Application of multi-phase particle swarm optimization technique to retrieve the particle size distribution

Hong Qi (齐 宏), Liming Ruan (阮立明), Shenggang Wang (王圣刚), Meng Shi (史 萌), and Hui Zhao (赵 辉)

School of Energy Science and Engineering, Harbin Institute of Technology, Harbin 150001

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The multi-phase particle swarm optimization (MPPSO) technique is applied to retrieve the particle size distribution (PSD) under dependent model. Based on the Mie theory and the Lambert-Beer theory, three PSDs, i.e., the Rosin-Rammer (R-R) distribution, the normal distribution, and the logarithmic normal distribution, are estimated by MPPSO algorithm. The results confirm the potential of the proposed approach and show its effectiveness. It may provide a new technique to improve the accuracy and reliability of the PSD inverse calculation.

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Particle size distribution (PSD) plays an important role in the field of production processes, product quality, and energy consumption, so it is highly required to on-line monitor the granularity to provide real-time measurements of both size distribution and particle concentration in the industrial fields. The retrieval of PSD with non-intrusive optical measurement has shown broad development space and huge potential gradually. Based on the absorption and scattering characteristics of the particle cloud, the PSD measurement by optical techniques has a lot of advantages, such as high measurement speed, well-repeated implement, wide measurement size range, easy automatization, etc.. The development trend of PSD measurement is to improve the measurement accuracy and modify the inverse algorithm. Nowadays, various optical techniques have been used to determine particle size, such as diffraction light scattering method, total light scattering method, angle light scattering method, dynamic light scattering method, and transmittance method[1]. Among them, the diffraction light scattering method and the total light transmittance method are the two most common experimental techniques in practice. However, in many cases of practical interest, the assumed PSD by these methods is necessarily inaccurate because of limitations imposed by the imperfect signal-to-noise ratio (SNR) of the raw data, coupled with the "ill-conditioned" nature of the deconvolution algorithms which need to calculate the first Fredholm integral equation. Theoretically speaking, the PSD inverse problem is actually a first Fredholm integral equation problem which is typically ill-posed and difficult to be solved directly. Thus, many random search intelligent algorithms have been introduced to inverse the PSD problems, such as genetic algorithm (GA), simulated annealing (SA), evolution strategies (ES), and artificial neural networks (ANN)[2−4]. Compared with the traditional gradient methods, the intelligent optimizations have some outstanding characteristics. Firstly, both linear and nonlinear or ill and non-ill inverse problems could be solved. Secondly, the inverse problem with complicated direct operator or without analytic expression could be solved. Thirdly, only the functional value is needed for the objective function, without explicit expressions. Fourthly, since the evaluation is carried out by the fitness value, the gradient information and the prior information about the unknown function are not needed. Reference [3] applied the particle swarm optimization (PSO) algorithm to solve the inverse problem for determining the PSD from a light transmittance technique, and obtained some reasonable results. In this paper, we apply the multi-phase particle swarm optimization (MPPSO) algorithm, which can guarantee the convergence of the global optimization solution with high accuracy, to the inverse problem of particle distribution under dependent model.

Among the optical measurement methods, the light transmittance technique is simple in principle and convenient for the optical arrangement and is a more useful diagnostic tool for spatially and temporally resolved measurement of PSD in a wide range of applications. The theoretical details of the transmittance technique are discussed in the following.

When a collimated laser beam passes through a suspension of particles, the transmitted light will be attenuated due to the absorbing and scattering of the particles. According to the Lambert-Beer theory, if the suspensions of particle cloud are polydisperse spheres and the multiple scattering and interaction effects can be neglected, the transmitted light intensity I may be expressed as

$$I = I_0 \exp\left[-\frac{\pi}{4}\int_{D_1}^{D_2} N_0 f(D) D^2 L Q_{\mathrm{ex}}(x,m)\mathrm{d}D\right], \quad (1)$$

where I_0 is the incident light intensity, L is the mean distance through which the laser passes, $N_0 f(D)$ is the number concentration of the particles with diameter D which is the PSD function to be measured, N_0 is the total number density of the particles, $Q_{\mathrm{ex}}(x,m)$ is the extinction efficiency factor which is dependent on the size parameter $x = \pi\lambda/D$ and the complex refractive index m and can be calculated by the Mie scattering theory[5].

For a particle cloud with fixed PSD function, assuming the amount of particles with diameter D_i is N_i, if there are several incident laser light beams with different wavelength λ_j $(j = 1, 2, \cdots, k)$, the following form equation

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may be obtained from Eq. (1):

$$\ln\left[I(\lambda_j)/I_0(\lambda_j)\right] = -\frac{\pi}{4}L\sum_{i=1}^{M} D_i^2 N_i Q_{\mathrm{ex}}(x_{i,j}, m), \qquad (2)$$

where M is the amount of D_i. When solving Eq. (2) practically, the range of diameters $[D_{\min}, D_{\max}]$ can be divided into M intervals $[D_i, D_{i+1}]$ $(i = 1, 2, \cdots, M-1)$. Each D_i in Eq. (2) could take the mean value or the maximum value of the ith interval, and Eq. (2) can be transformed into a group of linear equations. When the ratio of $I(\lambda_j)$ to $I_0(\lambda_j)$ is measured, the PSD function $f(D)$ $(f_i = N_i/N_0)$ could be obtained by solving the linear equations. However, Eq. (2) is seriously ill-posed and needs the inverse model, which is usually divided into two main categories, the dependent model and the independent one. Under the dependent model, a distribution function should be assumed *a priori*, such as normal distribution, logarithmic normal distribution, and the Rosin-Rammer (R-R) distribution (actually most of practical particulate systems often conform to some two-parameter size distribution functions). Under the independent model, by measuring the ratio of $I_0(\lambda_j)$ to $I(\lambda_j)$, the PSD function $[f_1, f_2, \cdots, f_M]$ could be obtained by solving the system of linear equations. In this paper, the MPPSO algorithm is used to retrieve different size distributions under the dependent model. The insight is put on the PSD inversion by using the MPPSO algorithm.

The PSO algorithm introduced in 1995[6], has been studied extensively by many researchers in recent years. It combines the concept of survival-of-the-fittest among string patterns with a regulated yet randomized information exchange. Generally, PSO is characterized to be simple in concept, easy to implement, and computationally efficient. Unlike other heuristic techniques, PSO has a flexible and well-balanced mechanism to enhance the global and local exploration abilities. It requires only primitive mathematical operators and is computationally inexpensive in terms of both memory requirements and CPU (central processing unit) time. Another feature of PSO is the use of the objective function instead of using derivatives (sensitivity analysis) or other auxiliary knowledge. During the last decade, as an important efficient concurrent optimization tool, the PSO algorithm has been successfully used in system identification, neural network training, function optimization, mode classification, fuzzy control, electrical equipment power feedback control, signal processing, robot technique, etc.[6−8].

However, the standard PSO algorithm has two weaknesses. 1) In order to avoid the local optima and extend the search range, it wastes a lot of calculation on the search of bad fitness. 2) The particle moves along towards a fixed direction until the direction changes, so it brings on the possibility to convergence to the local optima of adaptive values' difference. We introduce the MPPSO, which overcomes the deficiencies of the basic PSO algorithm. The MPPSO introduces the idea of 'grouping' and 'phase', so it increases population diversity and explores the search space more efficiently. Three main changes in the MPPSO algorithm are: 1) dividing particles into multiple groups, thereby increasing the diversity and exploration of the space; 2) introducing different phases, among which the direction of particle movement changes; 3) moving only to positions that will increase the fitness.

The fundamental parameters of the MPPSO are: phase amount N_{p}, phase changing frequency f_{p}, group coefficients C_{v}, C_{x}, and C_{g}, sub-length dimension sl, and velocity reinitializing frequency V_{c}. The details of these parameters are available in Ref. [9]. For each generation of particle i for some phase, the transient velocity and position change according to

$$\mathbf{V}_i(t+1) = C_{\mathrm{v}} \cdot \mathbf{V}_i(t) + C_{\mathrm{x}} \cdot \mathbf{X}_i(t) + C_{\mathrm{g}} \cdot \mathbf{P}_{\mathrm{g}}(t), \qquad (3)$$

$$\mathbf{X}_i(t+1) = \mathbf{X}_i(t) + \mathbf{V}_i(t+1), \qquad (4)$$

where $\mathbf{V}_i(t)$ is the present velocity, $\mathbf{P}_{\mathrm{g}}(t)$ is the global best position, and $\mathbf{X}_i(t)$ is the present position (or solution). The parameters' values are first determined and the first swarm is initialized. The swarm iterations are then executed until the termination conditions are met. During each iteration, the first step is to check the V_{c} variable to see if it is time to reinitialize the velocity. After that, the parameter sl is chosen, the current phase is set, and the particles are divided into groups. When the group is determined, the coefficients are determined. Using these, the new tentative positions for those dimensions encountered in the calculation are calculated and stored in the temporary memory. If the tentative position updates improve the fitness of the particle, the particle will accept the updates and move to the new position. Otherwise, computation of tentative updates is performed for the next group of sl dimensions. In the MPPSO iteration in this paper, the iteration stop criterion is the fitness function value F less than 1×10^{-10} or the iteration number larger than 1000. With the comparison of the MPPSO and PSO by using the three classic test function[9], the potential of MPPSO and its effectiveness and superiority over the standard PSO algorithm can be confirmed.

Using the MPPSO algorithm, this paper inverses the particle swarm diameter distribution for the R-R distribution, the normal distribution, and the logarithmic normal distribution. The corresponding distribution functions are defined as follows:

$$f_{\mathrm{R-R}}(D) = \frac{\sigma}{\overline{D}}\left(\frac{D}{\overline{D}}\right)^{\sigma-1}\exp\left[-\left(\frac{D}{\overline{D}}\right)^{\sigma}\right], \qquad (5)$$

$$f_{\mathrm{Normal}}(D) = \frac{1}{\sqrt{2\pi}\cdot\sigma}\exp\left[-\frac{1}{2}\left(\frac{D-\overline{D}}{\sigma}\right)^2\right], \qquad (6)$$

$$f_{\mathrm{Log-Normal}}(D) = \frac{1}{\sqrt{2\pi}\cdot D\ln\sigma}\exp\left[-\frac{1}{2}\left(\frac{\ln D-\ln\overline{D}}{\ln\sigma}\right)^2\right]. \qquad (7)$$

In Eq. (5), $\bar{D}$ is the characteristic diameter parameter, σ is the dispersion ratio. In Eqs. (6) and (7), $(\bar{D}, \sigma)$ represent the distribution parameters. The complex refraction index used in this paper refers to the practical situation. For instance, the real part and imaginary part of coal ash particle's typical complex refractive index are $n \in [1.18, 1.92]$ and $k \in [0.01, 1.13]$[10], respectively, and the complex refractive index is selected as $1.51 + 0.03i$, which is assumed to be independent of the wavelength. If not so, it is just needed to use the complex refraction index under each wavelength without increasing the calculation efforts. The incident wavelength is set as three wavelengths $\lambda = 0.4, 0.6, 0.8$ μm or six wavelengths $\lambda = 0.4, 0.6, 0.8, 1.2, 1.6, 2.0$ μm. For the R-R distribution, the true value of $(\bar{D}, \sigma)$ is set as (5, 2); for the normal distribution, is set as (5, 2); for the logarithmic normal distribution, is set as (5, 1.5).

The inverse PSD problem is solved through the minimization of a fitness function, which is expressed by the sum of square residuals between calculated and measured transmittance ratio as follows:

$$F = \sum_{j=1}^{k} \left\{ \frac{[I(\lambda_j)/I_0(\lambda_j)]_{\mathrm{e}} - [I(\lambda_j)/I_0(\lambda_j)]_{\mathrm{m}}}{[I(\lambda_j)/I_0(\lambda_j)]_{\mathrm{m}}} \right\}^2, \qquad (8)$$

where the subscript 'm' represents the measured value, 'e' represents the estimated value of every iteration, $j = 1, \cdots, k$ and k means the amount of incident laser wavelengths. The main parameters of the three diameter distribution function retrieved by the MPPSO are listed in Table 1.

Figures 1 – 3 show the PSD results of the MPPSO inverse calculation. Tables 2 and 3 show the statistic parameters of the inverse results. The parameters in the tables are defined as follows.

1) The span of particle swarm's diameter distribution

Table 1. Parameters of MPPSO Algorithm for Different PSD Functions

Parameter	R-R	Normal	Log-Normal
n	50	50	50
$\bar{D}$	$1 - 10$	$1 - 30$	$0.01 - 10$
σ	$1 - 30$	$0.1 - 10$	$0.01 - 10$
$V_{\max}$	5	5	3
V_{c}	10	10	10
f_{p}	50	50	50

n: the particle swarm size.

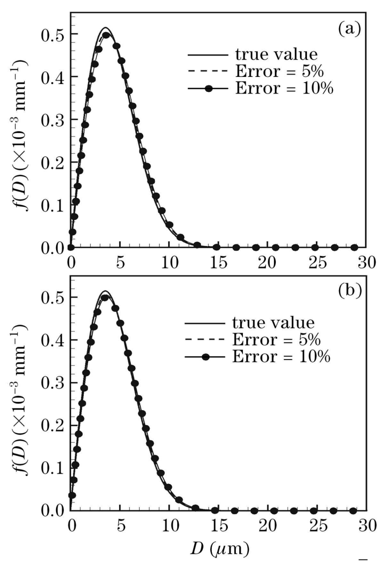

Fig. 1. Inverse results of R-R distribution $\bar{D} = 5$, $\sigma = 2$ with different measurement errors for (a) 3 and (b) 6 incident wavelengths.

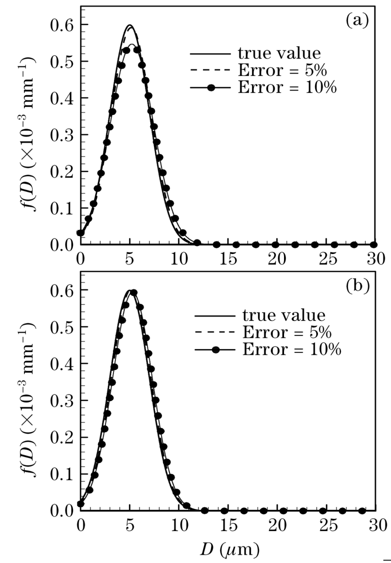

Fig. 2. Inverse results of normal distribution $\bar{D} = 5$, $\sigma = 2$ with different measurement errors for (a) 3 and (b) 6 incident wavelengths.

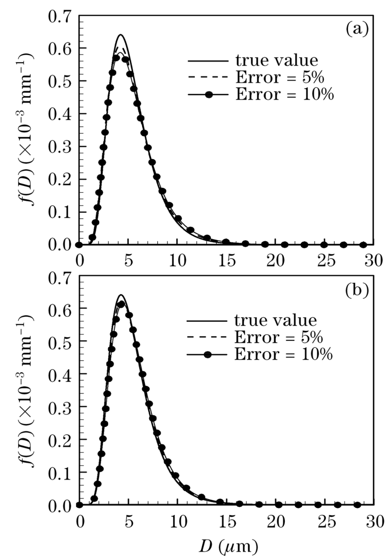

Fig. 3. Inverse results of logarithmic normal distribution $\bar{D} = 5$, $\sigma = 1.5$ with different measurement errors for (a) 3 and (b) 6 incident wavelengths.

is expressed as

$$\mathrm{SPAN} = \frac{D(W, 0.9) - D(W, 0.1)}{D(W, 0.5)}, \qquad (9)$$

where $D(W, 0.1)$ is the weight equivalent radius and the accumulated weight percent of the particles smaller than it occupies 10% of the total weight of particle system; $D(W, 0.5)$ is the weight intermediate radius and the accumulated weight percent of the particles smaller than it occupies 50% of the total weight of particle swarm. $D(W, 0.9)$ is defined similarly.

2) The absolute deviation of PSD ε means the deviation between the probability distribution obtained from the inverse calculation and the true PSD, which is defined as

$$\varepsilon = \frac{1}{2} \sum_{i=1}^{M} |f_{\mathrm{e}}(i) - f_{\mathrm{t}}(i)|, \qquad (10)$$

Table 2. Inverse Results of MPPSO for Different Distributions with Three Incident Wavelengths

Parameter	Error = 0%			Error = 5%			Error = 10%		
	F1	F2	F3	F1	F2	F3	F1	F2	F3
$\bar{D}$	5.000	5.001	5.000	5.137	5.138	5.021	5.238	5.215	5.100
$R_{\bar{D}}$ (%)	0.000	0.020	0.000	2.740	2.760	0.420	4.760	4.300	2.000
σ	2.001	1.998	1.499	2.010	2.024	1.538	2.042	2.190	1.555
R_σ (%)	0.050	0.100	0.067	0.500	1.200	2.533	2.100	9.500	3.667
SPAN	1.435	1.019	1.085	1.427	1.005	1.161	1.403	1.068	1.192
ε (%)	0.046	0.097	0.159	4.004	5.488	5.833	7.087	11.190	8.712

F1, F2, and F3 represent the R-R, normal and logarithmic normal distributions, respectively.

Table 3. Inverse Results of MPPSO for Different Distributions with Six Incident Wavelengths

Parameter	Error = 0%			Error = 5%			Error = 10%		
	F1	F2	F3	F1	F2	F3	F1	F2	F3
$\bar{D}$	5.000	5.000	5.000	5.101	5.150	5.099	5.170	5.285	5.203
$R_{\bar{D}}$ (%)	0.000	0.000	0.000	2.020	3.00	1.980	3.400	5.700	4.060
σ	2.000	2.000	1.500	2.003	1.990	1.508	2.015	2.016	1.502
R_σ (%)	0.000	0.000	0.000	0.150	0.500	0.533	0.750	0.800	0.133
SPAN	1.435	1.020	1.087	1.431	0.987	1.103	1.423	0.974	1.091
ε (%)	0.000	0.000	0.000	2.947	5.873	3.978	4.969	11.150	7.831

where f_t represents the true size distribution and f_e represents the estimated size distribution function resulting from the inverse calculation.

3) The relative deviations of eigenvalues $R_{\bar{D}}$ and R_σ. They are the deviations between the parameters obtained from the inverse calculation and the true values under the dependent model. The expressions are

$$R_{\bar{D}} = \frac{\bar{D}_\mathrm{c} - \bar{D}_\mathrm{t}}{\bar{D}_\mathrm{t}} \times 100\%, \quad (11)$$

$$R_\sigma = \frac{\sigma_\mathrm{c} - \sigma_\mathrm{t}}{\sigma_\mathrm{t}} \times 100\%. \quad (12)$$

The final consideration is the effect of measurement errors on the accuracy of estimation. In the practical experiment, the measurement error is unavoidable. Thus, the relative error is set as Error = 0, 5% and 10%, respectively. Tables 2 and 3 show that if there is no error introduced, the PSD function can be retrieved accurately by MPPSO algorithm. When the measurement error is 0%, 5%, and 10%, respectively, the inverse results of MPPSO algorithm are shown in Figs. 1 − 3 and Tables 2 and 3. Obviously, by increasing the relative error from 0% to 10%, the accuracy of the estimation decreases. As shown in Figs. 1 − 3, the retrieved results with six incident wavelengths are more accurate than that with three ones. For all the three PSD distributions, the estimated error is less than the measurement error. Usually in the laser extinction experiment, the total error caused by the measurement is less than 10%, so the MPPSO algorithm is suitable to retrieve the PSD in the practical industry processes.

In Conclusion, based on the Mie theory and Lambert-Beer theory, the MPPSO algorithm is proposed to determine the PSD under dependent model. Numerical simulation results can lead to the following conclusions. The PSD could be inversed accurately with the MPPSO algorithm under dependent model. The present MPPSO algorithm is robust and can obtain the satisfying estimation of radiation parameters even in the case of 10% measurement error. It does not require the PSD inverse problems to have a strict mathematical analytic model, and requires only primitive mathematical operators and is computationally inexpensive in terms of both memory requirements and CPU time, especially suitable to the *in-situ* measurement of PSD.

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