ACTUARIAL VALUATIONS

VERSION 4A

Remainder, income, and annuity examples for one life, two lives, and terms certain

For use in income, estate, and gift tax purposes, including valuation of pooled income fund remainder interests

TABLE OF CONTENTS

ACTUARIAL TABLES ASSOCIATED WITH PUBLICATION 1457

The actuarial tables associated with this publication are:

Table	Type of Factors
Table S	Single Life Factors
Table R(2)	2-Life Last to Die Remainder Factors
	Part 1: Interest from 0.2 to 4.0%
	Part 2: Interest from 4.2 to 8.0%
	Part 3: Interest from 8.2 to 12.0%
	Part 4: Interest from 12.2 to 16.0%
	Part 5: Interest from 16.2 to 20.0%
Table B	Term Certain Factors
Table H	Commutation Factors
Table K	Adjustment Factors for Annuities Payable at the End of Each Interval
Table J	Adjustment Factors for Term Certain Annuities Payable at the Beginning of Each Interval
Table 2010CM	Mortality Table

USE OF EXAMPLES AND TABLES

This publication provides examples for using actuarial factors for certain income, gift, and estate tax valuations of future interests. These examples use factors from actuarial tables associated with this Publication. The actuarial tables are located at

www.irs.gov/retirement-plans/actuarial-tables

The examples illustrate methods for using the factors in the associated tables for valuations, and for finding other factors not found directly in the tables. The examples in **Part A** show the use of last-to-die remainder factors in Table R(2), and show the method for finding last-to-die life estate and annuity factors. Life estate factors and factors for a term of years estate may also be referred to as income factors. **Part B and Part C** examples illustrate the use of factors from the single life Table S and from the last-to-die Table R(2) to obtain factors for such time as one person survives another, and to obtain first-to-die factors. In **Part D**, commutation factors from Table H are used to find factors for one life and a term of years. **Part E** provides similar explanations to compute factors for two lives and term of years. **Part F** demonstrates the use of the remainder factors for pooled income funds. In all cases, appropriate factors may also be computed with software using the same actuarial formulas from which these factors are computed, if the final factors so computed have, respectively, at least as many significant figures as are shown in these examples.

The factors and tables associated with this publication involving life contingencies are derived from the values of l_x taken from the Life Table for the Total Population appearing as Table 1, in U.S. Decennial Life Tables for 1999-2001 published by the U.S. Department of Health and Human Services, Public Health Service, National Center for Health Statistics. Values from Table 1 appear in the associated set of actuarial tables in Section 7, labeled as Table 2010CM. However, in Table 2010CM the values of l_x are stated to seven digits.

ASSOCIATED TABLES ON THE WEB

In the IRS <u>actuarial tables</u> on IRS.gov associated with this publication and these examples, the factors in Sections 1 through 6 are based on interest rates ranging from 0.2% to 20.0% in intervals of 0.2%. The wide range of rates is shown pursuant to Internal Revenue Code (IRC) Section 7520, which requires the use of an interest rate of 120% of the annual mid-term applicable federal rate for the month in which the valuation date falls. This rate is referred to in this publication as the "Section 7520 rate,", or simply the "7520 rate." All the factors associated with this publication reflect annual compounding of interest.

Table S contains factors for the present worth of a life annuity, a life estate and a remainder interest based on a single life.

Table R(2) contains factors for the present worth of the remainder interest in $1.00 payable at the death of the last to die of two persons.

Table B contains factors for the present worth of an annuity, an income interest and a remainder interest for a term certain.

Table H contains commutation tables for D_x, $\overset{o}{N}_x$ and $\overline{M}_x$.

Table K contains adjustment factors for annuities payable at the end of annual, semi-annual, quarterly, monthly and weekly periods.

Table J contains adjustment factors for term certain annuities payable at the beginning of annual, semi-annual, quarterly, monthly, and weekly periods.

Table 2010CM is the underlying mortality table used to calculate factors involving life contingencies.

HISTORICAL SYNOPSIS OF TABLES

Period	Table	Interest Rate	Publications
1-1-1951 to 12-31-1970	US1938	3.5%	11
1-1-1971 to 11-30-1983	Table LN	6%	723, 723A, 723B
12-1-1983 to 4-30-1989	Table CM	10%	723C, 723D, 723E
5-1-1989 to 4-30-1999*	80CNSMT	7520 rates	1457, 1458, 1459 (5-1989 release)
5-1-1999 to 4-30-2009	90CM	7520 rates	1457, 1458, 1459 (7-1999 release)
5-1-2009 to 5-31-2023**	2000CM	7520 rates	1457, 1458, 1459 (5-2009 release)
6-1-2023** to	2010CM	7520 rates	1457, 1458, 1459 (6-2023 release)

*On October 22, 1988, IRC Section 7520 was enacted. It requires the use of an interest rate equal to 120% of the midterm applicable federal rate, rounded to the nearest 0.2%.

**Transition period: For valuation dates from May 1, 2019, through June 1, 2023, you may rely on the actuarial tables based either on Table 2000CM or on Table 2010CM. However, you must be consistent in using the same set of tables to value each interest (income, remainder, partial, etc.) in the same property and with respect to all transfers occurring on the same valuation date.

EXAMPLES

A. Two Life Last-to-Die Factors

Example 1. Based on an interest rate of 4.2%, the present worth of $1.00 due at the death of the last to die of two persons age 60 and 65 is $0.35061. Find in Table R(2) Section 2 2 the older age 65 and the younger age 60. Across from those ages, under the column headed 4.2% is the remainder factor 0.35061.

A life estate factor or an annuity factor for the same ages and interest rate can be computed using examples 2 and 3.

Example 2. Based on an interest rate of 4.2%, the present worth of the right to receive the use of $1.00 until the death of the last to die of two persons age 60 and 65 is $0.64939, determined as:

$$\text{Remainder Factor from Example 1} = 0.35061$$
$$\text{Income Factor} = 1.00000 - 0.35061$$
$$= 0.64939$$

Example 3. Based on an interest rate of 4.2%, the present worth of an annuity of $1.00 per annum payable at the end of each year until the death of the last to die of two persons age 60 and 65 is $15.4617, determined as:

$$\text{Income Factor from Example 2} = 0.64939$$
$$\text{Annuity Factor} = 0.64939 \div 0.042$$
$$= 15.4617$$

B. Income or Annuity Payable for Such Time as One Person Survives Another

Example 4. Based on an interest rate of 4.2%, the present worth of the right to receive the use of $1.00 for such time as a person age 65 survives a person age 60 is $0.07110 determined as:

$$\text{Income Factor from Example 2} = 0.64939$$
$$\text{Single Life Income Factor from Table S at 4.2\%, age 60} = 0.57829$$
$$\text{Required Income Factor} = 0.64939 - 0.57829$$
$$= 0.07110$$

Example 5. Based on an interest rate of 4.2%, the present worth of an annuity of $1.00 per annum payable annually at the end of each year for such time as a person age 65 survives a person age 60 is $1.6928 determined as:

$$
\begin{aligned}
\text{Joint and Survivor Annuity Factor from Example 3} &= 15.4617 \\
\text{Single Life Annuity Factor, age 60, From Table S at 4.2\%} &= 13.7689 \\
\text{Required Annuity Factor} &= 15.4617 - 13.7689 \\
&= 1.6928
\end{aligned}
$$

C. First-to-Die Factors

Example 6. Based on an interest rate of 4.2%, the present worth of $1.00 due at the death of the first to die of two persons age 60 and 65 is $0.55816, determined as:

$$
\begin{aligned}
\text{Single Life Remainder Factor, age 60, from Table S at 4.2\%} &= 0.42171 \\
\text{Single Life Remainder Factor, age 65, from Table S at 4.2\%} &= 0.48706 \\
\text{Joint and Survivor Remainder Factor from Example 1} &= 0.35061 \\
\text{Required Remainder Factor} &= 0.42171 + 0.48706 - 0.35061 \\
&= 0.55816
\end{aligned}
$$

Example 7. Based on an interest rate of 4.2%, the present worth of the right to receive the use of $1.00 until the death of the first to die of two persons age 60 and 65 is $0.44184, determined as:

$$
\begin{aligned}
\text{First-to-Die Remainder Factor from Example 6} &= 0.55816 \\
\text{Income Factor} &= 1.00000 - 0.55816 \\
&= 0.44184
\end{aligned}
$$

Example 8. Based on an interest rate of 4.2%, the present worth of an annuity of $1.00 per annum payable annually at the end of each year until the death of the first to die of two persons age 60 and 65 is $10.5200, determined as:

$$
\begin{aligned}
\text{Income Factor from Example 7} &= 0.44184 \\
\text{Required Annuity Factor} &= 0.44184 \div .042 \\
&= 10.5200
\end{aligned}
$$

D. Factors Involving One Life and a Term of Years

Example 9. Based on an interest rate of 2.8%, the present worth of a temporary annuity of $1.00 per annum payable annually at the end of each year for 10 years or until the prior death of a person age 60 is $8.1791, determined as:

$$
\begin{aligned}
\text{Initial Age} &= 60 \\
plus\ \text{Term of Years} &= \underline{10} \\
\text{Terminal Age} &= 70
\end{aligned}
$$

$$
\begin{aligned}
\text{N-Factor, Table H(2.8), age 60} &= 271{,}994.3 \\
minus\ \text{N-Factor, Table H(2.8), age 70} &= \underline{133{,}677.8} \\
\text{Difference} &= 138{,}316.5
\end{aligned}
$$

$$
\begin{aligned}
\text{D-Factor, Table H(2.8), age 60} &= 16{,}911.03 \\
\text{Required Annuity Factor} &= 138{,}316.5\ /\ 16{,}911.03 \\
&= 8.1791
\end{aligned}
$$

Example 10. Based on an interest rate of 2.8%, the present worth of $1.00 due at the death of a person age 60 provided the death occurs within the first 10 years is $0.10392, determined as:

$$
\begin{aligned}
\text{Initial Age} &= 60 \\
plus\ \text{Term of Years} &= \underline{10} \\
\text{Terminal Age} &= 70
\end{aligned}
$$

$$
\begin{aligned}
\text{M-Factor, Table H(2.8), age 60} &= 9{,}295.187 \\
minus\ \text{M-Factor, Table H(2.8), age 70} &= \underline{7{,}537.826} \\
\text{Difference} &= 1{,}757.361
\end{aligned}
$$

$$
\begin{aligned}
\text{D-Factor, Table H(2.8), age 60} &= 16{,}911.03 \\
\text{Desired Remainder Factor} &= 1{,}757.361\ /\ 16{,}911.03 \\
&= 0.10392
\end{aligned}
$$

Example 11. Based on an interest rate of 2.8%, the present worth of the right to receive the income from $1.00 for 10 years or until the prior death of a person age 60 is $0.22901, determined as:

$$
\begin{aligned}
\text{Temporary Annuity Factor, Example 9} &= 8.1791 \\
\text{Required Income Factor} &= 8.1791\ \times\ .028 \\
&= 0.22901
\end{aligned}
$$

Example 12. Based on an interest rate of 2.8%, the present worth of the right of a person age 21, if living, to receive $1.00 upon attaining age 30 is $0.77336, determined as:

$$
\begin{aligned}
\text{D-Factor, Table H(2.8), age 30} &= 42{,}794.49 \\
\textit{divided by}\ \text{D-Factor, Table H(2.8), age 21} &= 55{,}336.02 \\
\text{Required Factor} &= 0.77336
\end{aligned}
$$

Example 13. The probability that a person age 21 will be alive at age 30 is 0.991558, determined as:

$$
\begin{aligned}
l_x\ \text{value, Table 2010CM, age 30} &= 97{,}989.90 \\
l_x\ \text{value, Table 2010CM, age 21} &= 98{,}824.20 \\
\text{Required Probability} &= 97{,}989.90 \, / \, 98{,}824.20 \\
&= 0.991558
\end{aligned}
$$

Example 14. Based on an interest rate of 2.8%, the present worth of a temporary annuity of $1.00 per annum payable in equal monthly installments at the end of each month for 10 years or until the prior death of a person age 60 is $8.2838, determined as:

$$
\begin{aligned}
\text{Annuity Factor from Example 9} &= 8.1791 \\
\text{Monthly Adjustment Factor, Table K at 2.8\%} &= 1.0128 \\
\text{Required Annuity Factor} &= 8.1791 \times 1.0128 \\
&= 8.2838
\end{aligned}
$$

Note: Factors from Table J are used in the same manner as factors from Table K, for application to situations where the annuity is a term certain annuity payable at the beginning of each period.

E. Factors Involving Two Lives and a Term of Years

Example 15. Based on an interest rate of 2.8%, the present worth of the right to receive $1.00 at the end of 10 years provided at least one of two persons age 60 and 65 alive is $0.74233, determined as:

$$
\begin{aligned}
l_x\ \text{value, Table 2010CM, age 60} &= 88{,}665.95 \\
l_x\ \text{value, age 70} &= 77{,}957.53
\end{aligned}
$$

$$
1 - \frac{77{,}957.53}{88{,}665.95} = 0.120773
$$

$$l_x \text{ value, Table 2010CM, age 65} = 84{,}221.59$$

$$l_x \text{ value, age 75} = 69{,}174.83$$

$$1 - \frac{69{,}174.83}{84{,}221.59} = 0.178657$$

$$\text{Remainder Factor, Table B(2.8), 10-Year Term} = 0.758698$$

$$\text{Required Remainder Factor} = (1 - 0.120773 \times 0.178657) \times 0.758698$$

$$\text{Required Factor} = 0.74233$$

F. Pooled Income Funds

Table R(2) may be used to obtain joint and survivor remainder factors for pooled income funds. The interest rate to be used to determine the remainder factor is the yearly rate of return for the fund as defined in IRS Notice 89-60. If the yearly rate of return falls between two interest rates for which the factors are given in Table R(2), a linear interpolation may be made.

Example 16. Based on a yearly rate of return of 3.636%, the present worth of the remainder interest in a pooled income fund of $1.00 payable at the death of the last to die of two persons age 60 and 65 is $0.40038 determined as:

$$\text{Remainder Factor, Table R(2), at 3.6\%} = 0.40374$$

$$\textit{minus} \text{ Remainder Factor, Table R(2), at 3.8\%} = 0.38507$$

$$\text{Difference} = 0.01867$$

$$\frac{3.636\% - 3.600\%}{3.800\% - 3.600\%} = \frac{X}{0.01867}$$

$$X = 0.00336$$

$$\text{Remainder Factor at 3.6\%} = 0.40374$$

$$\textit{minus} \ X = 0.00336$$

$$\text{Interpolated Remainder Factor at 3.636\%} = 0.40038$$

As an alternative to using an interpolation method, it is also acceptable to compute the remainder factor directly from the underlying actuarial formulas using an interest rate of 3.636%, provided that the resulting remainder factor is expressed to at least 5 decimal places, and that this method is applied consistently in valuing all interests in the same property.