	36,4	9,1

## 4 Discussion

Simple visual inspection reveals higher performance of Gauss series over gamma function. Regardless of the number of terms, Gauss series are better valued in terms of the

fitting performance. Additionally Gauss1 and Gauss2 have very low failing rates, as compared to gamma function or Gauss series of higher orders, an appropriate selection of initial is very difficult when the number of coefficients increases.

This superiority is consistent with the numerical results of two of the most used parameters to assess the goodness of fit: the Pearson's correlation coefficient and the RMSE value. For both parameters gamma function has a poorer performance with respect to Gauss series. Although the Pearson's correlation coefficient is above 0.95, which can be considered an expression of a good fitness for all the functions, there exist an considerable difference of gamma and one term Gauss series with respect to Gauss of 2 terms or higher. The lack of an enough number of records inhibits to do a consistent analysis of the behaviour of the fitting coefficients for the studied functions, this is more evident for Gauss series with a higher number of terms. However a comparison with a limited set of register for Gauss1 and gamma could be made, in order to evaluate the clustering ability of the coefficients taken separately. For gamma function only  $b$  seems to be able to identify between healthy subjects and patients of ataxia, presymptomatics are located in a middle overlap zone with respect to the two extreme groups.

Similar results were found for coefficients  $b$  and  $c$  of the Gauss1 series. The visual inspection and the analysis of mean and standard deviation were in coincidence to identify coefficient  $b$  of Gauss1 as the best to classify subjects. No data was available concerning other significant medical variables to correlates the condition of presymptomatics with the value of  $b$ , but it is reasonably to believe that  $b$  could be a significant marker of the progression of the disease, even before other symptoms are to be present. These results were not improved when more than one coefficient was used for clustering, in fact visual inspection revealed the presence of linear correlation between  $b$  and  $c$  for Gauss1.

## 5 Conclusions

In this contribution two fitting function has been evaluated for modelling of saccadic pulse component obtained by the application of ICA to electro-oculographic records, gamma function and Gauss series. Both models have achieved to describe the waveform of the signal in its active area, confirmed by the analytical results. Accordingly to the trade off between successful fittings and quality Gauss2 could be considered as the best choice, although the presence of two terms implies the need of 6 coefficients. On the other hand the use of only one of the coefficients of Gauss1 proved to be a good feature to classify subjects in the extreme categories of patients and healthy subjects, and as a probable indicator of the condition of presymptomatics, however these results must be considered preliminaries and further research is necessary to explain relationships between the condition of the subjects and the coefficients derived from modelling.

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