Electronic Supplementary Material (ESI) for Soft Matter.

SUPPLEMENTARY MATERIAL

Impact cratering in sand: Comparing solid and liquid intruders†

Rianne de Jong,[a] Song-Chuan Zhao,*[a,b], Diana García-González,[a] , Gijs Verduijn,[a] and Devaraj van der Meer [a]

In this Supplementary Material we compare, for the impact of a water droplet on a granular substrate, the final crater diameter with the maximum spreading diameter of the droplet.

In the main text, we claimed that the final crater diameter D_c^∞ is not related to the maximum droplet spreading diameter D_d^*. Here, we provide evidence for this statement.

In an earlier paper[1] we found that both the transient crater diameter D_c^*, measured at the moment that the maximum crater depth is reached, and the final crater diameter D_c^∞ collapse onto a single curve when plotted against the sand deformation energy E_s and definitely not when using the droplet deformation energy $E_d = E_k - E_s$ for variation in packing fraction. On the other hand[2], we used precisely the droplet deformation energy E_d to obtain a modified Weber number that collapses the maximum droplet spreading diameter D_d^*. In addition, we observed that droplet spreading is greatly reduced when mixing between liquid and grains becomes important, i.e., when wettability of the grains and their size are increased[3]. The above discussion alone already indicates that for droplet impact on a granular substrate droplet spreading and crater formation can be considered largely unrelated processes.

In addition, we are however able to directly compare the maximum droplet spreading diameter D_d^* with the final crater diameter D_c^∞ for a large subset of our earlier experiments[1–3] and the current paper. Here, we not only include the hydrophobic grains of 114 and 200 μm that were used in the main text of this article, but also hydrophilic ceramic beads of three sizes (98, 167 and 257 μm) and two types of wettabilities, for which we found[3] that the maximum droplet spreading diameter D_d^* is greatly reduced when mixing between liquid and grains is important, with no measurable effect on the crater diameter.

In Fig. 1*a* we plot the dimensionless final crater diameter D_c^∞/D_0 (with D_0 the droplet diameter) as a function of the dimensionless sand deformation energy E_s/E_g, defined as

$$\frac{E_s}{E_g} = \frac{Z_c^*}{\frac{1}{2}D_0 + Z_c^*}\frac{E_k}{E_g} = \frac{Z_c^*}{\frac{1}{2}D_0 + Z_c^*}\frac{\rho_i}{2\rho_g\phi_0}\frac{U_0^2}{gD_0}\,, \tag{1}$$

where U_0 is the impact velocity of the droplet, Z_c^* the maximum crater depth, $E_k = \frac{1}{12}\pi\rho_i D_0^3 U_0^2$ the kinetic energy of the droplet, $E_g = \frac{1}{6}\pi\rho_s\phi_0 D_0^4$ a gravitational potential energy scale based on the droplet diameter, ρ_i and $\phi_0\rho_g$ the (bulk) densities of impacting droplet and granular bed respectively, ϕ_0 the packing fraction of the bed, and g the gravitational acceleration. Clearly, this leads to a reasonable collapse of the data, regardless of properties like wettability and surface tension.

In Fig. 1*b* we plot the dimensionless maximum spreading diameter D_d^*/D_0 of the droplet versus the modified Weber number $\mathrm{We}^\dagger$, which is defined as

$$\mathrm{We}^\dagger = \frac{E_d}{E_\sigma} = \frac{\frac{1}{2}D_0}{\frac{1}{2}D_0 + Z_c^*}\frac{E_k}{E_\sigma} = \frac{\frac{1}{2}D_0}{\frac{1}{2}D_0 + Z_c^*}\frac{\rho_i U_0^2 D_0}{\sigma}\,, \tag{2}$$

where E_d is the droplet deformation energy, $E_\sigma = \frac{1}{12}\pi\sigma D_0^2$ a surface energy scale based on the droplet diameter, and σ the surface tension of the droplet air interface *. In this figure, the data do not collapse at all, which can be explained from the important role of wettability and grain size of the granular bed[3]. Note that the sand deformation energy E_s and the droplet deformation energy E_d decompose the impact kinetic energy E_k:

$$E_s + E_d = \left(\frac{Z_c^*}{\frac{1}{2}D_0 + Z_c^*}E_k\right) + \left(\frac{\frac{1}{2}D_0}{\frac{1}{2}D_0 + Z_c^*}E_k\right) = E_k\,.$$

If the crater diameter D_c^∞ would be related to the maximum droplet spreading diameter D_d^*, as postulated in the literature[4–6], we expect that when we plot these two length scales against

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[a] Physics of Fluids Group, Faculty of Science and Technology, University of Twente, PO Box 217, 7500 AE Enschede, The Netherlands.

[b] State Key Laboratory for Strength and Vibration of Mechanical Structures, School of Aerospace, Xian Jiaotong University, Xi'an, Shaanxi, China.

* Note that the second fraction on the right hand side of Eq (2) is the conventional Weber number $\mathrm{We} = \rho_i U_0^2 D_0/\sigma$

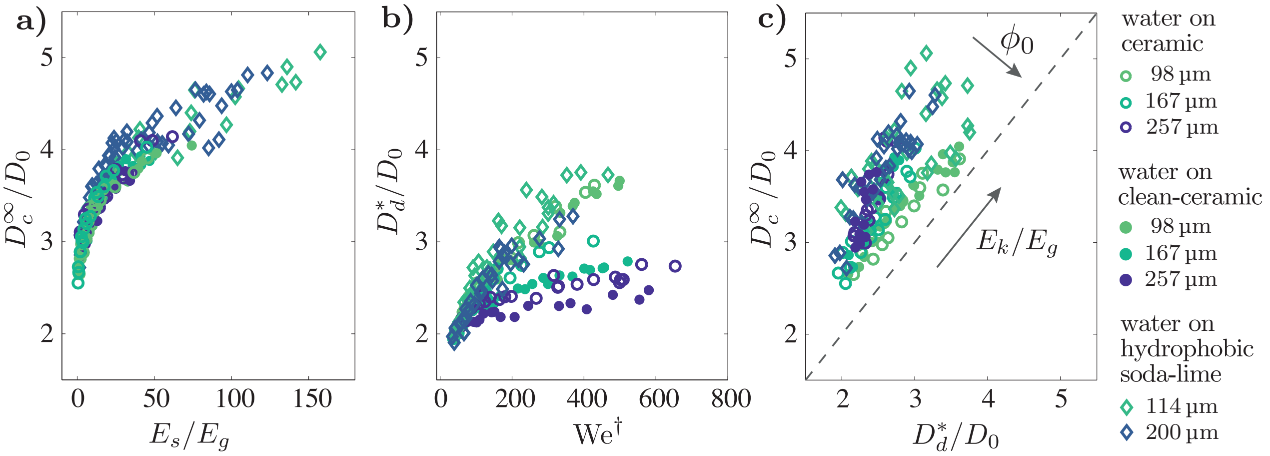

Fig. 1 (*a*) The dimensionless maximum crater diameter D_c^∞/D_0 plotted versus the dimensionless sand deformation energy E_s/E_g collapses all data for water droplet impact on various substrates onto a single curve. (*b*) The dimensionless maximum droplet spreading diameter D_d^*/D_0 is plotted against the modified Weber number We† for the same data set, which does not lead to data collapse. (*c*) The dimensionless final crater diameter D_c^∞/D_0 is plotted against the dimensionless maximum droplet spreading diameter D_d^*/D_0. The arrows indicate the directions of increasing packing fraction ϕ_0 and increasing dimensionless impact energy E_k/E_g, and the black dashed line indicates $D_c^\infty = D_d^*$. Clearly, D_c^∞ is always larger than D_d^*, but a simple relation between the two quantities does not exist.

each other, they would collapse onto a single curve. However, as clearly follows from Fig. 1*c*, the data of D_c^∞/D_0 versus D_d^*/D_0 presents considerable scatter. This is direct evidence that the hypothesis that maximum droplet spreading and final crater diameter are related, does not hold. In fact, the only thing that can be concluded from the data in Fig. 1*c* is that the final crater diameter is always observed to be larger than the maximum droplet spreading diameter, where equality of these two quantities is given by the dashed black line in the plot.

Notes and references

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