Wilhelm Weber's Main Works on Electrodynamics Translated into English

Volume V: Unipolar Induction, Galvanometry, Biographical Studies, and Weber's Electrodynamics Versus Different Field Theories

Edited by Andre Koch Torres Assis

Wilhelm Weber’s Main Works on Electrodynamics Translated into English

Volume V: Unipolar Induction, Galvanometry, Biographical Studies, and Weber's Electrodynamics Versus Different Field Theories

edited by André Koch Torres Assis

Apeiron
Montreal

Published by C. Roy Keys Inc.
4405, rue St-Dominique
Montreal, Quebec H2W 2B2 Canada
http://redshift.vif.com

First Published 2024

Library and Archives Canada Cataloguing in Publication

Title: Wilhelm Weber's main works on electrodynamics translated into English / edited by André Koch Torres Assis.
Names: Weber, Wilhelm Eduard, 1804-1891, author. | Assis, André Koch Torres, 1962- editor.
Description: Includes bibliographical references. | Content: Volume V: Unipolar induction, galvanometry, biographical studies, and Weber's electrodynamics versus different field theories.
Identifiers: Canadiana (print) 20210279079 | Canadiana (ebook) 20210279095 | ISBN 9781987980356 (v. 5 ; softcover) | ISBN 9781987980363 (v. 5 ; PDF)
Subjects: LCSH: Electrodynamics.
Classification: LCC QC631 .W4313 2021 | DDC 537.6—dc23

Front cover of Volume V: The picture on the cover of Volume 5 comes from a 1884 portrait of Wilhelm Weber taken by the German photographer Bernhard Petri (1840-1887). Source: H. Weber, "*Wilhelm Weber - Eine Lebensskizze*" (Eduard Trewendt, Breslau, 1893), frontispiece.

Wilhelm Weber's Main Works on Electrodynamics Translated into English

Volume V: Unipolar Induction, Galvanometry, Biographical Studies, and Weber's Electrodynamics Versus Different Field Theories

Edited by Andre Koch Torres Assis

Contents

Chapter 1

Acknowledgments

This is the 5th volume of the book *Wilhelm Weber's Main Works on Electrodynamics Translated into English*. The four earlier volumes were published in 2021.[1]

My gratitude goes mainly to my colleagues who understood the importance of this project, made the translations included in these 5 volumes and helped to edit the works presented in this book: Laurence Hecht, David H. Delphenich, Urs Frauenfelder, Joa Weber, Peter Marquardt, Hermann Härtel, Jonathan Tennenbaum, Peyman Ghaffari, Christof Baumgärtel, Mathias Hüfner and Frédéric Julian Linz. Without their support this book would never be published.

Roy Keys, the Editor of Apeiron, has been a supporter for many years. Without his encouragement some of my books might not have been published. He was very receptive to this particular project.

I wish to thank the Institute of Physics of the University of Campinas — UNICAMP, and the Teaching, Research and Extension Support Fund — FAEPEX of UNICAMP, which gave the necessary support for undertaking this work. I thank also the Alexander von Humboldt Foundation of Germany. This Foundation supported me to study German at the Goethe Institute in Göttingen (from April to July 2001). Humboldt Foundation also granted me four Research Fellowships to work at the Institute for the History of Natural Sciences of Hamburg University (from August 2001 to November 2002, and from February to May 2009), at the Dresden University of Technology (from April to June 2014) and at Augsburg University (from September to December 2023). During these stays I had the pleasure to work with Karin Reich, Gudrun Wolfschmidt, Martin Tajmar, Kai Cieliebak, Urs Frauenfelder and the late Karl-Heinrich Wiederkehr (1922-2012) who had written the main biography of Weber.[2] I also thank very much Hamburg University, Dresden University of Technology and Augsburg University for supplying excellent conditions during my stays abroad. I was always extremely well received, had all the necessary scientific support from these Universities, made many friends and important personal contacts. During these four research periods in Germany I could read and collect a huge amount of material which has been essential for the development of this project.

I would like to thank as well several other colleagues for their suggestions, references, ideas, support and encouragement: Simon Maher, Frederick David Tombe, Elena Roussanova, Elisabeth Becker-Schmollmann, Gilberto Orengo, Decio Schaffer, Robert W. Gray, Alan Aversa, Alexander Unzicker, Kjell Prytz, Jan Rak, Karel Janecek, Reiner Ziefle, Wal-

[1] [Ass21j], [Ass21k], [Ass21l] and [Ass21m].

[2] [Wie60] and [Wie67].

lace Thornhill, Tim Hooker, Lucy Wyatt, Thomas Herb, David de Hilster, John Lord, Karl-Heinz Glassmeier, Orges Leka, João Paulo Martins de Castro Chaib, Fabio Menezes de Souza Lima, Paulo Sakanaka, Thiago Pedro Mayer Alegre, Sandro Guedes de Oliveira, Mario Tamashiro, Pedro Raggio, Mikael Frank Rezende Junior, Sônia Maria Dion, Gildo Magalhães, Paulo Henrique Dias Menezes, Edson Eduardo Reinehr, Eloi Teixeira Cesar, Frederico Ayres de Oliveira Neto, Sérgio Roberto de Paulo, Arthur Baraov, Pietro Cerreta, Riccardo Urigu, Chuck Stevens, Wallace do Couto Boaventura, Danny Augusto Vieira Tonidandel, Marcos Cesar Danhoni Neves, Daniel Gardelli, Domingos Soares, Ivan Mingireanov Filho, Luciano Romenius Ferreira Guimarães, Luis Gustavo Vitti, Anderson William Mol, Ciro Lino Bellan, Cezar Cavanha Babichak, Arden Zylberstajn, Fernando Lang da Silveira, Moacir Pereira de Souza Filho, Lúcio Costa, Breno Arsioli Moura, José Emílio Maiorino, Elizabeth Silber, Hannes Täger, Amitabha Ghosh, Julian Barbour, Carlos Adriano Cardoso, Christine Blondel, Bertrand Wolff, Ana Paula Bispo da Silva, Daniel dos Anjos Silva, Douglas Soares da Silva, Frederico Ayres de Oliveira Neto, Mario Novello, Marcio Peron Franco de Godoy, Eduardo Luis Estrada, Ricardo Roberto Plaza Teixeira, Haroldo Fraga Campos Velho, Kathryn Olesko, Yuri Heymann, Joerg Fischera, Juan Muñoz Madrid, David Bower, Ismo V. Lindell, Ovidio Bucci, John Plaice, Klaus Hentschel, Tony C. Scott, Luis Carlos Malacarne, Renio dos Santos Mendes, Werner Martins Vieira, Ruy Hanazaki do Amaral Farias, Carlos Augusto Silva, Itala D'Otaviano, Marcio Antônio de Faria Rosa, Fabio Mibielli Peixoto, Alexandre Carlos Tort, Rolando Axt, Markus Wirz, John Eastmond, José Manuel Ferreirós Dominguez, Bernard Guy, Koen van Vlaenderen, Kirk McDonald, Rolf Laeuppi, Franz Pichler, Samer al Duleimi, Greg Volk, Mario Wingert, Michael D. Godfrey, Jocelyne Lopez, David Dameron, Mario J. Pinheiro, Hermann Borotschnig, Mischa Moerkamp, Mario Natiello, Júlio Akashi Hernandes, Giovana Trevisan Nogueira, José Luiz Matheus Valle, José Roberto Tagliati, Marco Antônio Escher, Simon Brunnquell, Andreas Otte, Max Tran, Konstantinos Kifonidis, Hans Günter Dosch, Albert Gerard Gluckman, Steve Hutcheon, Christian Ucke, Hartwig Thim, Steffen Kühn, Friedrich Steinle, Peter Heering, Rainer Müller, Sahand Tokasi, Cesar Pagan, Romis Attux, Joachim Schlichting, Matthias Heumesser, Stefan Suhr, Marcelo de Almeida Bueno, Dario Sassi Thober, Ademir Xavier Jr., Valter Cesar Montanher, Nelson Studart, Leandro de Paula, Jenner Barreto Bastos Filho, Olival Freire Jr., Dimas Guido Silva, Alexandre Alberto Visentin Ramos de Araújo, Thales Costa Soares, Mario Novello, Hugo Bonette de Carvalho, Valter Aparecido da Silva Junior, Wilson Bagni Jr., Manfred Pohl, Andreas Schuldei, Fritz A. Krafft, Wolfgang Engelhardt, Osvaldo Pessoa Jr., Filipe Pamplona, Alvaro Vannucci, Iberê Caldas, João F. N. Cortese, Ricardo Karam, Edmundo Capelas de Oliveira, Hector Torres Silva, Peter Puschnig, Michael de Carvalho, Pieter Jacqmaer, Matthias Dörries, Helmut Hansen, Juan Manuel Montes Martos, Manuel Filipe P. C. M. Costa, Roberto Machado Junior, João José Caluzi, Sergio Luiz Bragatto Boss, Ricardo Paupitz Barbosa dos Santos, Ceno Pietro Magnaghi, Fabio Ravanelli, Cesar Calderon, Juliano Camillo, Nivaldo Benedito Ferreira Campos, Alexander Montero Cunha, Alexandre Gomes Pinto, Alexandre Rodrigues, Karl-Heinz Schlote, Junichiro Fukai, Francisco Gonzalez Redondo, Olivier Darrigol, John Lewis Heilbron, Marco Mamone Capria, Andrew Chubykalo, Carlos Fiolhais, Wolfgang R. Dick, Neal Graneau, Klaus Reinsch, Daniel Steil, Jason Ross, Sepp Hasslberger, Hans-Joachim Linthe, Gustav Beuermann, Harry Hamlin Ricker III, Michael Matthews, Charbel El-Hani, Paulo Maurício, Jenaro Guisasola, Alexander N. Tarakanov, Franz Streibl, Chan Rasjid, Timothy Newton, Lei Zhao, Eugene Bagashov, Matthew Ehret, Michael Shilo, Anastasia Bendebury, Kirstie Finsterk, Michael Clarage and Alfonso San-Miguel Fuster.

This 5th volume, in particular, owes its existence to the invitation I received from Kai Cieliebak and Urs Frauenfelder to work at Augsburg University. I stayed there from September to December 2023 supported by a Research Fellowship given by the Alexander von Humboldt Foundation to undertake the project "Wilhelm Weber's Main Works on Electrodynamics Translated into English". The travel tickets for this project were supplied by the Teaching, Research and Extension Support Fund — FAEPEX of the State University of Campinas — UNICAMP.

Andre Koch Torres Assis
Institute of Physics Gleb Wataghin
University of Campinas — UNICAMP
Rua Sergio Buarque de Holanda 777
13083-859 Campinas, SP, Brazil
E-mail: assis@ifi.unicamp.br
Homepage: www.ifi.unicamp.br/~assis

Chapter 2

Introduction to Volume V of the Book *Wilhelm Weber's Main Works on Electrodynamics Translated into English*

A. K. T. Assis[3]

2.1 General Aspects of Volume V

The picture on the cover of Volume 5 comes from a 1884 portrait of Wilhelm Weber taken by the German photographer Bernhard Petri (1840-1887).[4]

This volume presents material which was not included in the 4 earlier volumes of the book *Wilhelm Weber's Main Works on Electrodynamics Translated into English*. Errata of these 4 volumes are available in my homepage.

I included here the Introduction of Gauss' 1840 paper on the general propositions relating to attractive and repulsive forces acting in the inverse ratio of the square of the distance. Then comes the full translations of Weber's two papers on unipolar induction, published in 1840 and 1841, preceded with introductory comments by the editor. After that comes Weber's large paper on galvanometry published in 1862. I also included Weber's 1874 paper on the equivalent of *vis viva*, preceded by the translator's introduction. Then comes Weber's 1875 treatise on the motion of electricity in bodies of molecular constitution.

Articles with biographical content were subsequently included: (a) Eduard Riecke's memorial speech given at the public meeting of the Royal Society of Sciences of Göttingen on December 5, 1891; (b) Heinrich Weber's 1893 biographical sketch of Wilhelm Weber; and (c) W. Voigt's 1899 speech on the unveiling of the monument to Gauss and Weber in Göttingen.

I also included the Prefaces to the six volumes of the collected works of Wilhelm Weber published between 1892 and 1894.

I then included the following treatises by Wilhelm Weber only published posthumously in his Collected Works: (i) on galvanometry (excerpt); (ii) comments on the paper: "investigation into the electric arc" by Prof. E. Edlund; (iii) electroscopic and electrodynamic actions of free electricity in closed conductors; and (iv) on electrothermism (on electricity and heat).

[3]Homepage: www.ifi.unicamp.br/~assis

[4]It appears, for instance, in [Web92e, frontispiece] and [Web93b, frontispiece].

Finally I included three chapters discussing additional topics:

- The *Weber* as an electrical unit of measure, in which I show the decisive role of Weber's main opponents, William Thomson (Lord Kelvin) and Hermann von Helmholtz, in suppressing the term *Weber* as the unit of electric current at the first International Electrical Congress held in Paris in 1881. It was their suggestion to replace the term *Weber* by *Ampère* as the unit of electric current.
- The velocity in Weber's electrodynamics versus the velocities in different field theories.
- Weber's electrodynamics versus different field theories.

The words between square brackets, [ ], in the middle of the text have been inserted by myself or by the translators in order to clarify the meaning of some sentences.

2.2 The Ratio Between the Numerical Values of the Same Magnitude Expressed in Two Different Units of Measure Is Equal to the Inverse Ratio of the Corresponding Units of Measure

I present here an important topic which helps to understand clearly Gauss and Weber's absolute system of units.

Let a distance d between two points be measured in centimeters (cm) and inches (in):

$$distance = d = 3.28\ cm = 1.29\ in\ . \tag{2.1}$$

Therefore,

$$\frac{3.28}{1.29} = \frac{in}{cm} = \frac{2.54}{1}\ . \tag{2.2}$$

This is a general result. Suppose there is a certain magnitude measured with two different units of measure:

$$magnitude = n_1 \cdot unit_1 = n_2 \cdot unit_2\ , \tag{2.3}$$

where n_1 and n_2 represent two dimensionless numerical values, while $unit_1$ and $unit_2$ represent the two units of measure. This magnitude can be a distance, a time interval, the value of an electric charge, a current intensity etc. In the specific case of Equation (2.1) we had $n_1 = 3.28$, $n_2 = 1.29$, $unit_1 = cm$ and $unit_2 = in$.

From Equation (2.3) we obtain

$$\frac{n_1}{n_2} = \frac{unit_2}{unit_1} = \frac{k}{1}\ . \tag{2.4}$$

In this equation k is a dimensionless numerical factor. It is used to convert the numerical value of the measurement of a magnitude expressed in one unit of measure into the numerical value of the same magnitude expressed in another unit of measure. It is also used to convert one unit of measure into the other unit of measure. In the case of Equation (2.2) we had $k = 2.54$, $1\ in = 2.54\ cm$, or $unit_2 = k \cdot unit_1$.

In other words, in general, the ratio between the numerical values of the same magnitude expressed in two different units of measure is the inverse of the ratio between the corresponding units of measure, as indicated by Equation (2.4), that is:

$$\frac{unit_1}{unit_2} = \frac{1}{k} \; . \tag{2.5}$$

That is, if $n_1/n_2 = k/1$, then $unit_1/unit_2 = 1/k$.

2.3 The Absolute Measure of Magnetic Force Converted to Magnetic Field Intensity

In Gauss and Weber's absolute system of units, all electrical and magnetic magnitudes can be measured in terms of length, mass and time. In particular, they used the millimeter (mm), milligram (mg) and second (s) as the fundamental or absolute units. Gauss explained some of his ideas in a letter to the German astronomer Heinrich Olbers (1758-1840) on February 18, 1832:[5]

> I am now working on geomagnetism, in particular the absolute determination of its intensity. My friend Weber is carrying out the experiments according to my instructions. Just as one can only give a clear concept of velocity, for example, by assuming a time and a space, so, I think, for the complete determination of the intensity of the Earth's magnetism one must specify (1) a weight $= p$, (2) a line $= r$, and then one can express the Earth's magnetism by $\sqrt{p}/r$, that is, a doubled Earth magnetism would result in a weight four times as great with the same r, or, with the same weight, half as great r.

In other words, the absolute unit of measure of the magnetic force is given by the unit of $\frac{\sqrt{p}}{r}$. Using Newton's second law, according to which a force or weight is equal to a mass (in milligrams, mm) times an acceleration (in millimeters per second squared, mm/s^2), we then have the following absolute unit of measure for the magnetic force:

$$\frac{\sqrt{mg \cdot mm/s^2}}{mm} = \frac{\sqrt{mg}}{s\sqrt{mm}} \; . \tag{2.6}$$

Measurements made by Gauss and Weber in May 21, 1832, indicated that the horizontal component of the Earth's magnetic force in the city of Göttingen was 1.7820 $\sqrt{mg}/(s\sqrt{mm})$, this quantity being represented by the letter T.[6] This was the first ever absolute measurement of the geomagnetic force:

$$T = 1.7820\frac{\sqrt{mg}}{s\sqrt{mm}} \; . \tag{2.7}$$

What they called the intensity of the magnetic force is represented by the magnetic field in current electromagnetic theory taught in textbooks. The magnetic field is represented by the letter, $B = |\vec{B}|$. The unit of measure of the magnetic field in the MKSA International

[5] See https://gauss.adw-goe.de/handle/gauss/990. See also [MB82], [Mal82] and [Sch88].

[6] [Gau32] with English translation in [Gau21b, p. 44] and [Gau94] with English translation in [Gau21d, p. 77].

System of Units is the Tesla ($1\ Tesla = 1\ kg/(As^2)$). The conversion between the absolute measurements of Gauss and Weber and the International System of Units is as follows:[7]

$$1\frac{\sqrt{mg}}{s\sqrt{mm}} \Longleftrightarrow 10^{-5}\ Tesla\ . \tag{2.8}$$

In other words, according to the measurements made by Gauss and Weber in Göttingen in May 21, 1832, the intensity of the horizontal component T of the Earth's magnetic force is equivalent to the horizontal component $B_{horizontal}$ of the Earth's magnetic field given by:

$$T = 1.7820\frac{\sqrt{mg}}{s\sqrt{mm}} \Longleftrightarrow B_{horizontal} = 1.7820 \times 10^{-5}\ Tesla\ . \tag{2.9}$$

[7]See footnote 5.

Chapter 3

[Gauss, 1840] Introduction of the Paper "General Propositions Relating to Attractive and Repulsive Forces Acting in the Inverse Ratio of the Square of the Distance"

C. F. Gauss[8,9,10]

Nature presents to us many phenomena which we explain by the assumption of forces exerted by the ultimate particles of substances upon each other, acting in inverse proportion to the squares of their distance apart.

Amongst these forces, the first to be noticed is that of universal gravitation, by virtue of which every ponderable molecule μ exercises upon every other such molecule μ', a moving force, which, if we call the distance r, is expressed by $\mu\mu'/r^2$, and tends to produce the approximation of the molecules in the direction of the straight line connecting them.[11]

If, in order to explain magnetic phænomena, we assume two magnetic fluids, one positive and the other negative, two magnetic elements μ, μ', will exert, each on the other, a moving force $\mu\mu'/r^2$, acting along the straight line which joins the two elements, repulsively if μ and μ', are of the same kind of fluid, attractively if they are of different kinds.[12]

The same is true of the mutual action of the particles of electric fluids upon each other.[13]

The linear element ds of a galvanic current exerts in like manner on an element of the magnetic fluid μ (if we allow the latter) a moving force, which is inversely proportional to

[8][Gau40] with English translation in [Gau43]. I present here only the Introduction of this paper.

[9]Edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

[10]The Notes by A. K. T. Assis are represented by [Note by AKTA:].

[11][Note by AKTA:] This law is due to Isaac Newton (1642-1727) in 1687. See [New34] and [New99]. Portuguese translation in [New90], [New08] and [New10]. See also [New79] with Portuguese translation in [New96].

[12][Note by AKTA:] This law is due to Charles Augustin de Coulomb (1736-1806). See [Cou88b] with complete German, Portuguese and English translations in, respectively, [Cou90b], [Ass22] and [AB23].

[13][Note by AKTA:] This law is also due to Coulomb, [Cou88a] and [Cou88b] with complete German, Portuguese and English translations in, respectively, [Cou90a], [Cou90b], [Ass22] and [AB23].

the square of the distance r; but there is now introduced a new and distinctive circumstance; the direction of the force is not in the connecting straight line, but is perpendicular to the plane passing through μ and the direction of ds; and the intensity of the force depends not on the distance alone, but also on the angle which r makes with ds. If this angle be called θ, then $\sin\theta \cdot \mu ds/r^2$ is the measure of the moving force which ds exerts upon μ; and an equal force in the parallel and opposite direction is exerted by μ on the element of the current ds, or on its ponderable carrier.[14]

If we assume with Ampère that the elements ds, ds' of two galvanic currents act attractively or repulsively on each other in the straight line which joins them, then the phænomena require us to consider this force as acting in the inverse ratio of the square of the distance; but as having, at the same time, a somewhat less simple dependence on the direction of the elements of the currents.[15]

We shall restrict ourselves in this treatise to the three first cases, or to those forces which are exerted by one element upon another in the straight line which joins them, and which are therefore simply in the inverse ratio of the square of the distance; although several propositions will be found applicable, with slight alterations, to the other cases also, the more detailed development of which must be reserved for another treatise.

[...]

[14][Note by AKTA:] In German: *dessen ponderabeln Träger*. That is, its ponderable carrier, or material conductor of the current element ds. This law is due to Jean-Baptiste Biot (1774-1862) and Félix Savart (1791-1841). See [Bio21]; [BS24] and [BS85] with English translations in [Far26] and [BS65a]; see also [AC15, Chapters 6, 16 and 17].

[15][Note by AKTA:] André-Marie Ampère (1775-1836). Ampère's masterpiece was published in 1826, [Amp26] and [Amp23]. There is a complete Portuguese translation of this work, [Cha09] and [AC11]. Partial English translations can be found at [Amp65] and [Amp69b]. Complete and commented English translations can be found in [Amp12] and [AC15]. A huge material on Ampère and his force law between current elements can be found in the homepage *Ampère et l'Histoire de l'Électricité*, http://www.ampere.cnrs.fr and [Blo05], at the homepage of the Friends of André-Marie Ampère, https://saama.fr, and at the homepage of the Ampère Museum, https://amperemusee.fr/en.

Chapter 4

Editor's Introduction to Weber's First Paper on Unipolar Induction

A. K. T. Assis[16]

4.1 Faraday's Experiment

Wilhelm Weber (1804-1891) presented in 1839 a paper on unipolar induction which was published in 1840.[17] We present here the first English translation of this paper.

Michael Faraday (1791-1867) was the first to perform this experiment in 1832.[18] He placed a copper disk above a cylindrical magnet and connected a galvanometer between the center and edge of the disk through sliding contacts. The magnet and the disk might rotate relative to the ground, while the galvanometer and conducting wires always remained stationary. The copper disk and magnet had the same radii and axis of symmetry, Figure 4.1.

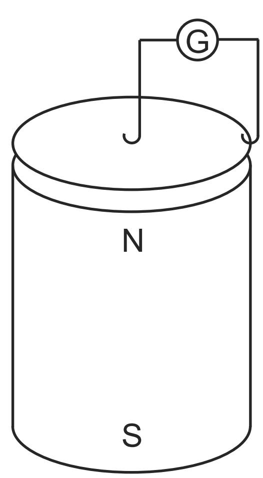


Figure 4.1: Unipolar induction experiment.

[16]Homepage: www.ifi.unicamp.br/~assis

[17][Web40].

[18][Far32a, §§ 217 to 230] with Portuguese translation in [Far11] and German translation in [Far32b] and [Far89].

Faraday observed the same electric current flowing through the galvanometer in two situations, namely, (a) when only the disk rotated relative to the ground with a certain angular velocity ω, and (b) when the disk and magnet rotated together relative to the ground with the same angular velocity ω. The direction of the current depends on the direction of rotation and also on the orientation of the magnet, that is, if the North pole is above or below the South pole.

In some experiments Faraday removed the disk and considered only a cylindrical magnet, with a galvanometer connected by sliding contacts to the center of the upper face and to a point along the edge of the magnet. When he rotated the conducting magnet relative to the ground, a current was indicated by the galvanometer.

This phenomenon became known as unipolar induction, a name coined by Weber in the paper which is being translated here. Other common names for this experiment are unipolar generator, homopolar induction, homopolar generator and Faraday generator.

4.2 Weber Knowledge of Faraday's Experiment

Weber seems to have developed the idea of this experiment quite independently from Faraday's earlier work of 1832. He performed the experiment and sent his text for publication without being aware of Faraday's paper, as pointed out by Wiederkehr:[19]

> When writing the article, Wilhelm Weber appears to have been unaware of Faraday's experiments on unipolar induction. The initially strange fact that Weber suddenly speaks of Faraday and his earlier experiments at the end of his treatise is explained by a letter from Weber to Gauss.

Wiederkehr then quoted Weber's letter to Gauss, number 15, from September 06, 1839:

> I saw Ettingshausen[20] in Leipzig on the way to Göttingen. He pointed out to me that the phenomenon which I had considered under the title of unipolar induction had also been noticed by Faraday. I found the passage easily and was able to refer to it, since Reimer[21] was willing to have the last page of this essay reprinted...

4.3 Weber's Experiment

One of Weber's original apparatus related to unipolar induction, his unipolar inductor of 1840, still exists at the Scientific Collections of the Georg-August-University Göttingen in Germany.[22] It appears in Figure 4.2.

[19] [Wie60, pp. 41-42, footnote 6]. See also [Gau d].

[20] Andreas von Ettingshausen (1796-1878) was an Austrian physicist.

[21] Karl August Reimer led the Weidmannsche Buchhandlung where Weber's work was originally published.

[22] https://sammlungen.uni-goettingen.de/sammlung/slg_1020/ and https://uni-goettingen.de/en/47114.html.

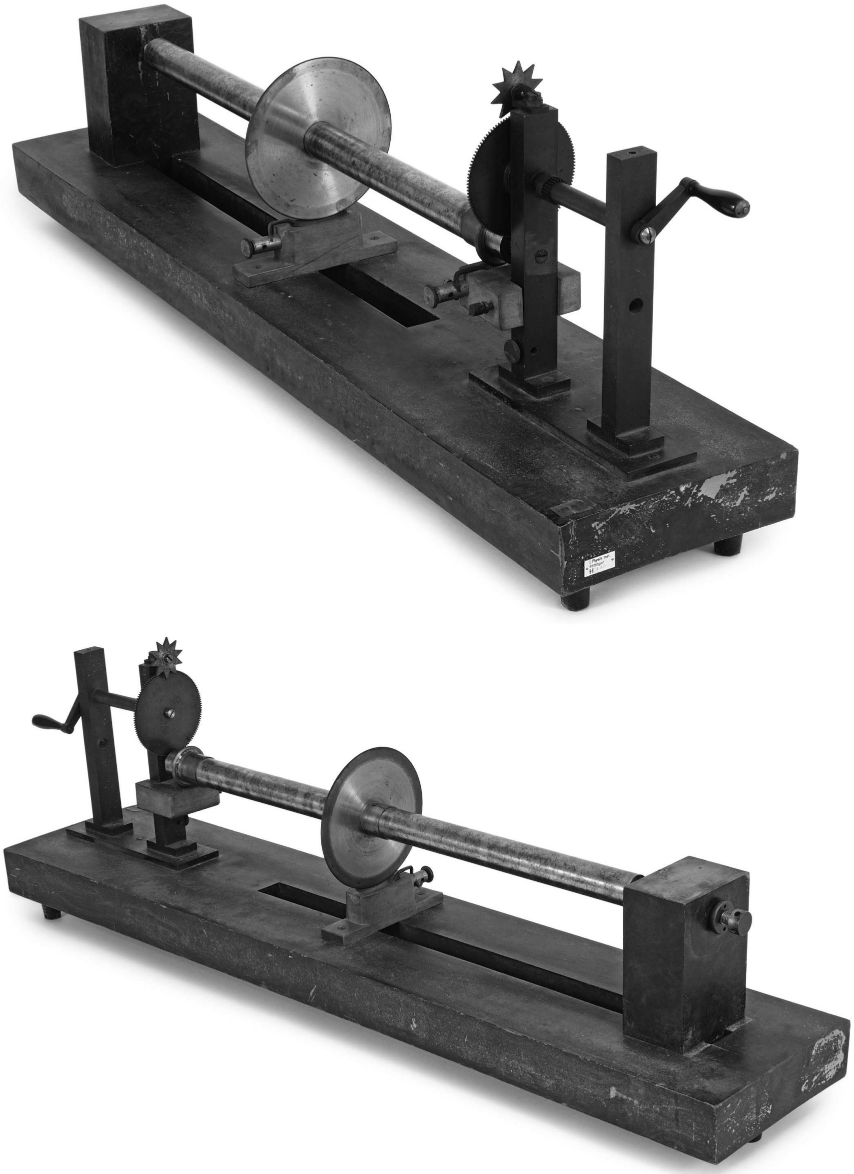


Figure 4.2: Weber's original instrument. Source: Physicalisches Cabinet, Georg-August-Universität Göttingen / CC BY-SA 4.0; Photo: Sauer Marketing, Gerhard und Maren Sauer.

Figure 4.3 presents the main components of this device.[23] A brass disk can be fixed at different points along the axis of a magnetized steel cylinder NS. The lower portion of the disk touches a mercury tray or trough. Hand gears allowed the rotation of the magnet around its horizontal NS axis at a known rate. A galvanometer G is connected through conducting wires to the mercury tray and to the center of one extremity of the magnet. When the disk and magnet rotate together relative to the ground, while the galvanometer and conducting wires remain stationary, a current flows through the galvanometer. One of his magnetized cylinders was 26.9 cm long with a diameter of 2.3 cm, while the other was 50.2 cm long with a diameter of 2.05 cm. This experiment is analogous to Faraday's unipolar induction experiment when the disk and magnet rotate together relative to the ground. It

[23][Bec79].

is also analogous to Faraday's experiment in which there was only a magnet which could rotate relative to the ground.

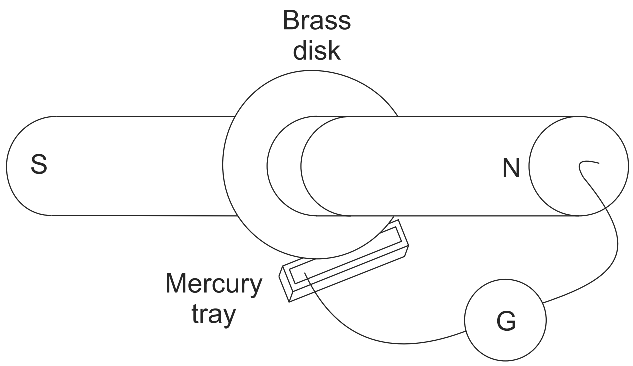


Figure 4.3: A representation of Weber's unipolar inductor.

4.4 On the Existence of Magnetic Fluids

In this paper of 1839 Weber based his explanation of the phenomenon on the existence of magnetic fluids, namely, the austral and boreal fluids (also called the northern and southern fluids, respectively). Later on he changed completely his point of view, rejecting the existence of these fluids. This was due to his experiments on diamagnetism published in 1852 in his third major memoir on Electrodynamic Measurements. In particular, he discussed this important topic in a Section 22 (On the Existence of Magnetic Fluids) of his 1852 paper.[24] André-Marie Ampère (1775-1836) had already argued against the existence of magnetic fluids in his masterpiece.[25]

In 1852 Weber concluded that the hypothesis of magnetic fluids in the interior of bodies had been refuted, while Ampère's hypothesis of the existence of electric molecular currents in the interior of bodies had been corroborated through diamagnetism. His conclusion runs as follows:[26]

> The *diamagnetic* phenomena discovered by Faraday[27] decide between these two theories in the same way as the phenomena of interference decided between the emission and wave theory in optics. This is the most essential and important meaning associated to this discovery. Thanks to the discovery of diamagnetism the hypothesis of *electric molecular currents in the interior of materials* gets affirmed and the hypothesis *of magnetic fluids in the interior of materials* gets disproved.

[24] [Web52a, Section 22] with English translation in [Web21h, Section 2.22].

[25] See Section 19 (The Magnetic Poles and Dipoles are Disposable Hypotheses) of [AC11] and [AC15]. See also footnote 15 on page 18.

[26] [Web52a] with English translation in [Web21h, Section 2.22 (On the existence of magnetic fluids), pp. 66-68]. See also [Web52d] with English translations in [Web53b], [Web66b] and [Web21l, Section 3.1.6 (On the existence of magnetic fluids)].

[27] [Far46b] and [Far46c].

4.5 On the Existence of Molecular Currents

Another aspect which should be emphasized here is that in this paper of 1839 Weber argued, based on the phenomenon of unipolar induction, against the existence of Ampère's molecular currents, see Subsection 5.4.1. Later on he also changed completely his point of view on this respect. For instance, in his paper of 1852 on diamagnetism he made the following comment:[28]

> Before in the "Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1839",[29] I tried to justify the conjecture that the phenomenon described by the name "unipolar polarity" could lead to such a decision.[30] However, this is not the case, since there can be given a different explanation for the phenomena described there, as soon as such a connection takes place between the electric fluids moving in the interior of the conductor and the ponderable parts of the conductor, that each force acting on the electric fluids completely or nearly is transferred to the ponderable parts, as I explained in more detail in the "Electrodynamic Measurements" (*Abhandlungen bei Begründung der Königlichen Sächsischen Gesellschaft der Wissenschaften* edited by the F. Jabl. Ges., Art. 19, p. 309).[31]

These molecular currents proposed by Ampère became the foundation of many researches which Weber developed in the following years.

4.6 Unipolar Induction Explained with Weber's Electrodynamics

Normally when most scientists considered unipolar induction in the last 190 years they concentrated their attention only on the rotations of the disk and magnet around their common axis. Let us represent by ω the clockwise angular velocity of the disk and/or magnet relative to the ground when seen from above, while $-\omega$ represents the anti-clockwise rotation. When we rotate only the disk relative to the ground, we can measure a current I flowing through the galvanometer connected by sliding contacts between the center and periphery of the disk, as represented in Figure 4.1. This current is linearly proportional to ω and to the intensity of the magnet. These facts suggest that the induced current depends on the interaction between the disk and magnet, an action originating from their relative rotation.

If we let the disk stationary in the ground and rotate the magnet in the opposite direction with an angular velocity $-\omega$, most people expect the same current I to be measured in the galvanometer. However, the galvanometer measures no current, as shown in Table 4.1

[28] [Web52a, footnote 1, p. 536 of Weber's *Werke*] with English translation in [Web21h, footnote 59, page 67].

[29] See [Web40, p. 171 of Weber's *Werke*] and Subsection 5.4.1 with the English translation of Weber's discussion.

[30] In 1839 Weber decided in favour of the existence of magnetic fluids and against the existence of molecular electric currents.

[31] [Web46, Section 19, p. 134 of Weber's *Werke*] with English translation in [Web21d, Section 5.19 (Development of a general fundamental law of electrical action), pp. 130-141].

Rotation of the Disk	Rotation of the Magnet	Current in the Galvanometer
ω	0	I
0	$-\omega$	0

Table 4.1: First apparent paradox.

There is an apparent paradox here. The relative motion between the disk and magnet is the same in both cases, but the measured effect indicated by the current in the galvanometer is completely different for these two cases.

Another apparent paradox originates when the disk and magnet are stationary, or when both of them rotate together relative to the ground. The result of this experiment is indicated in Table 4.2.

Rotation of the Disk	Rotation of the Magnet	Current in the Galvanometer
0	0	0
ω	ω	I

Table 4.2: Second apparent paradox.

In these two cases there is no relative motion between the disk and magnet. However, the current measured in the galvanometer is very different for these two cases.

The solution of these apparent paradoxes is that in unipolar induction we need to consider not only the disk and magnet, but also the closing circuit (composed of galvanometer and conducting wires connected to the center and periphery of the disk). We are here supposing that the magnetism of the magnet has a much larger intensity than the magnetism of the Earth, so that we can neglect the magnetic influence of the Earth on the outcome of this experiment.

In 1994 a theoretical prediction has been made of what would happen in this experiment, based on Weber's electrodynamics, if it were possible to rotate the closing circuit relative to the ground.[32] We now have 8 cases to consider, as indicated in Table 4.3.

From Table 4.3 we can see that the opposite of rotating only the disk clockwise is not to rotate the magnet anti-clockwise, but to rotate together the magnet and the closing circuit anti-clockwise, cases 2 and 6. In both cases the same current I should be measured in the galvanometer.

Likewise, the situation when everything is stationary in the ground is not equivalent to rotate together the disk with the magnet, but to rotate together the disk, magnet and closing circuit, cases 1 and 8. In both cases no current should be measured in the galvanometer.

Moreover, the situation when we rotate only the magnet is not equivalent to rotate the disk in the opposite direction, but to rotate together the disk and closing circuit in the opposite direction, cases 3 and 5. In both cases no current should be measured in the galvanometer.

And finally, the situation in which we rotate together the disk and magnet, should be equivalent to rotating only the closing circuit in the opposite direction, cases 7 and 4. In

[32] [AT94].

	Rotation of the Disk	Rotation of the Magnet	Rotation of the Closing circuit	Current in the Galvanometer
1	0	0	0	0
2	ω	0	0	I
3	0	$-\omega$	0	0
4	0	0	$-\omega$	I
5	ω	0	ω	0
6	0	$-\omega$	$-\omega$	I
7	ω	ω	0	I
8	ω	ω	ω	0

Table 4.3: Prediction based on Weber's electrodynamics.

both cases the same current I should be measured in the galvanometer.

These predictions of Weber's electrodynamics have been confirmed by an experiment performed in 2022:[33] Baumgärtel, C., Maher, S. Resolving the paradox of unipolar induction: new experimental evidence on the influence of the test circuit. Sci Rep 12, 16791 (2022). Available at `https://doi.org/10.1038/s41598-022-21155-x` and also at: `https://www.nature.com/articles/s41598-022-21155-x`.

[33][BM22] and [Bau22].

Chapter 5

[Weber, 1840] Unipolar Induction

Wilhelm Weber[34,35,36,37,38]

There are two sources responsible for magnetic phenomena, namely *terrestrial magnetism* and *bar magnetism*[39] which are differentiated not because there is a specific difference in the magnetism itself, but because the circumstances under which they act are different and the questions to be answered are different. This distinctness especially shows in the known general magnetic laws (which were found through experiments with bar magnets) often being applied *directly* to the action of bar magnets and give a straight-forward explanation to phenomena stemming therefrom; but being rooted and applied to terrestrial magnetism only *indirectly* through the general theory of terrestrial magnetism, which contains the principles of explanation for all terrestrial-magnetic phenomena. The latter theory was *first* developed by Privy Councillor Gauss[40] in the previous Volume of the *Resultate*;[41] the theory of bar magnetism is older[42] and, because it is essentially included in the general theory of magnetism, may in some respects be regarded as self-contained and completed for a long time, but this does not prevent the occurrence of individual problems which need to be solved, and through which even new light can be shed on the nature of magnetism. One of such tasks is the core of the present article. The phenomena to be regarded here are *induction phenomena* which consist in general of the excitation of galvanic currents through magnetism in motion. These induction phenomena are split into two categories, where the ones in the *first* category, which shall be titled *bipolar* induction, are sufficiently known and have been shown with both bar and terrestrial-magnetism; the ones of the *other* category, in contrast, which

[34] [Web40].

[35] Translated by C. Baumgärtel, Department of Electrical Engineering and Electronics, University of Liverpool, Liverpool, L69 3GJ, United Kingdom, ORCID: 0000-0002-0702-0480. Edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

[36] The Notes by E. Riecke, the editor of the second Volume of Weber's *Werke*, are represented by [Note by ER:]; the Notes by C. Baumgärtel are represented by [Note by CB:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[37] [Note by ER:] See Table VIII, Figures 1-4 [of Weber's *Werke*].

[38] [Note by ER:] Resultate aus den Beobachtungen des magnetischen Vereins, 1839, III, pp. 63-90.

[39] [Note by CB:] It is likely Weber is implying permanent ferromagnetism here.

[40] [Note by AKTA:] In German: *Herr Hofrath*. The title by which Weber addressed Gauss can also be translated as Mr. Court Councillor.

[41] [Note by AKTA:] [Gau39] with English translations in [Gau41a] and [GT14].

[42] [Note by AKTA:] See footnote 12 on page 17.

shall be titled *unipolar* induction, have been previously unknown and have only been shown to arise from bar magnetism. In addition to the many examples we have where essentially the same phenomena as with bar magnetism are also produced by geomagnetism (for example, almost all electromagnetic and magnetoelectric phenomena), it is interesting to learn of a case where this is not possible. That the reason for this impossibility lies not within magnetism itself, but in external circumstances, (for instance, the Earth is not as good a conductor as the steel of a bar magnet, and not all of Earth's parts are magnetic, — apart from the fact that the Earth itself prevents the execution of some experiments due to its size) is easily anticipated and proved through testing. — Before we move on to the experiments themselves, which have led to the investigation of unipolar induction, some general remarks shall be made about the nature, the method and the laws of unipolar induction, since this helps the understanding of the experiments and shortens their description.

5.1 General Remarks

5.1.1 Bipolar and Unipolar Induction

The existence of two magnetic fluids is presupposed, one northern and one southern, which exist in the molecules of a magnet in equal amounts, but separate from each other. If such a magnet is set in motion, a galvanic current is induced in a neighbouring conductor following known laws.[43] This current is such that it can be decomposed into two currents, of which one is caused by the motion of the *northern* fluid and the other by the motion of the *southern* fluid. This induction of two currents through the motion of *both* magnetic fluids shall be called *bipolar* induction. But it is also conceivable a kind of induction whereby either only one kind of magnetic fluid moves and the induced current of the other fluid is always zero, or the other fluid induces alternating positive and negative currents whose sum is zero, so that the only remaining current is the one induced by the first fluid. This induction of a current caused by the motion of one magnetic fluid shall be called *unipolar* induction.

5.1.2 *Method*

Imagine a horizontal circular or annular conductor and move a body containing only northern fluid downwards along the vertical axis, thus a galvanic current will show in the ring,[44] the direction of which is opposite to the diurnal motion.[45] For uniform velocity [of the northern fluid,] the current increases from zero during the motion from infinite height to the ring plane, and decreases similarly back to zero for motion from the ring plane to infinite depth. During this motion the magnitude of the current changes, but never the direction in the conductor. If eventually the body with the northern fluid is moved back from bottom to top, but not in a straight line and rather in a circular motion whose centre is located in the conductor and staying infinitely far away from the same, so that the body does not excite the conductor, the first motion can begin anew and the same current induced in the conductor a second time. In this manner the body containing only *northern* fluid could continue the

[43][Note by AKTA:] Weber is referring here to Faraday's law of induction from 1831. See [Far32a] with German translation in [Far32b] and [Far89], and Portuguese translation in [Far11].

[44][Note by CB:] In German: *im Ringe*. This expression can be translated as "in the ring" or "in the loop".

[45][Note by AKTA:] In German: *dessen Richtung der täglichen Bewegung entgegengesetzt ist*. That is, the current will flow anti-clockwise when viewed from above.

same induction arbitrarily, whereby the current's magnitude in the conductor changed, but never the direction. The same would be true for a body only containing *southern* fluid, but the current's direction would be opposite. In both cases the magnet's path can be shortened drastically, since on all paths where the magnet moves downwards through the ring and upwards around the ring to return to the initial position, the induction is the same. The essential criterion for a continuous homogeneous induction with a magnet containing only *one* magnetic fluid is, that this magnet moves downwards through the ring and upwards around it, or vice versa. On the contrary, if it is moved through the ring for both upwards and downwards motion or around it for both motions, the direction of the induced current changes and the total effect is zero.

It is simple to apply these laws to the *second* case, where a magnet is responsible for the induction that contains *both* fluids in equal amounts, where both move with the magnet at the same time. The current induced by both fluids simultaneously at every instant is the sum of the currents induced by each individual fluid at that instant, which leads to the conclusion that

1. if the magnet is moved back and forth from its original location and position, so that it passes through the ring either not at all (neither downwards nor upwards) or both times (downwards as well as upwards), the total effect is zero, as it vanishes in its parts;

2. if the magnet moves through the ring only once during this motion (downwards or upwards), the total effect is also zero, because the southern fluid induces an equal but opposite current to the northern one.

It does not follow from this, however, that a continuous homogeneous induction, such as can be produced by a magnet that contains only *one* fluid, is impossible with a magnet containing *both* fluids, rather, there remains a *third* case to be considered, which is not yet included in the previous two, and is possible if *really* magnetic fluids *exist* and are really *spatially separated* from one another in the molecules of the magnet, that is

3. a magnetic molecule[46] is moved in such a way that it does not pass through the ring completely or not at all, but *half* through it and *half* remains outside it, for instance that the half containing *northern* fluid goes down through the ring, upwards around the outside, or vice versa; but the other half containing *southern* fluid always remains outside. The total effect is then *non*-zero, since one fluid (which moved through the ring) has induced a current, which has *not* been nullified, since the other fluid (which did not move through the ring) has not induced any or only an inhomogeneous current whose combined effect vanishes in total. However, since the ring as well as the magnetic molecule are *solid* bodies, it appears that this third case is only possible if either one is *broken up*. Yet, a magnetic molecule cannot be broken in such a way that each part only contained *one* fluid, which would be necessary to move *one* single fluid through the unbroken ring; thus the *ring* has to be broken up, which is easily done: however, it must be noted that the galvanic circuit must not be interrupted while the ring is being broken. The ring can be broken without interrupting the circuit, if the inseparable magnetic molecule is such that the galvanic current can flow right through *between both fluids*; since that molecule can *conductively connect* both parts of the ring while it is being broken.

It is easy to create a setup which fulfills the conditions of the third case. It is sufficient to magnetize a steel cylinder in such a way that its magnetic axis coincides with its geometric one, and spin it around this axis. If both ends of a conducting wire then come into contact

[46][Note by AKTA:] Each magnetic molecule is made up of a North fluid and a South fluid of the same intensity, separated by a small distance, making it a magnetic dipole.

with the cylinder, one end to the rotational axis at A, Figure 1, the other to the periphery at B, the wire forms a conducting circuit $ABCDA$, which always remains closed during rotation of the cylinder.[47]

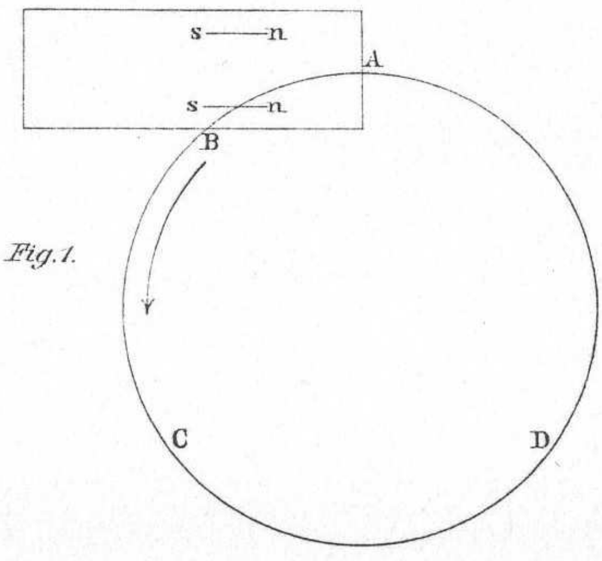
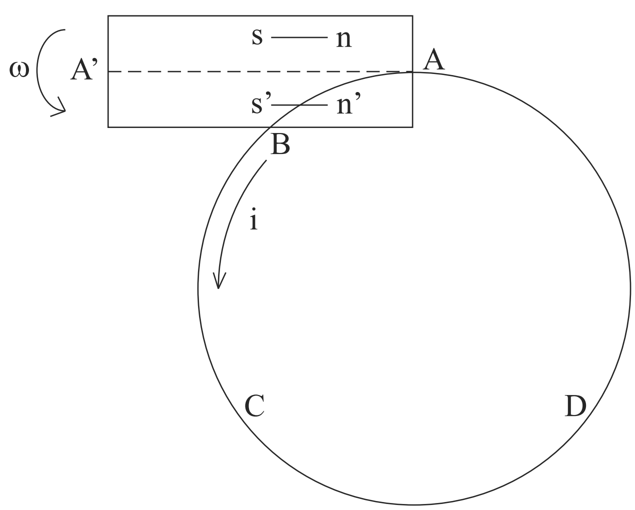


Fig. 1.

Let ns be a magnetic molecule in the cylinder, where the northern fluid is situated at end n and the southern at s. The molecule is such that a galvanic current can be passed right through it. If one imagines the conducting circuit $ABCDA$ containing the point where the centre of the molecule is located, it is easy to see that the northern fluid n is moved downwards through the ring of the circuit and upwards around it with each rotation, if we assume that in the Figure ns is moving downwards during rotation and after half a rotation

[47][Note by AKTA:] In Figure 1 the magnetized cylinder is represented by the horizontal rectangle. The arrow indicates the direction of the current in the wire $BCDA$. Weber considered the rotation of the magnet around its horizontal axis of symmetry. An experiment like this one was first performed by Faraday in 1832, see Chapter 4. In the Figure of this footnote I included the horizontal axis AA' of the magnet, its angular velocity ω around this axis, and the electric current i induced in the wire $BCDA$ due to the rotation of the magnet. I also replaced the lower letters sn by $s'n'$.



reaches $n's'$,[48] where it moves back upwards. In contrast, the southern fluid s always remains outside the circuit during rotation. These circumstances allow to speculate that a continuous homogeneous current will be excited, whose direction is indicated by the annotated arrow. This speculation has been confirmed by experience, as the experiments to be shown later will prove.

After explaining the underlying idea of the experiments to be described, a few theorems shall be developed which have guided the design of individual experiments.

5.1.3 Laws

1. The induction along all paths from the point of contact on the cylindrical surface to the point of contact at the end of the rotational axis is uniform if the magnetic fluids are separated uniformly everywhere.

It is presupposed that all magnetic molecules in the rotating cylinder are equal in strength and equally spaced, as if the cylinder was split into small identical cubes, with magnetic molecules sitting at the ends. The molecules may then form parallel rows to the rotational axis. No matter which path the current takes, it needs to traverse every row of molecules from the surface to the axis, and the probabilistic number of magnetic molecules being cut by the current on its way is proportional to the number n of these rows; furthermore it is directly proportionate to the length l of these molecules and indirectly to their distance a, so that $= nl/a$. Since all molecules are assumed equal and equally spaced (that is, l and a constant), it follows that the number of cuttings on all paths is expected to be equal. This theorem holds true even for those paths which exceed the rotational axis and cut many more rows of molecules beyond it, until eventually reaching the end of the axis; for it is obvious that such a path cuts each row beyond the axis twice, once moving away from the axis, the other time approaching the axis again, each time with equal probability to come across a magnetic molecule. The induction due to the cutting of a particle on the outbound path is cancelled by that on the return path, so that the probability of induction on such a detour is zero overall.

2. If the galvanic current passes simultaneously along several paths from the surface of the cylinder to the axis, on all of which the induction is the same, the induction is just as strong as if the current took only one path.

It is known that if you set up *multiple* equally strong galvanic piles and connect their poles of the same type to each other and to the ends of a long circuit of conductors (so that all currents emanating from the piles combine immediately after the piles and flow through the long circuit and eventually split immediately before the piles to complete their circuit),[49] then the current in the circuit is just as strong as if the ends of the circuit only made contact with the poles of *one* pile, presupposing that the resistance in the piles is vanishingly small compared to the resistance in the circuit. Applying this theorem to our case, every path through the cylinder can be compared to a path through a pile, from which the present theorem follows, since the resistance in the cylinder is vanishingly small compared to the rest of the circuit. From this it follows,

3. The induction is independent of the number of points on the surface of the cylinder being contacted.

[48][Note by AKTA:] In Figure 1 we should have at the lower portion of the magnet $n's'$ instead of ns, as shown in the Figure of footnote 47.

[49][Note by AKTA:] That is, the piles are connected in parallel.

4. The induction is independent of the length of the cylinder, whose molecules are equally strong magnetic.

5. The induction is proportional to the cross section of the cylinder under otherwise similar circumstances.

6. When there are different paths through the cylinder, some of which where the induction is larger, some of which where it is smaller, the current will be as strong as if it traversed the latter path alone through the cylinder.

This theorem stems from the comparison of our case with a circuit, which is split at the end and connected to multiple unequal piles. Because if such a current division occurs that some parts traverse weaker and some parts stronger piles, the current in the rest of the circuit will be just as strong as if there was no division and the current only traversed the weakest pile, presupposing the resistance of the piles vanishes in comparison to the resistance of the circuit. If one part was led only through a conductor, instead of through a pile, where too the resistance [of the conductor] disappears compared to the resistance of the entire circuit, the galvanic current would cease in the remaining undivided circuit. It is straightforward to apply this to our case. All induction would need to vanish if the surface of the cylinder was connected to the axis by a copper sleeve.

7. If the cylinder is equally strong magnetic in all parts, two rotations will induce a current which is equal to the current created by the same cylinder through *a single alternation* in an inductor coil consisting of a single winding,[50] presupposing that the diameter of the latter is very small compared to the length of the cylinder.

If M is the magnetic moment of the cylinder and L is its length, and if the magnetic fluids are spread across the end face of the cylinder, which is allowed under the previous condition that all particles of the cylinder are equally magnetic, then $\pm\frac{M}{L}$ is the amount of northern or southern fluid situated at one or the other end face. The induced current S by a *single alternation* is then equal to the current induced by *one* fluid $\pm\frac{M}{L}$ if it was moved twice along the same path in the same direction through the inductor ring (presupposing, that the diameter of the later is very small compared to the length of the cylinder), which allows to write

$$S = 2c \cdot \frac{M}{L} \ ,$$

where c is a constant only depending on the resistance of the circuit. If the inductor consists of multiple windings, c would need to be multiplied by the number of windings.

If the cylinder consists of equal and parallel molecules, each of which has a magnetic moment $= m$, a length $= l$ and whose distance is $= a$, then the number of molecules is equal to the volume of the cylinder divided by the cubed distance a, or $= \frac{\pi R^2 L}{a^3}$, where R is the radius of the cylinder. The sum of all molecules' moments is equal to the moment M, or

$$\frac{\pi R^2 L}{a^3} \cdot m = M \ .$$

If at one end of each molecule there is $+\frac{m}{l}$ (northern) fluid, at the other end $-\frac{m}{l}$ (southern) fluid, then the amount of northern (or southern) fluid which traverses the ring of the

[50][Note by AKTA:] In German: *der von demselben Cylinder durch einen Wechsel in einer aus einer Umwindung bestehenden Induktorrolle hervorgebracht wird*. The word *Wechsel* can be translated as alternation, change or rotation. The word *Umwindung* can be translated as winding, loop or turn. The word *Induktorrolle* can be translated as inductor coil. Gauss defined the meaning of the word "Wechsel" in his 1836 work "Erdmagnetismus und Magnetometer", [Gau36b, pp. 39-43 of the Jahrbuch and pp. 340-341 of Gauss' *Werke*] and [Web39b, pp. 108 and 112 of Weber's *Werke*].

circuit during each rotation, and induces a continuous homogeneous current, is obtaining by multiplying $\pm\frac{m}{l}$ by the number of molecule rows in the cylinder and by the ratio l/a (which measures the probability that the current cuts a molecule while traversing a molecule row). The amount of induction-causing fluid traversing the ring of the circuit during each rotation is then

$$= \frac{m}{l} \cdot \frac{\pi R^2}{a^2} \cdot \frac{l}{a} = \frac{\pi R^2 m}{a^3} \ ,$$

since the number of molecule rows in the cylinder is equal to the cross section πR^2 of the cylinder, divided by the square of the distance a between the molecules. According to this the current induced by every rotation is

$$s = c \cdot \frac{\pi R^2 m}{a^3} \ ,$$

where c has the same meaning as before. Comparing both currents, one finds

$$S = 2s \ ,$$

that is, the current induced by two rotations of the cylinder is equal to the current induced by a *single alternation*, presupposing that the wire of the inductor coil forms only *a single* winding.

8. If some parts of the cylinder are more strongly magnetized, some more weakly, the current induced by two rotations of the cylinder is weaker than the one by *a single alternation*, presupposing the wire of the inductor coil forms only one winding which is very small compared to the length of the cylinder.

There is one path among the many which the galvanic current takes through the cylinder, which traverses the most weakly magnetized parts. According to [theorem] (6.)[51] the current induced through rotation of the cylinder is not stronger than as if the cylinder was only weakly magnetized in all of its parts. In contrast, the current induced by a *single alternation* is increased, even if the magnetism of the cylinder is not amplified in all, but only in individual parts, from which the previous theorem follows by itself.

5.2 Instruments

The instruments used to cause and observe unipolar induction consisted of the following parts: *firstly* two axially magnetized steel cylinders; *secondly* gears with which the cylinders could be rotated around their axes with a known rate; *thirdly* a magnetometer equipped with a multiplier to measure the induced currents;[52] *fourthly* a device for conductively connecting one end of the multiplier wire to the end of the rotational axis, the other end [of the multiplier wire] to the cylindrical surface without impairing its rotation; *fifthly* a coil to perform induction experiments described in the previous Volume of the *Resultate*, page 98 and following,[53,54] with the same magnet.

[51][Note by AKTA:] See page 32.

[52]A multiplier is a galvanometer. It received this name "multiplier" from the German chemist, physicist and mathematician Johann Schweigger (1779-1857) who built the first galvanometer in 1820. He amplified the effect that Oersted first observed in 1820 by forming a coil with multiple windings looped around a rectangular frame, in the center of which the magnetized needle was suspended, [Sch20] and [Sch21d]; [Sch21c] with French translation in [Sch21a]; [Sch21b]. See also [Chi64] and [LSN21].

[53][Note by ER:] Wilhelm Weber's Werke, Vol. II, p. 115.

[54][Note by AKTA:] [Web39b, p. 115 of Weber's *Werke*].

5.2.1 The Cylindrical Magnets

Two hardened steel cylinders, one 269 mm long, 23 mm wide, the other 502 mm long, 20.5 mm wide, were fitted with a spike at one end (North end) and equipped with a nut at the other [end]. The latter was attached to a toothed wheel (40 teeth) whose shaft was pointed as depicted in Figure 2. The first steel cylinder was magnetized twice, first weakly, then strongly, so that its magnetic moment was first 65 [Million] and then 108 Million according to absolute measure.[55] The second cylinder was magnetized to 450 Million.

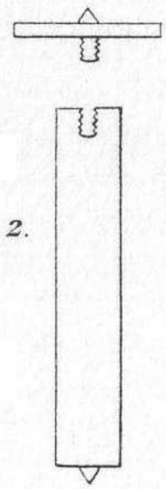


Fig. 2.

5.2.2 The Gears

The gear was the same as described in the second Volume of the *Resultate* (for 1837) in connection with the induction inclinometer.[56,57,58] Only an additional wheel with 60 teeth was added which meshed with the 40 tooth wheel fixed to the magnet. Each rotation of the crank equalled $8\frac{4}{7}$ of the steel cylinder. To connect the gears with the magnet a rack was used, Figure 3, which consisted of an iron clamp to which the gears were screwed, with a small notch at the end, in which the spike of the small wheel fixed to the magnet was

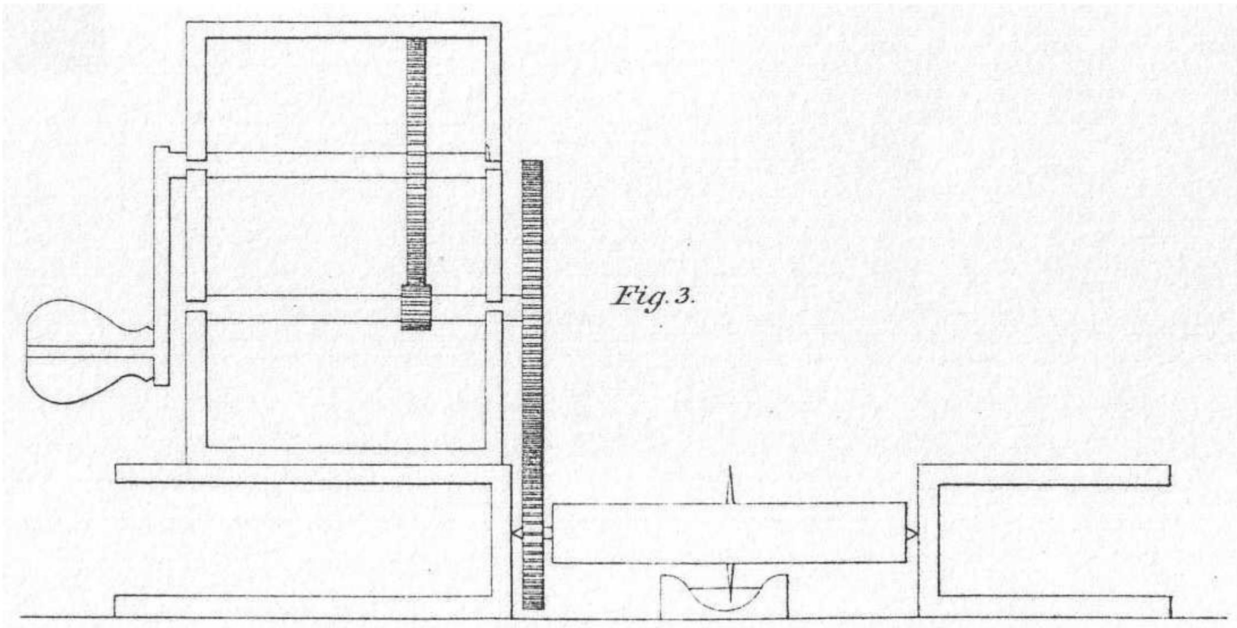
[55] [Note by AKTA:] Gauss introduced the absolute measure of bar magnetism in 1832, see [Ass21j]. Gauss' work on the absolute measure of the Earth's magnetic force was announced at the Königlichen Societät der Wissenschaften zu Göttingen in December 1832, [Gau32] with English translation in [Gau33a], [Gau37a] and [Gau21b], see also [Rei02, pp. 138-150]. The original paper in Latin was published only in 1841, although a preprint appeared already in 1833 in small edition, [Gau41b] and [Rei19]. Several translations have been published. There are two German versions, one by J. C. Poggendorff in 1833 and another one in 1894 translated by A. Kiel with notes by E. Dorn; a French version by Arago in 1834; two Russian versions, one by A. N. Drašusov of 1836 and another one by A. N. Krylov in 1952; an Italian version by P. Frisiani in 1837; an English extract was published in 1935, while a complete English translation by S. P. Johnson was published in 2003 and 2021; and a Portuguese version by A. K. T. Assis in 2003: [Gau33b], [Gau34c], [Gau36a], [Gau37c], [Gau94], [Gau35], [Gau52], [Gau75], [Gau03] and [Gau21d], and [Ass03].

[56] [Note by ER:] Ibidem, p. 77.

[57] [Note by AKTA:] [Web38, p. 77 of Weber's *Werke*].

[58] [Note by AKTA:] In German: *Induktions-Inklinatorium*. The dip circle, dip needle, inclinometer or inclinatorium is an instrument used to measure the angle between the horizon and the direction of terrestrial magnetism (the dip angle). It consists essentially of a magnetic needle pivoted at the center of a vertical graduated circle. Weber's induction inclinometer (*Induktions-Inklinatorium*) is a new instrument which he presented in 1837, [Web38]. It offered a novel way to circumvent the two main problems with dip circles: the effect of gravity, and the need to reverse the polarity of the needle, [WSH03].

inserted, while the spike of the North end of the magnet fit into a similar notch in the second clamp. The shape of the clamp was used to bring the ends of two large magnets close to the rotating magnet from two opposite sides, as done in some of the tests. The clamps were held by the weight of these magnets. In absence of the magnets the clamps were screwed tight to the table hosting the apparatus.



5.2.3 Magnetometer and Multiplier

The magnetometer used for these tests is the same transportable magnetometer as described in the previous Volume of the *Resultate*.[59] This smaller magnetometer was preferable over a larger one, because the multiplier wire had a larger number of windings (2000) over a shorter length (roughly 600 metres). Due to the shorter length, the resistance was smaller, and with this the induced current stronger; due to the larger number of windings, the current's intensity was multiplied: both leading to a larger deflection of the magnetometer's needle. To further increase the amplification, a 25 pound magnet-rod was erected about 2 metres South of the magnetometer, with its South-pole facing North. The magnetism of this rod balanced a large part of the Earth's magnetic force in the needle and thereby increased the sensitivity of the magnetometer, thus achieving the same purpose as an astatic device.[60] The needle's period of oscillation[61] was previously 10 seconds and was increased to about 20

[59][Note by AKTA:] [Web39a] with English translations in [Web41b], [Web66a] and [Web21k].

[60][Note by AKTA:] The adjective "astatic" is used in physics with the meaning of something having no tendency to take a definite position or direction. An astatic needle can be a combination of two parallel magnetized needles having equal magnetic moments, but with their poles turned opposite ways, that is, in antiparallel position. The arrangement protects the system from the influence of terrestrial magnetism. It was invented by Ampère, [Amp21] and [LA98]. An earlier system composed of a single magnetized needle had also been created by Ampère, [Amp20c, p. 198] with Portuguese translation in [CA09, p. 133], [Amp20a, p. 239] and [Amp20b, p. 2], see also [AC15, p. 57].

[61][Note by AKTA:] In German: *Schwingunsdauer*. Gauss and Weber utilized the old French definition of the period of oscillation t which is half of the English definition of the period of oscillation T, that is, $t = T/2$, [Gil71a, pp. 154 and 180]. For instance, the period of oscillation for small oscillations of a simple pendulum of length ℓ is $T = 2\pi\sqrt{\ell/g}$, where g is the local free fall acceleration due to the gravity of the

seconds through these means.

5.2.4 Connection of the Wire Ends with the Rotating Magnet

One end of the multiplier wire which was intended to be conductively connected to the rotational axis, was tied to the iron clamp on which the gears were screwed and into which ran the spike, which formed the end of the rotational axis. The other end of the multiplier wire was submerged in a tray of mercury which was placed underneath the rotating magnet. A brass disk was sitting around the centre of the magnet, rotating with it and its lower end running through the tray of mercury.[62] This way the magnet's rotation was not obstructed by being connected to both ends of the multiplier wire.

5.2.5 The Inductor Coil

A piece of the same sort of over-woven copper wire[63] as the multiplier was made of, was wound around a wooden ring of 44 mm diameter with 20 windings. This ring was used as an inductor coil. The resistance was small enough to be negligible compared to the larger resistance of the multiplier; therefore, the currents induced with the same magnet, sometimes by rotation, sometimes by the alternation of this coil, directly measure the magnitude of the induction.

5.3 Experiments

The magnets used for the following tests were, like all magnets, not equally magnetized throughout all their parts, but instead stronger in the middle and weaker towards the ends. Thus, they were not fulfilling the requirements laid out in the previous conditions. Similarly, no magnet can be manufactured that exactly fulfils these requirements. If, therefore, in these experiments, one has to be content with rods which are often very far away from a very uniform magnetization, one cannot expect that the previously-mentioned theorems will be directly and accurately applied to these experiments, and that the strength of the induced currents can be correctly and accurately predetermined from them. The previous laws can and should only be used under these circumstances to get an idea of the intensity of induced currents, or at least estimate the magnitude that can be expected. Only a *limit* of current intensity is given by these laws to which the induced currents come close but will never reach with an unevenly magnetized cylinder. Thus, the closest aim of the following tests is to check if a current can be generated by the described means at all, more so, if the current intensity is of the expected magnitude as the intensity of a current induced by described alternation, and finally if, as expected according to [theorem] (8.),[64] that current is exceeded by this one. If these questions are answered affirmatively by the following experiments, an attempt will finally be made to alter the external conditions of the *former* induction in such a way that the current produced approaches the limit indicated more closely, and even surpasses the other, hitherto stronger, current. The reason why the induced current is not

Earth, while $t = T/2 = \pi\sqrt{\ell/g}$.

[62][Note by AKTA:] See Figure 4.3.

[63][Note by AKTA:] In German: *übersponnenen Kupferdrahtes*. The coating insulates electrically the copper wire.

[64][Note by AKTA:] See theorem 8 on page 33.

reaching the previously specified limit, namely that the magnet is weaker towards the ends compared to its *centre*, can be partially or fully alleviated by approaching larger magnets, which will increase the magnetism of the *ends*, while almost not influencing the magnetism in the *centre*. Supposing that magnetism in the *centre* remains completely unaltered by this, and now being the weakest throughout the entire rod instead of previously the strongest, there would result a current which can never be lower than the previously specified limit; transforming the upper bound into a lower bound. It is easy to see that this largely depends on the length and initial magnetism and the softness of the steel cylinder. With short cylinders the magnetism would not only be amplified at the ends, but also in the centre, increasingly so the weaker the magnet initially was. With long cylinders the magnetism in the centre will be barely affected or not at all at considerable distance from the ends to the centre. From this we can expect that, (1) rotating a short, weakly magnetized cylinder between two fixed magnet-rods to reinforce its ends, will induce a current which *surpasses* the previously specified limit; however, (2) approaches it *more closely*, the *stronger* the cylinder gets magnetized; (3) if the same cylinder is rotated freely, *without* the presence of other magnets, the induced current will *not reach* the specified limit, but get ever closer, the stronger the cylinder gets magnetized; yet even at the highest point of saturation it cannot be reached, even if stronger magnetisation will smooth the unevenness of magnetism of centre and ends, however, it cannot remove the unevenness. (4) Rotating a very *long* cylinder, even if it is strongly magnetized, it is to be expected that the induced current will *never reach* the previously specified limit, and can only weakly be alleviated by external magnets slightly reinforcing the rod's ends; since it is to be expected that the area of influence of the latter is not reaching a respectable distance away from the ends and will not suffice to amplify the magnetism of all parts of the magnet so that they would be equal to the centre of the magnet. To verify this, the following sets of experiments are performed.

5.3.1 First Set

Rotation of a short and weakly magnetized cylinder. Its ends were reinforced through the presence of external magnets.[65]

The cylinder was 269 mm long and 23 mm thick, it's magnetic moment according to absolute measure = 65 Million.

[65][Note by AKTA:] In German: *Die Enden wurden durch magnetische Vorlagen verstärkt*. The expression *magnetische Vorlagen* can also be translated as magnetic templates or magnetic plates.

60 revolutions in 7 seconds					
Rotation forwards			Rotation backwards		
616.3			743.0		
	623.3			736.0	
626.8			732.5		
	622.1			737.0	
619.7			739.2		
	621.9	622.56		736.3	736.54
623.0			734.8		
	623.1			737.5	
623.2			738.8		
	622.4			735.9	
622.0			734.5		
617.0			734.2		
	623.1			736.7	
626.2			738.0		
	622.5			737.0	
620.7			736.5		
	622.2	622.02		737.2	737.12
623.0			737.5		
	621.0			737.2	
620.0			737.0		
	621.3			737.5	
622.0			737.8		

The *first* column shows the observations of maximum and minimum values of magnetometer readings during rotation; the *second* column shows the calculated real value consisting of two observations including damping considerations: the second observation is approximated towards the first by one third of the difference; the *third* column is the mean of the 5 readings in the previous column. If the values of the third column are put together, the differences of the readings, alternating between forwards and backwards rotation, give double the deflection caused by the induced current

forwards	622.56		
		113.98	
backwards	736.54		
		114.52	114.53
forwards	622.02		
		115.10	
backwards	737.12		

Through the same method, the double of the deflection for 30 rotations in 7 seconds is found as

$$= 56.52\ ,$$

which is nearly half of the previous. According to this we can assume on average 57.02 as the single deflection of 60 rotations or double deflection of 30 rotations in 7 seconds, giving

6.652 as single deflection of 1 rotation in 1 second, or 13.304 of 2 rotations in 1 second. For comparison this magnet was also used for the induction experiments described in the previous Volume of the *Resultate*, p. 98 ff.[66,67] It should be noted that the period of oscillation of the magnetometer's needle was 20.5 s and the coil had 20 windings. The external magnets had to be removed for these tests. It will suffice to collate the observation of elongations, without specifying the alternation of set-ups described *in the place cited*.

Elongations	a	b
643.0		
637.0	8.2	
651.2	...	17.0
654.0	9.2	
642.0	...	16.0
638.0	9.0	
651.0	...	15.2
653.2	8.0	
643.0	...	15.7
637.5	7.8	
650.8	...	16.7
654.2	8.6	
642.2	...	15.2

Elongations	a	b
639.0	8.5	
650.7	...	16.5
655.5	8.7	
642.0	...	17.7
637.8	8.5	
650.5	...	16.7
654.5	8.7	
641.8	...	16.3
638.2	8.2	
650.0	...	15.3
653.5	8.0	
642.0	...	15.5
638.0		

On average this yields

$$a = 8.5 \ ,$$
$$b = 16.15 \ ,$$
$$\frac{a^2 + b^2}{\sqrt{ab}} = 28.44 \ .$$

If the last value is multiplied by $\frac{t}{\pi n}$, where t is the oscillation period of the magnetometer's needle (= 20.5 s), n the number of windings in the inductor coil (= 20), the single deflection is

[66][Note by ER:] Wilhelm Weber's Werke, Vol. II, p. 115

[67][Note by AKTA:] [Web39b, p. 115 of Weber's *Werke*].

found which corresponds to 1 winding and 1 alternation in 1 second, = 9.279. If we compare the deflection obtained previously for 2 rotations in 1 second = 13.304; it is apparent that the induced current responsible for the deflection is stronger than the one causing the first deflection according to expectation (see the previous item (1)).[68]

5.3.2 Second Set

Rotation of a short, strongly magnetized cylinder.

Its ends were reinforced through the presence of extra magnets.

The cylinder was 269 mm long, 23 mm thick; its magnetic moment according to absolute measure = 108 Million. Because the experiments were carried out just like the previous ones, it suffices to present the results. The double deflection of 60 rotations in 7 seconds was found to be

$$= 152.50 \ ,$$

and of 30 rotations in 7 seconds

$$= 76.61 \ .$$

On average 76.37 can be assumed as single deflection of 60 rotations or double deflection of 30 rotations in 7 seconds, giving 8.91 as single deflection of 1 rotation in 1 second, or 17.82 for 2 rotations in 1 second.

For comparison the tests with the 20 windings inductor coil were repeated, where the oscillation period of the needle was $t = 21.44$ s. Which yields

$$\begin{aligned} a &= 14.22 \ , \\ b &= 26.94 \ , \\ \frac{a^2+b^2}{\sqrt{ab}} &= 47.412 \ . \end{aligned}$$

If the last value is divided by $\frac{n}{t}\pi = \frac{20}{21.44} \cdot 3.14159..$, the deflection corresponding to 1 winding and 1 alternation in 1 second is found as

$$= 16.178 \ .$$

Comparing this to the deflection found previously for 2 rotations in 1 second

$$= 17.82 \ ,$$

it is apparent, that the induced current causing the latter deflection is barely any stronger than the one causing the first deflection, in accordance with expectation (see the previous item (2)).[69]

[68][Note by AKTA:] Item (1) on page 37.
[69][Note by AKTA:] See item (2) on page 37.

5.3.3 Third Set

Rotation of a short, strongly magnetized cylinder without the presence of extra magnets.

The cylinder itself was unchanged from the second set.

The double deflection for 60 rotations in 7 seconds was found to be

$$= 64.33 \ ,$$

for 30 rotations in 7 seconds

$$= 31.83 \ .$$

On average 32.05 can be found as the single deflection for 60 rotations or double deflection for 30 rotations in 7 seconds, giving 3.74 as single deflection for 1 rotation in 1 second, or 7.48 for 2 rotations in 1 second.

If one compares this result with the deflection, which according to the previous series for the same magnet corresponded to 1 winding of the inductor coil and 1 alternation in 1 second,

$$= 16.178 \ ,$$

it can be seen that the induced current, causing that deflection = 7.48, is in accordance with expectations weaker than the one causing this deflection = 16.178 (see the previous item (3)),[70] however, it is still of the same magnitude, so that according to [Theorem] Number 8[71] it seems justified to deduce the difference in reading from the considerable difference which takes place between the magnetism of the middle and end parts in such a rod, the ends of which are not reinforced by any extra magnets.

5.3.4 Fourth Set

Rotation of a long, strongly magnetized cylinder.

Its ends are reinforced through the presence of external magnets.

The cylinder was 502 mm long and 20.5 mm thick, its magnetic moment according to absolute measure = 450 million. The double deflection for 60 rotations in 7 seconds was found to be

$$= 194.22 \ ,$$

for 30 rotations in 7 seconds

$$= 97.85 \ .$$

On average 97.36 can be found as a single deflection for 60 rotations or the double deflection for 30 rotations in 7 seconds, giving 11.36 as single deflection for 1 rotation in 1 second, or 22.72 for 2 rotations in 1 second.

[70] [Note by AKTA:] See item (3) on page 37.
[71] [Note by AKTA:] See theorem 8 on page 33.

For comparison the induction tests with the 20 windings coil were repeated as well with this magnet. The oscillation period of the needle was $t = 22.34$ s. Which yields

$$a = 28.76\ ,$$
$$b = 57.69\ ,$$
$$\frac{a^2 + b^2}{\sqrt{ab}} = 102.01\ .$$

If the last value is divided by $\frac{n}{t}\pi = \frac{20}{23.34} \cdot 3.14159..$, the deflection corresponding to 1 winding and 1 alternation in 1 second is found as

$$= 36.27\ .$$

Comparing this to the deflection found previously for 2 rotations of the cylinder in 1 second

$$= 22.72\ ,$$

it is apparent that the induced current causing the latter deflection with this long cylinder even in the presence of amplification of its outermost ends does not equal the current induced through the first method, causing a deflection = 36.27, as was postulated (see the previous item (4)).[72]

5.3.5 Fifth Set

In the experiments described so far, the brass disk running through mercury was always situated on the *centre* of the magnet; however, in the following experiments it was moved to the *end* of the cylinder to confirm that the length of the parallel path which the induced current has to travel inside the magnet parallel to the rotational axis has no influence on the current intensity. The current was first conducted at the end of the rotation axis that was further away from the brass disk and then at the end of the rotation axis closest to the brass disk.

Cylinder and extra magnets remained unchanged from the previous set.

1. Contacting the *far* end of the rotational axis.

Double deflection for 30 rotations in 7 seconds was found to be

$$= 57.12\ .$$

2. Contacting the *close* end of the rotational axis.

Double deflection for 30 rotations in 7 seconds was found to be

$$= 59.08\ .$$

Comparing these results it is self-evident that the induced current through the longer path it had to travel parallel to the rotational axis in the cylinder was at least *not* amplified. The difference between these results is too small to justify the conclusion of the opposite assertion.

[72][Note by AKTA:] See item (4) on page 37.

5.3.6 Sixth Set

Rotation of a long, strongly magnetized cylinder without external magnets.

The cylinder is the same as in the previous two sets; the brass disk running through mercury was situated in the centre of the magnet. The double deflection for 30 rotations in 7 seconds was found as

$$= 61.70 ,$$

yielding 7.20 as single deflection for 1 rotation in 1 second, or 14.40 for 2 rotations in 1 second.

Comparing this result with the deflection, which according to the fourth set for the same magnet corresponded to 1 winding of the inductor coil and 1 alternation in 1 second,

$$= 36.27 ,$$

one can see that the induced current, which produced that deflection = 14.40, is much weaker than the one causing this deflection = 36.27, as was also assumed under the prevailing circumstances (see above under (4)).[73]

5.3.7 Seventh Set

The experiments of the previous set were repeated by moving the brass disk running through mercury to the end of the cylinder, to confirm the result of the fifth set, where no external magnets were used.

1. Contacting the *far* end of the rotational axis.
Double deflection for 30 rotations in 7 seconds was found as

$$= 20.44 .$$

2. Contacting the *close* end of the rotational axis.
Double deflection for 30 rotations in 7 seconds was found as

$$= 21.66 .$$

Comparing these results again shows that the induced current through the longer path travelling parallel to the rotational axis in the first case was *not* amplified.

5.4 Applications

5.4.1 Application to Ampère's Electrodynamic Theory of Magnetic Phenomena

The phenomena of unipolar induction find an interesting application, first of all, to Ampère's electrodynamic theory of magnetic phenomena,[74] or to the question whether physical existence must be attributed to the two magnetic fluids, or whether the assumption of continuous galvanic currents inside the magnets is sufficient to explain the phenomena. To explain

[73][Note by AKTA:] See item (4) on page 37.
[74][Note by AKTA:] See footnote 15 on page 18.

unipolar induction the latter assumption does not seem to suffice, while the assumption of the physical existence of two magnetic fluids not only seems to provide this explanation, but also first led to the investigation of these phenomena.

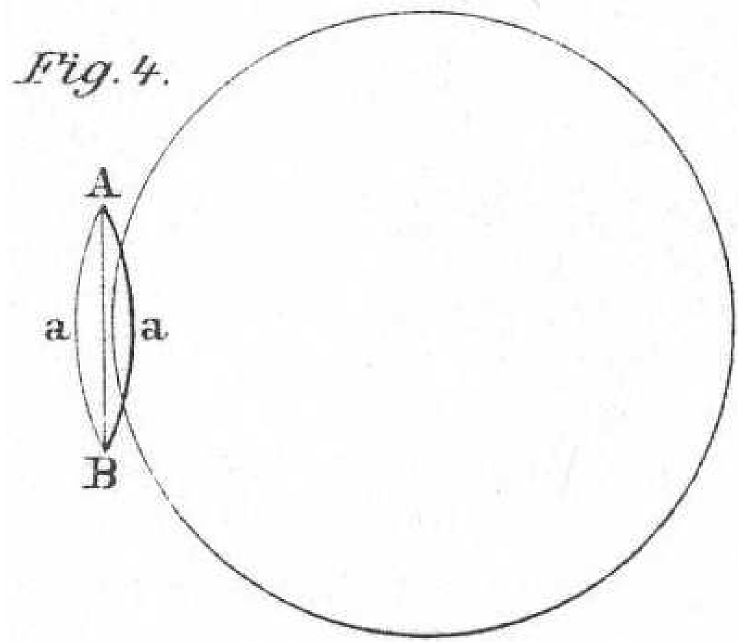
If one wanted to derive an explanation for phenomena titled with the name of unipolar induction from Ampère's electrodynamic theory of magnetic phenomena, the attempt would fail since according to Ampère galvanic currents can only be decomposed into such elements which attract or repel along the straight line joining those elements. It is evident from this that a current element in the plane of a ring cannot be moved perpendicularly against the ring by a current in the ring, and conversely, that such a movement of the current element cannot induce a current in the ring. The vital question of unipolar induction seems to consist in the fact that an induction is happening in the moment where the inducing element is present in the plane of the ring, because, if in this moment the induction is zero, a transition from positive to negative or vice versa takes place. The characteristic trait of unipolar induction, however, is rooted in the fact that such a transition never occurs. Thus, it seems futile to search an explanation for unipolar induction in Ampère's electrodynamic theory, at least so long as the decomposition of galvanic currents is limited to such elements that attract or repel along the straight line joining them.

The fruitlessness of this endeavour is made increasingly visible, if the beautiful theorem first proved by Ampère and stated in the previous Volume of the *Resultate*, p. 51,[75,76] is considered, with which the magnetic effects of galvanic currents can be defined. With this theorem — that instead of any linear current confining an *arbitrary* surface, a distribution of the magnetic fluids on both sides of that surface in immeasurably small distances from it can be substituted with the intended effect, — it should *first* be regarded that when a linear closed current is given, infinitely many confined surfaces can be thought of; *secondly* that what is true of the action of the current can only be true of the action of the magnetic fluids distributed on all those surfaces in common: in other words, that in this representation nothing may be inferred from the distribution of the magnetic fluids on one of those surfaces which does not also follow from the distribution on each of the other surfaces. Now, imagine the plane of a small circular conductor, through which a continuous galvanic current is flowing, which is replaced by a magnetic element according to Ampère's hypothesis, perpendicular to the plane of Figure 4, and let AB be the diameter of the circle; imagining to both sides at immeasurably small distances from the circular plane in AaB and $Aa'B$ the distribution of the northern and southern fluid; this conductor can be moved in such a way that the magnetism at a is moving through the inductor ring, while a' always stays outside.[77] It is easy to see, however, that something has been concluded from the distribution of the magnetic fluids on the plane of the circle which would not follow from the distribution of the same on any other surface bounded by the same circle, and which consequently cannot apply to the effect of the galvanic current in that circle.

[75][Note by ER:] Gauss' Werke, Vol. V, p. 169.

[76][Note by AKTA:] See footnotes 15 and 41 on pages 18 and 27, respectively. See, in particular, [Gau39, Section 37, page 51 of the *Resultate* and page 169 of Vol. V of Gauss' *Werke*] with English translations in [Gau41a, Section 37, pp. 229-230] and [GT14, Section 37, p. 39]. See also [AC15, Section 10.7: Equivalence between a Magnetic Dipole Layer and a Current-Carrying Closed Circuit].

[77][Note by AKTA:] In Figure 4 the left letter a should be replaced by a'. The northern fluid would be distributed on AaB, while the southern fluid would be distributed on $Aa'B$. The inductor ring is represented by the circle in the plane of Figure 4. This ring is orthogonal to the circular conductor with diameter AB where flows a constant current.



5.4.2 Application to the Distribution of Magnetism Inside Permanent Magnets

All effects of magnets which are usually observed are effects in external space, from which, as is well known, no definite result can be drawn about the distribution of magnetism in the interior [of the magnets]. There are rather countless ways in which internal distributions of magnetism can be assigned, which are all identical in relation to the effects. Among those is one, where no magnetism at all is situated inside the magnet, but all along the surface. There has been only *one* experiment by which something has been learned about the distribution in the interior and, in particular, it has been recognized that the latter type of distribution, namely on the surface, does not occur in nature; this is the experiment in which a magnet is *broken apart*.

But we have now gotten to know effects of a magnet through *unipolar induction* which act on the fluids on its *interior*, being set into flowing motion. It appears as an application from unipolar induction that the inner distribution of magnetism can be investigated without breaking the magnet. Even if it is not possible to know the distribution fully through this method, it is still very important to obtain only a few new insights about it.

From the point where the conducting wire is touching the cylindrical surface of the magnet to the end of the rotational axis in contact with the conducting wire, a path exists inside the magnet for galvanic current where induction is the *weakest*. If the cylinder is rotated, that path is changing in general and will describe a curved area over a whole rotation bisecting the cylinder into two parts like a cross section. The free magnetism within this area has a ratio to the average of free magnetism within an arbitrary cross section in the first investigated cylinder, according to the *third* set of 7.48:16.178, in the second cylinder according to the *sixth* set of 14.40:36.27. The induced current (which for the *shorter* magnet caused a deflection of = 16.178 scale divisions if repeated every second, for a *longer* magnet = 36.27) by *alternation* of an inductor (consisting of 1 winding) gives a measure of the average amount of free magnetism of all cross sections of the cylinder, while the current induced by 2 *rotations* of the cylinder (for the *shorter* magnet with rotations every 2 seconds caused a deflection of = 7.48 scale divisions and for the *longer* = 14.40) gives according to the sixth theorem, page 32,[78] a measure of the minimum of free magnetism which is contained in those curved cross sectional areas which can be described by the paths of the galvanic current in the rotating cylinder.

Looking at the results of the *fifth* or *seventh* set, where the current was diverted from the surface of the cylinder not in the middle, as in the other series of tests, but at the end, it is found (as pointed out previously) that the result is almost the same, for the galvanic current having to cross the entire length of the cylinder to reach from its point of entry to its point of exit, as well as not having to cross the length of the cylinder — that is, in other words, the two minima of free magnetism contained in those cross sections which can be described by the different paths during the cylinder's rotation, taken by the galvanic current from the contacted point on the surface either towards the *close* or the *far* end of the rotational axis are nearly the same, which leads to the conclusion that the galvanic current passes from the surface to the rotational axis only at its *entry* and *exit* (that is, here at the ends of the cylinder which are magnetized with nearly equal strength).

Comparing the results of the *fifth* and *seventh* set and considering that in the former magnetism in the ends (where the galvanic current crosses) was strongly reinforced through the use of external magnets, but not in the latter, the difference which one finds will not be noticeable, namely, that the measured deflection in the first case is nearly three times larger than the latter, or expressed by the ratio 58.10:21.05. It is interesting, however, to note that the former result, that is, 58.10, is close, but not identical to the one obtained in the sixth set, that is, 67.10, disregarding the amplification of the cylinder's ends (where the galvanic current crossed) through the use of external magnets, — a proof that this amplification is far from making the magnetism of those ends equal to the magnetism of the *centre* from which the galvanic current was conducted in the sixth series of experiments.

Further consideration of this application will need to be kept to a time in the future.

5.4.3 Application to the Distribution of Magnetism in Soft Iron

A special difficulty so far was the investigation of the distribution of magnetism in soft iron. The iron will only form a stronger magnetism if it touches a magnet or is brought at least into very close proximity, where we lack the means to distinguish the effects stemming directly from the iron and stemming from the magnet, all the more so because the latter cannot be regarded as constant because the magnet undergoes a change due to the reaction of the iron. However, unipolar induction now presents such a procedure. When the magnet is stationary and only the iron is rotated, an induction is caused solely by the magnetism of the iron, and if only the magnet is rotated an induction is caused solely by the magnet. Finally, when both are rotated together, it is possible to determine the magnetism of that cross section within the iron where magnetism is weakest (at the end facing away from the magnet).

5.5 Conclusion

It is known that nearly all magneto-electric experiments have electromagnetic counter experiments. One can therefore assume that there will also be such a counter experiment for our

[78][Note by AKTA:] Page 70 of the *Resultate* or page 158 of Weber's *Werke*, [Web40], which are equivalent to page 32 of this English translation.

experiment, which was first made by Faraday.[79] This is indeed the case. This counter experiment will not need to be performed, since it has already been performed and it has been known for a long time. This counter experiment obviously consists in the fact that, instead of rotating the magnetic cylinder and thereby inducing a galvanic current in the conducting circuit, a galvanic current is passed through the circuit in the opposite direction, where the magnet then begins to rotate by itself in the same direction in which it was previously rotated.[80] If one had researched this long-known phenomenon in more detail, this path would have easily led to the herein investigated *unipolar induction*, which to my knowledge has not been done. This long-known experiment seems also to contradict Ampère's hypothesis, that there are no magnetic fluids, but rather continuous galvanic currents exist inside the magnet; moreover this phenomenon also seems to only be explained by the real existence of two spatially separate magnetic fluids.

W.[81]

[79][Note by AKTA:] See footnote 18 on page 19, and Section 4.2.

[80][Note by AKTA:] This experiment was first performed by Ampère in 1822 and will be discussed in Chapter 6.

[81][Note by AKTA:] That is, written by Wilhelm Weber.

Chapter 6

Editor's Introduction to Weber's Second Paper on Unipolar Induction

A. K. T. Assis[82]

In his second paper on unipolar induction Weber made some additions and amendments related to his previous publication.[83]

In 1821 Faraday showed that the extremity of a straight piece of wire carrying a constant current could rotate around the vertical axis of a cylindrical magnet, while the other extremity of the piece of wire remained stationary along the axis of the magnet. He also showed that it was possible to rotate the extremity of a cylindrical magnet around a vertical piece of wire carrying a constant current, while the other extremity of the magnet was located along the axis coinciding with the vertical piece of wire, Figure 6.1.[84] This phenomenon is usually known as Faraday's motor and should not be confused with Faraday's unipolar induction experiment of 1832.

In 1822 Ampère showed experimentally and theoretically that it was possible to make a cylindrical magnet turn around its axis utilizing constant electric currents. He considered a magnet with its North and South poles along the vertical direction, with the N pole above the S pole. To turn the magnet, he connected a battery between the center of the upper face of the vertical magnet and a point in its lateral edge utilizing conducting contacts. When an electric current was made to flow through the magnet, it rotated around its axis. This phenomenon is known as Ampère's motor. Alternative names for this device are homopolar motor or the world's simplest motor.[85] Nowadays it can be easily reproduced utilizing a neodymium's magnet, a nail, a piece of wire and an ordinary battery, Figure 6.2.

In this paper of 1841 Weber compares Ampère's motor with Faraday's unipolar induction

[82]Homepage: www.ifi.unicamp.br/~assis

[83][Web40] and [Web41e], both of which are translated in this book.

[84][Far22b] and [Far52c] with French translation in [Far21], [Far22a] and [Far52a].

[85][Amp22a] with partial English translation in [Amp69a], [Amp22b] and [Amp22b]. See also [Blo82, pp. 114-115], [GVM01], [GVMA02], [GV02], [AGV07], [Cha09], [AC11], [AC12], [CA13] and [AC15, Section 7.2.3: Rotation of a Magnet around Its Axis].

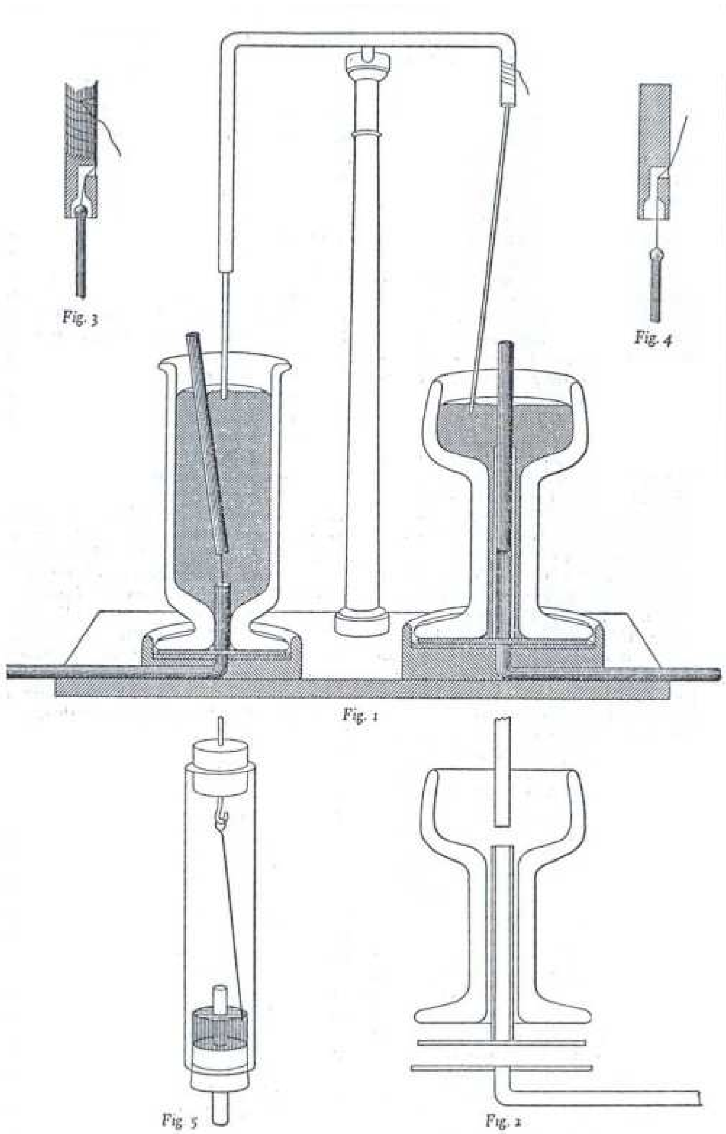


Figure 6.1: Faraday's motor.

experiment. In particular, he considers Faraday's experiment of unipolar induction in which a cylindrical magnet was rotated relative to the ground and this produced an electric current indicated by the galvanometer connected by sliding contacts between the center of the upper face of the vertical magnet and a point in its lateral edge.

Most portions of Weber's 1841 paper are identical with his earlier paper published in 1840. When Weber's second paper was reprinted in his Collected Works, the identical parts were not included.

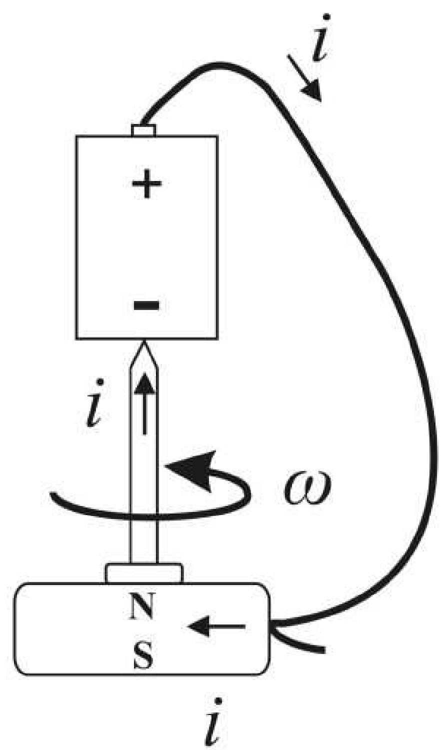


Figure 6.2: Ampère's motor.

Chapter 7

[Weber, 1841] Unipolar Induction

Wilhelm Weber[86,87,88,89]

[Excerpts.][90,91]

In a "Note about the interaction of a magnet and a galvanic conductor" contained in Volume 37 of *Annales de chimie et de physique*,[92] Ampère gave an account of the rotation of a galvanic conductor around the axis of a magnet as discovered by Faraday, and has tried to connect this to the explanation of the rotation of a magnet around its own axis when a galvanic current is passed through it as discovered by Ampère.[93]

Ampère first shows that according to the general laws he established, a closed current, which *is not in a fixed connection* with the magnet, cannot rotate it around its axis, nor vice versa (if all the parts of the current conductor are firmly connected to each other) can it be set in rotation by the influence of the magnet.[94] However, if a portion of the closed current lies within the magnet, he says this part will form a *fixed system* with the magnet, and action and reaction would have to cancel out. The remaining action is only the portion of the galvanic current which is not fixed to the magnet, and because that current is *not*

---

[86][Web41e].

[87]Translated by C. Baumgärtel, Department of Electrical Engineering and Electronics, University of Liverpool, Liverpool, L69 3GJ, United Kingdom, ORCID: 0000-0002-0702-0480. Edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

[88]The Notes by Johann Christian Poggendorff, the editor of the *Annalen der Physik und Chemie*, are represented by [Note by JCP:]; The Notes by E. Riecke, the editor of the second Volume of Weber's *Werke*, are represented by [Note by ER:]; the Notes by C. Baumgärtel are represented by [Note by CB:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[89][Note by JCP:] From the "Resultate des magnetischen Vereins" (Volume 4), with some additions and amendments by the author [that is, by W. Weber].

[90][Note by ER:] Annalen der Physik und Chemie, Vol. 52, p. 353-386.

[91][Note by AKTA:] [Web41e].

[92][Note by AKTA:] [Amp28].

[93][Note by AKTA:] In 1828 Ampère was comparing his own motor of 1822, the modern version of which is presented in Figure 6.2, with Faraday's motor of 1821 shown in Figure 6.1. Ampère was obviously not discussing Faraday's 1832 experiment of unipolar induction described by Figure 4.1.

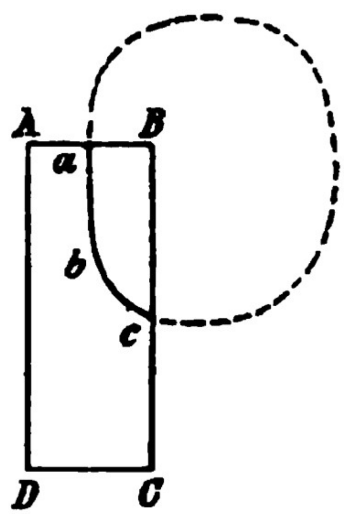
[94][Note by AKTA:] That is, under these conditions, the closed circuit cannot rotate the magnet around the axis of the magnet. Likewise, under these conditions, the magnet can also not rotate the closed circuit around one axis of the closed circuit.

closed, it will, in general, cause the magnet to rotate around its axis. He notes that it does not matter if the galvanic current flows *through the magnet* or if a portion of the conductor is rigidly connected with the magnet.

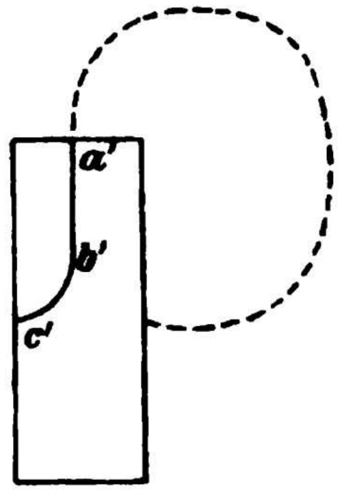
These two phenomena which Ampère tries to explain through the *same* reasons, are however, different in nature and each beg their own explanation. The explanation given by him only fits the rotation discovered by Faraday, but is not applicable to the one discovered by himself.

The difference between both phenomena is easily explained with the following.

Let $ABCD$, Figure 1, be the long cross section of a magnet, *abc* an insulated galvanic conductor guided through the magnet, the continuation of which outside the magnet is denoted by the dashed line.



In reality, this Figure depicts, as presupposed by Ampère, in the first place, the entire galvanic conductor as a closed curve, in the second place, the portion *abc* as a fixed system with the magnet. These two conditions which are fulfilled in Figure 1 are no longer valid, when the magnet is rotated around its axis. The insulated conductor *abc* will remain fixed within the magnet, but its endpoint c will describe a circle around the rotational axis and therefore be separated from the remaining circuit, as shown in Figure 2, at least if the magnet is not surrounded with a conducting belt, which the end c is constantly touching during its rotation. When such a connection is in place, there will be, additionally to the two conductor parts distinguished previously, a third part, that is, the piece of the belt which enables the connection between c and the dashed conductor, which is substantially different from the other two [parts] by its variable length. This conduction belt was utilised by Faraday through the use of mercury.



Figur 2.

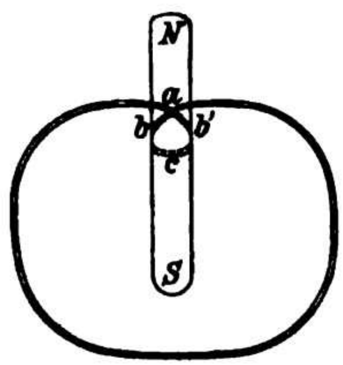
With Ampère's experiment, however, where *no insulated conductor abc* is present in the magnet, but instead the current is moving *freely* through the magnet from a to c, the condition which Ampère presupposes is not applicable, that the portion of the current moving through the magnet is acting exactly similar during rotation of the magnet as a current would, flowing through the insulated conductor *abc*, that is forming a *fixed system* with the magnet, and consequently would have to partake in the rotation, but cannot cause it. Magnetism and galvanic current, even if they are present in the same carrier (the steel cylinder) do not form a *fixed system*; since only the magnetism adheres to the steel molecules, and can only move *in conjunction* with them. The galvanic current, however, does not adhere to the steel molecules, but rather can move freely and independently inside the magnet *in all directions*. It is apparent from this, that this current and this magnetism are not allowed to be regarded as *rigidly connected bodies* rotating with the steel cylinder whose interaction would have to nullify each other, contrary to Ampère's claim. If however, the magnetism is bound to the steel molecules, but not the traversing galvanic current, it follows that the current could move the steel cylinder through magnetism, but not vice versa. This however, removes the reason attributed by Ampère to explain why a magnet rotates when a galvanic current *freely* flows through it, and this phenomenon, which exists without a doubt due to experimental proof, would according to Ampère even seem *impossible*, which is however, not the case if the *physical existence of magnetic fluids* is allowed to be presupposed in the steel molecules, instead of Ampère's hypothetical currents, as is to be demonstrated by the following experiments. The phenomena investigated herein are the *induction phenomena* discovered by Faraday, which form the analogue of the previous electromagnetic phenomena, and which can easily be applied to the latter.[95]

It does not follow from this, however, that a continuous homogeneous induction, such

[95][Note by AKTA:] The paper in the *Annalen der Physik und Chemie* continues here just as in the original publication in the *Resultate aus den Beobachtungen des magnetischen Vereins*. However, when this paper of the *Annalen der Physik und Chemie* was reprinted in Volume 2 of Weber's *Werke*, the common portion of both papers were not reproduced, as it had already appeared in the previous reprint of the paper published in the *Resultate aus den Beobachtungen des magnetischen Vereins*. We are here following this approach of Weber's *Werke*. The next 3 paragraphs should be compared with the corresponding paragraphs of page 29.

as can be produced by a magnet that contains only *one* fluid, is impossible with a magnet containing *both* fluids, rather, there remains a *third* case to be considered, which is not yet included in the previous two, and is possible if *really* magnetic fluids *exist* and are really *spatially separated* from one another in the molecules of the magnet, that is

3. a magnetic molecule is moved in such a way that it does not pass through the ring completely or not at all, but *half* through it and *half* remains outside it, for instance that the half containing *northern* fluid goes down through the ring, upwards around the outside, or vice versa; but the other half containing *southern* fluid always remains outside. The total effect is then *non*-zero, since one fluid (which moved through the ring) has induced a current, which has *not* been nullified, since the other fluid (which did not move through the ring) has not induced any or only an inhomogeneous current whose combined effect vanishes in total. However, since the ring as well as the magnetic molecule are *solid bodies*, it appears that this third case is only possible if either one is *broken up*. Yet, a magnetic molecule cannot be broken in such a way that each part only contained *one* fluid, which would be necessary to move *one* single fluid through the unbroken ring, thus the *ring* has to be broken up, which is easily done: however, it must be noted that while breaking the ring the galvanic circuit must not be interrupted. The conductor can (1) be broken without interrupting the circuit, if the inseparable magnetic molecule is such that the galvanic current can flow right through *between both fluids*; since that molecule can *conductively connect* both parts of the conductor while it is being broken; (2) the ring can be broken without interrupting the circuit if the copper wire forming the ring is wound once around the centre of the magnet before the break, and, after the break, behind the magnet at c, the connection is maintained at a, Figure 3. During this connection in a, the cut wire ends abc and $ab'c$ can be passed through a and finally their endpoints can be reconnected.



Figur 8.

Of theses two methods, the first is to be investigated further here. It is easy to devise a set-up that meets the required conditions.

To conclude this paper some words of explanation about the phenomenon first discovered by Ampère, which was discussed in the Introduction, are offered.[96] It is known that nearly all magneto-electric experiments have electromagnetic counter experiments. The herein in-

[96][Note by AKTA:] This final paragraph of the paper in the *Annalen der Physik und Chemie* should be compared with the final paragraph of the paper in the *Resultate aus den Beobachtungen des magnetischen Vereins*, see Section 5.5.

vestigated phenomenon of unipolar induction belongs to the class of magneto-electric experiments, whereas the phenomenon discovered by Ampère as discussed in the Introduction belongs to the class of electromagnetic experiments, and further investigation yields that this phenomenon[97] has to be seen as the counter experiment of that,[98] and therefore is to be explained by the same means, so that the counter experiment can be demonstrated as a proof of the physical separation of the magnetic fluids. After the well-known reversal, according to which the induction laws are derived from the electro-magnetic ones, the above considerations can easily be applied to this counter experiment, and it seems superfluous to elaborate further here.

[97][Note by CB:] Ampère's phenomenon.

[98][Note by CB:] Unipolar induction phenomenon.

Chapter 8

[Weber, 1862] On Galvanometry

Wilhelm Weber[99,100,101]

Presented to the Royal Society of Sciences [of Göttingen] on January 4, 1862.

In the ever-expanding technical applications of galvanism, various proposals have already been made for the introduction of *galvanic resistance units* (etalons or standards)[102] in order to meet the manifold needs arising therefrom, and it is likely that the serious efforts made by experts will succeed in not only identifying and establishing, to the widest extent and in the most perfect way, the measurement rules corresponding to this purpose, but also to bring them to a practical and successful implementation as soon as possible.

All *galvanic cells*[103] used for chemical analysis, galvanoplastic, telegraphic and other technical purposes, even if they are called constant,[104] are continuously subject to smaller and often larger changes, which one must get to know more closely in order to master them. But even if these cells were completely unchangeable, their *actions* would now be larger and sometimes smaller, depending on the variety of applications that are made of them. To control these *actions* therefore requires not only knowledge of the cell itself, but also of all bodies through which the current of the cell is to pass, namely, knowledge of their *resistance*. Therefore, *resistance measurements* have become indispensable, and the need for them has become most urgent for telegraphic use.

However, a *resistance unit of measure* is required for the resistance measurements. Without measuring with such units, the bodies through which the current is to be conducted can be described in various ways; but after a measurement made with such units, *a single*

[99] [Web62].

[100] Translated and edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

[101] The Notes by Wilhelm Weber are represented by [Note by WW:]; the Notes by Heinrich Weber, the editor of Volume 4 of Wilhelm Weber's *Werke*, are represented by [Note by Heinrich Weber:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[102] [Note by AKTA:] In German: *galvanischer Widerstandsmaasse (Etalons oder Standards)*. The word "Maass", nowadays written as "Maß", can be translated as: measurement, measure, dimension, unit, standard, unit of measure etc.

[103] [Note by AKTA:] In German: *galvanischen Säulen*. This expression can be translated as a galvanic cell, pile, battery or element.

[104] [Note by AKTA:] That is, a cell supposed to produce a constant electromotive force between its terminals, or to produce a steady electric current when connected to a closed circuit containing a constant resistance.

number is enough to express everything essential more completely and more precisely than is possible through all descriptions. This is because the resistance measurements often reveal differences and changes in the bodies that cannot be recognized even from their most precise description.

Basically, such a *standard* was put into use at an early stage by comparing the various bodies through which currents were to be conducted with copper wires, the length and cross-section of which were measured. It is obvious that this was based, even if only tacitly, on the resistance of a copper wire with a length equal to the linear unit of measure and a cross-section equal to the area unit of measure, as *resistance unit of measure*. However, the explicit determination of a certain standard of resistance was first raised by Jacobi in [Saint] Petersburg in 1846.

Jacobi said:[105]

> It is no less important than the absoluteness of the current measurements that the physicists express the magnitude of the conducting resistance in terms of a common unit. But there can be no absolute determination here, because it seems that there are differences in the resistances of even the chemically purest metals that cannot be explained by a difference in dimensions alone. All of these difficulties are eliminated if you let a copper wire or other wire of your choice wander around the physicists and ask them to refer their resistance measuring instruments to it and in the future to only give their measurements according to this unit.

Of such a *resistance standard* chosen by Jacobi (a copper wire 25 English feet long and weighing $22\,337\frac{1}{2}$ milligrams) a number of copies were actually made and used for resistance measurements. However, be it that the necessary care was not taken in the production, or be it that such resistance standards suffer changes over time, very significant differences have later emerged between these copies.

Therefore, in 1860, Siemens in Berlin,[106] with special consideration of the increasingly urgent needs of technical physics, tried to set up a *new resistance standard* that would meet all requirements and be easily produced by anyone and with the necessary precision, based on the use of the resistance of *mercury*, as that metal which can be obtained or produced anywhere with great ease in sufficient, almost perfect purity and, as long as it is liquid, does not assume any other molecular properties that modify its conductivity, and is also less dependent on temperature changes than other metals in its resistance, and finally offers particular convenience for use due to the size of its specific resistance.

With the creation of this new *resistance unit of measure*, Siemens also combined the presentation of *resistance scales*, as necessary and indispensable mediators between the standard and the objects to be measured, and has constructed them to such an extent and perfection that all resistances can be formed with the greatest ease and accuracy, which according to his unit of measure can be expressed by integer numbers from 1 to 10 000.

Finally, in England too, there are currently plans to establish a certain standard of resistance and it is hoped to ensure its general distribution and application, as well as all the scientific and technical purposes that can be achieved thereby, by establishing an institution under the combined protection of the British Association and the Royal Society,

[105][Note by AKTA:] Moritz Hermann von Jacobi (1801-1874). A French version of this letter can be found in [Jac51].

[106][Note by AKTA:] E. W. v. Siemens (1816-1892), [Sie60] with English translation in [Sie61]. See also [GT19].

from which every experimenter in the whole world should, at his request, be provided with a *resistance standard*, which is not only valid for a precisely determined temperature, but also with an indication of its variation for a specific change in temperature, and whose *galvanic significance is finally given by a precise indication of the [electromotive] force required to excite a specific current in it.*

A long time ago I dealt with more precise measurements for this latter purpose, namely, to investigate the *galvanic significance of a conductor*, by determining the [electromotive] force required to produce a certain current, under the title of *absolute resistance measurements*. For example, the galvanic significance of the Jacobian resistance standard was established by stating that in order to excite in it a current intensity equal to 1 according to the Gaussian units,[107] an electromotive force according to Gaussian measure of 5980 million units is required.[108,109,110] I have a similar determination from another copper circuit which I submitted to the Royal Society [of Göttingen] in 1853.[111,112,113] These previous provisions, however, had been more concerned with the method and the significance of the results to be obtained with them than with the extreme finesse of the quantitative execution, which had been achieved only on a trial basis with the tools and instruments available for other purposes.

But if these absolute resistance measurements are to find further application, if they are to be used to give a *lasting* expression to all quantitative results of important galvanic observations and research, then a similar situation arises as with other fundamental determinations, namely, the need to carry out at least one absolute resistance measurement according to the strictest methods, with the most perfect instruments and with all the art of the finest observation. This is a task which can only be completely solved by very skilful hands, with the most undisturbed leisure and with more complete facilities than are now available for physical research. The fact that only one such measurement is required, but one which must be carried out with the greatest precision, is readily apparent from the fact that the resistances of all bodies can be accurately compared with the resistances of *a single standard*, and that therefore only a precise knowledge of the *absolute value of this single standard* is required in order to transfer the advantages of all relations given by absolute values to all bodies in general.

Apart from these advantages which the knowledge of the absolute value of such a standard resistance can confer, the task of this measurement also offers interest in itself because of the influence which it has on the development of science. The development of almost all of *galvanometry* can be linked to this task, and all advances in galvanometry can be tested in solving this task. Once the goal to be achieved has been defined after gaining insight into the possibility of the solution, every more perfect solution is almost more important as proof of the progress of galvanometry than for its own immediate benefit.

A finer design of absolute resistance measurement not only fills essential gaps in galvanometry, but also brings many scattered investigations into a closer context. Conversely, if a higher level of galvanometry were to be achieved by other means, the result would be

[107][Note by AKTA:] That is, a current intensity equal to 1 according to the absolute system of units proposed by Carl Friedrich Gauss (1777-1855).

[108][Note by WW:] Abhandhingen der Königl. Sächs. Gesellschaft der Wissenschaften, I, p. 252.

[109][Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. III, p. 351.

[110][Note by AKTA:] [Web52b, p. 351 of Weber's *Werke*] with English translation in [Web21e, p. 333].

[111][Note by WW:] Abhandlungen der Königl. Gesellschaft der Wissenschaften zu Göttingen, Vol. 5.

[112][Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. II, p. 319.

[113][Note by AKTA:] [Web53d, p. 319 of Weber's *Werke*]. See also [Web53a] and [Web53c].

a finer execution of absolute resistance measurement. We will now take a closer look at some of these *galvanometric investigations* that serve to carry out the absolute resistance measurement in more detail.

A distinction is made between *galvanometers* and *galvanoscopes*. The former, which include tangent galvanometers,[114] are only used for stronger currents, the intensity of which is thus obtained in terms of a precisely known absolute unit of measure. The latter, on the other hand, serve to observe the slightest traces of currents, of which nothing else can be perceived. The great sensitivity of the latter is only achieved by very tightly surrounding the [magnetized] needle with its multiplier,[115] which means that the more precise knowledge of the scale is lost, which arises automatically from the construction of the tangent galvanometer. In order to use such a galvanoscope for measurements, therefore, in addition to observing the deflection produced by the current, some other observation is required as a yardstick for the sensitivity of the instrument. As a rule, this yardstick is determined once and for all by making corresponding observations on the galvanometer and galvanoscope in advance. Apart from the fact that such corresponding observations do not produce an exact result because of the very different sensitivity of the two instruments, the yardstick of sensitivity for very sensitive galvanoscopes is generally not constant at all and therefore cannot be determined in advance. On the other hand, the observation of the *deflection* can be combined with another observation, namely, that of *vibration damping*, which directly provides this yardstick.

This connection makes it possible to use the most sensitive galvanoscopes for the most precise measurements, which is the necessary condition for carrying out absolute resistance measurements. However, galvanoscopes for this use require a construction that differs from ordinary galvanoscopes and the theory of which must be specially developed. This development is also of interest because it paves the way for the use of the most sensitive galvanoscopes in many other fine researches.

The present purpose therefore requires such a construction which allows the *deflection* and *damping* to be observed simultaneously with the greatest accuracy, whereas with ordinary galvanoscopes only the finest observation of the deflection was decisive for the construction. But what increases the deflection does not always increase the attenuation and vice versa. In addition, the deflection and attenuation must not exceed certain limits if they are to be capable of the finest determination. It is now the consideration of damping that particularly requires the use of strong magnets as galvanoscope needles, to which is added the need for a longer period of oscillation and a less variable resting point for the galvanoscope needle. This establishes the use of an astatic system formed by two strong magnets,[116] the period of oscillation of which is regulated by the length and strength of the metal wire used for suspension.

The absolute measurement of a *standard resistance* does not only depend on the accuracy of the *galvanometric measurements*, but also on the accuracy of our knowledge of the *Earth's magnetism in absolute value at the place and at the time of those galvanometric measurements*. The ultimate goal of galvanometric measurements is therefore that the in-

[114][Note by AKTA:] In German: *Tangenten-Boussole*. The tangent galvanometer was invented by Johan Jakob Nervander (1805-1848) and the sine galvanometer by Claude Servais Mathias Pouillet (1790-1868), [Ner33], [Pou37] and [Sih21]. Friedrich Kohlrausch discussed measurement of currents with the tangent and sine galvanometers, [Koh83, Chapters 64 and 65, pp. 188-192].

[115][Note by AKTA:] See footnote 52 on page 33.

[116][Note by AKTA:] See footnote 60 on page 35.

evitable uncertainty in the *absolute value of the standard resistance*, which arises from the determination of the Earth's magnetism, is not noticeably increased by the galvanometric measurement. Explaining and examining how this goal is to be achieved is the main purpose of this treatise, which will be followed by some discussion of the *copying* of resistance standards and other questions relating to the determination and meaning of the resistance standard.

I - The Method of Absolute Resistance Measurement

8.1 Ratio of an Electromotive Force to a Current Intensity

A galvanic current i, which is moved with its ponderable carrier with respect to a conductor with the velocity v, exerts an *electromotive force* e on the conductor according to the induction law discovered by Faraday,[117] which is proportional both to the intensity of the inducing current i and to the velocity of the inducing movement v. The ratio of this electromotive force to the product of the intensity of the inducing current in the velocity of the inducing movement, e/iv, therefore has a value that is independent of both the intensity i and the velocity v, namely, this value is determined from geometrically given relationships between the current carrier and the conductor as a *pure numerical value*, that is, independent of the spatial units of measure used for the geometric dimensions, as well as of the units of measure of the electromotive forces, current intensities and velocities. If one considers a single length element α of the inducing current i, which is moved with the velocity v with respect to the length element α' of the conductor, at the moment when the distance of both elements from each other $= r$, and denote the four angles which are determined by the directions of the two elements, $[\alpha]$ and $[\alpha']$, on the direction of their connecting line $[r]$ and on the direction of movement of the current element $[v]$, with $\vartheta = [r,\ \alpha]$, $\vartheta' = [r,\ v]$, $\varepsilon = [\alpha,\ v]$, $\varphi = [r,\ \alpha']$, then according to the well-known law applicable to voltaic induction,[118] the electromotive force e, which is exerted by the element α of the inducing current i on the induced element α', [is given by:]

$$e = iv \cdot \frac{\alpha\alpha'}{r^2} (3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon)\cos\varphi \ ,$$

or it is the ratio

$$\frac{e}{iv} = \frac{\alpha\alpha'}{r^2} (3\cos\vartheta\cos\vartheta' - 2\cos\varepsilon)\cos\varphi \ ,$$

[117][Note by AKTA:] See footnote 43 on page 28.

[118][Note by AKTA:] The expression utilized by Weber, *Volta-Induktion*, had been first suggested by Faraday himself in paragraph 26 of his first paper on electromagnetic induction of 1831, [Far32a, § 26, page 267 of the *Great Books of the Western World*] with German translation in [Far32b] and Portuguese translation in [Far11, p. 159]:

> For the purpose of avoiding periphrasis, I propose to call this action of the current from the voltaic battery, *volta-electric induction*.

In this English translation of Weber's 1862 paper we utilized the expressions *Volta-induction* and *voltaic induction* for this class of phenomena which is nowadays called Faraday's law of induction.

the value of which is hereafter obtained expressed in a *pure number*, since the ratios of two lines α/r and α'/r are pure numbers, as are the cosines of the angles.

If we now call those ratios of the current carrier and the conductor to each other, under which this number is $= 1$, the *normal ratios*, it follows that under these normal ratios the ratio of the electromotive force to the current intensity, e/i, is equal to the velocity v with which the current carrier is moved, or that

$$\frac{e}{i} = v \ .$$

In general, one can see from this that the quotient of any electromotive force divided by any current intensity is equal to any velocity, which is expressed by the following theorem: *an electromotive force is to a current intensity as a distance is to a time.*

The same theorem also follows directly from the concepts which in the theory of galvanism are associated with *electromotive forces* e and *current intensities* i.

If ε denotes the quantity of positive or negative electricity in one unit of length of the conductor according to the electrostatic unit of measure (in parts of that quantity which exerts on an equal quantity in one unit of distance a force which would give one unit of velocity to one ponderable unit of mass in one unit of time), and u the velocity at which the electricity moves in the conductor, so i is proportional to εu and is obtained by multiplying it by the factor $[1/c] \cdot \sqrt{8}$, where c denotes a *constant velocity* known from the fundamental law of electric action, as shown in Volume 5 of the Treatises of the Mathematical-Physical Class of the Königl. Sächs. Gesellschaften der Wissenschaften [Royal Saxon Societies of Science],[119,120] page 264, which has been found $= 439\,450 \cdot 10^6$ millimeters/(second).

Furthermore, if f denotes the difference between the force acting on the positive electricity contained in the induced conductor in the direction of the conductor and the force acting on the negative electricity contained therein, expressed in parts of that force which would give one unit of velocity to one ponderable unit of mass in one unit of time, then the electromotive force e acting on the induced conductor is proportional to f and is calculated by multiplying it by the obtained factor $[c/\varepsilon] \cdot \sqrt{1/8}$.

These meanings of i and e are the same, according to which a current of the intensity $= 1$, when passing around one unit of area, exerts actions equal to one unit of the magnetic moment, and according to which, furthermore, one unit of the magnetic force on a closed conductor, while it is rotated in such a way that the projection of the area enclosed by it on the plane perpendicular to the direction of the magnetic force grows uniformly in one unit of time around one unit of area, exerts one unit of electromotive force. These meanings of i and e are to be used as the basis for all *absolute measurements* because of their relationship to magnetism.

According to these values of e and i, which must also be taken as a basis for the absolute resistance measurements, the ratio is

$$\frac{e}{i} = \frac{\frac{fc}{\varepsilon}\sqrt{\frac{1}{8}}}{\frac{\varepsilon u}{c}\sqrt{8}} = \frac{c^2}{8u} \cdot \frac{f}{\varepsilon^2} \ .$$

If the electrostatic force which the amount of positive or negative electricity contained in a piece x of the conductor, $= \varepsilon x$, exerts on an equal amount at the distance x, is called f', then it is known that

[119] [Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. III, p. 652.

[120] [Note by AKTA:] [KW57, p. 652 of Weber's *Werke*] with English translation in [KW21, p. 179].

$$f' = \frac{\varepsilon x \cdot \varepsilon x}{x^2} = \varepsilon^2 \ ,$$

consequently,

$$\frac{e}{i} = \frac{c^2}{8u} \cdot \frac{f}{f'} \ .$$

Now the ratio of two forces f/f', as well as the ratio of two velocities c/u, are expressed by pure numbers, from which it follows that

$$\frac{1}{8} \cdot \frac{c}{u} \cdot \frac{f}{f'} = n$$

is a pure numerical factor, and from this it follows that e/i is a velocity n times larger than the velocity c.

8.2 Representation of a Velocity Equal to the Resistance of a Conductor

According to Ohm's law of the galvanic circuit,[121] the current intensity i is directly proportional to the electromotive force e acting on the circuit, and inversely proportional to the resistance w of the circuit, and, if the resistance unit of measure is chosen accordingly,

$$i = \frac{e}{w}$$

can be set, from which it follows that the quotient

$$\frac{e}{i} = w$$

has a *constant* value for any given circuit, which is called its *resistance in absolute units*.

This *resistance*, because it is the quotient of an electromotive force divided by a current intensity, must, according to the previous Section, be equal to a certain *velocity*, and it is of interest not only to determine this velocity in terms of its magnitude, but also to actually represent it in such a way that it corresponds to the ratio e/i in all physical relationships.

Give the conducting wire the shape of a circle, which is placed parallel to the magnetic meridian plane and rotated around its horizontal diameter, while a small compass is placed in the center of the circle. This compass is then deflected from the magnetic meridian according to known laws, the more so the faster the circle is rotated; for the currents induced in the circle by the vertical component of the Earth's magnetism during this rotation act on the compass and exert on it a directive force[122] perpendicular to the meridian plane, the *mean value of which for the duration of half a revolution* increases proportionally with the angular

[121][Note by AKTA:] Georg Simon Ohm (1789-1854). Ohm's law is from 1826: [Ohm26a], [Ohm26c], [Ohm26d], [Ohm26b] and [Ohm27] with French translation in [Ohm60] and English translation in [Ohm66].

[122][Note by AKTA:] In German: *Direktionskraft*. This expression can be translated as directive force, directing force or directional force. This concept was introduced by Gauss in 1838, [Gau38b, p. 4] with English translation in [Gau41c, p. 254]. Consider, for instance, a compass needle of magnetic moment m. Utilizing Gauss and Weber's terminology, let T be the horizontal component of the Earth's magnetic force. The torque τ exerted by the Earth on the needle when it is deflected by an angle θ relative to the local magnetic meridian is given by $\tau = mT \sin\theta$. The so-called magnetic directive force is here given by mT.

velocity.[123] — During the duration of half a revolution, this directive force is of course variable, from which it follows that the [compass] needle cannot remain at rest, but must fluctuate within certain limits; however, the shorter the duration of half a revolution becomes during accelerated rotation compared to the oscillation period given to the needle by the Earth's magnetic directive force, the more those limits come closer to one another, and the above needle fluctuation can be reduced to such an extent that it becomes completely imperceptible and the needle appears completely still. — The *velocity* at which the conductor particles must be moved by the rotation, at a distance from the axis of rotation equal to the radius of the circle, so that this *mean value* is π^2 times greater than the vertical directive force exerted directly on the compass by the vertical component of the Earth's magnetism, is the *velocity equal to the resistance of the conducting wire.*

However, this *mean value* and this vertical directive force exerted directly by the Earth on the compass behave like the tangents of the deflections v and I they produce, where v is the horizontal deflection of the compass and I is the geomagnetic inclination observed during the rotation. So if one observes that at n revolutions in the unit of time, $\tan v/\tan I = \pi^2$, then the resistance of the circular conductor will be

$$w = 2n\pi r \ ,$$

if r denotes the radius of the circular conductor.

This velocity represented in this way, which is equal to the resistance, actually has the same physical relationships as the ratio of the electromotive force to the current intensity or the resistance of the conductor; because it can be proven that this velocity, as well as this resistance, is completely independent of both the strength and direction of the Earth's magnetic force, which acts inductively on the conductor, and also of the strength of the compass, on which the Earth's magnetism and the currents induced in the conductor act.

The following explanations serve to prove this relationship between the velocity and resistance just described.

If φ is the angle which the circular plane forms with the meridian plane, $d\varphi/dt$ is the angular velocity and r is the radius of the circle, then one obtains the electromotive force exerted on the circle by the vertical component T' of the Earth's magnetism, according to the meaning given in Section 8.1,

$$e = \pi r^2 \cdot T' \cdot \cos\varphi \frac{d\varphi}{dt} \ .$$

If the angular velocity $d\varphi/dt = \rho$ is constant, that is, $e = \pi r^2 T' \rho \cos\varphi$, proportional with $\cos\varphi$, then, according to Ohm's laws, the intensity of the current i induced in the conductor is also proportional to $\cos\varphi$, and can be set

$$i = i_0 \cos\varphi \ ,$$

where i_0 has a constant value.

According to electromagnetic laws, this induced current exerts a torque[124] on the needle m in the center of the circular conductor,[125] which, when the circular plane is vertical, or

[123][Note by AKTA:] In German: *Drehungsgeschwindigkeit.*

[124][Note by AKTA:] In German: *Drehungsmoment.* This expression can be translated as torque, rotatory action or moment of force.

[125][Note by AKTA:] That is, on the needle with a magnetic moment m.

$\varphi = 0$, and also when the deflection of the needle from the magnetic meridian $v = 0$, from the known theory of the tangent galvanometer would be represented by the quotient of the product of the length of the conductor $2\pi r$ into the current intensity i and into the needle magnetism m, divided by the square of the circle radius r^2, so $= 2\pi im/r$; but if φ and v are different from zero, then, as can be easily shown, this quotient must still be multiplied by $\cos\varphi\cos v$, whereby the torque exerted on the needle by the induced current is obtained[126]

$$= \frac{2\pi im}{r} \cdot \cos\varphi\cos v = \frac{2\pi m}{r} i_0 \cos v \cdot \cos\varphi^2 \ .$$

The *mean value* of this torque for the duration of half a revolution $= \pi/\rho$ results from this

$$\frac{2\pi m}{r} i_0 \cos v \cdot \frac{\rho}{\pi}\int_0^{\pi/\rho} \cos\varphi^2 dt = \frac{2\pi m}{r} i_0 \cos v \cdot \frac{1}{\pi}\int_0^{\pi} \cos\varphi^2 d\varphi = \frac{\pi m}{r} i_0 \cos v \ .$$

Now, further, the torque exerted by the Earth on the needle, when T denotes the horizontal component of the Earth's magnetism, is,

$$= Tm\sin v \ ,$$

which, if v suffers no perceptible change, can be taken as constant. This torque exerted by the Earth on the needle must then be equal to the *average value* of that exerted by the induced current, that is,

$$Tm\sin v = \frac{\pi m}{r} i_0 \cos v \ ,$$

consequently

$$i_0 = \frac{rT}{\pi}\tan v \ ,$$

$$i = \frac{rT}{\pi}\tan v \cdot \cos\varphi \ .$$

But now

$$e = \pi r^2 T'\rho \cdot \cos\varphi \ ,$$

consequently

$$\frac{e}{i} = \frac{\pi^2}{\tan v} \cdot \frac{T'}{T} \cdot r\rho \ ,$$

or, because $T'/T = \tan I$, if I denotes the geomagnetic inclination,

$$\frac{e}{i} = \frac{\tan I}{\tan v} \cdot \pi^2 r\rho \ .$$

Finally, if $2n\pi$ denotes the value of ρ for which

---

[126][Note by AKTA:] The next equation should be understood as:

$$= \frac{2\pi im}{r} \cdot \cos\varphi\cos v = \frac{2\pi m}{r} i_0 \cos v \cdot \cos^2\varphi \ .$$

The same meaning should be understood in similar equations.

$$\frac{\tan v}{\tan I} = \pi^2$$

is observed, then

$$\frac{e}{i} = 2n\pi r \ ,$$

that is, the *velocity* with which the conductor particle located at the distance r from the axis of rotation moves in its circular path represents the *resistance of the conductor* $w = e/i$.

If we denote the resistance of *one unit of length* of the conductor by the name of its *specific resistance*, then the specific resistance of a circular conductor is equal to a certain *angular velocity* of this conductor, namely, since $2\pi r$ is the length of the conductor, the angular velocity n, for which

$$\frac{\tan v}{\tan I} = \frac{\pi}{2}$$

is observed.

8.3 Determination of the Resistance from the Ratio $\int edt/\int idt$ for an Induction Surge

From the possibility of actually representing those velocities that are equal to the resistances of conducting wires, the possibility of *measuring* these velocities, and thus also the resistances equal to them, is recognized. These measurements are called *the absolute resistance measurements*.

However, even if the possibility of absolute resistance measurements is obvious from this, it is by no means the most accurate and finest method for the actual implementation, on which the practical significance of these measurements depends; rather, further discussions are required to determine them, which must be carried out before the implementation.

The conducting wire mentioned in the previous Section, which is brought into the shape of a circle and can be rotated around its horizontal diameter, together with the compass located in the center of the circle, essentially forms the same instrument that was already discussed under the name of the *induction inclinometer*[127] in the "Resultaten aus den Beobachtungen des magnetischen Vereins im Jahre 1837" (Results from the Observations of the Magnetic Association in 1837), pages 81-96.[128,129] Through this *induction inclinometer*, the resistance measurements can be reduced to velocity measurements. But an exact execution of these velocity measurements, as is self-evident, requires a completely uniform angular velocity, the realization of which, although not impossible, is associated with great practical difficulties. It is therefore of the greatest importance for the execution of an accurate absolute resistance measurement that it be made independent of the realization and measurement of such a completely uniform angular velocity.

The method of achieving this purpose is generally based on, instead of obtaining a certain average value of the electromotive force e and the current intensity i for a long time by continued uniform rotation, and measuring the same during this time, trying to produce

[127][Note by AKTA:] In German: *Induktions-Inklinatorium*. See footnote 58 on page 34.
[128][Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. II, pp. 75-88.
[129][Note by AKTA:] [Web38].

precisely determined and measurable integral values $\int edt$ and $\int idt$, but limited to a very short time, under circumstances in which the quotient e/i remains constant for all time elements dt, although e and i vary. Exact measurement of the integral values $\int edt$ and $\int idt$ then results in the quotient $\int edt / \int idt = e/i$, equal to the desired resistance of the conductor wire w, whereby it does not matter, whether the short period of time over which those integrals extend, which do not need to be measured at all, is slightly larger or smaller, since the result is completely independent of this.

8.4 Execution with the Induction Inclinometer

The method given in the previous Section could now be easily carried out using the *induction inclinometer* in the following manner. Instead of setting the circle formed from the conductor wire into a continuous, uniform rotation, you just turn it a little, for example halfway around, most expediently starting from the horizontal position of the circle up to the horizontal position again, and in a very short time, which is referred to by the name of an *induction surge*.[130] The integral value $\int edt$ for such an induction surge is easy to determine; because according to Section 8.2, $e = \pi r^2 T' \cos\varphi \cdot d\varphi/dt$, therefore the integral value of edt taken from $\varphi = -\pi/2$ to $\varphi = +\pi/2$ is

$$\int edt = 2\pi r^2 T' \ ,$$

if r denotes the radius of the circle and T' the vertical component of the Earth's magnetism.

The integral value $\int idt$ can also be determined very simply by determining the angular velocity at which the compass is set by such an induction surge; because if this angular velocity is denoted by γ, the magnetism and the moment of inertia of the compass are denoted by m and k,[131] then we have

$$\int idt = \frac{2rk}{\pi^2 m} \cdot \gamma \ .$$[132]

Now, with a needle set in vibration, the greatest angular velocity γ (at the moment when it passes through the equilibrium position) is related to the greatest deflection from the

[130][Note by AKTA:] In German: *eines Induktionsstosses*. This expression can be translated as induction surge, spike, impulse, shock, kick, blow or hit; or inductive surge, spike, impulse, shock, kick, blow or hit. That is, an induced voltage surge generated by electromagnetic induction, or a short-term induced voltage spike caused by electromagnetic induction.

[131][Note by AKTA:] That is, if the magnetic moment and the moment of inertia of the magnetized needle are represented by m and k, respectively.

[132][Note by WW:] According to Section 8.2, the horizontal torque exerted on the needle by the induced current $i = i_0 \cos\varphi$ was $= [2\pi m/r] \cdot i_0 \cos v \cos\varphi^2$, hence, if at the moment of the induction surge the needle is at rest and $v = 0$, [this expression becomes] $= [2\pi m/r] \cdot i_0 \cos\varphi^2$. This torque divided by the moment of inertia k gives the angular acceleration of the needle $d\gamma/dt = [2\pi m/rk] \cdot i_0 \cos\varphi^2$. From this we get, if the angular velocity of the circle $d\varphi/dt$ is denoted by ρ, $d\gamma = [2\pi m/rk] \cdot [i_0/\rho] \cdot \cos\varphi^2 d\varphi$, and the integral value thereof, between $\varphi = -\pi/2$ and $\varphi = +\pi/2$, $\gamma = [\pi^2 m/rk] \cdot [i_0/\rho]$, therefore $i = i_0 \cos\varphi = [rk/\pi^2 m] \cdot \rho\gamma \cos\varphi$, from which $idt = [rk/\pi^2 m] \cdot \gamma \cos\varphi d\varphi$, and the integral value thereof, between the limits $\varphi = -\pi/2$ and $\varphi = +\varphi/2$, $\int idt = [2rk/\pi^2 m] \cdot \gamma$ is obtained. The angular velocity ρ of the circle has been assumed to be constant; but one can easily see that the result would remain unchanged even if ρ were variable; because i_0 would then also be variable, but the ratio i_0/ρ would remain constant.

equilibrium position, that is, to the elongation width α, as [the ratio of] π to the period of oscillation t of the needle,[133] or it is $\gamma = [\pi/t] \cdot \alpha$, so

$$\int idt = \frac{2rk}{\pi mt} \cdot \alpha \ .$$

Since $\int edt = 2\pi r^2 T'$, from this we get the desired resistance of the conductor wire

$$w = \frac{\int edt}{\int idt} = \frac{\pi^2 mrtT'}{k\alpha} \ .$$

If T denotes the horizontal component of the Earth's magnetism and I the inclination, then it is known that $T'/T = \tan I$ and $mT/k = \pi^2/t^2$; consequently

$$w = \frac{\pi^4 r}{\alpha \cdot t} \tan I \ .$$

If the wire formed, instead of a simple circle, a ring composed of n windings of equal size, isolated from each other, one would find:

$$w = \frac{n^2\pi^4 r}{\alpha \cdot t} \cdot \tan I \ .$$

8.5 Separation of the Inductor from the Galvanometer

As simple as the method of absolute resistance measurement with the *induction inclinometer* described in the previous Section appears, it does not prove to be effective in practice. For, *in the first place*, the angular velocity imparted to the needle by a single half revolution of the circle (induction surge) and the elongation width thereby produced are far too small to be observed and measured with an ordinary compass; even the finest magnetometric observations would not be sufficient for this purpose if the compass could be replaced by a magnetometer equipped with a mirror and scale, the placement of which would, incidentally, be associated with great practical difficulties in the center of the rotating circle. *In the second place*, however, there is also the fact that with this method the horizontality of the needle axis would have to be completely guaranteed; otherwise, as one can easily see, when the circle rotates about its horizontal diameter, the induction of the vertical component of the Earth's magnetism would be mixed with the induction of the vertical component of the needle magnetism.

These reasons therefore make it seem far more expedient to form *two circles* from the conductor wire instead of one circle, one of which is used as an *inductor* and is rotated, the other serves as a *multiplier* and is fixed.[134] This separation gives freedom for the most expedient arrangement of the inductor as well as the multiplier required for the galvanometer, where each can then be constructed much more perfectly on its own, without having to take the other into account. On this separation of the inductor from the multiplier is based the

[133][Note by AKTA:] In German: *Schwingungsdauer*. Gauss and Weber utilized the old French definition of the period of oscillation t which is half of the English definition of the period of oscillation T, that is, $t = T/2$, [Gil71a, pp. 154 and 180]. For instance, the period of oscillation for small oscillations of a simple pendulum of length ℓ is $T = 2\pi\sqrt{\ell/g}$, where g is the local free fall acceleration due to the gravity of the Earth, while $t = T/2 = \pi\sqrt{\ell/g}$.

[134][Note by AKTA:] See footnote 52 on page 33.

method developed in the First Volume of the Treatises of the Königl. Sächs. Gesellschaft der Wissenschaften,[135,136] about which the following comment will suffice here.

The calculation of the resistance of the conductor wire from the observations is changed only slightly by the separation of the inductor from the multiplier, namely, only as a result of the fixed position in which the separated multiplier, which no longer takes part in the rotation of the inductor, remains, according to which, *firstly*, the horizontal torque $= [2\pi m/r] \cdot i_0 \cos \varphi$ exerted on the needle by the induced current $i = i_0 \cos \varphi$ is found (instead of that value $= [2\pi m/r] \cdot i_0 \cos \varphi^2$ stated in the Note to Section 8.4), from which follows $\int i dt = [rk/2\pi m] \cdot \gamma$; and *secondly*, the elongation width α can no longer be determined from the angular velocity γ according to the law $\gamma = [\pi/t] \cdot \alpha$ listed in Section 8.4, because this law is only valid for one freely oscillating needle that does not suffer any attenuation, which was the case in Section 8.4, because the multiplier connected to the inductor was always in a horizontal position before and after the induction surge. If, on the other hand, the multiplier, which is separated from the inductor, remains in its vertical position parallel to the meridian plane during the entire needle oscillation, the oscillating needle suffers damping and the elongation width α is then to be determined from the angular velocity γ according to the laws developed by Gauss in the "Resultaten aus den Beobachtungen des magnetischen Vereins in Jahre 1837" (Results from the Observations of the Magnetic Association in 1837).[137,138] If γ is determined according to these laws from the observed elongation width and from the simultaneously observed decrease in the oscillation arcs of the needle, the following equation results for the calculation of w, namely, either for simple circles of the same radius r, both as inductor as well as multiplier:

$$w = \frac{4\pi^4 r}{\gamma \cdot t^2} \cdot \tan I \ ,$$

or for a ring composed of n windings of radius r as an inductor and for a ring composed of n' windings of radius r' as a multiplier:

$$w = \frac{4nn'\pi^4}{\gamma \cdot t^2} \cdot \frac{r^2}{r'} \cdot \tan I \ .$$

8.6 Damping as a Measure of the Sensitivity of the Galvanometer

The freedom to make the radius r' of the multiplier windings smaller than the radius r of the inductor windings, and to increase the number n' of multiplier windings, which is obtained by the separation of the multiplier from the inductor discussed in the previous Section, then acquires greater importance in that, *firstly*, in the fixed position of the multiplier, the substitution of the compass with a magnetometer is no longer obstructed, *secondly*, that, in addition, the angular velocity γ imparted to the needle by an induction surge can be given a size appropriate for finer observation. For from the equation at the end of the previous Section it can be seen that if r' gets half the value and n' gets twice the value, otherwise under exactly the same conditions, with unchanged conducting wires, the angular velocity

[135] [Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. III, p. 301.
[136] [Note by AKTA:] [Web52b] with English translation in [Web21e].
[137] [Note by Heinrich Weber:] Gauss' *Werke*, Vol. V, p. 389.
[138] [Note by AKTA:] [Gau38a, p. 389 of Gauss' *Werke*].

γ obtained by an induction surge assumes a value four times greater. Only in this way is it possible, with an induction as weak as that offered by geomagnetism, to give the elongation width α of the needle, which is dependent on γ, the size necessary for accurate measurement.

But it is clear that if the multiplier encloses the needle tightly, instead of forming a wide circle around it as in a tangent galvanometer, the law valid for the tangent galvanometer, according to which it was determined the angular velocity γ given to the needle by an induction surge, namely, the equation $\int idt = [rk/2\pi m]\cdot\gamma$ listed in Section 8.5, according to which $\gamma = [2\pi m/rk]\cdot\int idt$ (or, for a plurality of windings n' of radius r', $\gamma = [2n'\pi m/r'k]\cdot\int idt$) no longer applies, because then the difference in the position of the various windings from which the multiplier is composed and the way in which the magnetism is distributed in the needle gain influence and must be taken into account more precisely; but even then γ remains proportional to $\int idt$ and the constant ratio $\gamma/\int idt$, which can be called the sensitivity coefficient of the galvanometer and can be denoted by f, can easily be determined for any given galvanometer by observation, by simultaneously measuring γ and $\int idt$. However, it should be noted that the constancy of the coefficient $\gamma/\int idt = f$ is necessarily linked to the immutability of the instrument, an immutability that cannot be attributed to such sensitive galvanoscopes with tightly surrounding multipliers *in the long term*, which is why, as already noted in the Introduction, the sensitivity of such instruments cannot be determined *in advance*. So the coefficient f, or the sensitivity of the instrument, must be determined for the moment of observation itself.

Such a determination is made by the observations combined according to the *throwback method*[139] (which was discussed in more detail in the First Volume of the Treatises of the Königl. Sächs. Gesellschaft der Wissenschaften, page 349),[140,141] which determine the *angular velocity* imparted to the needle by an induction surge and at the same time affect their *damping*; because this damping is proportional to the square of the coefficient f. If from such observations the damping resulting from the closure of the circuit is determined by the value of the logarithmic decrement (according to the natural system), then if w is the resistance of the circuit, k is the moment of inertia of the needle and τ denotes the period of oscillation under the influence of damping,

$$f^2 = \frac{2w}{k\tau}\cdot\lambda\ .$$[142]

[139][Note by AKTA:] In German: *Zurückwerfungsmethode*. See [Gau38a], [Web39b] and [WK68, p. 108, Note 13].

[140][Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. III, p. 441.

[141][Note by AKTA:] [Web52b, p. 441 of Weber's *Werke*] with English translation in [Web21e, p. 412].

[142][Note by WW:] According to the *electromagnetic* law, as stated before, the angular velocity imparted to the needle by the currents induced by an induction surge was

$$\gamma = f\cdot\int idt\ ,$$

so, during the induction surge, $d\gamma = fidt$. Therefore, the angular acceleration imparted to the needle by the current i in the multiplier is $d\gamma/dt = fi$, and consequently, if k denotes the moment of inertia of the needle, the torque exerted on the needle by the current i in the multiplier is $= kfi$.

However, if $i = 1$ is set in it, the expression of this torque gives, according to *magnetoelectric* law, the factor which, multiplied by the angular velocity γ of the needle, is equal to the *electromotive force* exerted by the moving needle on the multiplier, $= kf\gamma$, from which, according to Ohm's law, follows the current generated by the moving needle in the multiplier $i = kf\gamma/w$. If one now puts this value of i into the equation $d\gamma = fidt$, one finds that the damping caused by closing the circuit retards the angular velocity of the needle and that this retardation is $d\gamma/dt = [kf^2/w]\cdot\gamma$. Now the differential equation of the

But now $f = \gamma / \int idt$, and according to Sections 8.3 and 8.4, $w = \int edt / \int idt$ and $\int edt = 2n\pi r^2 T'$; consequently, if f, $\int edt$ and $\int idt$ are eliminated from these four equations, we get,

$$w = \frac{8(n\pi r^2 T')^2}{k\gamma^2\tau} \cdot \lambda \; .$$

With regard to the execution of the observations, it should be noted that, *firstly*, with strong attenuation, it may occur that the period of oscillation τ with a closed circuit cannot be directly determined precisely, and that it is therefore necessary to observe the period of oscillation t with an open circuit for this purpose; *secondly*, even with an open circuit a still perceptible attenuation occurs very often, which is determined by observing the logarithmic decrement λ_0. If the circuit is closed, λ is added to λ_0, and the observed logarithmic decrement is $\lambda_0 + \lambda = \lambda_1$. Under such conditions, $\lambda = \lambda_1 - \lambda_0$ and $\tau = t_0\sqrt{(\pi^2 + \lambda_1^2)/(\pi^2 + \lambda_0^2)}$ must be substituted in the above equation to represent the resistance w in its dependence on the observed values t_0, λ_0 and λ_1, namely:

$$w = \frac{8(n\pi r^2 T')^2}{k\gamma^2 t_0} \cdot (\lambda_1 - \lambda_0)\sqrt{\frac{\pi^2 + \lambda_0^2}{\pi^2 + \lambda_1^2}} \; .$$

8.7 Induction by the Horizontal Component of Terrestrial Magnetism

In a similar manner as the separation of the multiplier from the inductor specified in Section 8.5 can be used to obtain a galvanometer with a closely surrounding *multiplier* of the highest sensitivity for the measurement, in the same way this separation can also serve to give the *inductor* a more suitable and advantageous arrangement.

The radius of the inductor windings no longer needs to be limited by the galvanometer, but can be increased as much as is compatible with a rapid and slight rotation of the inductor, whereby the induction surges are significantly increased. Because the strength of the induction surge has been found $\int edt = 2n\pi r^2 T'$ and one can easily see that this value is increased m times, even with an unchanged wire length, if the radius r of the inductor windings is taken to be m times larger and consequently the number n of inductor windings is taken m times smaller.

In addition, as a result of the inductor being separated from the multiplier, the reason why the inductor rotation had to occur by the *horizontal* diameter of the inductor when the inductor and multiplier were combined no longer applies, namely, the reason that when the inductor rotates there is only an induction due to the Earth's magnetism, and not at the same time by the needle magnetism, because the latter would be difficult to determine or eliminate. As a result of the separation of the inductor from the multiplier, the rotation can also occur around the *vertical* diameter of the inductor, whereby the induction is made dependent on the *horizontal* component T of the Earth's magnetism, instead of on the *vertical*

---

oscillating needle (see *Resultate aus den Beobachtungen des magnetischen Vereins*, 1837, page 74) [[Note by Heinrich Weber:] Gauss' *Werke*, vol. V, page 389][[Note by AKTA:] [Gau38a, p. 389 of Gauss' *Werke*].] is $d^2x/dt^2 + 2\varepsilon dx/dt + n^2 x = 0$, where the angular velocity $\gamma = dx/dt$ and the rotation retardation resulting from the damping is set $[kf^2/w] \cdot \gamma = 2\varepsilon dx/dt$, hence $kf^2/w = 2\varepsilon$. — From this differential equation it follows $x = p + Ae^{-\varepsilon t} \sin\left(t\sqrt{n^2 - \varepsilon^2} - B\right)$, according to which the oscillation period is $\tau = \pi/\sqrt{n^2 - \varepsilon^2}$ and the natural logarithmic decrement is $\lambda = \varepsilon\tau$. According to this, $kf^2/w = 2\varepsilon = 2\lambda/\tau$, or $f^2 = [2w/k\tau] \cdot \lambda$, which had to be proven.

component T'. By substituting T with T', the equation at the end of the previous Section turns into the following:

$$w = \frac{8(n\pi r^2 T)^2}{k\gamma^2 t_0} \cdot (\lambda_1 - \lambda_0) \cdot \sqrt{\frac{\pi^2 + \lambda_0^2}{\pi^2 + \lambda_1^2}} \, .$$

This exchange offers the advantage that it is sufficient to measure the horizontal component T of the Earth's magnetism, while in the other case it was also necessary to measure the *inclination* I in order to be able to determine the *vertical* component $T' = T \tan I$.

II - Construction of the Galvanometer

8.8

From the overview of the method of absolute resistance measurement given in the previous Section, it is clear that the construction of the galvanometer is important for carrying out such a measurement. What is important is not just a high degree of sensitivity, but also that this degree of sensitivity can be precisely determined from the damping observations of the needle vibrations.

The theory of the galvanometer has often been discussed from different sides, according to the variety of purposes for which it was intended to serve. Closely related to the purpose of absolute resistance measurement, with which we are concerned here, is the use of the galvanometer, discussed in the Fifth Volume of these Treatises[143,144] for the measurement of *magnetic inclination* carried out with the help of *induction*, which also in the place mentioned has one application connected to the resistance measurement itself. The rules given there for the construction of the galvanometer also apply here, for example that the resistance of the multiplier should be almost the same as that of the rest of the circuit, to which the inductor belongs. However, in the case of the galvanometer there, it was mainly only a question of the sensitivity or size of the deflection of the galvanometer needle set in vibration by an induction surge, whereas here it is also about the size of the attenuation, which is to be used to precisely determine that sensitivity. This use of damping has already been discussed there, on the occasion of the application to resistance measurement; however, it still requires further discussion in order to determine what is the highest degree of accuracy in this *determination of sensitivity* and how it can be achieved.

8.9 Limits on the Size of the Deflection and Attenuation

Even if the *deflection* and *attenuation* could be increased at will, partly by tightly surrounding the galvanometer needle with the multiplier, partly by increasing the needle magnetism, certain limits should not be exceeded if the accuracy of the resistance measurement is not to be reduced instead of increased.

[143] [Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. II, p. 277.
[144] [Note by AKTA:] [Web53d].

For as far as the *deflection* is concerned, which must not go beyond the scale with which it is to be measured, its enlargement in all *magnetometric* observations is governed by the rule that it should always remain limited to small deflection angles, according to which the length of the scale length is determined. Otherwise the most important advantage of these observations would be lost, which is that small deflections are always needed and sufficient even for the finest measurements, which avoids many disturbing influences and greatly simplifies the calculation of the observations.

Likewise, with regard to the *attenuation*, which can be determined from the *difference* in the size ratio of two successive oscillation arcs from unity, it is clear that this *difference* must have a size sufficient for precise determination, but must not be so large that, on the other hand, the unity itself disappears because otherwise either the first oscillation arc would be too large to be measured with the scale, or the second would be too small for an accurate measurement. Between these two limits there must be a case where the accuracy that can be achieved in determining the damping is a maximum.

If the larger oscillation arc, which should be almost equal to the limit value set by the length of the scale, is denoted by a and the smaller one by x, then the size of the damping is found proportional to $\log[a/x] = \lambda$, and the accuracy that can be achieved in determining the damping is represented by the quotient of the smallest measurable change in x, divided by the corresponding change in λ expressed in parts of λ. The value of x for which the absolute value of this quotient is a maximum is given by the equation

$$\left(\frac{\lambda dx}{d\lambda}\right)^2 = x^2\left(\log\frac{a}{x}\right)^2 = \text{maximum},$$

from which follows $a : x = e : 1$ if $e = 2.71828$ denotes the base of natural logarithms. This results in the rule that for the determination of the damping it is most advantageous to construct the galvanometer in such a way that the ratio of two consecutive arcs of the vibrating needle is equal to or at least comes close to the ratio $e : 1$.

8.10 Unifilar and Bifilar Suspension of the Galvanometer Needle

The suspension of the galvanometer needle can be either *unifilar* or *bifilar* and only a closer consideration of the observations to be made can give preference to the choice of one or the other suspension.

If the galvanometer needle is set into vibration by an induction surge, that is, if it is given a certain angular velocity γ at the moment when it is in the rest position, it is well known that it is not enough to observe the deflection or the *first* elongation a of the needle, but rather it is necessary, especially to determine the attenuation, to also observe the *second* elongation b of the needle, to the opposite side from the rest position. However, for the purpose of accurate measurement, these two observations must be repeated more frequently. It is now clear that, instead of waiting until the needle has reached complete rest between every two repetitions, there is great advantage in carrying out a system of such repetitions without interruption in a continuous succession, which is useful if one considers that although the needle at the moment of each induction surge should be in the position where it could remain in equilibrium if it had no movement, it is not necessary for the purpose of these observations that it really be in equilibrium. Rather, the needle can have an angular velocity at this moment, if the latter is always the same in all repetitions at the moment of each induction surge. The method of arranging such observation system was given by Gauss and can be found in the First Volume of the Treatises of the mathematical-physical class of the Königl. Sächs. Gesellschaft der Wissenschaften, p. 349, discussed in more detail under the name of the *throwback method*.[145,146] It follows from this that the precise implementation of such an observation system requires, *firstly*, that the duration of an induction surge forms a very small fraction of the oscillation period of the needle, *secondly*, that the moment of each induction surge coincides as precisely as possible with the moment when the needle is in the position in which it could remain in equilibrium if its angular velocity were zero. However, it is clear that the fulfillment of these two requirements can only be achieved with a longer oscillation period of the needle, for example 20 to 30 seconds, which means that the construction of the galvanometer must be adjusted accordingly.

If such a longer period of oscillation is to be produced by *unifilar* suspension of the needle, and if the needle is to have the strongest possible magnetism in relation to its size for the purpose of damping, then the necessity of a larger needle is obvious, for example from 600 to 900 millimeters in length, which also requires a corresponding expansion of the multiplier in the direction of the needle. With such an extension of the multiplier, a sufficiently strong *damping* can be achieved through the associated increase in the magnetism of the needle; the size of the *deflection* caused by an induction surge, on the other hand, decreases so quickly as the needle and multiplier are lengthened, that it can happen that they are no longer sufficient for precise measurements. With larger needles, under normal circumstances one can calculate that the size of the deflection is inversely proportional to the length of the needle, for example with a 600 to 900 millimeter long needle, which would require an oscillation period of 20 to 30 seconds, the deflection would be 4 to 6 times smaller than with a 150 millimeter long needle.[147] If it were found that the deflection, under otherwise favorable conditions, remained of a size sufficient for finer measurements, there would be

[145][Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. III, p. 441.

[146][Note by AKTA:] See the footnote 139 and [Web52b, p. 441 of Weber's *Werke*] with English translation in [Web21e, p. 412].

[147][Note by WW:] For the present consideration it is sufficient to consider only a single winding of the multiplier in the vertical plane of the needle, and only two points of the needle, which can be called the North pole and the South pole, the distance from each other being $= l$. The multiplier winding forms a semicircle of radius r around each of these points, both connected by two parallel pieces of length l. The angular velocity which is imparted to the needle by an induction surge is then composed of several parts, namely, that which comes from the two semicircles acting on the needle poles located at their centers, which is obtained $= [\pi m/kr] \cdot \int idt$ when m denotes the magnetic moment and k the moment of inertia of the needle; further from that which comes from the two parallel connecting pieces, $= [2lm/rk\sqrt{l^2+r^2}] \cdot \int idt$, and finally from that which comes from each semicircle acting on the needle pole located at the center of the other semicircle, which, however, can be considered vanishing if r is very small compared to l. According to this, if r is very small compared to l, $\gamma = (\pi + 2)(m/kr) \int idt$ can be set. Now, for two homogeneous needles of similar shape, the largest magnetic moments they can assume, $m : m'$, behave like the cube [of their pole distances $l : l'$], their moments of inertia $k : k'$ behave like the fifth power of their pole distances $l : l'$, or it is $m : m' = l^3 : l'^3$ and $k : k' = l^5 : l'^5$, from which follows the ratio of their angular velocities

$$\gamma : \gamma' = l'^2 : l^2 .$$

And since their deflections $\alpha : \alpha'$ are in the composite ratio of this angular velocity and the period of oscillation proportional to the pole distance, we get from this

$$\alpha : \alpha' = l' : l .$$

no essential reason for its use to discard the *unifilar* suspension. However, if it should turn out that the reduced deflection was no longer sufficient, one would be forced to use *bifilar* suspension.

This *bifilar* suspension can then be set up in such a way that the resulting static directive force S is greater than the magnetic directive force D,[148] and that (if the needle poles are inverted) the oscillation period of the needle depends only on the difference $S - D$, which makes it possible to regulate and extend it as desired. A longer period of oscillation produced in this way, combined with a relatively strong magnetism of the needle, not only makes it possible to produce a galvanometer with very high sensitivity, but also to increase the damping so as to be able to determine it as accurately as the deflection of the needle. Finally, even in cases where the main purpose just described could be achieved by *unifilar suspension of larger magnets*, this *bifilar* suspension still offers the special advantage of making it possible to construct the galvanometer *on a smaller scale without compromising the accuracy of the measurements*, which is often of great importance for practical application.

8.11 Astatic Needle System with Unifilar Suspension

However, the same purpose for which, according to the previous Section, the *bifilar* suspension was particularly suitable and, especially when smaller needles were involved, seemed to deserve preference over the *unifilar* suspension, can also be achieved by the *unifilar* suspension if the simple needle is substituted with an *astatic needle system*, that is, with a system of two identical needles that are firmly connected to each other, one of which is enclosed by the multiplier, the other with the opposite position of the poles outside, either above or below the multiplier.[149] The magnetic directive forces of the two connected needles then cancel each other out, and the static directive force can be regulated by choosing a suitable suspension wire so that the most appropriate period of oscillation is obtained. In addition to all the advantages that could be achieved by *bifilar* suspension of a simple needle, this device offers a special advantage in that the influence of many otherwise unavoidable external disturbances is completely avoided. Such external disturbances are particularly due to the *declination variations of the Earth's magnetism.* Although these variations are generally small, even during the short duration of the observations, it should not be overlooked that they occur with a needle which is directed by the mere *difference between the static and magnetic directive force*, which was the case with a sensitive bifilar suspension, they are magnified as many times as that difference is contained in the whole magnetic directive force. As a result, the equilibrium position of the needle can often be subject to a rapid and considerably large change, which disrupts the precise execution of the induction surges and greatly reduces the certainty and agreement that otherwise characterize these observations. These disturbances are completely avoided in the described astatic needle system, if one pays attention to the exact equality of the needles and the parallelism of their axes; because it is obvious that the variations in the Earth's magnetism have no influence at all on the behavior of such a needle system. Thus, if only the *static* directive force is constant, the equilibrium position of such a system is completely unchangeable and allows the most precise execution of the observations.

[148][Note by AKTA:] See footnote 122.
[149][Note by AKTA:] See footnote 60 on page 35.

8.12 Theory of the Multiplier

After the preceding discussions about the construction of the galvanometer in general and in particular about the construction of the galvanometer needle and its suspension, we move on to the main task, namely, the theory of the multiplier, which for the present purpose requires a more complete development than has hitherto been given of it.

The main findings of previous discussions can be summarized briefly as follows. If, *firstly*, it is the multiplier of a tangent galvanometer, which is supposed to form a circular ring of a very large radius, against which the dimensions of the needle as well as those of its own cross-section are very small, then it is clear that the shape of this cross-section is of very little influence. If, however, this shape is not to be left to arbitrary determination, in the first place, for practical reasons, the expediency of giving the cross-section the shape of a rectangle is self-evident, and in the second place, for the ratio of the two sides of this rectangle the simple rule follows: if the area of the rectangle is given and is expressed in parts of the square of the ring radius $= 2\varepsilon^3$, the height of the rectangle to the base behaves like $\varepsilon : 2$.

If, *secondly*, it is a question of the multiplier of a galvanoscope, which is intended to enclose the needle as tightly as the free movement of the needle permits, then only the shape of the winding which initially encloses the needle is to be regarded as given. As a rule, this winding will form a figure enclosed by two parallel lines and two semicircles. For practical reasons, it may then be assumed that a number of coils of the same shape and size lie side by side on the surface of a column having that figure as its base and form the lowest layer of the multiplier coils over which all the other coils are wound in the form of parallel layers. The whole multiplier thus takes the shape of a ring, for which the shape of its cross-section has to be determined even more closely.

This determination is obtained by considering the torque exerted on the needle by any winding. When considering the torque in this way, it is sufficient (as in the Note to Section 8.10) to consider only two points on the needle, which can be described as the North pole and the South pole, and whose distance, $= l^0$, can be set equal to the length of the two parallel sides of the winding. The torque which the current i passing through the winding exerts on the needle is then composed of several parts, namely, *in the first place*, that which is exerted by the two semicircles on the needle poles lying in their axes, if the axis of a semicircle is understood to be the perpendicular to its plane erected in its center point; *in the second place*, from that which comes from the two parallel sides of the winding; *in the third place*, from that which is exerted by each semicircle on the needle pole located in the axis of the other semicircle. If r denotes the radius of the semicircles and x the length of the perpendicular drawn from a needle pole to the plane of the semicircle, so the *first* part is found $= \pi r^2 mi/(r^2 + x^2)^{3/2}$, the *second* [part] $= 2rl^0 mi/[(r^2 + x^2)\sqrt{l^{02} + r^2 + x^2}]$, the *third* [part] $= 2rmi \int (l^0 \cos\varphi + r) d\varphi/(l^{02} + r^2 + x^2 + 2rl^0 \cos\varphi)^{3/2}$, where the integral value is to be taken between the limits $\varphi = 0$ and $\varphi = \pi/2$. If we then prefer to consider the two *main cases*, namely, *in the first place*, where $l^0 = 0$ or the multiplier windings form circles, and *in the second place*, where l^0 is so large that x and r can be regarded as vanishing in comparison, then *in the first case* the second part is $= 0$ and the third is equal to the first, $= \pi r^2 mi/(r^2 + x^2)^{3/2}$, *in the second case* the third part is $= 2rmi/l^{02}$. If we now denote the quotient of the total torque of one winding divided by its length by the name of the specific torque, then all windings for which the specific torque is equal, in the case where $l^0 = 0$, that is, when the multiplier windings are circular, are given by the following equation:

$$\frac{1}{2\pi r}\cdot\frac{2\pi r^2 mi}{(r^2+x^2)^{3/2}} = \text{constant;}$$

in the case when l^0 is very large compared to r and x, that is, when the multiplier windings are very elongated, [the multiplier windings are given] by the following equation:

$$\frac{1}{2(l^0+\pi r)}\left(\frac{\pi r^2 mi}{(r^2+x^2)^{3/2}}+\frac{2rl^0 mi}{(r^2+x^2)\sqrt{l^{02}+r^2+x^2}}+\frac{2rmi}{l^{02}}\right) = \text{constant,}$$

for which can also be set

$$\frac{mi}{2l^0}\left(\frac{\pi r^2}{(r^2+x^2)^{3/2}}+\frac{2r}{r^2+x^2}\right) = \text{constant.}$$

In the former case, if one sets the constant $= mi/d^2$, or, by taking d as the unit of measure of length, $= mi$, one obtains

$$\frac{r}{(r^2+x^2)^{3/2}} = 1\ ;$$

in the latter case, if one sets the constant $= mi/2l^0 d$ or, by taking d as the unit of measure of length, $= mi/2l^0$, one obtains

$$\frac{\pi r^2}{(r^2+x^2)^{3/2}}+\frac{2r}{r^2+x^2} = 1\ .$$

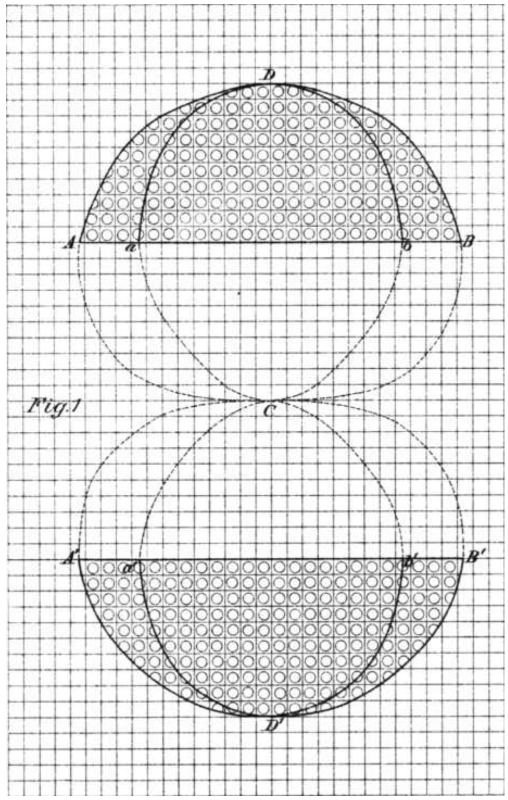
If you finally set $r^2+x^2=\rho^2$ in both equations, this results

for the first case, $r=\rho^3$,

for the second case, $\pi r = \rho(\sqrt{1+\pi\rho}-1)$.

But the sensitivity of the galvanometer increases in proportion to the torque exerted by the multiplier on the needle, and from the latter it is clear that if its value is to be a maximum, the specific torque of all the windings on the outer surface must be the same. It follows from this that the outer limit of the cross section of the multiplier must be determined according to the above equations if the galvanometer is to have the greatest sensitivity. The inner limit of the cross section of the multiplier is given by the space that must be left free for the needle.

Figure 1 represents the cross-section of the multiplier in both cases. The given inner boundary is indicated by the lines AB and $A'B'$; ADB and $A'D'B'$ are the outer boundaries in the former case, aDb and $a'D'b'$ in the latter case.



If, however, as with the multiplier of a tangent galvanometer, a *rectangular cross-section* is preferred, for which only the ratio of the two sides of the rectangle is to be determined, the following equation results for the former case, where the multiplier windings are circular, if the radius of the cylindrical space to be left free for the needle is set equal to 1 and the side of the rectangle parallel to the radius is denoted by a and the side perpendicular to it by $2b$:

$$\frac{2}{(1+a)^2-1}\int_1^{1+a}\frac{r^2dr}{(r^2+b^2)^{3/2}}=\frac{1}{b}\int_0^b\frac{(1+a)dx}{([1+a]^2+x^2)^{3/2}}\ ,$$

or when the integration is executed,

$$\log\frac{1+a+\sqrt{(1+a)^2+b^2}}{1+\sqrt{1+b^2}}=\frac{3(1+a)^2-1}{2(1+a)\sqrt{(1+a)^2+b^2}}-\frac{1}{\sqrt{1+b^2}}\ .$$

According to this,

for small values of a, $b=\sqrt{a}$,

for $a=1$, $b=1.1444$,

for $a=\infty$, $b=0.4413\cdot a$.

For the latter case, the following equation results in the same way:

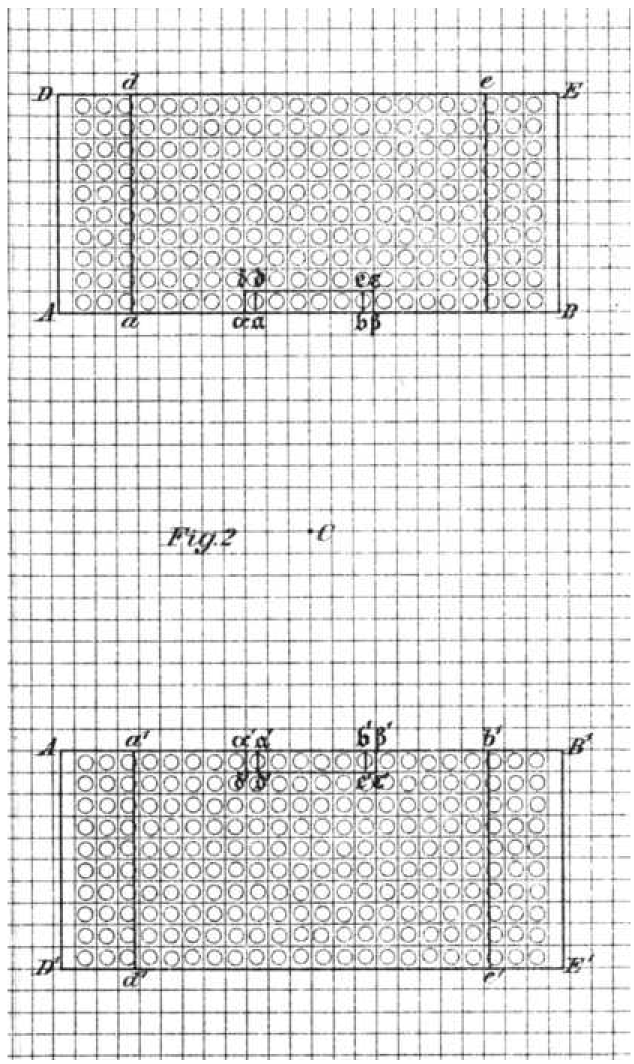
$$\frac{1}{a}\int_1^{1+a}\frac{\pi r^2dr}{(r^2+b^2)^{3/2}}+\frac{1}{a}\int_1^{1+a}\frac{2rdr}{r^2+b^2}=\frac{1}{b}\int_0^b\frac{\pi(1+a)^2dx}{([1+a]^2+x^2)^{3/2}}+\frac{1}{b}\int_0^b\frac{2(1+a)dx}{(1+a)^2+x^2}\ ,$$

from which is obtained

$$\log\frac{(1+a)^2+b^2}{1+b^2}+\pi\log\frac{1+a+\sqrt{(1+a)^2+b^2}}{1+\sqrt{1+b^2}}$$
$$=\frac{\pi(1+2a)}{\sqrt{(1+a)^2+b^2}}-\frac{\pi}{\sqrt{1+b^2}}+\frac{2a}{b}\arctan\frac{b}{1+a}\;.$$

According to this,

for small values of a, $b^2=\frac{3}{2}\cdot\frac{2+\pi}{4+3\pi}\cdot a=0.5745\cdot a$,
for $a=1$, $b=0.8322$,
for $a=\infty$, $b=0.3435\cdot a$.
Figure 2 represents these cross sections when $a=1$.



The given inner boundary against the space left free for the needle is indicated by the lines AB and $A'B'$; $ADEB$ and $A'D'E'B'$ represent the two rectangular cross sections of a circular [multiplier], $adeb$ and $a'd'e'b'$ those of an elongated multiplier. The former cross section $ADEB$ is more than twice as large as the square of the distance c of the needle axis from the multiplier, the latter $adeb$ is close to 5/3 of the same square. If a smaller multiplier is to be formed with the same size of the space left for the needle, then $\alpha\delta\varepsilon\zeta$ and $\alpha'\delta'\varepsilon'\zeta'$

represent the cross sections of the circular [multiplier], $\mathfrak{abcd}$ and $\mathfrak{a}'\mathfrak{b}'\mathfrak{c}'\mathfrak{d}'$ those of an elongated multiplier, of which one of the former is close to $1/16$ [of the square of c], one of the latter is close to $1/21$ of the square of c.

Finally, when designing the cross section of the multiplier, particular attention must be paid to ensuring that the *proportionality of the observed deflections with the current intensities* is maintained within the widest possible limits, which is very important for measurement purposes. In this respect it is sufficient to note that this proportionality is the more perfect the larger, with the same cross-section, the side $2b$ is in relation to a; but, given the small extent of deflections in magnetometric observation, such an increase in the side $2b$ compared to a need not occur at the expense of sensitivity, which would be the case if b exceeded the value determined above. On the other hand, it must be completely avoided, which often happens, that the multiplier breaks into two parts separated by a gap, in order to gain space for the suspension of the needle. Between the needle enclosed by the multiplier and the suspension thread or the needle to be connected astatically, it is always easy to establish a hook-shaped connection that is sufficiently firm and free and that runs around the cross section of the multiplier, which allows sufficient scope for the needle movement without bumping into it, so that there is no need to break through the multiplier to suspend the needle.

8.13

In many cases, for the galvanometer to be constructed, not only the room to be left free for the needle is given, which is, in the case of a circular shape, the radius c of the cylinder space enclosed by the multiplier, but also the wire itself to be used for the multiplier with its volume v. In all such cases, the provisions developed in the previous Section shall suffice; because from the given radius c and volume v both a and b can then be calculated, namely, for circular multipliers according to the two equations

$$\log \frac{1+a+\sqrt{(1+a)^2+b^2}}{1+\sqrt{1+b^2}} = \frac{3(1+a)^2-1}{2(1+a)\sqrt{(1+a)^2+b^2}} - \frac{1}{\sqrt{1+b^2}} \ ,$$

$$v = 2\pi(2+a)abc^3 \ .$$

However, the situation is different if, as in the present case, the choice of wire is left completely free in order to achieve the highest sensitivity and greatest attenuation.

This choice, apart from the specific nature (usually copper), refers only to the cross-section and volume of the wire. But since Section 8.8 already states that it is advantageous to make the resistance of the wire to be used for the multiplier the same as that of the rest of the circuit to which the inductor belongs, this freedom is reduced either to the choice of the cross-section from which the volume is determined, or the choice of volume from which the cross section is determined. It therefore remains to determine how this choice can be made most expediently to increase the *sensitivity* and *attenuation*.

For the selection of the volume, from which the wire thickness is to be determined in each case, it is first of all important to consider that the *sensitivity* also increases rapidly with increasing volume at the beginning, but that this growth is not uniform, but decreases until it disappears completely, whereupon even the case occurs where the sensitivity decreases with increasing volume. There is therefore a certain value of the volume for which the sensitivity is greatest.

In order to determine this value, the expression of sensitivity must be developed more precisely. The angular velocity f which is given to the needle by the unit of current in the time unit serves as a measure of sensitivity; therefore f must first be developed.

According to the electromagnetic law, the torque exerted on the galvanometer needle m by the element α of a multiplier winding, through which the current unit passes, when r is the radius of the winding and x is the distance of the axis of rotation of the needle from the plane of the winding, both expressed in parts of the radius c of the space left free for the needle, is, in the case of the circular shape,

$$= \frac{\alpha m}{c^2} \cdot \frac{r}{(r^2 + x^2)^{3/2}} \ .$$

If you multiply this expression by dx, the integral value of it from $x = -b$ to $x = +b$, divided by $2b$, gives the *mean* torque of all current elements corresponding to the same value of r

$$= \frac{\alpha m}{c^2} \cdot \frac{1}{r\sqrt{r^2 + b^2}} \ ,$$

from which the *average* torque of all multiplier windings corresponding to the same value of r is obtained by multiplying by $2\pi rc/\alpha$, namely,

$$= \frac{2\pi m}{c} \cdot \frac{1}{\sqrt{r^2 + b^2}} \ .$$

If one now multiplies this expression by dr, the integral value from $r = 1$ to $r = 1 + a$, divided by a, gives the *mean* torque of all windings of the entire multiplier

$$= \frac{2\pi m}{ac} \log \frac{1 + a + \sqrt{(1+a)^2 + b^2}}{1 + \sqrt{1 + b^2}} \ ,$$

from which the torque of the entire multiplier is obtained by multiplying by the length l of the entire multiplier wire and dividing by the average length of all its windings $2\pi c(1 + a/2)$.

The quotient of this torque, divided by the moment of inertia k of the needle, is the desired expression of f, or it is

$$f = \frac{2l}{(2+a)ac^2} \cdot \frac{m}{k} \log \frac{1 + a + \sqrt{(1+a)^2 + b^2}}{1 + \sqrt{1 + b^2}} \ .$$

If w now denotes the given absolute resistance of the multiplier wire, and $\varkappa$ the given specific resistance of the metal (copper) of which it is made, then the cross-section of the wire, according to Ohm's law, is $= [\varkappa/w] \cdot l$, that is, the volume of the entire multiplier

$$\frac{\varkappa}{w} l^2 = 2\pi(2+a)abc^3 \ .$$

If you put the resulting value of l into the above expression of f, you get

$$f = 2\frac{m}{k}\sqrt{\frac{2\pi w}{c\varkappa}} \cdot \sqrt{\frac{b}{(2+a)a}} \cdot \log \frac{1 + a + \sqrt{(1+a)^2 + b^2}}{1 + \sqrt{1 + b^2}} \ .$$

Since m, k, w, c, $\varkappa$ are given quantities, the sensitivity f only changes with the value of a and becomes a maximum if

$$\sqrt{\frac{b}{(2+a)a}} \cdot \log \frac{1+a+\sqrt{(1+a)^2+b^2}}{1+\sqrt{1+b^2}} = \text{maximum},$$

where b is given as a function of a by the first equation quoted at the beginning of the Section. If one then adds the second equation given there for v, and the equation resulting from Ohm's law, $l^2 = [w/\varkappa] \cdot v$, for the wire length l (from which the cross section $= v/l$ results), the four elements a, b, v, l can be determined, in which all regulations for the construction of the multiplier are completely contained.

If one initially only takes into account the equation

$$\sqrt{\frac{b}{(2+a)a}} \cdot \log \frac{1+a+\sqrt{(1+a)^2+b^2}}{1+\sqrt{1+b^2}} = \text{maximum},$$

but assumes $b/(1+a) = \beta$ as known or given, then, setting $r = 1+a$, one can write

$$\sqrt{\frac{r}{r^2-1}} \cdot \left(\log r + \log \frac{1+\sqrt{1+\beta^2}}{1+\sqrt{1+(\beta r)^2}} \right) = \text{maximum},$$

from which follows through differentiation[150]

$$\frac{1+r^2}{2r\frac{1}{2}\left(r^2-1\right)^{3/2}} \left(\log r + \log \frac{1+\sqrt{1+\beta^2}}{1+\sqrt{1+(\beta r)^2}} \right) - \sqrt{\frac{r}{r^2-1}} \cdot \left(\frac{1}{r} - \frac{\beta^2 r^2}{1+(\beta r)^2+\sqrt{1+(\beta r)^2}} \right) = 0 \ ,$$

which can be traced back to

$$\log \frac{r+\sqrt{r^2+b^2}}{1+\sqrt{1+b^2}} = \frac{3r^2-1}{1+r^2} \cdot \frac{1}{\sqrt{1+b^2}} - \frac{1}{\sqrt{1+b^2}} \ .$$

If one now also adds the equation given at the beginning of this Section, setting $r = 1+a$, namely,

$$\log \frac{r+\sqrt{r^2+b^2}}{1+\sqrt{1+b^2}} = \frac{3r^2-1}{2r\sqrt{r^2+b^2}} - \frac{1}{\sqrt{1+b^2}} \ ,$$

you get a third equation from these two equations

[150][Note by AKTA:] The next equation appeared in the original text as follows:

$$\frac{1+r^2}{2r\frac{1}{2}\left(r^2-1\right)\frac{3}{2}} \left(\log r + \log \frac{1+\sqrt{1+\beta^2}}{1+\sqrt{1+(\beta r)^2}} \right) - \sqrt{\frac{r}{r^2-1}} \cdot \left(\frac{1}{r} - \frac{\beta^2 r^2}{1+(\beta r)^2+\sqrt{1+(\beta r)^2}} \right) = 0 \ .$$

$$(1+r^2)\sqrt{1+b^2} = 2r\sqrt{r^2+b^2}\ ,$$

out of which follows

$$b^2 = \frac{3r^2+1}{r^2-1}\ ;$$

and if one substitutes this value of b^2 into the first equation, one finds for the determination of r

$$\log\frac{r\sqrt{r^2-1}+r^2+1}{\sqrt{r^2-1}+2r} = \frac{(r^2-1)^{3/2}}{r(1+r^2)}\ .$$

From this equation one finds $r = 3.0951$, from which $a = r-1$, $b = (3r^2+1)/(r^2-1)$, $v = 2\pi(2+a)abc^3$, $l = \sqrt{wv/\varkappa}$ can be calculated, namely,

$$\begin{aligned} a &= 2.095\,1\ , \\ b &= 1.861\,78\ , \\ v &= 100.364\cdot c^3\ , \\ l &= 10.018\,2\cdot\sqrt{\frac{w\cdot c^3}{\varkappa}}\ . \end{aligned}$$

All of these regulations for the construction of the multiplier result solely from the requirement for the *greatest sensitivity*, without any consideration of the *attenuation*, and it therefore remains to discuss in particular how the *attenuation* behaves in such a multiplier. If we now note in relation to the *damping* that it generally depends not only on the multiplier, but also on the needle magnetism, but that in our case, where we are only dealing with the *theory of the multiplier*, the needle magnetism should be considered as given, it can easily be proven that under these conditions, with a given needle magnetism, it increases with the sensitivity and at the same time reaches the highest value with it, so that through the same construction of the multiplier, through which the highest sensitivity is produced, the strongest damping will also be obtained. If the measure of sensitivity f given above is also referred to again, then, as already stated in Section 8.6, $f^2 = [2w/k\tau]\cdot\lambda$, where τ is the period of oscillation of a closed circuit (which is related to the oscillation period t of an open circuit as $\sqrt{1+\lambda^2/\pi^2} : 1$) and w is the resistance of the entire circuit, to which the multiplier and inductor belong, further $e^\lambda : 1$ was the ratio of two consecutive oscillation arcs, the exponent of which $\lambda = [k\tau/2w]\cdot f^2$, for a given oscillation period, can be taken as a measure of the damping. This measure of attenuation is therefore, with a constant value of the factor $k\tau/2w$, proportional to the square of the sensitivity, from which it follows that the highest sensitivity also corresponds to the strongest attenuation.

For the case of the highest sensitivity, however, according to page 83[151]

$$f = 2\frac{m}{k}\sqrt{\frac{2\pi w}{c\varkappa}}\cdot\sqrt{\frac{b}{(2+a)a}}\cdot\log\frac{1+a+\sqrt{(1+a)^2+b^2}}{1+\sqrt{1+b^2}} = 1.792\,27\cdot\frac{m}{k}\sqrt{\frac{w}{c\varkappa}}\ ,$$

[151] [Note by AKTA:] Page 83 of this book corresponds to page 49 of Vol. 4 of Weber's *Werke*.

where w denotes the *multiplier resistance*; the resistance of the *entire circuit* was twice as great, so that according to this designation

$$\lambda = \frac{k\tau}{4w} f^2 = 0.803\,056 \cdot \frac{\tau}{c} \cdot \frac{m^2}{k\varkappa} \; .$$

Now, according to Section 8.9, the ratio should be $e^{\lambda} : 1 = e : 1$, consequently

$$\lambda = 0.803\,056 \cdot \frac{\tau}{c} \cdot \frac{m^2}{k\varkappa} = 1 \; .$$

Therefore, if the attenuation corresponding to the highest sensitivity were too strong, only the galvanometer needle would need to be substituted with a weaker one, whereby it can easily be arranged so that m/k, and consequently the sensitivity f, remains unchanged with this substitution.

8.14

For the sake of simplicity, only the rules for constructing a multiplier with *circular windings* have been developed in the previous Section, although *elongated* multipliers are usually used for finer observations; however, it is clear that similar provisions for the latter can be derived from Section 8.12, which were also developed for *elongated* multipliers. For practical use, however, such a more precise development will rarely be necessary, but it will suffice to set up the cross-section of an *elongated* multiplier in accordance with the regulations given in Section 8.12, but otherwise to follow the regulations given in the previous Section for circular multipliers, only with the modification that the expression of the sensitivity f is reduced in the ratio of

$$1 : \frac{2+\pi}{2\pi} \sqrt{\frac{\pi c(1+a/2)}{\pi c(1+a/2)+l^0}} \; ,$$

after which the ratio of two consecutive oscillating arcs is also to be altered, namely, its exponent is to be set:

$$\lambda = 0.803\,056 \cdot \left(\frac{2+\pi}{2\pi}\right)^2 \cdot \frac{\pi c(1+a/2)}{\pi c(1+a/2)+l^0} \cdot \frac{\tau}{c} \cdot \frac{m^2}{k\varkappa} \; ,$$

consequently, because according Section 8.9, $e^{\lambda} : 1 = e : 1$,

$$0.803\,056 \cdot \left(\frac{2+\pi}{2\pi}\right)^2 \cdot \frac{\pi c(1+a/2)}{\pi c(1+a/2)+l^0} \cdot \frac{\tau}{c} \cdot \frac{m^2}{k\varkappa} = 1 \; .$$

It is sufficient here, as in the Note to Section 8.10, to consider only a single winding of the multiplier, whose torque is reduced for large values of l^0 in the ratio of $2\pi : 2+\pi$. In addition, by increasing the mass of the wire through the extension of the multiplier, with unchanged cross-section and resistance, in the ratio of $\pi c(1 + a/2) : \pi c(1 + a/2) + l^0$, the number of windings and, proportionally, both the torque and the sensitivity of the whole multiplier, decrease in the ratio of $\sqrt{\pi c(1 + a/2) + l^0} : \sqrt{\pi c(1 + a/2)}$.

Finally, it should be noted that in the theory of the multiplier developed here, also for the sake of simplicity, only a simple needle *enclosed by the multiplier* has been considered.

For an *astatic needle system*, according to Section 8.11, the results found can therefore only be directly applied if the needle not enclosed by the multiplier, which can easily be done, is kept so far away from the multiplier, that its action on the same needle disappears against the action on the enclosed needle. As a rule, however, this is not the case, but the two needles are usually suspended symmetrically at an equal distance from the upper side of the multiplier, which results in an increase in both sensitivity and attenuation. With an *elongated* multiplier, the sensitivity is increased by a ratio of almost 3 : 4 and the attenuation by a ratio of 9 : 16.

III - Galvanometric Observations

8.15

After discussing the method of absolute resistance measurement and the construction of the galvanometer, we move on to the consideration of the *observations*, which can be broken down into *galvanometric* and *magnetic*, but of which the latter only concern the intensity of the horizontal component of the Earth's magnetism, its determination in terms of absolute value has been dealt with so completely by Gauss that the necessary observations do not require any further consideration.

If the *galvanometric observations* were to be made with a galvanometer constructed according to the regulations of the previous Section, then for the production of it the *standard* whose resistance is preferred, namely, the etalon intended for general use and serving to compare the resistances of all bodies, must actually be available and given; for it is intended to provide the yardstick according to which the galvanometer is to be constructed, so that the resistance of the circuit formed by the multiplier of the galvanometer and the inductor becomes equal to its resistance, which is necessary so that both can be directly compared with each other.

If this *standard resistance* was really present and given and its absolute value was at least approximately known, namely, $W = 2w$, then, according to the rules developed in the previous Section, a more specific approach to the construction of the whole galvanometric apparatus, the galvanometer as well as the inductor, could easily be made in the following way.

For example, assume the radius of the space to be left free for the galvanometer needle is $c = 20$ millimeters. In the case of a *circular* multiplier, the volume of the multiplier would then be $= 802\,912$ cubic millimeters according to the equation $v = 100.364 \cdot c^3$. This wire would then have to be wound according to the rule that $b = 1.861\,78$ in the form of a ring with a clear diameter of $2c = 40$ millimeters and a height of $2bc = 74.471\,2$ millimeters, after which the outer ring diameter $2(1+a)c = 123.804$ millimeters. — In the case of an *elongated* multiplier, which is necessary in order to be able to use stronger needles and thereby produce stronger damping, the specified cross section would have to be changed slightly according to the rules developed in the previous Section, namely, to increase the outer diameter slightly, but to reduce the ring height slightly, whereby the size of the cross section would also suffer a small change. But since the size of the cross-section that is then precisely determined would correspond to a maximum value of the sensitivity, where small changes in the cross-section have only an insignificant influence on the sensitivity, it is sufficient for the practical purpose of the survey to adhere to the unchanged size of the cross-section calculated for a circular multiplier, for which it is necessary, however, to increase the volume of the multiplier in the ratio of $\pi c(1 + a/2) : \pi c(1 + a/2) + l^0$, if l^0 denotes the length of the parallel sides connected between the two half rings. Now $\pi c(1 + a/2) = 128.65$ millimeters, therefore, for $l^0 = 128.65$ [mm], the volume would be $= 1\,605\,824$ [mm^3], for $l^0 = 385.95$ [mm], the volume $= 3\,211\,648$ cubic millimeters. If one assumes the average density $= 6$ for copper, but taking into account the wrapping used for insulation and the gaps remaining in the cylindrical wire shape, the wire mass would be estimated at $9\,634\,944$ [mg] for $l^0 = 128.65$ [mm], while for $l^0 = 385.95$ [mm] it would be estimated at $19\,269\,888$ milligrams. This wire would therefore have to be wound on a multiplier frame, which would be formed by two semicircles, each of 20 millimeters in radius, and by two parallel sides, each of the length $l^0 = 128.65$ or $l^0 = 385.95$ millimeters, and that would leave a ring-shaped space for the wire, which should be slightly smaller in height than the value $2bc$ given for a circular multiplier, that is, a space of almost 70 millimeters in height. The length l of the wire to be formed from the given mass would be obtained from the equation $l^2 = [w/\varkappa] \cdot v$, where $\varkappa$ denotes the [specific] resistance of the wire with 1 millimeter length and 1 square millimeter cross section, for which the value $\varkappa = \frac{1}{6} \cdot 2\,000\,000$ can be calculated in round numbers, assuming a density $= 6$. This results in the value of $l = 2.1949\sqrt{w}$ for $l^0 = 128.65$, but $l = 3.104\sqrt{w}$ for $l^0 = 385.95$. If, for example, the given standard resistance was $W = 2w = 2 \cdot 10^{10}$, a wire length of $219\,490$ millimeters, which would form 426 windings, would have to be produced from the given mass for the case $l^0 = 128.65$, and a wire length of $310\,400$ millimeters, which would form 302 windings, for the case $l^0 = 385.95$.

As far as the manufacture of the *astatic needle system* is concerned, the moment of inertia k of the same can be divided into that of the two needles and that of their fixed connecting piece together with the mirror and other accessories. The latter can be considered as given since it is independent of the choice of needles and is assumed, for example, for millimeters and milligrams as units of measure of length and mass, $= 20 \cdot 10^6$. The length of the needles, which must depend on the length of the multiplier, can also be estimated at a maximum of 150 millimeters for $l^0 = 128.65$ and a maximum of 400 millimeters for $l^0 = 385.95$, after which can be calculated the moment of inertia of the two needles in the former case $= \frac{1}{12} \cdot 150^2 \cdot p$, in the latter case $= \frac{1}{12} \cdot 400^2 \cdot \left(\frac{8}{3}\right)^3 p$, where thin and homogeneous needles of similar shape are assumed and the mass of the smaller pair of needles is denoted by p. According to this, the moment of inertia of the entire astatic needle system, for $l^0 = 128.65$, is $k = 20 \cdot 10^6 + \frac{1}{12} \cdot 150^2 \cdot p$, for $l^0 = 385.95$, $k = 10 \cdot 10^6 + \frac{1}{12} \cdot 400^2 \cdot \left(\frac{8}{3}\right)^3 \cdot p$. The oscillation period of the system can be adapted to the needs of the observation by appropriately choosing the suspension wire, after which the oscillation period $\tau = 30$ seconds is assumed. Since the [specific] resistance of the wire with a length of 1 millimeter and a cross-section of 1 square millimeter has now been assumed to be $\varkappa = \frac{1}{6} 2\,000\,000$, then according to the equation of Section 8.14

$$0.803\,056 \cdot \left(\frac{2+\pi}{2\pi}\right)^2 \cdot \frac{\pi c(1 + a/2)}{\pi c(1 + a/2) + l^0} \cdot \frac{\tau}{c} \cdot \frac{m^2}{\varkappa k} = 1 \ ,$$

for $l^0 = 128.65$ [mm] the value of the magnetism m of a needle[152] is equal to the geometric mean of the two numbers $824\,880$ and $20 \cdot 10^6 + 1875p$; for $l^0 = 385.95$ [mm] is equal to that [geometric mean] of the two numbers $1\,649\,760$ and $20 \cdot 10^6 + 252\,840p$. So if the mass of a smaller needle is assumed to be $\frac{1}{2}p = 50\,000$ milligrams, its magnetic moment should be

[152][Note by AKTA:] That is, m is the magnetic moment of this needle.

= 13 083 000, hence for each milligram on average 261.66 units; the mass of the larger needle would then be 948 160 milligrams with a similar shape and its magnetic moment should be = 204 310 000, hence 215.48 units for each milligram. However, magnetic needles of this size and strength are easy to prepare.

According to the comment made at the end of Section 8.14, these provisions only apply to an *astatic needle system* if the outer needle is kept far enough away from the multiplier. However, if the arrangement is such that the upper side of the multiplier lies symmetrically between both needles, the change in the above equation is that the unit in the second term is substituted with the fraction 9/16, according to which the magnetic moments of the needles are obtained in the ratio 4 : 3 smaller, namely, for the smaller needle = 9 812 250 [units], for the larger needle 153 232 500 units.

Finally, as far as the *inductor* is concerned, it should be constructed in such a way that the length of wire $2\pi\Sigma r$ used for it has a resistance which is equal to the resistance w of the multiplier after subtracting the resistance of the connecting wires; so, if all wires are of the same type, it is $2\pi\Sigma r = l - l'$, if l' denotes the length of the two connecting wires. The integral value of the current produced by an induction surge is then, with inducing Earth magnetism T, $\int idt = [2\pi T/2w] \cdot \Sigma r^2$ and consequently the angular velocity imparted to the astatic needle system by such an induction surge, according to Section 8.13, is $\gamma = f \cdot \int idt = 2\pi T \cdot \Sigma r^2 \cdot \sqrt{1/wk\tau}$, if τ is the period of oscillation with a closed circuit and if in the ratio $e^\lambda : 1$ of two consecutive oscillation arcs, λ is $= 1$. This angular velocity γ, when given to the needle in the rest position, results in the size of the first subsequent deflection $\alpha = \gamma\tau \cdot \frac{1+e^2}{\sqrt{1+\pi^2}} \cdot e^{-\left(\frac{3}{2}+\frac{1}{\pi}\arctan\frac{1}{\pi}\right)}$, or, if we replace the value for γ,

$$\alpha = 2\pi T \cdot \Sigma r^2 \cdot (1+e^2)\sqrt{\frac{\tau}{(1+\pi^2)wk}} \cdot e^{-\left(\frac{3}{2}+\frac{1}{\pi}\arctan\frac{1}{\pi}\right)} .$$

If the radius of all inductor windings were the same and their number n, then the total wire length would be $l - l' = 2n\pi r$ and $\Sigma r^2 = nr^2 = [(l-l')/2\pi] \cdot r$. After substituting this value, one obtains from the previous equation

$$r = \frac{\alpha}{(1+e^2)(l-l')T} \cdot \sqrt{\frac{(1+\pi^2)wk}{\tau}} \cdot e^{\frac{3}{2}+\frac{1}{\pi}\arctan\frac{1}{\pi}} ,$$

where α can be taken as large as the scale with which the deflections of the needle are to be observed allows, since the deflections to be observed using the throwback method never quite reach the value α. So if, as is usually the case with magnetometers, a 1 meter long scale is needed at a distance of 5 meters from the mirror, $\alpha = 1/20$ can be set, and $T = 1.81$ (as currently in Göttingen),[153] so one obtains, since $\tau = 30$ has been assumed,

$$r = 0.009\,799 \cdot \frac{\sqrt{wk}}{l - l'} .$$

In the example already given, where $W = 2w = 2 \cdot 10^{10}$, this results in the *first* case, namely, with a multiplier for which $l^0 = 128.65$, $l = 129\,490$ and $k = 20 \cdot 10^6 + 1875p$, for $p = 100\,000$ and $l' = 10\,000$, $r = 67.38$ millimeters; in the *second* case, namely, with a multiplier for which $l^0 = 385.95$, $l = 310\,400$ and $k = 20 \cdot 10^6 + 252\,840p$, for the same values of p and

[153][Note by AKTA:] According to Section 2.3, this horizontal component of the Earth's magnetic force corresponds in modern field theory to a horizontal component of the geomagnetic field given by $B_{horizontal} = 1.81 \times 10^{-5}$ $Tesla$.

l', $r = 518.9$ millimeters. However, it should be noted that in the case of an astatic needle system it was assumed that the outer needle was so far away from the multiplier that its action on it disappeared compared to that on the enclosed needle. But if both needles lay symmetrically against the upper side of the multiplier, r would have to be reduced in almost the ratio $4 : 3$ and in the first case it should be $r = 50.53$, in the second case it should be $r = 389.18$ millimeters.

However, if these found radii do not fit correctly for the exact measurement or convenient rotation of the inductor, you only need to use a different type of wire to construct the inductor than the one used for the multiplier. For example, if you want the radius to have the value μr instead of r, you take a wire whose length and cross-section are related to those of the multiplier wire as $1 : \mu$; the resistance and the sum of the circular areas enclosed by all the windings, and consequently also the actions produced by an induction surge, remain completely unchanged.

With an apparatus manufactured in this way, in accordance with all the prescribed rules, the *galvanometric observations* would be made from which (in connection with the measurement of the horizontal geomagnetic force carried out with the instruments of a Magnetic Observatory and traced back to the place and time of the galvanometric observations) the given *standard-resistance* should be determined according to its absolute value. — At present, however, it is not possible to manufacture such an apparatus where there is still a lack of the standard, which is in general use and which serves to compare the resistances of all bodies.

If, as already noted, it is not a question of the definitive determination of such a *standard-resistance*, but only of testing the *galvanometric observations* required for this, which require an accuracy that is at least equal to that of the other observations, then observations made with an apparatus which does not conform to all prescribed rules are sufficient, if only the deviations either have little influence on the accuracy or if their influence can be easily and precisely determined. The following observations are therefore sufficient for this purpose.

8.16

The following observations were made with an apparatus that deviated from the given regulations only in that an already existing and precisely known inductor was used, the resistance of which was significantly greater than that of the multiplier. The resulting influence on the observations is easy to determine and will be discussed in more detail below. This deviation is therefore not of any significant disadvantage if it is simply a matter of testing the security and accuracy that can be achieved in *galvanometric observations*, and the following observations made with it can be used quite well for this purpose.

The following Table contains a set of such observations made by the *throwback method*,[154] arranged in five columns in such a way that, in order of time, the *first* column contains observations 1, 5, 9 etc., the *second* contains observations 2, 6, 10 etc., the *third* contains observations 3, 7, 11 etc., the *fourth* contains observations 4, 8, 12 etc. The observations in the first two columns are the elongations of the galvanometer needle that initially precede and follow the *negative* induction surges. The observations in the last two columns are the elongations that initially precede and follow the *positive* induction surges. The numbers are the scale divisions observed at the moment of greatest elongation.

[154] [Note by AKTA:] See footnote 139.

1.	2.	3.	4.
661.5	600.9	844.8	904.9
659.6	598.7	842.7	903.0
657.2	596.2	840.0	900.4
654.5	593.5	837.5	897.7
751.7	590.4	834.8	894.8
649.5	588.3	832.6	892.7
647.3	586.2	830.5	890.8
645.4	584.7	828.2	889.0
643.3	582.3	826.7	886.7
641.4	580.4	824.3	884.5
639.2	578.0	822.3	882.6
637.5	576.6	820.3	881.3
635.2	574.8	819.0	879.7
634.2	573.9	817.9	879.3
632.8	573.1	816.1	877.0

The resting position of the needle can be determined from four consecutive observations for the moment falling symmetrically in the middle between these observations. The average between two such successive periods of rest then gives the period of rest which corresponds to the moment of elongation in between. In this way, the resting positions corresponding to all individual elongation observations in the previous Table were calculated and the elongation widths resulting after their deduction were compiled in the following Table: the first two and the last two data in this Table are determined by adding two preceding and two following observations made solely for this purpose.

1.	2.	3.	4.
−92.49	−152.46	+92.01	+152.63
−92.14	−152.54	+92.00	+152.91
−92.24	−152.57	+91.89	+152.96
−92.29	−152.64	+92.05	+152.99
−92.29	−152.89	+92.15	+152.69
−92.07	−152.74	+92.10	+152.74
−92.14	−152.74	+92.04	+152.76
−92.16	−152.35	+91.64	+153.00
−92.21	−152.74	+92.19	+152.66
−92.10	−152.52	+91.93	+152.70
−92.05	−152.76	+91.99	+152.68
−92.00	−152.49	+91.66	+153.18
−92.54	−152.57	+91.95	+152.89
−92.36	−152.47	+91.75	+153.43
−92.75	−151.94	+91.49	+152.91
−92.255	−152.561	+91.923	+152.875

At first sight there is a very satisfactory agreement among these observations, which becomes even more clear when one considers that every small disturbance which slightly reduces or enlarges the elongation before an induction surge causes an enlargement or reduction of the next following elongation. The averages from the first two and the last two columns, in which these opposite influences almost balance each other, therefore provide an even more definite test of the accuracy of the results which can be obtained by this method of observation. These averages are as follows.

1. 2.	Difference from the mean.		3. 4.	Difference from the mean.
−122.475	+0.067		+122.320	+0.079
−122.340	−0.068		+122.455	−0.056
−122.405	−0.003		+122.425	−0.026
−122.465	+0.057		+122.520	−0.121
−122.590	+0.182		+122.420	−0.021
−122.405	−0.003		+122.420	−0.021
−122.440	+0.032		+122.400	−0.001
−122.255	−0.153		+122.320	+0.079
−122.475	+0.067		+122.425	−0.026
−122.310	−0.098		+122.315	+0.084
−122.405	−0.003		+122.335	+0.064
−122.245	−0.163		+122.420	−0.021
−122.555	+0.147		+122.420	−0.021
−122.415	+0.007		+122.590	−0.191
−122.345	−0.063		+122.200	+0.199
−122.408	+0.095		+122.399	+0.089

From this you can see that the difference between the individual values and their mean is on average less than 1/10 part of the scale.

8.17

According to the fineness of the observation method examined in the previous Section, it is sufficient for further consideration to stick solely to the four averages of the observed elongations, namely, corresponding to the four columns:

−92.255	−152.561	+91.923	+152.875

The first of the third and the second of the fourth elongation should now be oppositely equal. The reason why this is not exactly true is that as a result of the gradual decrease in the rest position of the galvanometer needle, which is immediately apparent from the observations cited in the previous Section, the individual induction surges do not occur exactly at the moment when the needle passed the rest position, but in a moment when the needle passed a slightly higher scale point (namely, the one which had been resting shortly before), from which it easily follows that with a uniformly decreasing resting point, the first and third elongations are almost as small as the second and fourth had to be found too big. This gradual decline in the rest position was a result of the elastic after-effect[155] of the recently suspended iron wire supporting the galvanometer needle, and would disappear in time. It did not seem necessary to wait for this time because, given the uniformity of the sinking, there would be no detrimental influence on the measurement. It is also easy to take

[155] [Note by AKTA:] In German: *der elastischen Nachwirkung*. This effect was discovered by Weber in 1835, [Web35] and [Dör91].

this influence into account by adding a small correction $+x$ to the observed elongations. So if, instead of the observed elongations, you put

$-92.255 + x$	$-152.561 - x$	$+91.923 + x$	$+152.875 - x$

and $x = 0.1615$, you get the corrected values:

-92.0935	-152.7225	$+92.0845$	$+152.7135$,

which do not deviate from the required symmetry by 1/100 part of the scale.

For further consideration, where only the difference of the fourth and second elongation $= 2a'$ and that of the third and first $= 2b'$ are taken into account, these corrections are not necessary at all because these differences are independent of this, namely:

$$2a' = 305.436 \ ,$$
$$2b' = 184.178 \ .$$

Let r now denote the distance of the mirror from the scale and set

$$a' = r \tan 2\varphi \ ,$$
$$b' = r \tan 2\varphi' \ ,$$

so φ and φ' are the deflection angles of the galvanometer needle and, if they are small, can be equated to the elongations labeled a and b in Section 8.10, from which the angular velocity γ and the logarithmic decrement λ should be calculated. For larger values of φ and φ', however, a and b, which should remain proportional to the angular velocities, grow like the sines of the half deflection angles, after which

$$a = 2 \sin \frac{1}{2}\varphi \ ,$$
$$b = 2 \sin \frac{1}{2}\varphi' \ ,$$

is to be set. Strictly speaking, these exact expressions[156] for needles oscillating without damping would have to be accompanied by a small correction resulting from the hitherto

[156][Note by WW:] For needles that oscillate without damping, according to Section 8.4, a is the product of the oscillation period t divided by π into the highest oscillation velocity. However, the greatest oscillation velocity is $= \gamma/2$, when γ denotes the entire change brought about by the induction surge, because without damping the needle velocity before and after the induction surge (in the throw-back method) should be oppositely equal, therefore $a = [t/2\pi] \cdot \gamma$. However, a needle whose period of oscillation is $= t$ oscillates just like a simple pendulum of length $l = gt^2/\pi^2$, which, when at the lowest point of its path, has the angular velocity $\gamma/2$, rises to the height $\frac{1}{4}\gamma^2 l^2/2g = l \sin \text{vers}\, \varphi$, from which $\gamma/2 = [2\pi/t] \cdot \sin\frac{1}{2}\varphi$ follows, so $a = 2\sin\frac{1}{2}\varphi$.

undeveloped theory of needle oscillations for larger oscillation arcs under the action of damping, which, however, may be neglected in the case of oscillation arcs as small as those observed with magnetometer needles.

If you now expand a and b into powers of a'/r and b'/r, you get

$$a = \frac{1}{2} \cdot \frac{a'}{r} - \frac{11}{64} \cdot \frac{a'^3}{r^3} + \dots ,$$

$$b = \frac{1}{2} \cdot \frac{b'}{r} - \frac{11}{64} \cdot \frac{b'^3}{r^3} + \dots .$$

In the observations described in the previous Section, from which $a' = 152.718$, $b' = 92.089$ were obtained in parts of the scale, the distance of the mirror from the scale was $r = 3245$ scale divisions; consequently,

$$a = 0.023\,503 ,$$
$$b = 0.014\,186 .$$

8.18

The galvanometer observations described in Section 8.16 must also be accompanied by the auxiliary observations with the *open* circuit, regarding the *period of oscillation* and the *decrease in the oscillation arcs*. These observations gave

Number of the oscillation	Time	Oscillation arc
0.	$11^h\ 34'\ 59.15''$	464.0
17.	$45'\ 23.20''$	376.3
34.	$55'\ 47.39''$	307.6

These observations yield the oscillation period $= 36.7109''$ reduced to infinitely small arcs according to clock time adjustment, or, after reducing the clock time adjustment to mean time, the oscillation period with the *open circuit* was

$$t_0 = 36.7061'' .$$

This also results in the logarithmic decrement for the decrease in the oscillation arcs when the circuit is *open*

$$\lambda_0 = 0.012\,09 .$$

To these observation results must be added the constants of the instruments, namely, the moment of inertia K of the galvanometer needle, and, for the inductor, the value of $S = \Sigma \pi r^2$, if r denotes the radius of the inductor windings, and finally the horizontal intensity of the Earth's magnetism found by *magnetic measurements* at the time and place of the galvanometer observations, namely,

$$K = 1\,132\,000\,000 \ ,$$
$$S = 39\,216\,930 \ ,$$
$$T = 1.816\,4 \ .$$

The inductor was the same as that described in the Fifth Volume of these Treatises[157,158] and that was used for inclination measurements, where the value of S was also given. The values of the moment of inertia K and the horizontal intensity T of the Earth's magnetism were determined according to the instructions given by Gauss in the *Intensitas*.[159]

These results of the auxiliary observations, added to those of the galvanometric ones, finally give in *absolute value* all the elements for determining the resistance of the circuit formed by the multiplier and inductor at the temperature of 17.5 degrees of the 100-divisions scale at which the galvanometer observations were made. Because first of all, according to the theory of the throwback method, it follows from the values listed

$$a = 0.023\,503, b = 0.014\,186, t_0 = 36.706\,1, \lambda_0 = 0.012\,09 \ ,$$

$$\gamma = \frac{\sqrt{\pi^2+\lambda_0^2}}{t_0}\left(a\sqrt{\frac{a}{b}} + b\sqrt{\frac{b}{a}}\right)\cdot\left(\frac{b}{a}\right)^{\frac{1}{\pi}\arctan\left(\frac{1}{\pi}\log\text{nat}\ \frac{a}{b}\right)} = 0.003\,330\,4 \ ,$$

$$\lambda_1 = \log\text{nat}\ \frac{a}{b} = 0.504\,87 \ ;$$

then according to Section 8.7, where $S = n\pi r^2$ was set, with the given values of K, S, T you get the resistance of the entire circuit

$$w = \frac{8S^2T^2}{K\gamma^2 t_0}\cdot(\lambda_1-\lambda_0)\cdot\sqrt{\frac{\pi^2+\lambda_0^2}{\pi^2+\lambda_1^2}} = 42\,855\cdot 10^6\frac{\text{millimeter}}{\text{second}} \ .$$

8.19 Thomson's Standard and Other Etalons

Even if there is not yet a generally accepted and used resistance standard to which the result derived from the previous observations could be related, there are several standards which have gained particular interest by the studies made with them or by the measurements reduced to them. The specific reason for the observations described above was to determine the absolute resistance value of such an etalon, which is referred to as Thomson's *standard*. It was available for this purpose in two copies, kindly communicated by Professor William Thomson of Glasgow,[160] one in copper wire, the resistance of which, according to Professor Thomson, increases by 36/10000 with each degree of the 100-divisions scale, and the other in

[157] [Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. II, p. 323.

[158] [Note by AKTA:] [Web53d, p. 323 of Weber's *Werke*].

[159] [Note by AKTA:] See footnote 55 on page 34.

[160] [Note by AKTA:] William Thomson (1824-1907), also known as Lord Kelvin, was a British mathematician, physicist and engineer. See [Llo80] with Portuguese translation in [Llo07].

nickel-silver wire,[161] the resistance of which increases by 36/100000 with each degree. Both were guaranteed to be exactly the same for the temperature of 16.3 degrees.

With this special purpose in mind, it was particularly important to ensure that, even if not all of the requirements made in the previous Section could be met in these observations, at least the resistance equality of the circuit formed from the multiplier and inductor with the standard was achieved as closely as possible, in order to put the result found for that circuit in the most direct and precise relationship with this standard resistance. This equality had been established to within 1/1850 of the standard resistance, as was shown by a very precise comparison with the copper copy made at 16.6 degrees.

This is followed by the resistance of the copper copy of Thomson's *standard* for 17.5 degrees of the 100-divisions scale from the results found in Section 8.18

$$= 42\,832\,000 \frac{\text{meter}}{\text{second}}$$

or for 0 degree temperature

$$= 40\,293\,000 \frac{\text{meter}}{\text{second}} \ .$$

Another standard was the Siemens' standard already discussed in the Introduction,[162] which formed the basis of a Siemens' resistance scale made of nickel-silver wires as a unit. This Siemens' *unit of resistance*, given in a nickel silver wire at 15 degrees of the 100-divisions scale, was, according to Siemens' data, exactly equal to the resistance of a mercury column with a length of 1 meter and a cross section of 1 square millimeter at 0 degree temperature. If this nickel-silver wire was connected to the circuit formed by the multiplier and inductor, the resistance of the circuit was increased in the ratio of 1 : 1.2395, from which the resistance of this nickel-silver wire is obtained at 17.5 degrees on the 100-divisions scale = 10 266 000 meter/second, hence at 15 degrees, that is, the Siemens standard resistance itself, = 10 257 000 meter/second. However, it is easy to see that the determination of the Siemens standard resistance derived from this comparison is not quite as accurate as that of the preceding Thomson standard resistance, because the influence of unavoidable observation errors on the ratio of two different resistances is much greater than when comparing two very close resistances.

Finally, Siemens also compared his standard resistance with Jacobi's *standard resistance*, whose absolute value, as already mentioned in the Introduction,[163] was found to be = 5 980 000 meters/second after an earlier, less precise measurement. Siemens found that the ratio of his standard resistance to the Jacobian would be 1000 : 661.8, from which the Jacobian standard resistance would follow = 6 788 000 meters/second. However, Siemens himself noted that he did not have the Jacobi normal standard at his disposal, but that he had obtained several copies of it, which, however, differed greatly from one another. This is probably the main reason for the deviation of over 12 percent, since the error in the earlier measurement can be estimated at a maximum of 2 to 3 percent. Anyway, this case can serve as evidence of how important an appropriately established Institution would be for the general dissemination of consistent, precisely tested resistance standards and scales.

[161][Note by AKTA:] In German: *Neusilberdraht*. Nickel-silver, German silver, argentan, new silver or alpacca is a range of alloys of copper, nickel, and zinc which are silvery in appearance but contain no silver.

[162][Note by AKTA:] See footnote 106.

[163][Note by AKTA:] See footnote 105.

8.20

If only the *galvanometric observations* referred to in Section 8.16 are considered, the mean error of a distance measured on the scale is 0.092 part of the scale, according to which the distances considered in Section 8.17 are obtained as:

$$
\begin{aligned}
2a' &= 305.436 \pm 0.092 \ , \\
2b' &= 184.178 \pm 0.092 \ .
\end{aligned}
$$

From this it follows that:

$$
\begin{aligned}
a &= 0.023\,503 \left(1 \pm \frac{1}{3320}\right) \ , \\
b &= 0.014\,186 \left(1 \pm \frac{1}{2000}\right) \ ,
\end{aligned}
$$

and finally the angular velocity γ caused by an induction surge of the needle, as considered in Section 8.18, and the logarithmic decrement λ_1 of the decrease in oscillation, [namely:]

$$
\begin{aligned}
\gamma &= 0.003\,330\,4 \left(1 \pm \frac{1}{2554}\right) \ , \\
\lambda_1 &= 0.504\,87 \left(1 \pm \frac{1}{865}\right) \ .
\end{aligned}
$$

Could the sensitivity of the galvanometer be increased according to the regulations developed in the previous Section, so that the distance

$$2a' = 1000 \pm 0.92$$

would be obtained, and if the damping could also be increased so that $\lambda_1 = 1$, then,[164]

$$2b' = 367.9 \pm 0.92 \ ;$$

in the same way, the mean error of γ would be

$$\pm \frac{1}{7127} \gamma \ ,$$

the mean error of λ_1 can be obtained

$$= \pm \frac{1}{3753} \lambda \ ,$$

[164] [Note by AKTA:] The next equation appeared in the original text as:

$$2b' = 367.9 \pm 092 \ ;$$

which may be regarded as the highest degree of accuracy with which these results of galvanometric observations can be determined, but only when the sensitivity and attenuation are regulated in the manner indicated. Errors in the scale division and in the measurement of the distance of the mirror from the scale are not taken into account at all because they are not to be counted as purely galvanometric observations, but as auxiliary observations.

Finally, the description of the apparatus with which the observations just considered were made shows how easy it would actually be to produce the required sensitivity and attenuation. For it has already been mentioned in Section 8.16 that this apparatus differed essentially from the rules prescribed in the previous Section only in that an already existing, exactly known inductor was used, the resistance of which was significant, namely, almost four times greater than that of the multiplier. Therefore, if only the cross-section of the inductor wire had to be increased in the ratio of 3 : 8, the resistance of the entire circuit would have been reduced in the ratio of $8 + 2 : 3 + 2$, whereby the sensitivity as well as the attenuation would have been increased in the opposite ratio, namely, in that of 1 : 2.

It even turns out that the sensitivity as well as the attenuation could easily be increased far beyond the prescribed limits, whereby the regulations given in Section 8.9 regarding these limits would actually come into effect.

Finally, it follows that it is not due to the *galvanometric observations* that the absolute value of a given standard resistance would not be obtained with great certainty and precision if the prescribed rules were followed; because the error arising from these observations alone would, according to the above information, only be about 1/2530 of the entire resistance; on the contrary, it will be difficult to carry out the other observations, especially the *magnetic* ones, to determine the intensity of the horizontal geomagnetic force at the place and time of the galvanometric observations, with corresponding accuracy, from which it follows that the unavoidable uncertainty in the *absolute value of the given standard resistance* resulting from the determination of Earth magnetism would not be significantly increased by the uncertainty of the *galvanometric measurement* carried out in accordance with the prescribed rules, so that the main purpose of the present Treatise, namely, to explain how to achieve that objective, may be regarded as fulfilled.

IV - Copying Methods

8.21

It follows from the two preceding paragraphs that a galvanometric apparatus for absolute resistance measurement can be manufactured with the greatest practicality only for the measurement of *one certain standard resistance*, which is sufficient because the comparability of the resistances of all bodies to one another only requires the exact knowledge of *one* such standard resistance in absolute terms in order to indirectly gain knowledge of the absolute resistance values of all bodies and to make all possible applications of it.

However, the same rules apply to the selection and determination of such a *standard resistance* as to the selection and determination of *fundamental units of measure*[165] *of other types of quantities*: only those types of quantities are suitable for the establishment of fundamental

[165][Note by AKTA:] In German: *Grundmaassen*. This expression can be translated as fundamental (or basic) units of measure, fundamental measures, fundamental dimensions or fundamental standards. See also footnote 102.

standards whose existing dimensions can be preserved unchanged, moved from one place to another at will, and reproduced using a method of the finest copying. Wherever these conditions can be met, the establishment of such a *fundamental unit of measure* would appear to be desirable because of the practical importance of the simplification of measurements that can thereby be achieved, which at the same time increase in finesse and accuracy. But where these conditions cannot be satisfied, the determination of an *unit of measure* is necessary, but this does not have to be a *fundamental unit of measure*, but can also be a *derived unit of measure*, namely, *an absolute unit of measure reduced to the basic units of measure of other types of quantities*. For example, velocities are one of those types of quantities that are not suitable for establishing a fundamental unit of measure because a velocity cannot be maintained unchanged, cannot be moved arbitrarily and cannot be copied exactly. On the other hand, straight lines and body masses are, as is well known, particularly suitable for establishing fundamental units of measure.

Whether *galvanic resistances* are suitable for establishing a *fundamental unit of measure* in a certain etalon or standard also depends only on whether an existing resistor can be preserved unchanged, moved from one place to another at will and reproduced using a method of the finest copying. If there is an unchanging resistance attached to a certain metal wire (like an unchanging length on a rod or an unchanging mass on a weight), then it is self-evident that the existing resistance of the wire can be maintained unchanged with the wire and moved from one place to another; it remains only to be proved that it can also be reproduced by the finest copying methods.

If one considers that the most important and essential thing about a *fundamental unit of measure*, in addition to its immutability for all times and places, is the *general use* of it, and if one considers the great difficulties which such a general introduction encounters, then for this reason it might seem expedient to limit the number of such *fundamental units of measure* as much as possible and to expand the use of the *derived units of measure*, namely, the *absolute units of measure* reduced to the fundamental units of measure of other types of quantities; but on closer examination one will easily see that, instead of substituting *absolute units of measure* for *fundamental units of measure*, it is more expedient to bring the *fundamental units of measure into quite exact agreement with the absolute units of measure* by which they could possibly be replaced because the *absolute unit of measure* can then represent the *fundamental unit of measure* where it is not widely used.

The *absolute unit of measure does not permit direct implementation*, as can be seen from the example of the absolute unit of resistance, but every absolute measurement is always *mediated* by certain laws of the interdependence of different types of quantities observed simultaneously in an object, and therefore requires a planned *combination of different observations*, the execution of which requires greater effort and work, and also sets stricter limits on accuracy, than if the results were based directly on simple observations. On the other hand, a *fundamental unit of measure is directly applicable* for types of quantities which are suitable for this purpose, combined with greater simplicity of measurement and greater fineness of results, whereby it should be particularly emphasized that the *freedom in the choice of such a fundamental unit of measure* also allows a really existing quantity that is equal to the *absolute unit of measure, or at least very close to it, to be taken as the fundamental unit of measure, or a higher or lower decimal unit of the same, or at least one which comes very close to it after the finest examination, and thus to preserve all the advantages connected with the use of absolute units of measure* which the laws of the dependence of different kinds of quantities considered simultaneously on one object confer on each other.

For example, the *derived unit of measure* we use for resistance is the *absolute unit of measure* traced back to the *fundamental unit of measure of space, time and mass*, namely, *millimeters*, *seconds* and *milligrams* have been assumed as fundamental units of measure for the latter types of quantities. If one persists with these latter fundamental units of measure, so the freedom available when choosing a *fundamental unit of measure for resistance* can very well be used, a really existing resistance which, after the most precise examination, is equal to *a higher decimal unit of that absolute unit of measure* or at least comes very close to it (which is the case with Siemens' measure, which is almost 10^{10} times larger than that absolute unit of measure, which actually takes place approximately), to be taken as the *fundamental unit of measure*, whereby all the advantages associated with the use of that *absolute unit of measure* would also be preserved for the use of this fundamental unit of measure.

But it must be possible to really apply such a fundamental unit of measure to *general or at least very extensive application*. However, the only way to do this is to *reproduce the standard by copying*, if a method is available which allows all copies made afterward to be considered *completely identical* for all practical purposes. Without a doubt, the establishment of a fundamental unit of measure for galvanic resistors is also very desirable, but above all it is necessary to check whether the *coping methods of the resistors* meet the stated purpose. This test becomes even more necessary in view of our investigation, because the wire circuit of our galvanometric apparatus itself is in no way suitable to serve as a fundamental unit of measure, since it cannot be moved from one place to another at will. This wire circuit should therefore also be a *copy of the fundamental standard*, which for all practical purposes may be regarded as completely identical to the fundamental unit of measure, so that all determinations obtained for this circuit also apply to the *fundamental unit of measure*.

8.22 Copying Methods Without Current Division

Copying is based on the judgment about the equality or inequality of two quantities. Directly from the definition of resistance arises a first method of judging the equality or inequality of two resistances, namely, according to the following principle: *the resistance of two conductors is the same if the same currents are excited in them by the same electromotive forces*. The accuracy of the method based on this principle is, however, limited: (1) by the accuracy that can be achieved when assessing the equality of two electromotive forces acting on two different conductors; (2) by the accuracy with which one can observe and compare the current intensities in two different conductors. Closer examination easily shows that this sets much narrower limits to the comparison of resistances than would appear permissible when copying unit-standards.[166]

A *second* method is that of *connection* according to the following principle: *if two conductors are connected successively in the same circuit, in which the same electromotive force always acts, the resistance of the two conductors is the same if the current intensities are the same*. — In addition to the definition of resistance, Ohm's law of summation of the resistances of conductors through which the same current passes is used. — Even according to this method, the accuracy of the comparison of two resistances is limited by the accuracy that can be achieved by observing the current intensities, which, even when using the

[166][Note by AKTA:] In German: *Maass-Etalons*. This expression can be translated as unit-standards or measure-standards.

finest galvanometers, generally does not meet the requirements to be made when copying unit-standards, even if it is sufficient for many practical purposes.

Finally, the *third* method to be discussed in more detail in the following Sections is that of *current division*, whereby two cases can be distinguished, namely, that of *single* and *double* division. The method implemented in Wheatstone *bridge* or *balance* is based on *double current division*, but a closer look at this method should be preceded by a brief discussion of the method based on *single current division*.[167]

8.23 Copying Methods With Simple Current Division

In order to determine the accuracy achievable by the *simple current division* method in comparing two resistors with each other, it is necessary to go back to the principle of this method. This principle is as follows: *if a current splits into two branch currents, and both branch currents, each through a multiplier through which it passes, act on the same magnetic needle but in opposite directions, then the resistance of two conductors passed through by these branch currents is equal if the total action observed at the magnetic needle is not altered by the exchange of the two conductors.* — The total action may be greater or smaller, or even zero, from which it is obvious that the accuracy achievable by this method in the comparison of two resistors is completely independent of the magnitude of the observed total action. — With this method, in addition to the laws listed in the previous Section, Ohm's laws of *current branching* are also used.

8.24

The principle stated in the previous Section can be easily proven in the following way. A current i produced by the electromotive force e passes through the conductor c, Figure 3,[168] which is divided into two branch currents i_1 and i', of which the former passes through the conductors which have the resistors a and α, the latter going through the conductors which have the resistors b and β.

[167][Note by AKTA:] In German: *Wheatstone'schen Brücke oder Waage*. The so-called Wheatstone bridge was invented by S. H. Christie (1784-1865) in 1833 and popularized by C. Wheatstone (1802-1875) in 1843, [Chr33], [Whe43, p. 325] with French translation in [Whe44b] and German translation in [Whe44a]. See also [Eke01].

[168][Note by AKTA:] In Figure 3 we should have $a(i_1)$ instead of $a(i)$.

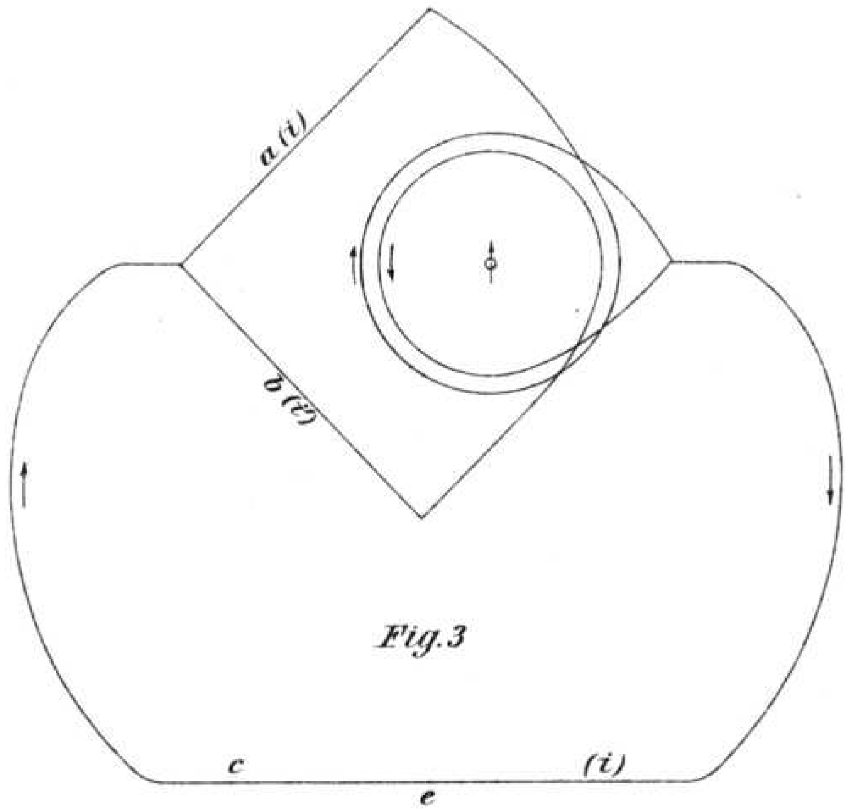


Fig. 3

a and b are the resistances that are to be compared with each other, which is why the arrangement is made so that the two conductors that have these resistances can be exchanged with each other. The conductors with the resistances α, β form multipliers for one and the same magnetic needle, which, however, is deflected by the branch current passing through the conductor α in the opposite direction than that passing through the conductor β. — A current in the conductor α of intensity $= 1$ keeps the needle in a deflection $= m$; a current in the conductor β of intensity $= 1$ keeps the needle in a deflection $= -n$. m and n can therefore be called the sensitivity coefficients of the two multipliers. — Finally, the two conductors α, β reunite with the conductor c, closing the circuit.

The following three equations result from Ohm's laws of current division:

$$\frac{e}{i} = c + \frac{(a+\alpha)(b+\beta)}{a+\alpha+b+\beta} \ ,$$

$$i_1 + i' = i \ ,$$

$$i_1 : i' = (b+\beta) : (a+\alpha) \ ,$$

in addition, there is the determination of the total deflection of the needle by the two branch currents, which may be denoted by A, namely:

$$A = mi_1 - ni' \ .$$

From these four equations, if i, i_1 and i' are eliminated,

$$A = \frac{m(b+\beta) - n(a+\alpha)}{c(a+\alpha+b+\beta) + (a+\alpha)(b+\beta)} \cdot e\ .$$

If one further denotes the total deflection of the needle after exchanging a and b with A', so is

$$A' = \frac{m(a+\beta) - n(b+\alpha)}{c(a+\alpha+b+\beta) + (b+\alpha)(a+\beta)} \cdot e\ .$$

From this it follows that if the deflection $A = A'$ is found,

$$(b-a) \cdot [(m+n)(a+b+\alpha+\beta)c + m(a+\beta)(b+\beta) + n(a+\alpha)(b+\alpha)] = 0\ .$$

However, since the second factor enclosed in brackets consists of a sum of positively given quantities and therefore cannot disappear, the first factor

$$b - a = 0$$

must be set, from which it follows that if the deflection $A = A'$ is found, the resistances a and b are equal, which had to be proven.

8.25

According to the method just considered, the accuracy of comparing the two resistances a and b with each other is completely independent of the size of the observed total action A, and A can therefore generally have a larger or smaller value, or be zero; however, carrying out such a comparison is much easier if A is quite small or zero, from which it follows for $a = b$ that the ratio of the sensitivity coefficients $m : n$ should be almost equal to the ratio of the resistances $a + \alpha : \alpha + \beta$ in the branch currents, which can best be achieved if both multipliers are made from exactly the same wires, which are wound together in such a way that they form exactly the same windings. The differences $m - n$ and $\beta - \alpha$ will then, if they do not disappear completely, at least be very small. If one now denotes the smallest value of the difference $A - A'$, which can still be observed with certainty, by $\triangle$ and the corresponding value of the difference $b - a$ by x, then the value of x/a can be developed, which gives the smallest fraction up to which the equality of the resistances a and b can be guaranteed using this method.

From the values of A and A' found in the previous Section, the following equation easily results:

$$\frac{\triangle}{ex} = \frac{A - A'}{e(b-a)}$$

$$= \frac{(m+n)[c(a+b+\alpha+\beta) + ab + \alpha\beta + \frac{1}{2}(a+b)(\alpha+\beta)] + \frac{1}{2}(a+b+2\beta)(m-n)(\beta-\alpha) + n(\beta-\alpha)^2}{[c(a+b+\alpha+\beta) + ab + \alpha\beta + \frac{1}{2}(a+b)(\alpha+\beta)]^2 - \frac{1}{4}(b-a)^2(\beta-\alpha)^2}\ ,$$

for which, considering that the differences $b - a$, $m - n$, $\beta - \alpha$ are always very small,

$$\frac{\triangle}{ex} = \frac{m+n}{c(a+b+\alpha+\beta) + ab + \alpha\beta + \frac{1}{2}(a+b)(\alpha+\beta)}\ ,$$

or even more simply

$$\frac{\triangle}{ex} = \frac{2m}{(a+\alpha)(a+\alpha+2c)}$$

can be set, from which is obtained

$$\frac{x}{a} = \frac{(a+\alpha)(a+\alpha+2c)}{2mea}\triangle \ .$$

8.26

After the determination found for the accuracy that is required for resistance comparisons using the method of simple current division, rules for the appropriate construction of the devices and the limits of the accuracy that can be achieved can easily be specified in more detail. In general, it is clear that the rules developed in the second Part apply to the construction of the galvanometer and especially the double multiplier required for it, according to which the multiplier space can be viewed as given, that is, the *product* of the length in the cross section of the multiplier wires. Since according to Ohm's law the *ratio* of length to cross section is proportional to the resistance α, this results in an n^2 value of the resistance α with n times the length. With n times the length, the number of multiplier windings, and thus also the sensitivity m, is increased n times. According to this, m and α can be determined in their dependence on n by the equations

$$m = nm_0 \ , \qquad\qquad \alpha = n^2\alpha_0 \ .$$

If you put these values of m and α into the equation of the previous Section, you get

$$\frac{x}{a} = \frac{\triangle}{2m_0ea} \cdot \frac{a(2c+a) + 2(c+a)\alpha_0 n^2 + \alpha_0^2 n^4}{n} \ .$$

It can be seen from this that the accuracy of the resistance comparison depends primarily on the choice of multiplier wires, by which the value of n is determined, and that there is a value of n and consequently of α for which that accuracy is greatest or the fraction x/a is smallest, namely,

$$n = \sqrt{\frac{a+c}{3\alpha_0}\left(2\sqrt{1-\frac{3}{4}\frac{c^2}{(a+c)^2}}-1\right)} \ ,$$

$$\alpha = \frac{1}{3}(a+c)\left(2\sqrt{1-\frac{3}{4}\frac{c^2}{(a+c)^2}}-1\right) \ .$$

In addition, the accuracy increases the smaller c/a becomes, which means that at the same time α and x/a approach certain limit values, namely,

$$\begin{aligned} \alpha &= \frac{1}{3}a \ , \\ \frac{x}{a} &= \frac{8}{9} \cdot \frac{a}{me}\triangle \ . \end{aligned}$$

Now $[3/4] \cdot me/a$ is the value to which, for $\alpha = [1/3] \cdot a$, the value of $me/[a + \alpha + c]$ approaches the more, the smaller c/a; but $me/[a+\alpha+c]$ is the deflection of the needle when the branch current passing through the conductors b and β is removed, and would be easily measured if at the high sensitivity m the length of the scale was sufficient. However, the high sensitivity m can be compensated for by a small electromotive force e. If, for example, one finds for an electromotive force $\varepsilon = e/100$ (if, for example, a thermomagnetic element is set for a Grove's element)[169] the deflection $m\varepsilon/[a + \alpha + c] = 1000\triangle$, then in the limit case $[3/4] \cdot me/a = 100000\triangle$, hence $x/a = [8/9] \cdot [a/me] \cdot \triangle = 1/150000$ is the smallest fraction up to which the equality of the resistances a and b can be guaranteed.

It follows from this that the copying method with simple current division allows a duplication of resistance etalons or standards, which may be considered completely identical for all practical applications.

8.27 Copying Method with Double Current Division

The same thing that can be achieved by simple current division according to the previous discussion can also be achieved by double current division, namely, with the Wheatstone *bridge* or *balance*.[170]

The Wheatstone bridge consists of a closed conductor, of which four points, Figure 4, A, B, C, D, are also connected crosswise.

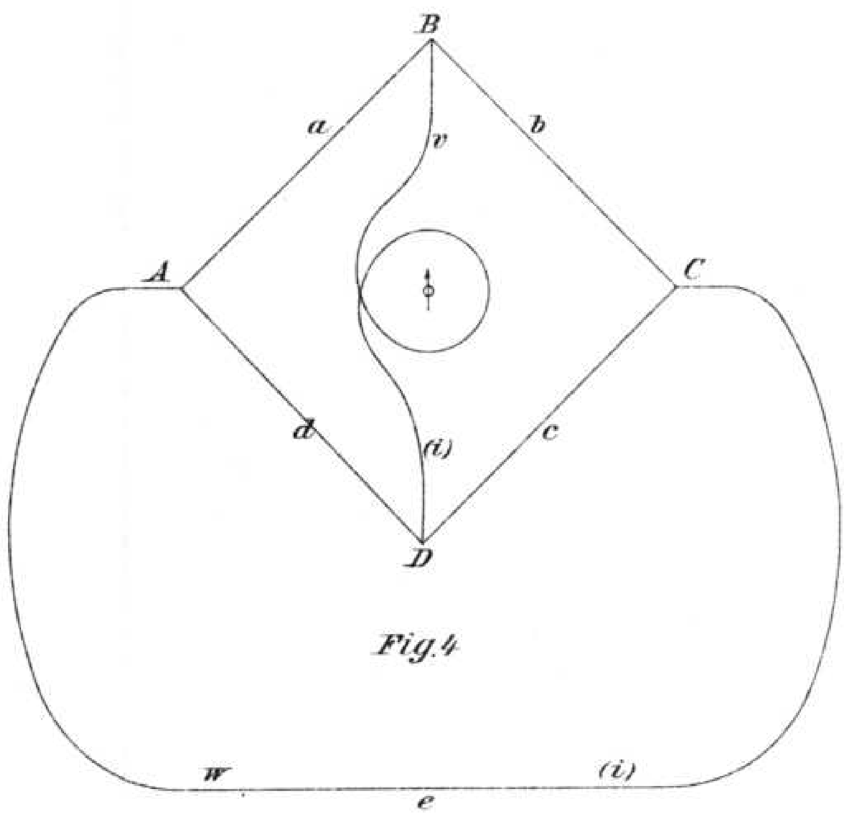


Fig. 4

[169] [Note by AKTA:] The Grove voltaic cell, element, battery or pile was named after its inventor, William Robert Grove (1811-1896), [Gro39].

[170] [Note by AKTA:] See footnote 167.

Let the resistors AB, BC, CD, DA be denoted in sequence by a, b, c, d; furthermore, by w the resistance of the conductor connecting the first point A with the third C, in which the electromotive force e (a voltaic pile)[171] acts and which may therefore be called the undivided conductor; by v the resistance of the conductor connecting the second point B with the fourth D, which is called the *bridge* and forms the multiplier of a galvanometer; let i denote the current intensity in the undivided conductor, i' denote the current intensity in the bridge. — If the bridge were missing, a current in the undivided conductor from A to C would form the two branch currents ABC and ADC and, in the whole, the resistance $w' = w + (a+b)(c+d)/[a+b+c+d]$; if the undivided conductor were missing, a current in the bridge from B to D would form the two branch currents BAD and BCD and, in the whole, the resistance $v' = v + (a+d)(b+c)/[a+b+c+d]$. — Finally, the real resistance which the current produced by e finds in its entire circuit is denoted by W.

It is known how, in the theory of the Wheatstone bridge, the ratio of the current intensity in the bridge i' to the intensity of the undivided current i is determined from the ratios of the resistances a, b, c, d to the resistance of the bridge v, namely, by the equation

$$\frac{i'}{i} = \frac{ac - bd}{(a+d)(b+c) + (a+b+c+d)v} = \frac{ac - bd}{(a+b+c+d)v'} \ .$$

It follows from this that if the current in the bridge i' disappears, $ac - bd = 0$ or $a : b = d : c$. If i' does not disappear, its value, and consequently that of $(ac - bd)$, should at least be very small. Assuming this, we add to that special equation for the Wheatstone bridge the general one given by Ohm's law, namely,

$$i = \frac{e}{W} \ ,$$

and develop the total resistance W in a series progressing after powers of $(ac - bd)$, where, however, with the assumed small value of $(ac - bd)$ all members containing a power greater than the square of this magnitude may be considered as vanishing. You then get

$$W = w + \frac{(a+b)(c+d)}{a+b+c+d} - \frac{b(c+d) + c(a+b)}{d(b+c) + v(c+d)} \cdot \frac{1}{b} \left(\frac{ac - bd}{a+b+c+d} \right)^2 \ ,$$

where $w + (a+b)(c+d)/[a+b+c+d] = w'$. From this it finally follows

$$i' = \frac{ac - bd}{(a+b+c+d)v'} \cdot \frac{e}{W} = \frac{(ac - bd)e}{(a+b+c+d)v'w'} \ .$$

If, as in the previous Section, the deflection of the needle produced by one unit of current intensity in the bridge is called m, then the deflection produced by the current i' is

$$A = mi' = \frac{(ac - bd)me}{(a+b+c+d)v'w'} \ .$$

If the two resistances a and b are now to be compared with each other, then one sets their very small difference $a - b = x$ and also $c - d = \delta$, which also has only a small value for small values of A. Then you get

$$A = \frac{a\delta + cx}{2(a+c)} \cdot \frac{me}{v'w'} \ ,$$

[171][Note by AKTA:] In German: *einer Säule*. See footnote 103.

and if a and b are swapped, we get

$$A' = \frac{a\delta - cx}{2(a+c)} \cdot \frac{me}{v'w'} \ ;$$

because the factor $me/[a+b+c+d)w']$ remains completely unchanged in this substitution, as you can see, if you set $w+(a+b)(c+d)/[a+b+c+d]$ for w'; v' remains unchanged at least for small values of x and δ, because $v' = v+(a+d)(b+c)/[a+b+c+d] = v+(a+c)/2-(\delta+x)/2$ then changes into $v + (b+d)(a+c)/[a+b+c+d] = v + (a+c)/2 - (\delta - x)/2$; finally $(ac - bd) = a\delta + cx$ becomes $(bc - ad) = a\delta - cx$. So it is

$$A - A' = \frac{mec}{(a+c)v'w'} \cdot x \ ,$$

consequently, if $A - A' = \triangle$ denotes the smallest value of the deflection difference that can still be observed with certainty, then one obtains the smallest fraction up to which the equality of the resistances a and b according to these observations can be guaranteed, namely,

$$\frac{x}{a} = \frac{(a+c)v'w'}{meac} \cdot \triangle = \frac{(a+c+2v)(2ac+(a+c)w)}{2meac} \cdot \triangle \ .$$

The smaller the resistances of the bridge v and of the undivided conductor w, the smaller this fraction is, and the smaller v and w become, the closer it approaches the value of

$$\frac{x}{a} = \frac{a+c}{me} \cdot \triangle \ .$$

Now $me/[a+c]$ is the value that $me/[a+c+v+w]$ approaches the smaller $v+w$ becomes; but $me/[a+c+v+w]$ is the deflection of the needle when the branch currents passing through b and d are removed, and can be easily observed and measured, even at high sensitivity m of the galvanometer, if the larger electromotive force e used in the observations A and A', as already stated in the previous Section, is substituted with a smaller electromotive force, for example, $\varepsilon = e/100$. If the deflection $m\varepsilon/[a+c+v+w]$ then has a measurable size, for example, $1000\triangle$, then in the limit case $me/[a+c] = 100000\triangle$, consequently $x/a = 1/100000$ the smallest fraction up to which the equality of the resistances a and b can be guaranteed.

It follows from this that the copying method with double current division allows an almost as accurate test of the equality of two resistances a and b as that with single current division, and therefore also enables a duplication of resistance etalons or standards, which for all practical applications can be considered completely identical; but in this respect the method of double division cannot be given any preference over the method of single division. — The method of double division only has a peculiar value when it is not a question of testing *equality*, but rather of determining the unknown *ratio* of two very different resistances $a : b$, which then, with disappearing deflection A, a known resistance ratio $d : c$ is recognized as equal; however, the accuracy of the result is made dependent on the exact knowledge of the resistance ratio $d : c$, which must be given.

V - On the General Principles of Resistance Measurement

8.28

The principles of galvanic resistance measurement were derived from the nature of galvanic resistance, which is a *property of ponderable bodies*, for example, a copper wire, and therefore had to be derived from the definition given by this property. Such a definition was first established on the basis of Ohm's law, which determines the dependence of the *current intensity* in a ponderable body on the *electrical forces* acting on the electricity contained therein. According to the principles derived from this definition, the method by which the resistance of a given body (a copper wire) can be determined most accurately has been developed in the first Parts of this Treatise. In the last Part it was finally discussed how the resistances of other bodies could be most accurately compared with the resistance thus researched.

All these investigations were based on the first definition of conduction resistance, which is based on the well-known Ohm's law of experience derived from related measurements of *electromotive forces* and *current intensities*, namely, that no matter how different the electromotive forces e and current intensities i, as long as the ponderable body remains the same to which these forces and these currents belong, the quotient e/i always has the same value, while it assumes different values for different bodies, according to which the *constant* value of the quotient e/i for each body is a *property of the body* which can serve to distinguish it from other bodies and is called its *conduction resistance*.

The *property of a ponderable body*, which is hereafter referred to as resistance, must have its *causes* in the peculiar nature of the ponderable body itself, and must in itself be independent of the forces that act on the electrical fluids contained in it, as well as of the movements, into which these fluids are thereby put; however, these *causes*, which lie in the nature of the ponderable body itself, have not yet been investigated. We therefore only know the *action* of its resistance *from experience*, and from this we only know that, *for a given electromotive force*, it consists of *a certain current intensity*.

But if resistance itself is a property founded in the nature of the ponderable body itself, then *other actions* can also exist that can be proven through experience; for example, the case could take place that such an action, which can be proven by experience, would be present in any *given* current that passes through the body, regardless of where it comes from or by what forces it is produced. Such a really existing action, which takes place with every *given* current passing through a body, is called *electrical work*,[172] and the only question is how this action can be observed and how its dependence on the conduction resistance of the body can be proven.

As experience shows, a current produces *heat* in the wire through which it passes, and heat is, according to the mechanical theory of heat, *vis viva* equivalent to *work*.[173] If one can

[172][Note by AKTA:] In German: *Stromarbeit*. This expression can be translated as electrical work or current work.

[173][Note by AKTA:] I am translating the German expression *lebendige Kraft*, literally "living force," by the Latin expression *vis viva* (plural *vires vivae*) also meaning "living force." Originated by Gottfried Leibniz (1646-1716) in the 17th century, the *vis viva* of a body of mass m moving with velocity v relative to an inertial frame of reference was defined as mv^2, that is, twice the modern kinetic energy. During the XIXth century many authors, including Hermann von Helmholtz (1821-1894) and Wilhelm Weber, defined the *vis*

now consider the heat generated by a current as *electrical work*, then this electrical work is measurable, as is the current that produces it. Joule and Lenz finally based an *empirical law* on these related measurements of the *intensity of the currents* and the *heat* they generate in the same way as Ohm's law on the related measurements of *electromotive forces* and *current intensities*, namely, the law that no matter how different current intensities i, and no matter how different heat productions A, as long as the ponderable body remains the same, to which those currents and these heat productions belong, the quotient A/i^2 always has the same value, which therefore also, as a *property of the ponderable body*, can serve to distinguish it from other bodies for which this quotient has different values.[174]

If this *second* property could now be considered identical to the *first one*, which was called *resistance* (experience really shows the proportionality of both quotients), then a *second definition of resistance* would be obtained, from which entirely new principles for the measurement of resistance would result, quite independent of those previously considered. The development of a method of resistance measurement based on these new principles would initially have to involve research into, *firstly*, the accuracy of the heat measurement methods to be used, *secondly*, the determination of the equivalence of heat with work, and *thirdly*, the examination of the prerequisite that all electrical work is converted into heat. However, before we go into this new, broad area of research, we still need to discuss in more detail what can be achieved independently of the consideration of heat, based solely on the known general electrical laws.

8.29 Electrical Work According to Electrical Laws

Work is only spoken of when the points of action of forces are moving. The work A of such a point is the product of the component of the force acting upon it, according to the direction of its motion, in the path it has travelled. However, work can be taken in two senses: it means either the work itself or the work that is done. According to the given definition, A is work in the latter sense, while work in the former sense is represented by the differential quotient of A with respect to time, that is, is expressed by dA/dt.

With a galvanic current i in a conductor element α, all particles of the electrical fluids contained in α are points of action of the electromotive forces, and these points of action move partly forward and partly backwards in the direction of element α.[175] The work A or dA/dt of all these points of application is the work of the galvanic current i in the conductor element α. The fact that the moving points of application of the forces in this case do not have a ponderable mass is of no importance for the work itself, according to the given definition.

The amount of *positive* electricity contained in the element α is denoted by $+\alpha\varepsilon$, and the force acting on it, which is proportional to it according to electrical law and directed forward in the direction α, is denoted by $+f$, where f is the numerical value which indicates how many times the force which gives the ponderable mass unit the unit of speed in the unit of

---

viva as $mv^2/2$, that is, like the modern kinetic energy.

[174][Note by AKTA:] James Prescott Joule (1818-1889) and Heinrich Friedrich Emil Lenz (1804-1865). See [Jou41b]; [Jou41a] with French translation in [Jou42]; [Len43c], [Len43a], [Len43b], [Len44b] and [Len44a]. See also [MS20] and [Mar22].

[175][Note by AKTA:] That is, according to the usual assumption at that time, positive particles would move in one direction relative to the body of the conductor, while negative particles would move in the opposite direction.

time is contained in it. — The amount of *negative* electricity contained in the element α is denoted by $-\alpha\varepsilon$, and the force acting on it, directed backwards in the direction α, is denoted by $-f$. — The velocity at which these electrical masses move forward and backward in the direction α shall be denoted by $\pm u$.[176] According to the given definition, the work of *positive* electricity in the element α, during time t, is

$$A' = (+f) \cdot (+ut) = +fut \ ;$$

the work of the *negative* electricity in the element α during the same time,

$$A'' = (-f) \cdot (-ut) = +fut \ ;$$

consequently the entire work of the galvanic current in the element α, during the time t,

$$A = 2fut \ .$$

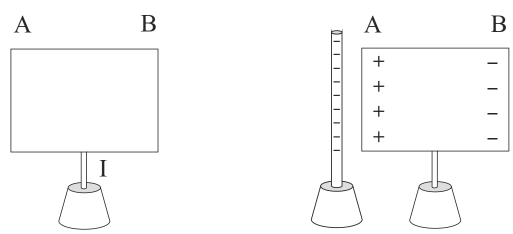
But for work, taken in the sense of working, you get

$$\frac{dA}{dt} = 2fu \ .$$

$2f$ is called the *absolute separating force*[177] acting on the electricity in the element α, u is the *absolute current velocity*,[178] both of which can neither be observed nor measured directly.

[176][Note by AKTA:] The velocity here should be understood as the velocity of the electrified particles in relation to the material body of the conductor, that is, the drift velocity.

[177][Note by AKTA:] In German: *Scheidungskraft*. This expression can be translated as "separating force", "force of separation" or "segregating force". I present here a simple example of a separating force. Consider a metal plate AB insulated from the ground by a dielectric support I as in Figure (a) of this footnote:



(a) (b)

If a negatively charged straw is placed close to side A of the plate, the charges on the plate become separated as illustrated in Figure (b). Side A of the plate becomes positively electrified, while side B becomes negatively electrified. This polarization of the plate is caused by the electric force of the negatively electrified straw acting on the free electrons of the plate. I presented several interesting experiments on this topic made with simple material, together with many quotes from original sources, in the 2 volumes of the book *The Experimental and Historical Foundations of Electricity* which is available in English, Portuguese, Italian and Russian: [Ass10a], [Ass10b], [Ass15b], [Ass17a], [Ass18a], [Ass18b] and [Ass19b].

Another effect of a separating force takes place in electrolysis. The electric forces in general are proportional to the charge q of the test particle on which they are acting. A positively electrified particle with $q > 0$ experiences a force in one direction, while a negatively electrified particle with $q < 0$ will be forced in the opposite direction. If these particles are free to move as in electrolysis, a double current will be produced due to this separating electric force. That is, the positive particles will move in one direction and the negative particles will move in the opposite direction.

[178][Note by AKTA:] In German: *die absolute Stromgeschwindigkeit*. From the context of Weber's discussion, this so-called absolute current velocity is the velocity of the electrified particles relative to the body of the conductor, that is, their drift velocity.

On the other hand, the so-called *electromotive force* e and the *current intensity* i acting on α are observed and measured according to the absolute units determined earlier.

If the electrical work in α is to be determined, the relationship between the separating force $2f$ and electromotive force e, and also the relationship between the current velocity u and the current intensity i, must be given, which has already been discussed in Section 8.1. It is, as stated there (where only f meant the absolute force of separation, denoted here by $2f$),

$$\frac{i}{u} = \frac{\varepsilon}{c}\sqrt{8} \ ,$$

$$\frac{e}{2f} = \frac{c}{\varepsilon}\sqrt{\frac{1}{8}} \ ,$$

where c is a *constant velocity* known from the fundamental law of electrical action, namely, $c = 439\,450 \cdot 10^6$ millimeters/second.[179]

This results in $2fu = ei$; consequently, the *resistance according to the second definition*, namely, the quotient of the electrical work dA/dt divided by the square of the current intensity,

$$\frac{1}{i^2} \cdot \frac{dA}{dt} = \frac{2fu}{i^2} = \frac{e}{i} \ ,$$

is identical to the *resistance according to the first definition*, namely, the quotient of the electromotive force e divided by the current intensity i,

$$\frac{e}{i} = w \ .$$

So the *electrical work* in a current conductor is $dA/dt = wi^2$, where i denotes the current intensity and w denotes the resistance of the conductor according to the absolute units determined earlier. Conversely, the resistance of a current conductor can be defined in absolute terms as the *work of the unit of current* in the conductor. If, in some way, electrical work wi^2 and current intensity i can be observed independently of one another and measured according to the established absolute units, then from these two measurements one can find the *resistance* according to absolute unit of measure $w = wi^2/i^2$, without any knowledge of the *electromotive force* e by which the current was produced. These principles result in an essentially new method of absolute resistance measurement.

It has already been noticed how the observation and measurement of the heat generated by a current in a conductor can be used to determine the work of the current independently of the intensity of the current; but there is another way where it is not necessary to use the assumptions of the mechanical theory of heat, but where the basic electrical law is sufficient, according to which measurable *work of ponderable bodies* can be converted into electrical work, so that electrical work can be determined by measuring the work of moving ponderable body. However, the closer discussion of this method of measuring the work of electricity should be preceded by a brief consideration of the *maximum of the electrical work*, which results directly from the determination of the work of the electricity given according to electrical laws.

[179][Note by AKTA:] See footnote 120 on page 64.

8.30 Maximum of Electrical Work

Let there be a voltaic pile or any other electric motor which, depending on the conductor through which it is connected, performs sometimes a greater, sometimes a smaller electrical work; we search the conductor for which this electrical work is a maximum.

If we denote the resistance of the conductor by w and the current intensity by i, then the electrical work in this conductor according to electrical laws, as shown in the previous Section, is $= wi^2$. According to Ohm's laws, if e denotes the electromotive force and w' denotes the resistance of the given electric motor, the current intensity $i = e/(w' + w)$, therefore, $wi^2 = e^2w/(w' + w)^2$. The conductor is then found for which the electrical work is a maximum if

$$\frac{e^2 w}{(w' + w)^2} = \text{maximum}$$

is set for a variable value of w, from which follows

$$\frac{[(w' + w)^2 e^2 - 2e^2 w(w' + w)]}{[w' + w]^4} = 0 \ ,$$

that is, $w = w'$. This means that the electrical work in the conductor is greatest when the resistance of the conductor is equal to the given resistance of the electric motor; but this greatest value itself is $= e^2/4w'$, while the entire electrical work, taken together in the conductor and in the electric motor, is $= e^2/2w'$, that is, twice as large. If $w > w'$, the work transferred to the conductor would be more than half of the total electrical work, but would still be smaller, with reduced total electrical work, than if $w = w'$.

However, the *maximum of the entire electrical work* takes place when no conductor is needed at the end of the circuit, and therefore no transfer of electrical work to such a conductor is possible, but the electric motor is closed in itself. This greatest value of the entire electrical work is $= e^2/w'$, which is four times greater than the electrical work that can be transferred to other conductors. This is related to the strong heating of self-contained cells, especially when these cells have a very low resistance in relation to their electromotive force, as is the case, for example, with Grove cells.

Incidentally, it is easy to see that the law previously established for galvanometers, namely, that their sensitivity, regardless of the size and shape of their multiplier, is always greatest when the resistance of the multiplier wire is equal to the resistance of the rest of the circuit, can be considered as individual case or special application of the more general law found for the maximum of the electrical work transferred to conductors.

8.31 Conversion of the Work of Moving Ponderable Bodies into Electrical Work through Electrical Interaction

If a closed conductor is moved against a solenoid, that is, against another closed conductor on which a given electromotive force e acts, then the fundamental law of electrical action results in partly electromotive forces which move the electrical fluids in their ponderable conductors (induction forces according to Faraday),[180] partly in forces that move the electrical fluids with their ponderable conductors (electrodynamic forces according to Ampère).[181]

The *former* or the inductive forces according to Faraday are

1. the electromotive force ε' acting on the *closed conductor* according to the law of *voltaic induction as a result of the movement* of the closed conductor against the solenoid;[182]

2. the electromotive force η' acting on the *closed conductor* according to the law of *voltaic induction as a result of the change in current [intensity]* in the solenoid;

3. the electromotive force ε acting on the *solenoid* according to the law of *voltaic induction as a result of the movement* of the closed conductor against the solenoid;

4. the electromotive force η acting on the *solenoid* according to the law of *voltaic induction as a result of the change in current [intensity]* in the closed conductor.

The *latter*, or the electrodynamic forces according to Ampère, are the attractive or repulsive forces exerted by all current elements of the solenoid on all current elements of the closed conductor.

According to this overview, one has to distinguish, *firstly*, the electrical work dA'/dt of the current i' excited by the electromotive forces $(\varepsilon' + \eta')$ in the *closed conductor*, *secondly*, the electrical work dA''/dt of the current i'' excited by the electromotive forces $(\varepsilon + \eta)$ in the *solenoid*, *thirdly*, finally, the work dA'''/dt carried out by the moving *ponderable* particles of the closed conductor, on which the attractive and repulsive forces exerted by the current elements of the solenoid act.

If the resistance of the closed conductor is denoted by w', then

$$\frac{dA'}{dt} = w'i'^2 = \frac{(\varepsilon' + \eta')^2}{w'} \ ;$$

if the resistance of the *solenoid* is denoted by w and if e is the *given constant electromotive force in the solenoid*, and $i = e/w$ is the intensity of the current excited by this force, then we have

$$\frac{dA''}{dt} = w(i + i'')^2 - wi^2 = \frac{(e + \varepsilon + \eta)^2 - e^2}{w} \ ;$$

finally, if we denote by f the sum of the components of all the attractive and repulsive forces exerted on a moving *ponderable* particle of the closed conductor by all the current elements of the solenoid, according to the direction of the movement, and [if we denote] by v the velocity of this movement, then we have

$$\frac{dA'''}{dt} = \sum fv \ .$$

If one now substitutes here the values of the electromotive forces ε, η, ε', η' known from the general fundamental electrical laws, as well as the electrodynamic forces f, then it must be proved that

[180][Note by AKTA:] See footnote 43 on page 28.
[181][Note by AKTA:] See footnote 15 on page 18.
[182][Note by AKTA:] See footnote 118 on page 63.

$$\int \left(\frac{dA'}{dt} + \frac{dA''}{dt} + \frac{dA'''}{dt} \right) dt = 0 \; ,$$

if the integration is extended over the entire period of time after which all ponderable particles of the closed conductor return to their previous positions with an unchanged velocity.

We restrict ourselves here to considering the simple case where the solenoid as well as the closed conductor are *circles* whose radii may be denoted by r and r'. The distance between the two circle centers is R and is so large that r and r' can be considered vanishing. The connecting line R is perpendicular to the solenoid plane, and the closed conductor rotates around its diameter perpendicular to R, with uniform velocity $d\alpha/dt = \gamma$, where α denotes the angle that the perpendicular to the plane of the closed conductor forms with R. If you then set $\pi^2 r^2 r'^2 / R^3 = a$, the following expressions for the electromotive forces can easily be derived from the fundamental laws of electrical action:[183]

$$\begin{aligned}
\varepsilon' &= -2a\gamma \frac{e + \varepsilon + \eta}{w} \cdot \sin\alpha \; , \\
\eta' &= -2a(1 - \cos\alpha) \frac{d\varepsilon + d\eta}{w dt} \; , \\
\varepsilon &= -2a\gamma \frac{\varepsilon' + \eta'}{w'} \cdot \frac{\sin\alpha}{1 + 3\cos\alpha^2} \; , \\
\eta &= -2a\sqrt{\frac{1}{3}} \left(\frac{\pi}{3} - \arctan\left[\cos\alpha \cdot \sqrt{3}\right] \right) \frac{d\varepsilon' + d\eta'}{w' dt} \; .
\end{aligned}$$

If one now expands $(\varepsilon' + \eta')$ and $(\varepsilon + \eta)$ in series according to increasing powers of a, one obtains the first members of these series, against which all subsequent members disappear,[184]

$$\begin{aligned}
\varepsilon' + \eta' &= -2a\gamma \frac{e}{w} \cdot \sin\alpha \; , \\
\varepsilon + \eta &= 4a^2\gamma^2 \frac{e}{ww'} \left(\frac{\sin\alpha^2}{1 + 3\cos\alpha^2} + \sqrt{\frac{1}{3}} \left(\frac{\pi}{3} - \arctan\left[\cos\alpha \cdot \sqrt{3}\right] \right) \cos\alpha \right) \; ,
\end{aligned}$$

and from this, likewise developed,

$$\begin{aligned}
\frac{dA'}{dt} &= 4a^2\gamma^2 \cdot \frac{e^2}{w^2 w'} \cdot \sin\alpha^2 \; , \\
\frac{dA''}{dt} &= 8a^2\gamma^2 \cdot \frac{e^2}{w^2 w'} \cdot \left(\frac{\sin\alpha^2}{1 + 3\cos\alpha^2} + \sqrt{\frac{1}{3}} \left(\frac{\pi}{3} - \arctan\left[\cos\alpha \cdot \sqrt{3}\right] \right) \cos\alpha \right) \; ,
\end{aligned}$$

or, since the differential coefficient is

$$\frac{d\left[\sin\alpha \cdot \arctan\left(\cos\alpha \cdot \sqrt{3}\;\right)\right]}{d\alpha} = \cos\alpha \cdot \arctan\left(\cos\alpha \cdot \sqrt{3}\right) - \frac{\sin\alpha^2 \cdot \sqrt{3}}{1 + 3\cos\alpha^2} \; ,$$

[183][Note by AKTA:] The expression $\cos\alpha^2$ should be understood as $\cos^2\alpha$.

[184][Note by AKTA:] The expression $\sin\alpha^2$ should be understood as $\sin^2\alpha$.

we have

$$\frac{dA''}{dt} = 8a^2\gamma^2 \cdot \frac{e^2}{w^2 w'} \cdot \sqrt{\frac{1}{3}} \left(\frac{\pi}{3} \cos\alpha - \frac{d\left[\sin\alpha \arctan\left(\cos\alpha \cdot \sqrt{3}\,\right)\right]}{d\alpha} \right) .$$

Finally, if one denotes the distance of any ponderable particle of the closed conductor from its axis of rotation by ρ, then the torque exerted by the solenoid on the closed conductor is $D = \sum f\rho$, and the velocity at which the ponderable particle moves in its circular orbit (the tangent of which coincides with the direction of the force f), $v = \rho\gamma$; consequently, at *constant angular velocity* γ,

$$\frac{dA'''}{dt} = \sum fv = \sum f\rho\gamma = \gamma \sum f\rho = \gamma D .$$

However, the torque D exerted by the solenoid on the closed conductor is according to Ampère's law

$$D = 2a\gamma \sin\alpha \cdot \frac{e+\varepsilon+\eta}{w} \cdot \frac{\varepsilon'+\eta'}{w'} ,$$

and if you insert the found values of $(\varepsilon+\eta)$ and $(\varepsilon'+\eta')$ here, and expand into powers of a, you get the first term against which the others disappear,

$$D = -4a^2\gamma \cdot \frac{e^2}{w^2 w'} \cdot \sin\alpha^2 ,$$

consequently

$$\frac{dA'''}{dt} = -4a^2\gamma^2 \cdot \frac{e^2}{w^2 w'} \cdot \sin\alpha^2 = -\frac{dA'}{dt} .$$

For dA''/dt this results in the integral value $\int [dA''/dt] \cdot dt$ for the time of a whole revolution of the closed conductor, that is, for the time after which all ponderable particles with unchanged velocity return to their previous positions, with *constant* angular velocity $d\alpha/dt = \gamma$,

$$\int \frac{dA''}{dt} dt = \int 8a^2\gamma \cdot \frac{e^2}{w^2 w'} \cdot \sqrt{\frac{1}{3}} \left(\frac{\pi}{3} \cos\alpha - \frac{d\left[\sin\alpha \cdot \arctan\left(\cos\alpha \cdot \sqrt{3}\,\right)\right]}{d\alpha} \right) d\alpha ,$$

which is equal to zero when taken between the boundaries α and $\alpha + 2\pi$. Since $dA'/dt + dA'''/dt = 0$, therefore also $\int (dA'/dt + dA'''/dt)\, dt = 0$, it follows from this, between the specified limits,

$$\int \left(\frac{dA'}{dt} + \frac{dA''}{dt} + \frac{dA'''}{dt} \right) dt = 0 ,$$

which was to be proved.

It can be seen from this, in relation to the *work of the ponderable particles* of the closed conductor, that at every time interval dt there is a loss of work due to the *damping* caused by the induction,

$$\frac{dA'''}{dt} dt = -4a^2\gamma^2 \cdot \frac{e^2}{w^2 w'} \cdot \sin\alpha^2 dt ,$$

which must be replaced by a *driving force*[185] acting on the closed conductor if the angular velocity γ is supposed to remain unchanged. On the other hand, at the same time interval dt there is a gain in *electrical work in the closed conductor*, namely

$$\frac{dA'}{dt}dt = +4a^2\gamma^2 \cdot \frac{e^2}{w^2w'} \cdot \sin\alpha^2 dt$$

of the same amount, from which it follows that through the mediation of electrical interactions, a pure conversion of *work from ponderable bodies* into *electrical work* has taken place.

If it were to result from observation that the angular velocity γ really remained completely unchanged, and if the *driving forces* were measured, which would have to act on the rotating closed conductor in order to keep this angular velocity unchangeable, both with the solenoid *open* (whereby the driving force required to overcome the resistance of the air and the friction is determined), as well as with the solenoid *closed* (whereby the driving force required to overcome the electrical damping is determined together with that required to overcome the resistance of the air and the friction), then the difference between the two measured driving forces, multiplied by the angular velocity γ, which is also easy to measure, would give the value of

$$-\frac{dA'''}{dt} = \frac{dA'}{dt} ,$$

that is, the value of the *electrical work in the closed conductor*, which the current i' induced therein performed in the unit of time.

If this measurement of the *electrical work* dA'/dt were finally combined with the measurement of the *current intensity* i', the *resistance of the closed conductor*, in absolute value, would result:

$$w' = \frac{1}{i'^2} \cdot \frac{dA'}{dt} .$$

8.32 Determination of the Electrical Work by Means of Heat Measurement, According to Experiments by Becquerel and Lenz

If the resistance of a conductor is to be determined in absolute terms, but not according to the previously used method, by measuring the *electromotive force* and the *current intensity*, but according to the last given method, by measuring the *electrical work* and the *current intensity*, then in general it is as shown, two ways open, depending on the difference in the method by which the *electrical work* is measured. *Electrical work* can be measured, *firstly*, by measuring the *work of moving ponderable bodies*, which is converted into electrical work, which was discussed in the previous Section, and *secondly*, by measuring the *heat* into which the electrical work is converted.

The *first method* was of particular interest because it was based solely on the known laws belonging to the pure theory of electricity. The way in which it is carried out was explained using a simple example in the previous Section, but in reality this would not lead to any practical results. At least the most favorable conditions for the observations required using

[185][Note by AKTA:] In German: *Triebkraft*.

this method should first be discussed in more detail, but this will not be discussed here because it is easy to overlook in advance that even then, the resistance of the air and the friction of solid bodies against each other are always dependent conditions under which all *ponderable bodies* that we observe move, the measurement of the work done by them, or the driving force necessary to maintain their movement, could not be carried out precisely enough even under the otherwise most favorable conditions.

The *latter method*, in which the laws of the mechanical theory of heat must be used, therefore seems practically the only one from which one may expect such precise determinations of *electrical work* as would be necessary to determine a conduction resistance from *electrical work* and *current intensity* as precisely as from *electromotive force* and *current intensity*. It is therefore of interest to take a closer look at what has been achieved in this way in recent times through the numerous experiments carried out on it, particularly by Becquerel and Lenz.

Edmond Becquerel states in his Treatise: *Des lois du dégagement de la chaleur pendant le passage des courants électriques à travers les corps solids et liquides* (The laws of heat production during the passage of electric currents through solid and liquid bodies) (Annales de Chimie et de Physique, 1843, Volume IX)[186] that, according to his experiments, a current which, if it were passed through water, would produce 3.383 cubic centimeters of oxyhydrogen gas[187] every minute, at a temperature of 0° and a barometer reading of 0.76 meter, would produce in a platinum wire 44 centimeters long and weighing 0.422 grams, as much heat per minute as 2.18523 grams of water need to raise its temperature by 1 degree.

If we add to this information the determination found by Joule,[188] based on the mechanical theory of heat, according to which the amount of heat which can warm 1 kilogram of water from 0° to 1° when converted into mechanical work has a work quantity of 423.55 kilogram-meters, it is found that the heat generated in *every minute* by the specified current in the platinum wire, when converted into mechanical work, has a work quantity of $2.18523 \cdot 0.42355$ kilogram-meters, so the heat generated *every second* gives the 60th part of this amount. This results in the *electrical work* according to the *absolute unit of measure of work*, which we reduce to millimeters, milligrams and seconds as the fundamental units of measure of length, mass and time (according to which the gravity $g = 9811$ millimeters/second^2) as given by

$$wi^2 = \frac{1}{60} \cdot 9811 \cdot 2.18523 \cdot 0.42355 \cdot 10^9 = 151340 \cdot 10^6 \ .$$

As far as the *intensity of the current* is concerned, we use the statement that the intensity of a current that decomposes 1 milligram of water in one second, is $106\frac{2}{3}$ times larger than the absolute intensity unit (see Treatises of the mathematical-physical class of the Königl. Sächs. Gesellschaften der Wissenschaften, Vol. 3, p. 224).[189,190] If you now calculate that 1 milligram of decomposed water produces 1.8568 cubic centimeters of oxyhydrogen at a temperature of 0° and a barometer reading of 0.76 meters, then the intensity of the current described, which produces 3.383 cubic centimeters of oxyhydrogen *every minute*, in absolute units, is

[186][Note by AKTA:] Edmond Becquerel (1820-1891) was a French physicist, see [Bec43].

[187][Note by AKTA:] In German: *Knallgas*. This expression can also be translated as explosive gas.

[188][Note by AKTA:] See footnote 174.

[189][Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. III, p. 614.

[190][Note by AKTA:] [KW57, p. 614 of Weber's *Werke*] with English translation in [KW21, p. 144]. See also [Web41d] and [Web42] with English translation in [Web21m, p. 199, footnote 357].

$$i = \frac{1}{60} \cdot \frac{3.383}{1.8568} \cdot 106\frac{2}{3} = 3.2391 \ .$$

From these determinations the *resistance of the described platinum wire in absolute units* finally emerges

$$w = \frac{wi^2}{i^2} = \frac{151340 \cdot 10^6}{3.2391^2} = 14425 \cdot 10^6 \ .$$

This resistance, multiplied by the mass of a millimeter-long piece of wire $= \frac{422}{440}$ and divided by the length of the wire $= 440$ expressed in millimeters, gives the resistance of a platinum wire of 1 millimeter length and 1 milligram mass according to Ohm's laws, that is, the *specific resistance of platinum*

$$p = 31\,443\,000 \ .$$

Lenz, in his Treatise: *Ueber die Gesetze der Wärmeentwickelung durch den galvanischen Strom* (On the laws of heat development by galvanic current) (Poggendorff's *Annalen*, 1843-44, Vols. 59, 61)[191] gives the time for heating 1 gram of water to 1° R[192] through a wire of resistance = 1, through which a current = 1 passes, as $57\frac{1}{2}$ minutes (due to a misprint, as it appears, $5\frac{3}{4}$ seconds is given), whereby *the unit of resistance* has been attributed to a copper wire of 6.358 feet length and 0.0336 English inches in diameter, at a temperature of 15°, and *the unit of intensity* has been attributed to a current whose electrolytic action = 41.16 cubic centimeters of oxyhydrogen per hour, at a temperature of 0° and a barometric pressure of 760 millimeters.

According to the mechanical theory of heat, in accordance with Joule's determination already mentioned, the heat generated by the assumed unit of current every second in the described copper wire, when converted into mechanical work, gives a *work quantity* $= [5/4] \cdot [1/(60 \cdot 57.5)] \cdot 0.42355$ kilogram-meter, that is, according to *absolute units of measure of work* (reduced to millimeters, milligrams and seconds as the fundamental units of measure of length, mass and time), the *electrical work*

$$wi^2 = 9811 \cdot \frac{5}{4} \cdot \frac{1}{60 \cdot 57.5} \cdot 0.42355 \cdot 10^9 = 1506 \cdot 10^6 \ .$$

Furthermore, for the *assumed unit of current*, the electrolytic action of which corresponded to 41.16 cubic centimeters of oxyhydrogen gas per hour, the value is found after reduction to *absolute unit of measure*

$$i = \frac{1}{3600} \cdot \frac{41.16}{1.8568} \cdot 106\frac{2}{3} = 0.65683 \ .$$

From these determinations the *resistance of the described copper wire according to absolute units of measure* finally results

$$w = \frac{wi^2}{i^2} = \frac{1506 \cdot 10^6}{0.65683^2} = 3490 \cdot 10^6 \ .$$

If you calculate the mass of the described 6.358 English feet = 1938 millimeter long copper wire to 9889 milligrams by assuming the density of the copper = 8.921, then according to

[191] [Note by AKTA:] See footnote 174.

[192] [Note by AKTA:] That is, 1° Réaumur.

Ohm's laws you get w by multiplying the resistance found by the mass of a 1 millimeter long piece, $= \frac{9889}{1938}$, and dividing by the length of the wire expressed in millimeters, $= 1938$, the resistance of a copper wire 1 millimeter long and 1 milligram mass, that is, the *specific resistance of the copper*

$$\varkappa = 9\,190\,000 \ .$$

This result, compared with that derived from Becquerel's experiments, would show that the resistivity of copper would be about $3\frac{1}{2}$ times smaller than that of platinum, while it is known from numerous direct comparisons that it is still much smaller, namely, according to Arndtsen's experiments, if one reduces the information given for equal wire lengths of the same cross-section to equal wire lengths of the same mass, and thereby assumes the density ratio of copper to platinum as 1 : 2.244, 15.22 times smaller, and after Matthiessen's experiments are 15.93 times smaller, on average 15.575 times smaller.[193] According to this, the specific resistance of copper

$$\varkappa = \frac{p}{15.575} = 2\,018\,800$$

would be calculated from Becquerel's experiments, which is quite close to the average of the values found so far in other ways for various types of copper, but is exceeded by $4\frac{1}{2}$ times in magnitude by the value derived from Lenz's experiments.

However, Lenz himself remarks at the place cited in relation to the *absolute* magnitude of the result derived from his experiments:

> This result is merely an approximation, and can only serve as a rough estimate, for neither the absolute quantity of the spirit nor its heat capacity have been determined with certainty. My present experiments had no other purpose than to determine the law of the heating of metal wires; for the exact determination of the absolute value of this heating, I intend to undertake special experiments.

It is therefore very likely that with the care Lenz otherwise took in all respects in these experiments, simply because the attention was less focused on *absolute* value determinations, some accidental confusion took place in the value of the reduction coefficients, which had no influence on the sole purpose of establishing the laws, which is to blame for the above great deviation in *absolute* value; for a closer examination of the experiments clearly shows that the determination of the resistance of a body using this method can be carried out, which also seems to be confirmed by the good agreement of the result derived from Becquerel's experiments with those found by other means; however, in order to obtain completely reliable and precise results in this way, the *heat measurement methods* would have to be very perfected and sharper determinations about the *equivalence of heat and work* than are currently available would have to be obtained, and even then, the absolute resistance measurement of a *conductor wire* using this method would not achieve the accuracy of the result that can be obtained by measuring electromotive force and current intensity.

But if one divides the galvanic conductors into *metallic ones, which cannot be decomposed by the current*, and *moist ones, which can be decomposed*, it follows that with moist decomposable conductors, for example with *water*, an inverse relationship to that with *conductor*

[193] [Note by AKTA:] Adam Frederik Oluf Arndtsen (1829-1919) was a Norwegian professor and physicist who was trained by Weber at Göttingen in 1857. Augustus Matthiessen (1831-1870) was a British chemist and physicist. See also [RA11].

wires takes place, namely, that a *direct* determination of the resistance of wet conductors by measured electromotive force and current intensity is almost impossible, and what's more, even an *indirect* determination by comparing the unknown resistance of the wet conductor with that known resistances of a conductor wire, because of the so-called *polarization* of the metal surface touching the wet conductor, finds great difficulties. It is known that, despite all the effort and care expended, the resistance conditions of wet conductors are still very poorly researched. For these reasons, the *other method of resistance measurement* gains the greatest importance for this research, namely, through measured electrical work (heat) and measured current intensity, because when applied to *wet conductors* it has just as great advantages over the former, as the *former*, when applied to *conductor wires*, owned before the second. These advantages are based not only on the more perfect heat measurement methods applicable to wet conductors (water), but primarily on the independence of the entire measurement from the consideration of the *electromotive force*, which must always be considered as variable in all circuits where wet conductors are connected, because the influences of polarization can be reduced, but not completely eliminated. However, the *electromotive force* cannot be precisely determined with such irregular changes.

This equally important and interesting application that this second method finds in the absolute resistance measurement of *moist, decomposable conductors*, should be reserved for a special discussion, since it has no closer connection with the subject of this Treatise.

8.33 On the Conversion of Electrical Work into Heat

The electrical work is linked to the movement of the electrical fluids; according to the mechanical theory of heat, heat is also linked to the movement of a body, which, however, is usually distinguished from electrical fluids. A closer insight into the way in which electrical work is converted into heat therefore first requires that the movements of the electrical fluids be followed closely to the end, in order to get to know the conditions under which the transition of the movement of the electrical fluids into the movement of another medium takes place. The ideal assumption of the superposition of several substances continuously and uniformly distributed in the space of the conductor, namely, the ponderable substance of the conductor, the two electric fluids and also that of a so-called heat medium, however appropriate it may be for many other purposes in the case of actions at a distance, would not seem to be permissible; rather, it is easy to see the need to assume that the ponderable substance of the conductor is concentrated in individual molecules, which are surrounded by electrical particles which, in the case of a current, move from one molecule to the other. The separation of an electrical particle from a molecule must then take place either more slowly or more quickly, depending on the different magnitude of the electromotive force by which the current is produced, on which the number of electrical particles separating from the molecule in a certain time depends. The work of each electrical particle in the separation movement, as a result of the forces exerted on it by the molecule, may or may not depend on the velocity of the separation; an oppositely equal work will always be done by the same particle in its union movement with the following molecule, so that these two work quantities compensate each other. But as soon as the electric particle is separated from the first molecule, it will, driven by the electromotive force f, pass through the space α to the second molecule and thereby perform the work $f\alpha$. The sum of all of these work quantities, $\sum f\alpha$, forms the total electrical work in the conductor. Each electric particle therefore enters the region of the following molecule with a *vis viva* that is greater than the value equivalent to $f\alpha$ when it left the region of the previous molecule, which means that the value of the *vires vivae* in the region of all molecules taken together must be increased by an amount equivalent to the entire electrical work. However, according to the mechanical theory of heat, such an increase in the *vires vivae* in all molecules taken together, which is equivalent to the electrical work, is also the heat generated by the current in the conductor, and the only question is whether it is completely identical to that, that is, whether it consists in the continuous movement of those electrical particles themselves, or whether the movement supplied to each molecule is transmitted from the electrical particles which they brought with them to other material particles, for example to the particles of a special medium located in the region of the same molecule and only after this transfer emerges as heat, where the laws of transfer would then have to be researched and a closer account given as to why the same *vis viva* only emerges as heat when it is linked to the particles of the heat medium instead of to electrical particles.

One can easily see that the assertion of such a transfer of the *vis viva* brought by electrical particles to the particles of another medium located in the region of the molecule does not encounter insignificant difficulties, especially because it would logically require the denial of any continuation of the motion of electrical particles within the range of such a ponderable molecule. If the electrical particles that carry the electrical work with them, upon entering the region of a ponderable molecule, must immediately give up the electrical work they have brought with them, and not just partially, but entirely, to other material particles (to the particles of the heat medium), for the same reason, any movement given to the electrical particles in the region of ponderable molecules, regardless of where it may come from, must be immediately withdrawn from them again, so that no *persistent movement of electrical particles* in the region of ponderable molecules would be possible. This would even make the possibility of electric current in the ponderable body doubtful; for an electric particle, even if it were driven by very great electromotive forces, could not get into any major movement if every movement that was created was immediately transmitted from it to the particles of the heat medium.

It is clear from this, that the assertion that all electrical work is transferred to the heat medium of ponderable molecules is, above all, in total contradiction with the assertion of the existence of *persistent electrical molecular currents*, as first put forward by Ampère.[194] So whoever denies with Ampère the real existence of two magnetic fluids and is thereby forced to assert *persistent electrical molecular currents*, must not admit this transfer, and he needs to admit it all the less because nothing can be cited that would be gained through such a transfer. At least according to the mechanical theory of heat, it is clear that in relation to heat, in principle, nothing else is *directly* relevant except the *vis viva* present in the molecules, for which the nature of their material support is indifferent. Only *indirectly*, according to the mechanical theory of heat, could the nature of the material carrier of the *vis viva* that forms the essence of heat come into consideration, namely, insofar as the forces of interaction of the particles of this carrier, partly with one another and partly with other particles, and consequently the transfer- or propagation laws (the laws of heat radiation, temperature communication and temperature equalization among different ponderable molecules) would depend on it.

If also the *heat ether in the empty space* is at least indirectly defined by the laws of wave propagation attributed to it, like the light ether, and its existence and distribution, also in the interior of the ponderable bodies, in the empty spaces between the molecules, cannot be abstracted without rejecting the whole wave theory of radiant heat, then there is no further

[194][Note by AKTA:] See footnote 15 on page 18.

relationship between the ponderable molecules of the body (with everything that lies in their region and belongs to it) and that ether than, *on the one hand*, the wave excitation in the ether (the heat radiation), *on the other hand*, the wave attenuation (heat absorption) must come from the ponderable molecules, for which a special heat medium is no more necessary in the molecules than air needs to be contained in the metal of the bell, which emits sound waves through the air medium.

All of these considerations can be briefly summarized as follows. Since, according to the mechanical theory of heat, an increase in the temperature of the ponderable molecules requires an increase in the *vis viva* in the molecules, since this increase in the *vis viva* is given by the electric particles which enter with greater velocity into the region of the molecules and exit with lower velocity, which form the current, since furthermore this increase in *vis viva remains* unweakened *according to the theory of persistent electrical molecular currents*, while the particles are in the region of the molecules, then it seems that there can be no question of a *conversion* of electrical work into heat, but that *electrical work* accumulated in the molecules then seems to itself have to be viewed as the *heat* contained in the molecules.

It is, of course, obvious that the laws of the relationships summarized under the name of *heat radiation* and *heat absorption* between the electricity in a persistent molecular flow around the individual molecules and the heat ether in the surrounding space still require a more detailed justification based on the nature of both media; but those laws would also require just the same justification if the so-called heat medium were substituted for electricity. While in the latter case such justification has not even been attempted, as far as the former case is concerned one can use the astute investigation carried out by C. Neumann: *Explicare tentatur quomodo fiat ut lucis planum polarizationis per vires electricas vel magneticas declinetur*, Halis Saxonum, 1858,[195] as such a first attempt; for it suggests that what Neumann says about the relationships between persistent electrical molecular currents and light ether will also apply in a similar way to the relationships between persistent electrical molecular currents and heat ether.

According to his premises, Neumann found that there could be no action of electrical molecular currents on *resting ether particles*; however, it should be noted that these premises, in accordance with the purpose of Neumann's investigation, which was limited to the influence of the molecular currents on the already existing wave trains propagated through the ether between the molecules, relate to the actions of the molecular currents in very small distances, but still allowed the admission of an *ideal* conception of molecular currents, according to which they are viewed as a *superposition of oppositely equal currents of positive and negative electricity*, which is apparently not permitted when it comes to the excitation of new wave trains by the electrical molecular currents, which can only take place in the *ether layer immediately adjacent* to the molecular currents. For this ether layer, the positive and negative electrical particles moving in opposite directions can no longer be considered as coinciding. If one then imagines, for example, that the negative fluid is firmly connected to the molecule, and only the positive fluid is conceived as in molecular flow, or vice versa (a way of thinking that is recommended because it can exist with the persistence of the molecular flows without electromotive forces), it is clear that the difference in the position and behavior of the two electrical fluids in the region of the molecule no longer needs to be taken into account even at very small distances (as Neumann considers them), on which the admissibility of that *ideal* conception of molecular currents is based, however, it can be of importance for the *immediately adjacent ether layer*, especially if the electrical fluid in the

[195][Note by AKTA:] [Neu58], see also [Neu63].

molecular flow was *not continuously and uniformly* distributed around the molecule.

But if there really is a disturbance of the equilibrium in the *immediately adjacent ether layer*, and consequently an excitation of ether waves, then it is obvious that this will be repeated with every revolution of the electricity around the molecule, that is, the *wave period* with the *revolution period of the electric particles* in the molecular current must match. In the case of *luminous molecules*,[196] however, the duration of the wave trains emitted by them is known precisely from optical experiments; if the assumed relationship between electrical molecular currents and the light ether, according to Neumann's idea, were confirmed, it would then be possible to obtain more detailed information from optical experiments about the behavior of the electricity forming the molecular currents. — In any case, Neumann's investigation was so successful in its initial development for optics, *to explain the rotation of the plane of polarization through galvanic and magnetic forces*, that one can hope that the further pursuit and development of the theory of persistent electrical molecular currents in their relationship to the light or heat ether and its wave movement will lead to many other insights concerning the important and still so little researched connection between *electricity*, *heat* and *light*.

[196][Note by AKTA:] In German: *Bei leuchtenden Molekulen.*

Chapter 9

Translator's Introduction to Weber's 1874 Paper

Laurence Hecht[197]

This paper by Wilhelm Weber (1804-1891) continues the unique and still largely unrecognized approach to the atomic and sub-atomic realm that Weber and C. F. Gauss (1777-1855) had pioneered in their collaboration, beginning in 1830.

Here Weber draws on the discovery, first reported in his *Sixth Memoir on Electrodynamic Measurements* (1871),[198] that there can exist a stable state of aggregation of two similarly charged electrical particles, which he refers to here as a *particle pair*. In that memoir, he is able to derive a *critical length*, below which the similarly charged particles no longer repel, but rather attract each other. Weber's formula for *critical length* is expressed such that it applies to pairs of either negative or positive particles and also takes into account the possibility of different masses:

$$r < \frac{2}{c^2}\frac{\varepsilon+\varepsilon'}{\varepsilon\varepsilon'}ee' \ , \tag{9.1}$$

where ε is the mass of the electrical particle, e is the charge in electrostatic units, and c the Weber constant.[199] (Weber later introduced the Greek letter ρ, rather than r, for his critical length.)

If one substitutes the values for electron and proton mass, which were derived many decades after Weber's death, into the Weber formula it results in a *critical length* of 5.6×10^{-13} *cm* (or twice the *classical electron radius*) for an electron pair, and a value 1836 times smaller for a proton pair. The later expression for the *classical electron radius* ($[e^2/(m_e c^2)]$

197 Email: larryhecht33@gmail.com.

198 [Web71] with English translations in [Web72] and [Web21g].

199 The symbol c, as used by Weber, came to be known in mid 19th century physics as the *Weber constant*. Its value was derived by Weber and Kohlrausch in experiments reported in 1855 to 1857 on electrical discharge currents, comparing the force produced by a static to a moving charge, [Web55] with English translation in [Web21i]; [WK56] with Portuguese translation in [WK08] and English translation in [WK21]; [KW57] with English translation in [KW21]. As Weber saw it, c was the relative velocity between two like electrical particles at which the force between them would reduce to zero. It had a value $\sqrt{2}$ times light velocity. The modern application of the symbol c for light velocity appears to have originated later with Maxwell, $c_{Weber} = \sqrt{2} \cdot c_{Maxwell} = \sqrt{2} \cdot v_L$, where v_L = light velocity in vacuum, see [Ass21h].

in the CGS system of units),[200] formulated well after Weber's death, is easily derived as a special case of Weber's *critical length*.

Within a sphere of this diameter, the particles of Weber's pair will approach and recede from each other at a *relative velocity* (defined as the derivative with respect to time of the ever-changing distance between the two moving particles), which shall never exceed the value then referred to as the *Weber constant* (c). Interestingly, as Weber first notes in Section 10.3, this defines a motion, and consequent energy, independent of any fixed frame of reference and thus expressible without the need for spatial coordinates.

9.1 The Equivalent of *Vis Viva*

In this paper, Weber draws attention to the fact that his velocity-dependent force law, and the consequent existence of a boundary defined by the *critical length* (ρ), introduces a new consideration into the laws of motion and energy. The concept of a potential, which arose in the consideration of the laws of gravitation and electrostatics requires modification.

Potential was defined as the work that would be done to bring two interacting particles from a given distance apart to an infinite distance, or vice versa. While it had been sufficient to consider the sum of the *potential* and the *vis viva* (similar to the modern kinetic energy) as a constant, Weber recognizes that something new is required. He thus defines a new quantity, the *work capacity*, which measures the work done to bring two interacting particles from the critical length, ρ, to an infinite distance of separation. Like the potential, it is "an equivalent of *vis viva*" in that as one increases the other decreases, but it is evaluated independently of the existing distance r.

As an illustration of such a force, Weber cites the example of the elastic bar, a subject of careful analysis by early 19th century physicists, including his colleague Franz Neumann (1798-1895). In the theory of the elastic bar, the bar possesses a "natural length," independent of whatever length is observed at a particular time. The "equivalent of *vis viva*," in this case, is the restoring force which acts to bring the bar back to the natural length when the tension on it is released.

Analogously, Weber's critical length, ρ, is a sort of "natural length" which defines the behavior of the particle pair. For the case of two particles outside the critical length, at distances greater than ρ, a hidden force (the equivalent of *vis viva*, or *work capacity*) is depleted as the force of repulsion moves them apart. He defines the work capacity as the integral

$$U = \int_{\rho}^{\infty} R dr \ , \tag{9.2}$$

where R is the repulsive force between the two particles at the moment when their distance apart is equal to r.

The consequences are far-reaching. As Weber shows in Section 10.8 below, for two like electric particles, his integral expressing *work capacity* has the value $mc^2_{Weber}/4$ ($= mc^2_{Maxwell}/2$).[201]

In his *Sixth Memoir*, Weber had shown that for a pair of bound particles inside the critical length their maximum relative *vis viva* (when $(dr/dt)^2 = c^2$) attains the value $mc^2_{Weber}/4$ as

[200] Or $[\mu_o e^2/(4\pi m)]$ in the International System of Units MKSA.

[201] Or, more precisely, $(1/4)mc^2_{Weber}\left[1 - v^2/c^2_{Weber}\right]$, where $v = dr/dt$.

well. Taken together, these two quantities are much like what came to be called the binding energy. But unlike the binding energy of the later nuclear physics, which was an *ad hoc* formulation to fit the experimental evidence, Weber's derives naturally from the velocity-dependent electrodynamic law first suggested by Gauss in 1835 and elaborated by Weber in his *First Memoir on Electrodynamic Measurements* of 1846.[202]

Interestingly, as the velocity c of Weber is $\sqrt{2}$ times greater than the velocity of light v_L (the c of Maxwell, Lorentz, and Einstein), or

$$c_{Weber} = \sqrt{2} \cdot c_{Maxwell} = \sqrt{2} \cdot v_L \ , \tag{9.3}$$

then

$$E = mc^2_{Maxwell} \tag{9.4}$$

becomes

$$E = \frac{mc^2_{Weber}}{2} \ , \tag{9.5}$$

retaining consistency between the mass-energy equation in Weber's formulation and the ordinary expression for kinetic energy or *vis viva*, $E = (mv^2/2)$. The two worlds, inner and outer, although distinct, obey the same natural laws.

The reader is warned that Weber's electrodynamics is an entirely different world than the one generally known today, which is based on the formulations of Faraday, Maxwell and their followers. It is not possible to simply carry over conceptions learned in college and graduate physics, and expect to grasp the alternative view that Gauss and Weber created. To have any deep comprehension of even this short paper requires a more thorough study of the fundamental conceptions presented in the First and Sixth Memoirs on Electrodynamic Measurements, than we can present here. Few are willing to undertake such a task. Yet we assure you, the hidden, and still scarcely explored, treasures that lie within are more than worth the effort.

Laurence Hecht
May 30, 2023

[202] See [Gau67] with English translation in [Gau21a], and [Ass21d]. See also [Web46] with a partial French translation in [Web87] and complete English translations in [Web07] and [Web21d].

Chapter 10

[Weber, 1874] On the Equivalent of *Vis Viva*

Wilhelm Weber[203,204,205,206]

Some of my earlier Treatises on electrodynamic measurements have already appeared in Volumes 73, 82, and 99 of the *Annalen*.[207,208] Here, only the principal subject of the last of these Treatises will be discussed in more detail, namely the connection of the established fundamental law of electrical action with the principle of the conservation of energy and with the equivalent living forces (*vires vivae*)[209] given by it.

As far as the objections raised against the aforementioned fundamental law of electrical action are concerned, lack of space compels me to confine myself here to the following general remarks.

The more recent *mathematical* investigations into electricity have primarily concerned *far-action effects* and have therefore, apart from electrostatics, mostly adhered to the *integral* and the *elemental laws*, in contrast to the point laws, as Neumann calls them.[210] From the

---

[203] [Web74].

[204] Translated by Laurence Hecht, larryhecht33@gmail.com, and edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

[205] The Notes by Heinrich Weber, the editor of Volume 4 of *Wilhelm Weber's Werke*, are represented by [Note by Heinrich Weber:], the Notes by Laurence Hecht are represented by [Note by LH:], while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[206] [Note by Heinrich Weber:] *Annalen der Physik und Chemie*, Jubilee volume dedicated to the Editor of the *Annalen der Physik und Chemie*, J. C. Poggendorff, Leipzig 1874, pp. 199-213.

[207] [Note by Heinrich Weber:] *Wilhelm Weber's Werke*, Vol. III, pp. 215, 276, 597.

[208] [Note by AKTA:] [Web48a] with English translations in [Web52c], [Web66c], [Web19] and [Web21p]; [Web51] with English translations in [Web61] and [Web21o]; and [WK56] with Portuguese translation in [WK08] and English translations in [WK03] and [WK21].

[209] [Note by LH and AKTA:] See footnote 173 on page 108.

[210] [Note by LH and AKTA:] In a paper in 1872, Carl Neumann (1832-1925), professor of mathematical-physics at Leipzig University, categorized the existing theories of electrodynamics based on far-action or action at a distance into three types. These were the point laws (*Punktgezetze* in German) of the type employed by Coulomb and Weber, based on the assumption of forces between electrical particles or between mass points; elemental laws (*Elementargesetze* in German) referring to Ampère's approach using mathematically described elements of current and the force between them; and integral laws (*Integralgesetze* in German) of the type developed by his father, physicist Franz Neumann (1798-1895), related to closed electric circuits, [Neu72].

physical point of view, on the other hand, the effects of electricity on the bodies through which it flows, particularly the *thermal* and *chemical effects*, have become so important and significant that the *physicist* cannot avoid turning his attention to the *point laws*, which alone can provide insight into the inner connection between electricity and heat, as well as into the inner mechanics of chemical processes.

These thermal and the chemical effects are, above all, what make it necessary to ascribe a *molecular constitution* to bodies in general. In its molecular constitution, however, every molecule, like a celestial body, is a world in motion by itself, whose inner relationships and motions are not directly observed. But these inner relationships and motions play a great part in all thermal and chemical processes, so that it has become necessary to assume a *molecular constitution for a body*, in both the *mechanical theory of heat* and the *chemical theory of the atom*.

In the case of a *molecularly constituted body*, however, as is easily seen, one cannot posit any arbitrary mass distribution; for the atomic masses in the molecules are masses of a *given magnitude*, which always remain *distinct*. They start out at definite distances from one another and in definite relative motions with respect to each other, and always maintain such mutual interactions so as to remain distinct and apart from each other. It is thus easy to understand that for such atoms *not every conceivable mass accumulation* will occur, and arbitrary assumptions about such mass accumulations can lead to contradictions. And even if all the laws of interaction of the atoms were known, these more precise details could not be determined so long as the three-body problem remains unsolved; because knowledge of the laws of motion of *two* mutually interacting bodies is not sufficient for this.

The foregoing objections must therefore be left unanswered for the time being, even if they are otherwise well-founded, because one does not know whether to place the blame for the contradictions on the contested Fundamental Law, or instead on those arbitrarily assumed mass accumulations.

10.1 Principle of Conservation of Energy

The law of inertia already prescribes that *if there is no external influence on a body, its vis viva remains unchanged.*

The principle of the conservation of *vis viva* follows from this: *for a system of bodies without external influence, the sum of its vires vivae remains constant so long as the relative positions of the bodies are the same.*

This principle of the conservation of *vis viva* was finally extended to the Law of the Conservation of Energy: *for a system of bodies without external influence, if the relative positions of the bodies vary, it is not the sum of their vires vivae alone, but rather this sum added to the work that would be done as a result of the interaction of the bodies if they were displaced to an infinite distance of separation, that remains constant at any given time.* That sum of the *vires vivae* is called the *kinetic energy*,[211] and the work that would be done is called the *potential energy* of the system of bodies, the sum of which always remains the same when external influences are excluded.

As can be seen, however, this statement of the law of conservation of energy contains various interconnected principles which are better separated from one another: *firstly* prin-

[211][Note by AKTA:] In German: *Bewegungsenergie*. This expression can be translated as kinetic energy or energy of motion.

ciples which are properties of individual bodies or particles, and *secondly* principles which relate to the properties of particle pairs.

10.2 Distinction of the Properties of an Individual Body or Particle, from Those of a Particle Pair

1. *Every body or particle* (every physical point or atom), considered by itself alone, possesses properties which completely determine its behavior in space and time, if the forces acting on it at any given instant and at all subsequent instants are given. These properties of the individual particles are *inertia* and *mass*, and there is no difference in these properties possessed by different particles other than the amount of their mass.

2. *Each pair of particles* has properties that are quite independent of the properties of the individual particles. These properties of the *pairs* are the reason that *work* is done with every change in the distance between the two particles. And the reason for a force of mutual repulsion or attraction lies in the properties of those *pairs*, which, multiplied by the change in distance between the two particles, gives the *work* done by the *pair*.

3. A system of *three* or more particles has *no* properties that are not already contained in the properties of the individual particles and the particle pairs.

This restriction of the nature of bodies to the properties of the *individual particles* and *particle pairs* leads to the important result that the investigation of the nature of all bodies can be greatly simplified by reducing them to the study of the individual pairs, which can be considered independently of one another. It would not be possible to so reduce the examination of the nature of all bodies, if the *totality of all particles* possessed unique properties that were not already contained in the properties of the *individual particles* or *particle pairs*. It is very important, in any case, that all the elements essential for the consideration of general processes of nature are already completely contained in the properties of *individual particles* and *particle pairs*.

10.3 Characteristics of a Fundamental Law of Interaction

Once the properties of *individual* particles were completely determined by *general mechanics*, *physics* essentially had only to determine the properties of particle pairs. For this determination nothing more need be considered than the *nature* and *mutual relations* of the particles forming a pair, and *the work produced by their interaction at every change in their distance of separation*. The *relationships* between these two particles are given by their *distance of separation* and the *velocity at which it changes*. — The fixed spatial *coordinate system*, which is required for a complete determination of the position and motion of particles, is completely unnecessary for the determination of the distance and relative velocity of the particles in a pair, and does not come into consideration at all here. It must therefore be possible to represent the *fundamental laws of particle interaction* for a *particle pair* without the aid of spatial coordinates.[212]

[212][Note by LH and AKTA:] The concept of representing *relative vis viva* in a system free of fixed coordinates is mentioned by Carl Jacobi (1804-1851) in his *Lectures on Dynamics*, Fourth Lecture, "The Principle of Conservation of *vis viva*," edited by A. Clebsch in 1866 from notes provided by C. W. Brockardt who attended

The only variables required to represent the *fundamental law of particle interaction* are the *relative distance* (r) between the two particles, their *relative velocity* (dr/dt) and functions of these two magnitudes. Functions of the spatial coordinates are not needed.

None of the propositions listed above under Section 10.1, not even the last one (the proposition advanced under the name of the principle of the conservation of energy), corresponds to these requirements for a fundamental law of particle interaction. This is because those propositions are not simply about the properties of a particle pair, but rather more generally about the properties of a system of bodies (a system of particle pairs), namely a property which this system does not always have, but only possesses so long as no outside influence is present. And further because the sum, which according to this theorem [in Section 10.1] should remain constant, is essentially a function of the space coordinates x, y, z.

10.4 Two Kinds of Equivalents of *Vis Viva*

A pair of particles, ε and ε', moving away from or approaching each other with the velocity dr/dt, possesses a *relative vis viva* which is determined by the product of the square dr^2/dt^2 multiplied by a factor dependent upon the masses ε and ε',[213] namely

$$\frac{1}{2}\left(\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'}\right) .$$[214]

We need not consider here the mean *absolute vis viva* of the pair (attributed to their center of gravity), namely

$$\frac{1}{2}\left[\frac{(\varepsilon\alpha+\varepsilon'\alpha')^2}{\varepsilon+\varepsilon'}+(\varepsilon+\varepsilon')\gamma^2\right] ,$$

where α and α' signify the velocities of the two particles in the direction r, and γ the velocity of the center of gravity perpendicular to r.

Nor do we need to consider the *vis viva* arising from the motion of the particles *around each other*, namely[215]

$$\frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'}\frac{ds^2}{dt^2} ,$$

where [ds/dt] represents the velocity with which the two particles move towards or away from each other in the direction perpendicular to r.

Jacobi's lectures at the University of Königsberg in the winter semester of 1842-43, [Jac66] and [Jac84] with English translation in [Jac09]. Jacobi's idea derives from earlier ideas of Lagrange (1736-1813) and Hamilton (1805-1865) describing a system by its conservation of *vis viva*.

[213] [Note by AKTA:] Weber's expression dr^2/dt^2 should be understood as $(dr/dt)^2$. For a detailed deduction of the following formulas of this Section see, in particular, Section 4 of Weber's Sixth Memoir on Electrodynamic Measurements, [Web71] with English translations in [Web72] and [Web21g].

[214] [Note by LH:] Weber here employs the concept of *reduced mass* ($mm'/(m+m')$), still used to describe a two-body system in celestial mechanics. In the case that the electrical particles, ε and ε', are of equal mass, the *reduced mass* is equal to one-half the mass of either particle.

[215] [Note by AKTA:] The next equation should be understood as

$$\frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'}\left(\frac{ds}{dt}\right)^2 .$$

However, the *relative vis viva* belonging to the system of both particles varies with time, such that as one part of it is lost, a new part is added. But such a change does not take place without something else simultaneously being changed, and when that other thing is restored, the lost part of the *vis viva* is also restored. The part of the *vis viva* which has been temporarily lost is said to have been replaced by something else, and this thing that replaces it is described as the *equivalent* of the part of the *vis viva* that was lost.

It has become a major focus of physics to establish *laws for this equivalent*, according to which it can be determined from *measurable quantities*.

Now, this *equivalent* can *either* be the motion of a body, *or* not. In the *first* case, the equivalent would also be a *vis viva*, like that which it replaces, and would differ from it only in belonging to other particle pairs. But it could also happen that it would become imperceptible to us as motion, as in the case of heat. In all these cases, where the equivalent of the lost *vis viva* is also a *vis viva* and, in fact, of the same magnitude, the sum of the *vires vivae* remains unchanged and it is merely a question of an altered distribution of it, the explanation for which is to be sought in the laws of motion.

The *other* case is completely different from this, where the lost *vis viva* is not directly replaced by *vis viva*, but by something else different from *vis viva*. The investigation of this later kind of equivalents for the lost parts of the relative *vis viva* of the particle pair requires closer examination, both as to their *nature* and as to the laws of their determination from other *measurable quantities*.

10.5 The Second Kind of Equivalent

1. *Vis viva* is something *real* and always *positive*, just like *mass*. It follows from this that the *potential of certain forces* (electrical forces, for example) which can be either positive or negative, *cannot be an equivalent of vis viva* in the real sense. Also, the potential is not something really *present*, as it is the work which would be done if the two particles were brought from an infinite distance to their distance of separation, r. Now, if instead of the work that would be done, one could put into the definition of potential the work that *was done* by moving the two particles from an infinite distance to the distance r, then *this work* itself would either consist of a change in the *relative vis viva of both particles*, or in the *cancellation of other work*. But *cancelled work* is also not something that really exists any more than a cancelled force. And a *change in the vis viva* is already contained as part of the existing *vis viva*, and therefore cannot be counted as being present alongside the existing *vis viva*.

2. Still, even such an *imaginary form of work*[216] as is the potential, which is the work that would be done if the two particles were brought *from a finite to an infinite distance of separation*, or vice versa, might serve as the *definition and magnitude* of something actually *existing*, namely as the definition of an actually existing *property* which the system of two particles possesses. Except that in the definition of such a property, the *presently existing* distance, r, should not be taken as the finite distance of separation from which the two particles are to be carried to an infinite distance of separation. Rather, that finite distance must be determined *completely independently of the existing distance*, r; for the property should hold *for all values of* r, without distinction.

As an illustration of how such *imaginary work* can serve to define a property that actually

[216] [Note by AKTA:] In German: *gedachte Arbeit*.

exists, the example of *elastic rods* can be cited, wherein a very specific finite distance between the two ends of the rod is determined by the so-called *natural rod length* ρ, *completely independent of the existing rod length* r. For such a rod, a merely *imaginary work* is used (namely, that which would be performed by the rod if it were brought from twice its natural length to its simple natural length, i.e. from 2ρ to ρ) as the *definition and measure* of something actually existing in the rod, namely its actually existing property of *elasticity*.

Thus, if the *tension* corresponding to the rod length r is denoted by R, then the *elasticity* of the rod is expressed by $\int_{2\rho}^{\rho} Rdr$. In the same way, the expression $\int_{\rho}^{\infty} Rdr$ can serve as a definition of the property of the *two electric particles* given above, if R designates their repulsive force at the moment when, during the transfer from the finite distance ρ[217,218] (determined *independently of* r) to an infinite distance, their distance has become *equal* to r.

— If this property of the two particles is to apply in general, then R must be a function of time t, in order to always correctly represent the repulsive force, which can be different at different times at the same distance.

— The property of the two electric particles defined by $\int_{\rho}^{\infty} Rdr$ shall be called their *work capacity*.[219,220]

3. Since the relative *vis viva* of two particles is something completely independent of their distance of separation, r, the equivalent of this *vis viva* must also be independent of r. If, therefore, the equivalent, like the potential, were represented by the integral of a function of r, this integral would have to be a *definite integral* taken between limits which are completely independent of r, just like the property just defined of two electric particles, which has been called its work capacity.

4. Finally, it is implicit in the notion of the equivalent of *vis viva* (because *vis viva* is unchangeable so long as no part of it is replaced by an equivalent), that the equivalent, expressed in proper units, of the lost portion of *vis viva* added to the existing part of the *vis viva* must form a *constant*. However, because of the *finite value* of this constant (as the sum of two positive magnitudes), a *limit* is given which cannot be exceeded by either of these two magnitudes, neither by the *vis viva*, nor by its equivalent. From this it is evident that wherever the relative *vis viva* of two particles could grow infinitely, beyond any limit, there could exist no equivalent of *vis viva*, and thus also no principle of conservation of energy for which the existence of an equivalent of *vis viva* is a condition.

But a *vis viva* constrained by such a *limit*, or a limited relative velocity of the particles, might seem to conflict with the principles of *general mechanics*, where all forces are taken as given without asking their *origin*, just as the initial distribution of the masses and their velocities are taken as given without asking how they came about. In fact, if in *general me-*

[217] [Note by LH:] Weber's use of the symbol ρ in this development of the illustration of an elastic rod draws on a conception developed a few years earlier in his *Sixth Memoir*. In his derivation there of the *critical length* for an electrical particle pair, ρ, Weber recognized that the force between two similarly charged electric particles would reverse at this length. In the case cited here ($\int_{\rho}^{\infty} Rdr$), Weber is discussing the repulsive force at distances greater than the critical length. However, within a sphere of diameter ρ, the particle pair would be in a bound state oscillating back and forth from a distance of separation $r = 0$ to $r = \rho$. (Compare Weber's *Sixth major Memoir on Electrodynamic Measurements*, Sections 2, 8, 10, and 11).

[218] [Note by AKTA:] See footnote 198.

[219] [Note by LH:] In German: *Arbeitsfähigkeit*. The more literal translation "ability to do work" is often used in science instruction as a first definition of energy. We take the term *work capacity*, from the first English translation of Weber's Sixth Memoir.

[220] [Note by AKTA:] [Web71, p. 267 of Weber's *Werke*], with English translations in [Web72, p. 19] and [Web21g, p. 83].

chanics by the "given forces" we were to understand *any forces*, the possibility of producing any relative velocity between two particles would be self-evident. This possibility is by no means plausible when one goes back to the *origin of the forces*, as in physics, and all forces are derived from the *lawful* interaction of bodies. Here one sees that, *in principle*, nothing stands in the way of such a *limit*, but the existence or non-existence of such a limit can only be decided from the *laws* of interaction of the bodies themselves.

10.6 The Law of Work Capacity under the Assumption that It Is the Equivalent of *Vis Viva*

It was explained at the beginning of the previous Section that only an actually existing and positive magnitude can be the sought for equivalent of *vis viva*. It then turned out that the potential of two particles $\int_\infty^r Rdr$, (i.e. *the work* that would be done if the two particles were brought from an infinite distance to the existing distance r) would not be such a magnitude and therefore could not be the sought-after equivalent of *vis viva*. On the other hand, $\int_\rho^\infty Rdr$, i.e. *the work* that would be done if both particles were brought from a finite distance ρ (which can be determined quite independently of the existing distance r) to an infinite distance, could probably serve as the *definition of such a magnitude*, namely as the definition of an actually existing property of the system of both particles in their current state, which was called the system's existing *work capacity*. *This work capacity* will now be determined more precisely, on the assumption that it is the sought for *equivalent of vis viva*.

If we call U the *work capacity* of two electrical particles (ε and ε') at any given time, and x the *relative vis viva of these particles* at this same time, then it follows that the *work capacity* U (*as the equivalent*), added to the *vis viva* x, is equal to a constant, a:

$$x + U = a \ , \qquad \text{or} \qquad U = a\left(1 - \frac{x}{a}\right) \ .$$

If now, following Section 10.4, we set the *vis viva* as:

$$x = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon + \varepsilon'} \cdot \frac{dr^2}{dt^2} \ ,$$

and, for the case of a disappearing U (where $x = a$), we set $dr^2/dt^2 = c^2$,[221] it follows that:

$$U = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon + \varepsilon'}c^2\left(1 - \frac{1}{c^2}\frac{dr^2}{dt^2}\right) \ .$$

[221][Note by LH:] Hence the *relative vis viva*, x, for two like electrical particles when $dr^2/dt^2 = c^2$ is $mc^2_{Weber}/4 = mc^2_{Maxwell}/2$. Weber here describes the case where the particles are beyond the critical length.

The relative velocity c occurs naturally inside the *critical length*, where the bound particle pair oscillates along the line connecting them from zero relative velocity at $r = \rho$ to a maximum relative velocity c at the center of the sphere, whence the *relative vis viva*, becomes also equal to $mc^2_{Weber}/4 = mc^2_{Maxwell}/2$. When $dr^2/dt^2 = c^2$, the sum of the values for *vis viva* both within and without the critical length becomes $mc^2_{Weber}/2 = mc^2_{Maxwell}$.

Note that Weber's c is a *relative velocity*, *i.e.* the rate of change of the distance *between* the particles, not their velocity as seen by a stationary observer in the laboratory frame of reference. For the observer in laboratory of two like, bound particles approaching each other at *relative velocity* $= c_{Weber}$, each particle will have the velocity $c_{Weber}/2 = (\sqrt{2}c_{Maxwell})/2$. There is no mass increase in Weber's formulation. Rather, the decrease in *charge-to-mass ratio* of high-velocity charged particles such as in the experiments reported by Kaufmann in 1901, had already been theoretically described in Weber's earliest electrodynamic work as due to a decrease in the *force* between electrical particles as their relative velocity increased.

In this expression for the work capacity U, however, the masses ε and ε' are, as is well known, quantities which cannot be determined by measurement. Therefore, two electric particles are commonly denoted not by the immeasurable masses, ε and ε', but rather by the measurable forces e^2 and e'^2, which each of these two particles exerts individually on the other equal one when *at relative rest*, at a distance $= 1$, or rather by the square roots of these values e and e', which are proportional to the immeasurable masses.[222]

In order to now substitute into the expression for *work capacity*

$$U = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon + \varepsilon'}c^2\left(1 - \frac{1}{c^2}\frac{dr^2}{dt^2}\right)$$

the *measurable* quantities e and e' for the *immeasurable* ε and ε', we must have reference to the limiting value of U for vanishing values of the velocity dr/dt, or for vanishing values of the *vis viva* $(x = [1/2][\varepsilon\varepsilon'/(\varepsilon + \varepsilon')][dr^2/dt^2])$ which may be denoted by U_0, namely:

$$U_0 = a = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon + \varepsilon'}c^2 \ .$$

In this case, the *electrostatic* law applies, according to which the work that would be done during a virtual change in distance dr is $= [ee'/r^2]dr$. If one understands by U_0 *that work* which would be performed as a result of this *electrostatic* interaction, if both particles were brought from a finite distance ρ (which can be determined quite independently of the existing distance r) to an infinite distance, then one obtains

$$U_0 = a = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon + \varepsilon'}c^2 = \int_\rho^\infty \frac{ee'}{r^2}dr = \frac{ee'}{\rho} \ .$$

From this it follows: *first*, that for every system of two electrical particles there really is a finite distance $\rho = ee'/a$ that can be determined quite independently of the existing distance r; *second*, that if one substitutes ee'/ρ in place of a in the equation $U = a(1 - [1/c^2][dr^2/dt^2])$, *the sought for work capacity* or the sought for *equivalent of vis viva becomes*:

$$U = \frac{ee'}{\rho}\left(1 - \frac{1}{c^2}\frac{dr^2}{dt^2}\right) \ .$$

10.7 Derivation of the Potential from the Work Capacity

In the previous Section we defined the work capacity, U, as the work that would be done if both particles were brought from the finite distance of separation, ρ, which can be determined quite independently of the existing distance r, to an infinite distance from each other, i.e.

$$U = \int_\rho^\infty R dr \ .$$

[222][Note by AKTA:] While ε and ε' are the values of the inertial masses of the two particles, e and e' are the values of their electric charges. The electrostatic force R between two point charges e and e' when they are separated by a distance r is given by $R = ee'/r^2$ in Gauss and Weber's absolute system of units. Therefore, when $r = 1$ *mm* in this absolute system of units, the electrostatic force between the two point charges e and e' will be given by $R = ee'$. When we have two equal charges e and e separated by one unit of distance, this force will be $R = e^2$. Likewise, for two equal charges e' and e' separated by one unit of distance, this force will be $R = e'^2$.

However, we found at the end of the previous Section

$$U = \frac{ee'}{\rho}\left(1 - \frac{1}{c^2}\frac{dr^2}{dt^2}\right) ,$$

which can also be written as

$$U = \int_{\rho}^{\infty} \frac{ee'}{r^2}\left(1 - \frac{1}{c^2}\frac{dr^2}{dt^2} + \frac{2r}{c^2}\frac{d^2r}{dt^2}\right) dr .$$

It then follows that the repulsive force resulting from the interaction of both particles is

$$R = \frac{ee'}{r^2}\left(1 - \frac{1}{c^2}\frac{dr^2}{dt^2} + \frac{2r}{c^2}\frac{d^2r}{dt^2}\right) .$$

According to the definition of the potential, V, as *the work* which would be done as a result of the interaction of both particles when they are brought from an infinite distance apart to the existing distance, r, we get, finally:

$$V = \int_{\infty}^{r} Rdr = \int_{\infty}^{r} \frac{ee'}{r^2}\left(1 - \frac{1}{c^2}\frac{dr^2}{dt^2} + \frac{2r}{c^2}\frac{d^2r}{dt^2}\right) dr ,$$

or,

$$V = \frac{ee'}{r}\left(\frac{1}{c^2}\frac{dr^2}{dt^2} - 1\right) .$$

10.8 General Application

Finally, if we consider the masses m and m' of *any* two particles whose relative distance apart is r, and whose relative *vis viva* is

$$\frac{1}{2}\frac{mm'}{m+m'}\frac{dr^2}{dt^2} ,$$

and if the interaction occurring between these two particles when *at relative rest* is such that the incremental change in distance, dr, corresponds to the work $[kmm'/r^n]dr$, then, under the same assumption about the existence of an equivalent of *vis viva* for the previously considered *electrical* particles, there results for these particles a *work capacity* U such that:

$$U = \frac{1}{2}\frac{mm'}{m+m'}c^2\left(1 - \frac{1}{c^2}\frac{dr^2}{dt^2}\right) .$$

Further, if the *constant sum* of the *vis viva* and work capacity, i.e.

$$\frac{1}{2}\frac{mm'}{m+m'}c^2 ,$$

for these particles is equated with the *limiting value* of the work capacity for the case of a vanishing *vis viva* (where the law of interaction valid for the case of *relative rest* applies), that is, if

$$\frac{1}{2}\frac{mm'}{m+m'}c^2 = \int_{\rho}^{\infty} \frac{kmm'}{r^n}dr = \frac{k}{n-1}\cdot\frac{mm'}{\rho^{n-1}} ,$$

it results that

$$U = \frac{k}{n-1} \cdot \frac{mm'}{\rho^{n-1}} \left(1 - \frac{1}{c^2}\frac{dr^2}{dt^2}\right) \ ,$$

which can also be written:

$$U = k \int_{\rho}^{\infty} \frac{mm'}{r^n} \left(1 - \frac{1}{c^2}\frac{dr^2}{dt^2} + \frac{2r}{(n-1)c^2}\frac{d^2r}{dt^2}\right) dr \ .$$

It follows from this that the *repulsive force* R resulting from the interaction of the particles (which, for the case of *relative rest*, is assumed to be $= kmm'/r^n$), becomes for the case of relative motion, that is when the *vis viva* $= [1/2][mm'/(m+m')][dr^2/dt^2]$:

$$R = \frac{kmm'}{r^n} \left(1 - \frac{1}{c^2}\frac{dr^2}{dt^2} + \frac{2r}{(n-1)c^2}\frac{d^2r}{dt^2}\right) \ .$$

Finally, we get from this the expression for the potential, V:

$$V = \int_{\infty}^{r} Rdr = \int_{\infty}^{r} \frac{kmm'}{r^n} \left(1 - \frac{1}{c^2}\frac{dr^2}{dt^2} + \frac{2r}{(n-1)c^2}\frac{d^2r}{dt^2}\right) dr \ ,$$

or,

$$V = \frac{k}{n-1} \cdot \frac{mm'}{r^{n-1}} \cdot \left(\frac{1}{c^2}\frac{dr^2}{dt^2} - 1\right) \ .$$

Chapter 11

[Weber, 1875] On the Motion of Electricity in Bodies of Molecular Constitution

Wilhelm Weber[223,224,225,226]

In my First Treatise on Electrodynamic Measurements in the year 1846,[227,228] I presented a general law of *electric force* that encompassed both electrostatics and electrodynamics, and I later showed how it implied some special consequences, namely: *first of all*, I published the general *potential law* of electric forces in these *Annalen*, Vol. 73, p. 229 (1848),[229,230] *secondly*, the *energy principle* and its connection with those general laws of electric force, in the last Treatise in Electrodynamic Measurements in the year 1871,[231,232] and *thirdly* and finally, on the *capacity for doing work*[233] for two electric particles, as an equivalent of *vires vivae*,[234] in the Jubilee volume of these *Annalen* in the year 1874, p. 199.[235,236] Some further discussions of those topics shall be appended here that could not find their place in the latter Treatises. In particular, after some prefatory remarks and addenda to the last two Treatises that are concerned with the objections that have been raised against the general

[223] [Web75].

[224] Translated by D. H. Delphenich, feedback@neo-classical-physics.info and `http://www.neo-classical-physics.info/index.html`. Edited by A. K. T. Assis, `www.ifi.unicamp.br/~assis`

[225] The Notes by Wilhelm Weber are represented by [Note by WW:]; the Notes by Heinrich Weber, the editor of Volume 4 of Wilhelm Weber's *Werke*, are represented by [Note by Heinrich Weber:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[226] [Note by Heinrich Weber:] Annalen der Physik und Chemie, edited by J. C. Poggendorff, Vol. 156, Leipzig, p. 1-61.

[227] [Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. III, p. 25.

[228] [Note by AKTA:] [Web46] with a partial French translation in [Web87] and a complete English translation in [Web21d].

[229] [Note by Heinrich Weber:] Ibidem, Vol. III, p. 245.

[230] [Note by AKTA:] [Web48a] with English translations in [Web52c], [Web66c], [Web19] and [Web21p].

[231] [Note by Heinrich Weber:] Ibidem, Vol. IV, p. 247.

[232] [Note by AKTA:] [Web71] with English translations in [Web72] and [Web21g].

[233] [Note by AKTA:] In German: *Arbeitsfähigkeit*. See footnote 219 on page 134.

[234] [Note by AKTA:] See footnote 173 on page 108.

[235] [Note by Heinrich Weber:] Ibidem, Vol. IV, p. 300.

[236] [Note by AKTA:] [Web74].

law of electric force that was presented in the First Treatise, *the motions of electricity in bodies of molecular constitution* shall be treated since the results of so many other researches lead to their consideration and necessitate that they themselves should be the subject of a thorough special investigation that can hardly be avoided, despite the narrow limitations that mathematics seems to presently impose on that investigation.

11.1 Remarks on the Basic Laws of Electricity that were Exhibited in the Treatise on Electrodynamic Measurements in the Year 1871, Section 4

In the second and third Section of the cited Treatise in the year 1871,[237] the *law of electric force*, which was presented in the Treatise from the year 1846, was considered, and in the context of its connection with the much-simpler *law of the electric potential* in these *Annalen*, Vol. 73, p. 229. However, since even the latter law is still lacking in the simplicity that one would desire in a *fundamental law*, that potential law was analyzed more precisely in the fourth Section of that same Treatise, in which an attempt was made to resolve that law into components that would possess the simplicity of fundamental laws, namely, the law of the dependency of the potential of two particles upon the distance between them when they are *in the same state of relative motion*, and the law of the dependency of the potential on the relative motion when they are *at a certain distance*, but in that way, the determination of that distance would essentially necessitate a third law, namely, the *law of electrostatics*, which already possesses the desired simplicity of a fundamental law.

The fundamental laws of electricity, including electrostatics, that are presented in the fourth Section [of the 1871 Treatise] are the following *three*:

First law: When two electric mass-particles ε and ε' at a distance of r from each other are in a state of relative rest, they will exert a force on each other that is directly proportional to the product of their masses $\varepsilon\varepsilon'$ and inversely proportional to the square of that distance r^2 in the direction of r, so it is equal to $\mu^2 \cdot [\varepsilon\varepsilon'/r^2]$. — If one sets $\mu\varepsilon = \pm e$ and $\mu\varepsilon' = \pm e'$, where the upper or lower sign will be valid according to whether the mass-particle carries positive or negative electricity, respectively, so the expression for the force ee'/r^2 will be positive or negative according to whether force is one of repulsion or attraction.[238]

Second law: When two electric particles e and e' are found in a state of relative rest or relative motion of equal magnitude (so they will possess equal relative *vires vivae*) at the distances r' and r'' at different points in time, they will do amounts of work V' and V'' by their reciprocal action when both particles can be brought from the given distances r' and r'' to infinity, that are in inverse proportion to the given distances, i.e.:

$$V' : V'' = r'' : r' \ .$$

Third law: The work U that would be done under the action of the forces that the particles e and e' exert upon each other when the particles are at a *certain distance apart*

[237][Note by AKTA:] See footnote 232.

[238][Note by AKTA:] Weber is considering two particles with masses ε and ε', while their electric charges are given by e and e', respectively. The masses ε and ε' are always positive. The charges e and e' can be positive or negative.

proportional to ee', i.e., $\rho = ee'/a$,[239] in which they possess a certain *vis viva* x, that will define a constant sum when combined with U, namely, a, when they are shifted to an infinite distance, i.e.:

$$U + x = a\ .$$

In regard to those laws, it should be noted that, *first of all*, the quantity U that was introduced into the third of them, which defines the constant sum a with the *vis viva* x, just like the other two quantities x and a, is *always positive*, regardless of whether the two electric particles e and e' have *the same* or *unequal* types.[240]

That *positive* value results from the equation that is implied by the second law $U = [r/\rho]V$, (in which[241]

$$V = \left(\frac{ee'}{r}\right)\left(1 - \left[\frac{1}{c^2}\right]\left[\frac{dr^2}{dt^2}\right]\right)$$

expresses the *potential* of the two particles e and e', and $\rho = ee'/a$ is a *distance that can be determined from the nature of the electricity and the particles* e *and* e'), since the quantities V and ρ will be either both positive or both negative according to whether e and e' have the same or different types, respectively, so the quotient V/ρ will then remain *always positive*. It should be pointed out that when one has any concerns about whether the calculation admits a *negative value of* ρ for the distance between two points, as was remarked in the Note on the third law,[242] instead of $\rho = ee'/a$, which is negative when the electric particles have unequal types, one can set $\rho = \mu\varepsilon \cdot \mu\varepsilon'/a$, which is *always positive*, but then at the same time, U must not be set equal to the *work* done

$$\left(\frac{ee'}{\rho}\right)\left(1 - \left[\frac{1}{c^2}\right]\left[\frac{dr^2}{dt^2}\right]\right)\ ,$$

which is *negative* for particles of unequal type, but must be equated to the *absolute value* of this work:

$$\pm\left(\frac{ee'}{\rho}\right)\left(1 - \left[\frac{1}{c^2}\right]\left[\frac{dr^2}{dt^2}\right]\right) = \mu^2\left(\frac{\varepsilon\varepsilon'}{\rho}\right)\left(1 - \left[\frac{1}{c^2}\right]\left[\frac{dr^2}{dt^2}\right]\right)\ .$$

Secondly, it should be noted that when the fundamental law of electrostatics is added to the electrodynamic one, as was done here, one of the electrodynamic laws, namely, the law of the dependency of the potential of both particles on the distance between them *for the*

[239][Note by WW:] In place of ee', one can set $\mu\varepsilon \cdot \mu\varepsilon'$ (namely, the *absolute value* of ee'), and in that way, one can ensure that r always has a *positive* value. However, one would then have to take the *absolute value* of the work done U in order for the stated law to be valid.

[240][Note by AKTA:] The electric particles e and e' are of the same type when $ee' > 0$, that is, when both are positive, or when both are negative. They are of unequal types when $ee' < 0$, that is, when one of them is positive and the other is negative.

[241][Note by AKTA:] The next equation should be understood as:

$$V = \left(\frac{ee'}{r}\right)\left(1 - \left[\frac{1}{c^2}\right]\left[\frac{dr}{dt}\right]^2\right)\ .$$

The same thing should be understood in some of the following equations.

[242][Note by AKTA:] See footnote 239.

same relative motion, can be dropped completely since it is, in fact, essentially included in the other two laws already and can be derived from them.

That is because the *first* law, namely, the fundamental law of electrostatics, will imply the *potential* V for $x = 0$ as a function of the three variable quantities e, e', r, and indeed is proportional to three factors E, E', and R, each of which includes only *one* of those quantities, so the value of V at $x = 0$ will be determined from the following equation:

$$V = A \cdot EE'R \; .$$

From the *third* law, however, it follows that the *potential* V for $r = \rho$ depends on the variable quantities e, e', x, and is proportional to three factors E, E', X, each of which contains only *one* of these quantities, according to which V, for $r = \rho$, is determined by the following equation:

$$V = B \cdot EE'X \; .$$

Here $E = e$, $E' = e'$, $R = 1/r$, $X = (1 - x/a)$, and furthermore the value of the constant A is equal to the value of X when $x = 0$, and the value of the constant B is equal to the value of R when $r = \rho$.

One can conclude from this that E, E', R, X, or e, e', $1/r$, $(1-x/a)$, are always factors of V, and that gives only the possibility of yet another factor, namely, the factor $(1 + f(r,\ x))$, in which $f(r,\ x)$ must be a function of r and x that vanishes for $r = \rho$, as well as for $x = 0$.

Thus, $V = EE'RX = (ee'/r)(1 - x/a)$ is, in any event, the *simplest* way of determining V that the first and third laws admit and is implied by those two laws independent of the second law, which can itself be derived from the determination of $V = (ee'/r)(1 - x/a)$ that was just achieved. That is because that determination will imply that for two values of V, namely, V' and V'', which are valid for equal values of x, but different values of r, namely, r' and r'', one has the following proportion:

$$V' : V'' = \frac{ee'}{r}\left(1 - \frac{x}{a}\right) : \frac{ee'}{r''}\left(1 - \frac{x}{a}\right) = r'' : r' \; ,$$

which is *entirely identical* to the second law.

However, a complication of the law, as would result from the addition of another factor $(1 + f(r,\ x))$, is in no way to be considered permissible without proven necessity.

That implies the *result* that instead of the three fundamental laws that were cited above, only two of them will already suffice, namely:

1. the fundamental law of electrostatics, and

2. the principle of energy;

since one easily sees that the fundamental law, which was previously placed last as the *third one*, is the *principle of energy* itself, whose essence consists of the fact that the *relative vis viva* x of two particles e and e' will indeed be bigger or smaller, but that in addition to that *vis viva* of the two particles, an *equivalent vis viva* will also be present that will experience a reduction with each increase in the *vis viva*, and conversely, such that the *sum of that vis viva and the simultaneously-present equivalent one will have a constant value* that will be denoted by a. At the same time, one sees that the quantity that was denoted by U in the statement of the fundamental law above is the *equivalent vis viva that is present in addition to the vis viva x of the particle-pair*.

C. Neumann arrived at a similar result to the one that was found here in his "Principles of Electrodynamics", Tübingen, 1868,[243] in such a way that quite the same thing that was achieved here by means of the *principle of energy*, in conjunction with the fundamental law of electrostatics, was achieved by the *law of propagation of the potential* that he posed, in conjunction with the fundamental law of electrostatics. In so doing, a connection between that principle of energy and the law of propagation of the potential was derived that would seem to lead to an explanation for the one in terms of the other. For that law of propagation of the potential, one can see also the *Mathematischen Annalen*, Vol. I, p. 317 and the *Abhandlungen der König. Sächs. Gesellschaft der Wissenschaften*, XVIII, p. 103 and the following pages.[244]

11.2 Remarks on the Essay in the Jubilee Volume of These *Annalen*, page 199

After distinguishing the properties of individual particles from the properties of particle-pairs, the theorem was expressed in the cited essay[245,246] that a system of *three* or more particles would possess no properties that were not included already in the properties of individual particles and pairs, and it was accordingly proposed that it would be a peculiarity of a true *fundamental law* that nothing should come under consideration in it beyond the *nature* and *mutual relationships* of the particles that form a pair, and the *work that is done by their interaction under any change of distance under those relationships*. For a *fundamental law* that is represented in that way, only the *time* t, the *relative distance* r between the two particles, their *relative velocity* dr/dt, and functions of those quantities should then come under consideration as variable quantities.

Having assumed that, one must infer the demand that must be imposed upon the *principle of energy* as a fundamental law, that it must be valid for a particle-pair under all relationships that they might be found in, regardless of whether they alone are present or arbitrarily-many particles besides them, and that in the latter case, neither the nature nor the relationships between the other particles can come under consideration in the expression of the principle.

Therefore, the *principle of energy*, as a fundamental law, can deal with only the particle-pairs themselves and the *energies* that are associated with them exclusively. One such energy is the relative *vis viva* of the particle-pair which is called its *energy of motion*.[247] However, since that energy of motion of a particle-pair changes, the principle of energy will necessarily assume the existence of *another energy* for the particle-pair in order for an *unvarying total energy* to be possible. That *other energy* must likewise change, and in such a way that a reduction in it will always be linked with an increase in the energy of motion, and conversely. The essence of the *second energy* therefore consists in the fact that *new vis viva is created or existing vis viva is annihilated* as a result of every reduction or increase of the second energy.

[243] [Note by AKTA:] [Neu68a] with English translation in [Neu21b]. See also [Neu68b] and [Neu69] with English translation in [Neu21a]. See also [Ass21c].

[244] [Note by AKTA:] [Neu69] with English translation in [Neu21a], and [Neu74, p. 103 and the following pages].

[245] [Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. IV, p. 300.

[246] [Note by AKTA:] [Web74].

[247] [Note by AKTA:] In German: *Bewegungsenergie*. See footnote 211 on page 130.

However, *new vis viva will be created or existing vis viva will be annihilated by work*, for example, by the work that is done by the interaction of the two particles themselves under any change in the distance between them. Nonetheless, the actual existence of such work assumes the *capacity to do work*[248] that is based in the interaction of the particles. That would imply the capacity to create or annihilate *vis viva*, and that *capacity to do work* that is to be measured according to the amount of *vis viva* that can be created or annihilated is the *second energy of the particle-pair*, which will be larger or smaller according to whether the *first energy*, namely, the *vis viva* of the particle-pair, is smaller or larger, respectively, such that the *sum of both energies*, namely, the *vis viva* and the *capacity to do work* of the particle-pair will remain unchanged.

We then infer the following detailed conditions for the definition of the *capacity to do work as a form of energy* from that: *First of all*, the capacity to do work of two particles e and e' is a *property that they always possess* for a *given energy of motion* (that is, for a given relative *vis viva* of the particles).

Secondly, the magnitude of that property will be determined by the work that would be done as a consequence of the interaction of both particles *under a certain change in distance between them that must be determined more precisely*. However, that *change in distance between them that must be determined more precisely* is not one that *actually* takes place or even can take place (which would have to start with the existing distance r), but only a *virtual change in distance* that begins with a distance ρ at which the particles must be thought of being shifted to, and which is determined completely independently of the existing distance r. That is because the work that would be done by a change in distance that begins with the existing distance r could not serve to determine the magnitude of the capacity to do work since it would depend upon r, and therefore would also assume different values with r *for an unchanged relative vis viva of the particles*, whereas for a fictitious shift of the particles to an always-equal distance ρ that can be determined independently of the existing distance r, a change in distance can be thought of as taking place between fixed limits ρ and ρ' at which, as a consequence of the interaction of both particles, *an always-equal amount of work would be done whose absolute value can serve as a measure of a property that belongs to the particle-pair (since positive or negative means the same thing for the capacity to do work). It is then self-explanatory that the relative vis viva of the particles must be thought of as remaining unchanged under that fictitious shift of the particles from the distance r to the distance ρ.*

If R denotes the force of repulsion that results from the interaction of the two particles e and e', and $\rho' - \rho$ denotes the imagined change in distance between the particles that results from the fictitious shift, then the *capacity to do work* U of those particles will be represented by the formula:

$$U = \pm \int_{\sigma=\rho}^{\sigma=\rho'} R d\sigma \ ,$$

in which the upper or low sign applies according to whether the two particles have the same or different types, respectively.

In that formula, R is a function of σ, but not always the same one, which would be the case only if $d\sigma/dt = 0$, so from the law of electrostatics $R = ee'/\sigma^2$, it would always be the same function of σ. When $d\sigma/dt$ is non-zero, R will be a function of σ and $d\sigma/dt$, and

[248][Note by AKTA:] In German: *Arbeitsvermögen*. This expression can also be translated as ability do to work, working capacity, work capacity, energy capability or energy capacity.

$d\sigma/dt$ will not always be the same function of σ, which is explained by the facts that, *first of all*, the *initial values* of σ and $d\sigma/dt$ can be given arbitrarily, so very different values of $d\sigma/dt$ can be given for the same value of σ; and *secondly*, for equal changes in distance, the relative velocity $d\sigma/dt$ can experience very different changes according to the variety of *external forces* that act upon the particles.

Now, if R is a function of σ and $d\sigma/dt$, then the *indefinite integral* $\int Rd\sigma$ will also be such a function. However, the *definite integral* $\int_{\sigma=\rho}^{\sigma=\rho'} Rd\sigma$ will depend upon merely the initial and final values of σ, namely, ρ and ρ', and the differential quotients that belong to those values.

Now, should the *definite integral* $\int_{\sigma=\rho}^{\sigma=\rho'} Rd\sigma$ express the *capacity for two particles to do work when they possess the relative velocity* r', then it was remarked already that the relative velocity r' existing at the distance r must be thought of as remaining unchanged under the fictitious shift of the particles to the distance ρ, such that $d\sigma/dt = 0$ is given for $\sigma = \rho$, which will determine the dependency of the capacity to do work U on r'.

However, *an equal dependency of the capacity to do work* U on the value of $d\sigma/dt$ that belongs to the final value $\sigma = \rho'$ would also exist, except for the case in which $\rho' = \infty$, in which case such a dependency would not need to exist. It would follow from this that one must set $\rho' = \infty$ since the formula for U, as the definition of the *capacity to work by two particles that possess a relative velocity* r', can depend upon no other relative velocity besides r', namely, the one that the particles actually possess at the moment at which their capacity to do work is considered.

When one sets $\rho' = \infty$, the *capacity to do work* U of the particles e and e' will then be expressed by the formula:

$$U = \pm \int_{\sigma=\rho}^{\sigma=\infty} Rd\sigma \ ,$$

in which R is a function of σ and $d\sigma/dt$, and $d\sigma/dt$ is a function of σ that possesses the value r' for $\sigma = \rho$, i.e., the relative velocity of the particles whose capacity to do work is to be determined.

Finally, as far as the quantity ρ is concerned, it will be determined in such a way that only *a finite distance between two electrical particles* can be given that can be determined by merely the nature of the electricity in general and of the two particles in particular, but quite independently of the existing distance r, namely, from the unvarying total energy a that belongs to the particle pair, and from the forces of repulsion $\mu^2\varepsilon^2$ and $\mu^2\varepsilon'^2$ that are exerted by each of the two mass-particles ε and ε' on an equal particle at a separation distance of one unit according to the fundamental law of electrostatics,[249] so according to the formula $\rho = \mu^2 \cdot [\varepsilon\varepsilon'/a]$. If one sets $\mu\varepsilon = \pm e$ and $\mu\varepsilon' = \pm e'$, where the upper or lower sign is true according to whether the mass-particle belongs to positive or negative electricity, respectively, then ρ, which is always positive, can be set equal to $\pm ee'/a$, in which the upper or lower sign will be true according to whether both particles have the same or different types, respectively.

Remark. Should U itself be expressed in the formula in such a way that R is a function

[249][Note by AKTA:] See footnote 222 on page 136.

of σ and $d\sigma/dt$, and $d\sigma/dt$ is a function of σ that possesses the value r' for $\sigma = \rho$, i.e., the *given* relative velocity of the particle whose *capacity to do work* is to be determined, then, *first of all*, R could be set equal to $R(\sigma,\ d\sigma/dt)$, *secondly*, in order for one to refer to R as a function of σ, $d\sigma/dt$ would be set equal to $f(\sigma)$, and *thirdly* and finally, in order for one to distinguish the function $f(\sigma)$ that should, in fact, assume *the given value* r' for $\sigma = \rho$ here, from any other function $f(\sigma)$ that can assume *the given value* r' for other values of σ, the value of σ for which the function f *assumes the given value* can be added to the function symbol f, in particular, so $f_\rho(\sigma)$ will be written in place of $f(\sigma)$ here. One can then express the *capacity to do work* as:

$$U = \pm \int_{\sigma=\rho}^{\sigma=\infty} R[\sigma,\ f_\rho(\sigma)]d\sigma \ ,$$

and the *given relative velocity*:

$$r' = f_\rho(\rho) \ .$$

However, when the two particles are thought of as being shifted to a separation distance $\sigma = \rho$ for the purpose of determining their capacity to do work U from that distance $\sigma = r$ at which they are actually found, while preserving the relative velocity r' that they actually possess, as a consequence of their interaction, *an amount of work would also be done under the actual shift from the existing distance* $\sigma = r$ to $\sigma = \infty$, *that will be called the potential of the particles* and which one cares to denote by V. With the notation that was given for U, one will get *the expression for that potential*:

$$V = \int_{\sigma=r}^{\sigma=\infty} R[\sigma,\ f_r(\sigma)]d\sigma \ ,$$

and the *given relative velocity*:

$$r' = f_r(r) \ .$$

From the given definition of the *capacity to do work* by two particles e and e', as the *energy of work*,[250] which will determine the energies of the two particles e and e' completely, in conjunction with the *relative vis viva as an energy of motion*, and from the *principle of energy* that was expressed as the *law of the unvarying sum of both energies*, one can now pose the problem:

> Determine the force R with which two arbitrarily-moving electric particles e and e' act upon each other reciprocally from the *principle of energy* in conjunction with the *fundamental law of electrostatics*.

For since the *fundamental law of electrostatics* determines the force with which two electrical particles e and e' act on each other *when their relative vis viva is zero*, and since the *principle of energy* further determines *what changes in the interaction of two electrical particles e and e'*, when their relative kinetic energy is not zero, but $= x$ (namely that the

[250][Note by AKTA:] In German: *Arbeitsenergie*.

increase in the energy of motion of the two particles by the amount x is connected with a *decrease in the energy of work* by the same amount x), it would seem to follow that from the *principle of energy, in connection with the fundamental law of electrostatics*, it must be possible to deduce the general law of the force with which two electric particles e and e' mutually act upon each other, a law which encompasses electrostatics and electrodynamics at the same time.

The *principle of energy* will then give the following formulas, namely, *first of all*, *the formula for the energy of motion* (or relative *vis viva*) of two particles that possess the masses ε and ε':[251]

$$\xi = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'} \cdot \frac{d\sigma^2}{dt^2} \ ,$$

from which it will follow that when the relative velocity at the existing separation distance $\sigma = r$ is denoted by r' and the relative *vis viva* by x, one will have:

$$x = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'} \cdot r'^2 \ ; \qquad (1)$$

secondly, the formula for the energy of work, namely:

$$U = \pm \int_{\sigma=\rho}^{\sigma=\infty} R[\sigma,\ f_\rho(\sigma)] d\sigma \ , \qquad (2)$$

in which $f_\rho(\sigma) = d\sigma/dt$ denotes a function of σ that possesses the given value r' for $\sigma = \rho$, and *thirdly*, the *law of constant total energy*, which is expressed by the following formula:

$$x + U = a \ . \qquad (3)$$

One must add a *fourth* formula to those *three* formulas that are given by the *principle of energy*, namely, the formula for the *fundamental law of electrostatics*, which is the law for the force of repulsion R by which two particles e and e' in a state of relative rest act upon each other at a separation distance of σ:

$$R = \frac{ee'}{\sigma^2} \ . \qquad (4)$$

For $\xi = 0$, when one also has $x = 0$, the expression for the electrodynamic force $R[\sigma,\ f_\rho(\sigma)]$ will go to the expression for the electrostatic force $R = ee'/\sigma^2$, and one will find from Equation (2) and Equation (3), for $x = 0$,

$$U = \pm \int_{\sigma=\rho}^{\sigma=\infty} \frac{ee'}{\sigma^2} d\sigma = \pm\frac{ee'}{\rho} = a \ .$$

Moreover, if one denotes the value of r' in Equation (1) for $x = a = \pm ee'/\rho$ by c, which will give

$$\pm\frac{ee'}{\rho} = \frac{1}{2}\left[\frac{\varepsilon\varepsilon'}{(\varepsilon+\varepsilon')}\right] \cdot c^2 \ ,$$

[251][Note by AKTA:] The following formula should be understood as:

$$\xi = \frac{1}{2}\frac{\varepsilon\varepsilon'}{\varepsilon+\varepsilon'} \cdot \left(\frac{d\sigma}{dt}\right)^2 \ .$$

and substituting the resulting value of

$$\frac{1}{2}\left[\frac{\varepsilon\varepsilon'}{(\varepsilon+\varepsilon')}\right] = \pm\frac{ee'}{\rho c^2}$$

in Equation (1), then one will find that:

$$x = \pm\frac{ee'}{\rho}\cdot\frac{r'^2}{c^2} .$$

If one now substitutes that value of x and the previously-found value of $a = \pm ee'/\rho$ into Equation (3) and appeals to Equation (2), then one will get:

$$U = \pm\frac{ee'}{\rho}\left(1-\frac{r'^2}{c^2}\right) = \pm\int_{\sigma=\rho}^{\sigma=\infty} R\,[\sigma,\ f_\rho(\sigma)]\,d\sigma .$$

Now, one has identically:

$$-d\cdot\frac{ee'}{\sigma}\left(1-\frac{1}{c^2}\frac{d\sigma^2}{dt^2}\right) = \frac{ee'}{\sigma^2}\left(1-\frac{1}{c^2}\frac{d\sigma^2}{dt^2}+\frac{2\sigma}{c^2}\frac{d^2\sigma}{dt^2}\right)d\sigma ,$$

from which the indefinite integral will follow:

$$-\frac{ee'}{\sigma}\left(1-\frac{1}{c^2}\frac{d\sigma^2}{dt^2}\right) = \int\frac{ee'}{\sigma^2}\left(1-\frac{1}{c^2}\frac{d\sigma^2}{dt^2}+\frac{2\sigma}{c^2}\frac{d^2\sigma}{dt^2}\right)d\sigma .$$

Now, when one substitutes $d\sigma/dt = f_\rho(\sigma)$ in that, which denotes a function of σ that possesses the given value r' when $\sigma = \rho$, then that will give:

$$-\frac{ee'}{\sigma}\left(1-\frac{1}{c^2}[f_\rho(\sigma)]^2\right) = \int\frac{ee'}{\sigma^2}\left(1-\frac{1}{c^2}[f_\rho(\sigma)]^2+\frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right)d\sigma ,$$

and if one considers that integral between the limits $\sigma = \rho$ and $\sigma = \infty$, one will get:

$$\frac{ee'}{\rho}\left(1-\frac{1}{c^2}[f_\rho(\rho)]^2\right) = \int_{\sigma=\rho}^{\sigma=\infty}\frac{ee'}{\sigma^2}\left(1-\frac{1}{c^2}[f_\rho(\sigma)]^2+\frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right)d\sigma ,$$

and as a result, since $f_\rho(\rho) = r'$, and one has

$$\left[\frac{ee'}{\rho}\right]\left(1-\frac{r'^2}{c^2}\right) = \int_{\sigma=\rho}^{\sigma=\infty} R\,[\sigma,\ f_\rho(\sigma)]\,d\sigma ,$$

then

$$\int_{\sigma=\rho}^{\sigma=\infty} R\,[\sigma,\ f_\rho(\sigma)]\,d\sigma = \int_{\sigma=\rho}^{\sigma=\infty}\frac{ee'}{\sigma^2}\left(1-\frac{1}{c^2}[f_\rho(\sigma)]^2+\frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right)d\sigma .$$

The simplest assumption for satisfying that formula consists of setting:

$$R\,[\sigma,\ f_\rho(\sigma)] = \frac{ee'}{\sigma^2}\left(1-\frac{1}{c^2}[f_\rho(\sigma)]^2+\frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right) .$$

In the expression for the *capacity to do work* U, [the expression] $f_\rho(\sigma)$ was set equal to $d\sigma/dt$ in order to designate a function of σ that possessed the given value r' for $\sigma = \rho$.

Now, in the expression for the *potential* V, [the expression] $f_r(\sigma)$ would likewise have to be set equal to $d\sigma/dt$ in order to designate a function of σ that possessed the given value r' for $\sigma = r$, one would then get in a similar way:

$$R\,[\sigma,\ f_r(\sigma)] = \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\,[f_r(\sigma)]^2 + \frac{2\sigma}{c^2}\cdot\frac{d\cdot f_r(\sigma)}{dt}\right) .$$

However, in the latter case, where $\sigma = r$ and $d\sigma/dt = r'$ are the separation distance and velocity that is actually present, one does not care to use the function symbol $f_r(\sigma)$ at all but leaves $d\sigma/dt$ unchanged in the formula. One can also drop the special notation for the force of repulsion between both particles as a function of σ and $d\sigma/dt$ that involves adding those variables under the function symbol R, namely, $R(\sigma,\ d\sigma/dt)$, and merely set it equal to R. Now, having done that, one can represent the common law for the force R with which two arbitrarily-moving electric particles e and e' act upon each other by the following formula:

$$R = \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\frac{d\sigma^2}{dt^2} + \frac{2\sigma}{c^2}\frac{d^2\sigma}{dt^2}\right) .$$

Finally, the *principle of energy* that was exhibited here shall be applied to the law that C. Neumann exhibited in his "Principles of Electrodynamics," Tübingen, 1868, p. 37,[252] and in the *Berichten der Königl. Sächs. Gesellschaften der Wissenschaften*, 1871, Section 20, p. 399,[253] since proving its agreement with the principle above is not without its own special interest.

Neumann expressed that law in the latter reference in the form:

> If a system of arbitrarily-many particles $M + \mu$ moves under the action of given external forces, then the following formula will be true for each time element dt:
>
> $$d(T + U^0 + U - V) = dS ,$$
>
> i.e., for every time interval, the increase in the *energy* of the system will be equal in magnitude to the work that is consumed by the system during that time interval. In that way, one understands the *energy* of the system to mean that expression $T+U^0+U-V$, which depends upon only its instantaneous state (i.e., the coordinates and velocities), and in which T denotes the *vis viva*, U^0 denotes the ordinary potential of the system, U denotes the electrostatic potential, and V denotes the electrodynamic one.

The *capacity to do work*, or the *energy of work* U, of two electric particles e and e' (which possess any well-defined relative *vis viva* x at any well-defined distance r from each other) was found to be:

$$U = \int_{\sigma=\rho}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\,[f_\rho(\sigma)]^2 + \frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right) d\sigma ,$$

[252] [Note by AKTA:] See footnote 243.
[253] [Note by AKTA:] [Neu71, p. 399].

in which $f_\rho(\sigma) = d\sigma/dt$ denotes a function of σ whose value for $\sigma = \rho$ is given by the existing *vis viva* x, namely, by the equation:

$$\pm\frac{ee'}{\rho c^2}\left[f_\rho(\rho)\right]^2 = x \qquad \text{or} \qquad f_\rho(\rho) = c\sqrt{\pm\frac{\rho x}{ee'}} \ .$$

The value of U above can now be represented as a sum of two terms, namely:

$$U = \int_{\sigma=\rho}^{\sigma=r} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right) d\sigma$$
$$+ \int_{\sigma=r}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right) d\sigma \ .$$

Since nothing more is generally determined here about the function $f_\rho(\sigma)$ than merely its value for $\sigma = \rho$, which results from the equation $[\pm ee'/\rho c^2][f_\rho(\rho)]^2$, namely $f_\rho(\rho) = c\sqrt{\pm\rho x/ee'}$, very different functions of σ can generally be set for $f_\rho(\sigma)$.

The function $f_\rho(\sigma)$ will *actually be defined precisely* only when one is dealing with an *actual displacement* of the particles e and e' for which all relationships upon which the function $f_\rho(\sigma)$ depends are actually given. However, one cannot speak of an *actual displacement* of the particles from ρ to ∞ when they are not even found at a distance ρ apart, but at a distance r. Nonetheless, for the purpose of defining U, it suffices to only imagine displacing the particles from ρ to ∞, once one has imagined *previously* displacing them from r to ρ, and indeed in such a way that the relative *vis viva* of the particles at the distance ρ would again be the same as it was when the distance was r, namely, $x = [\pm ee'/\rho c^2][f_\rho(\rho)]^2$, and in that way, the value of the function $f_\rho(\sigma)$ will be determined for $\sigma = \rho$.

If one would now like to imagine, moreover, that the further displacement of the particles, namely, initially from ρ back to r, takes places only under the reciprocal action of the particles, but *without the action of external forces*; then since $f_\rho(\sigma)$ is given for $\sigma = \rho$, namely, $f_\rho(\rho) = c\sqrt{\pm\rho x/ee'}$, the value of $f_\rho(\sigma)$ for a value of σ that is different from ρ (e.g., for $\sigma = r$) will be found to be equal to $c\sqrt{\pm\rho y/ee'}$, in which y is determined by the following equation:

$$y - x = \int_{\sigma=\rho}^{\sigma=r} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right) d\sigma \ ,$$

that is, the change in the relative *vis viva* during the change in distance from ρ to r is equal to the work done by the forces of interaction along the path that was taken.

However, when not only the forces that result from the interaction act upon the particles during their change in distance, but also other *external forces* P, and they likewise seek to get further apart from each other (or closer together), y would be determined by the following equation:

$$y - x = \int_{\sigma=\rho}^{\sigma=r} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{d\cdot f_\rho(\sigma)}{dt}\right) d\sigma + S \ ,$$

when $S = \int_{\sigma=\rho}^{\sigma=r} P d\sigma$ denotes the work done by the *external forces*.

Among all of the conceivable cases in this way, one also finds the case in which one has:

$$\int_{\sigma=\rho}^{\sigma=r} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_\rho(\sigma)}{dt}\right) d\sigma + S = 0 \ ,$$

for the value at $\sigma = r$, which is equal to the actual distance between the particles that possess the relative *vis viva* for which one desires to know U, so one has $y = x$. That means that the *vis viva* of the two particles at the end of the change in distance can be equal to the one at the beginning only when the work done by the *forces of interaction* during the change in distance is canceled by the work done by the *external forces*.

However, if the relative *vis viva* of the two particles at the distance $\sigma = r$, which is denoted by y, will be the same *at the end* of the change of distance that is given in the integral:

$$\int_{\sigma=\rho}^{\sigma=r} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_\rho(\sigma)}{dt}\right) d\sigma$$

as it was *at the beginning* (namely, it is equal to x), then that will explain the fact that the same value of the *vis viva* x will also be valid for the distance $\sigma = r$ *at the beginning* of the further change in distance from r to ∞ that is given in the integral:

$$\int_{\sigma=r}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_\rho(\sigma)}{dt}\right) d\sigma \ .$$

However, that explains the fact that the difference between the functions $f_\rho(\sigma)$ and $f_r(\sigma)$ vanishes, and:

$$\int_{\sigma=r}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_\rho(\sigma)}{dt}\right) d\sigma$$

denotes the same work as:

$$\int_{\sigma=r}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_r(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_r(\sigma)}{dt}\right) d\sigma \ ,$$

namely, the work that would be done as a consequence of the interaction of the particles that possess the given *vis viva* x under a change in distance from r to ∞. One will then have:

$$\begin{aligned}&\int_{\sigma=r}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_\rho(\sigma)}{dt}\right) d\sigma \\ = &\int_{\sigma=r}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_r(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_r(\sigma)}{dt}\right) d\sigma = V \ .\end{aligned}$$

Thus, one can now set the first part of U, namely:

$$\int_{\sigma=\rho}^{\sigma=r} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_\rho(\sigma)}{dt}\right) d\sigma = -S \ ,$$

and the second part of U, namely:

$$\int_{\sigma=r}^{\sigma=\infty} \frac{ee'}{\sigma^2}\left(1 - \frac{1}{c^2}\left[f_\rho(\sigma)\right]^2 + \frac{2\sigma}{c^2}\frac{df_\rho(\sigma)}{dt}\right) d\sigma = V \ ,$$

which will give:

$$U = V - S \ ,$$

and if one substitutes that value in the equation:

$$U + x = a \ ,$$

then one will get the following equation:

$$V + x - S = a \ .$$

When those particles are found at a distance of r_1 and possess the relative *vis viva* x_1, that will imply the following equation in the same way:

$$V_1 + x_1 - S_1 = a \ ,$$

from which one will get the following differential equation for small values of $r - r_1$ and $x - x_1$:

$$dV + dx - dS = 0 \ ,$$

which is the same equation that Neumann exhibited in the place cited, except that Neumann had denoted the *vis viva* by T and the potential by $U - V$, since it is composed of an electrostatic and an electrodynamic part, and finally, for the case in which the electric particles are endowed with ponderable masses, he added the potential U^0 that results from the interaction of those ponderable masses, so he expressed the same law in the following equation:

$$d(T + U^0 + U - V) = dS \ .$$

11.3 On the Objections that were Raised Against the Fundamental Law of Electric Action

When the fundamental law of electric action, which says that the interaction between two electric particles e and e' (expressed in electrostatic units) will result in the force of repulsion:

$$R = \left[\frac{ee'}{r^2}\right]\left(1 - \left[\frac{1}{c^2}\right]\left[\frac{dr^2}{dt^2}\right] + \left[\frac{2r}{c^2}\right]\left[\frac{d^2r}{dt^2}\right]\right) \ ,$$

is considered in regard to its connection with the principle of energy that was developed here, that will explain the fact that in all applications of that law that should be made in order to determine the later relationships between the particles from their *initial relationships*, those *initial relationships* cannot be assumed to be completely arbitrary. They cannot be assumed to be such that they would already contain contradictions to the principle at their basis, which would be the case, for example, when two electric particles are ascribed an *initial relative vis viva* that would already be greater in its own right than the total energy of the particles according to the principle of the unchanging total energy.

By the assumption of such *initial relationships* that contradict the principle that was posed, one can generally arrive at consequences whose admissibility one can rightfully object to, and which might seem to contradict the law, but that is not truly the case. One might return to some of the objections that Helmholtz raised against the law above.[254] For example,

[254][Note by AKTA:] [Hel73].

Helmholtz arrived at the consequence of the law above that two particles with a relative velocity that is initially finite, but greater than c (which would imply that the relative *vis viva* of the particles would be greater than the entire sum of energy, which is unchangeable in principle), would attain an infinite *vis viva* during a finite change in distance, and would therefore do an infinitely-large amount of work. One could also infer the possibility of a *perpetuum mobile* from that.[255]

Now, those consequences will generally go away by themselves, when *each energy* in the principle that was posed is coupled with an *essentially-positive quantity*, and when they are all taken together, they will define a *finite and unchanging sum*. However, even when one accepts a value for energy that is negative and increases to infinity, those consequences would not necessarily lead one to reject the law above, since the basis for declaring those consequences to be inadmissible would no longer exist, in fact. That would then explain the fact that when *one energy* is negative and becomes *negatively infinite*, *another energy* must likewise be present that would be positive and become positively infinite. Now, if that energy that increases to infinity were the *energy of motion*, then there would be an inexhaustible source of *vis viva*, so one would be able to product all of the effects that Helmholtz declared to be inadmissible.

The objections against the fundamental law of electric action that were just considered, which are based upon the fact that the principle of energy that was posed is not recognized and that the initial relationships between the electric particles that would contradict it must be assumed, can be combined with yet another objection that is based upon the fact that Helmholtz believed that he had proved that the distance ρ that he referred to as *critical* was not always a *molecular* distance. However, he had proved that only by assigning a meaning to the distance ρ that was completely different from the one that it was given for the purpose of defining the *energy of work* U for two particles e and e'. Namely, $\rho = ee'/a$ depended upon merely on the nature of electricity and on the two particles e and e', namely, on the three quantities a, e^2 and e'^2, which denoted the constant total energy of the particle-pair and the electrostatic forces of repulsion that the two particles will each exert upon a particle that is equal to it, at a unit distance.[256]

Helmholtz said (in the place cited, p. 43):[257]

> The value of the distance ρ is $\rho = 2ee'/c^2\mu$.

Therefore, Helmholtz was setting μ equal to $2a/c^2$. Helmholtz then continued with:

> If the electric particle is endowed with only its proper mass, then e/μ will have any well-defined value β. If μ also included ponderable mass then one will have $e/\mu < \beta$.

That explains why, according to Helmholtz, ρ is also a quantity that depends upon the ponderable mass that the electric particle e is endowed with, so it has an entirely different meaning from the one that it had in the law that I proposed. Helmholtz continued further:

> However, when $b = 2e/c^2\mu$ is also an exceptionally small quantity, ρ cannot depend upon b alone, but one will have $\rho = be'$, and e' can still have any arbitrary magnitude, and as a result, ρ as well. It should probably be noted in that regard that if we would

[255][Note by AKTA:] That is, the possibility of a perpetual motion.
[256][Note by AKTA:] See footnote 222 on page 136.
[257][Note by AKTA:] [Hel73, p. 43].

like to envision e' in the form of a spherical mass of a certain density that is an insulator that is carrying an electric fluid either through it or on it, when e' increases, the diameter of that ball will increase by either $\sqrt[3]{e'}$ or $\sqrt[2]{e'}$ according to whether e' is distributed throughout the interior or across the surface, respectively, but ρ will increase like e' itself, and that by appropriately increasing e' we can give the magnitude ρ any finite size and its end point any distance from the surface of the electrical mass e'.

The description given here by Helmholtz of the electric particle e' clearly shows how different it is, according to Helmholtz's conception, from every *atom* that actually exists in nature, given its size and mass. One easily sees that when one would like to imagine *atoms with planetary masses*, instead of the atomic bodies with immeasurably-small masses that actually exist in nature, as one is free to do, obviously, the molecular and atomic distances in that imaginary world would not be as immeasurably-small as they are in the real world. That such giant atoms would be producible in accordance with the fiction of solid compounds of ponderable atoms among themselves and with electric ones is self-evident. However, that could probably not be said about the fundamental law of electric action, which has no connection whatsoever to such fictions.

If the objections that Helmholtz cherished in relation to the possibility of a perpetuum mobile, as well as in regard to measurable magnitudes for the critical distance ρ, seem to originate mainly in differences in his fundamental concepts and pictures, then things will behave differently when one makes the following objection. One objection that Helmholtz raised consisted essentially of the fact that, as Helmholtz believed he had proved, it would follow from the fundamental law that I had proposed that

in certain cases, a force that acts forwards on the (driven) point μ will accelerate it backwards, and conversely.[258]

However, the proof rests essentially on the fact that in *Borchardt's Journal*, Vol. 75, p. 47,[259] as well as in *Monatsberichte der Akademie der Wissenschaften zu Berlin*, 1872, April 18, p. 253,[260] Helmholtz spoke of a *vis viva* equal to[261]

$$\frac{1}{2}\left(\mu - \left[\frac{1}{c^2}\right]\left[\frac{ee'}{r}\right]\cos\vartheta^2\right)q^2 \ ,$$

in which q is the velocity with which the mass μ moves, but the quantity $-[1/c^2][ee'/r]\cos\vartheta^2$ is not at all an *actually-existing mass*, much less a mass that moves with the velocity q. I have not been able to guess what Mr. Helmholtz has meant by saying of the magnitude

[258] [Note by AKTA:] [Hel73, p. 51].

[259] [Note by AKTA:] [Hel73, p. 47]. The *Journal für die reine und angewandte Mathematik* (Journal for Pure and Applied Mathematics) was founded by the German mathematician August Leopold Crelle (1780-1855) in 1826 and edited by him until his death. It was edited by the German mathematician Carl Wilhelm Borchardt (1817-1880) from 1856 to 1880, during which time it was known as *Borchardt's Journal*.

[260] [Note by AKTA:] [Hel72a, p. 253] with English translation in [Hel72b], see also [Hel82].

[261] [Note by AKTA:] The next equation should be understood as:

$$\frac{1}{2}\left(\mu - \left[\frac{1}{c^2}\right]\left[\frac{ee'}{r}\right]\cos^2\vartheta\right)q^2 \ .$$

$$\left(\mu - \left[\frac{1}{c^2}\right]\left[\frac{ee'}{r}\right]\cos\vartheta^2\right) \ ,$$

not that it *is* the mass moving with the velocity q, but that it *represents* the mass (*Borchardt's Journal*, Vol. 75, p. 48), or that it *stood in* for that mass (*Monatsbericht*, 1872, April 18, p. 253), and therefore I do not understand how Helmholtz, with the aid of this comparison, has managed to find that

> a consequence of Weber's law is that in certain cases, a force that points forward will accelerate the point μ backwards, and conversely.

It is just as hard for me to grasp how that quantity, which only *replaces* or *stands in* for a mass, can *collide* with another mass that actually exists and how its motion *after the collision* can be determined from the laws that would be valid if one were dealing with *actually-existing masses that move with velocity* q.

In *Borchardt's Journal*, as well as in the *Monatsberichte der Berliner Akademie*, Helmholtz had referred to the equation for *vis viva* that he developed from my fundamental law, and in the case of just *one* mass-point μ with the electric quantum e moving in a space that is bounded by a spherical surface with radius R that is uniformly endowed with electricity, it will reduce to the following equation, in which ε denotes the quantum of electricity per unit area of the spherical surface, namely:

$$\frac{1}{2}\left(\mu - \frac{4\pi}{3c^2}\cdot R\varepsilon e\right)q^2 - V + C = 0 \ .$$

V denotes the potential of the *non-electric* forces, and dV/ds denotes the *driving force* that Helmholtz referred to. Differentiating that equation will give:

$$\left(\mu - \frac{4\pi}{3c^2}\cdot R\varepsilon e\right)q\frac{dq}{ds} - \frac{dV}{ds} = 0 \ ;$$

so when dV/ds is positive, i.e., with Helmholtz's assumption, *with a forward driving force*, and when at the same time $(\mu - [4\pi/c]\cdot R\varepsilon e)$ is *negative*, q will decrease, that is, μ will be *accelerated backwards*.

However, in that way, Helmholtz was considering only *one part of the driving force*, namely, the one that was implied by the potential of the *non-electric* forces. Nonetheless, Helmholtz had not considered the *other part of the driving force at all*, which is implied by the *electric potential* $([4\pi/6c^2]R\varepsilon e\cdot q^2)$ and which Helmholtz combined with the *vis viva* $\frac{1}{2}\mu q^2$, merely because it had the common factor of q^2, since he said: "*q will decrease under a forward driving force, or μ will be accelerated backwards, when $(\mu-[4\pi/3c^2]R\varepsilon e)$ is negative.*" That should really read: μ will be accelerated backwards by a forward-pointing *non-electric* force when $(\mu - [4\pi/3c^2]R\varepsilon e)$ is negative. However, if the *total driving force* is included in the calculation, instead of merely one *part* of the driving force, then one will get:

$$\mu q\frac{dq}{ds} - \left(\frac{4\pi}{3c^2}R\varepsilon e\cdot q\frac{dq}{ds} + \frac{dV}{ds}\right) = 0 \ ,$$

upon differentiating the equation above, in which $([4\pi/3c^2]R\varepsilon e\cdot q[dq/ds]+dV/ds)$ is the *total driving force*, and it will follow from this that:

$$dq = \frac{ds}{\mu q}\left(\frac{4\pi}{3c^2}\cdot R\varepsilon e\cdot q\frac{dq}{ds}+\frac{dV}{ds}\right) \ ,$$

that is, when one recalls that $ds/\mu q$ is always positive, *for a forward-pointing total force* (electric and non-electric combined), μ *will always accelerate forwards*, and conversely, in which it is totally irrelevant whether $(\mu - [4\pi/3c^2]R\varepsilon e)$ has positive or negative value.

Once one has gotten around the apparent inconsistencies in the consequences of my fundamental law that Helmholtz inferred in that way, all that still remains will be a surprising result, namely that according to that law, a *non-electric* force that *retards* the motion of a particle μ and can be represented by a negative value of dV/ds, indirectly results in an electric force equal to $[4\pi/3c^2]R\varepsilon e\cdot q[dq/ds]$, which accelerates the particle μ in its motion, and indeed it will *accelerate* that particle more than it is retarded by the former force.

However, the *immediate basis* for that electric force $([4\pi/3c^2]R\varepsilon e\cdot q[dq/ds])$ does not lie in the force dV/ds, but, according to the fundamental law, it will lie in the existing *relative acceleration*, which is represented by $q[dq/ds]$ here and from which that force will be obtained in the specified manner by multiplying by $[4\pi/3c^2]R\varepsilon e$. However, according to the general laws of motion, that acceleration $q[dq/ds]$ will itself result, not from *one* force, but from *all existing forces*, so not merely the *non-electric* force dV/ds, but also the *electric* force $([4\pi/3c^2]R\varepsilon e\cdot q[dq/ds])$ itself, namely, upon dividing the sum of both forces by μ, that will give:

$$q\frac{dq}{ds} = \frac{1}{\mu}\left(\frac{dV}{ds}+\frac{4\pi}{3c^2}R\varepsilon e\cdot q\frac{dq}{ds}\right) \ .$$

Now, in general, the values of the *acceleration* $q[dq/ds]$ and the *electric force* $([4\pi/3c^2]R\varepsilon e\cdot q[dq/ds])$ will then be represented as also depending indirectly upon just the given *non-electric force* dV/ds, namely:

$$q\frac{dq}{ds} = \frac{1}{\mu - \frac{4\pi}{3c^2}R\varepsilon e}\cdot\frac{dV}{ds} \ ,$$

$$\frac{4\pi}{3c^2}R\varepsilon e\cdot q\frac{dq}{ds} = \frac{\frac{4\pi}{3c^2}R\varepsilon e}{\mu - \frac{4\pi}{3c^2}R\varepsilon e}\cdot\frac{dV}{ds} \ .$$

Therefore, when the given value of dV/ds is *negative*, for a very small negative value of $(\mu - [4\pi/3c^2]R\varepsilon e)$, the *given backwards-driving, non-electric* force that acts upon a mass μ that moves with a velocity of q will imply a much larger *forward-driving electric* force that acts upon that same mass.

That explains why the denominator $(\mu - [4\pi/3c^2]R\varepsilon e)$ can be zero or negative for only a positive value of εe, i.e., only when the electricity that the spherical surface is endowed with has the same type as the electricity that the moving ponderable mass is endowed with. That will then imply that when $\mu > [4\pi/3c^2]R\varepsilon e$, the electric force $([4\pi/3c^2]R\varepsilon e\cdot q[dq/ds])$ will have the *same direction* as the non-electric force dV/ds, and its *magnitude*, which is equal to the other force when $[4\pi/3c^2]R\varepsilon e = \frac{1}{2}\mu$, will increase with increasing values of $[4\pi/3c^2]R\varepsilon e$, until it becomes *infinite* when $[4\pi/3c^2]R\varepsilon e = \mu$ and then *changes sign*.

Such a jump in the *magnitude and direction* of the electric force, namely, from $+\infty$ to $-\infty$, can be truly undesirable when it actually follows from the law as a loss of continuity. However, such a jump will not actually occur at all, according to the law, since in fact the mass μ with its charge e cannot remain inside of the spherical space for so long as a result of the ever-increasing acceleration given to it until $[4\pi/3c^2]R\varepsilon e = \mu$, but before that, it must be driven up to the spherical surface that is composed of a *fixed insulator*, whose resistance will once more bring it to rest.

As one sees from this, those consequences include no inconsistencies whatsoever, and can even prove to be unexpected only under circumstances that are so highly exceptional that one cannot even think of them presenting themselves in reality. That is because if one imagines that electric charges that can actually exist in reality might amount to, say, 10 electrostatic units of charge per milligram of ponderable carrier, so $e/\mu = 10$, and one further imagines that the same charge is distributed over each square millimeter of the surface, so $\varepsilon = 10$, then the fact that $[4\pi/3c^2]R\varepsilon e > \mu$ will impose the demand that one must have a spherical insulator whose radius would be $R > 3c^2/400\pi > 46 \cdot 10^{19}$ millimeters, i.e., it would have to be 3 million times greater than the distance from the Earth to the Sun.

Furthermore, other even-more remarkable, but still not inconsistent, consequences of the fundamental law of electric action were already pointed out in the First Treatise on Electrodynamic Measurements in the year 1846,[262,263] in particular, the fact that the *interaction of two bodies* will depend indirectly on the *presence of a third body*, which will result in forces that Berzelius had referred to by the name of *catalytic*.[264]

If, however, these deductions find no analogies in the deductions drawn from other laws, the question may well be raised whether this lack of analogy is a disadvantage or an advantage, since all deductions from the law of gravitation and from all other laws established by analogy with it obviously cannot lead to the explanation of many phenomena, especially those which are more closely related to the molecular constitution of bodies; laws of another kind therefore seem necessary for this purpose.

11.4 Identity of the Moving Parts that are Contained in All Bodies, Whose Motion is Heat, Magnetism or Galvanism

One subdivides all ponderable bodies into solid, liquid, and gaseous ones and distinguishes between the statics and dynamics of those bodies according to whether one considers them to be in a state of rest or motion, respectively. However, when one speaks of the rest state in the statics of a body, one does not at all mean a state of rest for *all* of the parts that are enclosed within the boundary of that body, but only the *ponderable* bodies that are enclosed within that boundary. Without that restriction, one could never speak of the rest state of a body since any body will include not only its ponderable parts, but also other parts that will never be at rest.

That is because, *first of all*, the more precise study of all *electrical* phenomena that are observed in ponderable bodies has led to the fact that moving parts are present in the interior of all of those bodies (even so-called solid ones and ones that are found to be at rest), namely, electric ones, and that the motions of these parts in the interior of that body is the basis for all of the galvanic and electrodynamic phenomena and effects that exist in that body.

[262][Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. III, p. 212.

[263][Note by AKTA:] [Web46, p. 212 of Weber's *Werke*] with a partial French translation in [Web87] and a complete English translation in [Web21d, Section 5.32, p. 202].

[264][Note by AKTA:] Jöns Jacob Berzelius (1779-1848). See [Ber36c], [Ber36a] and [Ber36b].

Secondly, the closer study of all *magnetic* phenomena that are observed in ponderable bodies such as paramagnetism, as well as diamagnetism, likewise led to the fact that moving parts were present in the interior of all of those bodies that one had sought for a long time to distinguish from the electric fluids by calling them *magnetic fluids*. It was asserted that those magnetic fluids could be *distributed* in the interiors of those bodies in various ways according to the variety of the situations, but that they could come to rest and equilibrium under steady-state conditions. The basis for magnetic phenomena lies in the distribution of that magnetic fluid without requiring the continuing motion of it. However, further investigation implied that such magnetic fluids at rest could not be the basis for *all* magnetic phenomena (e.g., paramagnetism and diamagnetism), no matter how they might be distributed. Nonetheless, *all* of those phenomena can be explained by the presence of *continuously moving parts in the interiors of ponderable bodies*, and indeed the same parts whose motion were the basis for all galvanic and electrodynamic phenomena and effects, namely, the *electrical ones*.

Thirdly and finally, it must be added that the study of the *temperature* that is associated with any ponderable body has also led to the facts that moving parts are present in the interior of all of those bodies and that the basis for all of the temperature-related phenomena (i.e., *heat*) that are observed in those bodies consists of the motion of those parts.

Now, if the moving parts that are included in all ponderable bodies whose motions are the basis for all galvanic effects include no other parts beyond ones whose motions are the basis for all magnetic effects (e.g., paramagnetic and diamagnetic), then that would strongly suggest that the parts that are included in all ponderable bodies whose motion represents *heat* would also be identical to the parts that are included in all ponderable bodies whose motion represents *magnetism*, and as a result, they are also identical to the parts that are included in all ponderable bodies whose motion represents *galvanism*. For even if one must generally admit the existence of moving parts in the interior of bodies, while the ponderable parts remain at rest, one will have much more hesitation in assuming the existence of *several kinds of such parts*, namely in every smallest part of the body, which would have little prospect of being separated from each other and each of which would have to be investigated more closely. — This presumed identity is now confirmed by facts which will be considered in more detail below.

11.5 Identity of the *Vis Viva* that is Created in a Current by the Electromotive Force and the Heat that is Created by the Current in a Conductor

The creation of heat by the galvanic current in a current conductor has been the subject of many investigations that established the law that the *mechanical equivalent of the heat created* during the time element dt is equal to the product of dt with the square of the current intensity i and the resistance w of the conductor through which the current flows,[265] both of which are measured in absolute magnetic units.[266] However, it should be noted

[265][Note by AKTA:] Calling dq the amount of heat produced in the time interval dt we then have $dq = wi^2dt$. This result is due to James Prescott Joule (1818-1889), [Jou41b], and [Jou41a] with French translation in [Jou42]. A detailed analysis of Joule's paper can be found in [MS20] and [Mar22].

[266][Note by AKTA:] The German expression *absoluten magnetischen Maassen* is being translated here as absolute magnetic units.

that most of the measurements that have been performed in that regard were not actually reduced to absolute units at all and that this reduction, when it was even attempted, still did not attain the desired precision and certainty, since resistance scales with a precisely-guaranteed reduction to absolute units have been lacking up to now. That is because the only scales of resistance that are useful for such purposes up to now are the ones that were first implemented recently by Siemens,[267] and the only precisely-guaranteed reduction of those scales to absolute resistance units was first given by Kohlrausch (Supplementary Volume VI, 1873, p. 1).[268]

According to this, strictly speaking, only the law of *proportionality* of heat production with the product i^2wdt would have to be regarded as proven, and in order to be able to set *equality* for it, even finer absolute measurements would be required than could have been carried out so far. Meanwhile, we would like to assume that *equality* for the time being, as other physicists have done, even though it has still not been proven with sufficient precision, but only approximately.

However, according to the *magnetic units* used in the formulation of this law, the resistance of the conductor is now known as $w = e/i$, where e denotes the electromotive force and i denotes the intensity of the current produced by this [electromotive] force in the conductor. The mechanical equivalent of the heat generated by the current in the time element dt can therefore also be represented by $eidt$ instead of i^2wdt.

Furthermore, that implies that *according to magnetic units*, *first of all*, the current intensity is $i = 2Eu \cdot \sqrt{2}/c$,[269,270] where $2Eu$ denotes the sum of the product of the positive electricity $+E$ that is contained in a unit length of the conductor in electrostatic units with its velocity $+u$, and the product of the negative electricity $-E$ that is contained in a unit length of the conductor with its velocity $-u$.

Secondly, it follows that, *according to magnetic units*, the electromotive force is $e = [f/E] \cdot [c/\sqrt{2}]$,[271] in which f denotes one-half the difference between forces (expressed in

[267] [Note by AKTA:] E. W. v. Siemens (1816-1892), [Sie60] with English translation in [Sie61]. See also [GT19].

[268] [Note by AKTA:] [Koh73].

[269] [Note by WW:] See the Fourth Treatise on Electrodynamic Measurements, 1857, p. 264 [Note by Heinrich Weber: Wilhelm Weber's *Werke*, Vol. III, p. 652], in which the ratio of the magnetic unit of the current intensity to the mechanical one is given as $= c\sqrt{2} : 4$. — When only positive electricity is considered, as is also customary in the determination of the direction of the current, the current intensity in mechanical units will be expressed by Eu, where E denotes the positive electricity per unit length of the conductor in electrostatic units, and u denotes the velocity with which it moves. See the First Treatise on Electrodynamic Measurements, 1846, Section 21, p. 114 and the following [Note by Heinrich Weber: Wilhelm Weber's *Werke*, Vol. III, p. 152]. — That will imply the current intensity in magnetic units as $i = Eu \cdot 2\sqrt{2}/c$.

[270] [Note by AKTA:] [KW57, p. 652 of Weber's *Werke*] with English translation in [KW21, Section 7.17, p. 179]. See also [Web46, p. 152 of Weber's *Werke*] with a partial French translation in [Web87] and a complete English translation in [Web21d, Section 5.21, p. 144]. The velocity u should be understood as the drift velocity of the electrified particle relative to the body of the conductor, that is, relative to the copper wire.

[271] [Note by WW:] One understands the electromotive force that is exerted upon a conductor to mean the difference between forces that would act upon the positive and negative electricity in the conductor if each unit length of the conductor contained positive and negative electricity, which are expressed in mechanical units. Indeed, if each unit length contained the *electrostatic unit* of positive and negative electricity, the electromotive force acting upon the conductor would be expressed in *mechanical units*. By contrast, if it included a *magnetic unit* of positive and negative electricity, which would then be $c/[2\sqrt{2}]$ times bigger than the electrostatic unit, then the electromotive force acting upon the conductor would be expressed in *magnetic units*. — Now, if one lets $2f$ denote the difference between forces that actually act upon the positive and negative electricity in the conductor, as expressed in mechanical units, and lets E denote the number of

mechanical units) that act upon the positive and negative electricity in the conductor in the direction of the conductor, and E denotes the number of electrostatic units of positive or negative electricity that are contained in the unit length of the conductor.

The mechanical equivalent of the heat created by the current in the conductor will follow from that:

$$i^2 w dt = eidt = 2fudt = (+f) \cdot (+udt) + (-f) \cdot (-udt) \ ,$$

which is equal to the sum of the products of the forces that act upon each streaming particle with the path length from that particle in the direction of the force that acts upon it, i.e., it is equal to the *work done by the current*.

Now, if no other force acts upon the electricity that flows in the conductor than the electromotive force, then that would explain why the *vis viva* of the electricity that flows must increase and why the magnitude of that increase is given by the magnitude of the *work done by the current*. That increase in the *vis viva* of the current that flows will then further imply an increase in the velocity with which the flowing current moves. If, therefore, the flowing electricity in the conductor were to undergo no other motion than current motion, a constant increase in the intensity of the current would result, which would, however, contradict the *constant current* assumed here, for the production of which, according to Ohm's law, a constant electromotive force is required.[272]

All that will then remain in the case that was assumed here is that the electricity in conductor is *not always found in mere current motion*, but that this current motion will go to a *different* motion in the course of time, and conversely.

Now if this *other* movement is the movement of electricity around the ponderable molecules, which is always present in the conductor, and which is the cause of all magnetic (paramagnetic and diamagnetic) phenomena, and in which such a large quantity of electricity takes part that the quantity of flowing electricity disappears in comparison; it follows of itself that the flowing electricity must always have started from the preceding molecular currents at a lower velocity than it reaches the following molecular currents, as a result of the acceleration which it has suffered on its way through the electromotive force; but that the increase of *vis viva* thus gained by the flowing electricity is immediately given up again to the molecular currents at the next station, so that in the case of a persistent current only the *molecular currents* increase in *vis viva*. That increase in *vis viva* is nothing but the heat that is created by the current in the conductor, which can be inferred from the fact that it is equal to the mechanical equivalent of the heat created, which has been proved at least approximately, as was remarked before. — That confirms the suggestion that was expressed at the conclusion of the previous Section that the moving parts that are included in all ponderable bodies whose motion is one of *heat*, are *identical* to the parts that are included in all ponderable bodies whose motion is one of *magnetism*. There are no other moving parts in the interior of the body that are independent of the ponderable ones than those, namely, the *electric* parts.

---

electrostatic units of positive and negative electricity that are included in each unit length; then that will imply that the electromotive force that is exerted upon the conductor is equal to $2f/E$ in *mechanical units*, and it will be equal to $e = [f/E] \cdot [c/\sqrt{2}\,]$ in *magnetic units*.

[272][Note by AKTA:] See footnote 121 on page 65.

11.6 Motion of Electricity in Conductors

If the electricity in all bodies is found to be in continual motion, and especially around the ponderable molecules, and those motions are the basis for all *galvanic*, *magnetic*, and *thermal phenomena*, then that will also be true of electricity in *conductors*, particularly in *metallic conductors*, which are distinguished from all other bodies by their *galvanic* behavior, as well as in regard to *thermal conduction*,[273] and finally, some of them like iron and bismuth are also distinguished by their *magnetism* or *diamagnetism*, the basis for which is obviously to be sought in the special circumstances under which the electricity in those bodies is found.

Electrical current motions take place mainly in *metallic conductors*, and indeed *purely electric ones* (namely, ones for which only the electricity flows without the participation of the ponderable parts) take place *only* in metallic conductors. That is because in *non-metallic*, so-called moist or *decomposable conductors*,[274] no current will flow without *electrolytic* action, i.e., not without the participation of ponderable parts in the current, and indeed some ponderable parts will participate in the flow of positive electricity, while other ones will participate in the flow of the negative electricity.

Now, the *steadiness* of the electrical currents in metallic conductors requires a more detailed explanation. Namely, it is known from Ohm's law that a *steady current* can exist in a closed conductor only under the steady and ongoing action of a certain *electromotive force*, and from the previous Section, such an electromotive force must *accelerate* the electricity that flows in its direction, so the current intensity would also change.

However, if, as was stated in the previous Section, the current in the conductor consists of nothing but *current elements* in which the current motion is continuous only from one conducting molecule to another and when an electric particle arrives at another conduction molecule by way of that current, it will mix with the electricity that exists there and move around that molecule, so it *goes over* from a flowing motion to a rotational motion, while any other particle of the electricity that is present there will conversely *go over* from a rotational motion to a flowing motion, which will define a second current element, etc., then that will explain why acceleration of the electricity in each current element must indeed occur because of electromotive force, but that no increase in intensity of the total current needs to occur in that way when, in fact, the electric particles in all current elements will begin their flowing motion with a velocity that is always equal, *but smaller*, and conclude that motion in it with a velocity that is always equal, *but larger*.

It emerges from this that in metallic conductors, the *transition* from rotational motion to flowing motion and conversely must play a special role for electric particles. That is because that transition should mediate the conduction of electricity in its own right.

In addition, *electrical conduction* and *thermal conduction* are closely related in metallic conductors, and it is clear that if heat is really identical with the *vis viva* of the electricity constantly moving inside the ponderable bodies, then *thermal conduction in metallic conductors*, as well as electric conduction, must be mediated by the *transition* from rotational movement to current movement, and vice versa.

Now, if the basis for the capability of a metallic conductor to conduct electricity and heat lies in the fact that the electric parts that are found to be in rotational motion can be *converted* into current motion and conversely, then one must ask what that transition depends upon and why it takes place in *conductors*, but not in *insulators*. To that end, we shall go

[273][Note by AKTA:] In German: *Wärmeleitung*.
[274][Note by AKTA:] In German: *in nicht metallischen sogenannten feuchten oder zersetzbaren Leitern.*

on to the molecular motions of two electric particles of *different types* that were considered in the last Treatise on Electrodynamic Measurements (in Vol. 10 of the *Abhandlungen der Königl. Sächs. Gesellschaft der Wissenschaften*, 1871, Section 16)[275,276] and to the variety of molecular *constitutions of bodies* that are based upon them.

Namely, if we restrict ourselves to systems that consists of pairs of particles, one of which $-e$ is negatively electrical and bound to a ponderable mass, while the other one $+e$ is positively electrical and moving around the former, then such systems can differ from each other to varying degrees by the following properties:

First property: Every such system will assign a well-defined, and in fact negative, value to ρ (namely, when one sets $ee'/a = \rho$ and makes the signs of e and e' depend upon whether the particles that they denote belong to positive or negative electricity) that can be very different for different systems. It is then a *property* of such systems that each of them assigns a well-defined value to ρ, or to ρc^2, by which they can be distinguished from other systems.

Second property: From Section 11, in the place cited, one has that $r^2\alpha^2 = r_0^2\alpha_0^2$ (when r_0 and α_0 denote the initial values of the distance between the two particles and their relative velocity in the direction perpendicular to their connecting line, respectively, while r and α denote the current values) is a *constant* of the system, at least as long as no other forces act upon the particles than the ones that result from their interaction. That *constant* is a *second property* that can likewise serve to distinguish between different systems. However, this does not mean that *permanent* distinctions are made; instead, transitions from one system to another can take place as a result of *external* influences.

Third property: For a *steady-state* system, the distance between two particles can indeed vary, but there must be a finite least distance r_0, as well as a greatest one r^0, that depends upon the former. Now, the value of the least distance r_0 can be different for different systems and can therefore be considered to be a *third property* that will serve to distinguish between different systems but is likewise subject to variation as a consequence of external influences.

If one now denotes by $-n$ the quotient for such a system (in which $\rho = ee'/a$ has a negative value, as was remarked before) that is defined from the three constants ρc^2, $r_0^2\alpha_0^2$ and r_0 by dividing the second one by the product of the first and last ones, so one sets:

$$n = -\frac{r_0\alpha_0^2}{\rho c^2} \ ,$$

then according to Section 16, in the place cited, that will give the following equation of motion, in which u denotes the relative velocity of both particles, namely:

$$\frac{\rho - r}{\rho} \cdot \frac{u^2}{c^2} = \left(\frac{r}{r_0} - 1\right) \cdot \left(n\left[\frac{r_0}{r} + 1\right] - 1\right) \ .$$

It follows from this that for $u = 0$, one has either $r = r_0$ or $r = [n/(1-n)]r_0 = r^0$.

Moreover, that will imply the difference between *steady-state* and *non-steady-state* systems by way of the values of n. Namely, a *steady-state system* for which r_0 is the least value of r and r^0 is the greatest one will exist for only $1 > n > 1/2$, i.e., when the value of n lies between $1/2$ and 1. That is because for $n > 1$ and $n < 0$, no value at all will exist for r^0, which is essentially positive, and for $1/2 > n > 0$, one will get $r = r^0 < r_0$, i.e., the equation

[275] [Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. IV, p. 279.

[276] [Note by AKTA:] [Web71, p. 279 of Weber's *Werke*] with English translation in [Web72] and [Web21g, Section 9.16, p. 94]. By the molecular motions of two electric particles of *different types* we should understand the motions of a positive and a negative particle due to their mutual interaction.

would no longer allow one to find the larger of the two values of r for which $u = 0$ from the smaller one but conversely, the smaller from the larger.

All *steady-state* systems will then split into two classes, namely, the ones for which $1/2 < n < 1-\varepsilon$, which are *insulators*, and the ones for which $1-\varepsilon < n < 1$, which are *conductors*. The ε in that is determined in such a way that when $n = 1-\varepsilon$, the larger value of r for which $u = 0$, and which was denoted by r^0, is large enough that the moving particle will enter into the sphere of action of the neighboring system, and therefore go from one system to another. If one sets that value of r^0 equal to $(1+\mu)r_0$ and observes that in general, one will have $r^0 = [n/(1-n)]r_0$, then one will get the equation $1+\mu = n/(1-n)$ for $n = 1-\varepsilon$, and as a result $\varepsilon = 1/(2+\mu)$.

For the value $n = 1-\varepsilon$ at which the *transition from an insulator to a conductor* will take place, the conductance[277] will be equal to 0, and it will increase with n when the latter is greater than $1-\varepsilon$ and increasing.

11.7 Two Types of Heat Transfer in Ponderable Bodies

The considerations of the previous Section were essentially built upon the laws of motion of two electric particles that are left to only their own interaction. If other particles were present, then they would be assumed to be far enough away that their influence would almost vanish in comparison to that of the two particles under consideration. Only in the case where the two particles of a pair grow ever further apart must there be a limit beyond which the influence of other particles will become greater than the interaction of the particles in question. However, the laws of motion that apply to that transition are still not known completely and have not been developed in general. That is why the only result that was cited in the foregoing Section was that the two particles that had defined a pair up to a point in time will separate from each other and combine with other particles into new pairs.

If *heat* is the *vis viva* of moving particles inside ponderable bodies, and if these moving particles are positive electric particles which move around negative electric particles adhering to ponderable bodies, then that will explain why in *metallic conductors* (as they were defined in the previous Section), one will find *heat transfer by conduction*[278] at the boundary surface between two conducting elements, and indeed in opposite directions simultaneously, namely, individual positive electric particles will cross the boundary with the tangential velocity of their rotational motion about a molecule on one side of it and mix with the rotating electricity of a molecule on the other side of the boundary surface, and conversely. That *propagation of heat in metallic conductors*, which results from the transfer of *vis viva* through *all of its carrier*, is called the *propagation of heat by emission*,[279] or more briefly, *thermal conduction*.

However, *heat transfer* likewise takes place in *insulators*, i.e., the transfer of *vis viva* from a molecule on one side of the boundary surface between two elements of the insulator to a molecule on the other side, and conversely, but without the electrical particles that are the carriers of that *vis viva* crossing that boundary surface in their own right. That *second type of heat transfer*, as it is found in insulators, namely, by the *transfer of vis viva without the transfer of its carriers*, is called the *transfer of heat by radiation*, or more briefly, *thermal*

[277] [Note by AKTA:] In German: *Leitungsvermögen*.

[278] [Note by AKTA:] In German: *Wärmeverbreitung durch Leitung*. This expression can be translated as heat transfer by conduction or heat propagation by conduction.

[279] [Note by AKTA:] In German: *Wärmeverbreitung durch Emission*. This expression can be translated as heat transfer by emission or heat transfer by emission.

radiation.[280] It takes place from one ponderable body to another through empty space, e.g., in outer space.

It is known that the same thing is true for that transfer of heat by radiation in empty space or in insulators that is true for the radiation of light, namely, that it is mediated by the propagation of waves, which then assumes the existence of a medium that is capable of propagating waves. Up to now, one has sought to learn about the nature of that medium from the laws of wave propagation, as they were found from observations of light phenomena. However, if that medium consists of electricity, and one possesses more detailed knowledge about its constitution, then it would be possible to develop the laws of that wave propagation from the fundamental law of electric action and to explain the light phenomena in terms of it, which has actually being attempted in various ways, but we will not go further in that direction here.

11.8 On the Concept of Thermoelectricity Developed by Kohlrausch

We have distinguished between two types of heat transfer, namely, conduction and radiation, which coincide with two types of transfer for electrical motion, namely, the transfer of that motion either with or without its carrier. The *former* type of transfer takes place in *metallic conductors*, in which the electricity can also be set into current motion by *electromotive forces*.

The electric movements, however, which take place, even if the particles are not driven by electromotive forces, but merely by following the laws according to which they move around each other by virtue of their interaction, whereby they move away from each other until they exceed the molecular boundaries, differ substantially from the electric currents produced by electromotive forces in that, in the former, the same amount of electricity passes forwards through the interface of *two identical and equally hot* molecules as backwards, while in the case of currents produced by *electromotive forces*, a greater amount of electricity passes through the interface in the direction the force than in the opposite direction. Those oppositely equal movements cancel each other out [in the former case], so that no current in the narrower sense remains, because current in the narrower sense only means the difference between the two opposite movements.

For *equal, but unequally-warm* molecules in a metallic conductor on which no electromotive forces otherwise act and through which a steady-state current in a closed loop is produced, a larger amount of electricity can indeed go forwards through the boundary surface from the warmer molecule to the colder one during a moment than the amount that goes through it backwards, but that moment lasts only long enough for the excess of electricity that arrives at the colder molecules to generate a charge that exerts an electromotive force at the location of the boundary surface that will drive just as much electricity from the colder molecule backwards through the boundary surface as would go forwards without it, such that equality will once more be established in that way.

Once equality has been established, the *electric current* that goes through the boundary surface in a moment will vanish. By contrast, the *heat current* can still persist, namely, when the particles that come from the warmer molecules move *with greater velocity* than the ones that come from the colder molecules. One sees from this that the close connection

[280][Note by AKTA:] In German: *Wärmestrahlung*.

between heat movement and electric movement, which is based upon the fact that both of them originate in the electricity that goes through the boundary surface, *does not at all imply* that no heat current can exist without electrical current, or conversely.

However, from the concept of *thermoelectricity* that Kohlrausch developed in the *Nachrichten der Königl. Gesellschaft der Wissenschaft zu Göttingen*, 1874, p. 65,[281] such a connection between heat movements and electrical movements should *really* exist, in the same way as would be the case if electricity and heat were two bodies which were connected to each other by cohesive forces, where then there could very well be talk of a transfer[282] of heat through electricity, as well as of electricity through heat. However, heat is not a body, but the *vis viva* of a body, and as a result, *heat current* is the transfer of *vis viva* from one location to another, *either* by its carrier, as in metallic conductors, *or* without a carrier, as in insulators. Only in the *former* case, it is obvious, namely in metallic conductors, could the connection assumed by Kohlrausch possibly take place; in the *latter* case such a connection is not possible, because then no electric current exists at all.

In the unit time, an electric mass ε (in milligrams) with a velocity of α passes through an element f of the boundary surface between two metallic conducting elements from the warmer molecules on one side of it to the colder molecules on the other side. A mass ε' with velocity α' passes backwards through the same element of the boundary surface from the colder molecules to the warmer ones. In that way, an *electric current* that goes through f will be given, and likewise a *heat current* through f, each of which will have an intensity of $i = (\varepsilon - \varepsilon')$ in mechanical units (with the milligram as the unit of mass) and $W = (\varepsilon\alpha^2 - \varepsilon'\alpha'^2)$ as its mechanical equivalent.

The following cases are then possible, in general:

(1) $\varepsilon = \varepsilon'$,[283] in which a heat current of intensity $\varepsilon(\alpha^2 - \alpha'^2)$ would exist with no electrical current;

(2) $\varepsilon\alpha^2 = \varepsilon'\alpha'^2$, in which an electric current of intensity $(\varepsilon - \varepsilon')$ would exist with no heat current.

(3) When one has neither $\varepsilon = \varepsilon'$ nor $\varepsilon\alpha^2 = \varepsilon'\alpha'^2$, but a certain relationship exists between $(\varepsilon - \varepsilon')$ and $(\varepsilon\alpha^2 - \varepsilon'\alpha'^2)$ that remains constant under changes of temperature in the conductor, but which will vary with the different types of conductors.

The third case essentially agrees with the theory of thermoelectricity that Kohlrausch developed.

Namely, Kohlrausch made the assumption that the ratio of the intensities of the electric and heat currents $(\varepsilon - \varepsilon')/(\varepsilon\alpha^2 - \varepsilon'\alpha'^2)$ was constant for each conductor, but depended upon the nature of the conductor, and he denoted it by α, which made the current intensity $i = \alpha W$, if W denotes the intensity of the heat current. Kohlrausch deduced the law of thermo-electromotive forces from that assumption, namely, that the thermo-electromotive forces depend upon only the temperature at the location where contact exists and that they are proportional to the temperature difference, as well as Peltier's law of heat production,[284] according to which heat is developed or absorbed at the point of contact between two conductors, depending on whether the current there goes to a conductor with a smaller or larger thermoelectric constant.

The third case, namely, that a certain relationship exists between $(\varepsilon - \varepsilon')$ and $(\varepsilon\alpha^2 -$

[281] [Note by AKTA:] [Koh74].

[282] [Note by AKTA:] In German: *Fortführung*.

[283] [Note by AKTA:] In the original text: $\varepsilon = \varepsilon$.

[284] [Note by AKTA:] Jean Charles Athanase Peltier (1785-1845). See [Pel34].

$\varepsilon'\alpha'^2$), will obviously be satisfied when one sets $\alpha^2 = \alpha'^2$. However, under that restriction, the derivation of the law of thermo-electromotive forces that Kohlrausch gave found no application to thermopiles in which each of the conductors that define a closed loop possess different temperatures at its two ends since each temperature difference in a (homogeneous) conductor can have its basis in only the differences between the values of α^2 and α'^2. From the invariability of $(\varepsilon - \varepsilon')/(\varepsilon\alpha^2 - \varepsilon'\alpha'^2)$, which results from $\alpha^2 = \alpha'^2$, the *first* law, namely, the law of thermo-electromotive forces, cannot therefore be derived, but the *second* law, namely, Peltier's law of heat production or heat absorption, can.

If you have two different metallic conductors and the quotient $(\varepsilon - \varepsilon')/(\varepsilon\alpha^2 - \varepsilon'\alpha'^2)$, which is constant for $\alpha^2 = \alpha'^2$, is designated m for one conductor and n for the other, the amount of heat that passes through the boundary surface of the last two elements of the first conductor, $= m(\varepsilon - \varepsilon')$; the amount of heat that passes through the boundary surface of the first two elements of the second conductor, $= n(\varepsilon - \varepsilon')$. If a current of magnitude $(\varepsilon - \varepsilon')$ passes through the closed circuit formed by both conductors, then at the point where the first conductor touches the second, a total heat $(m-n)(\varepsilon - \varepsilon')$ will be *produced*. By contrast, at the other contact location, namely, the one where the second conductor contacts the first one, the total heat $(n - m)(\varepsilon - \varepsilon')$ will be produced, or what amounts to the same thing, the total heat $(m - n)(\varepsilon - \varepsilon')$ will be *absorbed* there.

However, in addition to the three cases that were cited above, there remains a fourth case to consider, namely, in addition to the cases in which one has either $\varepsilon = \varepsilon'$, $\varepsilon\alpha^2 = \varepsilon'\alpha'^2$, or $\alpha^2 = \alpha'^2$ in the quotient, there remains the case:

(4) One again has neither $\varepsilon = \varepsilon'$ nor $\alpha^2 = \alpha'^2$, but there is a dependency of the ratio α^2/α'^2 on the ratio ε/ε'; where, for example, α^2/α'^2 is equal to some power of ε/ε'.

Namely, in a metallic conductor, an increase in temperature will imply an increase in heat, i.e., from our assumption, that would be an increase in the *vis viva* of the moving electric parts in the conductor, which would imply an increase in the velocity of those parts, since their total amount or mass can suffer no change. Now, that increase in the velocity will also be true for the moment when they cross the boundary between two neighboring molecules whose velocity will be denoted by α. Therefore, α will increase with the temperature of the conductor. However, that increase in the velocity of all moving parts that is coupled with an increase in temperature assumes that the amount or mass of the particles ε that pass through the boundary surface per unit time increases in such a way that a simultaneous increase in α and ε will take place, which was assumed in the fourth case.

The equation:

$$\frac{\alpha^2}{\alpha'^2} = \left(\frac{\varepsilon}{\varepsilon'}\right)^n ,$$

will then give the following equation for the intensity of the heat current:

$$\varepsilon\alpha^2 - \varepsilon'\alpha'^2 = \varepsilon\alpha^2\left(1 - \frac{\varepsilon'}{\varepsilon} \cdot \frac{\alpha'^2}{\alpha^2}\right) = \varepsilon\alpha^2\left(1 - \left[\frac{\varepsilon'}{\varepsilon}\right]^{n+1}\right) .$$

If one divides that intensity of the heat current by the intensity of the electric current $(\varepsilon - \varepsilon') = \varepsilon(1 - \varepsilon'/\varepsilon)$, then one will get that the ratio of the two intensities is:

$$= \alpha^2\left(1 + \frac{\varepsilon'}{\varepsilon} + \ldots + \left[\frac{\varepsilon'}{\varepsilon}\right]^n\right) .$$

One sees from this that when $n = 0$, the fourth case will coincide completely with the third case that was considered already since the two cases will give:

$$\frac{\varepsilon - \varepsilon'}{\varepsilon\alpha^2 - \varepsilon'\alpha'^2} = \frac{1}{\alpha^2} \,.$$

The next case to consider is $n = 1$, for which one will get:

$$\frac{\varepsilon - \varepsilon'}{\varepsilon\alpha^2 - \varepsilon'\alpha'^2} = \frac{1}{\alpha^2\left(1 + \frac{\varepsilon'}{\varepsilon}\right)} \,.$$

However, the fact that the differences between two neighboring molecules in a conductor are always very small will further imply that the value of ε'/ε is only slightly less than 1, at least for weak currents in good conductors such that one will get approximately

$$\frac{\varepsilon - \varepsilon'}{\varepsilon\alpha^2 - \varepsilon'\alpha'^2} = \frac{1}{2\alpha^2}$$

in the fourth case that was just considered. The assumption that Kohlrausch made in this case that the intensity ratio of the electric and heat currents $(\varepsilon - \varepsilon')/(\varepsilon\alpha^2 - \varepsilon'\alpha'^2)$ was a constant whose value depends upon only the nature of the conductor is also true in this case, at least *approximately*. It will then follow in this fourth case that one can deduce the same consequences that Kohlrausch had deduced from his assumption *approximately*, and in particular, the law of thermo-electromotive forces that those forces depend upon only the temperature at the location where contact exists, and they are proportional to the temperature differences at those locations.

If the derivation of the law of thermo-electromotive forces that Kohlrausch gave also made no appeal to any *contact action*, then that would explain the fact that such a contact action is not completely excluded in that way either, but such a thing might possibly be added.

11.9 Resistance to Conduction and Maximum Current Intensity

If the electricity in metallic conductors with a molecular constitution are actually to be in the state of motion that was given in Section 11.6, namely, the positive electric parts rotate around the negative parts that are endowed with ponderable masses, although they do not always remain in the same circular orbit in that, but begin from a smallest circular orbit whose radius increases so they approach another molecule and finally go over to that molecule, then that will yield a dependency of the current intensity on the electromotive forces in such conductors that would not agree completely with the one that is given by Ohm's law, but deviate from it by the fact that the current intensity does not always increase uniformly with the electromotive force, but will finally approach a certain limiting value that it cannot exceed. However, that limiting value will be attained only when the directions of all of the parts that go over to a current motion, no matter how different they might be to begin with, are all brought into the direction of the electromotive force in the shortest time through ever-increasing values of that force. The intensity of the current would not be able to increase further then, so it would have attained its maximum. Attempts to decide whether the intensities of the currents that are excited by very large and small electromotive forces in the same conductor are always proportional to those electromotive forces would then be of greatest importance.

In Figure 1, let A be a molecule from which positive electric particles are ejected in all directions with the same velocity α. Let one such direction be AB, and let ξ be the length of the path that the particle would traverse in a time interval t due to its velocity α. However, a constant (electromotive) force acts upon that particle in a direction that is parallel to AC and subtends an angle of ψ with AB, so the particle would then traverse a path of length η in the time interval t that increases in proportion to t^2 or ξ^2 by that alone.

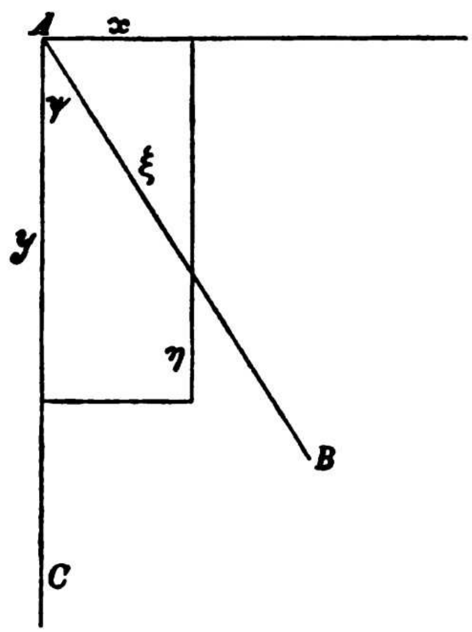


Fig. 1.

One then sets:

$$\eta = a\xi^2 \ ,$$

and furthermore:[285]

$$x = \xi \sin\psi \ ,$$

$$y = \xi \cos\psi + \eta = x \cot\psi + \frac{a}{\sin\psi^2} \cdot x^2 \ ,$$

$$r^2 = x^2 + y^2 \ ,$$

from which one will get:

$$y = \cot\psi \cdot \sqrt{r^2 - y^2} + \frac{a}{\sin\psi^2} \cdot (r^2 - y^2) \ .$$

[285] [Note by AKTA:] The equation for y should be understood as:

$$y = \xi \cos\psi + \eta = x \cot\psi + \frac{a}{\sin^2\psi} \cdot x^2 \ .$$

That ballistic motion[286] comes to an end when the distance of the particle from A has become equal to r, as the particle then reaches the neighboring molecule. That distance r is independent of the direction of the ballistic motion and can be taken to be equal for all particles that are thrown from A, so it will be referred to as the mean molecular distance.

First of all, because the greater the electromotive force proportional to a, the more all the other members of the equation above disappear against the member which has a as a factor, it follows that for a growing electromotive force, y^2 approaches a limiting value, namely

$$y^2 = r^2 \ ,$$

which will be the same for all particles that are ejected from A. It therefore follows that the distance traveled by all particles in the direction of the force would then be the same, namely $= r$.

If ε denotes the mass of the particles that are emitted from A per unit time, and n denotes the number of molecules that are contained in an element of the conductor of length r; then $n\varepsilon$ will be the mass of positive electricity that would go in the direction of the electromotive force through the boundary surface between two successive molecular layers per unit time when the electromotive force is increased to infinity, i.e., the limiting value of the current intensity in mechanical units when based upon the mechanical unit of mass (namely, the milligram), it should only be noted that because electricity cannot be determined in such mass units, when determining intensity according to so-called mechanical units, the quantities of electricity are not expressed in the mass units of mechanics (i.e., milligrams) but in *electrostatic* units.

Now, if σ denotes the number of *electrostatic* units that go to the unit mass of mechanics (milligram), then one will get the limiting value of the current intensity in so-called mechanical units equal to $n\varepsilon\sigma$, or when one adds the notation for the reduction of the mass unit to the three basic units of mechanics (namely, mass M, distance R, and time T), it is:

$$= n\varepsilon\sigma \cdot \left[\sqrt{\frac{MR^3}{T^4}}\right] \ .$$

In *electrostatic* units, an amount of electricity will be determined by a force (that this amount of electricity exerts on an equal amount of electricity) that is equal to $f \cdot [MR/T^2]$ and a distance (at which that force is exerted) equal to $r \cdot [R]$ and is expressed by:

$$r\sqrt{f} \cdot \left[\sqrt{\frac{MR^3}{T^2}}\right] \ .$$

However, the current intensity in mechanical units is the quotient of one such amount of electricity that goes through the cross-section of the conductor divided by the duration of that transfer, which equals $t \cdot [T]$, so that intensity will be equal to

$$r\frac{\sqrt{f}}{t} \cdot \left[\sqrt{\frac{MR^3}{T^4}}\right] \ .$$

In the present case, one has $r\sqrt{f}/t = n\varepsilon\sigma$.

[286][Note by AKTA:] In German: *Wurfbewegung*. This expression can be translated as ballistic motion or throwing motion.

Moreover, in that determination of the limiting value of the current intensity, it was assumed that the electromotive force itself had no effect on the number of electric particles that were emitted from the molecules.

If, on the other hand, the electromotive force or the quantity a proportional to it is very small, then in the equation found:

$$y = \cot\psi \cdot \sqrt{r^2 - y^2} + \frac{a}{\sin\psi^2} \cdot (r^2 - y^2) \ ,$$

in the last member, which has a as a factor, the approximate value resulting from the equation for $a = 0$ can be set for y^2, namely, $y^2 = r^2 \cos\psi^2$.[287] One will then get:

$$y = \cot\psi \cdot \sqrt{r^2 - y^2} + ar^2 \ ,$$

and that will likewise give approximately:

$$y = +r\cos\psi + ar^2 \sin\psi^2 \ .$$

That will then give, in the mean, for the two particles that were sent from A in the directions that were determined by the angles ψ and $\pi - \psi$:

$$y = ar^2 \sin\psi^2 \ .$$

The mean value over all of the paths that are traversed by the particles that are emitted from A in the direction of the force will then be:

$$\frac{1}{2\pi}\int_0^{\pi/2} 2\pi y \sin\psi d\psi = ar^2 \int_0^{\pi/2} \sin\psi^3 d\psi = \frac{2}{3}ar^2 \ .$$

If that value were equal to r, then the current intensity would be the same as the previously-considered limiting value, namely, $= n\varepsilon\sigma \cdot [\sqrt{MR^3/T^4}]$. Now, the actual current intensity amounts to only a very small fraction of that, namely, $2ar/3$, which will give that current intensity i^0 as:

$$i^0 = \frac{2}{3}ar \cdot n\varepsilon\sigma \cdot \left[\sqrt{\frac{MR^3}{T^4}}\right] \ .$$

Finally, in order to determine the coefficient a, it should be pointed out that when γ denotes the accelerating force that acts on the particles that are emitted from A, one will have:

$$\eta = \frac{1}{2}\gamma t^2 = \frac{1}{2}\gamma \cdot \frac{\xi^2}{\alpha^2} = a\xi^2 \ ,$$

which will give:

$$a = \frac{1}{2} \cdot \frac{\gamma}{\alpha^2} \ ,$$

and as a result:

[287][Note by AKTA:] This equation should be understood as $y^2 = r^2 \cos^2\psi$.

$$i^0 = \frac{1}{3} \cdot \frac{\gamma r}{\alpha^2} \cdot n\varepsilon\sigma \cdot \left[\sqrt{\frac{MR^3}{T^4}}\right] .$$

Upon dividing the current intensity i^0, when expressed in *mechanical* units, by $c/2\sqrt{2}$, namely, by the number of electrostatic units that go into a magnetic unit, one will get that same current intensity i, when expressed in *magnetic* units, namely:

$$i = \frac{2\sqrt{2}}{3c} \cdot \frac{\gamma r}{\alpha^2} \cdot n\varepsilon\sigma \cdot \left[\sqrt{\frac{MR}{T^2}}\right] .$$

Now, the *electromotive force* per unit length of the conductor, in *mechanical* units, e^0, is the product of the acceleration γ with the amount of electricity that flows per unit length of the conductor, which is equal to $n\varepsilon/r$, divided by the number of electrostatic units that are flowing in the unit length, which is equal to $n\varepsilon\sigma/r$, so one will have:

$$e^0 = \frac{\gamma}{\sigma} \cdot \left[\sqrt{\frac{M}{RT^2}}\right] .$$

That will then give the electromotive force per unit length of the conductor in *magnetic* units, e, by switching the number of electrostatic units $n\varepsilon\sigma/r$ with the number of magnetic units $n\varepsilon\sigma/r \cdot 2\sqrt{2}/c$, which will make:

$$e = \frac{c}{2\sqrt{2}} \cdot \frac{\gamma}{\sigma} \cdot \left[\sqrt{\frac{MR}{T^4}}\right] .$$

If one now substitutes the value of γ that this gives in the foregoing equation for determining i, then one will get:

$$i = \frac{8}{3c^2} \cdot \frac{er}{\alpha^2} \cdot n\varepsilon\sigma^2 \cdot \left[\sqrt{\frac{MR}{T^2}}\right] .$$

Now, if l denotes the length of the closed conductor, then el will be the total electromotive force that acts upon the closed conductor, and i will be the intensity of the current that it generates in *magnetic* units. That will then give the resistance w of the closed conductor:

$$w = \frac{el}{i} = \frac{3c^2}{8} \cdot \frac{\alpha^2}{n\varepsilon\sigma^2} \cdot \frac{l}{r} \cdot \left[\frac{R}{T}\right] ,$$

i.e., a *definition of resistance* that is completely independent of the determination of resistance according to Ohm's law that involves measuring the electromotive force and current intensity.

The *conductance* $= 1/w$ is therefore determined by its causes, which lie in the deflection of the particles from their trajectories along which they were ejected. Namely, that explains why the *conductance* must be proportional to:

1. The mass that is found to be in a ballistic motion in the conducting channel.

2. The rate of deviation that is produced by a certain force along a well-defined path in that mass.

In the conducting element r that mass is now $n\varepsilon$, and the deflection velocity caused by a certain force on the path r is *inversely proportional to the square of the ballistic velocity* α^2.

The *conductance* $1/w$ must then be proportional to $n\varepsilon/\alpha^2$, and as a result, the *resistance of the conductor* w must be proportional to $\alpha^2/n\varepsilon$. Since that is true for the resistance of the *conducting element* r, that will imply that the *resistance of a conductor of arbitrary length* l will be proportional to $\alpha^3/n\varepsilon \cdot l/r$, which coincides with the previous formula, according to which this resistance is equal to the product of this quantity with the *constant factor* $3c^2/8\sigma^2$.

That definition of the resistance of a conductor proves to be especially interesting due to the fact that it would then follow that when the resistance of a conductor w is constant, in addition to the values of l, n, r, the ratio of the two variables α^2 and ε, namely, α^2/ε, must likewise be constant, i.e., when α^2 and ε have changed into α'^2 and ε', respectively, so one would then need to have:

$$\frac{\alpha^2}{\varepsilon} = \frac{\alpha'^2}{\varepsilon'} \qquad \text{or} \qquad \frac{\alpha^2}{\alpha'^2} = \frac{\varepsilon}{\varepsilon'} \ .$$

Now, it follows from this that when the resistance w of a conductor is constant and it does not vary with the temperature of the conductor either, the value α^2/ε for that conductor will also be constant, so for such a conductor, according to Section 11.8, the view of thermoelectricity that Kohlrausch presented would be valid. However, since the resistance of metallic conductors varies more or less with their temperature, that would imply that the viewpoint that Kohlrausch presented could be true only approximately, and indeed mostly for metallic conductors whose resistance changes the least with the temperature of the conductor, and it would seem to follow from this that such a conductor would be most suited to the representation of thermomagnetic circuits.

11.10 Distribution of Electricity in Conductors

Electrostatics was founded and developed by Coulomb[288] and Poisson,[289] before the discovery of electromagnetism and electrodynamics, and therefore no account could be taken of these great discoveries by them. Indeed, the law of distribution for the electric fluids that are found in conductors at rest and in equilibrium that is developed in electrostatics, as well as the forces that are exerted by that distribution, have all been found to be in agreement with experiments, to the extent that observation and measurement allows. However, new discoveries, in particular, electromagnetism and electrodynamics, have shown that such a state of equilibrium in electric fluids, as Coulomb and Poisson assumed to exist in conductors, does not actually exist at all, but that all electric fluids in conductors are found to always be in *steady-state motion* around ponderable molecules, from which it would follow that, strictly speaking, the laws of distribution and the laws of action of *static* electricity that Poisson developed will find no application to the electricity that is found in conductors.

All of the phenomena that were considered in *electrostatics* up to now actually belong to *electrodynamics* accordingly, and it is in the laws of the latter that one must seek the complete explanation for the former. It would then seem that electrostatics, which previously defined the largest and most important part of the study of electricity, would have to experience

[288][Note by AKTA:] Charles Augustin de Coulomb (1736-1806). Coulomb's main works on torsion, electricity and magnetism have now been fully translated and commented in Portuguese and English, [Ass22] and [AB23]. See also [Pot84], [Gil71b] and [Gil71a].

[289][Note by AKTA:] Siméon Denis Poisson (1781-1840). See [Poi12a], [Poi12b] with English translation in [Poi19], [Poi13] and [Poi14].

a complete remodeling. However, one must make the demand of such a remodeling that it must be able to explain the entire sphere of phenomena that was explained by *electrostatics* up to now just as completely and precisely in terms of *electrodynamics*, but that has not happened up to now, nor has any attempt been made to do that yet.

Despite the inclination that one finds on the one hand to abandon the idea of *magnetic fluids* on which Coulomb and Poisson based the theory of magnetism, which was developed at the same time as electrostatics, there seems to be a certain fear of the consequences connected with this, namely of the then indispensable idea of the existence of persistent molecular currents in all magnetic and diamagnetic bodies, according to which electricity never and nowhere comes to rest and equilibrium. That comes down to saying that up to now any attempt to find an *electrodynamic* explanation for all previously considered phenomena of *electrostatics* has met with great difficulties, namely, due to lack of any assistance on the part of mathematics, which has proved to be powerless to deal with such complicated processes for some time now.

However, on the same grounds, just as little of an attempt has been made at the foundations of *electrostatics* to give a more precise justification for the constitution of the so-called *neutral fluid* and the *process for separating* that fluid in conductors, but one has restricted oneself to a general assumption about the mutual mobility of the two components of the neutral fluid, as well as their mixing, and in that way one has sought to make the development of the law of distribution of electricity independent of a more detailed knowledge of the constitution of the neutral fluid that is everywhere distributed inside the conductors.

However, on the basis of the assumption that one makes in electrostatics of the mutual mobility of the two components of a neutral fluid, nothing at all can be determined and established regarding the internal constitution of that fluid itself, and especially nothing regarding whether the two components are at rest and in equilibrium before their separation or whether they are found to be in motion with respect to each other, e.g., a rotating motion, such that the picture of steady molecular currents would still not seem to be excluded completely by the assumption that electrostatics makes of the mutual mobility of the two components of the neutral fluid, in principle. Rather, the picture of steady molecular currents could be regarded as an attempt to explain the assumed mutual mobility of both components. Therefore, electrostatics, as it was developed by Poisson, regardless of the fact that one cares to define it as the study of the distribution of the electricity in conductors when it is *at rest and in equilibrium*, is nonetheless not in direct contradiction with the existence of steady molecular currents of electricity in the interior of conductors. We have the *statics of solid bodies*, *hydrostatics*, and *aerostatics*, which also seem to be well-founded, and the same thing applies to them completely. Even when they are defined to be the study of rest and equilibrium for those bodies, that will not, however, contradict the fact that the interiors of those bodies are filled with particles that are not at rest, but are found to be in a state of constant motion, and indeed, large motions. That is because in just the same way that the *magnetic* and *diamagnetic* phenomena in bodies led to continual internal motions (electric molecular currents) in them, similarly, the *thermal phenomena* in those solid, fluid, and gaseous bodies for which statics, hydrostatics, and aerostatics are valid have led to such continual internal motions. That is because at each moment, every ponderable body will possess a temperature that is the effect of the heat that the body contains, and that heat is nothing but the *vis viva* that the moving particles in the interior of the body possess. However, the measurable magnitude of that *vis viva* (namely, the mechanical equivalent of heat) has shown that those motions are very large in the interiors of all such bodies.

If positive electric particles move in the *interior of the conductors* around the negative electric particles adhering to ponderable masses, but do not always remain in the same circular path during this movement, but approach the neighboring molecules with increasing radius, to which they finally pass over, and if such transitions take place from one molecule in the interior of the conductor indifferently in all directions to all neighboring conductor molecules, from which an equal emission of particles takes place simultaneously in all directions to all neighboring molecules, then, on the other hand, it results for those molecules of the conductor which lie closest to its surface that they have insulator molecules instead of conductor molecules as neighbors on their outer side, from which no such emission takes place, and which also do not receive the particles emitted by other molecules. It follows from this that if, as the radius of their orbit increases, individual particles have moved so far away from their center that they would pass over to the neighboring molecule if there were still other conductor molecules on the side where they are located, this will not happen if there are no conductor molecules at all on the side where they are located, but only insulator molecules. Those particles will then continue a little further in their circular orbit with increasing radius until they reach a side where there are other conductor molecules in the vicinity. This will be the case if the resultant of all electrical forces at this boundary between conductor and insulator is zero.

By contrast, when that resultant is non-zero and points *outward*, it can exert the same influence as a neighboring conductor molecule, namely, it can act in such a way that particles that are also on the side of the bounded insulator can leave the path that was previously followed around the conductor molecule and be fixed at the neighboring insulator particle. Therefore, such emitted particles of positive electricity can accumulate everywhere on the boundary surface between the conductor and the insulator according to the magnitude and direction of the outward-pointing resultant at each location on it.

If the resultant is different from zero and is directed *inwards*, its effect on the increase of the particles sent *inwards* by the nearest conductor molecules can only be compensated by slightly reducing the number of positively electric particles in these conductor molecules, while the number of negatively electric particles adhering to the ponderable mass, around which they rotate, remains unchanged.

Obviously, the distribution law that Poisson developed, which said that the excess of positive or negative electricity will exist only on the boundary surface between a conductor and an insulator, in any event, will be valid for the excess of positive electricity at some locations on the boundary surface between the conductor and the insulator and for the absence of positive electricity at the other locations that are close to the conductor molecules at the boundary of the insulator (and that lack of positive electricity is equivalent to an excess of negative electricity). For these laws of distribution of electricity on the surface make no difference whether there is a so-called separable, neutral fluid inside the conductor, as Poisson assumes, or whether there are conductor molecules with electricity moving around them, between which a continuous exchange of individual particles takes place. Such conductor molecules shall have just as little influence on the distribution of electricity on the surface as the neutral fluid that Poisson assumed, and conversely, the electricity that is distributed on the outer surface according to Poisson's law will have no effect on the conductor molecules since, according to Poisson, the distribution of electricity on the outer surface will be determined by just the fact that the resultant of the force that is exerted by all of the electricity that is distributed on the outer surface on any point in the interior should be zero, which is a demand that is entirely independent of whether a neutral fluid is found at the location considered in the interior of the conductor or a conductor molecule with electric molecular currents. — The true constitution of bodies and the true processes that depend upon that, although they are also very complicated and can be thought of as being represented by simpler processes only in part, will nonetheless remain the focus and ultimate goal of research, in spite of all obstacles.

Chapter 12

Editor's Comments on Riecke's 1891 Memorial Speech

A. K. T. Assis[290]

Eduard Riecke (1845-1915) was a German experimental physicist who studied under Wilhelm Weber at the University of Göttingen, where he received his doctorate in 1871. In 1881 he succeeded Weber as full professor at Göttingen University, taking over Weber's Laboratory and Institute.

Riecke's memorial speech was given at the public meeting of the Göttingen Royal Society of Sciences on December 5, 1891, and published in 1892.[291]

I don't agree with many of Riecke's points of view relating to action at a distance versus action mediated by a medium, atomic constitution of matter versus a continuous distribution of matter, and Weber's electrodynamics versus Faraday's and Maxwell's field theories. Despite these disagreements, I decided to include this memorial speech in the book. One reason is that it was one of the first biographical studies on Weber. Another reason is to show how quickly Weber's electrodynamics was replaced by the field theories of Faraday and Maxwell not only in Germany as a whole, but specifically at the University of Göttingen, where Weber worked for most of his scientific career.

[290]Homepage: www.ifi.unicamp.br/~assis

[291][Rie92b].

Chapter 13

[Riecke, 1891] Memorial Speech

Eduard Riecke[292,293,294]

Wilhelm Weber (born October 24, 1804, died June 23, 1891).

Speech given at the public meeting of the *K. Gesellschaft der Wissenschaften* (Royal Society of Sciences) on December 5, 1891.

When we are gathered today to honor the memory of Wilhelm Weber, who was part of our Society for six decades, we feel that with him, a time has come that will probably not appear a second time for our Society and our University. Because of that name Weber, we think of the man who brought his younger comrade to Göttingen, of Gauss,[295] who was a true king, who cultivated the areas of mathematics, astronomy, and physics in such a way that even today the Kärrners were unable to clean up the stones he had broken. We think of Wöhler,[296] who first composed an animal substance from inorganic substances and cleared the way for the development of physiological chemistry. The light emanating from these names will still remain on our University in the most distant times. They place Göttingen in the first row of the locations in which the development of natural sciences began in our century. The privileged position we enjoyed has fallen victim to higher goals; but the thought of the past will remain alive and the contact with the ground on which we stand will steel us to the extent of the power given to us to work on the promotion of science. In this sense, I would like to speak of Wilhelm Weber, a man to whom the entire scientific world commanded the admiring veneration of everyone who approached him and a deep affection.

Wilhelm Weber was born in Wittenberg on October 24, 1804, the son of the local professor

[292] [Rie92b].

[293] Translated by Mathias Hüfner, Email: mathias.huefner@t-online.de, homepage: http://mugglebibliothek.de/. Edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

[294] The Notes by Mathias Hüfner are represented by [Note by MH:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[295] [Note by AKTA:] Carl Friedrich Gauss (1777-1855).

[296] [Note by AKTA:] Friedrich Wöhler (1800-1882) was a German chemist. See [WW41c] with English translation in [WW21], [WW41a] and [WW41b].

of theology, Michael Weber.[297] He was the fifth of seven growing siblings. His childhood was a time of deepest humiliation for our fatherland, he saw his hometown in the hands of the French and experienced its siege by General von Bülow's Prussian army corps.[298] The bullets thrown into the city ignited a fire. His father's house was also robbed. The Weber family fled to the neighboring town of Schmiedeberg, and there, the thunder of the guns from the Battle of Leipzig reached the boy's ears. In 1815, the Wittenberg University was united with Halle, and the Weber family settled in the latter. Wilhelm Weber attended the orphanage's teaching facilities there, and later the University. Nothing is known about the influence of his teachers on his development. He mentions that only a few lectures were given in Halle, which would have been important to him. The case that in a theological house, three brothers devoting themselves to the study of natural sciences was probably a rare occurrence, and the question is how the scientific inclinations came into the house, in which the father was essentially devoted to theological and philological interests. First of all, it should be noted that the Weber family lived in Wittenberg in the house of a friend of theirs, Langguth,[299] a professor of natural science, whose scientific collections had a certain fame at the time. In addition, Chladni, the discoverer of sound figures,[300] the first researcher of meteoric masses that had fallen to earth, lived in the same house as a childhood friend of the owner of the house. He belonged to the very active circle in Wittenberg, which his lively and talented mother in particular knew how to bind to the Weber house. We can probably assume that Chladni, who was also a welcome guest at the Weber house in Halle, first aroused the desire for physical experiments in his older brother Ernst Heinrich.[301] However, he recognized early on the unusual talent of his brother Wilhelm, who was ten years his junior, and, as Weber himself reported, was until he received his doctorate almost his only teacher in the field of natural sciences. Therefore, throughout his entire life, Wilhelm Weber felt for him not only the deep love of his brother, but also a sense of piety that was directed towards the teacher and the almost fatherly friend. During Wilhelm's last years of study, the two brothers were engaged in experimental investigations, the results of which were published in the work "die Wellenlehre auf Experimente gegründet" (The Wave Theory Based on Experiments).[302] In 1826, Weber earned his doctorate with a dissertation "Ueber die Wirksamkeit der Zungen in den Orgelpfeifen" (On the effectiveness of reeds in organ pipes). The following year, he completed his habilitation in Halle with a paper "Ueber die Gesetze der Schingungen zweier Körper, welche so mit einander verbunden sind, dass sie nur gleichzeitig und gleichmässig schwingen können" (On the laws of vibrations of two bodies which are so connected that they can only vibrate simultaneously and evenly). An extraordinary professorship in Halle was awarded to him in 1828. In the autumn of that year he set off on foot from Halle to attend the natural scientists' meeting in Berlin because the first salary that the young professor had received was just enough to cover the contribution for the widow's fund and, in any case, the Weber family was used to making sacrifices, because their fortune had been lost in the storms of the war. His stay in Berlin became crucial for Weber because there he attracted the attention of Gauss with a well-organized and well-delivered lecture on the compensation

[297][Note by AKTA:] Michael Weber (1754-1833).

[298][Note by AKTA:] Friedrich Wilhelm Freiherr von Büllow (1755-1816) was a Prussian general of the Napoleonic wars.

[299][Note by AKTA:] Christian August Langguth (1754-1814) was a German physician and physicist.

[300][Note by AKTA:] Ernst Chladni (1756-1827) was a German physicist and musician. He is specially known for the so-called Chladni figures or Chladni patterns, that is, the modes of vibration on a rigid surface.

[301][Note by AKTA:] Ernst Heinrich Weber (1795-1878) was one of the founders of experimental physiology.

[302][Note by AKTA:] [WW25].

of organ pipes. When the full professorship of physics in Göttingen was finished due to the death of Tobias Mayer in 1830,[303] Gauss suggested him alongside Bohnenberger and Gerling to fill the chair,[304] emphasizing in particular the greater genius in the work to be expected for the Royal Society of Sciences as an important factor in Weber's favor. In 1837, Wilhelm Weber was removed from office as one of the Göttingen Seven.[305] Gauss and Alexander von Humboldt[306] tried to bring about his rehabilitation in Göttingen. However, the steps taken with this intention failed because Weber declared that he did not want to separate his fate in this matter from that of his comrades. But Weber was not exiled, and the salary he received from the Association founded to support the seven enabled him, who was always satisfied with little, to initially stay in Göttingen as a private citizen. However, he later refunded the transferred sums and handed them over to the Saxon Society of Sciences as a foundation for scientific purposes. What tied him to Göttingen was the desire to stay close to Gauss, and this led him to reject a professorship offered to him at the Polytechnic School in Dresden in 1841. But the following year, he was appointed to Leipzig in Fechner's place,[307] who had resigned from the professorship of physics due to severe suffering, and this time, he followed the call because he no longer wanted to be the only one, who accepted continued payment of his previous salary from the Leipzig Association. He also found himself in Leipzig reunited with his brothers Ernst Heinrich and Eduard,[308] who were so closely associated with him, and in living with them, he found a substitute for his dealings with Gauss. But when the turn of the times brought the callback of the expelled professors to Göttingen, he did not hesitate to break the bonds that bound him in Leipzig and returned to the old chair.

Let us now try to get an idea of Wilhelm Weber's scientific achievements. We begin with the already-mentioned investigation into the wave movement. The reason for this was a coincidence: one day, one of the two brothers poured mercury to clean it through a paper funnel from one bottle to the other. He observed on the surface of the mercury in this second bottle highly regular but intricate figures that were created by the inflow of the mercury, and he recognized these as an action of the waves regularly crossing each other at the same places. At the time when the Weber brothers began their investigations, wave theory had gained outstanding importance through the knowledge that the phenomena of light are based on wave movements in an elastic material, the ether, which permeates the entire space. A finely worked out theory of the waves traveling in such a medium had developed, which was in complete agreement with the phenomena of optics. In contrast to this, little was known about the waves that we create on the surface of a pond by throwing a stone into it, and the knowledge of the waves propagating in the air on which the sensations of sound and tones are based was incomplete in many respects. Filling in these gaps, the aim of the work undertaken by the Weber brothers was to give experimental research one lead over theory. The "Wellenlehre auf Experimente gegründet" (The Wave Theory Based on Experiments) will always remain one of the fundamental works of physical research, distinguished by a

[303][Note by AKTA:] Johann Tobias Mayer (1752-1830) was a German physicist.

[304][Note by AKTA:] Johann Gottlieb Friedrich von Bohnenberger (1765-1831) was a German astronomer, mathematician and physicist. Christian Ludwig Gerling (1788-1864) was a German mathematician who studied under Gauss.

[305][Note by AKTA:] See https://en.wikipedia.org/wiki/G%C3%B6ttingen_Seven.

[306][Note by AKTA and MH:] Alexander von Humboldt (1769-1859) was a German natural scientist and explorer, universal genius and cosmopolitan, scholar and patron.

[307][Note by AKTA:] Gustav Fechner (1801-1887) was a German physicist, philosopher, and experimental psychologist. See [Fec45] with English translation in [Fec21].

[308][Note by AKTA:] Eduard Friedrich Weber (1806-1871) was a German anatomist and physiologist.

wealth of the finest and most peculiar observations from classical science. Simplicity of the experimental tools, the ingenious and exact methods of measurement, as well as the attractive presentation through which the reader is drawn into a lively interest in the work of the two researchers. We see the brothers at their wave trough,[309] one of them letting the column of liquid sucked up in a glass tube fall back into the trough and thus creates the wave, while the other uses the watch to determine the speed of its progress; how they trace the image of the wave on a slate quickly dipped into the channel and use a microscope to follow the paths in which the particles suspended in the water move up and down, back and forth. The authors have also collected with great care the facts which relate to the calming of the waves by a thin layer of oil spread on the surface of the water and, through their observations, have amplified them. In the interest of shipping, they call for a repetition of the experiments on a larger scale that Franklin had unsuccessfully undertaken to moderate the surf of the sea.[310] They have significantly expanded our knowledge of the propagation of one liquid on the surface of another.

Through the perceptions made during the elaboration of the *Wellenlehre* (Wave Theory), Weber was led to a problem on which he wrote his dissertation, his habilitation thesis and a number of essays in the *Annalen der Physik*. The tone produced by a vibrating body, such as a violin string or an organ pipe, is under certain circumstances an extremely fine indicator of its physical properties. Strings are detuned by heating or cooling due to changed moisture conditions, and one can conclude from the changes in the tone the changes that have occurred in those external conditions. However, as often as you want to use the pitch in order to draw a conclusion about the nature of a body, you have to be able to compare the tone it produces with an absolutely unchangeable normal tone. But it is by no means easy to produce a body whose tone always remains at the same unchanging pitch. On closer examination, the tone of a tuning fork appears to be slightly lower if the fork is struck strongly, and slightly higher if it is struck lightly. Conversely, the tone of an organ pipe is higher if it is struck strongly and deeper if you blow on it weakly. Weber used this peculiar relationship to construct an instrument that produces the same tone when excited weakly and strongly. The same consists of the combination of a vibrating metal plate or reed with an organ pipe; neither the reed nor the organ pipe can do this. To carry out the oscillation that would be natural for each of them individually, one of the two bodies vibrating with each other must adapt its oscillations to those of the other, so that both oscillate at the same time. Weber now sets things up so that the tone of the pipe is increased by the resonating plate by just as much as, conversely, the tone of the plate is deepened by the resonating column of air. This relationship remains even if the reed and pipe are set into vibrations of greater width by blowing more strongly. The sound produced by such a "compensated pipe" retains its height regardless of the strength of the excitation.

If the *Wellenlehre* (Wave Theory) forms a monument to the intimate intellectual community that Wilhelm Weber shared with the older brother Ernst Heinrich, a similar relationship with the younger brother Eduard arose from the "Mechanik der menschlichen Gehwerkzeuge" (Mechanics of the Human Walking Apparatus),[311] in which the methods of physical research were applied to a physiological problem in an exemplary manner. The authors describe the appeal of the joint work in the Preface with the following characteristic words:

[309][Note by AKTA:] In German: *Wellenrinne*. This expression can be translated as wave trough, ripple tank or wave tank.

[310][Note by AKTA:] Benjamin Franklin (1706-1790).

[311][Note by AKTA:] [WW36] with English translation in [WW92].

> But even if we are convinced that the choice of our object does not require any defense, we do not want to hide the true reason that drove us to persistently pursue this object for a long time with combined forces. It was the joy that we found in a common activity, in an activity to which each of us brought our strengths and resources and which the other estimated and valued all the more highly because he lacked them. Humans are never more capable or more persistent in scientific research as with such mutual participation and stimulation that does not only take place after the work has been completed but throughout its entire course.

The *Mechanik der Gehwerkzeuge* (Mechanics of the Human Walking Apparatus) already belongs to Weber's first Göttingen period, but his scientific activity in this period was determined by his close relationships with Gauss. Above all, it was the profit that he expected from this that made the Göttingen professorship so desirable to him. Gauss had devised a general theory of earth magnetism,[312] through which the secure ground was prepared for all work aimed at researching this enigmatic force. With Weber, he won a comrade in following the newly opened track, who knew how to take up the given suggestion and develop it further in an independent and significant way. Weber played an outstanding part in the establishment of the Magnetic Society, which brought together a large number of observers scattered over a wide area for joint, systematic work according to a common plan, in the construction of instruments for measuring magnetic forces, in the development of new methods of observation, in the editing of the journal published by the Society, and the summarizing of the results of the Society's observations.[313] We also owe him an atlas of terrestrial magnetism, which illustrates the conclusions flowing from Gauss' general theory by means of a large number of magnetic maps.[314]

We owe a device to the joint investigations of Gauss and Weber that was destined to make an epoch in the history of telegraphy. It consisted of a galvanic circuit between the Astronomical Observatory and the Physical Cabinet with wires in the air over the houses up to the St. John's Tower and so pulled down again. The entire wire length was 8000′. At both ends, it was connected to multiplier wires,[315] which were led around one-pound magnetic rods suspended according to Gauss' arrangement. This magnificent device, the practical implementation of which is Weber's merit, was used for galvanic investigations, but it also proved very directly the feasibility of an electromagnetic telegraph and in fact provided convenient telegraphic communication for years, which was of great use for corresponding measurements at the Astronomical Observatory and the Physical Institute. Thanks to the device manufactured by Gauss and Weber, the problem of electrical telegraphy was solved for the first time safely and satisfactorily. The two researchers recognized without doubt that their invention contained the germ of a development that, as Gauss put it, almost frightened the imagination. But given the meager endowment of their Institutes, they were content with it only to satisfy their special purposes. They left the further exploitation of the idea for world traffic to others, and so it was from Göttingen that Steinheil received the inspiration for the work through which he so greatly influenced the development of electrical telegraphy.[316] Naturally, the popular appreciation and the bright sound that Weber's name enjoys in wide

[312] [Note by AKTA:] See footnote 55 on page 34.
[313] [Note by AKTA:] [GW37], [GW38], [GW39], [GW40b], [GW41] and [GW43].
[314] [Note by AKTA:] [GW40a].
[315] [Note by AKTA:] See footnote 52 on page 33.
[316] [Note by AKTA:] Carl August von Steinheil (1801-1870) was a German physicist, inventor, engineer and astronomer. See [Pri83].

circles are associated with the invention of the telegraph, as Weber was the only survivor of that memorable time. As much as we value the merit that lies in the first successful implementation of an idea that several outstanding physicists had tried in vain to realize, the invention of the telegraph is not Weber's most unique work. Rather, the information received from those times suggests that the original moving ideas are to be found on Gauss' side, while the credit for the practical implementation goes mainly to Weber.

When setting up the telegraph, Weber and Gauss made an ingenious application of the laws of magnetic induction which Faraday had recently discovered.[317] Weber's attention was thus turned to the discoveries of the great British researcher, and we find evidence of a sustained preoccupation with the new phenomena in several treatises which he has written down in the "Resultaten aus den Beobachtungen des magnetischen Vereins" (Results from the Observations of the Magnetic Association).[318] Among the subjects with which they are concerned, the use of the currents induced by the earth's magnetism for measuring inclination should be emphasized. The earth inductor constructed for this purpose later became of fundamental importance for absolute resistance measurements. Weber also applied the principle of determining the elements of the earth's magnetism through galvanic observations to the measurement of the horizontal intensity. From the magnetic work, which had formed the main subject of his activity since his employment in Göttingen, Weber was immediately led over to the field in which his genius was to develop in the freest and most original way, electrodynamics.

With his move to Leipzig began the series of treatises on *elektrodynamische Maassbestimmungen* (Electrodynamic Measurements), which are the main work of his life and a classic monument to him for all time.[319] Insofar as a theory is developed in these works which covers the entire area of electrical phenomena known at the time, they represent the completion of a great scientific development that goes back in its beginnings to Newton.[320] If we want to understand the meaning of Weber's electrodynamic theory in this context, we must first remind ourselves of the essential features of the earlier development.

Kepler had already conceived the notion that the planets were kept in their orbit by some force exerted by the sun.[321] He compared it to the attraction of a magnet to iron. He suspected that it diminished with distance, just like the effects of light. However, there

[317][Note by AKTA:] See footnote 43 on page 28.

[318][Note by AKTA:] See footnote 313.

[319][Note by AKTA:] Weber wrote eight major Memoirs between 1846 and 1878 under the general title *Elektrodynamische Maassbestimmungen* (Electrodynamic Measurements), [Web46], [Web52b], [Web52a], [KW57], [Web64], [Web71], [Web78] and [Web94b]. The eighth Memoir was published only posthumously in his complete works. These 8 major Memoirs and other of his main works on electrodynamics have now been fully translated and commented into English in the 4 volumes of the book *Wilhelm Weber's Main Works on Electrodynamics Translated into English*. Volume 1: Gauss and Weber's Absolute System of Units, [Ass21j], Volume 2: Weber's Fundamental Force and the Unification of the Laws of Coulomb, Ampère and Faraday, [Ass21k], Volume 3: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity, [Ass21l], and Volume 4: Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation, [Ass21m]. Included in these four volumes are also English translations of 5 papers by Gauss, translations of part of the correspondence between Gauss and Weber, 1 paper by Weber and Wöhler, 2 papers by Weber and Rudolf Kohlrausch (1809-1858), 1 paper by Fechner, 1 paper by Johann Christian Poggendorff (1796-1877), 1 paper by François Felix Tisserand (1845-1896), 2 papers by Carl Neumann (1832-1925), and 3 papers by Gustav Kirchhoff (1824-1887) related to Weber's electrodynamics.

[320][Note by AKTA:] See footnote 11 on page 17.

[321][Note by AKTA:] Johannes Kepler (1571-1630) was a German astronomer, mathematician, astrologer and natural philosopher.

was still a long way to go from such vague assumptions to Newton's theory of gravitation. First, a theory of motion and then a mathematical method had to be created to determine the resulting motion from the small changes that a given speed undergoes in a large number of successive time units. The creation of dynamics was the work of Galileo.[322] We owe the method of fluxions or differential calculus to Newton and Leibniz.[323] But then Newton achieved a big success. In a strict mathematical conclusion, he developed Kepler's laws from the assumption that the sun exerts a force on the planets that is inversely proportional to the square of the distance. He showed that this force is identical to the gravity that causes a stone to fall on the surface of the Earth. So Newton became the founder of the mechanics of the sky, which is still used today as the unrivaled model of mathematical physics. The same does not just reproduce the broad features of the phenomena, it rather follows the facts down to the finest details and every advance in observation is always just a new test for the perfection of the theory. The basis of Newton's theory, however, was formed by an assumption that was extremely strange to his contemporaries who were caught up in the Cartesian view,[324] which Newton himself seemed to consider to be little more than a mathematical fiction, but which his students soon turned into an unassailable dogma, the assumption of an immediate action at a distance between the bodies of the universe, as well as between the earth and the bodies on it, or finally [between] these latter themselves.[325]

The question of the nature of the actions that we observe in the physical world, whether direct action at a distance or mediation through pressure and impact, is now closely related to a conflict of views on the nature of matter, which we trace through the history of physics back to Democritus and Aristotle.[326] One view assumes that matter constantly fills space, while the other view is that matter is composed of small particles, molecules and atoms, and imagine these separated from each other by empty spaces. One can see how much the idea of an immediate action at a distance had to come to the aid of atomism, and one will therefore not be surprised if the French physicists, in particular at the end of the last century and the beginning of this century, combined the atomistic view with the idea of action at a distance to gain a path into the area of molecular phenomena. Laplace[327] had already remarked that a ponderable body could be compared with a nebula, which offers the appearance of a uniformly glowing disk in the night sky. Like a uniformly luminous disk consisting of an innumerable number of stars, between which wide spaces empty of stars extend, you can imagine ponderable bodies made up of molecules separated from each other by gaps, in comparison with which the dimensions of the molecules themselves disappear; and, just as the stars of a nebula attract each other with Newton's force, this would also be the case with the molecules of a body. But such an assumption is not suitable to explain the phenomena of elasticity or capillarity; rather, in the atoms of a body, Newton's attraction must be supplemented by other forces which have the property of only having a noticeable strength at very small distances, disappearing at larger distances. The introduction of these

[322][Note by AKTA:] Galileo Galilei (1564-1642) was an Italian astronomer, physicist and engineer. See [Gal14] with Portuguese translation in [Gal85].

[323][Note by AKTA:] Gottfried Wilhelm Leibniz (1646-1716) was a German mathematician, philosopher, scientist and diplomat.

[324][Note by AKTA:] René Descartes (1596-1650) was a French philosopher, scientist and mathematician.

[325][Note by AKTA:] For a discussion of Newton's points of view as regards action at a distance see, for instance, [Hen11], [Hen14], [Hen19] and [Hen20].

[326][Note by AKTA:] Democritus (around 460-370 BC) was a Greek philosopher. Aristotle (384-322 BC) was a Greek philosopher and polymath.

[327][Note by AKTA:] Pierre-Simon Laplace (1749-1827) was a French scientist and polymath.

so-called molecular forces led to a theory that was in agreement with the phenomena of elasticity and capillarity, but which celebrated its greatest triumphs in the wave theory of light. The idea that the aether has the properties of a solid elastic body with respect to the oscillations of light had already been developed by Fresnel in order to justify the possibility of transverse oscillations.[328] With such successes, the molecular theory had to become even more dominant in physics because, on the other hand, chemistry had also come to the assumption that bodies were made up of atoms or atom complexes, the molecules.

However, a question of fundamental importance was left open by the molecular theory or at least only touched superficially: the question of the stability of the assumed molecular systems. At the start, we compared such a system to a star cluster. But the fact that the similarity is not very extensive becomes clear when we look at our planetary system instead of a star cluster. As a result of the attractions that the planets exert on each other, their orbits continually deviate from Kepler's ellipses. However, the conditions of the system are such that the disruptions never add up to large amounts. The orbits actually traversed by the planets only carry out small oscillations around an unchanging position. The planetary system is stable as far as the orbits in which the individual bodies move are concerned. However, the systems' configuration is subject to the greatest changes as a result of these very movements. Similarly, the stability of a star cluster can only be that of movement. In contrast, molecular theory assumes that the individual molecules of a solid body are in stable equilibrium at certain points under the influence of mutually exerted forces and that the configuration of the system is completely determined and unchangeable as long as no external forces are acting on the body. It was noted that such an assumption only appears possible if the forces acting between the molecules contain both attractive and repulsive components. However, a real development of the stability conditions and a more precise formulation of the force law have not been attempted. For now, this assumption is only justified by the success with which it was introduced.

We can see how difficult it was for the idea of immediate action at a distance to gain more general significance despite the great success of Newton's theory of attraction from the fact that it was only around the year 1760 that action at a distance forces were introduced into the theory of frictional electricity and magnetism. But at the same time Euler,[329] an opponent of action at a distance, explained the electrical attractions and repulsions through changing pressure conditions in the air and developed a theory for the magnetic effects which is not too far removed from the views later developed by Faraday. Coulomb's measurements initially decided the alternative in favor of action at a distance.[330] To explain the electrical phenomena, he assumed the existence of two fluids corresponding to the electricity of glass and resin.[331] Particles of the same fluid repel each other. Particles of different fluids attract each other with a force that, like gravity, is inversely proportional to the square of the distance. A corresponding assumption was then transferred to the theory of magnetism and was confirmed here by the measurements of Gauss.

[328][Note by AKTA:] Augustin-Jean Fresnel (1788-1827) was a French engineer and physicist. See Section 5.2 (Fresnel's Contributions) of [AC11] and [AC15].

[329][Note by AKTA:] Leonhard Euler (1707-1783) was a Swiss mathematician, astronomer and physicist.

[330][Note by AKTA:] See footnote 288 on page 172.

[331][Note by AKTA:] Vitreous electricity is nowadays called positive electricity, while resinous electricity is called negative electricity. See Chapter 5 (Positive and Negative Charges) of [Ass10a] with Portuguese translation in [Ass10b], Russian translation in [Ass15b] and Italian translation in [Ass17a]. See also Section 4.3 (Differences between Old and Modern Glasses) of [Ass18a] with Portuguese translation in [Ass18b] and Russian translation in [Ass19b].

At the beginning of our century, the field of magnetic and electrical phenomena experienced a tremendous expansion with the discovery of electromagnetism by Oersted,[332] the interaction of galvanic currents by Ampère,[333] and induction by Faraday. All of these phenomena are effects of electricity, which is in the state of galvanic current in wires. This means that the laws that Biot, Savart and Ampère established for the discovered actions have a substantially different character than the earlier laws of action at a distance.[334] In the electromagnetic interaction, Oersted determined the law of action of a very short straight piece of wire, which is the carrier of the galvanic current, on a magnetic pole. Ampère's law determines the interaction between two such pieces of wire. So, unlike Newton's or Coulomb's law, it is not about the interaction of point masses or centers of force, but about the interactions between points and line elements, and the interactions of line elements among themselves.[335] We call laws that relate to such effects elementary laws in contrast to Newton's point law.[336] However, the fact that an element of a galvanic current cannot exist on its own is particularly noteworthy; it is only conceivable as part of a larger circle, of the closing arc of a galvanic voltaic cell[337] or a discharging Leyden jar.[338] This remark leads to the question of whether it is not possible to reduce these elementary laws to simpler actions. But if one further sees the basis of the electrical phenomena in the existence of the electrical fluids, then there can be no doubt that the same particles, which in the state of rest attract or repel each other according to Coulomb's law, in the state of a galvanic flow must give rise to the effects discovered by Ampère. This creates the task that Ampère himself had already set: to investigate how the electrostatic action at a distance of the particles could be modified through movement in such a way that Ampère's law results as a result of the various actions. This task is what Weber solved in the First Treatise on electrodynamic measurements.[339] The fact that his intention from the outset was not just aimed at theoretical speculations but also directly at fundamental tasks of measuring physics can be seen from the following words with which the mathematical part of the investigation is introduced:[340]

> Thus, if we take up the connection between the *electrostatic* and *electrodynamic* phenomena, we need not simply be led by its general scientific interest to delve into the existing relations between the various branches of physics, but over and above this, we can set ourselves a more closely defined goal, which has to do with the *measurement of Volta-induction*[341] *by means of a more general law of pure electrical*

[332][Note by AKTA:] Hans Christian Ørsted (1777-1851) was a Danish physicist and chemist. See [Oer20b] with English translations in [Oer20c], [Oer65] and [Ørs98]; French translation in [Oer20a]; German translation in [Oer20d] and Portuguese translation in [Ørs86]. See also [Fra81], [Mar86] and [Rei13].

[333][Note by AKTA:] See footnote 15 on page 18.

[334][Note by AKTA:] Jean-Baptiste Biot (1774-1862) and Félix Savart (1791-1841) were French physicists, astronomers and mathematicians. See [BS20] with English translation in [BS65b] and Portuguese translation in [AC06]. See also [BS24] with English translation in [BS65a].

[335][Note by AKTA:] The interaction between a point and a line element would represent the interaction between a magnetic pole and a current element. The interaction between line elements would represent the interaction between current elements.

[336][Note by AKTA:] See footnote 210 on page 129.

[337][Note by AKTA:] In German: *einer galvanischen Säule*.

[338][Note by AKTA:] See Chapter 12 (The Leyden Jar and Capacitors) of [Ass18b] with English translation in [Ass18a] and Russian translation in [Ass19b].

[339][Note by AKTA:] [Web46] with partial French translation in [Web87] and a complete and commented English translation in [Web21d].

[340][Note by AKTA:] [Web46, p. 134 of Weber's *Werke*] and [Web21d, p. 130].

[341][Note by AKTA:] See footnote 118 on page 63.

theory. These measurements of Volta-induction then belong to the *electrodynamic measurements* which form the main topic of this Treatise, and which, when they are complete, must also include the phenomena of *Volta-induction*. It is self-evident, however, that establishing such measurements is most profoundly connected with establishing the *laws*, to which the phenomena in question are subject, so that the one can not be separated from the other.

But if the general theory of electrical phenomena were to be based on the foundation of Ampère's law, it seemed necessary first of all to subject this itself to a new test through exact measurements. Weber carried out this test with the electrodynamometer he designed, which has since become an important measuring device in electricity theory. If he stated that the observations were in perfect agreement with Ampère's law, if he regarded it as the precise expression for a very extensive class of facts, he overlooked a circumstance whose significance was only recognized later. The object of observation is always only the action of closed circuits. However, how the overall action is distributed among the individual current elements is, to a certain extent, arbitrary, and this arbitrariness causes Ampère's law to appear as a possible, but not the only possible, expression of the electrodynamic interaction. By accepting the law, Weber introduced a somewhat hypothetical element into his theory. But he then brilliantly solved the task of uncovering the connection between the electrostatic and electrodynamic fundamental law through the law named after him, which determines the force acting between two electrical particles not only from their masses and their distance but also from their dependence on relative movement. With the establishment of this law, Weber had reached a point of view from which a uniform representation of electrical phenomena seemed possible. In the entire series of later treatises, he pursued his goal with great consistency and to an ever-increasing extent. He included the phenomena of magnetism in the circle of his views, and his last works also tried to connect gravitation and molecular actions with the law of electrical force. But true to the program set out from the beginning, his scientific activity was always twofold. Hand in hand with theoretical speculation came the electrical measurements, which have become of fundamental importance for the practical and technical side of physics.

First and foremost, Weber's law had to apply to the phenomena of voltaic induction discovered by Faraday, [namely,] to the creation of a current in a conductor wire when approaching an existing [current-carrying] circuit, [and also] the creation of a current when the current strength in the neighboring circuit changes. In fact, in both cases, forces are exerted on the neutral electricity resting inside the conductors, which drive the positive particles in one direction and the negative particles in the opposite direction. These forces will therefore not attempt to displace the conductors themselves, as is the case with the action discovered by Ampère. In fact, they only seek to move the electricity contained in the conductors, and we therefore call them electromotive. The application of Weber's law to the cases mentioned leads to elementary laws of voltaic induction, which are confirmed by observations of closed current and conductor circuits. The extension of the laws found to the phenomena of magnetic induction is made possible by the remark that the inducing action of galvanic spirals is subject to the same law as that of a magnetic rod.

While in the First Treatise on Electrodynamic Measurements the interest concentrates primarily on the development of the general basic law, in the Second Treatise[342] the practical side of the task comes to the fore and from this point of view, it has just as fundamental

[342][Note by AKTA:] [Web52b] with English translation in [Web21e].

significance, as the First for the development of the theory. Through the double interrelationship that exists between electricity and magnetism, through the peculiar distinction between electrostatic and electrodynamic phenomena, the number of quantities that form the object of observation and measurement is multiplied. The need therefore, becomes all the more urgent to have certain definitions for these quantities, a uniform system of units and convenient and precise methods of measurement. When founding his system of units, Weber started from an idea of great importance, which was first introduced into science by Gauss in his Treatise on the absolute measure of the Earth's magnetic force.[343] We want to imagine that an arbitrary agent is distributed in equal quantities between two identical ponderable bodies and that the result of this is a mechanical interaction, an attraction, repulsion, or rotation of the two bodies. The strength of the force exerted can be determined according to the general units of mechanics, for example, by weighing. This can only depend on the spatial conditions and the number of agents. If the dependence on the lines and angles to be measured is known, a measure for the quantity of the agent in question results, namely one that only requires the establishment of the units of measurement for lines,[344] time periods, and masses. In this sense, Gauss taught how to determine in absolute units the amount of magnetism contained in a steel rod. In the same sense, the amount of electricity imparted to two charged conductor balls can be calculated in absolute electrostatic units from the repulsion between them. When the principle is applied to galvanic currents, however, the peculiar circumstance arises that the strength of a current can be judged just as well by its action on a magnet as by its action on a second current. There are therefore two different absolute units for the strength of the galvanic current, and it makes sense to contrast these two with a third, which is particularly important because it establishes a direct relationship between the electrodynamic and electrostatic measurements. In accordance with the way we measure the strength of a flux, the unit of measure is represented by a galvanic current in which the total amount of electricity flowing through the cross-section of the conducting wire in one second is equal to the electrostatic unit as determined by the repulsion of two charged conductors. Accordingly, one can also now set up three different units for the electromotive forces. One can use the phenomena of magnet or voltaic induction for this purpose. But one can also be guided by the remark that the electromotive force of induction does not differ significantly from the forces of electrostatics, which also seek to cause a separation of the electrical fluids, so that the electromotive force can be expressed just as well as the electrostatic force in terms of the general units of mechanics. Finally, since the resistance of a conductor is equal to the ratio of the electromotive force to the strength of the galvanic current generated, the threefold possibility of determining the units also applies to this [magnitude, that is, to the resistance]. Of particular interest is the relationship between the electromagnetic and electrodynamic units on the one hand, and the electrostatic unit on the other, which is mediated by the so-called constant of Weber's law. According to this, the electrostatic repulsion between two similar particles is reduced by their movement and the constant mentioned indicates the relative speed at which the two particles no longer have any action on each other. At the same time, however, it also provides the factor by which one must multiply an electromagnetically measured current intensity to express it in mechanical units, i.e. in order to obtain the number of electrostatic units which the current carries through the cross-section of the conductor in one second. Weber carried out

[343][Note by AKTA:] See footnote 55 on page 34.

[344][Note by AKTA:] That is, measurement of distances.

the experimental determination of his constants in collaboration with Robert Kohlrausch.[345] The ratio of the electromagnetic unit of current to the electrostatic was 3.111×10^{10} cm per second, while according to the latest measurements, it is 3.012×10^{10} cm per second. Both values can be considered equal to the speed of light. With the determination of Weber's constant, the electrical system of units has reached its internal conclusion. The reason for this is that Weber has exerted a decisive influence on science to the greatest extent. In the present period of development, one will be inclined to look for the basis of his fame primarily in the works belonging to this subject. This is partly due to the ever-growing importance of electricity for technology and transport. Accurate measurements were not only a need of science but also of technology, and Weber had satisfied this need in advance. If the worker in an electrotechnical factory now operates with his [units of measure] Ampères, Volts, and Ohms in complete safety, then Weber deserves the first and foremost credit for this, and in this context, one should not hold back the regret that the Electrotechnical Congress in Paris suppressed the name Weber from the popular designation of electrical units.[346]

Let us return back to Weber's work, which is important for the further development of his theory. Ampère had already shown that the assumption of special magnetic fluids is superfluous, that the phenomena of magnetism are completely explained if, under the assumption of a molecular constitution of iron, one imagines each molecule surrounded by a ring-shaped galvanic current.[347] In a non-magnetic piece of iron, these so-called Ampère molecular currents will have all possible orientations. In a magnetic field, they are rotated consistently by the electromagnetic action and then exert the same external actions which, according to the earlier view, were explained by the separation of magnetic fluids. Based on very attractive considerations, Weber also included in this theory the diamagnetic repulsions discovered by Faraday, which many bodies experience in the vicinity of a magnetic pole.[348] If the molecules of a body are surrounded by paths in which the electrical fluids move without electromotive force, i.e. without resistance, then induction currents must be able to occur in these paths. These must persist until they are destroyed by an opposing cause. But the currents that are induced by approaching a magnetic pole have such a direction that they produce a repulsion between the pole and the approaching body, which would be the same repulsion discovered by Faraday. With this theory, Weber believed that he had decided the alternative between the assumption of separable magnetic fluids and Ampère molecular currents in favor of the latter. However, all phenomena of diamagnetism can also be explained by the assumption that the air and the ether filling the so-called empty space are capable

[345][Note by AKTA:] Although Riecke wrote "Robert Kohlrausch", the correct name is Rudolf Kohlrausch (1809-1858). This constant c would represent the uniform relative velocity at which Weber's force between the two particles would fall to zero. Weber's c (known throughout the 19th century as the *Weber constant*) is not the same as the modern light velocity in vacuum $v_L = 2.998 \times 10^8\ m/s$, but $\sqrt{2}$ times this last value (or, $c = \sqrt{2} \cdot v_L = 4.24 \times 10^8\ m/s$). The *Weber constant*, c, was first measured by Weber and Kohlrausch in 1854-1856. They obtained $c = 4.39 \times 10^8\ m/s$. See [Web55] with English translation at [Web21i]; [WK56] with English translations in [WK03] and [WK21], and Portuguese translation in [WK08]; and [KW57] with English translation in [KW21]. See also [Pog57] with English translation in [Pog21], and [Ass21h].

[346][Note by AKTA:] The main responsible for suppressing the name *Weber* in the popular designation of electrical units of measure in the 1881 Electrotechnical Congress in Paris were two of Weber's main opponents, namely, William Thomson (1824-1907), also known as Lord Kelvin, and Hermann von Helmholtz (1821-1894), as will be shown in Chapter 23. It was their suggestion to replace the term *Weber* by *Ampère* as the unit of electric current.

[347][Note by AKTA:] See Chapter 19 (The magnetic poles and dipoles are disposable hypotheses) of [AC11] and [AC15].

[348][Note by AKTA:] [Web52a] with English translation in [Web21h].

of magnetic polarization and to a higher degree than the so-called diamagnetic bodies. The existence of the Ampère molecular currents cannot therefore be claimed based on Weber's investigation. But we must not leave it without remembering that in it, for the first time, the magnetic excitation of bismuth in the interior of a galvanic spiral, the induction by the movement of a diamagnet, was not only demonstrated but also precisely measured and the ratio between the diamagnetic excitability of bismuth and the magnetic excitability of iron has been determined.

Weber's investigations, which we have reported above, are essentially related to the action at a distance of galvanic currents. The theory of the galvanic circuit, which Weber had already discussed in detail in his Treatise on resistance measurements,[349] should be considered as an area of electrodynamics that, in many respects, allows a deeper insight into the nature of electrical phenomena. To address this problem, knowledge of the electromotive forces exerted on the fluid contained in a conductor is not sufficient. This also requires knowledge of the molecular resistances with which the movement of electricity has to contend with inside the conductor. Finally, the inertial mass of the electricity set in motion must be given, if the movement is to be calculated according to the usual principles of mechanics. Kirchhoff was the first to give general equations for the movement of electricity in conductors,[350] assuming the general validity of Ohm's law.[351] To determine the laws of motion for conducting wires based on these equations, he introduced the assumption that every piece of such a wire that could still be considered straight was a million times longer than its thickness. It cannot be judged from the outset to what extent this requirement can be fulfilled in executable experiments and to what extent it is compatible with the general validity of Ohm's law. Only a little later than Kirchhoff, Weber submitted an investigation into the general laws of galvanic flow to the editor of the *Annalen für Physik und Chemie*. However, he withdrew it when he learned of the existence of Kirchhoff's work. Regardless of the general validity of Ohm's law, he then developed the equations of motion of electricity anew by starting from the general approach of mechanics and accordingly assigning to electricity a mass to be determined according to grams.[352] From the theoretical results of Weber's work, two are particularly interesting. He found that wave-like movements of electricity are possible in a linear conductor, similar to the progression of a wave in a tube filled with air. The propagation speed of waves could be expressed by the constant of Weber's law. It turned out that under certain conditions, the propagation velocity is equal to the speed of light. Weber was not inclined to attach any physical meaning to this result. Maxwell, however, based his theory of light on the relationship between Weber's constant and the speed of light when he found that the speed of propagation of electrical oscillations in the air corresponds to the speed of light.[353] A second comment relates to the inertia of electricity and the resulting deviations from Ohm's law. From the theory developed by Weber, it follows that the amplitude of fast electrical oscillations, which are excited in a closed conductor by a periodically changing force, depends on the inertial mass of the electricity. The formulas reveal, at least in principle, the possibility of determining, by measuring the amplitude, the

[349] [Web52b] with English translation in [Web21e].

[350] [Note by AKTA:] Gustav Kirchhoff (1824-1887) was a German physicist. See [Kir49] with English translations in [Kir50] and [Kir21a], and French translation in [Kir54]; [Kir57b] with English translations in [Kir57a] and [Kir21c]; and [Kir57c] with English translation in [GA94] and [Kir21b]. See also [Ass21a].

[351] [Note by AKTA:] See footnote 121 on page 65.

[352] [Note by AKTA:] [Web64] with English translation in [Web21c]. See also [Pog57] with English translation in [Pog21], and [Ass21b].

[353] [Note by AKTA:] James Clerk Maxwell (1831-1879) was a Scottish physicist. See [Max73a] and [Max73b].

ratio in which the quantity of electricity contained in the unit length of the conductor wire stands to the square root of its inertial mass, or [the ration in which] the current strength is to the square root of the kinetic energy of the current. The experimental part of the work, in which Robert Kohlrausch played a significant part, was interrupted right from the start by the latter's illness and death.[354] Hertz[355] later found that the kinetic energy of the electricity in one cubic millimeter of a conductor through which the unity of electromagnetic current flows, i.e., 3×10^{10} electrostatic units (g.cm.sec.) in one second, must be smaller than the living force of one five hundredth milligram, which is moved at the speed of 1 millimeter [per second].[356]

Weber attempted to develop the theory of galvanic flow even more fully in the treatise "Über die Bewegung der Elektricität in Körpern von molekularer Constitution" (on the motion of electricity in bodies of molecular constitution), published in the *Annalen der Physik und Chemie*.[357] In doing so, he replaced the dualistic idea recorded in his earlier work with a unitary one, assuming that the negative electrical particles adhere to the ponderable molecules, that the positive ones are in central motion around the molecules, whereby the Ampère rings then dissolve into systems of electrical satellites. Weber looks for the difference between conductors and insulators in the fact that in the former, the orbits of the positive particles extend into the spheres of attraction of the neighboring molecules, whereby a constant transition from one molecule to the other causes a constant change between central movement and flowing motion. If there is no external force, all directions in space will be equally represented in this flow movement. However, if an electromotive force acts on the conductor, the particles are deflected from the initial direction of movement, and the resulting joint displacement is the galvanic current. The electromotive force performs work, which finds its equivalent in the increased living force of positive electricity. Since, on the other hand, the electrical work is converted into heat according to Joule's law,[358] Weber comes to the conclusion that the heat energy of a body is nothing other than the kinetic energy of the positive electricity in central motion.

During the years in which Weber concentrated his efforts on the works on *elektrodynamischen Maassbestimmungen* — Electrodynamic Measurements, the principle of conservation of energy founded by R. Mayer, Joule, and Helmholtz had achieved its central position in the field of exact natural sciences.[359] No law could be considered permissible that did not correspond to the requirements of the energy principle. Given the peculiar character of Weber's law, it seemed doubtful from the outset whether it fulfilled that condition and whether the foundation of the entire theory was a legitimate one. Weber showed that the law of conservation of force applies to a system of particles that act on each other according to his law, i.e. that the sum of kinetic and potential energy is constant.[360] The difference from the

[354][Note by AKTA:] See footnote 345.

[355][Note by MH and AKTA:] Heinrich Rudolf Hertz (1857-1894) was a German physicist. He discovered electric waves propagating in air, see [Her00].

[356][Note by AKTA:] See footnote 173 on page 108.

[357][Note by AKTA:] [Web75] with English translation in Chapter 11.

[358][Note by AKTA:] See footnote 174 on page 109.

[359][Note by AKTA:] The principle of the conservation of energy by taking into account thermal energy had been established by Julius Robert von Mayer (1814-1878) in 1842 and by James Prescott Joule (1818-1889) in 1843, being also discussed by Hermann von Helmholtz (1821-1894), [May42] with English translation in [May62] and Portuguese translation in [May84]; [Jou43]; and [Hel47] with English translation in [Hel53] and [Hel66]. See also [Mar84] and [Ass21f].

[360][Note by AKTA:] [Web71] with English translations in [Web72] and [Web21g]; and [Web78] with English translation in [Web21f]. See also [Ass21f].

usual form in which the potential energy of a mechanical system appears is that in a system of electrical particles it also depends on the relative velocity. This now imposes a certain limitation on Weber's law. It turns out that its application to the movements of electrically charged bodies leads to worrying consequences if the density of the charge or the size of the bodies exceeds certain limits. A similar difficulty arises when one uses Weber's law to investigate the course of currents that have somehow been excited in a conducting body. Only for thin wires do the conclusions agree with the observed facts. In the case of bodies of larger dimensions, however, the equations of motion of electricity have, in addition to the integrals, which show a faster or slower disappearance of the excited movement, others that represent movements that increase to infinity. Helmholtz, who made these comments, has thus shown that Weber's law leads in certain cases to results that contradict the general views of mechanics. As long as these contradictions cannot be resolved, the law can only be attributed to the meaning of an interpolation formula. Within an area delimited by experience, it leads to correct results. But it cannot be applied beyond the same without coming into conflict with other facts of experience. Nevertheless, it will make a difference whether the conditions under which the law leads to contradictions are merely conceivable or whether they can also be realized experimentally. This point requires further clarification.

No matter how much weight one attaches to the concerns emphasized, Weber's construction still encompassed the entire field of observed facts; with its preliminary works it reached over into the field of molecular phenomena and opened up a view into the distant world of chemical affinities for its builder. One might therefore have expected that the breaches made in individual places in the walls would only serve as an incentive for redoubled work, and efforts would be made diligently to fill the gaps and strengthen the foundations. And if one was of the opinion that physical laws were ultimately nothing more than interpolation formulas that correspond to a given circle of facts, then one could expect that a formula that encompassed such an enormous circle could also be adapted through smaller additions to a somewhat enlarged circle. If this did not happen, if one left Weber's theory to erect a new building on a new foundation, then there are other reasons for this, which are not directed against individual gaps in the theory, but against the entire foundation of the same. And these we shall try to describe in the following, as best as time permits.

First, we have to mention a type of prejudice that relies on no less authority than that of Newton. In fact, Newton described gravitation, which he introduced into science, as merely a mathematical cause; the idea that one body could act on another through empty space without any mediation seemed absurd to him. But he left the question of whether the agent, which acts according to certain laws and creates gravity, is material or spiritual to his readers. Occasionally, he probably expressed the idea that the different tension of the aether filling space drives bodies from denser to less dense places and that gravity is based on this. He probably did not think much of such speculations and was satisfied that gravity exists and that the bodies of the sky and the sea tides move according to his laws.

Newton's vague suggestions gained firmer ground through Faraday, who, not accustomed to the formulaic language of mathematics, was looking for a clear means of being able to represent and understand the interactions of bodies in the areas of electricity and magnetism. Such a means presented itself to him in the lines of force, the system that we can so easily create with a magnet with the help of iron filings. If we look at such a chain connecting two friendly poles, we see that all its links are small magnets, which the opposite poles turn towards each other, which therefore attract each other and try to shorten the chain. If we imagine its ends soldered to the poles to which it connects, it will pull them towards each other, and the movement of the poles, which was otherwise seen as a consequence of their forces acting at a distance, now seems brought about by the tension of the chain. Faraday saw lines of force radiating out from an electrically charged body into the surrounding insulating space. Through processes of a hidden nature, a voltage was created along the lines of force, and this was the cause of the electrical phenomena observed. The wire in which a galvanic current moves is surrounded by ring lines of magnetic force and in these there is a voltage of the same kind as in that generated by a pole. The mutual disturbance of the voltages caused by two currents located next to each other in the same space is the cause of the apparent electrodynamic action at a distance. Faraday was also able to connect the facts of induction with the system of his lines of force by showing that an induced current always arises in a closed circuit when the number of lines of force passing through it changes. However, he found no clear mechanical picture of the relationship between the induced and the inducing circles. The theory developed by Faraday turned on its head the widespread and seemingly self-evident view that the conductors were the actual carriers of electrical forces and that the space surrounding them only played a passive role as long as it was impenetrable to the electrical fluids. According to him, the true cause of electrical actions lies in the insulators. The so-called conductors are incapable of conducting lines of electrical force and are only subject to the voltages of the insulator surrounding them. But this theory was much more than a clever game with possibilities and geometric lines because Faraday had shown that the insulators actually play an essential role in electrical phenomena and that a change in their electrical state really occurs along the lines of force. He had discovered that all bodies are capable of magnetic excitation, that there is actually a polarization of the surrounding space along the magnetic lines of force radiating from a pole. But if the dielectric and diamagnetic states assumed by Faraday have a real existence, then the attempt to consider them as the sole causes of the observed effects is also justified.

Mathematical physics, in particular the theory of potential, also led to views that contradicted the assumption of instantaneous action at a distance, but which were in contact with Faraday's theory in essential points. The safest and simplest means of representing the observed facts was increasingly considered not to be forces emanating from bodies, but rather to differential equations, which were satisfied by the quantities characteristic of the phenomena. But every differential equation can be understood as an instruction to calculate the state of any spatial element from that of a neighboring one. One can actually see from this the similarity of the mathematical concept with Faraday's idea of a dielectric or diamagnetic tension that progresses from element to element.

In mathematical physics, however, a development took place in another direction, moving away from the pursuit of atomistic theories and placing a new method of theoretical research in the foreground. Based on two general theorems of the principles of energy and entropy, it was possible to devise a theory of heat, which brought a wealth of new and surprising enlightenment. The peculiar advantage of this theory seemed to be that it was independent of any particular assumption about the nature of heat and that the change of ideas could not influence its unchanged and general validity. It made sense to apply the method thus provided in other areas and to connect the laws of phenomena not through special hypotheses about the nature of bodies but through those general principles. Thus, the principle of energy in the field of electricity made it possible to develop the laws of ponderomotive and electromotive actions of the galvanic current from one another.

Faraday's ingenious intuition of a physical existence of the lines of force, as fruitful as it had been for his own discoveries, had to stand back against the theory of action at a distance as long as it had not found a mathematical formulation. This was given to it by Maxwell. The battle of the theories was now fought with equal weapons and it initially became apparent that their results as a whole agreed to a surprising extent. However, Maxwell soon made a major and momentous discovery based on his theory by showing that transverse electric and magnetic waves can propagate in an insulator and that their speed of propagation in air space is equal to the speed of light. On this, he based his electromagnetic theory of light, which was confirmed, although not completely, by a series of later observations. Although Helmholtz succeeded in deriving the formulas of Maxwell's theory of light from the laws of electric and magnetic actions at a distance, Maxwell's developments remained simpler and more direct. It was also shown here that Faraday's method is superior to the theory of actions at a distance, when it comes to describing phenomena using differential equations. Maxwell's theory was not only important because it combined the phenomena of light with those of electricity into a unified whole, it also opened a new path for the study of electricity itself. Because if light is based on electrical vibrations, then conversely, electrical vibrations must also have the properties of light. Rays of electrical force must spread through space according to the same laws as light rays. This realization pointed the way to the decision between the theory of action at a distance and Faraday's view. Electrical oscillations occur when opposing electrical charges on two conductors balance each other out in the spark. According to the old theory, such a place is the origin of a double force and an immediate action at a distance, which requires no time to spread and which is to be regarded as the essential cause of the phenomena. In addition, there is a secondary effect as a result of the electrical and magnetic polarization of the surrounding air space, and this occurs at the speed of light from the spark gap. According to Maxwell's theory, the rays of electrical force that obey the laws of light are the only thing present. All effects produced by the spark gap are mediated by waves that travel through space at the speed of light. Now Hertz has shown through his work, which has developed so brilliantly from inconspicuous and painstaking beginnings, that from a spark gap effects spread at a finite speed, that their straight path is reflected and refracted through the intermediate media in the same way as the rays of light, and that the facts he observed, nowhere make it necessary to assume that, in addition to the mediated actions, there is also an immediate action at a distance of the spark gap. In accordance with Newton's principle, that in order to explain phenomena one should not allow more causes than are true and are sufficient to explain those phenomena, in the field of electricity the assumption of instantaneous forces acting at a distance will be dropped and Maxwell's theory must be viewed as that which corresponds to the current point of view of our experience.

What is proven by the development described above against the basic views of Weber's electrodynamics, and what is put in its place? Weber's theory was based on two different pillars, the assumption of immediate action at a distance and the idea of the atomistic constitution of matter. Of these, the first pillar has proven to be insufficient and superfluous compared to the phenomena. But the second pillar is in no way shaken by Maxwell's theory; because the mechanism, on which the propagation of the electrical force is based, makes no special assumptions. One can just as easily think of waves in a medium that continuously fills space in terms of tensions and pressures between the neighboring volume elements of such a medium, [or] as a transfer from particle to particle in an atomistically constituted medium. In the latter case, the action at a distance is then reintroduced into the theory, with the change that it is no longer considered to be present for arbitrarily large distances, but only for molecular distances. But if such an assumption proves to be useful and fruitful for the further progress of science, the prejudice existing against actions at a distance in general will not prevent it from being pursued. The conviction that actions exist through pressure and tension may be more immediate. Although their assumption may be closer to our perception, we know nothing about how they come about, and even with them, the body ultimately acts where it is not, i.e., in the distance. In this sense, the confirmation of Maxwell's theory did not result in a decision being made against the assumption of action at a distance.

The theory of action at a distance has two centuries behind it. We will not expect that the new methods that are to take its place will appear to us in an equally sophisticated and uniform manner. For the time being, the phenomena of gravity are separated from the other areas of physics by a deep gap, as long as it is not possible to explain Newton's attraction as an indirect effect caused by changes in the state of an ether-filling space. The attempts that have been made in this direction in recent times, from Riemann's metaphysical hydrodynamics to Isenkrahe's kinetic theory, do not have the character of a physical explanation.[361] They are based on a type of transcendental physics in that they attribute properties to the bodies that generate gravity that no physical body ever possesses. But even apart from this, we do not have a uniform method. Rather, an undeniable charm of current development lies in the diversity of perspectives from which attempts are made to bring connection and order into the realm of phenomena. The guiding ideas are not so separated from each other, that one excludes the other, but rather they can penetrate and complement each other in many ways, and we do not want to forget this relationship if in the following we separately highlight some points that are important in the recent developments of theoretical physics.

The first of these concerns the concept of energy, which has a fundamental meaning because it is the only thing that all areas of physics have in common. It therefore makes sense to place energy at the forefront of theory in each individual area and to connect the various areas with one another through the principle of conservation of energy. But people have gone even further by trying to view energy as a real substance and matter as the manifestation of energy. According to the different classes of physical facts, one has a mechanical, thermal, electromagnetic, and chemical form of energy. If it was previously considered a goal of science to reduce these different energies to a single form of mechanical or, more specifically, kinetic, the task of research is, on the other hand, limited to the investigation of the factors of energy in the individual areas, the paths, along which it moves and carries out its transformations. The demand made at the beginning to give the concept of energy a leading role in the development of theories has probably been fulfilled to a large extent. In its original form, Hamilton's principle of mechanics contains the difference between kinetic and potential energy.[362] In its further development, it reveals the possibility of replacing potential energy with the energy of hidden movements and of explaining action at a distance through movements in an intermediate medium. The mechanical theory of heat made the most important contribution to the development of the concept of energy. The more recent presentations of the theory of electricity also take their starting point from this. In no area, however, does the principle of conservation of energy provide a sufficient

[361][Note by AKTA and MH:] Bernhard Riemann (1826-1866) was a German mathematician and student of Gauss' non-Euclidean geometry, from which he developed his function theory of surfaces. He then became a temporary assistant to Wilhelm Eduard Weber. See [Rie67b] with English translation in [Rie67a] and [Rie77]. Caspar Isenkrahe (1844-1921) was a German mathematician, physicist and philosopher.

[362][Note by AKTA:] William Rowan Hamilton (1805-1865) was an Irish mathematician, astronomer, and physicist.

foundation for the development of theory. Rather, other facts completely independent of the same are added to the observation everywhere. It must also be emphasized that the practical interest which for us is associated with the establishment of general theories is rarely satisfied by the mere knowledge of energy and its conversions, so the energy principle is also inadequate in this direction. The view that energy exists independent of bodies and that these are only the vessels in which the movements of energy take place is difficult to implement, especially in mechanics. Finally, science will not be satisfied with the existence of the different types of energy and the fact of their convertibility; rather, it will always pursue the question of whether this can not be explained by the internal agreement of the forms of energy. Similarly, light, heat, electricity, and magnetism were previously explained by the actions of as many imponderable bodies, whereas at present, we only have to assume the existence of a single one.

Insofar as energetics goes against the methods of molecular physics, it subordinates itself to those theories that make use of the idea of a continuous filling of space. Based on the diverse facts, they assign properties to the volume elements of a body, which can undergo a constant increase or decrease with the location; they try to find mathematical relationships between the quantities given in this way, which reflect the observed connections. The equations which are given to us by the theories of the continuum have the great advantage of being valid, independent of the ideas which we associate with the quantities contained in them. They provide us with a description of the phenomena that is as complete and as simple as possible. Now our task is not to describe the phenomena but to explain them, i.e. to devise moving systems, which are images of the unknown real processes, so that every relationship that takes place between the bodies has a similar kind in the model, of every change, that we can do with it, corresponds to a real process in the world of appearances. This requirement is not satisfied by the mathematical formulas of the continuum theories. We will continue to search for a clear interpretation of the same in order to provide a guide for further research. In agreement with this, Maxwell said in his dynamic theory of gases:[363]

> Indeed the properties of a body supposed to be a uniform *plenum* may be affirmed dogmatically, but cannot be explained mathematically.

In the introduction to the treatise on Faraday's lines of force, Maxwell contrasts the representations of phenomena through mathematical formulas and physical hypotheses in an eloquent manner.[364] He says that in the first case, one loses sight of the phenomena to be explained and that the pursuit of mathematical consequences does not open up any new insight into the connection of things. On the other hand, physical hypotheses only show us the phenomena in a mirror. The successful explanation of a limited circle blinds us to the facts and leads us to hasty conclusions. Maxwell, therefore, seeks to discover a method of inquiry that will give the mind at every step the support of a clear physical view, without luring it away from appearances into the pursuit of analytical subtleties, and without drawing it in favor of any preconceived opinion beyond the facts respectively. He satisfies these conditions by the method of mechanical analogies, on which he based his theory of electrodynamics. The hypothesis on which it is based is that two galvanic currents have a concatenation of the same kind as the mechanisms that we now call bicyclic systems.[365] Under this assumption, the typical equations of the latter must also apply to

[363][Note by AKTA:] [Max67, p. 49].

[364][Note by AKTA:] [Max58] and [Max65b].

[365][Note by AKTA:] In German: *bicyklische Systeme*.

two galvanic currents. In this way, Maxwell actually arrives at the laws for the electromotive and ponderomotive actions of electrodynamics.

The method of mechanical analogies is not opposed to molecular theories like energetics and continuum theories. The natural connection which we subordinate to the typical form of a cyclic system, can be caused by an action exerted by molecule on molecule, as by an agent that continuously fills the space. However, it cannot be assumed that we will soon be able to dispense with the ideas of molecular theory. Especially in chemistry, the phenomena of chemical equilibrium accessible to energetics form only a part of the ideas to be explained. The question of why the chemical elements come together in certain proportions to form solid bodies of a certain crystalline form, is no more connected to the laws of chemical equilibrium than the theory of elasticity is to the laws of melting and evaporation. In optics, wherever the phenomena of light are connected with the chemical constitution of bodies, we are led to the assumption of the smallest particles, independent of one another, whose nature is so absolutely unchangeable that they carry out exactly the same oscillations in the most remote star in the flame of a Bunsen burner.[366] If one wants to accept the kinetic theory of gases as just a mechanical analogy, it would have made it very likely that there are tiny particles in a gas that, in a certain sense, move independently of one another. Biology in the fields of botany and zoology is based entirely on the ideas of molecular theory. The theory of the continuum itself did not attempt to view molecules and atoms as superfluous in the phenomena mentioned. It only asserts that the idea of them is not the last to which we can reach, and in this sense, William Thomson exploited the theory of vortices in a frictionless fluid.[367] With this turn, continuum theory no longer regards bodies as uniformly filling space. It only imagines an ideal fluid behind the bodies, on whose forms of movement the phenomena of the physical world are based.

We had come to the conclusion that the assumption of instantaneous action at a distance, as made in Weber's law, was inadequate and superfluous, but that the idea of the molecular constitution of bodies was not affected by Maxwell's theory. From the previous comments, it follows that the further development of science will not change this. What were Weber's own views on the questions discussed? He believed that he could maintain the correctness of his law despite the objections raised. But he was clear from the beginning about the possibility that this law was not the ultimate cause of electrical phenomena. At the end of the First Treatise on electrodynamic measurements, he said:[368]

> It is, however, possible to *conceive* that the forces included under the discovered fundamental law are also the kind of forces which two electrical masses *indirectly* exert upon one another, and which hence must depend, *first of all* upon the *transmitting medium*, and *further* upon all *bodies, which act on this medium*. [...] Another still undecided question is, however, whether the knowledge of the *transmitting* medium, even if it is not necessary for the determination of forces, would nevertheless be *useful*. [...] The *idea of the existence* of such a transmitting medium is already found in the *idea of the all-pervasive neutral electrical fluid*, and even if this *neutral fluid*, apart from conductors, has up to now almost entirely evaded the physicists' observations, nevertheless there is now hope that we can succeed in gaining more direct elucidation of this all-pervasive fluid in several new ways. Perhaps in other bodies, apart from

[366] [Note by AKTA:] Robert Bunsen (1811-1899) was a German chemist.

[367] [Note by AKTA:] See footnote 160 on page 95.

[368] [Note by AKTA:] See [Web46, pp. 213-214 of Weber's *Werke*] with English translation in [Web21d, p. 202].

conductors, no currents appear, but only *vibrations*, which will only be able to be observed more closely in the future. [...] Further, I need only recall Faraday's latest discovery of the influence of *electrical currents on light vibrations*,[369] which make it not improbable, that the all-pervasive neutral electrical medium is itself that all-pervasive ether, which creates and propagates light vibrations, [...].

Weber was particularly concerned with molecular theoretical investigations in the last period of his scientific activity, initially attempting to penetrate into the relationships of molecular movements using his law. He found that two different types of motion are possible for two similar electric particles.[370] In the one, a mutual reflection of two approaching particles takes place; in the second, the particles form a persistent system in which their distance periodically increases from zero to a certain amount and then decreases again to zero. He connects the first movement with the kinetic theory of gases, the latter with the stability of chemical compounds. He also continued to pursue Mossotti and Zöllner's assumption that ponderable molecules should be viewed as compounds of positive and negative electrical atoms and that gravity could be explained by the predominance of electrical attraction over repulsions.[371] He dealt with the problem of explaining the phenomena of light through waves in an electrical ether, assuming that the movements of its atoms correspond to the assumptions of gas theory. As long as he was allowed to work, he pursued the goal that he described in 1875 with the words:[372]

The true constitution of bodies and the true processes that depend upon that, although they are also very complicated and can be thought of as being represented by simpler processes only in part, will nonetheless remain the focus and ultimate goal of research, in spite of all obstacles.

With this outlook, we would like to conclude our consideration of Weber's scientific work. But to us, Weber is more than just a famous researcher who gave science new goals and new directions. Here he was at the peak of his life, here he enjoyed the peace of his old age. We experienced the friendliness and goodness of his nature and admired the character of rare greatness and purity in his undemanding appearance. So, as a student and younger friend of the deceased, I can try to bring the image of his personality back to our memories. The hours in which I, as an older student, listened to his lecture on experimental physics will always be among the most beautiful in my memory. Some people may have missed the smooth flow of speech, and the charm of effective experiments, but how quickly one forgot the external features that might have been noticeable at the beginning because of the wonderful art with which he knew how to develop the connection between phenomena and to expand the knowledge step by step. His lectures had a stimulating effect far beyond the circle of physicists thanks to the fine and apt remarks with which he illuminated the spirit and methods of precise research. I was soon fortunate enough to be able to personally get closer to the man I admired as a teacher. Anyone, who has ever visited Weber, will be familiar with the narrow room, the simple desk, will see him reading and working, his

[369] [Note by AKTA:] [Far46a].

[370] [Note by AKTA:] That is, for two particles electrified with charges of the same sign.

[371] [Note by AKTA:] Ottaviano Fabrizio Mossotti (1791-1863) was an Italian physicist. Johann Karl Friedrich Zöllner (1834-1882) was a German astrophysicist. See [Web94d] with English translation in [Web21b].

[372] [Note by AKTA:] [Web75, p. 357 of Weber's *Werke*]. See also page 175 of Section 11.10.

picture framed by the window through which the view fell onto the lawn and the towering trees of the garden. He will remember, not without emotion, the warm manner in which Weber greeted the visitor and the warm sympathy, he had for his concerns. It was a surprise of its own for the stranger when he came through the narrow, angled corridor between the houses on Jüdenstrasse to Wilhelm Weber's residence. In the middle of the city, separated from the noise and hustle and bustle of the day by few walls and yet peaceful and quiet, like the man who ended his great life in it. How pleased Weber was with the beautiful property, especially with the large, well-kept garden with its abundance of flowers and fruits and the secluded places that invited comfortable peace and quiet. How many a beautiful festival was celebrated there just a short time ago under his eyes, for he, who had retained the heart and faith of a child throughout his life, was full of joy when the garden echoed with the joy of a happy youth. When the older brother had retired from teaching, he and his family used to spend the summer in Göttingen in the Weber house, which had been enlarged for this purpose. A new life arose around the deceased.[373] Although he [Wilhelm Weber] was not married, he did not lack an attractive domesticity. When he returned to Göttingen [in 1849], his niece Sophie Weber accompanied him, and from then on, with a short interruption, she ran his household and took care of his esteemed uncle. More and more, however, the house in Göttingen became the center of the family and in this year the children and grandchildren of his brother Ernst Heinrich gathered around the already suffering man. And just as this house was a place of quiet work and joyful celebrations, it was also a place to which all those who had the privilege of frequenting it owed a great deal of inspiration and encouragement. Because Weber's interests were not limited to the circle of his science, he was a friend of philosophical reflection. He had an open sense of the beauty of poetry and knew and loved our classical music and also the things of this world. He followed the course of political events with wise judgment and patriotic spirit. When Weber comes before our inner eyes, we first think of his kindness and gentleness, his modesty in all the honors that fell to him in abundance without being asked for, and the amiable optimism that he maintained even when things did not go his way. But his kindness did not become weakness. Where he saw an injustice, the man, who was otherwise so calm, could flare up violently, whether the matter was great or small, and one might have smiled at the zeal with which he defended what he recognized as right, if it had not been for the reverence for the deep feeling for truth and justice that was expressed in it. He showed how serious he was about this on November 18, 1837, when the new king repealed the basic law of the state and released the civil servants from the oath they had taken to the constitution. In the idea drawn up by Dahlmann, it was said:[374]

> The entire success of our effectiveness does not rest so surely on the scientific value of our teachings as on our integrity. As soon as we appear before the studying youth as men who play a careless game with their oaths, just as soon the blessing of our effectiveness will be gone.

Weber knew what was at stake for him when he signed these words; although he did not have the care of a family, the dismissal from office was hard enough for him, because through it, all the conditions of his existence were profoundly shaken. For the natural scientist, more than for the representatives of the humanities, the possibility of successful work is tied to

[373] [Note by AKTA:] Weber's older brother, Ernst Heinrich Weber, retired from the University of Leipzig in 1871 and died in 1878.

[374] [Note by AKTA:] Friedrich Christoph Dahlmann (1785-1860) was a German historian and politician.

the possession of an academic chair, and the call to another university had to put an end to the intimate contact with Gauss and the joint work of the two researchers. But Weber clung to Gauss with a strong and deep feeling, which is expressed in the following words of a letter written after his dismissal:

> That I have had and will never have a higher wish in life than to always stay close to you, and you are certainly convinced that the dangers that now threaten the fulfillment of my wish are deeply shocking. — If only I am not exiled, I will remain close to you and will know how to adjust myself without a cabinet.

But it was not only on a big occasion and with a big decision that Weber put his eyes on his own advantage behind what he considered his duty. He demonstrated the same sense of duty towards the many small affairs which are connected with the position of professor and which so often disturb his circles at inopportune hours. Given his entire personality, Weber was not suitable to represent the University in a prestigious position. He also didn't like to show his personality in public. His influence on the affairs of the University and the share he took in them were therefore significant. He managed the deanship of the Philosophical Faculty three times; the reports on general affairs of the Faculty or the needs of the Institute he headed, which we have from his hand, are prepared with the same care as his scientific treatises and provide a wide range of instruction and inspiration.

Weber was a whole man, and whatever he did, he did it with all his strength and all his mind. He was pure and true and honest, and just as there was no falsity in himself, so he could not believe in any falseness in others. So his judgment could well be lacking, but the reason for the error was the inner goodness of his being. The work of his life, as it is handed down to posterity in his scientific treatises, unfolded with an admirable consistency from the beginning, without deviations, without regression, and with inner necessity. With the highest care in mathematical development, with the most unconditional reliability in the execution of the experiments, and with the most precise assessment of the secured ground go hand in hand with the broadest view of what is to be achieved. And he did not deceive Weber, because in all his work he did not seek his own, but, free from all selfishness and every touch of vanity, he placed himself in the service of the truth. When he became tired of working, he handed over one part of his official duties after another to younger hands, without complaint or bitterness. When the loss of his memory also made scientific work impossible, he laid down his pen, not without pain, but without the quiet peace of his soul ever being disturbed.

He had become lonelier over the years. His beloved brother went before him. The circle of friends, which used to gather every week for mutual instruction and informal exchange of ideas, had dissolved, and so it was more and more limited to the relationships that connected it with the nearby members of the family and with a few loyal ones friends from older times. So his mind gladly and often returned to days long past, and the present world appeared to him as if through a veil. What he experienced internally in those hours, when he seemed lost in dreams, is a secret that we remain in awe of. During the Pentecost days of this year, a change occurred in Weber's health, who had still retained admirable strength in his old age. Soon, one could no longer be mistaken that the denouement was coming. When, after misty days that forbade the enjoyment of the open air, the full sun shone again for the first time, he allowed himself to be led out into the garden, where he remained the whole day. After noon, he fell asleep sitting in the armchair. When the sun went down, his eyes opened clear and bright. He looked out into the distance, his gaze no longer directed at the things of this world, but up to a higher order that he had longed for a long time because he had grown tired of working in this world. Then he slumbered over into that long sleep, from which there is no awakening here, under the trees that he had once planted and which had been the witnesses of his blessed work for so long.

Chapter 14

[Heinrich Weber, 1893] Wilhelm Weber: A Biographical Sketch

Heinrich Weber[375,376,377,378]

14.1 Preface

The intention was to publish this biography already earlier, but the inspection of a large amount of unsorted letters caused some delay. I hope that the friends of Wilhelm Weber will like this little book. I would like to make here some corrections to the version of the biography which appeared in the "Deutsche Revue".[379] In the footnote on page 184 in the volume from August 1892 it should read: Both professorhips (for anatomy and physiology) E. H. Weber hold until 1865 (instead of 1870), when Professor Ludwig was appointed for physiology. He was professor for anatomy for 51 years (instead of 55 years). His younger brother Eduard was associate professor and prosector for anatomy.

The author.

---

375 [Web93b].

376 Translated by U. Frauenfelder, urs.frauenfelder@math.uni-augsburg.de. Edited by A. K. T. Assis, www.ifi.unicamp.br/~assis.

377 The Notes by Heinrich Weber are represented by [Note by Heinrich Weber:]; the Notes by Carl Friedrich Gauss are represented by [Note by Gauss:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

378 [Note by AKTA:] Heinrich Weber was born on January 1, 1839, in Leipzig and died on May 5, 1928, in Braunschweig. He was a German physicist and Professor at the Ducal Technical University Carolo-Wilhelmina in Braunschweig from 1866 until his retirement in 1906. He was appointed Privy Councilor, was the nephew of Wilhelm Eduard Weber (son of his older brother, the physiologist Ernst Heinrich Weber (1795-1878)) and was accepted as a full member of the German Academy of Natural Scientists Leopoldina on January 8, 1901.

379 [Note by AKTA:] [Web92a], [Web92b] and [Web92c].

Figure 14.1: This picture comes from a 1884 portrait of Wilhelm Weber taken by the German photographer Bernhard Petri (1840-1887). It appears, for instance, in [Web92e, frontispiece] and [Web93b, frontispiece].

14.2 Wilhelm Weber's Youth 1804-1825

Weber's family originally came from Gröben a little village in Thuringia close to the market town Teuchern located in between Weissenfels and Zeitz. There the grandfather of Wilhelm Weber owned a farm. According to the custom at that time his son Michael, the father of Wilhelm Weber, should have succeeded as a farmer. However the father of Wilhelm Weber already as a young man didn't show much ability and interest for farming. "Michael never will become a competent farmer, he just can study", his father said. After some reluctance he finally agreed that his son studies theology. Therefore in 1784 the just 30 year old Michael Weber was already professor for theology at the University of Wittenberg. A. M. Meyner in his History of the Town Wittenberg, Dessau 1854,[380] on page 147 attributes to him a thorough erudition, a strong sense for justice and good manners. Michael Weber's[381] religious view was based on the Revelation and until his death in 1833 he opposed Rationalism, which became more and more trendy. His serious, strongly religious but as well cheerful nature deeply influenced his children. Although Wilhelm Weber was undogmatic, he was religious and against materialism. Michael had a vast knowledge of Latin. Apart from theological writings, he translated a great number of hymns into Latin. Moreover, he wrote letters and poems in Latin to his children. His first spouse was Christiane Friederike Wilhelmine, née Lippold. Christiane is described as a gracious and delicate woman. She gave birth to twelve children. However, except Wilhelm, only four, three sons and a daughter, reached adulthood.[382] After she passed away Michael married Eleonore Friederike Henriette Pallas. Eleonore didn't give birth to children, but was a great mother for the children Michael had with his first wife. She took very good care of Michael until he passed away on August 01, 1833. It is said that Michael had a thundering voice which was very suitable for public speeches. He was president of the University of Wittenberg in 1802 during the celebration of its 300th anniversary and as well in 1807 at the peace festival on July 02.[383] After the merging of the universities of Halle and Wittenberg, Michael Weber moved to Halle and became a professor for theology. He diligently carried out his duties as a professor until his

[380] [Note by AKTA:] [Mey54].

[381] [Note by Heinrich Weber:] C. F. Fritzsche, Narratio de Michaele Webero primo nuper Halensi theologo, Halis Saxonum 1834.

[382] [Note by Heinrich Weber:] Gustav Weber was born in Wittenberg in February 07, 1790. He was pastor in Rackith close to Wittenberg and after retirement he lived in Niemegk close to Bitterfeld. He passed away in Klein-Wittenberg November 11, 1852.

Ernst Heinrich was born in Wittenberg in June 24, 1795. From 1806 until 1811 he went to school in Meissen. He studied from 1811 until 1813 in Wittenberg and after that in Leipzig from 1813 until 1815. In 1815 he became a doctor in Schmiedeberg, when the University was relocated to this town after the siege of Wittenberg. In 1817 after his State doctorate he became a private lecturer in Leipzig. Because of an offer from the newly founded University in Bonn he became associate professor in Leipzig. When Rosenmüller passed away just a year later in 1820 he became full professor of anatomy and after Kühn passed away he simultaneously became as well professor for physiology. Both professorships he hold until 1865, when after the appointment of Professor Ludwig for physiology a special professorship combined with an Institute was founded. The anatomy he later run jointly with his younger brother Eduard, who became an associate professor. Shortly after Eduard passed away, Ernst Heinrich resigned as well from this professorship, which he had hold for 51 years. Soon after he became ill and passed away January 26, 1878.

Eduard Friedrich was born in March 10, 1806 in Wittenberg. He went to the school of the Francke Foundations in Halle, became a doctor in Naumburg and since 1833 he was associate professor for anatomy in Leipzig. He passed away May 18, 1871.

Lina Weber was born April 06, 1802. She never married and passed away in July 01, 1881 in Halle.

[383] [Note by Heinrich Weber:] Meyner, Geschichte der Stadt Wittenberg, Dessau 1845, pp 147-148 and C. F. Fritzsche, narratio etc.

death.

Wilhelm Eduard Weber was born in October 24, 1804. At this time his parents were renting an apartment in the "Golden Ball" in Schloßstraße 5 (now number 15).[384,385] The owner of this house was Professor Langguth.[386] He had the same age as Michael. Since they lived together already more than ten years in this house — because his brother Ernst Heinrich, ten years older, was also born in the same house — the two families had become good friends. We do not have much information on the early boyhood of Wilhelm. Probably he played a lot with his younger brother Eduard under the careful protection by his mother in a peaceful environment. Without doubt the relations between the families Weber, Langguth and Chladni[387] had a strong influence not just on the life of Wilhelm, but as well on the life of his brothers. Chladni had as well an apartment in the house of Langguth from 1801 until 1813. Langguth was from 1782 until 1784 a doctor and an associate professor of medicine. He then became full professor of natural history until he passed away in February 09, 1814 in Wittenberg. He owned a geological, economical, physical as well as a medical collection. His collection was renowned for its real gems, its very rare pieces but as well its usefulness for instruction.[388] Chladni was a doctor of law without tenure. From 1756 until 1827 he mostly lived in Kemberg close to Wittenberg. Both he and Langguth were sons of professors in Wittenberg and so they were good friends since their teens.[389,390] In such an environment

[384][Note by Heinrich Weber:] According to an announcement of the mayor of Wittenberg Dr. Schild in the Göttinger Journal, August 13, 1891, No. 8634.

[385][Note by AKTA:] In German: *goldnen Kugel*. It might refer to an old-style shot ball. A modern picture of this house is shown in Figure (a) of this footnote, https://en.wikipedia.org/wiki/Wilhelm_Eduard_Weber and https://www.hmdb.org/m.asp?m=69820. In Figure (b) we show the memorial plaques. The upper one reads: Wilhelm Eduard Weber, professor of physics and inventor of electrical telegraphy, was born in this house on October 24, 1804. The lower one reads: Inventor of the electro-magnetic telegraph, Birthplace:

(a)

(b)

[386][Note by AKTA:] Christian August Langguth (1754-1814) was a German physician and physicist.

[387][Note by AKTA:] See footnote 300 on page 180.

[388][Note by Heinrich Weber:] Meyner, p. 43.

[389][Note by Heinrich Weber:] Bernhardt, Dr. Ernst Chladni, der Akustiker, Wittenberg 1856, p. 39.

[390][Note by AKTA:] [Ber56].

natural science had a lasting impact on the boys. Maybe this was as well influenced by the fact that the oldest brother Gustav already studied theology in the footsteps of their father and that another brother Fritz who unfortunately died very early became a soldier. Until his death Chladni exchanged letters with the Weber brothers. Due to his influence, Eduard and Wilhelm jointly studied the theory of waves, and acoustics was the field Wilhelm Weber started his physical career.

The peaceful family life didn't last too long. Napoleon finished his terrible retreat from Russia. August 26, 1813 Blücher won the fight at the Katzbach. On August 23 and September 6 Bernadotte, Bülow and Tauentzien ended up as victors in the battles at Gross-Beeren and Dennewitz. Everywhere people started ousting the French from their positions they had occupied. The fortress of Wittenberg as well had a French garrison. In September 1813 Bülow came with his army in order to liberate the city from the French. The French refused to withdraw and a terrible bombardment was the answer. Bernardt writes about this:[391]

> On the evening September 27, 1813, the Prussians under the leadership of Bülow started the second terrible bombardment on our unfortunate town. Ten Artillery batteries distributed from the rothen Mark over the Belziger and Bruchstreet until the river Elbe were shooting with 24 cannons. During this terrible cannonade the tower of the castle church as well as the rear part oft he castle caught fire. This fire spread from there to the houses of Dörffel, Meyner and Langguth. Several families and among them the one of Professor Weber could just save their life."[392]

After this dreadful night, Professor Weber like many other Professors left the fortress on siege and escaped to Schmiedeberg in the vicinity of Wittenberg, where he stayed until the end of September 1814. After the merger of the University of Wittenberg and the University of Halle he moved to Halle.[393]

In Halle Wilhelm went to the school of the Francke Foundations and diligently studied old languages. His interest for the classical writers shows a list of Latin and Greek writers which he bought from his own pocket money. This was much more then he was obliged to do by the school. When he still went to school, he was called in by his older brother Ernst Heinrich, who already was a professor in Leipzig, to carry out experiments on wave motion. The results of these studies later formed the jointly published *Wellenlehre*.[394,395] In order to carry out these researches undisturbed, Ernst had arranged for Wilhelm to be completely exempted from attending school for a longer time. By sharing common interests, the brothers who already had a strong family relation got so close, that they promised each other to always hold together firmly "whatever destiny will bring". Rarely people were commuting on foot so often from one city to the other on the country road between Leipzig and Halle than during the time where Wilhelm and Eduard lived in Halle and Ernst in Leipzig. They didn't care about the distance of eight hours. At this time, Easter 1821, Wilhelm probably made his first longer travel together with his brother Ernst to the Bohemian baths about which he made manifold records.

[391][Note by Heinrich Weber:] Bernhardt, Wittenberg vor 50 Jahren, Wittenberg 1863, p. 33. — Meyner, pp. 69 and 70, and mayor Dr. Schild, Göttinger Zeitung from August 13, 1891.

[392][Note by Heinrich Weber:] The house of Langguth, the golden ball, did not completely burn down, but could be restored after a major repairing. It survived until today and is now endowed with a plaque.

[393][Note by Heinrich Weber:] Meyner, p. 68.

[394][Note by Heinrich Weber:] Die Wellenlehre auf Experimente gegründet, Leipzig 1825.

[395][Note by AKTA:] [WW25]. Original title in German: *Die Wellenlehre auf Experimente gegründet* (Wave Theory Founded on Experiments).

In Easter 1822 Wilhelm finished school and began studying mathematics in Halle. It is not possible to trace back which lecturers he liked most. Probably he studied mathematics with Johann Friedrich Pfaff and physics with Johann Salomo Christoph Schweigger.[396] During his studies Wilhelm made several times smaller and longer travels on foot. Immediately in the summer of 1822 he was travelling with his brother Ernst for three months to Switzerland and Northern Italy. In later terms he actively joined the physical seminar of Schweigger. Due to his preference for acoustics, he published excerpts of the papers of Savart on sound and tone in Schweigger's *Journal für Chemie und Physik*, volume 14, p. 385.[397] Meanwhile his joint studies on the propagation of waves came to an end and could be published in the year 1825 in a treatise entitled "Wellenlehre auf Experimente gegründet".[398] This work made him quite famous in the world of science and encouraged him to proceed his career in experimental research. The two editors dedicated the work to their old friend Chladni. After having received the paper, Chladni wrote to Wilhelm in August 20, 1825:

> Dear friend, thank you very much for the nice dedication of your joint treatise with your brother in Leipzig on the theory of waves. I am very honoured since it is a really groundbreaking work in which it is explained much clearer and much more coherent than before what happens during this kind of movement. Moreover, it contains many new contributions enhancing our knowledge. What distinguishes your work is that you describe only the immediate results and observations in the most simple way. This is much more useful than a philosophical approach which often has not much in common with actual nature.

Not only scientists were excited, but as well the government. Altenstein which at that time was minister of education in Prussia wrote in March 14, 1826:

> I acknowledge the great merit the rich paper, which you wrote jointly with your brother, has for the advancement of science. If you need more funding just tell me.

The minister added to this letter a report of an unnamed "outstanding physicist" who refereed the paper.

The great success of his first paper[399,400] had the most advantageous consequences for Wilhelm Weber. From this time on until he moved to Göttingen, the Ministry not only funded very generously the acquisition of devices and instruments Wilhelm Weber needed for his research, but as well paid him quite well, even before he became a lecturer at the University. When he later became an associate professor, the Ministry continued its favorable treatment of Wilhelm Weber and awarded him several wage increases.

---

[396] [Note by AKTA:] Johann Friedrich Pfaff (1765-1825) was a German mathematician. Johann Salomo Christoph Schweigger (1779-1857) was a German chemist, mathematician and physicist.

[397] [Note by AKTA:] Felix Savart (1791-1841) was a French physicist and mathematician. See [Web25a] and [Web25b].

[398] [Note by AKTA:] See footnote 395.

[399] [Note by Heinrich Weber:] The Royal Society of Sciences in Göttingen is currently editing the collected works of Wilhelm Weber in Julius Springer's publishing company in Berlin. The first and second volume of six volumes in total will appear this autumn.

[400] [Note by AKTA:] The six Volumes have already been published: [Web92e], [Web92d], [Web93c], [Web94g], [WW93] and [WW94]. Most of his papers have already been translated into English in the 4 Volumes of the book *Wilhelm Weber's Main Works on Electrodynamics Translated into English*: [Ass21j], [Ass21k], [Ass21l] and [Ass21m].

14.3 Further Years of Study 1824-1831

Wilhelm Weber obtained his PhD in August 26, 1826. Its title was "Theoriam efficaciae laminarum maxime mobilium arcteque tubas aerem sonantem etc. continens".[401] However, Wilhelm Weber felt that Halle could not offer him what he desired for his further education. Therefore he wished to go for one year to Göttingen in order to continue his studies in mathematics and exact sciences with the famous mathematician Gauss. After that he planned to go for one year to Paris where mathematics and physics flourished. In view of the favourable treatment he received so far from the Ministry of Education, he took courage to ask it for funding. However, in a letter from April 29, 1826, the Ministry refused any support. The minister of education wrote:

> For several reasons the Ministry thinks it is better for you that you first work for some time as a private lecturer at your current university and visit Paris later. In order to obtain advice and support from the Privy Counsellor Gauss for your hydraulic and related researches, it is not really necessary that you go for one year to Göttingen. Instead of that you can exchange letters with Gauss and maybe visit him during your vacation. In this way you can achieve the same result in a cheaper way.

Therefore Wilhelm Weber gave up his travel plans. He continued his researches in acoustics and published some small papers in Schweigger's *Journal für Chemie und Physik*. In 1827 he submitted his habilitation thesis "Leges oscilliationis oriundae etc." (The laws of vibration generation and so on).[402] It is interesting to note that among others Johann Tobias Mayer congratulated him without knowing that four years later Wilhelm Weber will become his successor in Göttingen.[403] Wilhelm Weber was private lecturer only for a short time. Already in the autumn of the following year he became associate professor after having received shortly before his first international honour by becoming a corresponding member of the Royal Academy of Science of Turin. In September he visited the meeting of natural scientists in Berlin where he made his first acquaintances with the participating professors and lecturers. Wilhelm Weber himself gave a lecture on organ pipes published in Poggendorff's *Annalen*, volume 14, 1828.[404] His lecture impressed Alexander von Humboldt and Gauss who participated in the meeting.[405]

Wilhelm Weber still wished very much to continue his studies outside of Halle. Immediately after the meeting of natural scientists he visited Berlin for a longer period starting in October 17, 1828. This visit had a lasting impact on his future career. The personal intense exchange with colleagues of his age like Dirichlet, Dove, Magnus or Wöhler but as well senior people like Mitscherlich, Heinrich and Gustav Rose, Poggendorff, Enke, Seebeck,

[401] [Note by AKTA:] The theory of the efficiency of the most movable plates and the trumpets closely containing the sounding air etc.

[402] [Note by AKTA:] Complete title in Latin: Leges oscillationis oriundae si duo corpora diversa celeritate oscillantia ita conjungutur ut oscillare non possint nisi simul et synchronice exemplo illustratae tuborum linguatorum — The laws of oscillation arising if two bodies oscillating at different speeds are joined in such a way that they can only oscillate simultaneously and synchronously are illustrated by the example of tongued tubes. See [Web27].

[403] [Note by AKTA:] Johann Tobias Mayer (1752-1830) was a German physicist.

[404] [Note by AKTA:] [Web28].

[405] [Note by AKTA:] Alexander von Humboldt (1769-1859) was a German polymath, geographer, naturalist and explorer.

Steiner, Weiss, Ehrenberg, Ermann, Crelle and others had a stimulating effect on him. Especially valuable for him was that he could visit Alexander and even Wilhelm von Humboldt.[406] Alexander von Humboldt had a great interest on the young ambitious researcher. Proof of this is a note Wilhelm Weber made November 12. He wrote:

> Alexander von Humboldt invited me for lunch. He advised me to continue physical measurements and promised to help me with the acquisition of a balance, a measuring device and a monochord. For this purpose he sends me to Dr. J. Schulze who is working for the Ministry of Education. He will talk to him in person and write to the Ministry in case this is needed. He recommends me to travel to Hamburg to Schumacher and will write me a letter of recommendation. He further suggests I should attend Dirichlet's lectures on Fourier's theory of heat. He further invites me to visit with him his brother Wilhelm the following day at eight o'clock in the evening. According to him, I have some chance to get a position in Göttingen if there is some vacancy there.

Concerning the visit the following day Wilhelm Weber writes:

> Around 8 o'clock I went to Humboldt who showed me two new volumes of the Philosophical Transactions in which Barlow used objectives for binoculars with liquid in order to study the change of refraction of this liquid. We were riding a carriage without back seat and picked up an Englishman so that the three of us were squeezed like herrings on the only seat of the carriage. We were driving to the minister Wilhelm von Humboldt. He himself was not at home but there we met his wife, his daughter and two guests. All rooms were decorated with paintings. We drank tea, looked at some of the paintings and talked a lot in French, since the Englishman didn't speak German. After that Wilhelm von Humboldt came. He looks much older than Alexander and talks more precise and focused but less fluent than his brother. During the whole evening one held his hat in his hand. The conversation was very free and easy. Topics were the quarrel if the largest vase in Europe made of granite should be put inside or outside the museum, how Cotta could be used for the enhancement of science and so on.[407] At 10 o'clock we went home.

Another time, in December 03, Wilhelm Weber writes:

> I had an appointment to meet Wilhelm von Humboldt in the evening together with Dirichlet. Since it was beginning to thaw, Dirichlet could not come. At Wilhelm von Humboldt's house a letter of the crown prince was read and we talked about the war.[408] Alexander von Humboldt came together with Ms. von Cotta.

Wilhelm Weber had a lot of interaction with Alexander von Humboldt. Apart from Dirichlet and Poggendorff he often kept company as well with Mitscherlich.[409] Although

[406][Note by AKTA:] Wilhelm von Humboldt (1767-1835) was a German philosopher, linguist and diplomat.

[407][Note by AKTA:] Probably they were referring to the German publisher Johann Friedrich von Cotta (1764-1832) who published several works of Humboldt.

[408][Note by Heinrich Weber:] Probably the Greek War of Independence.

[409][Note by AKTA:] Peter Gustav Lejeune Dirichlet (1805-1859) was a German mathematician. Johann Christian Poggendorff (1796-1877) was a German physicist. He edited the *Annalen der Physik und Chemie* from 1824 to 1876, where many of Weber's papers were published. The modern *Annalen der Physik* is the successor of this Journal. Eilhard Mitscherlich (1794-1863) was a German chemist.

Mitscherlich was Professor for Chemistry he was very much interested in physical researches. In his house he hosted several casual evening parties dedicated to science and people like Alexander von Humboldt, the two Rose's, Dove, Karsten, Dirichlet, Wöhler, Ehrenberg, Poggendorff, Magnus and others were participating. At such parties Mitscherlich showed new instruments, or one repeated and discussed new experiments like the one of Arago concerning rotational magnetism.[410] Every Tuesday evening Weber was invited to Mitscherlich. Apart from that, Weber and Mitscherlich made joint experiments in his laboratory on the tones of the chemical harmonica. For that purpose Weber ordered to send him his monochord from Halle. Moreover, they studied the speed of propagation of acoustic noise in different gases and other things. Mitscherlich had a large audience in his lectures which Wilhelm Weber attended as well. Wilhelm Weber wrote in a note:

> Mitscherlich is very much dedicated to his lectures. He discussed the main ingredients of our nutrients, gum, gluten, albumin and sugar. He speaks fluently but softly. His experiments are prepared with great care. Mitscherlich has a lot of pleasure on natural phenomena. He teaches all the little tricks for producing them safely. One can profit a lot from the arrangement of his instruments.

The advice of Alexander von Humboldt to attend the lectures of Dirichlet was not in vain. Weber attended Dirichlet's lectures assiduously and soon became a good friend of him. This friendship was later 1855 a major reason for Dirichlet's move from Berlin to Göttingen.[411] Wilhelm Weber wrote in November 21 about Dirichlet:

> He sometimes stumbles in his lectures and is not as well prepared as Scherk. But one can learn a lot from him since he knows very well how to explain the subtle points of higher calculus.[412] Steiner and Dr. Scheibler are attending his lectures as well.

The lectures took place thrice a week from noon until 1 pm. The lectures were usually followed by a walk, which Dirichlet often joined. It became the habit to go in the afternoon to the coffee party "Dirichlet".

> One of us is inviting the others after the lecture to the coffee shop Dirichlet, where we enter at 2 pm or 3 pm and stay until around 6 pm in a very enjoyable atmosphere.

Steiner was from 1825 until 1863 teacher of mathematics at the vocational school and therefore a colleague of Wöhler.[413] Weber writes:

[410][Note by AKTA:] François Arago (1786-1853) presented his discovery at the Academy of Sciences of Paris in the meetings of 22 November 1824, 7 March 1825 and 3 July 1826, [Ara24], [Ara25] and [Ara26]. These papers were also included in his complete works, [Ara54].

[411][Note by Heinrich Weber:] Gustav Peter Lejeune Dirichlet was born on February 13, 1805, in Düren close to Aachen. At this time Düren was French. He was the son of the commissionaire de poste. After attending high school in Cologne he went to Paris for his studies. He was a tutor in the house of General Foy but later moved to Prussia, probably at the instigation of Alexander von Humboldt. In 1827 he obtained his PhD in Bonn and got a job in Breslau. After he became in 1828 an associate professor, he obtained as well an offer from the military school in Berlin which he accepted. However, he kept his position in Breslau until 1831, when he became an associate professor at the university in Berlin. In 1839 he became full professor. In 1855 he moved to Göttingen as the successor of Gauss. He died in Göttingen in 1859.

[412][Note by Heinrich Weber:] This remark concerns the first year of Dirichlet's lectures. Later on his lectures became the role model for teaching mathematics. The author can confirm this, since he had the luck to attend some of them.

[413][Note by AKTA:] Jakob Steiner (1796-1863) was a Swiss mathematician. Friedrich Wöhler (1800-1882) was a German chemist. See [WW41c] with English translation in [WW21], [WW41a] and [WW41b].

> Steiner who made excellent geometrical contributions to Crelle's *Journal* seems to be born in Swabia[414] and has a lot of talent for jokes. He made fun a bit of the great Ohm and told his stories about the oval.[415]

The new friends introduced Weber to more specialized communities. Dirichlet invited him to the Mathematical Society in the English House and Poggendorff to the Humanitarian Society in which Leopold von Buch gave a lecture. But despite his busy schedule with younger people, Weber still found time to communicate with older scientific authorities socially and scientifically, like the mineralogist Weiss, the two Roses, Ehrenberg, Schaffrinsky or the academician Seebeck.[416] Seebeck told him in a conversation about mathematics an interesting story about Gauss. Gauss once told Pfaff that he had his most brilliant ideas at times of greatest despair. Weber also took the opportunity to visit the mechanical workshop of Pistor, where he met Schieck, the Müller brothers, the watchmaker Tiede and huge establishments like the iron foundry, the gasworks and so on. Schleiermacher also wanted to hear Weber preach, but he was prevented from holding the service.

Wilhelm Weber left Berlin in January 22, 1829, at 7 pm having received a lot of experiences and impressions. He was well endowed with letters of recommendation from Alexander von Humboldt, Enke, Ermann and Poggendorff in order to visit Repsold and Schumacher in Hamburg and Altona. He himself describes the cold, boring, uncomfortable ride for which the carriage at this time needed 38 hours. However, the ride got much more pleasant since he met Rosenberger in Klötze probably by appointment. Rosenberger was professor of mathematics and astronomy in Halle and came via Magdeburg to Klötze in order to travel jointly with Wilhelm Weber to Hamburg and Altona. Schumacher was very friendly to his two young colleagues from Halle and showed them in detail his facilities and his instruments. Moreover, Repsold introduced them to all items and specialities of his famous workshop for astronomical devices. How long Wilhelm Weber stayed in Hamburg cannot be reconstructed, probably the two colleagues travelled from there jointly back to Halle.

Weber stayed in Halle until his appointment to Göttingen. It seems that he intended to spend the summer term of 1829 again in Berlin. He already received the approval from the Ministry of Education. As far as we can tell, the plan did not materialize since all official letters to him from Halle as well as from Berlin were sent to his address in Halle. Weber now dedicates himself especially to his lectures and researches for which the Ministry as already mentioned before offered gorgeous funding. He stayed in contact with his friends from Berlin, in particular with Poggendorff. Poggendorff's letter from January 30 and March 1830 clearly show, that Weber was not forgotten in Berlin.

> Please send in order to advance your cause your latest treatise to Humboldt. Leopold von Buch is eagerly reading your papers and talked very favourably about you in the Society.

Humboldt also asked Poggendorff to motivate Weber to write up a summary of his researches in French, which he wanted to send himself to Arago.[417] On the other hand Alexan-

[414] [Note by Heinrich Weber:] In fact Steiner was born in Switzerland.

[415] [Note by AKTA:] Crelle's Journal is the common name for a mathematics journal, the *Journal für die reine und angewandte Mathematik* (Journal for Pure and Applied Mathematics). It was founded by August Leopold Crelle (1780-1855) in 1826 and edited by him until his death.

[416] [Note by AKTA:] Thomas Johann Seebeck (1770-1831) was a Baltic German physicist. See [See25] and [See26] with partial English translation in [See69] and partial Portuguese translation in [FS16].

[417] [Note by Heinrich Weber:] Humboldt as well wrote directly to Weber. However, probably Weber donated these letters as autographs.

der von Humboldt sent him via Enke a paper by Poisson. Encouraged by such requests, Weber sent his papers as well to Gauss. Gauss answered in a letter from April 02, 1830, which is probably the first written exchange between Gauss and Weber, as follows:[418]

> Dear Professor, I received your kind letter together with your valuable papers on acoustics in a very busy moment, so that I had to reserve my reading of them until I found some spare time. Now there are vacations or more correctly for me the time of vacations. I started reading your papers. I was fascinated by the originality of your work, but realized that to understand their rich content in depth one has to spend more time than I have at the moment. Therefore I cannot postpone it for longer to thank you sincerely for your letter and to express my joy that you dedicate yourself to this interesting researches with such diligence and success.
>
> I always had the opinion that the acoustics belonged to these parts of mathematical physics in which still the most splendid progress can be made. Indeed, acoustics is concerned with relations in time and space and therefore it should be possible to treat it mathematically. But, alas, how little, how very little we know! The things which at first sight seem obvious we do not know yet how to attack. Our previous researches just treat the speed of propagation and the thickness of acoustic waves. The tone pitch depends on the ratio of these two quantities. But the distinctive specific features of a sound which is sometimes referred to as tone color and manifests itself in the most miraculous way in the articulation is until now a completely unknown continent. It seems that this can only depend on the shape of the waves of sound. What huge field of study is there to be explored by us. I am convinced that the human mind once will open this field and bring it to the same clarity which optics enjoys. One should believe that it is not too hard to bring light into this darkness for somebody who has enough diligence, mathematical strength, experimental skills and time. One only needs to understand what are the specific distinctions for different tones, not in the body which emits the sound and not in the ear, but in the elastic medium in between. Concerning the body which emits the sound the ingenious work by Kempelen[419] is a good starting point and it is astonishing that after fourty years there was not much progress in this matter.
>
> I hope that you are the one which opens up for this problem a new field of research. With my very best regards,
>
> Göttingen, April 20, 1830.
>
> Sincerely
>
> C. F. Gauss.

In November 30, 1830, Tobias Mayer, full professor for physics at the University of Göttingen, passed away. Motivated by Alexander von Humboldt's encouragement, Weber started to think about moving to Göttingen. He asked Gauss for advice on this matter. Gauss wrote:

[418][Note by AKTA:] See also [Gau d].

[419][Note by Heinrich Weber:] Wolfgang von Kempelen, Mechanismus der menschlichen Sprache, nebst Bechreibung einer sprechenden Maschine, Wien, 1791.

Dear Friend, the reason that I did not answer your friendly letter immediately is not surprising in view of the events in our town. On top of that my activity was disturbed by heavy personal distress. However, I cannot wait longer with answering your letter. I think I showed you several times since I first met you, how much I appreciate your skills as well as your personality.

Gauss than writes that he would support Weber's appointment as soon as he has an opportunity and that he thinks it is a good idea to contact the public authority.

Unfortunately, often one overlooks a modest meritorious person in favor of a writing maniac. I do not know if this can happen in the current case.

With my most sincere regards,

Göttingen, January 27, 1831,

C. F. Gauss.

Probably later on the government in Hanover asked Gauss for advice concerning the appointment of the professorship. In any way Weber was appointed full professor at the University of Göttingen by the secret counsellor Hoppenstedt on April 29, 1831. Weber accepted the offer in May 14, although his wish to become a member of the Society was not granted at the moment "since already two mathematicians Thibaut and Gauss are members of the Society, but just one philologist and one orientalist." On July 05, 1831, Gauss congratulated him on his appointment:

I am very pleased that you accepted the offer. I sincerely wish you all the best, but I as well wish the best to Göttingen and myself, since I expect that the scientific and cordial interchange with you will be a major improvement of my life. If I can help you with something I am happy to do so.

Gauss added, that he is convinced, that Weber will soon become a member of the Society.

As discussed before, Wilhelm Weber had a very close relationship with his brothers Ernst Heinrich and Eduard. The brothers discussed intensely every personal circumstance and every scientific examination. Therefore the decision of Wilhelm to move to Göttingen had a deep significance. Many letters show how much both sides felt the separation. We only show some excerpts. Shortly after having moved to Göttingen, Wilhelm writes to his brother Ernst Heinrich:

In view of the current danger,[420] I repeat our promise to always keep the same interests and consider us as a single family even if we are living now far from each other. I hope that this brings you some relief. You always kept our promise. Now I have for the first time the opportunity to follow your example. Since Eduard will soon face the same situation, I believe that our alliance guarantees us complete safety. I am prepared, dear Ernst, to do everything I can, even at a great distance, so that we can stay in touch from now on.

[420][Note by Heinrich Weber:] Ernst Heinrich was seriously ill.

The brothers kept their promise. Instead of the road between Halle and Leipzig, they faced now the road Leipzig - Halle - Sangershausen - Nordhausen - Heiligenstadt - Göttingen. Sometimes the brothers met at a place in between which they decided before. Sometimes they went on foot the whole distance, since there did not exist trains at that time. Of course as often as before such visits could not take place anymore. In April 1832 Ernst Heinrich writes to Wilhelm:

> I am busy with editing the book by Hildebrandt. There are only three folios left, but it has to appear before the fair. On top of that there are my obligations as a dean. Therefore I cannot make it now to Göttingen. Instead of that I intend to come to Göttingen during my vacation so that we can spend a longer time together. If you can manage to come from time to time to Leipzig our separation will be less painful. During the semester let us each work on his own, so that during the semester break we can exchange our results and plan new researches.

Weber had the obligation to start with his lectures in Göttingen in the winter term of 1831. First he had to officially give back the instruments which he had bought with funds from the government of Prussia. The accounting of this required a lot of his time. He first sent his furniture to Göttingen after he had the guarantee from the government in Hanover that they are exempt from customs duties. Then he went to Göttingen on foot via Weimar, Erfurt and Mühlhausen. A little adventure he recounted with pleasure. He just arrived happily in Weimar and decided to go to the theater in the evening. By chance at this day a larger crowd of students from Jena visited as well the theater. It happened that an unbiased member of a students' corps took a seat next to Weber. He thought that the slim, inconspicuous-looking newly appointed professor in Göttingen was a high school student from Weimar. Looking down self-confidently on him he meant: "Well, you probably will soon enter university." If Weber clarified the matter, we do not know.

One cannot be too much surprised about the remark of the member of the students' corps. In fact, as an eyewitness confirmed, Wilhelm Weber in between his two handsome brothers made the impression of the weakest and most unimpressive one. Nobody was expecting at that time that Wilhelm would outlive his two brothers and reach quite an old age.

Wilhelm Weber was at an important turning point in his life. A new period of his scientific exchange and his scientific researches started.

14.4 First Period in Göttingen 1831-1837

The first matter Wilhelm Weber had to deal with after arriving in Göttingen was to furnish the Physical Cabinet.[421] Moreover, he was busy with preparing his lectures in experimental physics. In connection with them, he intended to re-edit the compendium of physics by Tobias Mayer, as he was requested from the booksellers. However, this never materialized. Immediately after arriving in Göttingen, Weber started his scientific collaboration with Gauss. Despite the age difference of 27 years, the two of them started a true friendship. Some of the handwritten notes of Gauss inviting Weber for lunch are often ending with the words:

[421][Note by Heinrich Weber:] The former Physical Cabinet in Göttingen was situated opposite to the current one. In the same building was the Museum of Natural History. When recently the library got a new building, the old house was teared down and at its place a wing of the library was constructed. It was only after the reappointement of Wilhelm Weber 1849 that he moved to the current Physical Cabinet.

> Nobody else is coming,
>
> sincerely
>
> C. F. Gauss.

On the other hand, Gauss as well often visited Weber. His sister, which did his household during the first years of his stay in Göttingen, complained in a letter to Ernst Heinrich written in June 02, 1832, about the often unprepared invitations Wilhelm made.

> Wilhelm enjoys Gauss every day as long as he wants. Gauss is very lonely and Wilhelm is welcome at every time. Gauss is such a social, educated person that he never talks about science in my presence. He discussed with us many topics from noon until 5 pm. Recently the Privy Counsellor Gauss was here for lunch for three days in a row.

In view of the enormous interest Gauss had at that time for magnetic phenomena, especially terrestrial magnetism, his friend got interested in these phenomena as well. The first outcome of these magnetic studies is the famous paper: Intensitas vis magneticae ad mensuram absolutam revocata. Gauss talked about this work on the meeting of the *königlichen Societät* (Royal Society [of Sciences of Göttingen]) December 15, 1832.[422,423]

Since that time Weber was completely excited about magnetic researches. Partly he did these researches on his own, partly he did it together with Gauss. To walk from the Astronomical Observatory to the Physical Cabinet took about 15 minutes. When working on the same topic this was very annoying, in particular, since some researches on the change of magnetic declination required simultaneous observations. Therefore many times comparisons were needed. Short notes of Gauss concerning magnetic observations show that often messengers were commuting between the two Institutes. In order to improve this unpleasant situation, the first telegraph was invented in 1833.[424] Galvanic currents were used

[422] [Note by Heinrich Weber:] *Göttingische gelehrte Anzeigen* of the year 1832, Vol. 205, p. 2041 and Vol. 206, p. 2049.

[423] [Note by AKTA:] See footnote 55 on page 34.

[424] [Note by AKTA:] Figure (a) of this footnote shows the Astronomical Observatory (Sternwarte) of Göttingen, where Gauss lived and worked, https://de.wikipedia.org/wiki/Sternwarte_G%C3%B6ttingen. Figure (b) show the commemorative plaque. It reads: First electric telegraph of Gauss and Weber, Easter 1833:

(a)

(b)

to transmit messages. Gauss reported in the *Göttingischen gelehrten Anzeigen* in August 9, 1834, volume 128, page 1265 on the recently newly built Magnetic Observatory in Göttingen, after giving details about the establishment of the Observatory and the observations made there:[425]

> We have to mention a fantastic new device which we owe to Professor Weber. Already last year he connected the Physical Cabinet with the Astronomical Observatory by two wires spanned over the roofs of the town. Now this connection gets extended to the Magnetic Observatory. If one includes the galvanometers at both ends, the galvanic current is running a distance of almost nine thousand feet. — The certainty and easiness how one controls by the commutator the direction of the current and the motion of the needle depending on it led already last years to experiments on applications of telegraphic signals. Even with whole words or little sentences they succeeded completely satisfactory. There is no doubt that it would be possible to build a telegraphic connection between two places several miles apart. Of course here is not the place to discuss these ideas further.[426]

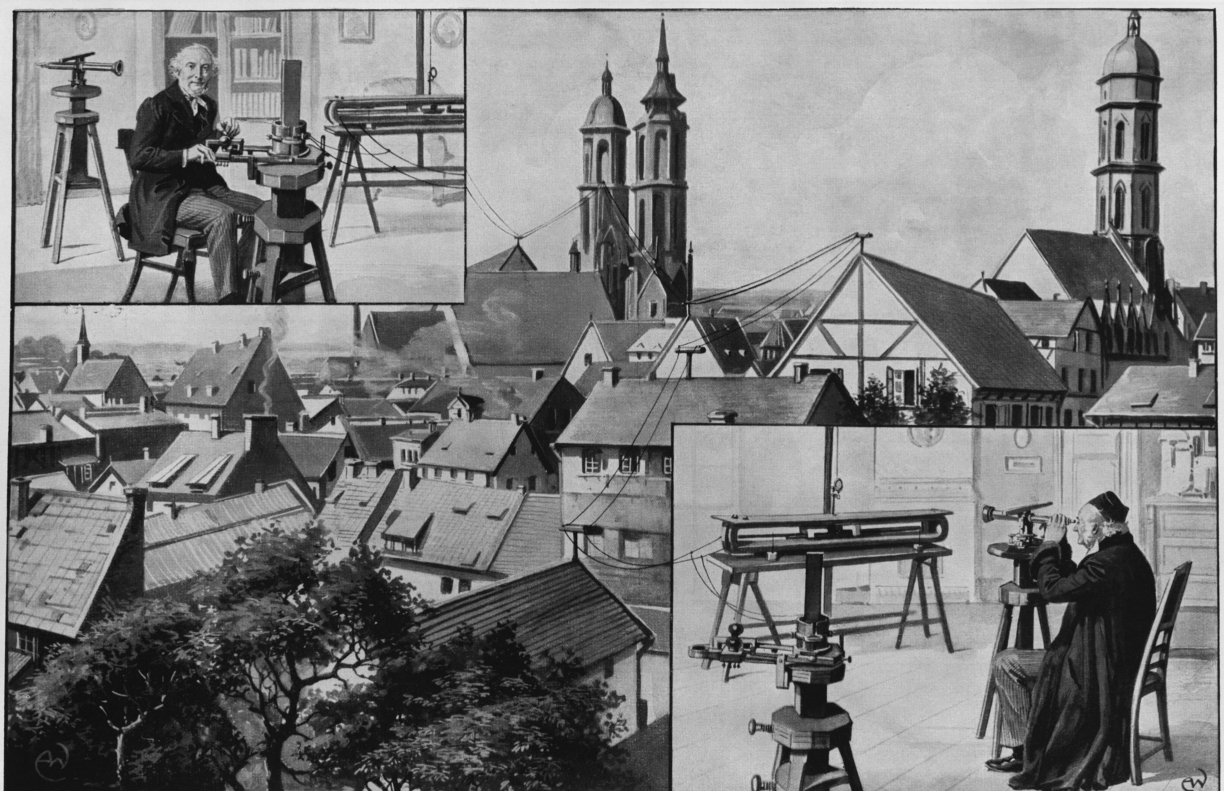
One cannot say that Gauss and Weber invented the electromagnetic telegraphy. Its base are the actions of galvanic currents discovered by Örstedt in 1819 and Arago in 1820. The devices to amplify these actions (Schweigger's galvanometer 1820) and the methods to make them visible and subject to precise observations (Poggendorff's mirror galvanometer 1826)

[425][Note by AKTA:] [Gau34a, pp. 524-525 of Gauss' *Werke*].

[426][Note by AKTA:] Wilhelm Weber and Gauss created in 1833 the world's first operational electromagnetic telegraph, [LB67, Section 66: Gauss and Weber's telegraph, pp. 41-42], [Ano89], [Fey33a], [Fey33b], [Wie60, Chapter 5, pp. 17-20], [Wie67, pp. 85-90], [Tim05], [Wol05], [MRGL10] and https://www.uni-goettingen.de/de/historische-sammlung/47114.html. It was a 3 km long twin lead connecting Göttingen University, where Weber was Professor of Physics, with the Astronomical Observatory (*Sternwarte*), directed by Gauss. This telegraph worked based on Faraday's law of induction discovered two years earlier, [Far32a] with German translation in [Far32b] and [Far89], and with Portuguese translation in [Far11]:

were already well-known in the year 1833.[427] Even before Gauss and Weber, it was suggested several times to base telegraphy on the electromagnetic actions of galvanic currents (Ampère, Barlow, Green, Triboaillet, Fechner, Schilling von Cannstadt), and even some experiments were carried out on a smaller scale. However, in these experiments the galvanic current passed through only rather small distances and one generally believed that the galvanic current gets damped by running through longer distances in such a way that his actions could not be applied anymore for the above mentioned purpose. Only few, in particular Fechner,[428,429] advanced the view that electromagnetic telegraphy is possible through large distances. But there were no experiments confirming this view. Barlow reported that he had carried out experiments with a wire 200 feet long, but had obtained negative results.

There is no doubt that Gauss and Weber deserve the great credit for first constructing a telegraphic connection on a longer distance with just two wires. This proved that galvanic currents can as well be applied for long range telegraphy. Only later one got interested in the history of telegraphy and the first telegraphic experiments became known to a larger audience. Therefore there is no doubt that Gauss and Weber developed their device without knowledge of the experiments carried out before.[430,431]



Friedrich Gauß und Wilhelm Weber beim Arbeiten mit dem von ihnen erfundenen ersten elektrischen Nadel-Telegraphen
Links Weber am Gebe-Apparat, an dem er die stromweckende Spule bewegt; rechts Gauß am Empfänger, durch das Fernrohr auf den Spiegel des Galvanometers schauend. In der Multiplikatorspule unter dem Spiegel die Magnetnadel, ein schwerer Eisenstab. Die Verbindungsdrähte sind über die Stadt Göttingen gespannt und auch am Johannisturm befestigt. Nach einer Zeichnung von A. Wald.
(Zu Seite 18)

[427] [Note by AKTA:] See footnote 52 on page 33. See also [Pog26].
[428] [Note by Heinrich Weber:] Fechner, Lehrbuch des Galvanismus und der Elektrochemie, Leipzig, 1829, p. 269.
[429] [Note by AKTA:] See footnote 307 on page 181 and [Fec29].
[430] [Note by Heinrich Weber:] A detailed history of telegraphy can be found in Zetzsche, Handbuch der elektrischen Telegraphie, Vol. 1, Julius Springer. Berlin, 1877.
[431] [Note by AKTA:] [Zet77].

Weber in particular made a lot of effort to connect the two Institutes, the Astronomical Observatory and the Physical Cabinet, by a telegraphic connection. The following letters of the municipal authorities of Göttingen deal with the permission given to them to run the two telegraphic wires over the roofs of the town as far as the tower of the Johannis church and from there further to the Astronomical Observatory.

> The principal Ebell forwarded the helpful letter of Your Excellency addressed to him to the municipal authorities on the 15th/16th of this month. Your letter anticipated a request to explain the purpose of the equipments on the church tower you did without our previous knowledge. Although we are always willing to do our best for promoting scientific institutions, it is our duty to ask you to kindly provide an explanation on the following points. 1) Are the equipments just for a test or are they supposed to stay there forever? 2) Will it be necessary to give certain persons permission to access the tower at every time and, if yes, who are these persons? 3) Will you supplement the cords by wires and from which metal are they made of? 4) Is it necessary that the blinds you removed from the windows of the tower in future will not come back and the windows have to stay open?
>
> While asking you about your kind answer as soon as possible, we would already like to remark concerning the last point, that the tower is too much exposed to the weather such that we cannot allow to keep the windows open, since this is causing too much damage to the building.
>
> We would like to use this opportunity to express our perfect respect for Your Excellency.
>
> Göttingen, April 18, 1833.
>
> The municipal authorities to His Excellency, the Professor Weber in person.
>
> Signed G. Ebell

> After the explanations from Your Excellency, we happily allow you to leave your equipments on the tower of the Johannis church in the near future in order to carry out magnetic-galvanic experiments. It is fine as well if Your Excellency together with an assistant stays from time to time on the tower.
>
> With our deepest respect.
>
> Göttingen, May 06, 1833.
>
> The municipal authorities of the town Göttingen
>
> G. Ebell

In order to tell the prehistory of the telegraph in Göttingen as complete as possible, we disclose as well the documents which were published in 1887 in the supplement of the *Allgemeine Zeitung*, No. 248, by an expert. They show that Weber had to face many difficulties in order to be able to set up his equipment:

> Documents concerning the prehistory of the invention of the electric telegraph by Gauss and Weber.
>
> I. Record of the board of trustees from the University of Hanover.

"Professor Weber from Göttingen told us that the room used to carry out optical[432] experiments in the first floor of the Academic Museum is very limited and too dark. He asks for a different room in this building like the one where up to now the collection of paintings of Flügge were stored. Before we can make a decision, we wait for the report of the Academic Museum if and how this is possible under the current conditions.

Hanover, February 10, 1832.

Royal British-Hanoverian board of trustees of the University.

Arnswaldt to the Academic Museum in Göttingen."

II. Anwer of Blumenbach. Handwritten draft.

"To the Royal board of trustees of the University. Report of the senior medical officer of health Blumenbach, February 27, 1832, concerning the rooms of the museum dedicated to Natural History. Immediately after having received on the tenth of this month the document of Your Excellency concerning a wish of Professor Weber to get a room in the middle floor of the museum, I immediately discussed this with the other custodians of the museum Privy Counsellor Hausmann, Privy Counsellor Osiander and the assistant Dr. Herbst. Dr. Herbst then wrote the following report which the two other custodians and myself strongly support."

III. (Handwritten) report of Dr. Herbst.

"1) The paintings of Flügge were not stored in a separate room, but were kept in two rooms which could be used less at various places, some of them beside each other, some of them in front of each other. 2) In the rooms where the paintings are located, a large part of the ichthyological collection, as well as parts of the osteological and botanical collections, was always on display. In recent times, due to the limitation of the remaining space, a part of the highly valuable collection of specimens has also had to be added. 3) Many of the objects incorporated little by little to the Academic Museum are piled up in the other rooms waiting until the display cabinets and showcases needed are set up. 4) In addition, one of the two rooms was used for the experiments and works of the Academic Museum. 5) By separating one of the two rooms, the communication between the middle and upper floor of the Academic Museum would be disturbed. Moreover, the visit of the museum would become complicated. 6) However, maybe a rather large, very bright hallway in front of the two rooms in the middle floor of the Academic Museum or a similar hallway in the upper floor of the museum might offer enough room and light for carrying out physical experiments."

IV. Handwritten votes.

"I checked the rooms in the museum and completely agree with the report of Dr. Herbst. Your most obedient Hausmann, Göttingen, February 15.

The museum would loose one of its best rooms, if one separated the hall requested by Professor Weber. The hallway of 16 feet length and 12 feet width has two windows and might be very suitable for physical experiments. I therefore completely agree with

[432][Note by Heinrich Weber:] This expression instead of "electric" is used several times due to insufficient knowledge of the matter.

Dr. Herbst and Privy Counsellor Hausmann. Your most obedient Osiander, February 20, 1830 [sic]."

V. Letter from the board of trustees of the University.

"A copy is sent to the Academic Museum in Göttingen. Referring to our writing from the 10th of last month, we inform Professor Weber that no room in the Academic Museum can be offered to him in order to carry out optical experiments, since no room in this building can be dispensed with. However, a hallway in the middle floor of the museum which is said to be quite spacious, very bright and well-equipped, as well as a similarly designed room on the upper floor, can be used by Professor Weber for carrying out optical experiments in the future. We therefore leave it to him to make use of it according to circumstances. The Academic Museum is informed."

Hanover, March 06, 1832, Royal British-Hanoverian board of trustees of the University.

Several years ago, the above documents were saved from a package of documents thought to be of no value and incorporated into the library of the University. They appeared in an article in the supplement of this newspaper of August 12 of this year with the title "The prehistory of the introduction of the electric telegraph in England", celebrating the fiftieth anniversary of this introduction (July 25, 1837) on a long stretch of the North West Railway in London using the signaling of Gauss and Weber.

Blumenbach, Hausmann and Osiander had no idea that the matter concerned an invention which now rules the world. They didn't show much cooperation by delegating the report to their assistant Dr. Herbst, in order to avoid to give a completely negative answer. Professor Herbst is now the oldest member of the University of Göttingen. Thanks to him, one found the above solution. By his modesty, Weber was satisfied with the scanty hallway. Only few people now still know where this hallway was located. Hence many errors spread around. Therefore we give a brief description where and how the facility was made. Usually one says "in the Physical Cabinet", and in the current Physical Cabinet there is as well attached a marble plaque inscribed:

First electric telegraph, Weber-Gauss 1833.

However, at this time the head of administration von Werthoff lived in this house and even later it was known by his name. The older modest Physical Cabinet where Tobias Meyer and Weber in his first years worked and lectured was located at the northern part of the lower floor of the Museum of Natural History. After Weber moved to the current Physical Cabinet, Hausmann stored there the mineralogical collection. The museum was demolished in 1881 in order to enlarge the library. At the place of the former Physical Cabinet lies now the northwestern corner of the library. In the above mentioned hallway in the upper floor of the museum in former times hanged the paintings of the professors in Göttingen, which are now in the assembly hall. The hallway was crudely arranged for the electric experiments. The wire was spanned from there over the old library to the tower of the Johannis church going down on the other side to the pharmacy of the University, then over the roofs and streets to the most southeastern corner of the wall at the Geismar gate, where it was attached to a pole on a tree, descending to the little house erected for this purpose in front of the Astronomical Observatory. No iron nail was used for this little house. Instead of that, all nails and even the key were made of copper. Originally one was afraid that the several hundreds of rifles in the barracks nearby would exert an attracting force. However, this was not the case. Gauss was operating in this little house and Weber at the other end in the museum. The experiments were continued until the most splendid success was reached and one of the most crucial inventions of this century was made.

Strictly speaking, the marble plaque at the new Physical Cabinet does not indicate the precise location. However, since the actual location is floating in the air, so to speak, above the little door in the empty space of the library, it might stay were it is. In particular, after the wire was removed which was laid from here over the street to the corner of the house of Benfey. Several people might remember how, after many years, the old wire was smashed to pieces in a heavy thunderstorm. In fact, it is no fairy tale that one fiery piece hit the dress of the wife of Professor Grisebach in the Johannis street.

There are no written records about experiments in this time. Only little letters of Gauss could be found in the manuscripts left behind by Weber. Although they have no real value, we mention them here in view of the interest of the matter. In a little undated letter Gauss writes:

> Unfortunately I can only send messages to you, but not receive yours, since the magnetic bar was removed yesterday and replaced by a 20 pound heavy wooden bar. However, I send you the message in order that you have in advance a date for the position of the clocks. I hope that tomorrow the magnetic bar is restored again. G.

In a letter from August 28 at 7:30 am without specification of the year one can read:

> A lady who would like to admire our actions at a distance and is already leaving this noon has an appointment this morning in the Astronomical Observatory. Dear Weber, could you please send a galvanic current through the wire at 10 am? Since you know precisely the position of the clock at the observatory compared to your one, you could start the current exactly at 10:00:00 am. Some commutator changes in an interval of time of 43 seconds could probably increase the admiration. The comet I have seen and observed last night.
>
> Sincerely yours, Gauss.

In the year 1834 "words and little sentences" could be sent by wire. A problem was that through the use of Voltaic plate couples the needles started oscillating wildly, in particular, since at this time the set-up of a damper was not known. This unpleasant situation motivated Gauss to apply induction currents by displacing wire coils with respect to the poles of heavy magnets. In a report of a lecture by Gauss held at the *Königl. Societät* — Royal Society [of Sciences of Göttingen][433,434] one reads:

> The original procedure (for telegraphing) was replaced in 1835 by a different one which Privy Counsellor Gauss discovered motivated by the laws of induction and which was much better than the previous one.

The new method was indeed a great success. In a little letter which Gauss seems to have written after the first experiments based on this new method were carried out, one can read:

[433] [Note by Heinrich Weber:] *Göttingische gelehrte Anzeigen*, October 30, 1837, No. 173, p. 172.

[434] [Note by AKTA:] [Gau37b, p. 356 of Gauss' *Werke*].

> To my great joy, the induction by inserting the red wire into the coil appears clearly, amounting to about 12 scale parts.

This new device allowed the needles in both Institutions to make simultaneously small oscillations to the right and to the left as much as one liked, without causing them to vibrate too much. By combining these oscillations to the left and right, letters could be constructed. The equipment at that time transmitted 4 letters per minute, later on 8 letters. Weber connected the telegraphic device as well with an alarm clock which, with the help of several consecutive current pulses, started to ring and indicated the start of the telegraphing.

It is fair to say that from this moment on the telegraphic devices achieved maturity so that they could start to spread to the public. In the first great attempt at submarine telegraphy, the telegraphic connection between England and North America, the telegraphic procedure described was used for a long time because the usual method with stronger currents resulted in disturbing charging phenomena.

The wish to make the telegraph system, which had only been tested on a small scale, useful for public life, had already a chance of being fulfilled in 1835. In this year one started to build the Leipzig-Dresden train. Ernst Heinrich Weber, Wilhelm's brother, suggested to the board to build a telegraphic connection among the stations of the train according to the Göttinger scheme. The report ends with a clear insight on the current development of electric telegraphy with the words:

> If once the earth will be connected by a net of railways covered by telegraphic lines, this net will serve similar purposes as the nervous system in the human body transmitting partly movement, partly the propagation of perception quickly as a flash.

Gauss and Weber had only accumulated experience with the telegraphs constructed by themselves. At this time they erroneously believed that for telegraphic devices the earth should be considered as an insulator. In fact experiments on the resistance of conductors showed that the earth possessed a resistance several million times bigger than the one of iron. This seems to refer to the specific resistance. In a letter to Erdmann,[435] member of the Leipzig-Dresden Railway Society from July 12, 1835, Weber proposes to use the tracks without further isolation for the ingoing and outgoing wire. Gauss however in a letter[436] from September 15 of the same year, thinks that this proposal is difficult to carry out in practice, since the axis of the wheels of the car will serve as a conductor. For this reason, he proposes to connect the stations through a separate 1.6 mm thick copper wire or 3.8 mm thick iron wire and use the tracks only as return wire. The matter got delayed since in the board one started to doubt the feasibility of the project. In order to remove these doubts, Weber wrote in March 1836 in a second report to the board:[437]

> Gauss completed the theory of electric telegraphy.[438] The distance of the action, the strength of the wires, the currents and things like that can nowadays be computed with the same precision as a lunar eclipse.

[435] [Note by Heinrich Weber:] Records of the Leipzig-Dresden Railway Society, Vol. I, p. 2.

[436] [Note by Heinrich Weber:] Records of the Leipzig-Dresden Railway Society, Vol. I, p. 5.

[437] [Note by Heinrich Weber:] Records of the Leipzig-Dresden Railway Society, Vol. I, p. 13. See also M. M. Freiherr von Weber, Das Telegraphen- und Signalwesen der Eisenbahnen 1867. Here the proposal of Weber was criticized severely from a technical point of view.

[438] [Note by Heinrich Weber:] Insofar as it related to the experiments carried out in Göttingen.

He meant that the facility is "not an experiment but a warranted enterprise." He claimed it for sure that

> two copper wires of a diameter of three quarters of an inch laid through the ocean to East India or America can serve as a telegraphic connection to these countries.

The facility between Leipzig and Dresden was computed by him to cost only 500 thalers, since no special wires were needed. Although Weber was wrong at this time when he deduced the behavior of the ground from the specific resistance, today's telegraphy proved his last assertions to be true.

The board started doubting the matter even more, after telegraphic experiments carried out by Steinheil showed a negative result.[439] Steinheil was urged by Gauss to develop further the electric telegraphy from a technical point of view. This led him to the discovery that the earth behaves like a conductor and can be used as the return path. He also was the discoverer of the first ingeniously devised printing telegraph, which is the reason that his name will always be mentioned at one of the first places in the history of telegraphy. The whole issue came to an end in October 26, 1837. The board decided against installing an electric telegraph. This was due that shortly before Master Hülsse, the later director of the Dresden University of Technology which had examined the telegraphic facilities of Göttingen in depth, made a very high cost estimate for a telegraphic connection with two insulated copper wires.

The magnetic researches[440,441] for which Gauss, with his nature-penetrating mind, built a completely new base gained a particular significance since they inspired the foundation of a Magnetic Society whose intention was to examine the effects of terrestrial magnetism at different places of the surface of the earth. At the beginning, except Göttingen, only Enke in Berlin participated. However, soon Leipzig, Copenhagen and Braunschweig joined and the number of places at which corresponding observations were made grew so quickly that in the Society not only geographically conveniently distributed places in Europe were represented, but as well places outside Europe. Already in October 23 and 24 in Göttingen and Leipzig corresponding observations were made. They were repeated with great care later in October 01 and 02 by Weber himself who spent his vacations in Leipzig.[442,443] Referring to them, Gauss writes in October 03.

> Dear Friend. Thank you very much for the report on the magnetic observations in Leipzig from September 23 and 24. The records of your brother and Professor Möbius are still missing whose inspection this time at some places is desirable since only very weak oscillations happened during these two days. Here the observations are rather painful since my son is not back yet and Professor Ulrich did not participate due to illness of one of his children. — I am looking forward very much to your return to Göttingen, dear friend. My loneliness increased since my youngest daughter is not here either. She is on the countryside since eight days.
>
> With my very best regards, C. F. Gauss.

439[Note by AKTA:] See footnote 316 on page 183.

440[Note by Heinrich Weber:] Gauss reports on these in the *Göttingischen gelehrten Anzeigen*, March 07, 1835, Vol. 36, p. 345 and October 30, 1837, Vol. 173, p. 1721.

441[Note by AKTA:] [Gau35] and [Gau37b].

442[Note by Heinrich Weber:] Poggendorff's *Annalen*, Vol. 33, p. 426.

443[Note by AKTA:] [Gau34b].

In order to keep track of the different observations, Gauss and Weber published annually the most important findings under the title "Resultate aus den Beobachtungen des magnetischen Vereins" ("Results from the Observations of the Magnetic Society").[444] The first volume appeared in 1836 and the last one in 1841. At the same time the *Resultate* offered the opportunity to discuss topics closely connected to the magnetic observations. Gauss himself published fourteen papers, among them the famous foundational ones on the potential and the general theory of terrestrial magnetism.[445] Weber contributed twenty three papers among them the one on the tangent galvanometer, the electrochemical equivalent of water, unipolar induction and the rotation inductor.[446] These papers became all seminal for later researches. Closely connected to these researches is the "Atlas of terrestrial magnetism" published by Gauss and Weber 1840, which was mainly edited by Weber and Goldschmidt.[447]

Despite Weber's enormous interest on the new field of research on magnetism opened up by Gauss, he simultaneously was busy with physical-physiological experiments together with his younger brother Eduard in the years 33-36. These results were published in Göttingen in 1836 in the book "Mechanik der menschlichen Gehwerkzeuge" (Mechanics of the Human Walking Apparatus).[448] Several times the younger brother Eduard stayed for longer time in Göttingen, in order to carry out experiments on walking. Also from the military side Gersdorf in Berlin provided valuable data on the size of steps, the rhythm, the size of a usual march and a fast march, and so on. The data were growing so quickly that Poggendorff wrote to Weber February 10, 1833:

> That your theory on walking grew so much that it fills a whole book is an additional obstacle for the next generation of this world who has to learn how to move the legs according to art.

The years 1831 until 37 passed for Weber under arduous scientific activity. Then a new event occurred which profoundly changed Weber's life. This event had not just a huge impact on science, but as well on politics and led to fervent debates even outside Germany. This event was the dismissal of the seven Göttinger Professors.

14.5 The Interim Period 1837-1843

Under the rule of William IV, Hanover was united with England. Shortly after Ernst August as a German prince moved to his capital in June 20, 1837, he declared in July 5 that the constitution from the year 1833 is not valid anymore, but instead of that the old constitution from the year 1819 has to be reintroduced again. First everybody cheered about independence, however soon educated people especially at the University of Göttingen in connection with the hundred years anniversary of the University in September 17 started raising some

[444][Note by AKTA:] See [GW37], [GW38], [GW39], [GW40b], [GW41], [GW43] and [GW40a].

[445][Note by AKTA:] [Gau39] with English translations in [Gau41a] and [GT14], and Spanish translation in [Gau21c]; and [Gau40] with English translation in [Gau43].

[446][Note by AKTA:] [Web39c]; [Web40]; [Web41a] with English translation in [Web21j]; and [Web41d] with English translation in [Web21m].

[447][Note by AKTA:] Carl Wolfgang Benjamin Goldschmidt (1807-1851) was a German astronomer, mathematician, and physicist. See [GW40a].

[448][Note by AKTA:] [WW36] with English translation in [WW92].

doubts. First Weber didn't care much. In a letter from August 13 in which he invites his brothers to the anniversary he writes:

> The political circumstances will not affect the anniversary except that a few fewer princes will come here. Just now, where the enemies of higher education make plans against the universities and would like to make them national schools as in Austria, this celebration arranged with royal pomp will have a political impact on the whole of Germany. It will testify loudly that the universities are not obsolete institutions, but that they are advancing with the times, leading the banner of civilization that all of Germany is following.

In fact, the secular ceremony was celebrated with great splendor in the days from September 17th to 19th. Even the king Ernst August participated at the first day at the festivities. Apart from the ministers from Hanover, Alexander von Humboldt and many deputies of other universities took part. At this occasion Gauss gave his above mentioned talk on magnetic experiments at a solemn meeting of the Society in September 19.

In October 30, 1837, the parliament was dismissed and the "Royal Servants" released from their oath to the constitution of the year 1833. In November 01 the civil servants and later on the advocates as well were obliged to take a new oath. Hereupon the seven Professors from Göttingen, Albrecht, Dahlmann, Ewald, Gervinus, Jakob and Wilhelm Grimm, and Weber filed a joint petition written by Dahlmann to the board of trustees of the University.[449] In this petition they declared that they are bond by their oath to the constitution from 1833. Therefore they cannot accept a different parliament as legal, and consequently they are not able to take the new oath. This famous entry reads as follows:

> To the Royal board of trustees of the University.
>
> Göttingen, November 18, 1837.
>
> Most subservient petition of some members of the University concerning the Royal Edict from November 01.
>
> The undersigned humbly feel compelled in their conscience to submit their respectful declaration to the High University Board of Trustees regarding the content of the Royal Edict of the 1st of this month.
>
> Despite their due awe to the Royal word, the petitioners cannot convince themselves that the constitution is illegal and therefore invalid since the previous king did not found the whole content of it on contract, but at its proclamation did not authorize

[449][Note by AKTA:] Wilhelm Eduard Albrecht (1800-1876) was a German constitutional lawyer, jurist, and docent. Friedrich Christoph Dahlmann (1785-1860) was a German historian and politician. Georg Heinrich August Ewald (1803-1875) was a German orientalist, Protestant theologian, and Biblical exegete. In 1830, Ewald married Wilhelmine Gauss (1808-1840), the daughter of Gauss. Georg Gottfried Gervinus (1805-1871) was a German literary and political historian. Jacob Ludwig Karl Grimm (1785-1863) was a German author, linguist, philologist, jurist, and folklorist. He was the older brother of Wilhelm Carl Grimm (1786-1859), a German author and anthropologist. The Germanist brothers were famed fairy tale and folk tale writers and storytellers, known together as the Brothers Grimm. A painting of the so-called Göttingen Seven appears in this footnote, taken from https://en.wikipedia.org/wiki/G%C3%B6ttingen_Seven. It is a Lithograph from Friedrich Eduard Ritmüller, 1837/1838, after a drawing by the Swedish artist Carl Gustav Adolph Rohde (1806-1873). Top row: Wilhelm Grimm, Jacob Grimm. Middle Row: Wilhelm Eduard Albrecht, Friedrich Christoph Dahlmann, Georg Gottfried Gervinus. Bottom Row: Wilhelm Eduard Weber, Heinrich Georg August Ewald:

some propositions of the parliament and made some changes not approved by the parliament. The accepted legal rule that the valid thing cannot get destroyed by the invalid one implies that not the whole constitution is invalid, but only some parts of it. The same case would occur if in the constitution rights of the agnates were violated. The principle that each amendment of the constitution is subject to agnatic approval could not be established without causing a big threat to Royal rights. The most subservient petitioners would like to remember that concerning the claim that Royal rights were violated, the Royal edict from 1833 aimed in particular to ensure these rights. Furthermore the German Federal Assembly composed a committee to deal with these kind of questions. This committee never had objections but instead of that applauded to the constitution of this kingdom as an example of prudent moderation and circumspection. Therefore the most subservient petitioners after pondering in earnest on the importance of this case came to the conclusion that the constitution is still valid. Hence they cannot let it tacitly happen without violating their conscience that the constitution perishes just by way of power without further examination and vindication. Instead of that, their duty in view of their oath to the constitution is neither to participate on the election of a deputy nor accept an election or a parliament on a different legal base.

The reason that the most respectful petitioners of the University sign as individuals, is that they would like to avoid conflicts which every hour can bring as soon as possible. They do not doubt in the least that their colleagues share the same opinion. In their official duty they always warned students from political extremes and tried their best

to foster their allegiance to the government. However, their success is not only based on their scientific value, but as well on their integrity. If their students experience that they do not take their oaths seriously, the bless of their effectiveness will be seriously damaged. What value has an oath of allegiance and obeisance for your majesty the king, if it is taken from subjects who just violated their previous oath?

F. G. Dahlmann, E. Albrecht, Jakob Grimm, Wilhelm Grimm, G. Gervinus, H. Ewald, Wilhelm Weber.

In November 28, 1837, Ernst Heinrich writes to Wilhelm:

All sympathizers read the public declaration of the seven professors with great joy. This is brave. I am eagerly awaiting the effect of this step.

One did not need to wait long to see the effect. Already in December 14 the seven Professors were dismissed and Dahlmann, Jakob Grimm and Gervinus even had to leave the country. The later ones almost had no time to bring their affairs into order. They first fled to the little town Witzenhausen in Hessia and later on they moved to Kassel.

The disturbances in Hanover already led before to protests in Baden, Bavaria, Saxonia, Hessia, Braunschweig and Württemberg. But now the whole of Germany took part on the destiny of the "Seven". Scholars and commoners as well wrote letters of sympathy and appreciation and poems celebrated their deed. But beyond just moral support, in the whole of Germany committees were built whose purpose was to collect money so that the seven professors could earn their previous salary until they found a new employment. Especially Leipzig took the lead. Berlin, Hamburg and other towns followed its example. First the seven hesitated to accept this offer, so that Ernst Heinrich Weber wrote to Wilhelm in January 07, 1838:

I am sorry that you have concerns to accept the donated money. I am afraid that this will slow down the donation. I think it is quite honorable that such a donation takes place not just from party members, but from moderate people who often are not inclined to politics. I strongly advise you against accepting to teach as private lecturers in Göttingen or to finish your lectures. It would be a different thing if one rehabilitated you and abstained from taking the new oath. It would be a disaster if one allured you and drove you crazy. In my opinion, the seven professors can only carry out steps together.

To remember the magnanimous deed, we cite part of the document, the committee wrote in December 1842 in Leipzig,[450] when it dissolved after having achieved its goals.

When we reported in the beginning of this year about the status of the Göttinger issue, we could not hope yet that so soon our goal would be achieved. Meanwhile Privy Counsellor Dahlmann, the only one left for which our Association was still taking care of, found a secure new employment. The goal of our Association is therefore completely achieved. We have to be proud of our common home country that it was possible to achieve our goal without the need of turmoil. In the worst case we had

[450][Note by Heinrich Weber:] The committee of the "Göttingen Association" was built by Dr. M. Crusius, chair, E. Hirzel-Lampe, secretary and cashier, A. Dufour Féronce, Professor Erdmann, Gustav Harkort, Dr. Härtel, S. Hirzel, Karl Reimer, G. L. Preußer, Professor E. H. Weber, Otto Wigand.

> been still able to continue the honorable national support of the Göttinger gentlemen for a longer time.
>
> Many people supported us generously and promised continuous assistance. The total amount donated to the committee added up to 22357 Reichsthaler. Most of the supporting documents concerning the use of this money is at your disposal. As you know, Professor Wilhelm Weber waived a lot of the money granted to him in favour of his colleagues. However, after the Association reached its goal, the reason for this waiving disappeared and therefore we thought it appropriate to donate him the 1400 Reichsthaler he would have received before his employment at Leipzig.[451] After the deduction of this sum, 2404 Reichsthaler remain. — We propose to donate the 2404 Reichsthaler to the seven professors, so that they can freely use them maybe to build a foundation to remember the event at Göttingen, maybe to grant stipends at one or several of the universities were they found protection and employment after their dismissal from Göttingen. We mention in passing that strictly speaking only the professors Ewald and Gervinus have a legal right to this money, since this money only heaped up by their waiving in favour of their colleagues. — After having fulfilled this last obligation, we decided to end our activity as the committee of the Göttinger Association and to store the documents of our negotiations at the library of the local university.
>
> We take the opportunity to thank you for your trustfulness and kindness and sign with great respect.
>
> The Committee of the Göttinger Association, Leipzig, December, 1842.

Wilhelm Weber as well received financial support from private persons. Professor Fritzsche in Halle wrote in January 17, 1838, to Weber:

> Friends and former colleagues from Halle, most respected Professor, offer you 200 Thaler per annum. Every quarter 50 Thaler are paid to your disposition as a proof of our respect for you and your unforgettable father, which all of us called Father Weber. Please tell us how we can transfer this money to you. We do not know if you are still in Göttingen.

How far the sympathy for the Seven reached, little events show clearly. When Dahlmann moved from Kassel to Leipzig, he stayed overnight in the guesthouse "Stadt Rom". It turned out that the three daughters of the innkeeper were all subscribers of the Göttinger Association. Mr. Ries from Leipzig immediately offered Dahlmann free accommodation. As long as Weber stayed in Göttingen unemployed, he became an honorary member of the Literary Museum having the rights of an ordinary member without the obligation to pay fees. The Dieterich family in whose house Weber lived at this time sent back his rent and so on. During this time (December 28, 1837) Weber received the honorary appointment of a *Doctor medicinae* from the medical faculty in Königsberg, "since thanks to his deserving researches the physiology as well received treasurable enrichment."

From the many letters Weber received, we only mention a few of them. Shortly after his dismissal, E. H. Weber wrote to Wilhelm:

[451][Note by Heinrich Weber:] Weber donated his money to a foundation at the University of Leipzig stipulated to support scientific researches with its interests.

> We were prepared to the blow you wrote. For you it is very hard to get torn away from Göttingen with Gauss and your physical laboratory to whom you dedicated so much effort in many years. But you cannot regret to have acted as an honest man faithful to his oath. The example of the Seven has an effect to the whole of Germany and the results of this exemplary deed cannot be foreseen yet. It will have a huge impact. It would have been outrageous if justice without defending herself gave in to power. What concerns the subscription, it makes a lot of progress. Also in Berlin it started. Let me know when the money is to be sent for the three expelled persons. As soon as you come to Leipzig you will be very welcome.
>
> With kind regards, Ernst.

Similarly Steinheil wrote from Munich in December 25, 1837.

> Dear Weber, it is impossible to tell you how deeply and sadly we all are moved by the calamity which hit Göttingen and you. Let us hope, that God rewards what humans do out of their most sincere conviction and that the future will heal wounds stroke by the present. What will do Gauss? Is he staying in Göttingen?
>
> Your sincere friend, Steinheil.

Many people thought that Gauss, the most brilliant star of Göttingen, and other professors due to intolerable political circumstances would leave Göttingen as soon as they have the opportunity to do that. Concerning this matter, E. H. Weber wrote to Wilhelm:

> Dahlmann got news that the donations in Hamburg are quite gorgeous. The same happens in Berlin and Leipzig. In the Vogtland and the Ore Mountains there are donations by the poorest people like stocking knitters and so on. — I am very much against half-hearted measures. Therefore I think it is best you leave Göttingen soon. I hope that Gauss, you and Ewald can find a new employment in Berlin, Bonn or somewhere else, as soon as one knows for sure that Gauss will accept an offer.

All this kind of opinions were disproved by later events.

Concerning Gauss, he led a secluded life. Always pondering about mathematical and physical problems, he was hardly interested in politics. Moreover, as a sexagenarian, he had a strong antipathy to move to a different place. In a letter to A. von Humboldt, Gauss writes:

> To start somewhere else afresh is in my age quite difficult. At least I would lose several years which in my age are quite precious.

Probably in the scientific conversations between Gauss and Weber politics only played a minor role. In fact, when Gauss tried to undo the dismissal of Weber and keep him in Göttingen, he did not judge Wilhelm Weber correctly in political respect by describing him in letters to influential people as a harmless person not interested in politics. On the contrary, Weber was quite interested in politics during all his life and could become quite agitated by discussing political questions. The situation for Weber was quite different than the one of Gauss. As an unmarried young man at the age of 33, he often kept company with the families of other professors where pressing questions of the day were discussed. In the houses of Ewald, Grimm, and Kraut, he always was welcome as a guest. Many letters addressed to Weber from Dorothea Grimm or Gauss daughter Minna Ewald prove this.[452] The letters from this time also contain lively political discussions with his brother Ernst Heinrich. For example in June 09 (1838?) Wilhelm writes concerning current politics in Hanover:

> The news from my last letter on the situation here needs some correction. Here the elections turned out quite badly and did not stir any interest at all, the opposite happened in the provinces. The Chamber seems to support with vast majority the constitution as proved by the presidential election, as you probably read in the journals. Concerning the negotiations you will read little in the journals since the Hanoverian Journal has to be quiet about that and according to federal law other journals are only allowed to inform about the negotiations in the Chamber what the journals of the state contain. The silence about it is therefore a clear sign what the matter is. One conjectures that the Chamber will carry out such decisive steps that it will soon be dissolved. Unfortunately the first Chamber will do nothing and therefore the effort of the second Chamber is useless. Several people think it would be better if under such circumstances the second Chamber would act less wildly. Anyway it is quite a relief that the unethical behaviour is felt by many so deeply and is outspoken. By this the Ministry will get inclined to carry out new violent measures and the sickness of the present gets even more increased. The hope for a future betterment is postponed. There are news about a petition from the president to the Chamber concerning the constitutional issues. Jacobi, the previous president of the Chamber, who established the principle that one has to have the courage to ignore any legal issue, this time only got 9 votes. The current president is senator Meier from Lüneburg.

Often the question was raised why only seven professors participated at the protest. This question Weber himself answered at the age of 85 on the request of the superintendent Schuster in Hanover.[453] Schuster discovered a letter of the philosopher Ritter from the year 1837 in which he complained that he got to know about the protest just after its publication. Weber writes:

> Among friends one often discussed this on common walks. There was no secret concerning our opinions and they spread around so that nobody can complain not to have known about them. The matter was in the air at this time. Who wanted to participate, could do it. But one knew that one was observed and therefore one avoided to take the lead to further negotiations, which could have been interpreted as propaganda. We were aware of the consequences of a possible declaration and could not take the responsibility to force others to join. It was a protest from conscience which developed in the heart. When I signed I did not know who and how many would do this as well. I myself in particular discussed this with Albrecht whose opinion as a professor of law I joined. He informed me about the protest of Dahlmann with which I agreed and therefore signed. Ritter joined the faculty in Göttingen just a year before and it would have been strange to force him to join these disputes which were alien to him. It was up to each individual to seek connection with his or her own convictions.

[452] [Note by AKTA:] See footnote 449.

[453] [Note by Heinrich Weber:] Zeitschrift des Historischen Vereins für Niedersachsen (1889).

Moreover, soon after the protest of the Seven, on December 13, 1837, when the matter was presented in public journals as if the Seven had been abandoned by their colleagues, six other teachers of the university, Müller, Kraut, Ritter, Thöl, von Leutsch and Schneidewin, declared that they had never expressed disapproval of the action of their colleagues who had been removed from office, and in addition a considerable number of citizens either did not take the new oath or only signed it with reservations. The government of Hanover did not take notice of these events probably because of the great stir the dismissal of the Seven already had in Germany.

Naturally the dismissed professors got even closer to each other than they already were before. Albrecht, Ewald and Weber first stayed in Göttingen, while Dahlmann, Jacob and later as well Wilhelm Grimm with his wife, the latter for a longer period, took residence in Kassel. A busy exchange between Göttingen and Kassel developed. Often Weber went there on foot or by carriage. One was afraid at this time to exchange political views by mail and preferred to discuss them orally. Weber wrote to his mother:

> You will easily understand that I could not cut immediately the friendly relations which became even stronger due to the imminent danger. As you know I still have various collaborations with Gauss which keep me here.

Since the Seven could not accept their dismissal as legal, they decided to sue the Ministry of the king to pay their salary further. However, their lawyer Dr. Grefe in Göttingen lodged the complaint not until December 1838. After several years of negotiations, no result could be achieved. Although the complaint was accepted by the judiciary of Hanover, the Ministry of the king pointed out that the issue is not a legal but a governmental matter and requested the judiciary to declare itself incompetent. However, the judiciary refused to do that. Due to this conflict which was strongly related to the constitutional dispute, the process had to be postponed. As a consequence some persons involved published rather personal political pamphlets. Therefore E. H. Weber wrote to his brother:

> The whole of Europe is following this issue. A pamphlet written by one of the seven professors should express the character of the seven professors but should not be too personal. It is fine if it contains the key idea that the reason for this step is not to get the leadership of a political party, but to provide an example for those who due to moral reasons do not want to perjure themselves. Dahlmann is working on a treatise, too.

First one could hope that a compromise between the parliament and the government could be found so that it became possible to reinstall the dismissed professors. But as longer the conflict continued this hope waned more and more. Already the government of Hanover started to find solutions for the annoyances the university faced due to the dismissal of the Seven. Weber wrote to his brother:

> The issue of the Physical Cabinet will probably soon find a solution. How this solution looks like is at the moment completely unclear. The board of trustees required a report from the senate. The senate unanimously requested from the board of trustees that the Cabinet stays devoted to the joint experiments of Gauss with myself. Ewald wrote from London. At evenings I in turn visit Gauss, Ewald, Grimm's, Albrecht, Kraut, Müller.

Ewald which at the beginning of the year 1838 had made a travel to London accepted around Easter an appointment in Tübingen. Weber therefore lost the direct intercourse with one of his friends. However, the friendship was kept assiduously by mail and later in 1849 after the return of both of them to Göttingen continued until Ewald passed away in 1875. Weber became lonely after all of the Seven except him and Albrecht left Göttingen. The old plan to visit scientists abroad in London and Paris materialized in March 1838. Weber was traveling via Leipzig to Berlin. Here Poggendorff joined, to first visit London and later Paris. Both cosmopolitan cities were very stimulating for Weber, so that he only returned to Göttingen in August 1838. The company of his close friend Poggendorff was very precious for Weber. However, this travel almost had a bad impact on their friendship. As important the relationship of Weber with Gauss was, not everybody in Berlin appreciated it. One thought that it might be an obstacle for Weber as a scientist of his own. This opinion is also expressed in the following letter Poggendorff wrote to Weber in April 06, 1839, after a longer break.

> I am sorry for the late reply to your letter, in which you restarted the exchange between us after a long break. - - - You will surely believe that I as well after our separation in Cambridge thought quite often on you and was several times about to stop the long silence. - - - That you spend your efforts on promoting science despite the difficult exile even increases our esteem for you. I myself and all your friends here sincerely hope that this brings you some recompense for all the privations you have to suffer. You can also rest assured that none of your local friends' interest in you and your activities has diminished in any way. Also the disagreement between you and myself and others is just due to our great interest in you. As others, I have the opinion that you harm yourself by just assisting Gauss in his research instead of doing your own one. So Gauss got the Copley Medal, where you were barely mentioned.[454] However, I cannot deny it that among the current hopefully temporary circumstances your alliance with Gauss is quite fruitful. But it would be good, that you can again be a researcher on your own, if the circumstances have changed.

There are no notes about the travel itself. Only a letter of Dirichlet shows, that Weber had discussions with Dirichlet's friends, when he was in Paris.

During this travel Gauss took a lot of effort to undo the dismissal of Weber in order to keep him in Göttingen. For this purpose Gauss contacted people close to the king. At the beginning he was quite successful as the following letter from March 12, 1838, clearly shows. Gauss wrote to Weber:

> My dear Weber, the Legation Councillor von L. already two days after your departure arrived here. He wanted to come earlier but first the election issues in the county Hohnstein had to be finalized. As far as I can see it seems quite possible to re-employ you but for that you have to be here in person. It is my great wish that this happens. It is completely compatible with your honour. However, I arranged with G. v. L. that a third person got involved. This is Müller. The two of them had a conference and L.

[454][Note by AKTA:] The Copley Medal is the most prestigious award of the Royal Society of London. Gauss received it in 1838 for his inventions and mathematical researches in magnetism, together with Michael Faraday for his researches in electrical induction. Weber only received it in 1859 for his investigations contained in his Memoirs on *elektrodynamische Maassbestimmungen* (Electrodynamic Measurements) and other researches in electricity, magnetism and acoustics.

> explained after visiting me again that the conference was quite successful. However as mentioned before, details have to be fixed in oral negotiations. I would blame myself heavily for not having dissuaded you from departing from Leipzig if this travel would not help to release you from a certain state of unfreedom. However, I have to mention two points: 1) in the letter attached from my daughter there is mentioned a travel to London you are planning to do. You never told me about this travel and I am therefore not sure if you really intend to go there. However, I have to admit that I am afraid that if you travel to London, the negotiations have to be postponed or stopped. 2) if I remember correctly you showed me a letter of attorney *in blanco* for a lawyer. I have forgotten if Albrecht, you or Ewald have to fill in this letter. However, I do not see how such a lawsuit is compatible with a peaceful coming back. Apart from it that such a lawsuit will lead to no result, it seems that neither the material object, i.e., the sum of money, nor the true object, i.e., to annoy the enemy, is in any reasonable relation to what you put at risk, the coming back to the true element, i.e., the scientific one, from the false one, i.e., the political action. The well-being of the Georgia Augusta [Göttingen University] and the preservation of dear friendly relations is closely connected to it. I therefore wish that Privy Counsellor Albrecht suspends the beginning of the trial as long as there is hope for a peaceful coming back. - - - Please give my best regards to Möbius and Enke, as well as to your two brothers, to Humboldt and Dirichlet.
>
> Sincerely yours, C. F. Gauss.

The favorable opinion of this letter soon changed to its opposite. Probably due to political events the government got very irritated, so that Gauss saw the only possibility to achieve the goal by directly talking to the king Ernst August. The travel of the king to Berlin offered a great opportunity for that. Gauss knew that in such a case Alexander von Humboldt would get in touch with the king and hoped that he could make some impression on the king by addressing the issue. He therefore wrote in Mai 13, 1838, a detailed letter to Alexander von Humboldt which is of great interest since Gauss explains his opinions in greater detail. Gauss writes:

> My most dear friend, the kind interest you showed on the new tools to understand terrestrial magnetism during my talk on September 19 urges me to report what happened since then. - - - On march this year in Göttingen were carried out observations on both devices, but this time other places joined. This is due to the great effort of our indefatigable friend Weber. The three other places on which observations were carried out as well are the following. 1) In Munich, Weber instructed Professor Steinheil on the installation of a magnetometer, who had built a precise copy of the one in Göttingen. 2) In Leipzig, Weber instructed Professor Fechner on its use, although he had just brought a rough model of such an apparatus. 3) In Berlin, where he had built a similar one in great haste, he supervised the measurements himself. I have the results in front of me. All four curves show an amazing agreement, I would like to say military uniformity, that we have long known from the declination curves. There are no large movements anywhere, so it is all the more striking to recognize the harmony in the many small ones. The last rather agitated hour from 11 am until 12 am in April 01, I spent drawing declination and intensity for each kind of curve according to the method well-known to you. With nostalgic delight I enjoyed the agreement between them.

Why do I say with nostalgic delight? I feel like having discovered a new world, having smoothed the entry to it and then at a sudden the door get shut. The continuation of our publication, the "Resultate",[455] through which at least the collective activity of the participants is held together for now, indeed the continuation of my entire scientific activity in Göttingen, crucially depends on being able to keep Weber in Göttingen.

Before I had great hopes to keep Weber in Göttingen. Now they are almost dwindled away. Basically you are my last hope. As I read in the journal our king left the day before yesterday for Berlin. Could you find an opportunity to talk to him to prevent the destruction of all my hopes and keep Weber in Göttingen? In this respect I tell you confidentially all the relevant circumstances, so that you can judge more easily the previous events and the current state.

I only mention briefly what you already know. In particular, that Weber has to be distinguished from the other Seven. I do not even exempt Ewald, my own son in law, just maybe the younger Grimm brother. Weber did nothing else than putting the five letters of his name to the petition to the board of trustees on November 18. He did not mingle into politics. He did not let print anything for justification. The topics of his lectures have no connection at all with politics. At the beginning I thought that his re-employment is possible, even easy supposed that the other side shows some goodwill. For reasons not suitable to a letter which you can guess easily, I did not address the king directly nor people most close to him. However, I tried many indirect ways. Some of the intermediaries might not have had enough power or interest to help. But once I seemed to be close to the goal.

The representative of our university Legation Councillor Lassert, who however does not live in Göttingen but in Ilefeld, discussed the matter with count Münster and told me that the later is very dedicated to it and has great hopes. But more details I got to know later when H. L. von L. was personally in Göttingen, several days after the departure of Weber to Leipzig. Count Münster did not appeal directly to the king but to the minister von Scheele, and the king seemed to agree to re-employ Weber "if Weber takes the first step and makes an apology of the so-called protestation". Count Münster himself meant that this condition makes a lot of sense.

I myself did not think that the cause is lost. The two expressions seemed to be rather flexible. To "take the first step" did not really imply to go to Hanover, but could mean to make a declaration for example to me, that he is willing to get re-employed. In fact this was really necessary since the king could not risk to get a rejection at his re-employment. And even "making an apology" did not seem to exclude the possibility of a version that would be compatible with the finest sense of honor. However, also according to the opinion of H. v. Lassert the matter was so delicate, that it could not be discussed in letters. In particular, after Weber went from here to the atmosphere in Leipzig whose neutralization one could expect in Berlin. I therefore could do nothing else than to wait until he comes back and to write a rather general letter to him in which I warned him from any foreclosing step.

Weber's return delayed about 3 to 4 weeks. In this time matters dramatically changed. The board of trustees informed me that the king is extremely angry at the Seven

[455] [Note by AKTA:] See footnote 444.

> and very reluctant to re-employ one of them. He would be hardly satisfied with an explanation Weber would or could give. Instead of that, he was pressing for all positions to be filled again soon, and so on. I do not want to get into details concerning many aspects of the last point.
>
> After Weber's return I thought it too risky to continue the matter by letter. I informed Weber about all essential points. However, my sensitivity does not allow me to ask him if he were able to write a "declaration" which looks like a revocation. In fact, I myself would not do it if I were him, even having the feeling that I acted too rashly. The question if he feels like that my sensitivity did not allow me ask. But apart from subtle sense of honour as well common intelligence forbids to do such a thing. How would such a person be considered by his colleagues and his students. There is no need to recall that I here just consider the circumstances how they are in the agitated time without expressing my own point of view about our public matters.
>
> In this situation I got despaired to achieve something by way of letters. To do something orally was not possible for several reasons. You are my last hope. You know now completely the state of matters as far as I know it myself. If you can find an auspicious moment you surely will take advantage of it. Convince the king to give back to Göttingen an outstanding, harmless scholar! Appeal to the generosity of the king, to forget about the past in recognition of Weber's great scientific value. I am convinced that this would make a favorable impression. I add that it is not too late for sure.
>
> The day before yesterday my daughter departed with Ewald for Tübingen. I did not try anything to do for Ewald, since it is against my principles to take personal favors. Ewald's departure is a great loss for Göttingen. The separation from my daughter is even infinitely more painful to me. I hope that the last tie connecting me to Göttingen is not torn. But neither place, time nor circumstances can weaken my feelings of affection and trust to you.
>
> With deep respect, C. F. Gauss.

As this try did not bring a result either, Gauss used the opportunity as the board of trustees asked him to make proposals for the re-appointment of the position to try a last time to get Weber's rehabilitation. However, this try failed as well. As a response to his report, Gauss received a letter in which all conditions were listed which were a prerequisite for the re-employment of Weber. These conditions clearly show how at this time in Hanover the "royal servant" believed to be able to behave against his citizens. Gauss himself informed Weber about this in a letter to London from July 18, 1838. However, Weber just received this letter after his return from his travel. In the letter it is written:

> Dear friend, even before having received your letter from June 18, I got one from H. (probably Humboldt), which roughly contains the same things on the negotiations as the one addressed to you. After carefully pondering all circumstances, I still could not consider the result he has given as the last word for the following reasons. 1) Since one seems to be less flexible with respect to a person one considers hostile. 2) I have some doubts if the intermediaries took a sincere interest in the matter. At least concerning one of the intermediaries there are some reasons for these doubts. In view of this state of affairs, I did not yet comply with the request of Privy Councillor

List to tell him the person most suitable to get your position, although according to your letter you almost seem to wish that I suggest such a person. I answered to P. L. that I could, and would, if he insisted, name several qualified persons with whom the Magnetic Association could continue to exist "as long as Weber continues honoring Göttingen with his presence". I pointed out however, that I have the duty to explain him first the whole state of the matter. I did this in such a way that you would surely agree with it, just in the way I explained it in my previous letter. I received an answer yesterday which I copy here so that you get all the information I have.

> "Your Excellency, I thank you very much for the detailed letter from January 28[456] you had the kindness to send me. I heavily deplore the loss of Professor Weber for my[457] Georgia Augusta and surely everybody will do that who knows the great merits of this man. I am happy to believe you that the political nuisance of our time was alien to Professor Weber and that other people induced[458] him to sign the unfortunate petition, as well that he believed that the petition was not for the public but just for the Royal board of trustees of the University. But according to the state of affairs this is not enough to re-employ him in Göttingen.
>
> Without authorization of his Majesty the King, I will tell you my thoughts about a re-employment of Professor W. [Weber]. I hope very much that Mr. W. [Weber] will be responsive to these ideas.
>
> I think that Mr. Weber has to write a detailed report to his Majesty, the King, in which he explains the following points. a) That he did not take part in political activities and disputes, but just lived for his science. b) That the question of the abolition of the constitution whose content he did not know was of no interest for him at all. c) That the agitation among his colleagues after the Royal Edict from November 01, 1837, mislead him[459] to sign the petition which was the reason for his dismissal. d) That he would have never signed the petition if he would have known that it gets public while he was convinced that it should be just sent to the Royal board of trustees of the University. e) That he feels very sorry for everything which happened afterwards since it is against his principles on the relation between subject and king.[460]
>
> The letter to the king has to be written in such a way that one could publicize a major part of it. I expect that you inform me, if Professor Weber agrees to write a letter of the kind mentioned above to his Majesty the King. In every case I wish that you have the great kindness to tell me the scholars most suitable for this position in case Weber cannot be re-employed and point out who is the most worthy among them. It is a pleasure .... L."

Dear friend, I do not want to anticipate your decision but just mention that despite

[456][Note by Gauss:] The date is wrong. I have written the letter earlier but I do not know precisely at what day.

[457][Note by Gauss:] Sic!

[458][Note by Gauss:] I did not write such an assumption.

[459][Note by Gauss:] There is no need to remark that my letter did not contain such a statement.

[460][Note by Gauss:] It is not quite clear to me what is meant by e).

the sentence "without authorization of his Majesty" I have no doubts that L. showed my letter to the king and wrote his answer with his previous knowledge. The answer is written very neatly so that for sure there was a concept before. It would be helpful if you could send me your answer, but I do not expect this from you. However, I ask you to enable me to answer in a way fitting with your plans. ... Please, dear Weber, make me the pleasure to write a letter soon, best with the message of your return in the near future.

Yours sincerely, C. F. Gauss

Obviously the answer of Weber was negative and therefore his dismissal final. His position was subsequently transferred to Listing.[461] From this time on Weber lived in Göttingen as a private citizen. Partly he was busy with the edition of the *Resultate*,[462] partly he did his own research. Concerning his own research it was quite painful for him to have no funding from the state anymore. On the advice of Dirichlet, he asked the physical class of the Academy in Berlin, when he was in need of 8000 meter of copper wire for his experiments. The Academy provided him the necessary money and lent him the wire.

The situation for Weber at this time was definitely not an easy one. However, he still had the hope to get a position somewhere else, if such a one got free. Already in January 1841 he got the offer of a directorship at the Technical School in Dresden. Although the Ministry in Saxony tried to fulfill his wishes as good as possible, he decided against it after pondering for a long time "since he felt that his scientific career would take a completely different turn if he accepted."

All relations between me and Gauss would be cut, since an exchange by mail could not serve as a replacement. My share in the magnetic investigations and undertakings would have to pass into other hands, if they were to proceed. Immediately I have to resign from the co-authorship of the edition of the *Resultate*. The wire I received from the Academy in Berlin I have to give back immediately. All this sacrifices would be deplored heavily, and I do not know, if I am safe from reproaches. I suppose that Humboldt would remark, if I wanted to separate from Gauss, what he does not recommend in the interest of science, another option would be to ask directly in Berlin for a position.

Much more suitable seemed to be a position in Braunschweig at the *Collegium Carolinum*, the current Technical University Carola Wilhelmina. According to Blasius, which led the negotiations, the college was organized according to academic standards. However, the negotiations did not progress above their initial state in Braunschweig, since almost simultaneously Weber received offers from Halle and Leipzig. In Halle, Kämtz had decided to move to Dorpat and Weber was offered Kämtz's position in Halle officially in April 1842. However, already before Weber was negotiating with the Ministry in Saxonia about the professorship in physics at the University of Leipzig. The reason why it took such a long time was that the holder of the professorship, Professor Fechner, a common friend of all three Weber brothers, who suffered from a heavy eye complaint, had the chance to recover. In this case he would be able to continue later his own position.

[461][Note by AKTA:] Johann Benedict Listing (1808-1882) was a German mathematician. In 1839 he succeeded Wilhelm Weber as professor of physics.

[462][Note by AKTA:] See footnote 444.

Albrecht, whom Weber contacted because of Fechner's health condition, wrote in April 18, 1842, to Weber:

> To wait one year before accepting is definitely a good way to know better about Fechner's condition.

Of course the brothers in Leipzig encouraged Wilhelm. Ernst Heinrich writes:

> I agree that it is a great luck to work so closely with one of the greatest mathematicians which you like and admire so much. I can understand how painful it is for you to leave Göttingen. But once you have to separate. Half a year earlier or half a year later does not matter, if your current situation can be put on a more solid foundation. You did everything you could, to prolong your stay in Göttingen as much as possible.

The negotiations finally reached an end in June 18, 1842, with the appointment of Weber, under the condition, that if Fechner recovers, he can continue his former position as director of the Physical Institute and in the Philosophical Faculty, while Weber would get in this case a Physical Laboratory and a Magnetic Observatory. Although Weber missed the personal exchange with his friend Gauss, whom he admired so much, he received compensation for that through the company with the professors in Leipzig, in particular his brothers.[463]

14.6 Leipzig Period 1843-1849

In Easter 1843 Weber started his new position in Leipzig. His beloved magnetic investigations first built the main part of his research. A stimulus for that was the construction of an

[463] [Note by AKTA:] In 1998, the bronze monument of the Göttingen Seven, created by the Italian artist Floriano Bodini (1933-2005), was erected on the forecourt of the Lower Saxony State Parliament at the Square of the Göttingen Seven in Hanover. Figure (a) of this footnote shows the image of this monument, https://en.wikipedia.org/wiki/G%C3%B6ttingen_Seven and https://www.myheimat.de/hannover-mitte/c-freizeit/die-goettinger-sieben-ein-landesdenkmal-in-hannover-an-der-leine_a54843. Figure (b) shows the detail of the statue representing Wilhelm Weber:

(a)

(b)

isolated, iron-free, Magnetic Observatory for which the Ministry in Saxony provided funding. Moreover, he had the intention to publish a seventh volume of the *Resultate* as a follow-up to the already existing six volumes.[464] However, this intention never materialized, since the number of magnetic observatories increased in short time so drastically, that a single person could not deal anymore with all the observations. The assiduous collaborator Goldschmidt was now separated from Weber. Moreover, after Weber left, Gauss lost interest in magnetism and turned his attention to purely mathematical researches. A letter Gauss wrote to Weber in May 21, 1843, shortly after Weber moved to Leipzig, indicates this change.

> With many thanks I send you back, dear friend, the letter of Repsold and the drawing. - - - In addition to the painful loss I can never overcome due to your separation, recently an additional one occurred. Major Müller, to whom I have been quite close during the last 25 years, some weeks ago suddenly has passed away. He had the intention to collaborate this summer in the trigonometric measurements. The last half of April I had some correspondence with him about that. Two or three days before his death I sent him a pair of heliotrops. The sad thing is that one gets more and more isolated as one grows older. - - - In the last two months I was quite busy with my own mathematical speculations. This took a lot of time without that I have actually reached my first goal. Always I got lured from one direction into another one, sometimes as well by a will-o'-the-wisp, how this is often the case in mathematical speculations. I am very sorry that this activity and the need to write after that a treatise for the Society prevent me from contributing to a further volume of the *Resultate*, in case you actually plan to continue them. Generally after your separation and the loss of your help, I lost interest in magnetism.
>
> With my best wishes also to your brothers, C. F. Gauss

Other letters from Gauss from that time, which he wrote when sending magnetic observations to Weber, show his reluctance to continue the magnetic researches that he had previously pursued so diligently. Especially a letter from January 08, 1844, in which he states:

> Since a long time, dear friend, I did not receive any direct news. I hoped so much that in the autumn vacation you would visit me, until Ewald destroyed this hope and brought instead the other one, that you will come for sure at Easter. I am looking forward very much to that. The other reason for writing this letter is a paper by Herschel for the board of the English Association. I attached a copy, so that you do not need to send it back. After you left Göttingen, I got alienated from magnetism so that I am unable to answer the three questions. Even if I did not get alienated, I could not answer the first and third question without having a detailed knowledge about the English foreign establishments, which I am completely missing at the time. — Since you stayed much more familiar in this field and will stay so as long as I do not have the pleasure to collaborate with you again, I hope that you know how to answer the second question. I ask you therefore about your opinion on the whole matter.

[464][Note by AKTA:] See footnote 444.

The English took more and more the lead in the terrestrial magnetic measurements. They had huge funding.[465] The way they proceeded did not appeal quite to Gauss and Weber. For these reasons and the above mentioned circumstances, Weber moved more and more to electrodynamics. In this field he continued working until the end. The field was not alien to Weber. Already when he was in Göttingen he did important contributions to it, like the measurement of intensity with the help of the tangent galvanometer, the electrochemical equivalent of water, or works connecting magnetism with electrodynamics, like the induction inclinometer, the rotational inductor, unipolar induction or magnetic friction.[466]

The fruits of these electrodynamical researches are seven treatises published by the Royal Saxonian Society under the title "Elektrodynamische Massbestimmungen" ("Electrodynamic Measurements").[467] It is fair to say that together with the treatises of Weber's friend F. E. Neumann in Königsberg[468,469] they were until recently the major generally recognized base of electrodynamics. Only recently have views on the nature of electricity other than those held by Weber gained general acceptance, after Maxwell had summarized the views which Faraday had gained from his experiments in a mathematical theory.[470] Apart from their content, these treatises of Weber are also characterized by their form, by their clear, scientific formulation and by the consistency in the implementation of the individual questions, which make all seven treatises appear as a coherent whole. Already in the year 1844 Weber was occupied with electric researches by approaching the fundamental experiment of Alessandro Volta by a more qualitative and quantitative perspective.[471] This fundamental experiment is the base of galvanism. Weber was using for this a sphere built from two hemispheres of different metal, which he suspended in a thread in such a way that the interface was parallel to the direction of the thread. An electrical body brought close to the sphere would then cause a rotation of the sphere. However, to conclude from the magnitude of the rotation the electrical charges which the hemispheres acquire as a result of their contact, a more refined knowledge of its distribution was needed. This problem caused a lot of trouble. Gauss was very interested, when Weber told him about it. Gauss writes in January 27, 1844:

> Your experiment, which makes the electrical difference of zinc and copper directly visible, interests me a lot. I am inclined to believe that this is the starting point for very important progress in this field. Again I sadly feel how nice it were, if I could work with you directly on it.

Gauss then explains his opinions how to continue such experiments in a useful way. Weber

[465] [Note by Heinrich Weber:] Sartorius was informed by Herschel, when visiting in 1845 the English assembly of Natural Scientists, that every year 34000 Pound Sterling were spent for magnetic purposes.

[466] [Note by AKTA:] [Web38], [Web39b], [Web39c], [Web40], [Web41a] with English translation in [Web21j], [Web41d] with English translation in [Web21m], [Web41e], [Web41c] and [Web42].

[467] [Note by AKTA:] The 8th treatise was published posthumously. These 8 treatises have already been fully translated into English: [Web46] and [Web21d]; [Web52b] and [Web21e]; [Web52a] and [Web21h]; [KW57] and [KW21]; [Web64] and [Web21c]; [Web71], [Web72] and [Web21g]; [Web78] and [Web21f]; [Web94d] and [Web21b].

[468] [Note by Heinrich Weber:] F. C. Neumann, "Die mathematischen Gesetze der induzierten elektrischen Ströme," Abhandlungen der Berliner Akademie, 1845, as well as "Über ein allgemeines Prinzip der mathematischen Theorie induzierter elektrischer Ströme," Abhandlungen der Berliner Akademie, 1847.

[469] [Note by AKTA:] [Neu46] and [Neu47] with French translations in [Neu48].

[470] [Note by AKTA:] See footnote 353 on page 191.

[471] [Note by AKTA:] Alessandro Volta (1745-1827) was an Italian physicist and chemist. He was the inventor of the electric battery. See [Vol00a] with English translation in [Vol00b] and [Vol64]; Italian translation in [Vol23]; Spanish translation in [Col00]; and Portuguese translation in [MA08].

at this time was quite devoted to the laws of galvanic currents. When visiting Poggendorff in Berlin in the autumn 1844, Poggendorff asked Weber "to explain everything about the galvanic bridge, i.e., about a conducting wire in the shape of a Wheatstone bridge", since he was very interested in it.[472] The scientific researches came to a first conclusion with the publication of the first treatise on the Electrodynamic Measurements, "Über ein allgemeines Grundgesetz der elektrischen Wirkung" ("On a General Fundamental Law of Electric Action"). This treatise appeared at the foundation of the Royal Saxonian Association of Science in Leipzig in 1846.[473]

In this treatise, Weber starts from the interaction which two currents or current elements exert on each other, for which Ampère had already formulated his famous law in his classical treatise "Mémoire sur la théorie mathématique des phénomènes électrodynamiques uniquement déduite de l'expérience. Mémoire de l'académie royale des sciences de l'Institut de France. Année 1823" (Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience. Memoirs of the Royal Academy of Sciences of the Institut de France. Year 1823).[474] The law of Ampère nowadays familiar to anybody, at this time was almost forgotten. Weber himself tells that he had a lot of trouble to find the treatise of Ampère. Ampère justified his law in an ingenious way by a couple of carefully devised experiments, which however, due to the lack of exact measuring devices did not meet the requirement of exact quantitative measurements. Weber earns the credit to have invented sensible devices to give a thorough justification of Ampère's law according to measure and number. Among these instruments is the electrodynamometer, which is even used nowadays in technical measurements.

We cannot enter here into details of this treatise. Therefore some general remarks are in order. At the time when Wilhelm Weber was engaged in his work, the actions of resting electrical quantities on each other (Coulomb's law), as well as the actions which constant currents or the individual current elements of these exert on each other (Ampère's law), were known, as were the induction actions discovered by Faraday and an empirical law established by Lenz, through which Ampère's phenomena are related to the induction phenomena.[475] F. E. Neumann had developed all induction laws qualitatively and quantitatively shortly before Weber's treatise appeared as a necessary consequence of Ampère's law in view of his refined version of Lenz law.[476] Hence at this time there was no connection between the actions of charges at rest on each other and moving charges building a galvanic current on each other. However since all actions were due to electricity at rest or moving, one could conjecture that there is a law describing actions of electric charges on each other in the resting case as well as in the moving case. Weber managed to find such an "electric fundamental law", from which all known actions could be deduced as a necessary consequence. This law however is rather different from other laws of nature. Whereas in other laws of nature the actions of force depend only on the quantities of materials acting on each other and the distance between them, according to this fundamental law the force also depends on the state of motion in which these quantities are, that is, on their relative velocity and relative acceleration. Although later on one found different laws predicting the phenomena of resting and moving electricity, Weber's law has the advantage that it only depends on quantities

[472][Note by AKTA:] See footnote 167 on page 101.

[473][Note by AKTA:] [Web46] with English translation in [Web21d].

[474][Note by AKTA:] See footnote 15 on page 18.

[475][Note by AKTA:] See footnotes 15, 43 and 288, on pages 18, 28 and 172, respectively. Heinrich Friedrich Emil Lenz (1804-1865). See [Len34] with partial English translation in [Len69].

[476][Note by AKTA:] See footnote 469 on page 241.

at a given time, namely the amount [of charge], the distance, the relative velocity and the relative acceleration. This law led to scientific discussions. One got convinced that all laws of nature should be subject to a most general, superior law of nature. This superior law was first formulated by Robert Mayer and then worked out in detail by Helmholtz and Joule.[477] It is the law of conservation of energy which should provide a criterion on the correctness of all other laws of nature. In fact under usual circumstances the fundamental law complies with conservation of energy but under certain assumptions one could construct cases were this agreement is violated. However, to conclude from this that the fundamental laws does not hold one had to show that such assumptions actually occur in nature. But recently for other reasons one started to consider Weber's law not as a fundamental law anymore. In our time the opinion on the essence of forces is changing. One comes back to points of view popular before Newton that the action of two spatially separated centers of force is not a direct immediate one, but requires a medium. Just as the light and heat actions which the sun produces on the earth are mediated by vibrations of the light ether which is spread everywhere, so in general every remote action should require a propagating medium, an *action in distans* [action at a distance] should not exist in nature. Faraday reintroduced this point of view, Maxwell formulated it mathematically and Hertz recently provided further evidence to it.[478] Neither the gravitational law of Newton nor Weber's fundamental law comply with it since they have *action at a distance* as hypothesis. However, it is conceivable that in the future it is possible to replace all actions at a distance by moderating media,[479] so that these laws still keep a certain significance.

During his stay at Leipzig, Wilhelm Weber lived partly with his younger brother Eduard, who was not yet married at this time, but partly as well with his older brother Ernst Heinrich. The social interchange among the professors in Leipzig at this time was rather simple, free from the luxury of our days. A special peculiarity was the open-minded character of the different social classes in Leipzig so that people from quite different classes like merchants, booksellers, scholars, people from the municipal administration and so on came in close contact to each other. This had the effect that the public interests were quite manifold. When Ernst Heinrich Weber as a member of the first Chamber represented the interests of the University and the town of Leipzig, the city parliament welcomed him with quite honourable ovations upon his return from Dresden. Eduard wrote to his brother Wilhelm:

> Since some days Ernst is back. His return was celebrated quite impressively. Lord Mayor Dr. Müller and two distinguished merchants met him halfway in Vorsdorff in a carriage drawn by four horses. They invited him to leave the express carriage and join them. At the green tavern at the end of the cabbage gardens around 60 horsemen and many carriages welcomed them. Among them were the most distinguished merchants. After a solemn speech and thanksgiving, people cheered and one returned back to town. There 300 people gathered. At the gate one cheered again.

Wilhelm Weber, who was of a quieter nature than his brothers, was particularly attracted by the intimate intercourse in a close circle, which met regularly on a certain day of the week under the name "Fechner-Kränzchen", and which, in addition to Fechner,[480] Dr. Härtel

[477][Note by AKTA:] See footnote 359 on page 192.
[478][Note by AKTA:] See footnote 355 on page 192.
[479][Note by AKTA:] In German: *vermittelnde Medien*. This expression can be translated as moderating, mediating or intermediate media.
[480][Note by AKTA:] See footnote 307 on page 181.

(Breitkopf & Härtel) and the Weber brothers, later also had the philosopher Professor Weise as a participant. Family members participated as well and questions from politics, philosophy and science were discussed in a lively way. Especially Fechner who liked to discuss very much contributed a lot thanks to his brilliancy. Due to his arduous experiments on subjective colour phenomena, Fechner suffered from a heavy eye disease, which forced him to stay in dark rooms for many years and to avoid every activity demanding his eyes. In later years luckily he could use his eyes again, albeit to a very limited extent.

The politically so important year of 1848 came and had a big impact on the German people and its government. In view of these events the step of the seven professors in Hanover ten years ago was judged now quite differently and Ernst August had the intention to re-employ again the seven dismissed professors. On April 16, 1848, Weber received from Hanover the inquiry if he were interested to accept again a professorship in Göttingen. Almost simultaneously he got as well a letter from Gauss in which Gauss wrote:

> I now address the most important point already suggested by Sartorius to you. It seems that Sartorius made a lot of progress to let it become true. All I know about it is the following: The university and the town as well sent a petition to the Ministry to reappoint the Seven. The current director of the board of trustees already answered that the reasons responsible for the well-known reprimand 10 years ago is now ineffective and no obstacle exists anymore that these men teach again at the University. He already received the authorization from the king to start negotiations to gain them anew. So far the rescript which I cite from memory. Unofficial news from Hanover tell that first of all you and Albrecht are meant. If this news is correct, probably you already know more about it than I do. I guess that you can make your requirements yourself. How my dear Weber can I disclose my feelings! I will follow your letter. I do not think that the main purpose is to attract many students, but the moral rehabilitation of justice. This is indeed the first condition to raise Göttingen again. That you will attract many students is obviously not to be expected, although I have no doubt that you will contribute as well. For example some weeks ago Quintus Icilius, a previous student of mine who got his doctoral degree last year, left Göttingen for Leipzig solely because of you in order to study physics under your supervision. You would bring new life to the mathematical branch of our Society. That we will publish as much as in these happy former years I doubt however. This is not due to my many other important works, but since I am 10 years older, rather fragile, and on the descending branch. I have to add that mainly due to that former events my enthusiasm dwindled away. I felt so lonely the last years. However, I can tell you that I will try everything I can to promote your research. Living with you and your constant attachment would be the most precious things in my life. But you have to make the decision by your own. Ewald was here 8 days ago and among us said, it seemed to me, that he will accept an offer to get re-employed if he gets one. In order to be able to bring this letter to the postal office today I have to finish quickly. I just add that I am looking forward very much to see you here soon and that you are very welcome in my house, if you do not dislike its monasterial seclusion.
>
> Yours sincerely, C. F. Gauss.

Again Weber had to make a tough decision. On the one hand the friends in Leipzig, his brothers and the thousands of contacts he had made during his stay attached him to Leipzig.

He could not deny either that Gauss at the age of 72 was not able to offer him in scientific respect the same he did in former times. The ten years of separation could not be replaced. Several people urged Weber to stay in Leipzig. Poggendorff wrote:

> Do you want to follow my advice? Don't go! What do you gain. Gauss indeed. But apart from him nothing at all. Gauss is an old man who might die any day and even if he does not die any time soon, he gets every day older and duller. In Leipzig people like you much. You have your brothers and rich resources which you do not need to obtain again from scratch. In my opinion there is no question at all, since gratitude you owe to the people of Leipzig as much as to the people of Göttingen, who would now have to offer you at least twice as much. Leipzig helped you in hard times. This is a heavy argument. In good times it is easy to find friends.

But on the other side the personality of Gauss, "the greatest mathematician since Archimedes", had a very strong appeal to Weber. Moreover, he met there his old friends Wöhler, Sartorius and his fellow sufferers. A little town more fitted the temper of Weber than a big one. Already in June 26, 1848, Gauss could write to Weber:

> With the greatest pleasure I read in your letter of the 21st of this month, which I received yesterday, that your affair, my dearest friend, or as I would rather say, our affair, has now almost reached the desired conclusion. I immediately asked Sartorius to discuss the issue confidentially with Listing. However, Listing himself came to me. He declared or better repeated his declaration that from the beginning he considered the position only as one which he would one day like to hand back to you. I am convinced that he is waiting for your return with sincere joy. - - - It seems certain that Ewald comes back in autumn, although I have no direct confirmation from Hanover nor from himself.

The position of Listing, who held Weber's previous position in Göttingen as associate professor, caused some difficulties. Weber's suggestion that Listing continues to lecture on experimental physics while he himself will lecture on mathematical physics was not accepted by the board of trustees. After several negotiations, Weber received in October 16 the official offer, in which his conditions were accepted, that Listing got promoted to full professor for mathematical physics and received a special fund to acquire measuring devices. Weber himself obtained in the Philosophical Faculty the same position he left in 1837. At Easter 1849 he started it. In a letter from October 20, 1848, Gauss expresses his joy about the successful end of the negotiations by writing:

> Although today I suffer from a heavy headache, I cannot refrain from expressing my joyful congratulation to you, my dear friend, in the same hour, where I received the complete certainty of your return. I just read in a rescript of the Faculty the great news. - - - All the best for you, dear friend. I will count the weeks until you are back in Göttingen.
>
> Sincerely, C. F. Gauss.

14.7 Second Period in Göttingen 1849-1891

After moving to Göttingen in Easter 1849 Weber had the opportunity to join the fiftieth anniversary of Gauss doctorate in July 16 not just as friend but again as colleague. Although Gauss, like Weber, had a dislike of all kinds of festivities concerning his person, there was a big celebration at this day. Apart from speeches, to which friends and deputies appeared, among them Jacobi and Dirichlet from Berlin, the award of diplomas and the appointment as honorary citizen of Göttingen and Braunschweig, Gauss was celebrated by a solemn meeting of the Royal Society of Science in which Gauss presented his paper "Contributions to a theory of algebraic equations" coming back to the topic of his thesis. This was the last paper Gauss wrote. Weber soon realized that a collaboration as in former years was not possible again. Gauss devoted most of his time to lighter pursuits, like reading fiction. In particular, he became quite interested in the Russian language. He learned Russian so diligently that he became almost fluent in it.

In contrast to Gauss, Weber was still young and he was about to publish some of his most important works in quick succession. Following the first treatise on electrodynamic measurements, there appeared in the year 1851 in the records of the Royal Saxonian Society of Sciences the second treatise on measurements of resistance and in the following year the third on diamagnetism.[481] In the second treatise Weber introduced absolute units of measure for three quantities, which are connected by Ohm's law, namely intensity of current, electromotive force and resistance. By absolute units of measure he means units of measure just depending on the units of length, mass and time, independent of the random and varying circumstances how the measurement takes place. Gauss pioneered the introduction of absolute units of measure for magnetic quantities in his treatise *Intensitas vis magneticae*, which appeared in the year 1832.[482] Weber recognized already at this time the great advantage of absolute units of measure not just for electrodynamics, but for physics in general. In fact in his posthumous writings one can find the development of absolute units of measure for many areas of physics. Since the International Congress of Electricians (Paris 1881), recognizing the great advantages of the absolute system of units, proposed the general introduction of this system of units in electricity, it has also been used in other physical areas, and it seems only a question of time before it becomes the dominant system in physics and even in technology. In electric engineering this is already the case now. The units used to measure current intensities, electromotive forces and resistances, namely, Ampères, Volts and Ohms, are directly related to the absolute system of units. It must only seem strange that, while the units were named after men who had made outstanding contributions to the theory of electricity, the founders of the system, Gauss and Weber, were completely ignored.[483]

In the third treatise on diamagnetism, Weber, based on his experiments, arrives at precise ideas on the nature of magnetism and diamagnetism. Already Ampère showed that the so called molecular currents, namely little galvanic currents revolving without friction around the molecules of magnetic materials, can be used to explain all magnetic phenomena. In particular, there is no need to assume the existence of magnetic fluids. Weber showed that if one takes into account the diamagnetic phenomena, only Ampère's hypothesis is adequate. He arrived at the idea that the molecules of magnetic bodies, such as iron, nickel, etc., are surrounded by molecular currents whose paths are firmly connected to the molecules,

[481][Note by AKTA:] See footnote 467 on page 241.
[482][Note by AKTA:] See footnote 55 on page 34.
[483][Note by AKTA:] See footnote 346 on page 190.

while the molecules themselves can rotate. On the other hand the molecules of diamagnetic materials including all nonmagnetic materials cannot rotate. They contain current-free orbits in which molecular currents can develop in case the material gets into a diamagnetic state due to magnetic forces. While Weber drew these final conclusions from his investigations, it was far from him to attribute to them a real existence in nature without further ado. The main focus was to make the new ideas fruitful for future research. Weber writes in his Treatise:[484]

> And even if we now associate to *the electric molecular currents* in the interior of materials *reality*, same as to the ether in optics responsible for the propagation of waves, it can happen in the future by further development of science that they have to be transferred to the class of *ideal* notions.

In the year 1857 Weber published jointly with his friend Rudolph Kohlrausch the Fourth Treatise on electrodynamic measurements, whose content is strongly related to the previous Treatises.[485] One and the same intensity of current can be measured according to three different absolute units of measure namely electrostatic, electromagnetic and electrodynamic. The question was to find the ratio among these different units of measure. With respect to the electromagnetic and the electrostatic units of measure, one obtained a number quite close to the speed of light. This fact is of great significance for the recently proposed electromagnetic theory of light. In addition, this ratio determines as well the numerical value of the constant in Weber's fundamental law. There were additional treatises on electrodynamic measurements published in the years 1864, 1871, and 1878. Their content however is rather theoretical and therefore beyond the scope of this article. A last treatise in this series Weber began shortly before his death. He could not finish it anymore.[486]

A few years after his return to Göttingen, Weber had the opportunity to buy a property with a small house originally built only for summer stays. In the middle of the town, however surrounded by gardens this small house provided Weber the peace of mind he needed to carry out his works.[487] Here in quiet privacy the drafts of his papers took shape. Experiments he mostly did during vacation, when he was not disturbed by academic obligations and could fully concentrate. Weber proceeded quite systematic. All major experiments were already prepared with great care and could be carried out in rather short time. During these events the Physical Cabinet was full of life. Not caring about his usual daily schedule Weber brought the prearranged series of experiments to a definite end thanks to his peculiar energy.

Dramatic incidents which Weber experienced often in his younger years did not occur anymore. However, Weber had to cope with the death of many of his friends. Not only the high age of 87 years he attained was responsible for that. He also lost younger friends. For only six years after his return he could enjoy the company of Gauss. In February 23, 1855, Gauss passed away. After a long fight with illness and the dwindling of his life force in the last half a year, this outcome was to be expected. Only a few years later Weber suffered a new loss. In March 9, 1858, Rudolph Kohlrausch passed away in Erlangen. Together with Kohlrausch, Weber wrote the treatise "Zurückführung der Stromintensitätsmessung

[484][Note by AKTA:] [Web52a, p. 538 of Weber's *Werke*] with English translation in [Web21h, p. 68].

[485][Note by AKTA:] See footnote 467 on page 241.

[486][Note by AKTA:] See footnote 467 on page 241.

[487][Note by AKTA:] The image of this footnote shows Weber's house on Jüdenstrasse in Göttingen, in the middle of a large garden. Small drawing in the possession of the Göttingen City Museum, [Wie67, pp. 143 and 221]:

auf mechanisches Mass" mentioned before.[488] With Kohlrausch Weber lost a friend of a similar scientific spirit combined with a free view of life, with whom he had hoped very much to collaborate in further projects.

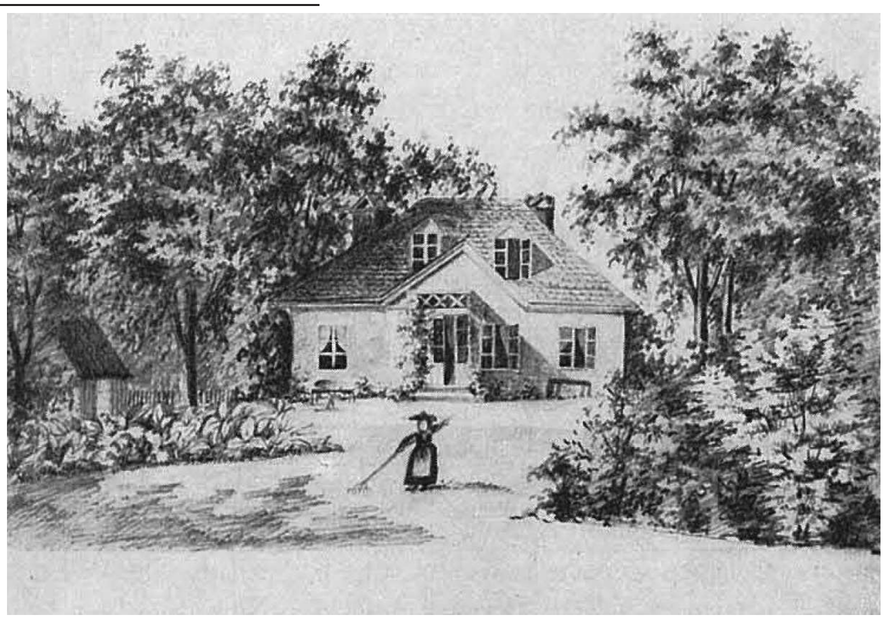
After Gauss passed away, probably the best thing Weber could do to recover from this blow was to attract Dirichlet to come to Göttingen. The unexpected happened. Already in autumn 1855 Dirichlet started his lectures at the Georgia Augusta and mathematical science boomed again. Since his stay in Berlin in 1828 Weber and Dirichlet exchanged letters. Weber visited Poggendorff regularly and each time he met Dirichlet as well in his house or the one of his brother in law Paul Mendelsohn. In December 1852 Dirichlet wrote:

> I hope that you comply with the urgent requests of your friends here to come to Berlin. They think that according to tradition they have the right, that you visit them. Unfortunately, I could not carry out your electric question since I was busy with two papers. One of them is in print soon and I am thinking about the other one. As soon as this is done, I will try to focus on your problem.

The problem mentioned was to determine the movement of two equal masses attracting each other according to Newton's law of gravitation but as well subject to two parallel, opposite and equal forces, constant in intensity and direction. Later on Dirichlet wrote to Weber about the solution of the problem:

> I am sorry that I did not try to solve your electric question earlier. I thought it is quite hard and expected that to solve it would require more time than I had available this winter, if a solution actually could be found. After pondering about the matter some days ago, I got convinced that the problem is indeed hard but luckily almost a century ago Euler and Lagrange already found a solution.

Personal friendship and common scientific interests led to a daily exchange which also involved the family members. But for how short time this lasted!



[488][Note by AKTA:] See footnote 467 on page 241.

In autumn 1858 Dirichlet returned back ill from vacation. It is said that due to a too warm bath he got a heart disease which resulted in a severe disease ending with Dirichlet's death on May 05, 1859. During Dirichlet's illness Poggendorff wrote on March 11, 1859:

> If one talks here about Göttingen, the first question always is how Dirichlet does. I know, it is sad, very sad. It would be a relief if you could write soon more details about him. It would be a misery if a man in his best age already succumbed to human destiny. The almost 90 years old Humboldt is comparably well. The day before yesterday he joined the Academy and 8 days ago the party at the University. Destinies of human beings are so strange!

Dirichlet's illness and his death soon later however were not the only hard blow the unhappy family had to suffer. Already during Dirichlet's illness his wife Rebekka, née Mendelsohn, passed away. A stroke, caused by inner excitation, suddenly put an end to her life. In short time the lively and stimulating exchange came to an end. Dirichlet's house became deserted and Weber as one of his closest friends had the sad duty to care about his younger children as legal guardian.

Apart from the obligations as legal guardian of Dirichlet's children, Weber at this time had other tasks to do which he carried out dutifully. After Gauss passed away, he became director of the Astronomical Observatory. Already when Dirichlet was still alive the edition of Gauss' Collected Works by the Royal Society of Science in Göttingen was planned. A committee consisting of Stern, Riemann, Dedekind, Schering and Weber was built.[489] In this committee Weber diligently cared about details of the format of the printing and so on. Riemann's illness, which soon occurred and other circumstances led to the dissolution of the committee, and Schering alone edited the works of Gauss.

Quite early Weber recognized in Bernhard Riemann the outstanding mathematical talent and promoted him as much as he could. After Dirichlet passed away, he was convinced that Riemann would be the most suitable successor of Dirichlet. But Riemann could only take the position for a short time. Soon he got ill. On Weber's request the government of Hanover provided Riemann the means to go to Italy in order to cure his lung. The stay in Messina in the winter of 1862 was a great success. However, unfortunately Riemann went back over the Simplon. Unexpected cold led to a relapse and Riemann returned to Göttingen ill. Once more one tried to preserve the extraordinary man for science by a stay in the south, but without success. In July 20, 1866, Riemann passed away in Selasca at the Lago Maggiore.

Since a long time Göttingen was a leading place for mathematics and science. Therefore one was eager to find soon an outstanding successor for Riemann after his death. Clebsch from Giessen was appointed, but only a few years later Weber had to mourn the loss of this younger friend as well. An unusually strong attack of diphtheria brought the activity of the man in his best age to a sudden end. At this time Weber got in closer touch with Zöllner, who was teaching at the University of Leipzig.[490] Zöllner was a brilliant, warmhearted character full of ideals. He became an ardent supporter of Weber and had a strong impact on him due to his fresh mind and his enterprising scientific spirit. At first Zöllner was motivated by the most noble ideas, but in later years irritated by strong attacks he went much too far. On top of this, there were the well-known spiritualistic experiments with the American Slade at

[489][Note by AKTA:] Moritz Abraham Stern (1807-1894), Bernhard Riemann (1826-1866), Richard Dedekind (1831-1916) and Ernst Christian Julius Schering (1833-1897) were German mathematicians.

[490][Note by AKTA:] Johann Karl Friedrich Zöllner (1834-1882) was a German astrophysicist.

which Weber, Fechner and others participated and which led to a great stir. Weber had a honest character not depending on the opinions of others. Hence he was willing to examine all phenomena for which he had no explanation. But despite his fine power of observation, he was not the right person for this kind of experiments for which knowledge of human nature were much more important than power of observation. Just the experiments and not the experimenter were checked and neither the fervent Zöllner nor the other participants were suitable for checks of the later.

Zöllner had the localities of the Astronomical Observatory on the Pleissenburg at his disposal. This circumstance offered Weber who was already in his seventieth the opportunity to carry out once more major experimental work. At this time he had already given over the directory of the Physical Cabinet in Göttingen to Riecke who as well substituted his lectures at the Georgia Augusta.[491] In Leipzig, Weber not only had the opportunity to live together for a longer time with his brother Ernst Heinrich (the younger brother Eduard had died already), but could meet as well Fechner, Scheibner and Karl Neumann, with whom he already had carried out scientific exchange since several years.[492]

In the same way as one needs a normed unit of weight, one needs a universally agreed scale of resistance and a precisely guaranteed resistance standard. At the time of the French Revolution in 1789 the kilogram became a unit for weight. For resistance Jacobi first proposed a certain standard in Saint Petersburg (Jacobi's wire), later proposals came from Werner Siemens (mercury unit) and the British Association (British Association unit). However, one realised soon that a standard resistance made of metal wire changed with time and could not meet the requirements of permanence. Weber had therefore already intended in earlier years to produce a standard conductor wire of such a quality and arrangement that both its resistance and its spatial conditions could be checked at any time by direct measurements in absolute units. With such a device, one was then independent of any changes in the resistance of the standard conductor and had a means of checking the accuracy of resistance scales at any time by comparing them with the standard conductor. If, following Siemens' suggestion, the resistance standard was based on mercury, whose specific resistance can be considered to be unchanging, then one had the advantage of being able to measure its resistance precisely in absolute units. In order to carry out this plan, two large multipliers were mounted at the old Astronomical Observatory.[493] Both of the multipliers had the same size. Their diameter was more than a meter. One served as galvanometer, the other as earth inductor. Weber himself participated with a youthful spirit at the coiling of the wire, the setup and mounting of the devices and the measurements. In order that the appliance would be used again in the future and not just for a single measurement, the wire could be winded off easily from the multipliers and the numbers of the windings and the size of the winding areas could be controlled anytime. Unfortunately Weber could only finish the first part of this work, namely the determination of the resistance of the wire. The second part, namely the comparison of this wire with other units of resistance, he had to leave unfinished, since Zöllner in 1881 suddenly passed away and the rooms and devices from the University at his disposal could not be transferred to other people for a longer time. Later on Privy Counselor Wiedemann obtained the devices and carried out the determination of the Ohm according to absolute measure. Although Zöllner did not participate much in these experiments, Weber later published the results jointly with him in a treatise "Über die Einrichtung zum

[491][Note by AKTA:] See Chapter 12.
[492][Note by AKTA:] See footnote 243 on page 143.
[493][Note by AKTA:] See footnote 52 on page 33.

Gebrauche absoluter Masse in der Elektrodynamik von W. Weber und F. Zöllner", which appeared in the *Berichte der Königlichen Sächsischen Gesellschaft der Wissenschaften*, vol. 32, 1880.[494]

This was Weber's last experimental work. He was now old and many anniversaries were waiting for him. After he celebrated in October 24, 1873, his 70th birthday among a smaller group of people, in August 26, 1876, the 50th anniversary of his doctorate was celebrated officially. But Weber due to his modest character got irritated by personal ovations and escaped to Karlsbad. There he met his friend for many years and fellow traveller, the economist Privy Counselor Hanssen and celebrated the important day among a few close friends. But as well there a large amount of official letters from friends and former students reached him. After the celebration of the fiftieth anniversary of his promotion to full professor in Göttingen in September 24, 1883, and the 80th birthday in October 24, 1883, a dignified completion was the 60th anniversary of his doctorate, August 26, 1886. Quite extraordinary was the affection the still lusty old man enjoyed. He was honoured by deputies from the University of Göttingen, whose senior he was since a longer time, from the Royal Society of Science itself, as well as from the town, which made him a honorary citizen. Weber obtained many honorable letters from scientific associations whose member he was, from corporations and from private persons. The postal office as well as the telegraph were extremely busy to bring all these letters to Weber. Even the director of the imperial institutions, state secretary Dr. von Stephan, who already at opportunities before showed his warm esteem to Weber, sent his congratulation. But Weber received as well splendid recognition from outside by obtaining the Grand Cross of the Order of Henry the Lion by his Royal Highness prince Albrecht of Prussia. Moreover, he was appointed as Royal Prussian real Privy Counselor with the title excellency by his Majesty the Emperor. This was a special honour since Weber was the first professor to receive this distinction without having any connection to the Court in Berlin. Count Bismarck arranged it. After he got this most splendid mark of respect for which he had not looked for, Weber retreated from the public and spent the last years in quiet seclusion.

Weber was a member of 19 academies and scientific societies and around 23 associations spreading over Germany, Austria, England, France, Italy, the Netherlands, Sweden, Denmark and Russia. He was holder of the medal from the Leopoldina (Acad. Caesar. Leopold. Caroli Herm.), the English Copley Medal,[495] the French Becquerel Medal and he owned many orders of merit, like the *Pour le Mérite* and the Maximilian Order for Science and Art. When Justus Liebig sent him the later one, he added to the official letter on November 25, 1858, the following private lines:

> Dear Friend, I cannot refrain from adding the following private lines to express how deeply I feel that the honour you receive from the king of Bavaria is much below the merit you deserve for your amazing work. According to the rules of the foundation, we could not take into account your previous researches before the year 1856/57. In order to spare you further trouble, I divided the amount and sent half of it already to Kohlrausch. The poor guy is very ill, one is afraid that he might die. Please receive once more my congratulation and visit me soon in Munich.
>
> With kind regards Justus Liebig.

[494][Note by AKTA:] [WZ80].

[495][Note by AKTA:] See footnote 454 on page 233.

Often one experiences that recognition and honour are changing the character. This was not the case with Weber! Everybody who entered his simple office found the same friendly, sympathetic man, independently if he just had the title of Professor or the one of Excellency. His work and endeavour were concerned with the cause and not the person. Therefore he shunned priority disputes and was always happy to share his ideas and experiences with others to promote science. Free of needs, he led a frugal life and avoided the limelight. After he suffered in July 1871 for a longer time from smallpox, he also gave up smoking. He lived quite moderately and followed a strict daily routine. Until old age he went in the early morning for a walk. Part of his vacations he usually went hiking. His fellow travellers had often the opportunity to experience the energy and endurance Weber developed when hiking. In every undertaking, even a secondary one, Weber was always careful and followed a well-devised plan. On the other hand he was not pedantic and could as well change his plans, which he referred to as "policy of the free hand". His own scrupulousness he expected as well from others and this explains the recognition of authority of all those he once gave trust. Although benignity and tolerance were the main feature of Weber's character especially when he got old, he as well had enormous energy combined with a lively and easily excitable temper. Weber could throw a tantrum and since this was against his natural disposition, such a fit could happen for quite accidental reasons.

Although he was not married, Weber led a keen family life. Since he moved to Göttingen for the second time in the year 1849 his oldest niece with the exception of just a few years did his household. Cheerful younger and older guests frequented his house and garden. Weber always had a strong sense for friendly exchange, as is shown by the personal letters he wrote to the wives of his colleagues especially Ms. Dorothea Grimm, Ms. Poggendorff, the first and second wife of his friend Ewald, Ms. Dirichlet and so on. Almost everywhere he was a good friend and was consulted for advice. The closest exchange however he had with his brothers and their families. The relation between the brothers was almost ideal, especially the one to Ernst Heinrich, since Ernst Heinrich, 10 years older than Wilhelm, was his closest advisor in every circumstance since childhood. Often he mentioned full of gratitude how much he owed to him. It was quite pleasant to hear how each of them recognised the merits of the other fully. Between Göttingen and Leipzig there was therefore a lively family exchange and it was always an exception if there was not a hospitable house, which accommodated another family member. Especially when in the early 1870s Ernst Heinrich stopped teaching, the two brothers often spent most of the summer together in Göttingen. When Wilhelm as well in 1874 stopped lecturing partly and in 1876 completely, the opportunity to meet each other was used even more. The winter of 74/75 Ernst Heinrich had to spend at the Riviera and in Naples, since he suffered from asthma. Wilhelm accompanied him. It was the last travel together. Wilhelm especially enjoyed the family gatherings, which took place in Göttingen every year at Pentecost. Then Weber became young again and took part at all undertakings.

Weber was not a good speaker. However, in his lectures he fascinated his students by the peculiarity of his teaching. Starting from simple facts he developed the building of his science in a strong logical order. He focused on the flow of ideas and the methods of scientific research. He did not talk about religion and religious points of view. Although he had his religious convictions, he was quite tolerant to others. That he concerned himself quite with religion is shown by the fact that among his scientific notes there can be found as well some religious ones. Several times he mentioned the followings:[496]

[496][Note by AKTA:] See [Web94b] with English translation in [Web97] and [Web21a]. See also [Ten97].

> Each thought, each perception and each memory I am aware of are my property. That I can think, perceive and remember is a gift of god.
>
> Human beings are aware of every thought, every perception and every memory they have and call them their own. There is nothing what one could call his own with greater right. However, human beings are aware as well that they did not receive from themselves their thinking, their perception and their memory. They say that these are granted to them by god. Having not received their awareness, their thinking, their perception and their memory by themselves is their religious conviction.
>
> Dream is interruption of thinking. A faint memory is the interruption of matching ones thoughts, i.e., interruption of intellectual work. Our life is an intermittent existence.

To reconstruct Weber religious conviction out of these isolated sentences, is hardly possible without bringing in one's own opinion. But how could one not be touched by these sentences? As Weber was completely free of vain ambition of honour and recognition, he did not care about money and property at all, either. What would make others happy was a burden to him. His brother Ernst Heinrich took care of his small income and when Ernst Heinrich died, Wilhelm distributed all his property among the children of his brother.

When his memory started to become faint, it became hard for Weber to bear old age. He used to say:

> My memory is so faint, that when reading a book I have already forgotten the previous page, when I start reading a new one.

Like Gauss, Weber started to read fiction, especially Goethe and Shakespeare interested him since a long time, but as well Chamisso, the letters of Bismarck, the correspondence of Dahlmann, Wöhler, Grimm and so on.

At the beginning of the year 1891 Weber got jaundice. He had nutrition problems and his powers were dwindling away. Although he got better once more and there was hope that he could recover completely, a new attack of this illness consumed his powers slowly so that he passed away in June 23. During this longer illness Weber did not always had to stay in bed. But he was not able anymore to get impressions from outside and therefore was alone with his own thoughts. His mind more and more detached from this world and he transferred the thoughts which absorbed him to reality. He often mentioned that he is travelling to Göttingen. If one replied that he is already in Göttingen, he said:

> This is not my Göttingen. I mean Göttingen, in which Gauss lives.

Already several days before his death, he felt that the end is near and in his selflessness he pointed out that there will be a great stir in the house, that many people will come and if one already has taken measures to deal with this. He had completed his mundane path and longed for the end. "I do not want to work anymore on this dark world", he said and asked, if the expected people are already coming. He often thought about his brother Ernst Heinrich, who already had passed away.

> God is calling. I have to go away, my brother Ernst is waiting for me.

In the morning of June 23, the first sunny day after a longer period of rain, he went supported by his niece to the garden, he liked so much, and sat in an armchair. He peacefully slept there almost the whole day in the warm rays of the sun. When the sun was setting, he raised his head and his eyes wide open were shining in a peculiar way. For a long time he watched the sky and the setting sun, like looking already to the hereafter. A few minutes later Wilhelm Weber passed away one day before the birthday of his beloved brother Ernst Heinrich, to join him, his younger brother Eduard and all his other siblings forever.

The sad news was spread by the telegraph to all directions. All who knew the deceased mourned him. The funeral took place in June 26. Many people attended. Apart from the professors, the students and the deputies of the citizens from Göttingen, many external people came to show the last respect to the beloved man, if they managed to come to Göttingen at such short notice. A large number of wreaths, palms and flowers were not just sent by scholars, but as well by individual persons and corporations. The citizens of Göttingen considered the simple, unsophisticated man since a long time as one of themselves and while still alive the deceased barely could imagine how he managed to have so many friends among all classes of people. The emperor Wilhelm let express his condolence by the trustee and prince Albrecht of Prussia let lying down a wreath in front of the grave by his adjutant. Also the secretary of state Dr. von Stephan expressed his commiseration. The preacher of the University councilor Dr. Schulz gave a moving speech in front of the coffin, which stood under the open sky among large trees.

So he left, the great and nevertheless modest man, beloved by everybody who was close to him, honoured the more he did not aspire recognition in his life. May Wilhelm Weber be a role model for the academic youth! Free of arrogance and vanity, free of social prejudice, a true German scholar. Germany will always be proud of him.

Now he rests in one of the first graves of the cemetery of Göttingen. Rest in peace![497]

[497][Note by AKTA:] Weber's grave appears in this footnote, https://en.wikipedia.org/wiki/Wilhelm_Eduard_Weber. It reads: Called by God to a higher activity, here rests the mortal remains of Wilhelm Weber, born in Wittenberg on October 24, 1804, and died in Göttingen on June 23, 1891:


Von Gott zu einer höhe-
ren Thätigkeit abberufen
ruh't hier die sterbliche
Hülle von
WILHELM WEBER
geb. in Wittenberg
am 24. Oct. 1804
gest. in Göttingen
am 23. Juni 1891.

Chapter 15

Prefaces to the Collected Works of Wilhelm Weber (1892-1894)

Translated and edited by A. K. T. Assis.[498] I also included all footnotes of these Prefaces.

15.1 [Voigt, 1892] Preface to the First Volume

W. Voigt[499]

This First Volume of W. Weber's Collected Works contains treatises from the fields of acoustics, mechanics, optics and thermodynamics. The majority of these works date from the time *before* Weber was led by Gauss into the area of work in which his strength was to fully develop and his scientific personality was to grow to the importance that is generally recognized today.

The treatises collected here are primarily of historical interest; they only partly contain independent works, and even in these some of them must be rejected as incorrect. But in an edition which is primarily intended as a memorial to the great physicist, these landmarks of the beginnings of his development must not be missing.

The chronological order of the treatises could not be the sole criterion for their arrangement, as it would often have separated related works from one another. Rather, it seemed appropriate to form groups of works with related content, to introduce a chronological order within them, and to arrange the groups themselves in the two Parts of the Volume approximately according to the date of the treatise that opened them.

The First Part (Acoustics) is opened by the treatises I-XIII, written under Chladni's direct influence, which partly report critically on the acoustic work of others, partly repeat and continue experiments carried out by them.[500] The series concludes with a biography of Chladni and a paper on his acoustics.

A second series is formed by the treatises on the tones of reed pipes, in which Weber shares truly new and fundamental observations on an area that had hardly been dealt with before, and by a number of short notes on individual questions of acoustics; it is concluded

[498]Homepage: `www.ifi.unicamp.br/~assis`

[499][Voi92]. Woldemar Voigt (1850-1919) was a German physicist.

[500]See footnote 300 on page 180.

by the short overview of the whole of acoustics written in 1835 for the *Universallexikon der Tonkunst* (Universal Lexicon of Music).

Attached to this Part is one of the reviews that Chladni gave to his young friend's habilitation thesis, which is printed under article XIV.

You will search in vain for W. Weber's *doctoral dissertation* among the papers compiled here; according to the results of my inquiries, it was not printed and has not been found in manuscript form in his estate. The title "Theoria efficaciae laminarum maxime mobilium arcteque tubos aerem sonantem continentes claudentium" (The theory of the efficiency of the most movable plates and the trumpets closely containing the sounding air) suggests with certainty that it served in an extended form as a habilitation thesis.

If the treatises collected in the First Part bear witness to the continuous occupation with a closed area of physics, the treatises compiled in the Second Part (mechanics, optics, thermodynamics) provide information on problems that are dealt with more occasionally and for a shorter period of time.

The seeds for a greater development lie here especially in the treatises V-VII, in which the phenomena of the elastic aftereffect are subjected to an exact investigation for the first time according to an ingenious method by Gauss.[501]

Of particular interest is also treatise XVI, in which the author presents and applies a very useful method for observing the difference between the adiabatic and isothermal dilatation of solid bodies, which he, however, still based on the standpoint of the older heat theory, utilizes in a substantially different way than is currently the case.

The present Collected Edition adheres strictly to the original publications — apart from the correction of individual obvious arithmetical and printing errors. With regard to form, it should be noted that citations which refer to a passage in the *same* treatise have simply been changed in accordance with the new edition, — citations which refer to other treatises by Weber have been left in their original form, except for the introduction of a consistent numbering of the volumes of Poggendorff's *Annalen*;[502] in the latter case, a reference to the corresponding passage in the Collected Edition has been added in square brackets below the page. Other additions by the editor are identified as such by the same symbol.

Göttingen, July 1892.

W. Voigt.

15.2 [Riecke, 1892] Preface to the Second Volume

E. Riecke[503]

The Second Volume of Wilhelm Weber's Collected Works contains his treatises in the area of magnetism. Gauss had devised new and highly sophisticated means of observation for the study of the Earth's magnetism and had shown how the strength of terrestrial magnetism could be expressed in the general units of length, mass and time independently of the variable strength of the oscillating magnetic needle.[504] After his appointment to the chair of physics in Göttingen, Weber took an eager and successful part in the further pursuit of the path

[501] See footnote 155 on page 92.
[502] See footnote 409 on page 210.
[503] [Rie92a]. See Chapter 12.
[504] See footnote 55 on page 34.

thus opened up. He joined forces with Gauss in the publication of "Resultate aus den Beobachtungen des magnetischen Vereins" (Results from the Observations of the Magnetic Society), of which 6 volumes appeared from 1836-1841.[505] These were not only to contain the corresponding observations of the numerous places that had joined the Magnetic Society, their graphic representations and the explanations attached to them; they were also to include all work relating to the broad field of geomagnetism and thus provide the impetus for new advances in science. The significance of what Gauss and Weber created in the decade of their joint work does indeed go far beyond the goal they initially pursued. A large part of our present-day art of observation has developed from the problems they dealt with, and it is above all to Weber's credit that he extended to the measurements of galvanism the strict principles introduced by Gauss. Weber's contributions to the *Resultate* form the greater part of the treatises combined in this Second Volume of the Collected Works. The explanations of the term-observations, which were written by Weber for the last four volumes, serve the direct purposes of the Magnetic Society. These are only reproduced in excerpts, containing comments that characterize the entire activity of the Society or refer to important results of the observations. These explanations would be followed by the essays devoted to the description of the Göttingen Magnetic Observatory and the apparatus installed there. They are not only of great historical interest, but are still of immediate practical importance today due to the theories of the instruments, their errors and corrections they contain. Weber endeavored with great success to construct instruments for determining the elements of terrestrial magnetism while traveling that were both comprehensive and highly accurate. Several essays are devoted to the description of these instruments, some of which we still use today in a barely modified form, and to the presentation of the observations made with them.

The requirements and experiences that arose from this work inevitably led to questions of more general importance. This includes the study of the dependence of bar magnetism on temperature, the results of which have since been confirmed but hardly extended. Of the treatises relating to the doctrine of induced magnetism (IX, XX and XXIII of this Volume), we may regard the first as the forerunner of the investigations described in the treatise XXVIII of this Volume and in the third treatise on *elektrodynamische Maassbestimmungen* (Electrodynamic Measurements).[506] This earlier work leads to even less general results, as Weber seems not to have known the points of view of Poisson's theory when he wrote it.[507] In the second treatise, effects of the coercive force are described which are closely related to those recently called "hysteresis". In the Leipzig invitation paper, the magnetic properties of steel are explained by the assumption that it is composed of parts of greater and lesser coercive force. The treatise on magnetic friction still contains almost everything we know about this subject today.

The treatises in the area of magneto-electricity are of particular interest, not only because of their own importance, but also because of their relationship to the [series of works on] *elektrodynamische Maassbestimmungen* (Electrodynamic Measurements). They form the bridge from magnetism to electrodynamics, the field to which Weber gave a new form through a series of epoch-making, truly classical works. Some of the treatises belonging to this field can in themselves be counted with the same right to galvanism or electrodynamics; they have been included in the present Volume because of their close relationship to others of purely

[505] See footnote 313 on page 183. See also [GW40a].
[506] See footnote 467 on page 241.
[507] See footnote 289 on page 172.

magnetic content.

The treatise on unipolar induction forms the starting point for its own extensive literature and numerous controversies.[508] Weber himself gave rise to these. He had first explained the phenomenon with the help of a peculiar idea, which was, however, linked to the assumption of the real existence of magnetic fluids; he later emphasized the contradiction of his [earlier] theory [of unipolar induction] with the laws of electrodynamics and the possibility of another explanation in a Note to the third treatise on *elektrodynamische Maassbestimmungen* (Electrodynamic Measurements), but without developing it himself. The first application of magnetic induction to the measurement of galvanic resistances can be found in the seventh treatise of this Volume. The induction inclinometer[509] described by Weber as early as 1837 is also of interest because the ingenious principle underlying its construction was later used in one of Weber's methods of absolute resistance measurement; and when he later gave the Earth inductor the form in which it has remained essentially unchanged ever since as one of the physicist's most important instruments, he did not limit himself to magnetic applications in the illustration of the new method of measurement, but added the important proof of how attenuation can be used as a measure of the sensitivity of a multiplier,[510] which then provided a new solution for the fundamental task of galvanometry, the absolute resistance measurement. It is hardly necessary to recall the importance of this work for practical physics. The fact that Weber recognized the importance of magneto-electric forces in this direction early on is evident from his essays on the rotation inductor and magneto-electric machines, which cannot be regarded without interest as the forerunners of our electrotechnical literature, which is now so widespread.

Apart from two rearrangements required by the context, the treatises which we have attempted to characterize briefly in the foregoing have been printed in chronological order in the present Volume; a separation into individual groups did not seem expedient in view of the multiple relationships between them. As in the First Volume, the essays are printed strictly according to their first publication. Citations that refer to a passage in the same treatise have been changed in accordance with the new edition, while citations that refer to other treatises by Weber have been left in their original form. In the latter case, a reference to the corresponding passage in the Collected Works is added below the page. In the same way, the frequent citations to treatises by Gauss are supplemented by a reference to the edition of Gauss' Collected Works. These and all other additions by the editor are indicated by square brackets.

Göttingen, July 1892.

Eduard Riecke.

15.3 [Heinrich Weber, 1893] Preface to the Third Volume

Heinrich Weber[511]

---

[508]See Chapters 4, 5, 6 and 7.

[509]In German: *Induktionsinklinatorium*. See footnote 58 on page 34.

[510]See footnote 52 on page 33.

[511][Web93a]. See footnote 378 on page 203.

The Third Volume contains the treatises published by Wilhelm Weber up to the end of 1857 in the areas of galvanism and electrodynamics in the same order in which they were published. As a result of this arrangement, the seven treatises to which Weber gave the common title "Elektrodynamische Maassbestimmungen" (Eletrodynamic Measurements) appear here separately from one another, and the first four of them are printed in this Volume under articles numbers V, X, XI and XV, while the later three are included in the Fourth Volume.[512]

It would have been desirable to combine these seven treatises of fundamental importance, which follow on harmoniously from one another and appear as a rounded whole, in one Volume. However, the division of the treatises into two Volumes would have been so uneven that it seemed advisable to refrain from such a combination of all seven treatises. Instead, the strictly chronological arrangement has ensured that, with only a few exceptions, treatises of related content follow one another, giving the reader an insight into the development of W. Weber's work in the areas of galvanism and electrodynamics.

Treatises I to IV deal specifically with the galvanic current; in particular, in treatise II, page 9, the absolute unit of measure for current intensities is first precisely specified, which Weber probably used for the first time in the treatise on magnetic friction (Volume II, pages 202 and 203). Treatises II and IV contain the theory of the tangent galvanometer, and in treatise III the electrochemical equivalent of water is subjected to precise measurement for the first time on the basis of the introduced absolute unit of [current] intensity.

The simultaneous occupation with galvanic currents and magnetic phenomena soon led Weber to a precise study of Ampère's investigations, the result of which is set down in the first treatise published under the title *Elektrodynamische Maassbestimmungen* (Electrodynamic Measurements), which is printed here under treatise number V.[513] With the aid of a number of instruments newly invented by him, Weber proves the correctness of Ampère's law in the sharpest possible terms, and then establishes his fundamental law of electrical action, by which the electrostatic, electrodynamic and induction phenomena are traced back to a common basis.

In the treatises designated here as VII and XI, Weber turns to the study of diamagnetism, which shortly after its discovery by Faraday in 1846 became of outstanding importance for the theory of magnetism, insofar as the explanation of diamagnetic phenomena simultaneously brought about a decision between the hitherto equally valid hypotheses about the nature of magnetism.[514] Weber not only succeeded in comparing the magnitude of diamagnetic and magnetic forces, but also in generating diamagnetic induction currents and determining the relationship between their intensity and that of magnetic induction currents. These investigations then led to a precise conception of the nature of magnetism and diamagnetism.

The construction of the Earth inductor, which was initially only used for inclination measurements, led Weber to establish an absolute unit of measure for electromotive forces. At the same time, however, this also provided the absolute unit of measure for galvanic resistance with the aid of the previously established absolute unit of measure of [current] intensity according to Ohm's law.[515] The units of measurement are discussed in depth in the treatise X, and the methods of measuring resistance according to absolute units are discussed in detail. This system of units, which is based on magnetic actions, has become generally

[512]See footnote 467 on page 241.

[513][Note by AKT:] See footnotes 15 and 467 on pages 18 and 241, respectively.

[514]Michael Faraday (1791-1867). See [Far46b] and [Far46c].

[515]See footnote 121 on page 65.

accepted today with only minor modifications.

The fundamental law established by Weber contains a constant whose numerical value he determined jointly with Rudolph Kohlrausch and published in the treatise listed under number XV.[516] Knowledge of this constant makes it possible to express quantities measured in magnetic or electrodynamic units in mechanical units or vice versa.

These few words may suffice to give an indication of the wealth of intellectual work contained in the following treatises, which, in addition to their theoretical importance, also form a model for experimental research. As in the first two Volumes, remarks and citations that do not originate from Weber himself are indicated by square brackets; only in the case of some citations has the page number of the original text been directly replaced by the corresponding page number of the present Volume for the convenience of the reader. The treatises by Gauss frequently cited in the text have been accompanied by a location reference in Gauss' Collected Works.

Brunswick, January 1893.

Heinrich Weber.

15.4 [Heinrich Weber, 1894] Preface to the Fourth Volume

Heinrich Weber[517]

The Fourth Volume follows on directly from the Third Volume in terms of content. It contains all the treatises and essays in the areas of galvanism and electrodynamics that Wilhelm Weber published in the period 1858-1880, as well as a number of treatises and essays that have been found in his estate. An essay with remarks on the Munich Magnetic Observatory, which was only discovered after the publication of the first three Volumes and could therefore no longer be included in the Second Volume, to which it belongs in terms of content, is also included as an Appendix.

The treatises, with the exception of those forming the handwritten posthumous works and the Appendix, have been arranged in chronological order, for the same reasons that prompted the same arrangement in the Third Volume. The last three of the seven treatises published under the joint title "Elektrodynamische Maassbestimmungen" (Electrodynamic Measurements), the first four of which were printed in the Third Volume, can be found under treatises numbers V, VIII and XII and an eighth treatise belonging to them, which has not yet been published, is included as the first treatise in the handwritten posthumous works.[518]

While the first four treatises of the *Elektrodynamische Maassbestimmungen* (Electrodynamic Measurements) mainly deal with the investigation of the reciprocal forces that electric particles exert on each other or on other bodies, the following treatises, which are included in this Volume, are primarily concerned with the movements of the electric particles caused by these forces. It was a curious coincidence that when Wilhelm Weber was about to publish his first paper on this subject, Kirchhoff, who had been working on the laws of galvanic currents at the same time, had already presented the editor of the *Annalen für Physik und*

[516] See footnote 467 on page 241.

[517] [Web94a].

[518] See footnote 467 on page 241.

Chemie with a treatise on the same subject a short time earlier.[519] The remark by J. C. Poggendorff included under number VI, to which Wilhelm Weber himself refers (Wilhelm Weber's *Werke*, Vol. IV, p. 130), refers to this.[520] It was not until six years later that Wilhelm Weber published his treatise on electrical oscillations (number V), in which he then combined the content of the previously withdrawn treatise with a comprehensive experimental investigation, in which Rudolph Kohlrausch participated until his death in 1858. In this treatise, Wilhelm Weber introduces the mass of electric fluids for the first time and then develops the equations for the movement of electricity in wires, especially in those of circular shape, according to the general laws of mechanics, without assuming, as Kirchhoff did, the validity of Ohm's law even in cases where the current intensity in the individual current elements is different and subject to rapid change.

In the following treatises VIII and XII of the *Elektrodynamische Maassbestimmungen*, to which treatises VII, IX and above all treatise X "Ueber die Bewegung der Elektricität in Körpern von molekularer Konstitution" (On the motion of electricity in bodies of molecular constitution) published in Poggendorff's *Annalen* are to be counted as connecting links, W. Weber enters into investigations for which the energy principle forms the basis.[521] In treatise number VIII, in particular, the relation of the fundamental law of electric action to the energy principle is examined more closely, and it is shown that there is no contradiction between the fundamental law and the latter principle, as has been asserted by others, unless assumptions are made about initial states whose compatibility with existing nature requires special proof. At the beginning of treatises X and XII, Wilhelm Weber then proceeds to detailed considerations of the energy principle by showing that the energy principle formulated in the usual way, to which Carl Neumann had drawn attention, is capable of extensions which, depending on their nature, lead to different results.[522] The extension which Wilhelm Weber gives to this principle consists in the assumption that the energy of interaction and the relative living force of two particles are homogeneous quantities whose sum is always equal to a constant. If this extension is valid, then the initial conditions can no longer be assumed to be completely arbitrary; on the contrary, all assumptions which in themselves contradict the underlying principle are excluded from the outset. Wilhelm Weber also derives the ordinary energy principle from the principle of the conservation of energy formulated by him and shows how the law of electrodynamic potential results from his principle and the law of electrostatic potential.

Following on from the laws of motion of two particles that are only subject to their interaction, which have already been developed in treatise VIII, special cases are then discussed in detail in treatise XII. Of particular importance are also the investigations carried out in treatise X, on the motion of electricity in bodies of molecular constitution, in which the galvanic, magnetic and thermal actions, which can occur simultaneously in ponderable bodies, are brought into the field of consideration. In doing so, Wilhelm Weber abandons his earlier view of the galvanic current as a double current and, taking up earlier considerations (Wilhelm Weber's *Werke*, Vol. III, p. 403), gives a clear idea of the generation of thermal energy by the galvanic current and of the cause of resistance in metallic conductors.

The above-mentioned treatises are now joined organically by the first treatise included in

[519]See footnotes 350 and 409 on pages 191 and 210, respectively.

[520]See footnote 345 on page 190.

[521]See Chapter 11.

[522]Carl Gottfried Neumann (1832-1925) was a German mathematician, the son of the mineralogist, physicist and mathematician Franz Ernst Neumann (1798-1895).

the handwritten posthumous works, which has not yet been published and which, according to the title given to it by W. Weber himself, is the eighth of the seven treatises published under the title "Elektrodynamische Maassbestimmungen". Wilhelm Weber may well have suspected that he would not be able to go into the details of the immense field of phenomena covered by the observations in this treatise, and so individual sections appear as pointers to the paths leading to a further expansion of the field of research.

On the basis of the assumption that all ponderable molecules are mere compounds of equal quantities of positive and negative electricity, and that the attractive force of equal quantities of dissimilar electricity is greater than the repulsive force of the same quantities of similar electricity, Wilhelm Weber moves from the field of the pure theory of electricity to that of ponderable bodies. He shows how the various kinds of ponderable molecules, especially those of the elements, can be thought of as composed of electric molecules, furthermore how metallic conductors differ from glassy and crystalline bodies, and how, on the basis of this difference, the propagation of electricity and heat in the former takes place in a different way, namely by ballistic motion, than that of light and heat in the latter, namely by wave motion. According to Wilhelm Weber's view presented here, the luminiferous aether is a static medium made up of positive electric molecules.

The original manuscript of this treatise contains, in addition to the Sections included here, four other Sections whose titles are given in a Note added at the end of the treatise. Since Wilhelm Weber himself later excluded these Sections from the treatise, it was necessary to refrain from publishing them. It should be noted, however, that the entire estate has been handed over to the Royal Library in Göttingen for safekeeping, thus preserving the possibility of gaining insight into the content of these Sections.

In addition to these treatises on the nature of electricity and ponderable bodies, the present Volume contains two more of particular importance, namely treatise number III "Zur Galvanometrie" (On galvanometry) and treatise number XIV, published in collaboration with Zöllner, of which the latter is Wilhelm Weber's last experimental work.[523] In the former, the methods of absolute resistance measurements are discussed and a complete theory as well as the most advantageous construction of galvanometers are given and the copying methods are subjected to a detailed examination, while the latter deals with the establishment of a standard conductor whose resistance can be determined at any time according to absolute units, which should facilitate the general application of absolute units in electrodynamics. Wilhelm Weber was no longer able to carry out the originally planned comparison of the resistance of this standard conductor with today's generally accepted practical unit of resistance, the Ohm.

In addition to the above-mentioned treatise on *elektrodynamischen Maassbestimmungen*, the handwritten posthumous works also contain essays and treatises, some of which originate from very different periods of Wilhelm Weber's life. Some of them were undoubtedly intended for publication, although perhaps in a different form, but for unknown reasons were not published; others, on the other hand, are transcripts, interwoven with various remarks which Wilhelm Weber quickly threw in to support his memory, the compilation of which could often only be accomplished by overcoming great difficulties. Of these, the larger treatise "Über Maassbestimmungen" (On Measurements) should be emphasized here, which has not lost its importance even today after the general acceptance of the absolute system of units. Although this treatise was not written until after 1864, Wilhelm Weber's surviving notes from 1834 show that at that time he had already reduced the most important physical

[523] See Chapter 8 and [WZ80]. Johann Karl Friedrich Zöllner (1834-1882) was a German astrophysicist.

quantities to absolute units utilizing the basic units of length, time and mass. The essay "Über die Einrichtung des Bifilargalvanometers" (On the setup of the bifilar galvanometer) is in any case of much earlier origin than the addition added to it in 1864, which deals with the simultaneous measurement of geomagnetism and current intensity in absolute units. Finally, the two essays "Bemerkungen zu der Abhandlung: Untersuchungen über den galvanischen Lichtogen, von Edlund" (Comments on the paper: "Investigation into the electric arc" by Prof. E. Edlund) and "Elektroskopische und elektrodynamische Wirkung der freien Elektricität geschlossener Ketten" (Electroscopic and electrodynamic actions of free electricity in closed circuits) lead to a different field, because both refer to the charge on the surface of conductors in which the electricity is in motion.[524]

We have refrained from publishing some of the essays that were also found in the estate, partly because they date from very early in Wilhelm Weber's life and no longer offer anything new today, and partly because their content has been included in later treatises. To confirm this, the documents may be listed and their essential content indicated.

The first essay belonging here, which is entitled "Der Dämpfer" (The damper), was intended to be included in the *Resultate aus dem Beobachtungen des magnetischen Vereins* (Results from the Observations of the Magnetic Society), 1837. After explaining the purpose of a damper, he considered in more detail the action of a closed metal ring as a multiplier, then as an inductor and finally, when both actions are combined, as a damper. The paper concludes with a calculation of the reduction in vibration of the [magnetized] needle and a comparison of the damping action with the action of a damping rod. A second essay, "Über die Einrichtung der Multiplikatoren" (On the setup of multipliers), is the forerunner of the detailed considerations given later in the treatise "On galvanometry". The third essay "Über ein neues Galvanometer" (On a new galvanometer) deals with the setting up, installation and testing of the bifilar galvanometer and its use for absolute [current] intensity measurements and for determining the electrochemical equivalent of oxygen and hydrogen, an application which Wilhelm Weber discusses in detail in the essay on the electrochemical equivalent of water (Wilhelm Weber's *Werke*, Vol. III, p. 13). The fourth essay, comprising only a few pages, deals with "Die absolute Messung der in einer Leidener Flasche vorhandenen freien Elektricität" (The absolute measurement of the free electricity present in a Leiden jahr). The measurement is based on oscillation and deflection experiments of a movably suspended, charged Franklin's plate, on which another charged Franklin's plate acts from a distance. This task has been solved in a more perfect way in the treatise, *Elektrodynamische Maassbestimmungen, insbesondere Zurückführung der Stromintensität auf mechanisches Maass* (Electrodynamic measurements, specially attributing mechanical units to measures of current intensity) (Wilhelm Weber's *Werke*, Vol. III, p. 618).[525] Finally, the resolution is a document comprising 11 quarto pages without a heading, which contains the proposal to produce an arbitrary standard of measure for galvanic resistance to facilitate galvanic measurements, especially for technical applications, and to distribute a large number of copies of this unit. However, this practical unit of measure must be measured precisely in absolute units. This is followed by the derivation of the oscillation equation of a magnetic needle within a multiplier, if self-induction is taken into account.

The writings compiled in this Volume under the title "Nachlass" (Posthumous works) can only be handed over to the public with all reservations, since it must appear doubtful whether Wilhelm Weber intended them for printing, especially in the form in which they

[524]These papers are translated in Chapters 17 and 19. See also Chapter 18.
[525]See footnote 467 on page 241.

were found in the estate. Although it was obvious to make changes to the wording and presentation in various places, as Wilhelm Weber would undoubtedly have done himself in the event of publication, this was not done in all cases, as the heading *"handschriftlicher" Nachlass* ("handwritten" posthumous works) already sufficiently indicates. All quotations and additions which do not originate from Wilhelm Weber himself are, as in the previous Volumes, also indicated in this Volume by square brackets.

Brunswick, January 1894.

Heinrich Weber.

15.5 [Riecke, 1893] Preface to the Fifth Volume

Eduard Riecke[526]

Three of the sons of the Wittenberg theologian Michael Weber devoted themselves to the study of the natural sciences and the close intellectual community that bound them together throughout their lives was commemorated in two epoch-making works, one of which, the *Wellenlehre* (Wave Theory), arose from the joint work of Wilhelm Weber and his older brother Ernst Heinrich, the other, *die Mechanik der Gehwerkzeuge* (Mechanics of the Human Walking Apparatus), from the joint work of Wilhelm and his younger brother Eduard.[527] In the preface to the *Wellenlehre*, the brothers report on a coincidental external reason for their investigations. However, it can be assumed that their attention had already been drawn to problems of wave theory at an earlier stage. In the first place, through their friendly relationship with Chladni, the founder of experimental acoustics, to whom the work is dedicated.[528] The older brother, however, who was already a professor in Leipzig at the time of the joint work, also had the physiological applications of wave theory in the background of his thoughts. Two years after the publication of the joint work, he published the first of the treatises that made him the founder of an exact physical theory of blood circulation.

The *Wellenlehre* (Wave Theory) is one of the classic works of physical literature, above all because of its beautiful and fundamental investigations into the waves of incompressible fluids. What is said about this in the first main part of the work must still be read today by everyone who wants to become more familiar with this part of hydrodynamics. The numerous observations, the measurements carried out with such simple means in the most meaningful way still contain enough stimulation for further experimental and theoretical investigations. The peculiar attempt to build a bridge from the discovered laws of wave motion to the phenomena of vortices was, of course, doomed to fail, since the later progress of science revealed an essential difference between the two types of motion. In the context of the whole, however, the three paragraphs on the formation of vortices could not be suppressed.

The second main part of the work deals with waves in relation to sound and light; it is less extensive than the first and is no longer as directly relevant as the first part. The allusion in § 249 to a connection between the sound of a string and the shape it alternately assumes as it vibrates has been replaced by a comprehensive theory of overtones and vocal sounds;

[526][Rie93].

[527]Ernst Heinrich Weber (1795-1878) was a German physician. Eduard Friedrich Weber (1806-1871) was a German anatomist and physiologist. See [WW25] and [WW36] with English translation in [WW92].

[528]See footnote 300 on page 180.

new methods of observation, new instrumental aids of great perfection have been created, and the results of research have been made more generally accessible through excellent works. Weber's *Wellenlehre* (Wave Theory), however, always contains a series of fundamental observations in its part devoted to acoustics, to which every presentation of acoustics still refers. With reference to this, we recall the measurements of the speed of rope waves, the studies on reed pipes and the occurrence of silence in the vicinity of vibrating tuning forks.

Only a few paragraphs are devoted to the wave theory of light; optics was far removed from the authors' own experimental work and Fresnel had only opened the series of his works a few years earlier; in the same year as the *Wellenlehre*, the translation of a Fresnel treatise appeared for the first time in the Poggendorff's *Annalen der Physik*.[529]

The same principles were applied to the present reprint of the *Wellenlehre* as to the reprint of the collected treatises in the earlier Volumes. The sign for the beginning of a new paragraph had been omitted in some places in the original work, for example in § 3 and § 143, the sign for § 219 was used twice; these oversights could easily be corrected using the detailed table of contents printed before the work. There were a number of discrepancies between the text and the figures, with parts of the figures or individual letters missing. In most cases, there could be no doubt about the small changes and additions that had to be made to ensure consistency. In all these cases, the text or the figure was changed or supplemented according to the circumstances. A Note on pages 96 and 98 draws attention to a major deviation, which could only be completely eliminated by a more radical change to the text. In Figure 15, the missing letters have not been added, as the necessary indications are not given; incidentally, the understanding of the relevant § 94 is not impaired by this. We have refrained from transferring the length measurements based on Paris feet to the metric system. The observations are not concerned with the determination of absolute physical constants, but with the measurement of phenomena whose course depends on the specific conditions of the experiment. On the other hand, the value in grams is added for weights that are given in the medicinal weight, which is foreign to physicists; an overview of the units of the medicinal weight used by the authors is given in a Note on page 128. The column titles of the original had to be changed because of the differences in the typesetting. As a rule, the content of the entire section is given on the left-hand page, while the heading on the right-hand page follows the original version as closely as possible. The extracts from the works of Laplace, La Grange, Gerstner, Poisson and Cauchy contained in the third section have been compared with the originals and the text and formulas were subsequently improved in some places. Additions and comments by the editor are indicated by square brackets. In the table of contents, the numbers in brackets indicate the relevant pages of the original.

Göttingen, March 1893.

Eduard Riecke.

15.6 [Merkel and Fischer, 1894] Preface to the Sixth Volume

F. Merkel and O. Fischer[530]

[529] Augustin-Jean Fresnel (1788-1827). See [Fre25].

[530] [MF94]. Friedrich Sigmund Merkel (1845-1919) was a German anatomist. Otto Fischer (1861-1916) was a German physiologist and mathematician.

The publication of this Sixth Volume of Weber's Collected Works was undertaken by W. Braune in Leipzig, who was not only closely related to the Weber family, but also had to be regarded as the most qualified editor of the "Mechanik der menschlichen Gehwerkzeuge" (Mechanics of the Human Walking Apparatus) in terms of his entire line of work.[531] As with numerous other works, he had invited Otto Fischer in Leipzig to collaborate with him. However, before both could begin their work, W. Braune was taken away from science by an untimely death. He was replaced by a full member of the *Königl. Gesellschaft der Wissenschaften* (Royal Society of Sciences [of Göttingen]), Fr. Merkel in Göttingen, who was now responsible for the edition together with O. Fischer.

October 1893.

Fr. Merkel (in Göttingen) and O. Fischer (in Leipzig).

[531]Christian Wilhelm Braune (1831-1892) was a German anatomist. He was son-in-law to Wilhelm Weber's older brother, Ernst Heinrich Weber. See footnote 527.

Chapter 16

[Weber, 1894c] On Galvanometry (Excerpt)

Wilhelm Weber[532,533,534]

(From the 10th volume of the Abhandlungen der Königl. Gesellschaft der Wissenschaften zu Göttingen — Treatises of the Royal Society of Sciences in Göttingen)[535,536]

16.1 First Part

The First Part of the present Treatise has the object of precisely determining the resistance of a given *resistance standard* in absolute units.

The determination of a *resistance* according to absolute units is based on the determinations of an *electromotive force* and a *current intensity* according to absolute units; because according to Ohm's law,[537] the resistance of a circuit is to be equated to the quotient of the electromotive force acting on the circuit, divided by the intensity of the current produced by this force in the circuit.

The *electromotive force* which is exerted by the Earth's magnetic force T on a closed conductor while it is moving can now be determined most accurately by absolute units. If one denotes by S the surface area which encloses the projection of the closed conductor on the normal plane of T, and by dS the change in this surface space in the time element dt as a result of the movement, then the electromotive force exerted by T on the closed conductor

[532] [Web94h].

[533] Translated and edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

[534] The Notes by Wilhelm Weber are represented by [Note by WW:]; the Notes by Heinrich Weber, the editor of Volume 4 of Wilhelm Weber's *Werke*, are represented by [Note by Heinrich Weber:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[535] [Note by Heinrich Weber:] This essay is an excerpt from the treatise "On Galvanometry", Wilhelm Weber's *Werke*, Vol. IV, p. 17, which was probably intended to be published in Poggendorff's *Annalen*.

[536] [Note by AKTA:] [Web62]. This paper is translated in Chapter 8.

[537] [Note by AKTA:] See footnote 121 on page 65.

in absolute units is

$$e = \frac{TdS}{dt} \ .$$

It is not necessary that all parts of the closed conductor take part in the movement; a part of the conductor can remain at rest, if only the movement of the remaining part is such that the value of S can always be precisely determined.

Furthermore, the *intensity of the current produced by such an electromotive force in the closed conductor* would be easily determined in absolute terms using a *tangent galvanometer*[538] if the effect of the current on the galvanometer were strong enough to be accurately measured. But because this is not the case, the tangent galvanometer, in which the multiplier forms a wide circle around a very small compass needle, must be replaced with a *very sensitive galvanometer*, where the multiplier encloses the compass needle very closely.[539]

Such a sensitive galvanometer can now be constructed in such a way that, as with the tangent galvanometer, *in equilibrium*, the tangent of the deflection of the compass needle from the meridian is proportional to the current intensity; but the *factor* by which that tangent must be multiplied in order to give the current intensity according to absolute units, which for the tangent galvanometer has the known value $rT/2\pi$, if r is the radius of the multiplier circle, is *unknown* for a sensitive galvanometer, where the multiplier windings are very close to the compass needle.

But should this factor also have been determined, which is possible by measuring the *damping* exerted by the multiplier on the moving compass needle when the circuit is closed;[540,541,542] it would not be possible to make use of this determination, because the compass needle never reaches the assumed *equilibrium* under the influence of the current produced by the electromotive force $e = TdS/dt$, but because of the variability of e, *always oscillates*.

The *regularity* of these oscillations depends, however, on the concentration of the action of the current produced by the electromotive force $e = TdS/dt$ on the compass needle over a time interval in the middle of the oscillation, which is only a very small part of the oscillation period. In order to increase this momentary action as much as possible, the closed conductor

[538][Note by AKTA:] See footnote 114 on page 62.

[539][Note by AKTA:] See footnote 52 on page 33.

[540][Note by WW:] If e denotes the base number of the natural logarithms and $e^\lambda : 1$ denotes the ratio of two consecutive oscillation arcs of the compass needle under the influence of the *damping* exerted on the compass needle by the multiplier when the circuit is closed, where λ is called the logarithmic decrement and is determined from the observations of the oscillating compass needle; then, if t denotes the oscillation period of the compass needle without damping, and k denotes its moment of inertia, the desired factor is

$$= \pi \sqrt{\frac{k\sqrt{\pi^2 + \lambda^2}}{2w\lambda t^3}} \ .$$

Here w now has the meaning of the resistance of the circuit to which the multiplier belongs, in absolute units, the value of which is precisely the task of measuring the resistance, which requires the determination of the current intensity in absolute units. But if this current intensity i can also be represented as a function of w using this factor, then equating the quotient e/i according to Ohm's law with the resistance w leads to an equation, in which w is the only unknown quantity whose value is determined thereby. But after w is found in this way, the above factor is also determined and can then be used to measure all current intensities in the same circuit.

[541][Note by AKTA:] The logarithmic decrement is defined as the logarithm of the ratio of any two successive peaks.

[542][Note by AKTA:] See footnote 61 on page 35.

is moved during the short period of time in such a way that the value of S either goes from the minimum S_0 to the maximum S^0 or vice versa. Such a movement of the closed conductor is called an *induction surge*,[543] and the sum of the electromotive force exerted by it is

$$\int edt = \pm \left(S^0 - S_0\right) T \ .$$

According to Ohm's law, if w denotes the unknown constant resistance of the closed conductor, the sum of current produced by it is

$$\int idt = \frac{1}{w} \int edt = \pm \frac{S^0 - S_0}{w} T \ ,$$

after which is found the unknown resistance

$$w = \frac{e}{i} = \frac{\int edt}{\int idt} = \pm \frac{(S^0 - S_0)T}{\int idt} \ .$$

If one now denotes the change in the angular velocity[544] of the compass needle caused by such an induction surge as γ, then with a sensitive, appropriately constructed galvanometer as well as with the tangent galvanometer, γ would be proportional to the current sum $\int idt$ produced by the induction surge; but the *factor* by which γ must be multiplied in order to give $\int idt$ according to absolute units, which for the tangent galvanometer has the known value $[r/2\pi] \cdot [k/m]$, is *unknown* for such a sensitive galvanometer, where the multiplier windings are very close to the needle; however, this factor can also be determined by measuring the *damping* exerted by the multiplier on the moving compass needle when the circuit is closed. If, as in the previous Note, the logarithmic decrement resulting from this damping is denoted by λ, then this factor is

$$= \sqrt{\frac{k\tau}{2w\lambda}} \ ,$$

if τ denotes the oscillation period of the compass needle under the influence of damping, or, if t denotes the oscillation period with the open circuit,

$$= \sqrt{\frac{kt\sqrt{1 + \frac{\lambda^2}{\pi^2}}}{2w\lambda}} \ .$$

However, with such a sensitive galvanometer as is required for these experiments, it is of great importance that this factor applies at exactly the same time as the other galvanometer observations necessary for resistance measurement are made. It is therefore particularly important to have such an *arrangement of the induction surges* so that the two quantities γ and λ can be determined at the same time from the observed oscillations of the compass needle.

The simplest method that achieves this is the *throwback method* given by Gauss,[545] which was described in the "Treatises on electrodynamic measurements, resistance measurements,

---

[543][Note by AKTA:] In German: *Induktionsstoss*. See footnote 130 on page 69.

[544][Note by AKTA:] In German: *Drehungsgeschwindigkeit*.

[545][Note by AKTA:] In German: *Zurückwerfungsmethode*. See [Gau38a], [Web39b] and [WK68, p. 108, Note 13].

Appendix C".[546,547] According to this, the compass needle is set in such oscillations that a larger elongation a always alternates with a smaller b, where a and b can be observed very precisely. It then emerges

$$\lambda = \log \frac{a}{b} \ ,$$

$$\gamma = \frac{\pi}{t} \left(\frac{a^2 + b^2}{\sqrt{ab}} \right) \cdot e^{-\frac{\lambda}{\pi} \arctan \frac{\lambda}{\pi}} \ ,$$

or more precisely, if one takes into account the value of the logarithmic decrement λ_0, which remains even when the circuit is open, and sets $\lambda_0 + \lambda = \lambda_1$, $t_0 = t\sqrt{1 + \lambda_0^2/\pi^2}$, where t_0 denotes the oscillation period observed at the logarithmic decrement λ_0,

$$\lambda_1 = \log \frac{a}{b} \ ,$$

$$\gamma = \frac{\sqrt{\pi^2 + \lambda_0^2}}{t_0} \cdot \left(\frac{a^2 + b^2}{\sqrt{ab}} \right) \cdot e^{-\frac{\lambda_1}{\pi} \arctan \frac{\lambda_1}{\pi}} \ .$$

Adding the equations found above, namely, taking λ_0 into account,

$$\int idt = \gamma \cdot \sqrt{\frac{kt\sqrt{1 + \frac{\lambda_1^2}{\pi^2}}}{2w(\lambda_1 - \lambda_0)}} \ ,$$

$$w = \frac{(S^0 - S_0)T}{\int idt} \ ,$$

this makes it easily to calculate the resistance w from the observed quantities:

$$a, b, \lambda_0, t_0, k, \frac{1}{2}(S^0 - S_0), T.$$

However, as is self-evident, the certainty and accuracy of the results from the observations made according to these regulations depend primarily on the *arrangement* of the galvanometer used for this purpose and the remaining part of the closed circuit which is movable for the purpose of the induction surges. The solution of this subtler problem of galvanometry, which concerns the most practical arrangement of such a measuring apparatus, therefore forms the main subject of this Part [of the paper].

It is easy to understand that it is not only important to have a very large *sensitivity* of the galvanometer, which should be so large that a corresponds to a very large number of parts of the scale, so that the value of a from the observations made on the scale is reliably obtained up to a very small fraction of it; but it is also important that b stands in an appropriate relationship to a, so that the value of λ is also reliably obtained to within a very small fraction of it. It is also considered that the accuracy of the observation by the throwback method requires that, since the duration of an induction surge cannot be reduced to less than 1 second, the duration of the oscillation of the compass should be approximately

[546][Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. III, p. 441.

[547][Note by AKTA:] [Web52b, p. 441 of Weber's *Werke*] with English translation in [Web21e, Appendix C, p. 404].

20 to 30 seconds, and finally that the compass should be fitted with a transverse beam[548] and appropriate weights in order to be able to determine its moment of inertia with great finesse and precision.

If only the sensitivity of the galvanometer were taken into consideration, it would be important to produce the smallest and yet precisely observable compass needle, which could be enclosed quite closely by the multiplier, and then to determine the strength of the wire and the cross-section of the multiplier appropriately; but then the *damping* would not be large enough to determine λ exactly.

Since $\lambda = \log(a/b)$, and a can be regarded as determined by the demands made on the sensitivity of the galvanometer, it is clear that in order to obtain the value of λ to the smallest fraction of it with certainty, the following condition must be fulfilled, namely

$$\left(\frac{\lambda db}{d\lambda}\right)^2 = b^2\left(\log\frac{a}{b}\right)^2 = \text{maximum},$$

hence $\lambda = 1$ or $a/b = e = 2.718...$

However, the requirement for such a strong damping can only be met with *stronger magnetism of the compass needle*, which can be easily achieved, without a significant reduction in sensitivity, by increasing the dimensions of the compass needle in proportion to the length and thickness.

However, with regard to the sensitivity, this increase in the size of the compass needle should not go further than is necessary for the purpose of attenuation; such a strongly magnetic needle would still have a period of oscillation that is far too short. In order to bring this oscillation period to 20 to 30 seconds, it is most expedient to form an *astatic system* by firmly connecting an identical magnet with an oppositely directed axis above the multiplier with the compass needle in the multiplier, and to hang it on an elastic metal wire that is so strong that the oscillation period of the system is thereby established in a prescribed manner.[549]

After this, it is essentially only important to establish appropriate rules for the *wire strength* and for the *cross-section of the multiplier*.

It easily turns out that it is most advantageous if the resistance of the multiplier is made equal to the resistance of the remaining part of the closed circuit, where according to Ohm's laws, the *ratio* l/s of the multiplier wire length l to its cross section s can be considered as given.

Furthermore, with regard to the regular winding of the wire, a *rectangular shape of the multiplier cross section* may be assumed to be the most expedient, which is determined by the two sides of the rectangle a and b, of which the latter is the horizontal one, which is bisected by the meridian of the compass needle.

However, the values of a and b should now satisfy the condition that for a given area ab any change in the ratio of a to b would weaken the sensitivity, assuming that the position of the inner rectangle facing the compass needle remains unchanged and only the outer rectangle sides may be moved. This condition equals the mean value of the torque[550] exerted on the compass needle by all current elements on the outer surface of the multiplier, corresponding to the horizontal rectangular side, with the mean value of the torque exerted on the compass

[548][Note by AKTA:] In the original: *virga transversalis*.
[549][Note by AKTA:] See footnote 60 on page 35.
[550][Note by AKTA:] In German: *Moment*.

needle by all current elements on the side surfaces of the multiplier, corresponding to the vertical rectangular side.

To simplify the equation between a and b resulting from this condition, consider the case where the compass needle occupies only a very small space in the center of the multiplier and the outer and inner surfaces of the multiplier form concentric cylinders around it; if one sets the given radius of the smaller cylinder $= 1$, the following equation results between a and b, namely:

$$\log \frac{1+a+\sqrt{(1+a)^2+b^2}}{1+\sqrt{1+b^2}} = \frac{3(1+a)^2-1}{2(1+a)\sqrt{(1+a)^2+b^2}} - \frac{1}{\sqrt{1+b^2}} \ .$$

Finally, you get another condition if, while fulfilling the specified relation between a and b, and with the position of the inner side of the rectangle facing the compass needle remaining unchanged, you let a and b grow at the same time, and calculate the associated growth of the multiplier volume $ls = v$. We then designate the ratio l/s, given above as constant, by c, after which the wire cross-section $s = \sqrt{v/c}$ is found; this results in the growing *number of multiplier windings* $2ab/s = 2ab\sqrt{c/v}$. According to this, the magnitude of the torque exerted on the compass needle by a certain current passing through the multiplier can be calculated as a function of the value a or b, and it follows that as a or b increases, this torque also initially grows, but then becomes a maximum, and if a or b continued to grow, it would even decrease again. This also results in the condition of taking the value for a or b for which that torque is a maximum.

If one sticks to the case of a circular multiplier described above to simplify the resulting equation, the following formula results for a:

$$\log \frac{(1+a)\sqrt{(1+a)^2-1}+(1+a)^2+1}{\sqrt{(1+a)^2-1}+2(1+a)} = \frac{[(1+a)^2-1]^{3/2}}{(1+a)[(1+a)^2+1]} \ ,$$

from which $a = 2.0951$ follows, and then, according to the previous relations,

$$b = 1.86178 \ ,$$

$$v = 100.364 \ ,$$

$$l = 10.0182 \cdot \sqrt{c} \ ,$$

is found. If the radius of the smaller cylinder, which was set $= 1$, is denoted by ε, one obtains

$$a = 2.0951 \cdot \varepsilon \ ,$$

$$b = 1.86178 \cdot \varepsilon \ ,$$

$$v = 100.364 \cdot \varepsilon^3 \ ,$$

$$l = 10.0182 \cdot \sqrt{c\varepsilon^3} \ .$$

The observations given as examples in the treatise finally prove what great accuracy of results can be achieved with instruments whose setup corresponds, even if only approximately, to the given regulations. In order to meet all the regulations exactly, all instruments would have had to be completely redesigned. It seemed sufficient to have shown that the *galvanometric* part of the observations can be carried out so precisely according to these regulations, that it was in no way inferior to the *magnetic* part of the observations for the determination of the Earth's magnetism T, and that, on the contrary, the unavoidable uncertainty in determining the absolute resistance resulting from that part of the observations turns out to be even smaller than that resulting from the latter part.

However, the entire *measuring apparatus* required for the absolute determination of resistance would deserve to be manufactured in the most complete and perfect way for the long term, if it is a definitive determination of a *normal standard of resistance*, with generally widespread and used *standard copies*, of which the most precise knowledge of their value in *absolute units* would be required in order to be able to transfer this knowledge to all other resistances compared with them. It would then be most expedient to set up the *measuring apparatus* itself in such a way that the closed circuit of it formed the *normal standard*, because only by repeating the *absolute measurement* from time to time can full security be achieved that the normal standard really remained unchanged. — However, the achievement of the main purpose of such a statement would depend on the *standard copies*, in particular on the general distribution and application that they would find, as well as on their guaranteed equality with the normal standard. For the latter purpose, the fineness of the copying methods and the most appropriate addition to the measuring apparatus were discussed.

16.2 Second Part

The second Part of the treatise discusses the possibility of whether absolute resistance measurements could be carried out *in various ways*, according to very different principles.

The *resistance* is a property of the ponderable body through which the current passes. This property must have its basis in the peculiar nature of the body itself, and should therefore, with complete knowledge of this nature, be directly determinable from this, completely independent of the consideration of all circumstances that do not directly affect the nature of the body, that is, independent *firstly* from the consideration of the variable forces which act on the electrical fluids contained in the body, *secondly* from the consideration of the movements into which these fluids are set by those forces, *thirdly* by considering the actions produced by these movements.

It is only because such a direct determination of resistance from its basis in the nature of the body itself is not possible, owing to lack of knowledge of this nature, that this resistance can only be known *indirectly from experience*, through careful observation of the behavior of bodies (to electricity) under different conditions, and by determining what is *constant* to it.

If the body forms a ring or a closed circuit in which a current i is *generated*, — an electromotive force e acts on the electrical fluids contained in it, and the current i is created in the body, — the precise observation of that electromotive force e and this current intensity i shows, even in cases where both have very different values, that every body has a specific and constant value of the *ratio* e/i. — The property of the body that gives it this constant value of e/i is called its *resistance*.

But if the body now forms a closed circuit in which *is present* the current i, the continuation of the current is associated with certain actions, which are called the *electrical work* A,[551] and the precise observation of the current intensity i and the electrical work A shows that every body has a certain and constant value of the ratio A/i^2. The property of the body by virtue of which it has this constant value, could now equally rightly be called its *resistance*, but the question is whether this constant value is *identical* to the previous one.

If this were the case, — which presupposes that the two properties of the same name have the same basis in the nature of the body, — then it would be possible to carry out resistance measurements in two different ways, according to two completely different principles, their agreement between each other would then serve to confirm that both properties were essentially identical. However, the latter method would first require a more detailed discussion *of the actions associated with the continuation of the current*, which are given the name *electrical work*.

For these actions of the continuous current can be partly *direct*, partly *indirect*, both of which can be useful for the purpose of measuring resistance, but should not be confused with each other.

Through careful observation, one now knows the *heat generation*[552] in the body as an action associated with the continuation of the current. According to the mechanical theory of heat, however, heat generation is viewed as *work* and it is therefore obvious to recognize the *electrical work* A in this heat generation. The question, however, is whether this thermal action of the ongoing current is *direct* or *indirect*. Because if it were an indirect action, there could be other indirect actions besides it, which would have to be taken together in order to obtain the *whole electrical work* A.

For example, if there were a movable magnet nearby, the movement of the magnet would also be an action associated with the continuation of the current,[553] which seems to be equally rightly described as electrical work.

An attempt has therefore been made to, *in the first place*, precisely define the action that is directly and therefore necessarily associated with the duration of the current, which, because it is the original, it deserves to be called *simply the electrical work*, and then *secondly* to particularly research the relationship of every action of the continuous current that has become known to us from experience to that electrical work.

It is assumed the *molecular constitution of the ponderable body* and the *existence of two electrical fluids* between the molecules of the body; on the other hand, the presence of two magnetic fluids is disregarded and instead, as is well known, such a nature of the ponderable molecules is assumed, thanks to which the electric fluids can form *persistent molecular currents* around them, from which all *magnetic* and *diamagnetic* phenomena of the body can be explained.

According to this, the process of a continuous current is that electrical fluid is pulled out of the molecular current of a molecule of the body, driven to the next molecule of the body and drawn into the molecular current there.

The extraction of electrical fluid from a molecular current occurs through *electromotive force*. If such an electromotive force is present, it also continues to act on the drawn-

[551][Note by AKTA:] See footnote 172 on page 108.

[552][Note by AKTA:] In German: *Wärmeentwickelung*. This expression can be translated as heat generation or heat development.

[553][Note by AKTA:] Weber is referring here to the torque exerted by a current carrying wire on a nearby magnet.

out electrical fluid and *increases its velocity* until it re-enters the next molecular current, from which it follows that, with a continuous current, the electric fluid withdrawn from the previous molecular currents enters into all molecular currents at the same time *at a greater velocity* than it had left them.

According to this, the work directly and necessarily associated with the continuation of the current i consists in the *amplification of the molecular currents*, and if we call A this work, which can be determined in absolute units, then $A/i^2 = e/i$ is constant for every body, that is, the two principles according to which the resistance is either the ratio of the electromotive force e to the current intensity i produced thereby, or the ratio of the work A done by the current to the square of the intensity of the current i, by which it is carried out, are basically completely *identical*.

So all that remains is to determine the ratio of the *heat generation*, as the action of the ongoing current, to that *immediate electrical work*.

Experience has now shown that the work equivalent of the heat generation associated with the continuation of the current is equal to the immediate electrical work; for it has been shown that the work equivalent of the heat generation associated with the continuation of the current, according to absolute units, divided by the square of the current intensity determined according to absolute units, is equal to the resistance of the body determined according to absolute units by the ratio e/i.

But this leads to the alternative that either the generation of heat itself is nothing other than the direct work of electricity, that is, *amplification of the molecular currents* in the body, or that all immediate electrical work disappears and is replaced by heat generation, perhaps through a still unknown interaction between electrical and heat fluids, every amplification of the molecular currents is converted into a heat generation equivalent to the work.

However, according to the assumed molecular constitution of the ponderable bodies and the *persistence* of the electrical fluids in their molecular current movements, the *latter alternative* is now inadmissible, because then all immediate electrical work, *consisting in amplification of the molecular currents*, *disappear* and should be replaced by heat development, that is, because then the increased molecular currents *would not be persistent*, as was assumed.

This produces an interesting result, that with the mentioned prerequisites of the *molecular constitution* of ponderable bodies and of the *persistence of molecular currents*, as they form the basis of the doctrine of *magnetism* and *diamagnetism*, only the *former alternative* is compatible, namely that *the heat generation associated with the permanence of the current is itself nothing other than the immediate electrical work*, which is only conceivable, if *all heat generation in ponderable bodies consists in the strengthening of the molecular currents of the electrical fluids in these bodies*, after which a special *thermal fluid* would be eliminated in the ponderable bodies, just as was the case with the *magnetic fluids* under the assumption of persistent molecular currents.

The presupposition of an ether distributed in all empty spaces (even between the ponderable molecules of the body) would remain independent of this; only then would the mediation of heat transfer from one ponderable body to another distant body through this ether, according to the laws of *radiation* and *absorption*, be attributed to an *interaction of the electrical fluids with this ether*, as C. Neumann already tried to justify for the purpose of his theory of the rotation of the polarization plane of light through galvanic magnetic forces.[554]

Finally, all other actions that experience has shown to be associated with the duration of

[554][Note by AKTA:] Carl Neumann (1832-1925). See [Neu58] and [Neu63].

the current in a ponderable body, namely all *electromagnetic*, *electrodynamic* and *induction actions* on distant bodies, result more precisely not as actions of the *current duration*; but as actions of the *current decrease* in the body; because even in cases where the current strength is maintained unchanged during such actions, a *decrease in current* takes place in view of these actions, which is only not observed because in these cases, in addition to the electromotive force *necessary* to maintain current in the body, there is another electromotive force which would otherwise produce an *increase in current*, but in these cases it is only used to compensate with that *decrease in current*.

Chapter 17

[Weber, 1894d] Comments on the Paper: "Investigation into the Electric Arc" by Prof. E. Edlund

Wilhelm Weber[555,556,557,558]

Professor Edlund in Stockholm reported interesting experiments and measurements relating to the *electric arc* in the 131st volume of Poggendorff's *Annalen*.[559] Using a rheostat specially set up for these experiments, he was able to keep the current strength of a Bunsen cell of 70 to 80 elements,[560] measured by a tangent galvanometer,[561] constant, even if the carbon tips[562] used to close the circuit were brought out of contact and at different distances from one another, whereby the electric arc was created between them. Every time the distance was increased there had to be a reduction in the rheostat resistance, every time the distance was reduced there had to be an increase in the rheostat resistance; apart from the sign, the measurements showed that the magnitude of the change in distance was very closely proportional to the magnitude of the change in resistance of the rheostat. According to this, Mr. Edlund equates the real resistance in the current section associated with a change in distance of the two carbon tips to the corresponding change in resistance of the rheostat.

However, when the two carbon tips gradually approached each other until contact was made, there was, in addition to this, a sudden reduction in resistance, which had to be compensated for by a considerable increase in the rheostat resistance, which Mr. Edlund distinguishes from the *real* resistance of the current piece between the two carbon tips, because it has no relation to the length of this current piece. Mr. Edlund considers this only

[555] [Web94c].

[556] Translated and edited by A. K. T. Assis, `www.ifi.unicamp.br/~assis`

[557] The Notes by Heinrich Weber, the editor of Volume 4 of Wilhelm Weber's *Werke*, are represented by [Note by Heinrich Weber:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[558] [Note by Heinrich Weber:] This essay, which W. Weber wrote down after reading Edlund's treatise, was found in his posthumous works without a heading.

[559] [Note by AKTA:] [Edl67], see also [Edl86]. Erik Edlund (1819-1888) was a Swedish physicist. The German expression *galvanische Lichtbogen* can be translated as electric arc, voltaic arc or galvanic arc.

[560] [Note by AKTA:] In German: *Bunsen'-schen Säule*. The Bunsen voltaic cell or element was named after its inventor, Robert Wilhelm Eberhard Bunsen (1811-1899).

[561] [Note by AKTA:] See footnote 114 on page 62.

[562] [Note by AKTA:] In German: *Kohlenspitzen*.

an *apparent* change in resistance, which is due to a *change in the electromotive force equivalent to the measured current intensity*. It is obvious that this change in the electromotive force must occur in the carbon tips at the moment when the contact is removed.

The investigation of this electromotive force occurring in the carbon tips at the moment of breaking the contact now forms the main goal of the further investigation carried out by Mr. Edlund, using as a guide that if E is the electromotive force of the Bunsen's cell and L the resistance of the whole circuit, the electrical work[563] or its equivalent is represented by $E \cdot [E/L]$. If now, at the moment when the contact is removed, the force D opposite to the electromotive force E, which has its seat in the carbon tips, is added, the current, which was $= E/L$ at the contact, would be reduced by D/L, and in the same way the actual electrical work would drop from $E \cdot [E/L]$ to $(E - D) \cdot [(E - D)/L]$.

On the other hand, Mr. Edlund rightly states facts and laws according to which such a reduction of the work of the current does not take place when the current continues after the contact is removed and therefore compares this phenomenon with the phenomena of damping, where the damper reduces the work of the original current by the electromotive force it exerts on the closed circuit of the original current, but for which the work of the current induced in the damper itself gives an equivalent. Such an equivalent, says Mr. Edlund, is the work required to tear off the carbon particles[564] from which the arc originates.

I now completely agree with Mr. Edlund on the two main points, namely, *firstly*, that at the moment the contact is removed, a separating force D opposite to the electromotive force of the cell E is added to the carbon tips, and *secondly*, that the sum of the work done in the arc and the electrical work of the rest of the circuit is equal to the electrical work $E \cdot [E/L]$, which the Bunsen cell performs during the contact, then in order to avoid any contradiction a third point seems to deserve closer consideration.

If the electromotive force of the Bunsen's cell is the only one that has worked in the *closed* circuit of resistance L up to now, but the electromotive force $= -D$ is added at the moment the contact is removed, then if the circuit also had the same resistance L after the contact has been removed, the value of the previous electrical work $E \cdot [E/L]$ would from now on necessarily fall to $(E - D) \cdot [(E - D)/L]$, without any equivalent for this loss, just as if a cell[565] with infinitesimally small resistance of force D had been inserted *upside down* into the previous circuit of force E.

But since, if we combine the work in the circuit and in the arc, such a reduction in work does not actually occur, it is clear that in order to eliminate the contradiction it must be discussed in more detail whether the circuit is really still *closed* after the contact has been removed and whether, therefore, the piece of current lying between the two carbon tips has a *real* resistance, as Mr. Edlund has determined it, in that the *real resistance of a conductor* is understood to be the constant ratio of the electromotive force acting in this conductor to the strength of the current thereby produced in the conductor.

In my opinion, after the contact has been removed, there is no closed circuit and no real resistance for the current section between the carbon tips that are out of contact, because such resistance only applies to a real conductor that is not present between the carbon tips.

I distinguish between a *closed electrical current* and a *closed galvanic circuit*, in that every closed galvanic circuit also forms a closed current, but not vice versa every closed current

[563][Note by AKTA:] See footnote 172 on page 108.

[564][Note by AKTA:] In German: *Kohlentheilchen*.

[565][Note by AKTA:] In German: *Becher*. That is, a voltaic cell or element.

forms a closed galvanic circuit. Ohm's law,[566] according to which the current strength is represented by the quotient E/L (when E denotes the electromotive force, L the resistance), only applies to *closed galvanic circuits* (i.e. for electrical currents in closed conductors) but in no way to closed currents *without a closed conductor*.

Assuming this, it is essentially a question of what Mr. Edlund described and measured as the *real* resistance of the section of current between the carbon tips. I also consider it only as an *apparent* resistance, which is based on an *electromotive force equivalent to the measured current intensity*.

Mr. Edlund denotes the *real* resistance (which is proportional to the distance of the carbon tips) for one unit of distance with b, the *apparent* resistance with a and sets, because a has its basis in an electromotive force $= -D$, the current intensity $= (E - D)/(M + nb)$ when n denotes the distance of the carbon tips, instead of putting it $= E/(a + nb)$ as it should be if a were a real resistance, like nb. Instead of this I set the current intensity $= (E - D)/M$ because I am looking for the source of all the *apparent* resistance $a + nb$ in the electromotive force $-D$.

According to his calculation, Mr. Edlund finds the electromotive force D, expressed in parts of the electromotive force of the cell, to be almost constant, namely

$$D = 0.3239 \ .$$

On the other hand, according to my calculation, I find that it increases almost proportionally with the length of the electric arc, namely

Length of the electric arc in scale divisions	D
5	0.377
4	0.371
3	0.362
2	0.355
1	0.349

The essential point to be considered in this different calculation is that Mr. Edlund, according to his calculation, has to look for a new, unknown source for the electromotive force D; according to my calculation, the source of this electromotive force arises automatically and necessarily from the laws of distribution of electricity in an unclosed galvanic circuit.

For in a closed galvanic circuit, the current path is surrounded by a surface on which free electricity is distributed in such a way that the ratio of the electromotive force to the resistance is balanced in all current elements;[567] in an unclosed galvanic circuit, on the other hand, the surface charged with free electricity also envelops the two ends of the circuit which have been brought out of contact, just as in a self-discharge jar[568] the positive and negative charge also extends over the surfaces of the two buttons of the spark micrometer which face each other and are separated by a small intermediate space.

However, this electricity distributed on the surface plays a completely different role when the circuit is not closed than when the circuit is closed; in the latter it has no influence

[566][Note by AKTA:] See footnote 121 on page 65.

[567][Note by AKTA:] For a discussion of surface charges in resistive conductors carrying steady currents, see [AH07] with Portuguese translation in [AH09] and German translation in [AH13].

[568][Note by AKTA:] In German: *Selbstentladungsflasche*. That is, a self-discharge Leyden jar, see Chapter 12, The Leyden Jar and Capacitors, of [Ass18a] with Portuguese translation in [Ass18b] and Russian translation in [Ass19b].

at all on the value of the entire electromotive force E, because the integral value of the electromotive forces exerted by each particle on all elements of a *closed* line in the direction of the same is known to be zero. However, the integral value of the electromotive forces exerted by each particle on all elements of an *unclosed* line (which forms the unclosed circuit) in the direction of the same is different from zero and therefore has a very large influence on the value of the entire electromotive force E, so that at electrical equilibrium (when the unclosed galvanic circuit is surrounded by a perfect insulator), the value of E is completely canceled.

Just as in a self-discharging jar the charges on the buttons can disappear at the moment of self-discharge, but are always recreated as the jar is continuously connected to the electrostatic machine,[569] so is the case with the galvanic circuit when sparks flash over between the ends that are out of contact and form the arc. Even if there were a *continuous* withdrawal of electricity from both ends in some way other than through a *conductor closing the circuit*, these charges and their influence on the value of the entire electromotive force E would not disappear.

If an amount of electricity were continuously withdrawn from the two ends (in any other way than through a conductor closing the circuit), which would be in the ratio of $n : 1$ to the amount passing through the cross-section of the circuit when contact was made, then such a distribution of electricity would be formed on the surface of the unclosed circuit that in all elements of the circuit the ratio of the electromotive force to resistance would be like $nE : L$, so that the total electromotive force in the whole length L of the circuit would be equal to nE, only a fraction of the electromotive force of the cell, while the rest of the latter, namely $(1-n)E$, would be completely canceled out by the influence of the electrical distribution on the surface of the unclosed conductor. The electrical charge on the surface of the galvanic circuit after contact has been removed therefore exerts the electromotive force $-(1-n)$, the same as that described above as $-D$.

Of the process in the arc, by which the two ends of the galvanic circuit which have been brought out of contact are continually deprived of electricity, only so much is known that it is not mediated by any conductor closing the circuit. Now imagine this process consisting of a small conductor oscillating between the ends that have been brought out of contact. Provided that no other force acts on this conductor than that resulting from the interaction of its electrical charge and the charges of the two ends, and that when it hits the immovably held ends, this conductor is thrown back according to elastic laws in such a way that it loses nothing of its living force,[570] its speed would increase to infinity, and the position of a conductor closing the circuit would be more and more perfectly represented by it.

But if the conductor were not thrown back according to elastic law, or if living force were to be withdrawn from the pendulating conductor in some other way, and indeed the same fraction of the total living force it possessed with each repetition of its movement, then its velocity would soon approach a limit where it would remain constant. The renewal of the living force withdrawn from the oscillating conductor at this speed forms the work A done by the electricity between the two ends (in the arc) and the equation used to determine this limit state is:

$$A + nE \cdot \frac{nE}{L} = E \cdot \frac{E}{L} \ .$$

[569][Note by AKTA:] In German: *Elektrisirmaschine*. That is, an electrostatic generator or electrostatic machine.

[570][Note by AKTA:] See footnote 173 on page 108.

It is easy to see that if the determinations of $(1-n)E = D$, as given by Mr. Edlund, could be combined with determinations of the periodicity of the discharges through the electric arc, in the manner given by Mr. Feddersen[571] for the discharge spark of a Leyden jar, considerably more insight into the still little known process in the electric arc would be gained.

If the two carbon tips were suddenly completely isolated from each other with the removal of the contact, the value of $-D$ would suddenly jump from zero to $-E$. Due to the unknown process between the carbon tips, by which electricity is still transported from one carbon tip to the other even after the contact is removed, this sudden jump in the value of $-D$ is somewhat reduced, namely from 0 to $-(1-n)E$. But it is obvious that by gradually increasing the distance of the carbon tips from each other, the value of $-D$ must also gradually change from $-(1-n)E$ to $-E$, thus D must increase with the distance of the carbon tips, as has also been shown by Edlund's measurements according to my calculation above.

[571][Note by AKTA:] Berend Wilhelm Feddersen (1832-1918) was a German physicist.

Chapter 18

Editor's Introduction to Weber's Posthumous Paper on Electroscopic and Electrodynamic Actions of Free Electricity in Closed Circuits

A. K. T. Assis[572]

Wilhelm Weber (1804-1891) discussed in Sections 28 to 36 of his second major Memoir of 1852 on *Electrodynamic Measurements* two situations in which a current can flow along a resistive conductor.[573]

The first situation appears in Figure 18.1.

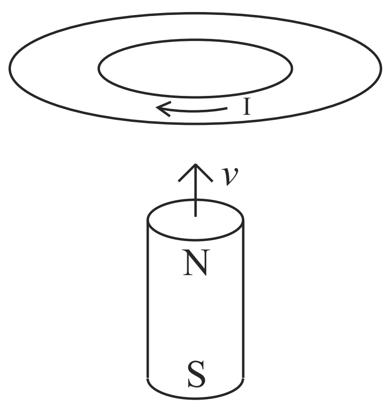


Figure 18.1: A magnet moving with velocity v along the axis of a resistive ring, perpendicular to its plane, and inducing an azimuthal current I.

Consider a cylindrical magnet NS with its axis orthogonal to the plane of a circular homogeneous copper ring whose cross-section is the same everywhere. The ring is kept at

[572]Homepage: www.ifi.unicamp.br/~assis

[573][Web52b] with English translation in [Web21e]. These Sections have been discussed in detail in Appendix A of the book *The Electric Force of a Current: Weber and the Surface Charges of Resistive Conductors Carrying Steady Currents*. It is available in English, [AH07], Portuguese, [AH09], and German, [AH13]. See also [Ass21e].

rest in the laboratory. When the magnet moves with velocity v along the axis of the ring, it exerts the same electromotive force on all elements of the ring as a result of that motion. Since all elements of the ring are also endowed with the same resistance, an equal electric current will be generated simultaneously in all of the elements of the ring by that motion, flowing along its azimuthal direction. The value of the current may depend on the distance between the magnet and the ring. But it will have the same value along the azimuthal direction, that is, it will not depend on the azimuthal angle θ along the ring. From this fact it will follow that a greater or smaller accumulation of positive or negative electricity, varying along the azimuthal angle θ, cannot arise at any location on the ring. This first case is that of a current in a closed resistive conductor which exists without variation of free electricity along the azimuthal direction in the circuit.

The second situation appears in Figure 18.2.

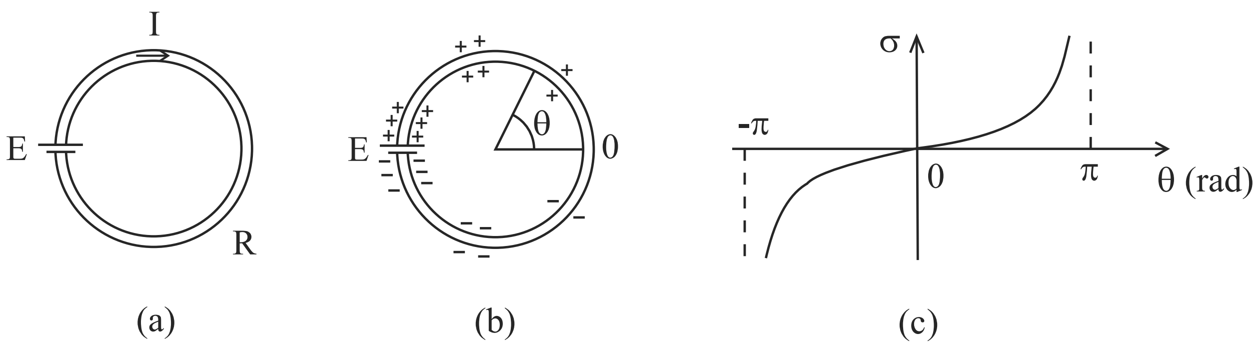


Figure 18.2: (a) Resistive ring connected to a battery and carrying a steady current I. (b) Charges distributed along the surface of the ring. (c) Surface charge density σ as a function of the azimuthal angle θ.

We have a battery of negligible internal resistance and generating a constant voltage or electromotive force E between its terminals. The ring has a total resistance R and a small opening at a certain point. When the two terminals of the battery are connected to the two extremities of the opening of the ring, there will be a distribution of charges along the surface of the ring. Soon there will be a steady state configuration in which a constant current I flows everywhere along the azimuthal θ direction of the ring. This current will satisfy Ohm's law $E = RI$.[574] In this steady state situation, the surface charge density σ (amount of charge per unit area) along the surface of the ring will be constant in time, but its magnitude will vary along the azimuthal direction θ. It will be zero in the opposite side of the battery, that is, at $\theta = 0$ in Figure 18.2. The amount of positive charges will increase towards the positive terminal of the battery and the amount of negative charges will increase towards the negative terminal of the battery, as indicated in Figure 18.2 (b) and (c).

In his second major Memoir on *Electrodynamic Measurements* published in 1852 Weber performed pioneering calculations of the distribution of the surface charge density σ for different resistive circuits carrying constant currents. In particular, he considered a straight cylindrical wire and a resistive ring. He calculated how σ varied along the length of the cylindrical wire. He also calculated how σ varied along the azimuthal direction of the ring.

The present paper was published posthumously in 1894 in Volume 4 of Weber's Collected Works.[575] Weber's goal was to obtain an equation describing the total amount of positive

[574]See footnote 121 on page 65.
[575][Web94e].

and negative charges distributed along a closed conductor of total resistance R carrying a steady current I when this conductor had a small opening which was connected to the two terminals of a battery generating a voltage or electromotive force E between its terminals. In particular, Weber considered the situation in which the current had a numerical value = 1 in a certain system of units. In this case, from Ohm's law $E = RI$, the electromotive force E and the resistance R had equal numerical values in the same system of units. This system of units might be, for instance, the absolute system of units created by Carl Friedrich Gauss (1777-1855) and W. Weber.[576]

We can illustrate Weber's goal with Figure 18.2, although in this posthumous paper he did not consider specifically a conducting ring. If Weber were considering this ring in this posthumous paper, he would be interested in the total amount of positive charge distributed between $\theta = 0$ and $\theta = \pi$, together with the total amount of negative charge distributed between $\theta = 0$ and $\theta = -\pi$, as shown in Figure 18.2 (b) and (c).

After this introduction, we present the English translation of Weber's paper.

[576] [Ass21g].

Chapter 19

[Weber, 1894e] Electroscopic and Electrodynamic Actions of Free Electricity in Closed Circuits

Wilhelm Weber[577,578,579]

Approximate calculation of the amount of electricity, which needs to be distributed over a closed circuit, if an electromotive force, equal to the resistance of the circuit, is acting on one position of the circuit, such that the intensity of the current is $= 1$ *everywhere in the circuit.*

In a closed circuit, a steady current without charge can only persist, if the electromotive forces of all circuit elements are proportional to the resistance of the elements.[580] If this proportionality does not exist, the intensity of the current differs in different elements [of the circuit], and, as a consequence of this inhomogeneity, in some parts of the circuit (where the intensity of the current decreases) positive electricity, and in other parts (where the intensity of the current increases) negative electricity will be released, or the circuit will be charged. From this charging of the circuit, additional electromotive forces are formed, from which higher intensities of the current will be reduced, while lower [intensities of the current] will be enhanced. The remaining inhomogeneities produce additional charges and therefore additional electromotive forces, resulting in a compensation of the remaining inhomogeneities of the current intensity, and this [process] continues, until a particular amount of charge has been generated in the circuit, at which an equal amount of current intensity can persist in all elements [of the circuit].[581]

[577][Web94e].

[578]Translated by F. J. Linz, fjlinz@posteo.de, and edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

[579]The Notes by W. Weber are represented by [Note by WW:]; the Notes by F. J. Linz are represented by [Note by FJL:], while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

[580][Note by AKTA:] An example of this situation was discussed by Weber in 1852, as illustrated in Figure 18.1. In this case there is no variation along the azimuthal direction of the amount of free charges distributed on the surface of the ring. That is, the surface density of free charges does not depend on the azimuthal angle θ.

[581][Note by AKTA:] An example of this situation appears in Figure 18.2 in which the two terminals of a

This charge of the circuit is of great importance for the theory of the current formation, but has been of little importance in application so far, because it [the charge of the circuit] always adjusted itself instantly, and their peculiar actions, independent on the current [of the circuit], are so small, that the most accurate means of observation can only hardly detect them. These hardly observable actions of the charges of a closed circuit are called electroscopic actions, and to their exploration, only few and imperfect experiments have been performed so far.

These electroscopic actions recently gained greater practical meaning, e.g. in Rühmkorff's *machines*,[582] for which the requirements to the insulation of the wire winding, to avoid sparks arcing over, depends on this charge [which causes the electroscopic actions], as does the striking distance of the sparks from one end of the induced wire[583] to the other [end of the wire], furthermore in *electrical telegraphy*, for which the speed of signaling depends significantly on the speed of current formation. This current formation is strongly delayed if the charging of the circuit required for a steady current is to be formed, for example, under the influence of a capacitor, as is the case with submarine telegraphs, where the metal tube enclosing the insulated wire, together with the surrounding water, forms such a capacitor.

To avoid this delay of the charge and hence the accompanying delay of the signals, it will be necessary to return the current, instead of through the earth or through the sea, through a second conducting wire insulated from the first [conducting wire] and enclosed with it in the same cable. The opposite charges of equal configuration,[584] which thereupon must be preserved by these two wires in each position during formation of the current, compensate each other in their action on the encircling metal tube, the most perfect [compensation], if the distance of the two wires to each other is very small compared to their distance to the metal tube. The metal tube will thereupon not take any appreciable charges and consequently will not exert any noticeable reaction on the formation of the charge in the enclosed wires. — Thus, at the same time, the induction of currents in the circuit by variations of the terrestrial magnetism will be avoided.

Thereby, the study of electroscopic actions of a closed circuit and of the laws of the charge [distribution], which they [the electroscopic actions] are depending on, gained practical interest. For this purpose, the experiments will no longer be sufficient upon which the investigation was limited so far, namely, to use these charges of a closed circuit to charge fine capacitors and sensitive electroscopes, from which the name of the *electroscopic actions* of the circuit originates.

Very important to these investigations are the data which will be obtained with the aid of submarine telegraphs themselves on the duration of the formation of steady currents; yet it is easy to overlook the fact that even the most accurate observations of such telegraphs will not give an accurate determination of this duration of the formation [of steady currents], because

battery are connected to the two extremities of a small opening in a resistive ring. For a steady current to exist in this configuration, it is necessary a distribution of free charges along the surface of the ring. The surface density of charges is constant in time for a constant current, but its magnitude varies along the length of the ring. That is, the surface charge density σ varies along the azimuthal angle θ, as shown in Figure 18.2 (b) and (c).

[582][Note by FJL and AKTA:] In the original text: "Rhumkorff'sche Maschinen". Rühmkorff's machines or induction coils were named after Heinrich Daniel Rühmkorff (1803-1877), a German instrument maker, [Nor96].

[583][Note by FJL:] In the original text: "induzierter Draht", so the wire in which the induced current flows.

[584][Note by FJL:] In the original text: "entgegengesetzt gleichen Ladungen", meaning that in the two wires, the charges have the same magnitudes, but opposite signs, such that their actions cancel.

of the manifold other influences which are involved, on which the time between signaling and signal observation also depends. In addition, the most influential conditions during current formation in such telegraphs are invariable and therefore provide only incomplete data on which no laws can be based, if they have not already been found by other means.

A different approach is featured by the electrical oscillations, when steady currents are formed in a closed circuit, but only of short duration and with changing direction. Thereby, those electrical charges [mentioned above] are also generated, but alternately quite different charges. The transition from one charge to another, or the formation of a different [charge] distribution, is associated to a movement distinct from the equal amount of current in the whole circuit, which [movement] differs in the individual parts of the circuit.

It is clear from this, that in all parts of the circuit the same synchronous oscillation can, strictly speaking, not occur alone, if the excitation is originating from one position of the circuit, but it [the synchronous oscillation] is necessarily connected to another oscillation, which [the second one] can be considered approximately synchronous, but very inhomogeneous in different parts of the circuit. According to Ampère's laws,[585] this latter oscillation results directly in *electrodynamical forces*, which modify the observed deflection of the dynamometer; furthermore from the laws of electrostatics and induction result additional electromotive forces, which modify the intensity of the current and hence directly act on the dynamometer and need to be observed and studied by accurate observations of the dynamometer.

The element of a closed circuit, in which the electromotive force ω acts, is enclosed between the cross-sections aa and bb (Figure 1).

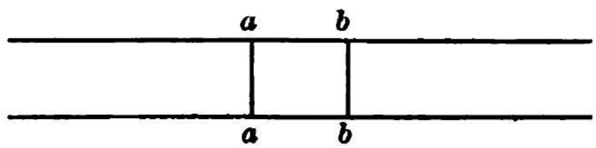


Fig. 1.

Suppose the electromotive force being spread in a way that it is increasing from both boundary surfaces towards the center, proportionally to the distance from the boundary surface. Suppose aa and bb are charged with positive electricity on both sides, such that the potential is constant within the (infinitely small) gap between both coatings[586] of aa and bb. Let the average density of this coating be $= +e$.[587] Since $[dV/dx]_{-\varepsilon} = 0$ between the two coatings,[588] beyond the coatings we have[589]

$$\left(\frac{dV}{dx}\right)_{+\varepsilon} - \left(\frac{dV}{dx}\right)_{-\varepsilon} = \left(\frac{dV}{dx}\right)_{+\varepsilon} = 4\pi e \ ,$$

if the repulsive forces are assumed to be positive.

One shall think of the charge directed towards bb being distributed uniformly until the center between aa and bb, and call the density of positive electricity uniformly distributed

[585][Note by AKTA:] See footnote 15 on page 18.

[586][Note by AKTA:] In German: *Belegungen*. This word can be translated as coatings, occupancies, allocations, assignments, surfaces, layers etc.

[587][Note by AKTA:] That is, e is the surface density of charge at the point under consideration.

[588][Note by AKTA:] Here ε represents an extremely small distance.

[589][Note by AKTA:] The next equation is the boundary condition at an interface and V represents the electric potential.

in half the space of the element under consideration by k,[590] and call the halfway distance bb from aa by α, then

$$k\alpha = e \ .$$

In this region of space then

$$\frac{d^2V}{dx^2} = 4\pi k \ ,$$

if $d^2V/dy^2 = d^2V/dz^2 = 0$ is assumed, and the repulsive forces are considered to be positive.

From this it follows that

$$\frac{dV}{dx} = 4\pi kx \ ,$$

if x is calculated from aa, where $dV/dx = 0$.[591] Consequently, the integral value of the electromotive force is[592]

$$\int_0^\alpha \frac{dV}{dx} dx = \int_0^\alpha 4\pi kx dx = 2\pi k\alpha^2 = 2\pi e\alpha \ ,$$

moreover suppose bb being charged with negative electricity on both sides, such that the potential is constant within the (infinitely small) gap between both coatings [of negative electricity]. Let the average density of this coating be $= -e$. As $[dV/dx]_{+\varepsilon} = 0$[593] (where $+\varepsilon$ is located in the gap), it follows that

$$\left(\frac{dV}{dx}\right)_{-\varepsilon} - \left(\frac{dV}{dx}\right)_{+\varepsilon} = \left(\frac{dV}{dx}\right)_{-\varepsilon} = 4\pi e \ .$$

One shall think of the charge directed towards aa being distributed uniformly until the center between bb and aa, and call the density of negative electricity uniformly distributed in half the space of the element under consideration by $-k$, and call the halfway distance from aa to bb by α, then

$$k\alpha = e \ .$$

In this region of space then

$$\frac{d^2V}{dx^2} = -4\pi k \ ,$$

if $d^2V/dy^2 = d^2V/dz^2 = 0$ is assumed, and the repulsive forces are considered to be positive.

From this it follows that

$$\frac{dV}{dx} = 4\pi k(\alpha - x) \ ,$$

if x is calculated from the center, such that for $x = \alpha$, $dV/dx = 0$. Consequently, the integral value of the electromotive force is

[590][Note by AKTA:] That is, k is the volume density of charge at the point under consideration.

[591][Note by AKTA:] That is, $x = 0$ at aa. At this point $x = 0$ we have $dV/dx = 0$.

[592][Note by FJL:] In the original text, there is an equal sign in between, $\int_0^\alpha \frac{dV}{dx} = dx = ...$, which has been removed in this English translation.

[593][Note by FJL:] In the original text, two different brackets are used here: $[dV/dx)_{+\varepsilon} = 0$.

$$\int_0^\alpha \frac{dV}{dx}dx = \int_0^\alpha 4\pi k(\alpha - x)dx = 2\pi k\alpha^2 = 2\pi e\alpha \ .$$

From this results the electromotive force for the whole element under consideration

$$= 4\pi e\alpha \ .$$

One shall set this electromotive force oppositely equal to the given [electromotive force] $(-E)$, hence

$$4\pi e\alpha = E \ .$$

Then, instead of the outer coating of aa, the surface of the wire is now covered [with electricity] from aa to the point which bisects the entire length of the wire, and denote this half length by L. The density ε of this coating[594] grows from the halving point to aa from 0 to $[8\alpha/\pi r] \cdot e$[595,596] proportionally with the length of the wire from the halving point to the point under consideration, then, if the wire is very long compared to its thickness, the electromotive force $\left(-\frac{1}{2}E\right)$ will act in a distributed manner over the length L of the wire, and its distribution can be considered to be approximately uniform.

It follows thereupon that the charge of the length L of the wire, which causes this uniform distribution of the half given electromotive force $\left(-\frac{1}{2}E\right)$, if r is the radius of the wire,

$$= \int_0^L 2\pi r\varepsilon dx \ ,$$

wherein

$$\varepsilon = \frac{8\alpha e}{r\pi L} \cdot x \ ,$$

thus

[594] [Note by AKTA:] Weber is now utilizing the symbol ε to represent the surface density of charge, that is, the amount of charge per unit area.

[595] [Note by WW:] According to Clausius (Poggendorff, *Annalen* 1852, Vol. 86, p. 203), for a capacitor, with radius a [of the plates] and distance c between the plates, the charge of one plate can be expressed by

$$M = -\frac{a^2}{4c} \cdot F \left[1 + \frac{c}{a\pi}\left(\lg\frac{17.68a}{c} + 2\right)\right] \ .$$

The charge of the second plate by

$$N = \frac{a^2}{4c} \cdot F \left[1 + \frac{c}{a\pi}\left(\lg\frac{17.68a}{c} - 2\right)\right] \ .$$

If one adds to the bound electricity N so much free electricity N', such that $N + N' = -M$, then $N'/(N + N')$ gives the free electricity in parts of the total charge a. Yet it is

$$\frac{N'}{N+N'} = \frac{4c}{a\pi\left[1 + \frac{c}{a\pi}\left(\lg\frac{17.68a}{c} + 2\right)\right]} \ ,$$

or in case of very small c/a, $N'/(N + N') = 4c/a\pi$. If one substitutes in our case 2α for c and r for a, then $N'/(N+N') = 8\alpha/r\pi$ is the fraction of the free electricity from the total charge. If the density e corresponds to the total charge, then the density $[8\alpha/r\pi] \cdot e$ corresponds to the free charge.

[596] [Note by AKTA:] [Cla52].

$$= \int_0^L \frac{2\pi re}{L} \cdot \frac{8\alpha}{r\pi} x dx = \pi r \frac{8\alpha}{r\pi} eL = \frac{2LE}{\pi} \ .$$

The same amount of negative electricity is found distributed on the other half of the wire. Thus, if we denote the whole length of the wire to be $l = 2L$, then

$$\pm \frac{lE}{\pi}$$

is the amount of positive or negative electricity distributed in the closed circuit; consequently, if

$$E = w \ ,$$

where w is the resistance of the circuit, then

$$\pm \frac{lw}{\pi}$$

is that amount of positive or negative electricity which must be distributed in a closed circuit, in which the current = 1 flows, when the current is generated from one point of the circuit.[597]

[597][Note by AKTA:] Weber is here writing Ohm's law as $E = wi$, where E is the electromotive force acting at one point of the circuit. The circuit has a total resistance w. This electromotive force produces a constant current i along the circuit. If $i = 1$ in a certain system of units, then $E = w$ in this system of units. The total amount of positive electricity distributed in half of the circuit will be given by $lE/\pi = lw/\pi$, while the total amount of negative electricity distributed in the other half of the circuit will be given by $-lE/\pi = -lw/\pi$, where l is the total length of the circuit under consideration.

Chapter 20

[Weber, 1894f] On Electrothermism. (On Electricity and Heat)

Wilhelm Weber[598,599,600]

On Electricity and Heat

After the first fundamental phenomenon of *electromagnetism* was discovered by Oersted,[601] other fundamental phenomena were also discovered very soon and hand in hand with these discoveries the theory of electromagnetism was developed, so that the theory often anticipated and provided the guide to the discovery of those fundamental phenomena.

Things were completely different with the fundamental phenomena and with the theory of *electrothermism*. Despite the fact that most of the fundamental electrothermal phenomena were discovered earlier than the electromagnetic ones, the theory of electrothermism has still remained completely undeveloped.

The fundamental electrothermal phenomena discovered so far are as follows:

1. A tourmaline, as it cools, is positively charged at one end and negatively charged at the other. Vice versa while it is being heated. Hankel, Elektrische Untersuchungen (Electrical Investigations). Ninth Treatise (Abh. d. Königl. Sächs. G. d. W. math.-phys. Klasse, Vol. X, No. IV, 1872).[602]

2. A current in a closed conductor generates *heat* in a piece of the latter which has resistance w in the time element dt, the mechanical equivalent of which is $= wi^2dt$,[603] with w and i expressed in absolute units.

3. A current occurs in a closed conductor made of two different metals if the conductor has unequal temperatures at the two points where the different metals touch each other.

598[Web94f].

599Translated and edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

600The Notes by Heinrich Weber, the editor of Volume 4 of Wilhelm Weber's *Werke*, are represented by [Note by Heinrich Weber:]; while the Notes by A. K. T. Assis are represented by [Note by AKTA:].

601[Note by AKTA:] See footnote 332 on page 187.

602Wilhelm Gottlieb Hankel (1814-1899) was a German physicist. See [Han74].

603[Note by AKTA:] This result is due to James Prescott Joule (1818-1889). See [Jou41b]; [Jou41a] with French translation in [Jou42]; [Len43c], [Len43a], [Len43b], [Len44b] and [Len44a]. See also [MS20] and [Mar22].

(Seebeck's fundamental phenomenon).[604]

4. If a current passes through a conductor made up of two different metals, which has the same temperature everywhere, then (apart from the heat generated by the current in the individual pieces of conductor in proportion to their resistance) heat is generated in one surface of contact between the metals and disappears in the other. (Peltier's fundamental phenomenon).[605]

A basis for a theory of electrothermism (10th volume of the Treatises of the Königl. Ges. d. Wissenschaften zu Göttingen — Royal Society of Sciences in Göttingen, 1862, "Zur Galvanometrie" — On Galvanometry, Section 33,[606,607] and 10th volume of the Treatises of the math.-phys. Klasse der Königl. Ges. der Wissenschaften, Leipzig, 1871, "Elektrodynamische Maassbestimmungen, insbesondere über das Princip der Erhaltung der Energie" — Electrodynamic measurements, sixth memoir, relating specially to the principle of the conservation of energy, Sections 19 and 20)[608,609] has now been sought in the adoption of Ampère's hypothesis of molecular currents[610] and in a further development of this hypothesis.[611]

As one can easily see, the assumption of molecular currents in all magnetic and diamagnetic bodies consistently leads to a completely new view of the behavior of the electrical fluids in conductors in the so-called electrostatic equilibrium states. One is led to the conclusion that the electrical fluids are never at rest, but are in constant motion in circular orbits around the individual molecules, according to which the entire electrostatics developed by Poisson must be reorganized.[612]

Furthermore, one is also led to a new view of the galvanic currents, namely that the entire electrical fluids in the current conductor, with so-called constant currents, do not move at a constant speed at all, but that only individual particles of the electrical fluids are torn away from the molecular currents to which they belong, pass jerkily to the next molecular currents, to which they then belong until they are torn away again, and so on.

The electromotive force that causes the breakaway, accelerates the movement of the broken-away particles until they enter the next molecular current. This results in an increase of living force,[613] which is equivalent to the mechanical work determined by the product of the electromotive force and the distance between the two molecular currents.

But now it has emerged that the mechanical work determined in this way is the mechanical work equivalent of the heat generated by the current in the conductor, from which it can be concluded that the *heat and living force of the electrical fluids moving in the molecular currents are identical*. This is essentially the theory of the second fundamental phenomenon of electrothermism, which is based on Ampère's molecular currents.

---

[604] [Note by AKTA:] Thomas Johann Seebeck (1770-1831). See [See25] and [See26] with a partial English translation in [See69] and a partial Portuguese translation in [FS16].

[605] [Note by AKTA:] Jean Charles Athanase Peltier (1785-1845). See [Pel34].

[606] [Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. IV, p. 91.

[607] [Note by AKTA:] [Web62, Section 33, p. 91 of Weber's *Werke*]. This paper is translated in Chapter 8. See, in particular, Section 8.33 on page 120.

[608] [Note by Heinrich Weber:] Wilhelm Weber's *Werke*, Vol. IV, p. 291 and 294.

[609] [Note by AKTA:] [Web71] with English translation in [Web72] and [Web21g].

[610] [Note by AKTA:] In German: *Molekularströmen*.

[611] [Note by AKTA:] Weber is referring to the hypothesis of molecular currents developed by André-Marie Ampère (1775-1836) between 1820 and 1827, see Chapter 5 (Ampère's Conception of Magnetism) of [AC15]. See also footnote 15 on page 18.

[612] [Note by AKTA:] See footnote 289 on page 172.

[613] [Note by AKTA:] See footnote 173 on page 108.

But if, according to this theory of the second fundamental phenomenon of electrothermism, *heat* is the living force of the electrical fluids moving in the molecular currents, then it follows that the different degrees of temperature *in one and the same body* must be due to different *speeds* of the electrical fluids moving in the molecular currents, while the *masses* of the electrical fluid moving in the molecular currents are not different. Because the mass of the moving electrical fluid is given by the always equal mass of the fluids firmly connected to the core of the molecular current.

However, *different bodies* will differ from each other by the difference in the fluid that is firmly connected to the cores of the molecular currents and therefore, if there is *no charge*, also by the different *masses* of the moving electrical fluid.

If various such bodies come into contact at the same temperature and the large masses of electrical fluids moving in them are denoted by e and e' and their velocities by u and u', then according to the definition of the temperature equality of different bodies

$$\varepsilon u^2 = \varepsilon' u'^2 \ ,$$

where ε denotes the electrical mass torn from the first molecular current and passed to the other, ε' denotes the electrical mass torn from the other [molecular current] and passed to the first. So if the velocities u and u' belonging to two different bodies at the same temperature are different, then the masses ε and ε' are also different. So there is a charging on the molecules, namely a *positive* of one and a *negative* of the other, that is, a separating force[614] acts on the contact surface, which is the *principle of galvanism*.

If we designate that temperature with t, further with a and a' the specific heats of the two bodies and with b and b' the number of temperature degrees below zero for which the living force of the molecular currents in these two bodies would be reduced to zero in proportion, so one has

$$t = \frac{e}{a}u^2 - b = \frac{e'}{a'}u'^2 - b' \ , \tag{1}$$

for a temperature t' one obtains in the same way:

$$t' = \frac{e}{a}w^2 - b = \frac{e'}{a'}w'^2 - b' \ ,$$

or putting

$$w^2 = u^2 + v^2 \ , \qquad\qquad w'^2 = u'^2 + v'^2 \ ,$$

[one obtains:]

$$t' = \frac{e}{a}\left(u^2 + v^2\right) - b = \frac{e'}{a'}\left(u'^2 + v'^2\right) - b \ . \tag{2}$$

Equation (1) subtracted from (2) gives

$$\frac{e}{a}v^2 = \frac{e'}{a'}v'^2 \ ,$$

or

[614] [Note by AKTA:] See footnote 177 on page 110.

$$v'^2 = \frac{e}{e'}\frac{a'}{a}v^2 \ . \tag{3}$$

From Equation (1) it follows

$$u'^2 = \frac{a'}{e'}\left(\frac{e}{a}u^2 + b' - b\right) \ , \tag{4}$$

so if you add (3) and (4), you get

$$u'^2 + v'^2 = \frac{a'}{e'}\left[\frac{e}{a}\left(u^2 + v^2\right) + b' - b\right] \ ,$$

or

$$w'^2 = \frac{a'}{e'}\left(\frac{e}{a}w^2 + b' - b\right) \ .$$

But from the temperature equality of both bodies, namely $= t'$, it follows

$$\eta w^2 = \eta' w'^2 \ ,$$

where η denotes the electrical mass torn from the first molecular current and passed to the other, η' denotes the electrical mass torn from the other and passed to the first [molecular current]. From this it follows

$$\frac{\eta}{\eta'} = \frac{a'}{e'}\left(\frac{e}{a} + \frac{b' - b}{w^2}\right) \ ,$$

or, since $w^2 = [a/e](t' + b)$,

$$\frac{\eta}{\eta'} = \frac{e}{e'}\frac{a'}{a}\left(1 + \frac{b' - b}{t' + b}\right) \ .$$

The choice of the ratio η/η' (i.e. the magnitude of the separating force at the contact surface of both bodies) depends on the difference in temperature t', if the value $b' - b$ for the two bodies is different from zero, i.e. if the number of temperature degrees below zero, for which the living force of the molecular currents would disappear in proportion, is different for one body than for another body.

It is obvious that for all bodies for which the value of the specific heat changes more or less with temperature, the value of $b' - b$ must be different from zero. This probably occurs to a particularly high degree for bismuth and antimony.

This dependence of the magnitude of the separating force at the contact surface on the temperature contains the principle of Seebeck's fundamental phenomenon, or the third electrothermal fundamental phenomenon.

The principle for Peltier's fundamental phenomenon or for the fourth electrothermal fundamental phenomenon follows itself from this.

Let E be the amount of electricity that passes through the cross section of the circular conductor in a unit of time, and let u and u' be the speeds at which the electrical fluids in the molecular currents of the two bodies (metallic conductors), which make up the circular conductor, move. E then flows from the first conductor into the second conductor at speed u, and E flows further away from the boundary layer of the second conductor into the following layer at speed u'. The living force of the molecular currents of the second conductor at the boundary layer is therefore increased by $E\,(u^2 - u'^2)$ per unit of time, or heat is developed in this boundary layer, whose mechanical equivalent is $= E\,(u^2 - u'^2)$.

Further, E flows into the first conductor from the second conductor with velocity u', and E flows further away from the boundary layer into the first conductor with velocity u. Thus, the living force of the molecular currents of the first conductor at this boundary layer is decreased by $E\,(u^2 - u'^2)$ in the unit time, or heat disappears at this boundary layer, whose mechanical equivalent is $= E\,(u^2 - u'^2)$. This is the principle of Peltier's fundamental phenomenon.

If, after this theoretical consideration, we move on from the second, third and fourth fundamental electrothermal phenomenon to the first, we must initially observe that tourmaline is an insulator. The molecular currents in an insulator are such that, even under the influence of a separating force, no particle is torn off from the fluid in the molecular flow[615] and driven over to the following molecular current. There is also very weak heat conduction in such an insulator, but there is not a complete lack of heat conduction, since the different temperatures of the different parts of a tourmaline balance themselves out over a moderately long time. This heat conduction must therefore be based on a different principle than the separation of individual particles of the fluids in molecular flow and their transfer to neighboring molecular currents.

Without such a migration of electric particles ε at the speed u to which they are subject in the molecular currents (i.e. with the heat whose mechanical equivalent is $= \varepsilon u^2$), heat can only be transferred from one molecular current to another by induction. This induction arises when one considers that the radius of the molecular currents sometimes increases, sometimes decreases, and is always changing. However, if the cooling state of the tourmaline were to be explained in this way, the associated positive and negative charges on the opposite end surfaces of the tourmaline would still not be explained. It is also obvious, because this electrical charge occurs as a result of cooling only in tourmaline and similar crystals, that its explanation has to be more closely related to the crystalline structure, i.e. with the positional relationships of the molecular currents in these crystals.

Although crystallography gives determinations about these situational conditions, it does not give any determinations about the forces through which they came into existence and are maintained. If one describes the reason for the preservation of these layer relationships by the name of the solidity (or elasticity)[616] of the aggregate state, then one has no knowledge of the laws of interaction between the molecular currents (molecules) with one another, from which such a solid aggregate state results.

In general, one would have to take three types of interactions into account, namely, (1) the electrical interactions, the laws of which are given, (2) interactions of electricity with the ponderable cores of the molecular currents. If one maintains that such an interaction does not take place at a distance, all that remains is the interaction by means of which the ponderable cores of the molecular currents remain firmly connected to a certain amount of one electrical fluid. In addition, (3) the interactions between the ponderable cores of the molecular currents, which they exert on each other from a distance, must be taken into account. These latter, as far as they arise from the law of gravitation, would hardly come into consideration because of their small size. Although no such interactions are known in detail that are independent of the law of gravity, there is reason to suspect that such interactions

[615][Note by AKTA:] In German: *Molekularströmung*.

[616][Note by AKTA:] In German: *Festigkeit (oder Elasticität)*. The expression "Festigkeit" can be translated as solidity, firmness or strength.

exist and that they are so large that they are of great importance for the formation of solid crystalline aggregate states.

In the absence of complete knowledge of this type of interaction, the best way to give an account of the formation of solid crystalline aggregate states is to investigate in more detail everything that could arise from the *electrical interactions of the molecular currents*.

Theorem. Two Ampère's molecular currents whose aligned axes lie in a straight line attract each other at greater distances and repel each other at smaller distances.

Theorem. Every relative movement of two such Ampère's molecular currents is dampened by interaction, i.e. the mechanical movement, which consists of displacements of the molecular currents relative to one another, is transformed into molecular flow (heat energy).

The law of electrical interaction teaches *how* the conversion of mechanical kinetic energy[617] into thermal energy is accomplished. If such Ampère's molecular currents form the molecules of solid bodies, then, according to the first stated theorem, the *solidity* of these bodies is explained by the interaction of these molecular currents at certain positions and distances.

Briot, *Théorie de la Chaleur*, Introduction, p. 2, traces the explanation back to an attractive force between the ponderable particles $= mm' \cdot a/r^n$ and to a repulsive force between their two aether atmospheres $= mm' \cdot b/r^{n+p}$.[618] However, this only determines the distance between the particles in equilibrium, but not the orientation of the straight line connecting them. According to the former explanation, this orientation is given by the axis of the first molecular current, and the orientation of the second molecular current is also determined by the fact that its axis should be parallel to that of the first.

A series of such molecular currents, all in a straight line and equally spaced, form a column. A second, identical column of molecular currents is held at a certain distance from the first, so that oppositely directed currents come to lie next to each other in the two columns. A series of such columns, all at equal distances from one another and all in one plane, form a layer, which in turn holds an equal layer at a certain distance, etc.

In this way one obtains a fixed system of molecular currents extending over the entire space, not only with certain *distances* of the molecules from one another, but also with certain directions and orientations of the molecules relative to one another. This solid system would be a crystal system with three mutually perpendicular elasticity axes, two of which had the same elasticity moduli. However, it does not seem that the explanation of the other crystal systems could be found in this way.

[617][Note by AKTA:] In German: *mechanischer Bewegungsenergie*. See footnote 211 on page 130.

[618][Note by AKTA:] Charles Auguste Albert Briot (1817-1882) was a French mathematician. See [Bri69].

Chapter 21

Editor's Introduction to Voigt's 1899 Paper

A. K. T. Assis[619]

I present here an English translation of W. Voigt's 1899 speech on the unveiling of the monument to Gauss and Weber in Göttingen.[620]

The keynote speech was given by Woldemar Voigt (1850-1919), the successor to Johann Benedikt Listing (1808-1882). Listing had completed his doctorate under Carl Friedrich Gauss (1777-1855) in 1834 with a mathematical thesis and became the successor of Wilhelm Eduard Weber (1804-1891) in Göttingen in 1838. When Weber was called back to Göttingen in 1849, a new chair was created for Weber. From then on, Weber represented experimental physics and Listing, like his successor W. Voigt, held the chair of theoretical physics.

The model of the Gauss Weber Monument was made by the German sculptor Carl Ferdinand Hartzer (1838-1906). The bronze casting was made by the foundry Aktien-Gesellschaft Gladenbeck located in Berlin/Friedrichshafen. Sources of the image: https://denkmale.goettingen.de/portal/seiten/gauss-weber-denkmal-900000619-25480.html and https://www.flickr.com/photos/unigoettingen/15178191062/in/photostream/.

[619]Homepage: www.ifi.unicamp.br/~assis

[620][Voi99].

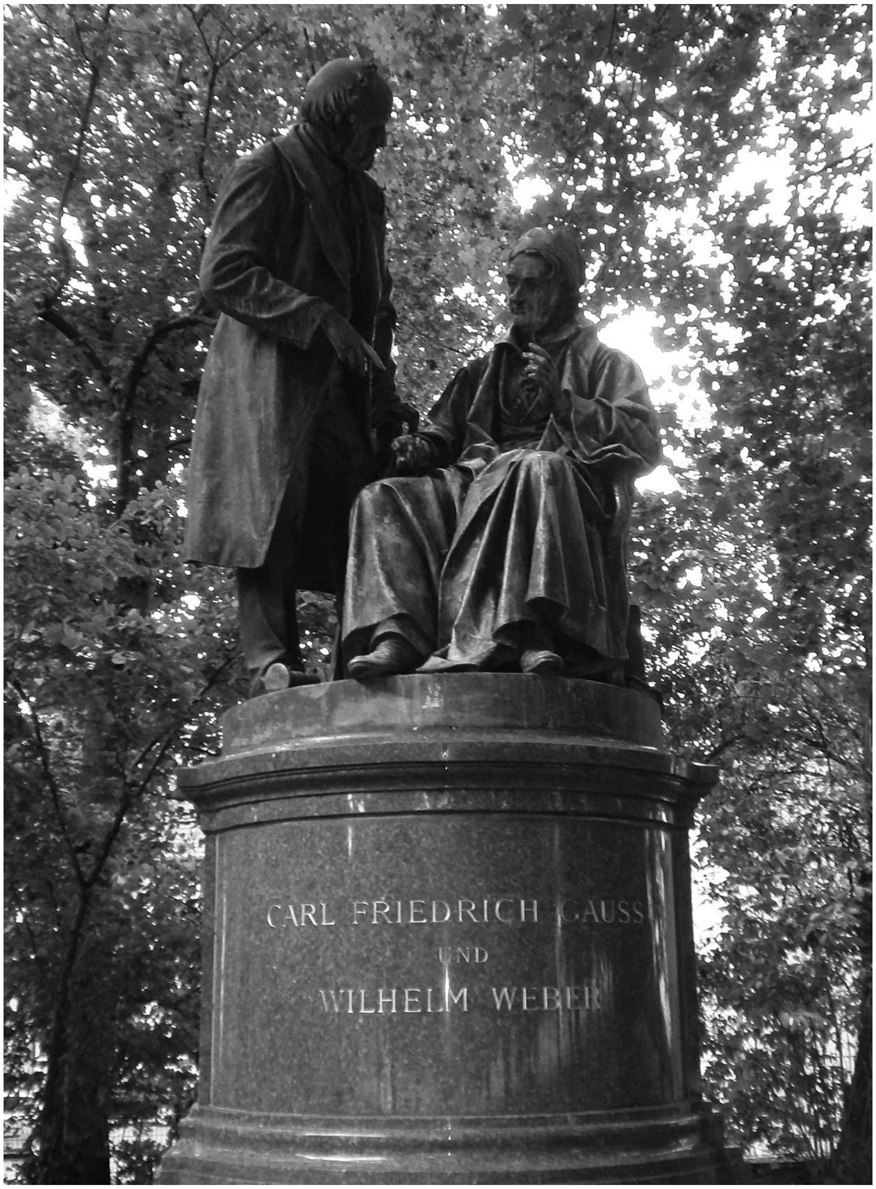


Figure 21.1: Gauss-Weber Monument in Göttingen.

Chapter 22

[Voigt, 1899] Gauss-Weber Monument

W. Voigt[621,622,623]

On June 17 of this year [1899] the monument for

Carl Friedrich Gauss and Wilhelm Weber

was unveiled in Göttingen, whose construction was supported by the Association of German Engineers. We publish the speech, which was kindly provided to us by Professor W. Voigt, dean of the Faculty of Philosophy of Göttingen University. The celebration came along with an exhibition on Gauss and Weber, which showed a wealth of keepsakes, instruments and documents from the busy life of the two researchers, but as well amazing treasures of their previously unknown handwritings. The speech was as follows.

Dear festive gathering,

it is a great pleasure to welcome you at the feet of the monument. Seven years we struggled to achieve it. This festive hour symbolizes the goal of our wishes. The University and the town of Göttingen join the festivity and many of their representatives gather here. Honored guests from various places came to Göttingen to celebrate the monument and the memory of the men, whose names are well-known and highly esteemed, wherever the sciences find love and care.

It was in the winter of 1891 to 1892, a few months after Wilhelm Weber passed away, when the representatives of the topics taught by Gauss and Weber gathered wishing to build a joint monument for the two eminent researchers at the place of their major achievements. This idea existed secretely already before, since the time when Gauss passed away. It was the reason that our Association accepted it that Braunschweig, the native city of Gauss, before us honoured him. Then it was clear to everybody that in the history of the University and city of Göttingen the name of Gauss for all times is intimately connected with the name of Weber.

[621] [Voi99].

[622] Translated by U. Frauenfelder, urs.frauenfelder@math.uni-augsburg.de. Edited by A. K. T. Assis, www.ifi.unicamp.br/~assis

[623] All Notes were written by A. K. T. Assis.

Now after the work of Weber found an end, the idea of a joint monument naturally manifested itself. At the place of their work we are connected to one of them by a living tradition and to the other by a shorter or longer personal relationship.

What signify Gauss and Weber to us?

Already after his first successes a century before, the scientific world felt clearly that with Gauss a first grade discoverer entered the scene. There was mourning for the death a king who mastered the broadest areas of science when he passed away 45 years ago.

Even more fantastic things than the published works teach in their ice-cold, crystal-clear beauty, his inconspicuous diaries reveal us. Just recently the grandson of the great master gave us to our use a little diary covering for many years the records of his discoveries.

Restless, as if guided and ruled by a demon, he is digging tunnels into unexplored areas. A demon shows him the way to countless veins of gold. But only little of it he brings to light elaborated for the use of others. Most of it he hides from the world happy to possess it secretly.

Only decades later, when other people unlocked these veins from different sides, the traces of his powerful hand were revealed.

Still almost a boy he successfully attacks problems on which the brightest minds of previous times failed. 16 years old he devotes himself to the most difficult questions on the foundations of geometry. 17 years old he finds that invaluable method to exploit measurements rationally and unbiased. 19 years old he constructs the 17-gon by ruler and compass, one of the first great leaps forward since the time of Euclid. 22 years old he is anticipating the basic ideas of the theory of quaternions developed much later by Hamilton. With 23 years he discovers many results on elliptic functions rediscovered later. In his 25th year he devotes himself suddenly to astronomy and achieves a major breakthrough by determining the orbit of Ceres. In the following he caused so much sensation by further theoretical and observational results obtained with limited resources that he was appointed professor of astronomy in Göttingen at the age of 30. With 32 years he publishes *Theoria motus corporum coelestium* which puts him on equal footing with the first theoretical astronomers of all times.[624]

After getting the offer from Göttingen, Gauss gains on the one hand an honorable, secure position but he looses the precious freedom to choose his ways as he likes. He needs to deal with a lot of time-consuming work of little scientific significance.

Despite of that he devotes himself energetically to two new scientific topics, namely, geodesics and mathematical physics, where he attains many outstanding achievements. To it belong as well two major scientific endeavours, which occupy Gauss until his death. These are the arc measurements in Hanover and the organization of the geomagnetic observations. Not only directly thanks to the results obtained, but as well indirectly by opening up new research directions, these were of great importance and benefit for science. To help him with his geomagnetic plans Gauss brings W. Weber to Göttingen.

Next to the almost uncanny brilliance of the mighty meteor Gauss, Weber's beautiful, pure star shines with a calm, friendly ray, guiding the way, promising the goal.

The first scientific stimuli which Weber received in his hometown Wittenberg from the well-known musician Chladni, and from his older brother Ernst Heinrich, led him to the topics of wave theory and acoustics.[625] After obtaining important results in this field he

[624] [Gau09] with English translation in [Gau57].

[625] Ernst Chladni (1756-1827) was a German physicist and musician. Ernst Heinrich Weber (1795-1878) was a German physician, one of the founders of experimental psychology.

earned an associate professorship in Halle at a young age, and he gained the interest and confidence of Gauss.

To appreciate the relationship between the two men one should as well take into account their age difference. Gauss was 53 years old, when the 26 year old Weber moved to Göttingen. That the 27 years younger Weber first was led and stimulated by Gauss does not belittle him. Heinrich Hertz as well was first influenced heavily by the much older Helmholtz and nevertheless achieved outstanding results.[626]

Gauss devised the general theory of terrestrial magnetism. The task was to make systematic measurements in order to apply the theory. The organization of the newly founded Magnetic Association, the planning of the observations, and the exploitation of the results is mainly Weber's enormous, unforgettable merit.

An outcome of this magnetic collaboration was the invention which mainly popularized the names of Gauss and Weber, namely, the installation of the first electromagnetic telegraph in the year 1833. By the practical realization of an idea with a great impact on communication, they became the pioneers of the flourishing field of Electrical Engineering.

The collaboration between Gauss and Weber was disrupted in the year 1837 when Weber lost his position due to his dauntless advocacy of his conviction. We do not blame that the 60 year old Gauss stayed neutral in the dispute. That the 33 year old Weber sacrificed his position to his conviction, we will never forget. In particular, by taking into account how important this position was for Weber because of its connection to Gauss.

It might be that the temporary separation from Gauss actually was an advantage for the scientific performance of Weber. Anyway in Leipzig his work found strong support from the Royal Scientific Society there and Weber's research took a quite independent, significant turn.

In Leipzig Weber began to write his "Elektrodynamischen Massbestimmungen" (Electrodynamic Measurements) on which he was working until shortly before his death.[627] Of course one can find inspirations from Gauss in them as in every successor the achievements of his predecessor live on. But Weber there pursued quite new directions. He discovered his fundamental law of electricity and determined in an ingenious way his remarkable constant which turned out to have the same value as the speed of light. This inspired Maxwell to put the theoretical electrodynamics on a new foundation.[628] We further mention the fruitful basic ideas on the movement of ions in electrolytes, the amazing realization of the absolute system of measurement, as well as the brilliant construction of measuring devices for science and technology. These are all undisputed and everlasting achievements of Weber.

This was our vivid image of the men when we gathered seven years ago with the aim of building a monument for them.

Our first task was to find the rather large necessary sum of money for which a wide network seemed essential. For this reason we asked in the spring of 1892 many scientists, technicians, and civil servants in several countries to join and support a further committee in Göttingen for the construction of the monument for Gauss and Weber.

Almost everybody we asked was supportive and excited which increased our hope to bring the project to a good end.

With this confidence we dared to ask the illustrious president of our University, his Royal Highness Prince Albrecht of Prussia, prince regent of the Duchy of Braunschweig, to take

[626] See footnotes 355 and 359 on pages 192 and 192.

[627] See footnote 467 on page 241.

[628] See footnote 353 on page 191.

over the protectorate of our organization.[629] That his Royal Highness accepted our request greatly boosted our project, not just directly by providing a large sum of money from the budget of the Duchy of Braunschweig but as well indirectly by the force of his princely name which often was more effective than our modest request.

Unfortunately, our hope that his Royal Highness could join today's event did not materialize. By the advise of his doctors his Royal Highness cannot attend. Therefore we express remotely our sincere gratitude to our illustrious protector.

The collection for the monument of Gauss and Weber in Göttingen was very successful. We did not just receive many large and small contributions, but our supporters build as well collecting points in various towns from the near Braunschweig to the remote Tiflis and sent us the collected money.

However, two years ago this golden stream dried out before the basin was filled. At least we had collected enough money that success was guaranteed and we could dare to make bolder steps to bring our project to an end.

By a request which gained the approval of prominent supporters of our project we managed to get His Majesty the Emperor and King interested in the monument for Gauss and Weber. His most gracious donation not only increased our assets but as well inspired others to donate as well.

We are extremely grateful to His Majesty for this favor.

After this great help similar ones soon followed from the administration of Hanover and the town of Göttingen, to whom we would like to express as well our gratitude.

To overcome the last remaining difficulty we got help thanks to the intervention of two splendid friends from Berlin, secretary of state Sydow and president Boedicker. They managed to get the big electrotechnical companies in Germany interested in the monument for Gauss and Weber. This blew new wind in our sails and our little ship finally reached its destiny a few weeks ago.

We would like to thank sincerely all our supporters.

Simultaneously with the financial aspects the artistic aspects of our project were addressed.

Professor Hartzer in Berlin who is well-known in Göttingen thanks to his monument for Wöhler and several felicitious busts of famous professors provided a model which found the vivid acclamation of the committee.[630] I would like to express our gratitude and admiration to the highly esteemed sculptor, who unfortunately due to illness could not join us today.

One agreed to indicate only very slightly the age difference between Gauss and Weber and represent the two men in their prime. The theme which unites the two figures to a group should have a connection to their most popular achievement, the invention of the telegraph.[631] The company Gladenbeck in Friedrichshafen carried out the ore casting.

The place where the monument is erected was kindly offered and prepared by the city council of Göttingen. It agrees with the wishes of the sculptor and we are very pleased with it, since it will be located near the mathematical, physical and astronomical Institutes of the University when the new buildings are erected in the locations we desire.

[629] Prince Albert of Prussia (1837-1906) was a Prussian general field marshal and regent of the Duchy of Brunswick from 1885 until his death.

[630] Carl Ferdinand Hartzer (1838-1906) was a German sculptor. Friedrich Wöhler (1800-1882) was a German chemist.

[631] The monument shows Weber with a roll of wire in front of his feet and leaning on the telegraph signal transmitter next to his seated colleague Gauss, [Ker13].

Now the statues of Gauss and Weber are ready to enter the town where the two men carried out their beneficial work. Fluttering flags and green wreaths are greeting them. The dear, eagerly awaited sun waits to gild them. Esteemed guests, representatives of the town, the University, and the student body are waiting to pay homage to them.

Down then with the veil! And let Händel's eternally young song of victory resound to those to come![632]

> (While the covers drop slowly the orchestra intones the melody "See, the conquering hero comes" from Händel's Judas Maccabaeus.[633] The participants stand up from their seats, the students from the academic deputations have drawn their rapiers and greet with them as with the flags of the corporations.)

A song of victory is greeting those to come!

They deserve heroic honors, because intellectual heroes are the men who fought for the fatherland to become a great power in the realm of science. After the dreadful devastation of the great war, it seemed for centuries as if Germany's soil could no longer produce intellectual greatness. At the same time that Italy, France, England and the Netherlands were gaining immortal fame in the exact sciences, here, with a few isolated exceptions, everything was dead. With C. Fr. Gauss a new, better time dawned, and Bessel, Jacobi, Dirichlet, Weber, Neumann, Helmholtz, Clausius and Kirchhoff joined him in forming the intellectual army that fought for our powerful scientific position in the world.[634]

My fellows! You are entrusted with the heritage of these intellectual giants. Honor it together with the heritage of our political and war heroes!

Whoever attended a scientific meeting of Italian, French, English or even Dutch scientists was surely moved by the feeling of the glorious scientific history of their countries which exalts these men. Germany entered centuries after these countries into the competition. There is still a long way to go until it achieves parity with their predecessors or even gets ahead of them.

Genius is a present which is thrown into the lap of the people, as well as the individual. But what the individual, as the people, can do for science through their will is to prepare the ground so that the precious seed of genius, when planted in it, can develop. Here lies a sacred task for each of you. Foster the scientific spirit in whatever position your life leads you, so that when a fortunate circumstance sends genius among us, it can spread its wings freely and powerfully.

I would also like to say a word to you at this time, my esteemed fellow citizens, whose collaborators I am happy and proud to work with.

If the name of Göttingen is more famous than the one of other towns of the same size, if Göttingen appears as a noble blossom in the wreath of German towns, then this is due to the scientific work which is done inside its walls. Take care that such work can be continued here. Do not consider it as a waste of public funds to host the University and the growing number of Institutes. The wealth of this town is strongly connected to science and the University. The monument which we just uncovered may bring this to mind.

[632] George Frideric Händel (1865-1759) was a German-Britisch composer.

[633] https://www.youtube.com/watch?v=XAKzwmYdcQ4.

[634] Friedrich Wilhelm Bessel (1784-1846), Carl Gustav Jacob Jacobi (1804-1851), Peter Gustav Lejeune Dirichlet (1805-1859), Franz Ernst Neumann (1798-1895), Rudolf Clausius (1822-1888) and Gustav Kirchhoff (1824-1887) were German scientists.

Thus I hand over in the name and mandate of the committee the monument of Gauss and Weber to the town of Göttingen as a memorial to two of its most splendid citizens, as a symbol of scientific work which joins countries and people, as an ornament and jewel of our beloved town which is our cozy home.

Chapter 23

The *Weber* as an Electrical Unit of Measure

A. K. T. Assis[635]

The first International Electrical Congress was held in Paris in 1881. It established international electrical units of measure. It endorsed the 1873 proposal of the British Association for the Advancement of Science for defining the *Ohm* and the *Volt* as practical units and also defined the *Ampère*, *Coulomb* and *Farad* as units for electrical current, quantity of charge and capacitance, respectively. It was a tribute to the works of Georg Simon Ohm (1789-1854), Alessandro Volta (1745-1827), André-Marie Ampère (1775-1836), Charles Augustin de Coulomb (1736-1806) and Michael Faraday (1791-1867). The Chairman of the congress was Adolphe Cochery (1819-1900), Minister of Posts and Telegraphs of the French Government. The foreign vice-presidents were William Thomson (1824-1907), also known as Lord Kelvin, Hermann von Helmholtz (1821-1894) and Gilbert Govi (Italy).

It was a surprise that the names of Carl Friedrich Gauss (1777-1855) and Wilhelm Weber (1804-1891) were not included in this list of electrical units. After all, they were the creators of the absolute system of units. In this system, the electric and magnetic units could be defined solely in relation to absolute units of length, mass, and time. Their original proposal was based on a millimeter-milligram-second system of units. The CGS system of units is based on the same idea, although it utilized a centimeter-gram-second system of units.[636]

At that time the term *Weber* enjoyed some use, especially in England and Germany, for the unit of electric current.[637]

Two of Weber's main opponents were Helmholtz and William Thomson.[638] The decision to suppress the term *Weber* as the unit of electric current was taken on purpose by Helmholtz and Thomson in this congress of 1881. At that time there was a bitter dispute between Weber and Helmholtz on the foundations of electrodynamics. In 1881 Kelvin and Helmholtz suggested to replace the term *Weber* by *Ampère* as the unit of electric current. This fact has been registered by Éleuthère Mascart (1837-1908), the secretary of the section of the

[635] Homepage: `www.ifi.unicamp.br/~assis`

[636] [ARW04] and [Ass21g].

[637] [Con82, pp. 44-45], [Wie60, Chapter 2, Section 7: Die praktischen absoluten elektrischen Einheiten und ihre Namengebung, pp. 102-107], [Wie67, Die internationalen elektrischen Einheiten und ihre Namensgebung, pp. 135-137], [Woo68, footnote 20, p. 305] and [Woo81, p. 205].

[638] See [Ass21f].

congress dealing with electrical units. He recounted how agreement was finally reached on international electrical units.[639]

> As we still only had two units, the *Ohm* and the *Volt*, and it was necessary to complete the system, I asked the president, M. Cochery, if the commissions could at least meet.
>
> I had to bow to his negative response, and we stayed, with von Helmholtz, near Lord and Lady Kelvin who, having neglected to eat lunch, were having a chocolate in the Chiboust restaurant, located near the Congress hall. It was in this small committee, around a common white marble table, that the following three units were agreed: *Ampère* (instead of *Weber*), *Coulomb*, and *Farad*.
>
> I was responsible for reading the text the following day, September 21, in the general session. Many members of the commission, who only knew about the Saturday session, were a little surprised, but the comments of Lord Kelvin and von Helmholtz no longer allowed any hesitation. The practical system of units was thus born.

It was only in the 1930s that the terms *Gauss* and *Weber* were officially introduced for the practical unit of magnetic flux by the International Electrotechnical Commission (IEC). On the one hand, this was a late tribute to Carl Friedrich Gauss and Wilhelm Weber. On the other hand, it must be pointed out that Gauss and Weber never worked with the magnetic field concept, nor with the magnetic flux concept.

[639] See: [Lan09], [Jan09], [Woo68, footnote 20, p. 305], [Fri82, p. 209], [Tun92, p. 35], [Bor08], [Bor09] with English translation in [Bor12] and [Bor13]:

> Comme nous n'avions encore que deux unités, l'ohm et le volt, et qu'il était nécessaire de compléter le système, je demandai au président, M. Cochery, si les commissions au moins pouvaient se réunir.
>
> Je dus m'incliner devant sa réponse négative, et nous restâmes, avec von Helmholtz, auprès de Lord et Lady Kelvin qui, ayant négligé de déjeûner, prenaient un chocolat dans le restaurant Chiboust, installé près de la salle du Congrès. C'est dans ce petit comité, autour d'une vulgaire table en marbre blanc, que furent convenues les trois unités suivantes: Ampère (au lieu de Weber), Coulomb et Farad.
>
> J'étais chargé d'en lire le texte le lendemain 21 septembre en séance générale. Nombre de membres de la commission, qui ne connaissaient que la séance du samedi, en furent bien un peu surpris, mais les commentaires de Lord Kelvin et de von Helmholtz ne permirent plus aucune hésitation. Le système pratique d'unités était fondé.

Chapter 24

The Velocity in Weber's Electrodynamics Versus the Velocities in Different Field Theories

A. K. T. Assis[640]

In this Chapter I compare the velocity which appears in Weber's electrodynamics with the velocities which appear in different field theories.[641]

Before that I show that when one gives different interpretations to the magnitudes appearing in the same mathematical formula, then one has different theories.

24.1 Different Theories Described by the Same Mathematical Formula

Suppose a force proportional to the following expression:

$$\frac{mm'}{r^2} \, . \tag{24.1}$$

Is this force a mathematical representation of Newton's law of gravitation? The answer to this question depends on the meanings assigned to the magnitudes m, m' and r.

If m and m' refer to the masses of the interacting particles, and r is the distance between them as given by Figure 24.1 (a), then one can say that this formula is a mathematical representation of Newton's law of universal gravitation.

If, on the other hand, m and m' indicate the surface areas of the two interacting bodies, then Equation (24.1) no longer represents Newton's law of gravitation. Likewise, if m and m' indicate the volumes of the two interacting bodies, then this Equation will not represent Newton's law of gravitation. With these new interpretations of magnitudes m and m'

[640] Homepage: `www.ifi.unicamp.br/~assis`

[641] [Ass92, Appendix (A): The Origins and Meanings of the Magnetic Force $\vec{F} = q\vec{v} \times \vec{B}$], [AP92], [Ass94, Appendix A: The Origins and Meanings of the Magnetic Force $\vec{F} = q\vec{v} \times \vec{B}$], [SP13], [Ass13, Section 14.5], [Ass14b, Section 15.5 (Origins and Meanings of the Velocity $\vec{v}$ which Appears in the Magnetic Force $q\vec{v} \times \vec{B}$) and Appendix A (Relational magnitudes)] and [Ass15a, Appendix B: Origins of the electromagnetic force $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ and the different meanings given to the velocity $\vec{v}$].

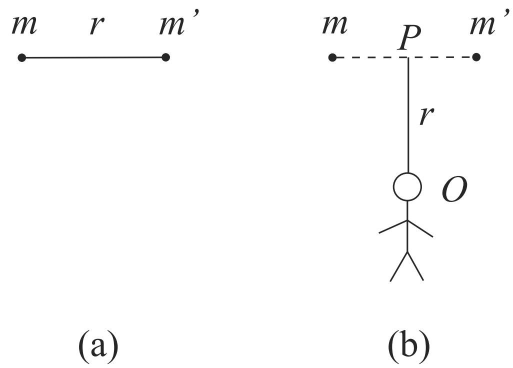


Figure 24.1: Different definitions of the magnitude r. (a) As the distance between the particles m and m'. (b) As the distance between P and the observer O, where P is the middle point between m and m'.

one would have theories which are different from Newton's theory of gravitation, although represented by the same algebraic formula.

In Figure 24.1 (b) I consider an observer O and the point P midway between m and m'. If the magnitude r to be utilized in Equation (24.1) is defined as the distance between O and P, as indicated in Figure 24.1 (b), then this Equation is no longer a mathematical representation of Newton's law of universal gravitation, even if m and m' indicate the masses of the two bodies. In this case Equation (24.1) would be a mathematical representation of a different theory and should not be called *Newton's* law of gravitation.

24.2 Force Acting on an Electrified Body based on Electromagnetic Fields

Newton presented the universal law of gravitation in terms of a force acting between material bodies. He did not utilize the field concept.

In gravitational field theory, on the other hand, the gravitational force is usually expressed in terms of a gravitational field $\vec{g}$ generated by a source gravitational mass M_g. When this field reaches another test gravitational mass m_g, it generates a force $\vec{F}$ on this mass given by:

$$\vec{F} = m_g \vec{g} \ . \tag{24.2}$$

Analogously, in electromagnetic field theory, the force $\vec{F}$ acting on an electrified body which has a charge q in the presence of an electric field $\vec{E}$ and a magnetic field $\vec{B}$ is given by:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \ . \tag{24.3}$$

The history of this force and the different meanings of the velocity $\vec{v}$ which appears in this equation will be discussed in Section 24.3.

24.3 Origins and Meanings of the Velocity $\vec{v}$ which Appears in the Classical Electromagnetic Force Law $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$

Classical electromagnetism is composed of two main portions, namely, Maxwell's equations and the electromagnetic force acting on an electrified particle with charge q in the presence of an electric field $\vec{E}$ and a magnetic field $\vec{B}$. Maxwell's equations relate the fields $\vec{E}$ and $\vec{B}$ with the sources of these fields, namely, the volume charge density ρ and the volume current density $\vec{J}$.

The electromagnetic force $\vec{F}$, on the other hand, specifies how the fields $\vec{E}$ and $\vec{B}$ act on a test particle with charge q. In the International System of Units (SI or MKSA) this so-called Lorentz force is normally expressed by the following equation:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \ . \tag{24.4}$$

Most textbooks when presenting for the first time this force express themselves as Feynman, Leighton and Sands:[642]

> We can write the force $\mathbf{F}$ on a charge q moving with velocity $\mathbf{v}$ as
>
> $$\mathbf{F} = \mathbf{q}(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \ . \qquad (\mathbf{1.1})$$
>
> We call $\mathbf{E}$ the *electric field* and $\mathbf{B}$ the *magnetic field* at the location of the charge.

Likewise:[643]

> The force on an electric charge depends not only on where it is, but also on how fast it is moving. [...] The total electromagnetic force on a charge can, then, be written as
>
> $$\mathbf{F} = \mathbf{q}\,(\mathbf{E} + \mathbf{v} \times \mathbf{B}) \ . \qquad (\mathbf{13.1})$$
>
> This is called the *Lorentz force*.

Statements such as this one by Feynman, Leighton and Sands are found in most textbooks on electromagnetism.[644] However, these statements have no meaning because they do not specify what is the velocity which appears in Equation (24.4). Velocity is not an intrinsic property of any body. It is a relation between the electrified particle and another body relative to which the charge is moving. For this reason one and the same electrified body can have several different velocities simultaneously. For instance, it can be simultaneously at rest relative to the ground, moving towards a second electrified particle, moving away from the observer, moving away even faster from the magnetic field detector etc. Which of these different velocities should one apply when utilizing Equation (24.4)? My goal in this Chapter is to discuss the meaning of this velocity $\vec{v}$ which appears in this equation.

Consider for instance the situation of figure 24.2, where all velocities are relative to the ground. The test particle with charge q moves with velocity $\vec{v}_q$, the magnet NS moves with velocity $\vec{v}_m$, the circuit c carrying a current I moves with velocity $\vec{v}_c$, the observer O moves with velocity $\vec{v}_o$ and the magnetic field detector d moves with velocity $\vec{v}_d$. In particular, the velocity $\vec{v}$ which appears in Equation (24.4) should be understood relative to what object, body or entity? That is, $\vec{v}$ is the velocity of the particle q relative to what?

---

[642] [FLS64, p. 1.2].

[643] [FLS64, p. 13.1].

[644] Several examples are quoted in [AP92].

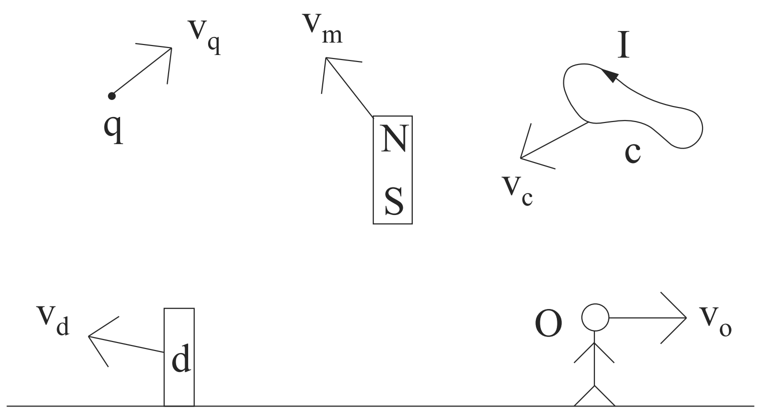


Figure 24.2: Velocities of several bodies relative to the ground.

Some possible answers to this fundamental question:

- Relative to Newton's absolute space.
- Relative to the laboratory, relative to the ground or relative to the Earth.
- Relative to the frame of fixed stars.
- Relative to the universal frame of distant galaxies.
- Relative to the macroscopic source of the magnetic field (that is, relative to the magnet or relative to the current carrying wire).
- Relative to the average velocity of the microscopic charges (normally electrons) which generate the magnetic field.
- Relative to the magnetic field itself.
- Relative to the detector of the magnetic field.
- Relative to the observer.
- Relative to any inertial frame of reference.
- Relative to an arbitrary frame of reference, which does not need to be inertial.
- Relative to the ether.
- Etc.

In this Section I discuss the history of this force and the different interpretations which have been given to the velocity appearing in this formula along the years by several authors.

24.3.1 Meaning of the Velocity According to Maxwell

The force given by Equation (24.4) is usually called Lorentz force in the textbooks. However, it seems that Maxwell was the first to obtain it.[645]

Maxwell presented this force in 1861-1862 in his article on physical lines of force.[646] Moreover, he discussing it in 1864-1865 in his paper with a dynamical theory of the electromagnetic field,[647] and also in his main book of 1873, *A Treatise on Electricity and Magnetism*.[648] He was considering the force acting on an electrified body. Sometimes he referred to this test body as a conductor, as a dielectric or insulator, as a particle, as a current element of an electric circuit, or simply as electricity.

Maxwell interpreted this velocity as being the velocity of the test body relative to the magnetic field.

On page 166 of his paper of 1861-1862 he said:[649] "μ is a quantity bearing a constant ratio to the density." The "density" here refers to the supposed density of vortices in the medium. On page 283: "To determine the motion of a layer of particles separating two vortices. Let the circumferential velocity of a vortex, multiplied by the three direction-cosines of its axis respectively, be α, β, γ, as in Prop. II." On page 288: "Let P, Q, R be the forces acting on unity of the particles in the three coordinate directions, these quantities being functions of x, y, and z." On page 342: "[...] F, G, and H are the values of the electrotonic components for a fixed point of space, [...]" The force per unit charge, analogous to $\vec{F}/q$ of Equation (24.4), was originally written as follows in his paper of 1861-1862:[650]

$$\left.\begin{array}{l} P = \mu\gamma\frac{dy}{dt} - \mu\beta\frac{dz}{dt} + \frac{dF}{dt} - \frac{d\Psi}{dx}\ , \\ Q = \mu\alpha\frac{dz}{dt} - \mu\gamma\frac{dx}{dt} + \frac{dG}{dt} - \frac{d\Psi}{dy}\ , \\ R = \mu\beta\frac{dx}{dt} - \mu\alpha\frac{dy}{dt} + \frac{dH}{dt} - \frac{d\Psi}{dz}\ . \end{array}\right\} \ (77) \qquad (24.5)$$

Soon after this equation he wrote, our emphasis:[651]

> The first and second terms of each equation indicate the effect of the *motion of any body in the magnetic field*, the third term refers to changes in the electrotonic state produced by the alterations of position or intensity of magnets or currents in the field, and Ψ is a function of x, y, z, and t, which is indeterminate as far as regards the solution of the original equations, but which may always be determined in any given case from the circumstances of the problem. The physical interpretation of Ψ is, that it is the *electric tension* at each point of space.

In the paper of 1864-1865 the force per unit charge, analogous to $\vec{F}/q$ of Equation (24.4), was presented as follows, our emphasis:[652]

> The complete equations of *electromotive force on a moving conductor* may now be written as follows:

[645] [Mar90, p. 31], [Rib08], [Cur09, Section 4.6: On the paternity of Lorentz force, pp. 122-128], [Hur10, p. 22], [Tom12a], [Tom12b] and [Yag20].
[646] [Max62, p. 342, Equation (77)] and [Max65c, Equation (77)].
[647] [Max65, p. 484, Equation (D)] and [Max65a, Equation (D)].
[648] [Max54, Vol. 2, §§ 598-599, pp. 238-241, equations (B) and (10)].
[649] [Max62].
[650] [Max62, p. 342, Equation (77)].
[651] [Max62, p. 342, soon after Equation (77)].
[652] [Max65, p. 484].

Equations of Electromotive Force.

$$\left.\begin{array}{l} P = \mu\left(\gamma\frac{dy}{dt} - \beta\frac{dz}{dt}\right) - \frac{dF}{dt} - \frac{d\Psi}{dx}\ , \\ Q = \mu\left(\alpha\frac{dz}{dt} - \gamma\frac{dx}{dt}\right) - \frac{dG}{dt} - \frac{d\Psi}{dy}\ , \\ R = \mu\left(\beta\frac{dx}{dt} - \alpha\frac{dy}{dt}\right) - \frac{dH}{dt} - \frac{d\Psi}{dz}\ . \end{array}\right\} \text{(D)} \tag{24.6}$$

> The first term on the right-hand side of each equation represents the electromotive force arising from the motion of the conductor itself. This electromotive force is perpendicular to the direction of motion and to the lines of magnetic force; and if a parallelogram be drawn whose sides represent in direction and magnitude the velocity of the conductor and the magnetic induction at that point of the field, then the area of the parallelogram will represent the electromotive force due to the motion of the conductor, and the direction of the force is perpendicular to the plane of the parallelogram.
>
> The second term in each equation indicates the effect of changes in the position or strength of magnets or currents in the field.
>
> The third term shows the effect of the electric potential Ψ. It has no effect in causing a circulating current in a closed circuit. It indicates the existence of a force urging the electricity to or from certain definite points in the field.

In his *Treatise on Electricity and Magnetism* Maxwell defined the electric field $\vec{E}$, which he represented by the German letter $\mathfrak{E}$, on articles 44 and 68. He also called this electric field by the name "electromotive intensity:"[653]

The Electric Field.

> 44.] The Electric Field is the portion of space in the neighbourhood of electrified bodies, considered with reference to electric phenomena. [...]
>
> Let e be the charge of the body, and F the force acting on the body in a certain direction, then when e is very small F is proportional to e, or
>
> $F = Re$,
>
> where R depends on the distribution of electricity on the other bodies in the field. If the charge e could be made equal to unity without disturbing the electrification of other bodies we should have $F = R$.
>
> We shall call R the Resultant Electromotive Intensity at the given point of the field. When we wish to express the fact that this quantity is a vector we shall denote it by the German letter $\mathfrak{E}$.

Analogously, on article 68 he mentioned that:[654]

Resultant Intensity at a Point.

> 68.] In order to simplify the mathematical process, it is convenient to consider the action of an electrified body, not on another body of any form, but on an indefinitely small body, charged with an indefinitely small amount of electricity, and placed at

[653] [Max54, Vol. 1, § 44, pp. 47-48].
[654] [Max54, Vol. 1, § 68, p. 75].

any point of the space to which the electrical action extends. By making the charge of this body indefinitely small we render insensible its disturbing action on the charge of the first body.

Let e be the charge of the small body, and let the force acting on it when placed at the point (x, y, z) be Re, and let the direction-cosines of the force be l, m, n, then we may call R the resultant electric intensity at the point (x, y, z).

If X, Y, Z denote the components of R, then

$X = Rl, \quad Y = Rm, \quad Z = Rn.$

In speaking of the resultant electric intensity at a point, we do not necessarily imply that any force is actually exerted there, but only that if an electrified body were placed there it would be acted on by a force Re, where e is the charge of the body.[655]

This force not only tends to move a body charged with electricity, but to move the electricity within the body, so that the positive electricity tends to move in the direction of R and the negative electricity in the opposite direction. Hence the quantity R is also called the Electromotive Intensity at the point (x, y, z).

When we wish to express the fact that the resultant intensity is a vector, we shall denote it by the German letter $\mathfrak{E}$. [...]

In Maxwell's *Treatise* the vector magnetic induction was represented by $\mathfrak{B}$ and its components along the x, y and z direction by a, b and c, respectively.[656] In modern vector notation this vector would be written as $\vec{B}$ and its components as B_x, B_y and B_z. The vector-potential of magnetic induction was represented by $\mathfrak{A}$ and its components by F, G and H, respectively.[657] In modern notation this magnetic vector potential would be written as $\vec{A}$ and its components as A_x, A_y and A_z. The vectors $\vec{B}$ and $\vec{A}$ were related by:[658]

$$\vec{B} = \nabla \times A \ . \tag{24.7}$$

The electromotive force E due to induction acting on the secondary circuit was written as follows:[659]

$$E = \int \left(P\frac{dx}{ds} + Q\frac{dy}{ds} + Rdzds \right) ds \ . \quad (5) \tag{24.8}$$

Chapter VIII of Volume 2 of Maxwell's *Treatise on Electricity and Magnetism* was devoted to an exploration of the field by means of the secondary circuit. He mentioned on page 229 that:[660] "[...] the electromagnetic action between the primary and the secondary circuit depends on the quantity denoted by M, which is a function of the form and relative position of the two circuits." He wished to study the electrokinetic momentum of the secondary circuit depending on the primary current i_1, which he denoted by $p = Mi_1$. On page 230 he mentioned that "the part contributed by the element ds of the circuit is Jds, where J is a

[655] [Note by Maxwell:] The Electric and Magnetic Intensities correspond, in electricity and magnetism, to the intensity of gravity, commonly denoted by g, in the theory of heavy bodies.

[656] [Max54, Vol. 2, § 400, p. 25].

[657] [Max54, Vol. 2, § 405, p. 29].

[658] [Max54, Vol. 2, § 405, p. 29, Equation (21) and § 591, p. 233, Equation (A)].

[659] [Max54, Vol. 2, § 598, p. 239].

[660] [Max54].

quantity depending on the position and direction of the element ds." On page 232 he said that the electrokinetic moment at the point $(x,\ y,\ z)$ was identical to the vector-potential of magnetic induction.

The force per unit charge representing the equations of electromotive intensity, analogous to $\vec{F}/q$ of Equation (24.4), was expressed in the *Treatise* as follows:[661]

$$\left.\begin{array}{l} P = c\frac{dy}{dt} - b\frac{dz}{dt} - \frac{dF}{dt} - \frac{d\Psi}{dx}\ , \\ Q = a\frac{dz}{dt} - c\frac{dx}{dt} - \frac{dG}{dt} - \frac{d\Psi}{dy}\ , \\ R = b\frac{dx}{dt} - a\frac{dy}{dt} - \frac{dH}{dt} - \frac{d\Psi}{dz}\ . \end{array}\right\} \text{(B)} \tag{24.9}$$

He summarized these equations, which he denoted by the letter (B), as follows:[662]

> The electromotive intensity, as defined by equations (B), may therefore be written in the quaternion form,
>
> $\mathfrak{E} = \mathrm{V}.\mathfrak{G}\mathfrak{B} - \dot{\mathfrak{A}} - \nabla\Psi\ .$ (10)

Maxwell's Equation (10) can be written in modern vector notation as follows:

$$\vec{E} = \vec{v} \times \vec{B} - \frac{\partial \vec{A}}{\partial t} - \nabla\Psi\ . \tag{24.10}$$

Maxwell's Equation (24.10) is then analogous to Equation (24.4), expressing the force per unit charge, namely, $\vec{F}/q$.

In his *Treatise*, soon after presenting his equations (B) for the electromotive force, that is, our Equation (24.9), he said the following, our emphasis:[663]

> The terms involving the new quantity Ψ are introduced for the sake of giving generality to the expressions for P, Q, R. They disappear from the integral when extended round the closed circuit. The quantity Ψ is therefore indeterminate as far as regards the problem now before us, in which the electromotive force round the circuit is to be determined. We shall find, however, that when we know all the circumstances of the problem, we can assign a definite value to Ψ, and that it represents, according to a certain definition, the *electric potential* at the point $(x,\ y,\ z)$.
>
> The quantity under the integral sign in equation[664] (5) represents the electromotive intensity acting on the element ds of the circuit.
>
> If we denote by $T.\mathfrak{E}$, the numerical value of the resultant of P, Q, and R, and by ϵ, the angle between the direction of this resultant and that of the element ds, we may write equation (5),
>
> $$E = \int T.\mathfrak{E} \cos\epsilon ds\ . \quad (6)$$
>
> The vector $\mathfrak{E}$ is the electromotive intensity at the moving element ds. Its direction and magnitude depend on the position and motion of ds, and on the variation of the magnetic field, but not on the direction of ds. Hence we may now disregard the circumstance that ds forms part of a circuit, and consider it simply as a portion of a *moving body*, acted on by the electromotive intensity $\mathfrak{E}$. The electromotive intensity

[661] [Max54, Vol. 2, § 598, p. 239, Equation (B)].
[662] [Max54, Vol. 2, § 599, p. 241, Equation (10)].
[663] [Max54, Vol. 2, § 598, pp. 239-241].
[664] That is, our Equation (24.8).

has already been defined in Art. 68. It is also called the resultant electrical intensity, being the force which would be experienced by a unit of positive electricity placed at that point. We have now obtained the most general value of this quantity in the case of *a body moving in a magnetic field* due to a variable electric system.

If the body is a conductor, the electromotive force will produce a current; if it is a dielectric, the electromotive force will produce only electric displacement.

The electromotive intensity, or the force on a particle, must be carefully distinguished from the electromotive force along an arc of a curve, the latter quantity being the line-integral of the former. See Art. 69.

Maxwell continued his book as follows, our emphasis:[665]

599.] The electromotive intensity, the components of which are defined by equations (B), depends on three circumstances. The first of these is *motion of the particle through the magnetic field*. The part of the force depending on this motion is expressed by the first two terms on the right of each equation. It depends *on the velocity of the particle transverse to the lines of magnetic induction*. If $\mathfrak{G}$ is a vector representing the velocity, and $\mathfrak{B}$ another representing the magnetic induction, then if $\mathfrak{E}_1$ is the part of the electromotive intensity depending on the motion,

$$\mathfrak{E}_1 = \mathrm{V}.\mathfrak{G}\mathfrak{B}\ , \qquad (7)$$

or, the electromotive intensity is the vector part of the product of the magnetic induction multiplied by the velocity, that is to say, the magnitude of the electromotive force is represented by the area of the parallelogram, whose sides represent the velocity and the magnetic induction, and its direction is the normal to this parallelogram, drawn so that the velocity, the magnetic induction, and the electromotive intensity are in right-handed cyclical order.

Maxwell's Equation (7) would nowadays be written in vector notation as follows:

$$\vec{E} = \vec{v} \times \vec{B}\ . \tag{24.11}$$

It is important to emphasize some aspects here. Maxwell's equations (B) of the *Treatise*, our Equation (24.9), is analogous to the force given by Equation (24.4). Maxwell's seems to have been the first to write down this equation, publishing his results between 1861 and 1873.

The magnetic component of this force, namely, $\vec{F}_m = q\vec{v}\times\vec{B}$, seems to have been obtained by Maxwell after considering Ampère's electrodynamic force exerted by a closed circuit C_2 acting on a current element $i_1 d\vec{\ell}_1$. Consider a current element $i_1 d\vec{\ell}_1$ located at $\vec{r}_1$ relative to the origin O of an inertial coordinate system and another current element $i_2 d\vec{\ell}_2$ located at $\vec{r}_2$. The distance between them is given by $r = |\vec{r}_1 - \vec{r}_2|$. Let $\hat{r} = (\vec{r}_2 - \vec{r}_1)/r$ be the unit vector pointing from 2 to 1.

In modern vector notation and in the International System of Units, Ampère's force exerted by a closed circuit C_2 acting on a current element $i_1 d\vec{\ell}_1$ can be written as follows:

$$d\vec{F}_{21} = \frac{\mu_o}{4\pi} i_1 i_2 \oint_{C_2} \frac{\hat{r}}{r^2}\left[3(\hat{r}\cdot d\vec{\ell}_1)(\hat{r}\cdot d\vec{\ell}_2) - 2(d\vec{\ell}_1\cdot d\vec{\ell}_2)\right] = i_1 d\vec{\ell}_1 \times \oint_{C_2} \frac{\mu_o}{4\pi}\frac{i_2 d\vec{\ell}_2 \times \hat{r}}{r^2}\ . \tag{24.12}$$

[665][Max54, Vol. 2, § 599, pp. 240-241].

Maxwell, but not Ampère, then defined the magnetic field $\vec{B}$ at the location $\vec{r}_1 = (x_1,\ y_1,\ z_1)$ of the test element $i_1 d\vec{\ell}_1$ and being due to the closed current carrying circuit C_2 as follows:

$$\vec{B}(\vec{r}_1) \equiv \oint_{C_2} \frac{\mu_o}{4\pi} \frac{i_2 d\vec{\ell}_2 \times \hat{r}}{r^2} \ . \tag{24.13}$$

With this definition, Ampère's Equation (24.12) exerted by the closed circuit C_2 on the current element $i_1 d\vec{\ell}_1$ could then be written as follows:

$$d\vec{F}_{21} = i_1 d\vec{\ell}_1 \times \oint_{C_2} \frac{\mu_o}{4\pi} \frac{i_2 d\vec{\ell}_2 \times \hat{r}}{r^2} = i_1 d\vec{\ell}_1 \times \vec{B} \ . \tag{24.14}$$

Maxwell then finally replaced this current element $i_1 d\vec{\ell}_1$ by $q\vec{v}$, where q is the charge of the electrified body and $\vec{v}$ its velocity. The magnetic force $\vec{F}_m$ acting on this charged body moving in a magnetic field would then be written as:

$$\vec{F}_m = q\vec{v} \times \vec{B} \ . \tag{24.15}$$

Moreover, Maxwell interpreted that this velocity $\vec{v}$ which appears in equations (24.4) or (24.15) was the velocity of the particle with charge q moving relative to the magnetic field $\vec{B}$. As shown above, in his paper of 1861 and in article 598 of his Treatise of 1873, he mentioned explicitly the force acting on "a body moving in a magnetic field."

24.3.2 Meaning of the Velocity According to Thomson

In 1881 J. J. Thomson (1856-1940) obtained theoretically the magnetic force acting on a particle with charge q as given by $q\vec{v} \times \vec{B}/2$.[666] This velocity $\vec{v}$ in his theory was interpreted as the velocity of the particle relative to the medium through which it was moving, a medium whose magnetic permeability was μ. For Thomson this was not a velocity of the particle relative to the magnetic field, nor relative to the luminiferous ether, nor relative to the magnet which generated the magnetic field $\vec{B}$, nor the velocity of the electrified particle relative to the observer. He called this velocity the "actual velocity" of the electrified particle. He said the following on page 248 of his original article:[667]

> It must be remarked that what we have for convenience called the actual velocity of the particle is, in fact, the velocity of the particle relative to the medium through which it is moving [...], medium whose magnetic permeability is μ.

24.3.3 Meaning of the Velocity According to Heaviside

In 1889, O. Heaviside (1850-1925) deduced theoretically the magnetic force acting on a particle with charge q as given by $q\vec{v} \times \vec{B}$, where $\vec{B}$ was the magnetic field at the location of the charge. This is the same value obtained earlier by Maxwell and twice the value obtained by Thomson. The interpretation of this velocity $\vec{v}$ for Heaviside is clear from the title of his paper, namely:[668]

[666] [Tho81] and [Whi73, pp. 306-310].
[667] [Tho81, p. 248].
[668] [Hea89].

On the electromagnetic effects due to the motion of electrification through a dielectric.

This title shows that for him this $\vec{v}$ was the velocity of the particle relative to the dielectric material through which it was moving.

24.3.4 Meaning of the Velocity According to Lorentz

In 1895 H. A. Lorentz (1853-1928) presented the force acting on a particle with charge q as follows:[669]

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \ . \tag{24.16}$$

His deduction contains only two pages which belong to Section I (The Fundamental Equations for a System of Ions Located in the Aether) of the paper. I will quote his full deduction taken from the English translation:[670]

> **The second part of the force acting on ponderable matter**
>
> § 12. A current element as the one previously considered, may be located in a magnetic field generated by external causes. According to a known law it suffers an electrodynamic force
>
> $$[\mathrm{ids}\cdot\mathfrak{H}],$$
>
> for which we also can write now
>
> $$[\Sigma\ \mathrm{e}\mathfrak{v}\cdot\ \mathfrak{H}],$$
>
> or
>
> $$\Sigma\{\mathrm{e}[\mathfrak{v}\cdot\ \mathfrak{H}]\}.$$
>
> This action results (according to our view) from the forces, which will be exerted by the aether upon the ions of the current element. It is thus near at hand, to assume for the force acting on a single ion
>
> $$\mathrm{e}[\mathfrak{v}\cdot\mathfrak{H}],$$
>
> a hypothesis, which we still want to extend in a way, so that we *generally* assume a force acting on ponderable matter of the volume element $d\tau$
>
> $$\rho d\tau[\mathfrak{v}\cdot\mathfrak{H}]$$
>
> In unit charge this would be
>
> $$\mathfrak{E}_2 = [\mathfrak{v}\cdot\mathfrak{H}].^{671}$$
>
> By putting this vector together with $\mathfrak{E}_1$ that was considered earlier (§ 9), we obtain for the total force exerted on the unit charge, *i.e.* for the *electric force*,
>
> $$\mathfrak{E} = 4\pi V^2\mathfrak{d} + [\mathfrak{v}\cdot\mathfrak{H}]. \qquad \text{(V)}$$

[669] [Lor95, Section 1, § 12, pp. 21-22 and table after page 138], [Pai82, p. 125] and [Pai86, p. 76].

[670] [Lor95, Section 1, § 12, pp. 21-22 and table after page 138]. Translation available at https://en.wikisource.org/wiki/Translation:Attempt_of_a_Theory_of_Electrical_and_Optical_Phenomena_in_Moving_Bodies

[671] [Note by Lorentz:] If we don't want to consider an ordinary electric current as a convection current, then we must substantiate this formula by the assumption, that a body in which a convection takes place, experiences the same electrodynamic actions as a corresponding current conductor.

> We refuse to express the thus stated law by words. By elevating it to a general fundamental-law, we have completed the system of equations of motion (I)—(V), since the electric force, in connection with possible other forces, determines the motion of ions.
>
> Concerning the latter, we still want to introduce the assumption, that the ions never rotate.[672,673]

Lorentz presented a similar deduction in a course of lectures delivered in Columbia University, New York, in March and April, 1906. They were published in his famous book *The Theory of Electrons*, first published in 1909.[674] I quote from the second edition of 1915. Passages in square brackets are our words and the modern rendering of some of his formulas (for instance Lorentz' $[\mathbf{a}\cdot\mathbf{b}]$ is nowadays usually represented by $\vec{a}\times\vec{b}$). He utilized the cgs system of units. What he called "electron" represented a generic electrified particle (the specific particle called nowadays the "electron," with a charge of $q = -1.6\times10^{-19}\ C$ and mass $m = 9.1\times10^{-31}\ kg$, was only discovered in 1897). Here are his words with our emphasis:[675]

> However this may be, we must certainly speak of such a thing as the force acting on a charge, or on an electron, on charged matter, whichever appellation you prefer. Now, in accordance with the general principles of Maxwell's theory, *we shall consider this force as caused by the state of the ether, and even, since this medium pervades the electrons, as exerted by the ether on all internal points of these particles where there is a charge*. If we divide the whole electron into elements of volume, there will be a force acting on each element and determined by the state of the ether existing within it. We shall suppose that this force is proportional to the charge of the element, so that we only want to know the force acting per unit charge. This is what we can now properly call *the electric force*. We shall represent it by **f**. The formula by which it is determined, and which is the one we still have to add to (17)-(20) [Maxwell's equation's], is as follows:
>
> $$\mathbf{f} = \mathbf{d} + \frac{1}{c}[\mathbf{v}\cdot\mathbf{h}]. \qquad \left[\vec{f} = \vec{d} + \frac{\vec{v}\times\vec{h}}{c}\right] . \qquad (23)$$
>
> Like our former equations, it is got by generalizing the results of electromagnetic experiments. The first term represents the force acting on an electron in an electrostatic field; indeed, in this case, the force per unit charge must be wholly determined by the dielectric displacement. On the other hand, the part of the force expressed by the second term may be derived from the law according to which an element of a wire carrying a current is acted on by a magnetic field with a force perpendicular to itself and the lines of force, an action, which in our units may be represented in vector notation by

[672][Note by Lorentz:] In an earlier published derivation of the equations of motion (La théorie électromagnétique de Maxwell et son application aux corps mouvants), I have discussed the necessary conditions.

[673][Lor92].

[674][Lor09, pp. 14-15] and [O'R65, p. 561].

[675][Lor15, pp. 14-15].

$$\mathbf{F} = \frac{s}{c}[\mathbf{i} \cdot \mathbf{h}], \qquad \left[\vec{F} = \frac{i\vec{s} \times \vec{h}}{c} \right] ,$$

where $\mathbf{i}$ is the intensity of the current considered as a vector, and s the length of the element. According to the theory of electrons, $\mathbf{F}$ is made up of all the forces with which the field $\mathbf{h}$ acts on the separate electrons moving in the wire. Now, simplifying the question by the assumption of only one kind of moving electrons with equal charges e and a common velocity $\mathbf{v}$, we may write

$$s\mathbf{i} = Ne\mathbf{v} , \qquad [i\vec{s} = Ne\vec{v}] ,$$

if N is the whole number of these particles in the element s. Hence

$$\mathbf{F} = \frac{Ne}{c}[\mathbf{v} \cdot \mathbf{h}], \qquad \left[\vec{F} = \frac{Ne\vec{v} \times \vec{h}}{c} \right] ,$$

so that, dividing by Ne, we find for the force per unit charge

$$\frac{1}{c}[\mathbf{v} \cdot \mathbf{h}]. \qquad \left[\frac{\vec{v} \times \vec{h}}{c} \right] .$$

As an interesting and simple application of this result, I may mention the explanation it affords of the induction current that is produced in a wire moving across the magnetic lines of force. The two kinds of electrons having the velocity $\mathbf{v}$ of the wire, are in this case driven in opposite directions by forces which are determined by our formula.

9. After having been led in one particular case to the existence of the force $\mathbf{d}$, and in another to that of the force $\frac{1}{c}[\mathbf{v} \cdot \mathbf{h}]$, we now combine the two in the way shown in the equation (23), going beyond the direct result of experiments by the assumption that in general the two forces exist at the same time. If, for example, an electron were moving in a space traversed by Hertzian waves, we could calculate the action of the field on it by means of the values of $\mathbf{d}$ and $\mathbf{h}$, such as they are at the point of the field occupied by the particle.

O'Rahilly made very important comments related to this deduction of Lorentz. These comments can also be applied to Maxwell's earlier and similar deduction presented in Subsection 24.3.1. It is difficult to disagree with O'Rahilly, when he noted that:[676]

The ordinary proof of the formula is extremely unsatisfactory. Here is Lorentz's effort: [...]

There are two overwhelming objections to this alleged generalization. (1) The two 'particular cases' he 'combined' are quite incompatible. In the one case we have charges at rest, in the other the charges are moving; they cannot be both stationary and moving. (2) Experiments with a 'wire carrying a current' have to do with *neutral* currents, yet the derivation contradicts this neutrality.

[676] [O'R65, p. 561].

O'Rahilly then quotes several current textbooks showing that the proofs they present of Lorentz force are equally unsatisfactory.

I can add a third very unsatisfactory aspect of Maxwell and Lorentz' deductions of their force laws, namely, they do not specify clearly the meanings associated with the velocity of the electrified particle. In order to grasp the meanings they gave to this velocity, one needs to study carefully their papers, paying close attention to their words.

They begin with the force acting on a current element $Id\vec{\ell}$ and replace it by $q\vec{v}$. As pointed out by O'Rahilly, when Ampère obtained his expression for the force exerted by a closed circuit acting on a current element of another circuit, he was dealing with neutral current elements. Therefore it is wrong to replace $Id\vec{\ell}$ by $q\vec{v}$.

Moreover, when there is a current flowing along a metal wire, the electrified particles are moving relative to the matter of the conductor. This conductor can be, for instance, a copper wire. Therefore, the first idea would be to consider this velocity $\vec{v}$ of the electrified particle as the drift velocity of the charge relative to the metal wire. But this is not what Maxwell and Lorentz did. As was shown in Subsection 24.3.1, Maxwell considered it as the velocity of the particle relative to the magnetic field.

Lorentz, on the other hand, considered it as the velocity of the particle relative to the ether. This can see from the above quotation ("[...] force as caused by the state of the ether, and even, since this medium pervades the electrons, as exerted by the ether [...]"). Therefore, for Lorentz the velocity $\vec{v}$ meant originally the velocity of the electrified particle relative to the ether and not, for instance, relative to the observer or frame of reference. He did not interpret this velocity as being the velocity of the test electrified particle relative to the magnetic field, nor relative to the observer or frame of reference. In Lorentz theory the ether was in a state of absolute rest relative to the frame of fixed stars.[677]

A conclusive proof of this interpretation of the velocity which appears in the magnetic force $q\vec{v}\times\vec{B}$ can be found in another work of Lorentz, *Lectures on Theoretical Physics*. This work is based on a course of Maxwell's theory presented in 1900-1902 and on another course on the principle of relativity for uniform translations presented in 1910-1912 which were first published in 1925 and 1922, respectively. Figure 24.3 shows our representation of the two situations he was considering. Lorentz considered three bodies, namely, the Earth E, the circuit carrying a constant current I, and the test particle with charge q. In situation (a) these three bodies were considered at rest relative to the ether and relative to the frame of fixed stars. In situation (b) these three bodies are moving together relative to the ether and relative to the frame of fixed stars with a common velocity $\vec{v}$.

In this work Lorentz said:[678]

> 8·9. There is yet one problem worth of attention. Imagine an electric current flowing in a closed circuit without resistance. Will this current act upon a particle carrying a charge e which is placed in its neighbourhood? We purposely speak of a circuit without resistance. For, if it had a resistance, a certain electromotive force would be necessary to sustain the current, and this would unavoidably give rise to a potential gradient and to charges (no matter how small) spread over the conductor which would act upon the electrified particle. In fine, our question concerns the effect of the current *as such* upon the particle.

[677] [Pai82, p. 111].

[678] [Lor31, Volume 3, p. 306] and [O'R65, Volume 2, p. 566].

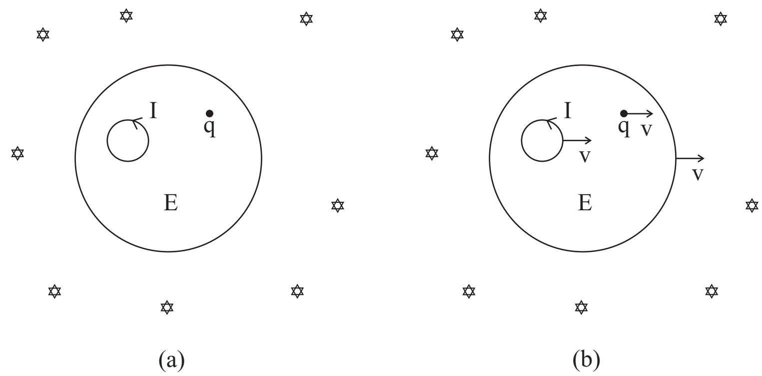


Figure 24.3: Earth E, circuit carrying a constant current I, and test particle with charge q. In configuration (a) these three bodies are at rest relative to the ether and relative to the fixed stars. (b) These three bodies move together with velocity $\vec{v}$ relative to the ether and fixed stars.

> The answer to this question was, of course, that the current did not act upon the particle. It would act upon a magnetic needle placed in the neighbourhood, since it is surrounded by a magnetic field, but there is no trace of an electric field. This is certainly correct so long as the current and the electrified particle are at rest.[679] Suppose, however, that both share in some motion, *e.g.* the Earth's motion.[680] What then? To begin with, the charged particle will move with a certain velocity through the magnetic field of the current and it will thus be acted upon by some force.

In this work Lorentz said that when a current carrying wire and an external electrified particle are at rest relative to one another, and also at rest relative to the ether, then no magnetic force would act on the particle. In his words: "the current did not act upon the particle."

On the other hand, if the current carrying wire and the electrified particle were at rest relative to one another, but if both were moving with the same velocity $\vec{v}$ relative to the ether, then there would be a magnetic force acting on the particle. In his words: "the charge particle will move with a certain velocity through the magnetic field of the current and it will thus be acted upon by some force." In this last situation there was no motion between the electrified particle and the current carrying circuit, nor between the particle and the Earth or laboratory, nor even between the particle and the observer or detector of magnetic field (who are supposedly at rest in the laboratory). But to Lorentz, even in this case there would be a magnetic force acting on the particle. He could only consider this possibility because he supposed $\vec{v}$ to be the velocity of the electrified particle relative to the ether or relative to the fixed stars. As the fixed stars did not cause any electromagnetic net force on the particle with charge q, all that remained was the force acting on the particle and being exerted by the ether.

In conclusion, to Lorentz the velocity appearing in his force given by Equation (24.16) was the velocity of the electrified particle relative to the ether. Moreover, he assumed this ether to be at rest relative to the frame of fixed stars.[681]

[679][Note by O'Rahilly:] At rest in the aether.
[680][Note by O'Rahilly:] That is, assuming that the laboratory *has* a motion through the aether.
[681][Pai82, p. 111].

24.3.5 Meaning of the Velocity According to Einstein

The velocity $\vec{v}$ of the test particle with charge q which appears in the force $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ had several meanings according to different authors. (a) According to Maxwell, it was the velocity relative to the magnetic field; (b) according to Thomson, it was the velocity of the electrified particle relative to the medium of magnetic permeability μ; (c) according to Heaviside, it was the velocity relative to the dielectric; while (d) according to Lorentz, it was the velocity of the particle relative to a very specific medium, the ether. These four authors considered that this force did arise due to an interaction of the test electrified particle with four different entities, namely, (a) the magnetic field, (b) with a material medium of permeability μ, (c) with the dielectric, or (c) with a very specific medium, namely, the ether. As the electrified particle was supposed to be interacting with these material media, it was natural to interpret $\vec{v}$ as the velocity of the particle relative to these media.

Einstein changed all this with his paper of 1905 on the special theory of relativity. What Einstein proposed in this paper was that the velocity $\vec{v}$ which appears in Equation (24.4), should be interpreted as the velocity of the electrified particle relative to the observer.[682]

He initially obtained Lorentz transformations for the spatial coordinates and for time. These transformations relate the magnitudes in one inertial frame to another inertial frame moving relative to the first frame with a constant linear velocity. Einstein then obtained these transformations also for the electric and magnetic fields. He applied these transformations for the electric and magnetic fields in the electromagnetic force given by Equation (24.4). In this way Einstein began to utilize the velocity $\vec{v}$ as being the velocity of the electrified particle relative to the observer, or as a velocity of the particle relative to the inertial frame of reference. For instance, in this paper Einstein gave the difference between the old paradigm of electromagnetism and the new one based on his theory of relativity (passages in the footnotes are our words):[683]

> As to the interpretation of these equations[684] we make the following remarks: Let a point charge of electricity have the magnitude "one" when measured in the stationary system K,[685] i.e. let it when at rest in the stationary system exert a force of one dyne upon an equal quantity of electricity at a distance of one cm. By the principle of relativity this electric charge is also of the magnitude "one" when measured in the moving system. If this quantity of electricity is at rest relatively to the stationary system, then by definition the vector $(X,\ Y,\ Z)$[686] is equal to the force acting upon it. If the quantity of electricity is at rest relatively to the moving system (at least at the relevant instant), then the force acting upon it, measured in the moving system, is equal to the vector $(X',\ Y',\ Z')$. Consequently the first three equations above[687] allow themselves to be clothed in words in the two following ways:
>
> 1. If a unit electric point charge is in motion in an electromagnetic field, there acts

[682] See footnote 641.

[683] [Ein52, p. 54] and [Ein78, p. 71].

[684] [Note by AKTA:] Equations for the Lorentz transformations of the electric and magnetic field components in two different inertial frames of reference which move relative to one another with a constant velocity.

[685] [Note by AKTA:] A system of coordinates in which the equations of newtonian mechanics hold good.

[686] [Note by AKTA:] This vector $(X,\ Y,\ Z)$ represents the electric force per unit charge. That is, it is the vector electric field, which nowadays would be expressed as: $\vec{E} = (E_x,\ E_y,\ E_z)$.

[687] [Note by AKTA:] That is, equations for the transformation of the electric and magnetic field components in two different inertial systems which move relative to one another with a constant velocity.

upon it, in addition to the electric force,[688] an "electromotive force" which, if we neglect the terms multiplied by the second and higher powers of v/c, is equal to the vector-product of the velocity of the charge and the magnetic force, divided by the velocity of light.[689] (Old manner of expression.)[690]

2. If a unit electric point charge is in motion in an electromagnetic field, the force acting upon it is equal to the electric force which is present at the locality of the charge, and which we ascertain by transformation of the field to a system of co-ordinates at rest relatively to the electrical charge. (New manner of expression.)

Following Einstein, let us call K the stationary inertial system and k the inertial system which is moving relative to K with a constant velocity v. I will utilize primed symbols for the magnitudes expressed in k. According to Einstein, the charge of the electrified particle has the same value in both coordinate systems, $q' = q$. Moreover, the particle moves with velocity $\vec{v}$ relative to K and is stationary relative to k, that is, $\vec{v}\,' = 0$. Therefore, according to Einstein, the net force acting on the electrified particle in K would be given by $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$. On the other hand, the net force acting on the electrified particle in the frame k would be given by $\vec{F}' = q'\vec{E}' + q'\vec{v}\,' \times \vec{B}' = q\vec{E}'$, as $q' = q$ and $\vec{v}\,' = \vec{0}$.

This is the crucial passage in which Einstein introduced frame-dependent forces, that is, forces whose value depend on the motion between the test body and the observer. He presented here a completely new meaning for the velocity $\vec{v}$ which appears in Equation (24.4), namely, velocity of the electrified particle relative to the observer or relative to the frame of reference.

The introduction of physical forces which depend on the state of motion of the observer has created many problems and paradoxes for the explanation of several simple phenomena of nature. Unfortunately it has been part of theoretical physics ever since that time. No experiment has suggested or forced this new interpretation. This whole interpretation arose from Einstein's mind. The usual expression for the magnetic force might have been maintained, interpreting $\vec{v}$ as the velocity of the test electrified particle relative to the magnet or relative to the current carrying wire, without any contradictions with experimental data. Instead of adopting this reasonable point of view, Einstein decided to change the interpretation of this velocity. He created an enormous confusion with this new interpretation which has plagued theoretical physics ever since 1905.

24.4 These Different Meanings Given to the Velocity in the Classical Electromagnetic Force Law Imply *Different* Field Theories

The so-called Lorentz force can be written as follows:

$$\vec{F} = q\vec{E} + q\vec{v} \times \vec{B} \ . \tag{24.17}$$

[688][Note by AKTA:] That is, beyond the force $q\vec{E}$.

[689][Note by AKTA:] This "electromotive force" would then be given by $q\vec{v} \times \vec{B}$ in the International System of Units.

[690][Note by AKTA:] Therefore, in this old manner of expression, the net force on the test particle with charge q would be given by $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$.

I have been dealing with the meaning of the velocity in this force since my undergraduate years:[691]

> During my undergraduate and graduate studies in physics I was introduced to the so-called Lorentz force. As usually mentioned in the textbooks, if an electrified particle with charge q is moving with velocity $\vec{v}$ in the presence of an electric field $\vec{E}$ and a magnetic field $\vec{B}$, then the Lorentz force $\vec{F}$ acting on this charge is given by $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$. The textbooks usually do not specify the meaning of this velocity. Is it the velocity of the charge q relative to what? This force can only be applied or utilized when we know the meaning of this velocity. When I discovered that nowadays this velocity $\vec{v}$ is interpreted as the velocity of the electrified particle relative to the observer, I did not accept it. This interpretation was against my physical intuition, after all the charge q is not interacting with the observer. It is interacting with other electrified particles.

I then began to make a historical search related to the meaning of this velocity in the so-called Lorentz force.[692] I concluded my 1994 analysis of this topic with the following words:

> This change in the meaning of $\vec{v}$ in $\vec{F} = q\vec{E} + q\vec{v} \times \vec{B}$ is very strange, confusing and unusual in physics.

In the last few years I have been finally aware that this is not only strange, confusing and unusual in physics. As I show in Section 24.1, by changing the meaning of the magnitudes appearing in the same mathematical formula, one changes the underlying theory. In particular, Equation (24.1) is a mathematical representation of Newton's law of gravitation when one interprets the distance r in this equation as given by Figure 24.1 (a), but it represents a different theory when one interprets the distance r as given by Figure 24.1 (b). Likewise, it is now clear to me that all of these different meanings given to the velocity appearing in Equation (24.17) imply different theories.

Lorentz, in particular, as shown in Subsection 24.3.4, interpreted the velocity appearing in Equation (24.17) as the velocity of the electrified particle relative to the ether. As in modern electromagnetic textbooks the velocity appearing in this Equation is interpreted as the velocity of the charge relative to the observer, it is **wrong** *to call it Lorentz force, as is the case in every textbook on electromagnetism.*

After I was convinced that Maxwell himself had deduced a similar formula before Lorentz, I decided to call this equation the Maxwell-Lorentz's force.[693] *However, as shown in Subsection 24.3.1, Maxwell interpreted the velocity appearing in this equation as the velocity of the electrified particle relative to the magnetic field. As in modern electromagnetic textbooks the velocity appearing in this equation is interpreted as the velocity of the charge relative to the observer, it is* **wrong** *to call it the Maxwell-Lorentz force, as I did in 2013 and 2014.*

The five different interpretations of the velocity discussed in Subsections 24.3.1, 24.3.2, 24.3.3, 24.3.4 and 24.3.5 imply five different theories. These different theories should be called by different names. Maxwell, Thomson, Heaviside, Lorentz and Einstein expressed the force acting on an electrified particle by the same formula, Eq. (24.17). However, as they

691 [Ass21i, pp. 16-17].

692 See footnote 641.

693 [Ass13, p. 240] and [Ass14b, p. 274].

gave different interpretations for the meanings of the velocity appearing in this equation, each of these force laws should receive a different name. In particular, if one interprets this velocity as the velocity of the particle relative to the observer or frame of reference, as is the case in all modern textbooks on electromagnetism, the names of Maxwell, Thomson, Heaviside and Lorentz should not be associated with this force law.

24.5 Forces Depending on Velocity in Newtonian Mechanics

In newtonian mechanics there are also forces which depend on the velocity of the test body. However, these forces depend only on the *relative velocity between the interacting bodies*. That is, these forces do not depend on the velocity of the test body relative to the observer, nor on the velocity of the test body relative to the frame of reference. I present here two examples, one in mechanics and the other in electromagnetism.

Suppose a parachutist falling to the ground after leaving an airplane which was flying horizontally. Its initial vertical velocity relative to the ground was zero. Due to its weight, the person is initially accelerated downwards. An upward force due to air resistance begins to act on the body. This dragging force increases with the velocity of the body relative to air. The vertical velocity increases until it reaches a terminal constant velocity relative to the ground. In this last situation the weight is balanced by the dragging force exerted by the air. This dragging force depends only on the *relative velocity* $\vec{v}_r$ between the test body and the air around it. This relative velocity for a rigid body is given by $\vec{v}_r = \vec{v} - \vec{v}_f$, where $\vec{v}$ is the velocity of the body relative to the ground, while $\vec{v}_f$ is the velocity of the surrounding fluid relative to the ground. Suppose air is at rest relative to the ground, $\vec{v}_f = \vec{0}$. Let $\vec{v}_t$ represent the terminal velocity of the parachute relative to the ground. Let us analyze the problem from the terrestrial point of view when the body is falling with this terminal constant velocity relative to the ground. The dragging force acting on it will depend on the relative velocity $\vec{v}_r = \vec{v} - \vec{v}_f = \vec{v}_t - \vec{0} = \vec{v}_t$. By equating the weight of the body with the upward dragging force, one can relate the terminal velocity of the body with its weight, shape, air density etc.

Let me now analyze the problem from the parachutist point of view when he is falling with the terminal velocity $\vec{v}_t$ relative to the ground. Although the parachutist is not moving relative to himself, the dragging force acting on it will not be zero from his own point of view. Let S' be the frame of the parachutist. As the parachutist is at rest relative to himself, he has a zero velocity, $\vec{v}\,' = \vec{0}$. The air around him, on the other hand, is moving upwards and has a velocity $\vec{v}_f\,'$ different from zero. When the parachutist is falling at terminal velocity $\vec{v}_t$ relative to the ground, the air around him is moving upwards relative to him with a constant velocity given by $\vec{v}_f\,' = -\vec{v}_t$. Therefore, the relative velocity between the parachutist and the fluid around him will be given by $\vec{v}_r\,' = \vec{v}\,' - \vec{v}_f\,' = \vec{0} - (-\vec{v}_t) = \vec{v}_t$. That is, this relative velocity has the same value it had in the terrestrial frame of reference. This means that one can solve the problem not only in the terrestrial frame of reference, but also in the frame of reference of the parachutist falling with its terminal velocity. No paradoxes appear here and it is not necessary any transformation of the gravitational field. It is also not necessary any transformation of the dragging force in two different inertial frames of reference.

The second example is that of Ohm's law. When a potential difference $\Delta\phi$ is applied between the terminals of a circuit with a resistance R, a constant current I will flow along the circuit as given by $\Delta\phi = RI$. Microscopically this Ohm's law can be written at a certain point inside the conductor as $\vec{J} = -\sigma\nabla\phi = \sigma\vec{E}$, where $\vec{J}$ is the volume current density, σ is the conductivity of the medium and $\vec{E}$ is the force per unit charge acting at this point. In the case of metals, only conduction electrons move relative to the wire. The volume current density can then be written as $\vec{J} = \rho_-\vec{v}_{r-}$, where ρ_- is the volume density of negative charges and $\vec{v}_{r-}$ is the *relative velocity* between the electron and the wire. This relative velocity can be written as $\vec{v}_{r-} = \vec{v}_q - \vec{v}_w$, where $\vec{v}_q$ represents the velocity of the conduction electron relative to the ground, while $\vec{v}_w$ represents the velocity of the wire relative to the ground. As in the previous case of the parachutist, one can analyze Ohm's law in a frame of reference at rest relative to the wire. In this case $\vec{v}_q = \vec{v}_d$ and $\vec{v}_w = 0$, where $\vec{v}_d$ is the drift velocity of any specific conduction electron, that is, its velocity relative to the wire. Therefore $\vec{v}_{r-} = \vec{v}_q - \vec{v}_w = \vec{v}_d - 0 = \vec{v}_d$.

One can also analyze this problem in a frame of reference S' which is at rest relative to a specific conduction electron. In this case $\vec{v}'_q = 0$ and $\vec{v}'_w = -\vec{v}_d$. Therefore the relative velocity in this frame S' is given by $\vec{v}'_{r-} = \vec{v}'_q - \vec{v}'_w = 0 - (-\vec{v}_d) = \vec{v}_d$. That is, the relative velocity between any conduction electron and the wire is the same as in the wire frame of reference.

In both cases there will be the same force exerted by the wire on the electron, as the relative velocity between them is the same, no matter the inertial frame of reference.

24.6 The Velocity in Weber's Force Law

Weber proposed in 1846 a *fundamental law of electric action*.[694] It was a force combining Coulomb's law for the interaction between electrified particles separated by a distance r, together with a component depending on the relative velocity dr/dt between the particles and another component depending on their relative acceleration d^2r/dt^2.

In 1852 he presented his law utilizing a constant c representing the uniform relative velocity at which the force between the particles would fall to zero.[695] Weber's c (known throughout the 19th century as the *Weber constant*) was first measured by Weber and Kohlrausch in 1854-1856. They obtained $c = 4.39 \times 10^8\ m/s$.[696]

In 1857 Weber and Kirchhoff were the first to derive theoretically the complete telegraph equation working independently from one another and arriving simultaneously at the same result. Utilizing the modern concepts and usual terminology of circuit theory, we can say that they were the first to take into account not only the capacitance and resistance of the wire, but also its self-inductance. For a circuit of negligible resistance, they concluded that the velocity of propagation of an electric wave along the wire would be given by $c/\sqrt{2} = 3 \times 10^8\ m/s$. This value coincided with the known light velocity in vacuum, v_L, as deduced from astronomical observations and from terrestrial optical experiments. That is,

$$\frac{c}{\sqrt{2}} = v_L \qquad \text{or} \qquad c = \sqrt{2} \cdot v_L \ . \tag{24.18}$$

This result was independent of the cross section and conductivity of the wire, and also independent of its surface density of electricity. Kohlrausch, who was collaborating with

[694] [Web46] with partial French translation in [Web87] and a complete English translation in [Web21d].
[695] [Web52b, p. 366 of Weber's *Werke*] with English translation in [Web21e, p. 346].
[696] [Web55] with English translation at [Web21i]; [WK56] with English translation in [WK21] and Portuguese translation in [WK08]; and [KW57] with English translation in [KW21]. See also [Ass21h].

Weber on some experiments related with the propagation of electromagnetic waves, died in 1858. Weber's work has been delayed in publication and appeared only in 1864.[697]

Weber's 1846 fundamental force can then be expressed in terms of his 1852 constant c and also in terms of light velocity in vacuum, v_L, as follows:

$$\frac{ee'}{r^2}\left[1-\frac{1}{c^2}\left(\frac{dr}{dt}\right)^2+\frac{2}{c^2}r\frac{d^2r}{dt^2}\right]=\frac{ee'}{r^2}\left[1-\frac{1}{2v_L^2}\left(\frac{dr}{dt}\right)^2+\frac{1}{v_L^2}r\frac{d^2r}{dt^2}\right] . \tag{24.19}$$

The force between two particles was thus dependent upon their *relative velocity*, dr/dt, and *relative acceleration*, d^2r/dt^2. Velocities of the particles relative to the observer or frame of reference are not relevant here. Weber's force points along the straight line connecting the interacting particles and follows the principle of action and reaction.

24.7 Comparison Between Lorentz and Weber's Deductions of Their Force Laws

In the last 130 years Weber's force (1846) has been replaced by the so-called Lorentz force (1895) in electromagnetic textbooks. The so-called Lorentz force has been presented side by side with Maxwell's equations as the basis of classical electromagnetic theory. In this Section I will compare the deduction of these two force laws. A comparison with similar conclusions might be made between Maxwell and Weber's deductions of their different force laws.

Lorentz presented the deduction of his force in 2 pages of his 1895 paper, and similarly in 2 pages of his book *The Theory of Electrons*.[698] Weber, on the other hand, presented the deduction of his force in 25 pages, Sections 18-21, of his 1846 treatise.[699]

24.7.1 Experimental Proof of Ampère's Force

To my knowledge Lorentz never performed a single experiment to arrive at his force law of 1895 presented in Subsection 24.3.4.

Weber, on the other hand, utilizes the first 50 pages of his paper of 1846 presenting a detailed experimental proof of Ampère's force for the interaction between electric currents.[700] To this end he introduced the famous electrodynamometer which he created. According to Maxwell:[701]

> The experiments which he made with it furnish the most complete experimental proof of the accuracy of Ampère's formula as applied to closed currents, and form an important part of the researches by which Weber has raised the numerical determination of electrical quantities to a very high rank as regards precision. Weber's form of the electrodynamometer, in which one coil is suspended within another, and is acted on by a couple tending to turn it about a vertical axis, is probably the best fitted for absolute measurements.

[697] [Kir57b] with English translation in [Kir57a] and [Kir21c], [Pog57] with English translation in [Pog21], and [Web64] with English translation in [Web21c]. See also [Ass21h].

[698] [Lor95, Section 1, § 12, pp. 21-22 and table after page 138], [Lor09, pp. 14-15] and [Lor15, pp. 14-15].

[699] [Web46, pp. 132-157 of Weber's *Werke*] with partial French translation in [Web87] and a complete English translation in [Web21d, pp. 129-149].

[700] See footnote 694.

[701] [Max54, Vol. 2, Article 725, p. 371].

Weber then utilized his electrodynamometer to test Faraday's law of induction. He also used it to many other applications.

24.7.2 Lorentz Utilized the Force Exerted by a Closed Circuit on a Current Element, While Weber Utilized the Force Between Two Current Elements

In order to obtain the magnetic component of his force, Lorentz considered a current element $id\vec{s}$ in a magnetic field $\vec{B}$ generated by external causes and said that according to a known law it suffers an electrodynamic force given by

$$id\vec{s} \times \vec{B} \ . \tag{24.20}$$

The external causes which Lorentz did not specify could be a magnet or a closed circuit. André-Marie Ampère (1775-1836) obtained in 1822 the final expression of his fundamental force law between two current elements. He then proved for the first time, experimentally and theoretically, that a closed circuit of arbitrary shape carrying a steady current exerts a force on an external current element which is always orthogonal to this element and also orthogonal to a certain straight line which he called directrix. The direction of this directrix coincides with the direction of the magnetic field of modern theories, although it should be emphasized that Ampère himself never worked with the magnetic field concept. Later on Ampère also proved for the first time the equivalence between a magnetic dipole layer and a closed circuit carrying a steady current, so that any magnet can be replaced by a system of closed loops carrying steady currents.[702]

In essence, Equation (24.20) represents the result first obtained by Ampère that the force acting on a current element due to a magnet (or due to a closed circuit of arbitrary shape carrying a steady current) is always orthogonal to this element and to the straight line given by Ampère's directrix. That is, Equation (24.20) is essentially equivalent to Ampère's result expressed with modern vector notation and in the International System of Units by Equation (24.12).

Weber, on the other hand, in order to arrive at his force law decided to begin directly with Ampère's force between two current elements. In modern vector notation and in the International System of Units, Ampère's force exerted by a current element $i_2 d\vec{\ell}_2$ acting on a current element $i_1 d\vec{\ell}_1$ can be written as follows:

$$d^2\vec{F}_{21} = \frac{\mu_o}{4\pi} i_1 i_2 \frac{\hat{r}}{r^2} \left[3(\hat{r} \cdot d\vec{\ell}_1)(\hat{r} \cdot d\vec{\ell}_2) - 2(d\vec{\ell}_1 \cdot d\vec{\ell}_2) \right] = -d^2\vec{F}_{12} \ . \tag{24.21}$$

Here $d^2\vec{F}_{12}$ is the force exerted by current element $i_1 d\vec{\ell}_1$ on current element $i_2 d\vec{\ell}_2$.

Ampère's force always complies with Newton's action and reaction law. Moreover, it always points along the straight line connecting the two current elements, no matter their orientations in space.

Equation (24.21) is more basic or fundamental than Equation (24.20). After all, Equation (24.12), which is analogous to Equation (24.20), was deduced by Ampère from his force between current elements (24.21) after integrating it over a closed circuit C_2. Moreover, from Ampère's force between current elements one can also obtain other results, like the

[702] See footnote 15 on page 18.

force exerted by a portion of a circuit on another portion of the same circuit; the force exerted by a portion of one circuit acting on a portion of another circuit; etc.

According to Maxwell, Ampère's force between current elements should always remain the most important formula of electrodynamics:[703]

> The experimental investigation by which Ampère established the laws of the mechanical action between electric currents is one of the most brilliant achievements in science. The whole, theory and experiment, seems as if it had leaped, full grown and full armed, from the brain of the 'Newton of electricity.' It is perfect in form, and unassailable in accuracy, and it is summed up in a formula from which all the phenomena may be deduced, and which must always remain the cardinal formula of electro-dynamics.

Moreover, in the deduction of his force law Weber began not only with Ampère's force between current elements, but also with Faraday's law of induction.

24.7.3 Lorentz Replaced a Neutral Current Element by Particles with a Net Charge, While Weber Replaced a Neutral Current Element by Oppositely Charged Particles

Lorentz then replaced the current element $id\vec{s}$ in Equation (24.20) by a sum of particles, each with a net charges e and moving with velocity $\vec{v}$, namely, $\sum e\vec{v}$. He then assumed that this would also be valid for a single ion, namely:

$$e\vec{v} \times \vec{B} \ . \tag{24.22}$$

O'Rahilly correctly pointed out that this proof is extremely unsatisfactory because there is an overwhelming objection to this generalization. After all, experiments with a wire carrying a current have to do with neutral currents, yet Lorentz derivation contradicts this neutrality. He replaced a neutral current element $id\vec{s}$ with $e\vec{v}$, which has a net charge e.[704]

Weber did not make this obvious mistake in his deduction of his own force law. He began with Ampère's force between two current elements ids and $i'ds'$. He then replaced the first current element by two equal and opposite electrified particles with charges e and $-e$ moving with velocities u and $-u$, while the second current element was replaced by two equal and opposite electrified particles with charges e' and $-e'$ moving with velocities u' and $-u'$. That is, each current element remained electrically neutral with Weber's replacement, which was not the case with Lorentz replacement.

24.7.4 The Different Meanings of the Velocities of the Electrified Particles Composing a Current Element

In order to obtain their forces between electrified particles, Lorentz and Weber began with the force acting on a current element ids of length ds. This current element belonged to a wire carrying a current i.

[703] [Max54, Vol. 2, article 528, p. 175].
[704] [O'R65, p. 561].

Lorentz replaced the current element with an electrified particle having charge e and moving with velocity v. That is, he replaced ids with ev. However, he did not consider this velocity as the drift velocity of the particle with charge e relative to the wire. As shown in Subsection 24.3.4, he considered it as the velocity of the charge relative to the ether.

Weber, on the other hand, replaced ids with particles with charges e and $-e$ moving with velocities u and $-u$, respectively. He called u and $-u$ the absolute velocities of the electrified particles. He considered these velocities as the velocities of the electrified particles relative to the material body of the conductor, as it is clear from the context of his detailed discussion in his 1846 paper. This conductor might be, for instance, a copper wire. Therefore the velocities u and $-u$ in Weber's deduction meant the drift velocities of the electrified particles relative to this copper wire.

Three further aspects should be emphasized here.

- In 1846 Weber assumed a double current in each piece of wire carrying an electric current. In particular, he assumed Fechner's hypothesis of 1845, namely, he supposed in each current element equal and opposite electrified particles moving relative to the wire with equal and opposite drift velocities.[705] This was a common assumption at that time. However, it can be shown that one also deduces Ampère's force between current elements from Weber's force between electrified particles without imposing any restrictions on their velocities. That is, one can assume current element ids composed of charges e and $-e$ moving relative to the wire with arbitrary velocities u_1 and u_2, while current element $i'ds'$ is composed of charges e' and $-e'$ moving relative to the wire with arbitrary velocities u'_3 and u'_4. These four velocities are each one of them arbitrary and independent from one another. This means that, beginning with Weber's force, one arrives at Ampère's force between current elements even in metallic circuits in which the positive charges are fixed in the lattice ($u_1 = u'_3 = 0$), while only the moving electrons are responsible for the current. This will also happen when the positive and negative charges move in opposite directions with velocities of different magnitudes (as in the situations of electrolysis).[706]

- Later on Weber changed his mind. Instead of assuming a double current in each wire, he assumed a single current. In particular, he began to suppose that the particles electrified with charges of one sign remained at rest relative to the lattice of the conductor, while only the particles electrified with charges of the other sign were responsible for the current, moving with a drift velocity relative to the wire. He was not sure if the positive charges should be assumed at rest, while only the negative charges would be moving relative to the wire; or if the negative charges should be assumed at rest, while only the positive charges would be moving relative to the wire. A detailed discussion of this topic with many quotations can be found in Section 1.5 (The Evolution of Weber's Conception of an Electric Current: From a Double Current to a Simple Current) of our book Weber's Planetary Model of the Atom.[707]

- The third fact to emphasize is that the electron was only discovered in 1897, six years after Weber's death (1891). In any event with this assumption of a single current Weber was much ahead of his time.

705 [Fec45] with English translation in [Fec21].
706 [Ass90], [Wes90], [Ass94, Section 4.2: Derivation of Ampère's force from Weber's force] and [Ass15a].
707 [AWW11] with Portuguese translation in [AWW14] and German translation in [AWW18].

24.7.5 The Final Velocities Appearing in Lorentz and Weber's Force Laws

According to Lorentz, the velocity of electrified particle appearing in his force law is the velocity of the particle relative to the ether, as was shown in Subsection 24.3.4.

We now consider Weber's force law between electrified particles. In order to arrive at his force he began with Ampère's force law between two current elements. He then replaced the first current element ids by oppositely electrified particles with charges e and $-e$, moving relative to the first piece of wire with drift velocities u and $-u$. The second current element $i'ds'$ was similarly replaced by electrified particles e' and $-e'$ moving relative to the second piece of wire with velocities u' and $-u'$. However, in the development of his force law, he ended up with the *relative velocities*. That is, if r is the distance between any electrified particle e_1 of one current element and any electrified particle e_2 of the other current element, Weber's final force law depends only on the product of the values of these charges, e_1e_2, on their distance r, on their *relative velocity* dr/dt and also on their *relative acceleration* d^2r/dt^2.

Newton and Coulomb's force laws depend on the distance r between the interacting particles. Weber's force is a generalization of Coulomb's law depending also on the *relative* velocity, dr/dt, and *relative* acceleration, d^2r/dt^2, between the interacting particles.[708] These are *relational magnitudes* which have the same values for all observers and for all frames of reference. These magnitudes r, dr/dt and d^2r/dt^2 have the same value even when comparing an inertial frame of reference with a non inertial frame of reference. They are intrinsic magnitudes related only to the interacting bodies.[709] Weber's force always complies with Newton's action and reaction law. Moreover, it always points along the straight line connecting the two interacting particles, no matter their motion in space. It complies with conservation of linear momentum, conservation of angular momentum and conservation of energy. These were the main reasons which made me work with Weber's electrodynamics ever since I encountered it in my undergraduate studies.[710]

24.7.6 Analysis and Synthesis in Weber's 1846 Treatise

After breaking down Ampère's force between current elements and Faraday's law of induction in order to deduce his force law between electrified particles, Weber reverts the arguments. In the final 57 pages of his 1846 treatise, Sections 22-32, he postulates his force law and deduces a series of consequences.[711] He begins Section 22 with the following words:

> Having attained the *fundamental electrical law* expressed in the previous Section, we can place it at the head of the theory of electricity, and from it synthetically derive a system of consequences, which is the ultimate purpose of such a law.
>
> [...]
>
> For moving electricity, first the uniform motion of the electricity of galvanic currents in conductors at rest is to be considered, to which Ampère's law relates. Now, since

[708] See footnote 694.

[709] [Ass13, Section 2.8] and [Ass14b, Section 2.8 (Weber's force between electrified bodies) and Appendix A (Relational magnitudes)].

[710] [Ass21i, pp. 16-17].

[711] [Web46, pp. 157-214 of Weber's *Werke*] with English translation in [Web21d, pp. 149-203].

> the above fundamental electrical law was developed analytically from Ampère's law, Ampère's law must in turn follow synthetically from this fundamental law. This derivation is actually to be given here.

He then deduces several consequences from his force law:

- The laws of Coulomb and Poisson when there is no motion between the interacting particles.
- Ampère's force between two current elements.
- The law of the electrodynamic action of a closed circuit on a current element.
- The law of the electromagnetic action of a magnet on a current element.
- Ampère arrived at his force between two current elements by considering the interaction between wires carrying steady currents. Weber now shows that Ampère's force will also be valid with variable current intensities.
- Weber also deduces mathematically Faraday's law of induction from his force between electrified particles.
- Etc.

All of this reminds us of Newton's approach to inverse problems.[712]
In the last Query of his book *Opticks* Newton expressed himself as follows:[713]

> As in mathematicks, so in natural philosophy, the investigation of difficult things by the method of analysis, ought ever to precede the method of composition. This analysis consists in making experiments and observations, and in drawing general conclusions from them by induction, and admitting of no objections against the conclusions, but such as are taken from experiments, or other certain truths. For hypotheses are not to be regarded in experimental philosophy, and although the arguing from experiments and observations by induction be no demonstration of general conclusions; yet it is the best way of arguing which the nature of things admits of, and may be looked upon as so much the stronger, by how much the induction is more general. And if no exception occur from phaenomena, the conclusion may be pronounced generally. But if at any time afterwards any exception shall occur from experiments, it may then begin to be pronounced with such exceptions as occur. By this way of analysis we may proceed from compounds to ingredients, and from motions to the forces producing them, and from particular causes to more general ones, till the argument end in the most general. This is the method of analysis: and the synthesis consists in assuming the causes discovered, and established as principles, and by them explaining the phaenomena proceeding from them, and proving the explanations.
>
> In the first two books of these Opticks, I proceeded by this analysis to discover and prove the original differences of the rays of light in respect of refrangibility, reflexibility, and colour, and their alternate fits of easy reflexion and easy transmission, and the

[712] [Ass98], and [Ass11] with Portuguese translation in [Ass17b].
[713] [New79, pp. 404-405] with Portuguese translation in [New96, pp. 292-293].

properties of bodies, both opake and pellucid, on which their reflexions and colours depend. And these discoveries being proved, may be assumed in the method of composition for explaining the phaenomena arising from them: an instance of which method I gave in the end of the first book.

Newton formalized his general approach in science at the Preface of the first edition of his book *Mathematical Principles of Natural Philosophy*:[714]

> I offer this work as the mathematical principles of philosophy, for the whole burden of philosophy seems to consist in this—from the phenomena of motions to investigate the forces of nature, and then from these forces to demonstrate the other phenomena; and to this end the general propositions in the first and second books are directed. In the third book I give an example of this in the explication of the system of the world; for by the propositions mathematically demonstrated in the former books, in the third I derive from the celestial phenomena the forces of gravity with which bodies tend to the sun and the several planets. Then from these forces, by other propositions which are also mathematical, I deduce the motions of the planets, the comets, the moon, and the sea.

In the third book of the *Principia*, Newton presented six phenomena comprising Kepler's laws. From these phenomena he deduced that the force of gravitation is inversely proportional to the square of the distances. After arriving at this result, he begins the opposite process. That is, starting from a force of gravitation varying as $1/r^2$, he deduces Kepler's laws and other results. On example:[715]

> Proposition XIII, Theorem XIII. *The planets move in ellipses which have their common focus in the centre of the sun; and, by radii drawn to that centre, they describe areas proportional to the times of description.*
>
> We have discoursed above on these motion from the phenomena. Now that we know the principles on which they depend, from those principles we deduce the motions of the heavens *a priori*.

24.7.7 Comparison Between Lorentz and Weber's Force Laws

In the previous Subsections I compared Lorentz and Weber's deductions of their force laws.

In earlier works I made a direct comparison of their final expressions. There I showed, for instance, situations in which the so-called Lorentz force does not comply with Newton's action and reaction law; the prediction of a missing torque utilizing Lorentz force which has never been observed experimentally, etc.[716]

Now anyone can compare Lorentz and Weber's deductions and final expressions of their force laws by reading their original German texts or their English translations.[717]

Everyone can then make their own decision. Which of these two forces is better? Which should be developed theoretically and explored experimentally? Which of these two theories will you dedicate your time and effort to?

---

[714] [New34, p. xvii] with Portuguese translation in [New90, pp. I-II], [New08] and [New10].

[715] [New34, p. 420] with Portuguese translation in [New08, p. 210].

[716] [Ass92], [Ass94, Chapter 6: Forces of Weber and of Lorentz], [Ass95] and [Ass15a].

[717] [Lor95] with English translation in https://en.wikisource.org/wiki/Translation:Attempt_of_a_Theory_of_Electrical_and_Optical_Phenomena_in_Moving_Bodies, [Lor09] and [Lor15]; and [Web46] with English translation in [Web21d].

Chapter 25

Weber's Electrodynamics Versus Different Field Theories

A. K. T. Assis[718]

In this Chapter I will make a comparison between Weber's electrodynamics and different field theories. Although modern textbooks on electromagnetism pretend that there is a single field theory as developed by Faraday, Maxwell and Lorentz, I will show that there are many contradictory definitions of the field concept. This means that there are in reality many different field theories.

25.1 Multiple Definitions of the Field Concept

Faraday explained his findings utilizing different abstract concepts. Some of these concepts were the magnetic curves, lines of force, the electrotonic state and the field. It is not easy to understand the definitions and relations between these concepts according to Faraday. Maxwell also utilized the concepts of lines of force, electrotonic state, field and ether. As it happened with Faraday, it is very complex to understand the definitions, meanings and relations between these concepts according to Maxwell.

In this Section I discuss some of the definitions, meanings and properties of the field concept.[719] The main difficulty is the polysemy associated with the field concept, that is, it has several meanings. These multiple meanings associated with the field concept appear not only in the original works of Faraday and Maxwell, but also in modern textbooks. Moreover, these several meanings are usually contradictory with one another, although the authors do not seem to be aware of these contradictions.

[718] Homepage: www.ifi.unicamp.br/~assis

[719] [O'R65, Vol. 2, Chapter 13, Section 4: The 'Field', pp. 645-661], [Lar82], [Gar04], [KS05], [And07], [SK07], [KS08], [Rib08], [RVA08], [ARV09], [AC11], [Ass13], [Ass14b] and [AC15].

25.1.1 The Field Is a Region of Space Around Gravitational Masses, Electrified Particles, Magnets, and Current Carrying Wires

In this Subsection one will see that Faraday, Maxwell and others originally defined the field as a region of space, equating it with space.[720] That is, they did not define it as something material, like a real entity, located in the space.

Faraday utilized the word "field" for the first time in November 7, 1845, in his Diary.[721] But much before this time he already utilized analogous expressions such as "magnetic curves," "lines of magnetic force," or "lines of force." In a paper published in 1851 he defined the *field* as a *region of space* around the bodies he was investigating:[722]

> 2806. I will now endeavour to consider what the influence is which paramagnetic and diamagnetic bodies, viewed as conductors (2797), exert upon the lines of force in a magnetic field. Any portion of space traversed by lines of magnetic power, may be taken as such a field, and there is probably no space without them.

The same concept was adopted by Maxwell, as can be observed from his definition of the electric field in his *Treatise*:[723]

> 44.] The electric field is the portion of space in the neighbourhood of electrified bodies, considered with reference to electric phenomena.

Maxwell and Jenkin presented a similar definition of the magnetic field, my emphasis:[724]

> It is clear that the presence of a magnet in some way modifies the surrounding space, since any other magnet brought into that space experiences a peculiar force. *The neighbourhood of a magnet is, for convenience, called a magnetic field*; and for the same reason the effect produced by a magnet is often spoken of as due to the magnetic field, instead of to the magnet itself.

Maxwell presented in the *Treatise* a similar definition of the magnetic field when interpreting Oersted's discovery of the deflection of a compass placed in the neighborhood of a long wire carrying a steady current, my emphasis:[725]

> 476.] It appears therefore that in the space surrounding a wire transmitting an electric current a magnet is acted on by forces depending on the position of the wire and on the strength of the current. *The space in which these forces act may therefore be considered as a magnetic field*, and we may study it in the same way as we have already studied the field in the neighbourhood of ordinary magnets, by tracing the course of the lines of magnetic force, and measuring the intensity of the force at every point.

[720] [Lar82, p. 16].
[721] [Ner89, Nota 17].
[722] [Far52b, § 2806, p. 690].
[723] [Max54, Vol. 1, § 44, p. 47].
[724] [MJ64, p. 133] and [MJ65].
[725] [Max54, Vol. 2, § 476, p. 139].

In his article of 1864-1865 containing a dynamical theory of the electromagnetic field he had already expressed similar views:[726]

> (3) The theory I propose may therefore be called a theory of the *electromagnetic field*, because it has to do with the space in the neighbourhood of the electric and magnetic bodies, and it may be called a *dynamical* theory, because it assumes that in that space there is matter in motion, by which the observed electromagnetic phenomena are produced.
>
> (4) The electromagnetic field is that part of space which contains and surrounds bodies in electric and magnetic conditions.

This definition was also followed by J. J. Thomson (1856-1940). After describing the basic triboelectric phenomena he said:[727]

> The sealing-wax and the flannel are said to be *electrified*, or to be in a state of *electrification*, or to be charged with *electricity*; and the region in which the attractions and repulsions are observed is called the *electric field*.

James H. Jeans (1877-1946) also followed the definitions of Faraday and Maxwell:[728]

> 30. The space in the neighbourhood of charges of electricity, considered with reference to the electric phenomena occurring in this space, is spoken of as the electric field.

Several other authors presented a similar definition of the field, as quoted by O'Rahilly.[729] Heilbron combined these definitions as follows:[730]

> *Field* in general signifies a region of space considered in respect to the potential behaviour of test bodies moved about in it; the electricians of 1780 lacked the word but not the concept, which they called 'sphere of influence', *sphaera activitatis*, or *Wirkungskreis*. [...]

Later on several authors presented other definitions for the field concept, some of which are presented in the sequence.

25.1.2 The Field Is a Real Physical Entity Located in Space

Many modern physicists consider the field as some real physical entity located in space, or filling the space. Albert Einstein (1879-1955), for instance, said the following:[731]

726[Max65, p. 460] and [Max65a, p. 527].
727[Tho21, p. 1].
728[Jea41, p. 24].
729[O'R65, Vol. 2, p. 651].
730[Hei81, p. 187].
731[Ein20b, p. 43, § 19] with English translation in [Ein20a, p. 74].

"If we pick up a stone and then let it go, why does it fall to the ground?" The usual answer to this question is: "Because it is attracted by the Earth." Modern physics formulates the answer rather differently for the following reason. As a result of the more careful study of electromagnetic phenomena, we have come to regard action at a distance as a process impossible without the intervention of some intermediary medium. If, for instance, a magnet attracts a piece of iron, we cannot be content to regard this as meaning that the magnet acts directly on the iron through the intermediate empty space, but we are constrained to imagine—after the manner of Faraday— that the magnet always calls into being something physically real in the space around it, that something being what we call a "magnetic field." [...] The effects of gravitation also are regarded in an analogous manner.

Feynman, Leighton and Sands expressed themselves as follows:[732]

We can write the force $\mathbf{F}$ on a charge q moving with a velocity $\mathbf{v}$ as

$$\mathbf{F} = q\left(\mathbf{E} + \mathbf{v} \times \mathbf{B}\right) \; . \qquad (1.1)$$

We call $\mathbf{E}$ the *electric field* and $\mathbf{B}$ the *magnetic field* at the location of the charge.

[...] It is precisely because $\mathbf{E}$ (or $\mathbf{B}$) can be specified at every point in space that it is called a "field." A "field" is any physical quantity which takes on different values at different points in space.

According to Griffiths:[733]

What exactly *is* an electric field? I have deliberately begun with what you might call the "minimal" interpretation of $\mathbf{E}$, as an intermediate step in the calculation of electric forces. But I encourage you to think of the field as a "real" physical entity, filling the space in the neighborhood of any electric charge.

25.1.3 Field Is a Vector Quantity (with Magnitude and Direction)

Most authors consider the field as a vector quantity, having magnitude and direction. The gravitational field is normally represented nowadays by $\vec{g}$ or $\mathbf{g}$, the electric field by $\vec{E}$ or $\mathbf{E}$, while the magnetic field is represented by $\vec{B}$ or $\mathbf{B}$.

Maxwell emphasized this point as follows in his *Treatise*:[734]

Let e be the charge of the body, and F the force acting on the body in a certain direction, then when e is very small F is proportional to e, or

$$F = Re \; ,$$

where R depends on the distribution of electricity on the other bodies in the field. If the charge e could be made equal to unity without disturbing the electrification of other bodies we should have $F = R$.

[732] [FLS64, pp. 1-2 and 1-4].
[733] [Gri89, p. 64].
[734] [Max54, Vol. 1, § 44, p. 48].

> We shall call R the resultant electromotive intensity at the given point of the field. When we wish to express the fact that this quantity is a vector we shall denote it by the German letter $\mathfrak{E}$.

Instead of Maxwell's $\mathfrak{E}$, nowadays the electric field is normally represented by the symbol $\vec{E}$. The force $\vec{F}$ acting on a particle with charge e would then be written as

$$\vec{F} = e\vec{E} \ . \tag{25.1}$$

Maxwell also considered the magnetic field as a vector. For instance, Chapter II of Volume 2 of his *Treatise on Electricity and Magnetism*, devoted to the magnetic force and magnetic induction, has the following statement:[735]

> The three vectors, the magnetization $\mathfrak{J}$, the magnetic force $\mathfrak{H}$, and the magnetic induction $\mathfrak{B}$, are connected by the vector equation
>
> $\mathfrak{B} = \mathfrak{H} + 4\pi\mathfrak{J}$. (7)

Nowadays this equation connecting the field $\vec{B}$ (called magnetic field, magnetic induction or magnetic flux density), the field $\vec{M}$ (called magnetic dipole moment per unit volume) and the auxiliary field $\vec{H}$ (called magnetic intensity by some authors, while other authors call it magnetic field), in the cgs-Gaussian system of units and in the International System of Units, is written as, respectively:[736]

$$\vec{B} = \vec{H} + 4\pi\vec{M} \ , \tag{25.2}$$

and

$$\vec{B} = \mu_o\vec{H} + \mu_o\vec{M} \ . \tag{25.3}$$

25.1.4 The Electromagnetic Field Propagates in a Material Medium According to Maxwell

Maxwell presented his electromagnetic theory of light in Chapter 20 of his book *Treatise of Electricity and Magnetism* of 1873. He defended the existence of a material medium, the ether, existing in the space between material bodies. This was an elastic medium that had a finite density of matter. According to Maxwell, light would be an electromagnetic perturbation in this medium, propagating relative to it:[737]

> 781.] In several parts of this treatise an attempt has been made to explain electromagnetic phenomena by means of mechanical action transmitted from one body to another by means of a medium occupying the space between them. The undulatory theory of light also assumes the existence of a medium. We have now to shew that the properties of the electromagnetic medium are identical with those of the luminiferous medium.
>
> [...]

[735] [Max54, Vol. 2, § 400, p. 25].
[736] [Jac75, p. 188], [Gri89, p. 258] and [HM95, p. 26].
[737] [Max54, Vol. 2, § 781, p. 431].

> But the properties of bodies are capable of quantitative measurement. We therefore obtain the numerical value of some property of the medium, such as the velocity with which a disturbance is propagated through it, which can be calculated from electromagnetic experiments, and also observed directly in the case of light. If it should be found that the velocity of propagation of electromagnetic disturbances is the same as the velocity of light, and this not only in air, but in other transparent media, we shall have strong reasons for believing that light is an electromagnetic phenomenon, and the combination of the optical with the electrical evidence will produce a conviction of the reality of the medium similar to that which we obtain, in the case of other kinds of matter, from the combined evidences of the senses.

25.1.5 The Electromagnetic Field Propagates in Empty Space According to Einstein

In his paper of 1905 introducing the special theory of relativity, Einstein made the ether superfluous and considered that light and the electromagnetic waves propagate in empty space, my emphasis:[738]

> Examples of this sort, together with the unsuccessful attempts to discover any motion of the Earth relatively to the "light medium," suggest that the phenomena of electrodynamics as well as of mechanics possess no properties corresponding to the idea of absolute rest. They suggest rather that, as has already been shown to the first order of small quantities, the same laws of electrodynamics and optics will be valid for all frames of reference for which the equations of mechanics hold good. We will raise this conjecture (the purport of which will hereafter be called the "Principle of Relativity") to the status of a postulate, and also introduce another postulate, which is only apparently irreconcilable with the former, namely, that *light is always propagated in empty space with a definite velocity* c which is independent of the state of motion of the emitting body. These two postulates suffice for the attainment of a simple and consistent theory of electrodynamics of moving bodies based on Maxwell's theory for stationary bodies. The introduction of a "luminiferous ether" will prove to be superfluous inasmuch as the view here to be developed will not require an "absolutely stationary space" provided with special properties, nor assign a velocity-vector to a point of the empty space in which electromagnetic processes take place.

Later on Einstein and Infeld expressed themselves as follows:[739]

> Our only way out seems to be to take for granted the fact that space has the physical property of transmitting electromagnetic waves, and not to bother too much about the meaning of this statement.

[738][Ein52, pp. 37-38] and [Ein78, p. 48].
[739][EI38, p. 159].

25.1.6 The Field Stores Energy, Linear Momentum and Angular Momentum

Several authors consider that there is a density of energy in the electromagnetic field.[740] One can say that the electric field stores electric energy, that is, it contains energy. Likewise, the magnetic field stores magnetic energy.

J. J. Thomson, for instance, expressed himself as follows:[741]

> If, as I do, we believe with Faraday and Clerk Maxwell that the properties of charged bodies are due to lines of force which spread out from them into the surrounding ether, we must place the energy of the electron in the space outside the little sphere which is supposed to represent the electron.

According to Einstein:[742]

> For if the magnet is in motion and the conductor at rest, there arises in the neighbourhood of the magnet an electric field with a certain definite energy, producing a current at the places where parts of the conductor are situated.

These authors also consider that there is a density of linear momentum in the electromagnetic field.[743]

J. J. Thomson expressed himself as follows:[744]

> To take an example, according to Newton's third law of motion, action and reaction are equal and opposite, so that the momentum in any direction of any self-contained system is invariable. Now, in the case of many electrical systems there are apparent violations of this principle; thus, take the case of a charged body at rest struck by an electric pulse, the charged body when exposed to the electric force in the pulse acquires velocity and momentum, so that when the pulse has passed over it, its momentum is not what it was originally. Thus, if we confine our attention to the momentum in the charged body, i. e., if we suppose that momentum is necessarily confined to what we consider ordinary matter, there has been a violation of the third law of motion, for the only momentum recognized on this restricted view has been changed. The phenomenon is, however, brought into accordance with this law if we recognize the existence of momentum in the electric field; for, on this view, before the pulse reached the charged body there was momentum in the pulse, but none in the body; after the pulse passed over the body there was some momentum in the body and a smaller amount in the pulse, the loss of momentum in the pulse being equal to the gain of momentum by the body.

[740] [O'R65, Vol. 1, Chapter 8, Section 4: Localized energy, pp. 281-290], [Gri89, Section 7.5: Energy and momentum in electrodynamics, pp. 320-333, and Subsection 8.2.2: Energy and momentum of electromagnetic waves, pp. 358-360], [HM95, Section 4.6: Energy in the electromagnetic field, pp. 143-147, and Section 14.12: Energy-momentum tensor of the electromagnetic field, pp. 522-527] and [CS02, Section 23.5: Energy and momentum in electromagnetic radiation, pp. 854-859].

[741] [Tho29, p. 12] and [O'R65, Vol. 1, p. 281].

[742] [Ein05], with English translation in [Ein52, p. 37] and Portuguese translation in [Ein78].

[743] [O'R65, Vol. 1, Chapter 8, Section 5: Electromagnetic momentum, pp. 291-304], [Gri89, Section 7.5: Energy and momentum in electrodynamics, pp. 320-333, and Subsection 8.2.2: Energy and momentum of electromagnetic waves, pp. 358-360], [HM95, Section 4.6: Energy in the electromagnetic field, pp. 143-147, and Section 14.12: Energy-momentum tensor of the electromagnetic field, pp. 522-527] and [CS02, Section 23.5: Energy and momentum in electromagnetic radiation, pp. 854-859].

[744] [Tho04, pp. 24-25] and [O'R65, Vol. 1, p. 294].

Jackson expressed himself as follows:[745]

> [...] electromagnetic fields can exist in regions of space where there are no sources. They can carry energy, momentum, and angular momentum and so have an existence totally independent of charges and currents.

Griffiths expressed himself as follows:[746]

> When a charge undergoes *acceleration*, a portion of the field "detaches" itself, in a sense, and travels off at the speed of light, carrying with it energy, momentum, and angular momentum. We call this **electromagnetic radiation**. Its existence invites (if not *compels*) us to regard the fields as independent dynamical entities in their own right, every bit as "real" as atoms or baseballs.

25.1.7 The Field Mediates the Action between Gravitational Masses, Electrified Particles, Magnets, and Current Carrying Wires

This has been the point of view expressed by Maxwell, against the theories of action at a distance.[747]

This idea has also been presented by Griffiths as follows:[748]

> The Field Formulation of Electrodynamics
>
> The fundamental problem a theory of electromagnetism hopes to solve is this: I hold up a bunch of electric charges *here* (and maybe shake them around)—what happens to some *other* charge, over *there*? The classical solution takes the form of a **field theory:** We say that the space around an electric charge is permeated by electric and magnetic **fields** (the electromagnetic "odor," as it were, of the charge). A second charge, in the presence of these fields, experiences a force; the fields, then, transmit the influence from one charge to the other—they "mediate" the interaction.

25.1.8 The Field Is a Magnitude with Dimensions

In the International System of Units, for instance, the unit of the gravitational field $\vec{g}$ is that of acceleration, m/s^2. The unit of the electric field $\vec{E}$ is that of $V/m = kgmC^{-1}s^{-2} = kgmA^{-1}s^{-3}$. The unit of the magnetic field $\vec{B}$ is that of $T = Wm^{-2} = kgC^{-1}s^{-1} = kgA^{-1}s^{-2}$.

25.1.9 The Field as the Lines of Force Taken Together

Einstein and Infeld presented a definition of the field as the lines of force taken together:[749]

[745] [p. 3] jackson75.

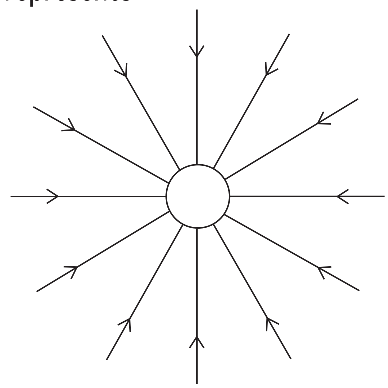
[746] [Gri89, p. 4].

[747] [Max54, Vol. 1, Preface to the first edition, pp. v-xii, Vol. 2, §§ 641-646, pp. 278-283 and Chapter 23, §§ 846-866, pp. 480-493].

[748] [Gri89, p. 4].

[749] [EI38, pp. 129-131].

[...] We know that two particles attract each other and that this force of attraction decreases with the square of the distance. We can represent this fact in a new way, and shall do so even though it is difficult to understand the advantage of this. The small circle in our drawing represents



an attracting body, say, the Sun. Actually, our diagram should be imagined as a model in space and not as a drawing on a plane. Our small circle, then, stands for a sphere in space, say, the Sun. A body, the so-called *test body*, brought somewhere within the vicinity of the Sun will be attracted along the line connecting the centres of the two bodies. Thus the lines in our drawing indicate the direction of the attracting force of the Sun for different positions of the test body. The arrow on each line shows that the force is directed toward the Sun; this means the force is an attraction. These are the *lines of force of the gravitational field*. For the moment, this is merely a name and there is no reason for stressing it further. There is one characteristic feature of our drawing which will be emphasized later. The lines of force are constructed in space, where no matter is present. For the moment, all the lines of force, or briefly speaking, the *field*, indicate only how a test body would behave if brought into the vicinity of the sphere for which the field is constructed.

Later on in the same book:[750]

In this way, the lines of force, or in other words, the field, enable us to determine the forces acting on a magnetic pole at any point in space.

25.1.10 The Field as a State of the Space

According to Einstein, the field might be considered a particular state of the space:[751]

If we are here going to talk about the ether, we are not, of course, talking about the physical or material ether of the mechanical theory of undulations, which is subject to the laws of newtonian mechanics, to the points of which are attributable a certain velocity. This theoretical edifice has, I am convinced, finally played out its role since the setting up of the special theory of relativity. It is rather more generally a question of those kinds of things that are considered as physically real, which play a role in the causal nexus of physics, apart from the ponderable matter that consists of electrical

[750] [EI38, p. 135].
[751] [Ein24, p. 85] with English translation in [Ein91, p. 13].

> elementary particles. Therefore, instead of speaking of an ether, one could equally well speak of physical qualities of space. Now one could take the position that all physical objects fall under this category, because in the final analysis in a theory of fields the ponderable matter, or the elementary particles that constitute this matter, also have to be considered as 'fields' of a particular kind, or as particular 'states' of the space.

25.1.11 The Field Is Generated or Produced by Source Bodies Like Gravitational Masses, Electrified Particles, Magnets and Electric Currents

Some scientists assume that "field" is a magnitude which is generated, created or produced in space by certain bodies. The bodies producing the fields are called "source bodies." Gravitational source masses, for instance, are supposed to generate or produce gravitational fields. Electrified bodies or electric charges are supposed to generate electric fields. Moving electrified particles, magnets or current carrying wires are supposed to generate magnetic fields.

I present here some quotations showing this point of view.

Landau and Lifshitz expressed themselves as follows:[752]

> The interaction of particles can be described with the help of the concept of a *field* of force. Namely, instead of saying that one particle acts on another, we may say that the particle creates a field around itself; a certain force then acts on every other particle located in this field.

Feynman, Leighton and Sands expressed this idea as follows:[753]

> We then have two rules: (a) charges make a field, and (b) charges in fields have forces on them and move.

25.1.12 The Field Due to Source Bodies Generates or Produces a Force on Other Test Bodies like Gravitational Masses, Electrified Particles, Magnets and Electric Currents

According to most physicists, the fields due to source bodies can affect other bodies called "test bodies." Gravitational test masses, for instance, are affected by gravitational fields due to other source masses. This field generates or produces a force on these test masses, accelerating them relative to an inertial frame if they are free to move. Likewise, test electrified particles are affected by electric fields due to other source electrified particles. Moving charges, magnets or current carrying wires are affected by magnetic fields due to other sources, being accelerated by these fields if these test bodies are free to move relative to an inertial frame of reference.

This idea has been expressed clearly by Feynman, Leighton and Sands:[754]

[752] [LL75, p. 46].
[753] [FLS63, p. 2-5].
[754] [FLS63, p. 2-5].

More was discovered about the electrical force. The natural interpretation of electrical interaction is that two objects simply attract each other: plus against minus. However, this was discovered to be an inadequate idea to represent it. A more adequate representation of the situation is to say that the existence of the positive charge, in some sense, distorts, or creates a "condition" in space, so that when we put the negative charge in, it feels a force. This potentiality for producing a force is called an *electric field*. When we put an electron in an electric field, we say it is "pulled."

25.1.13 Condensations of the Electromagnetic Field Are the Elementary Particles of Matter

Einstein presented this view in an address delivered on May 5th, 1920, in the University of Leyden, as follows:[755]

> Since according to our present conceptions the elementary particles of matter are also, in their essence, nothing else than condensations of the electromagnetic field, our present view of the universe presents two realities which are completely separated from each other conceptually, although connected causally, namely, gravitational ether and electromagnetic field, or—as they might also be called—space and matter.

He expressed the same point of view in other publications:[756]

> In the present situation we are de facto forced to make a distinction between matter and fields, while we hope that later generations will be able to overcome this dualistic concept, and replace it with a unitary one, such as the field theory of today has sought in vain.

Likewise, in his joint book with Infeld, *The Evolution of Physics*, there are similar statements:[757]

> Field and Matter
>
> [...]
>
> We have two realities: *matter and field*. There is no doubt that we cannot at present imagine the whole of physics built upon the concept of matter as the physicists of the early nineteenth century did. For the moment we accept both the concepts. Can we think of matter and field as two distinct and different realities? Given a small particle of matter, we could picture in a naive way that there is a definite surface of the particle where it ceases to exist and its gravitational field appears. In our picture, the region in which the laws of field are valid is abruptly separated from the region in which matter is present. But what are the physical criterions distinguishing matter and field? Before we learned about the relativity theory we could have tried to answer this question in the following way: matter has mass, whereas field has

[755] [Ein22, p. 22] and [O'R65, Vol. 2, p. 655]. See also [Ein24, p. 85] with English translation in [Ein91] and [EI38, pp. 255-258].
[756] [Ein24, p. 85] with English translation in [Ein91, p. 13].
[757] [EI38, pp. 255-258].

not. Field represents energy, matter represents mass. But we already know that such an answer is insufficient in view of the further knowledge gained. From the relativity theory we know that matter represents vast stores of energy and that energy represents matter. We cannot, in this way, distinguish qualitatively between matter and field, since the distinction between mass and energy is not a qualitative one. By far the greatest part of energy is concentrated in matter; but the field surrounding the particle also represents energy, though in an incomparably smaller quantity. We could therefore say: Matter is where the concentration of energy is great, field where the concentration of energy is small. But if this is the case, then the difference between matter and field is a quantitative rather than a qualitative one. There is no sense in regarding matter and field as two qualities quite different from each other. We cannot imagine a definite surface separating distinctly field and matter.

The same difficulty arises for the charge and its field. It seems impossible to give an obvious qualitative criterion for distinguishing between matter and field or charge and field.

[...]

We cannot build physics on the basis of the matter-concept alone. But the division into matter and field is, after the recognition of the equivalence of mass and energy, something artificial and not clearly defined. Could we not reject the concept of matter and build a pure field physics? What impresses our senses as matter is really a great concentration of energy into a comparatively small space. We could regard matter as the regions in space where the field is extremely strong. In this way a new philosophical background could be created. Its final aim would be the explanation of all events in nature by structure laws valid always and everywhere. A thrown stone is, from this point of view, a changing field, where the states of greatest field intensity travel through space with the velocity of the stone. There would be no place, in our new physics, for both field and matter, field being the only reality. This new view is suggested by the great achievements of field physics, by our success in expressing the laws of electricity, magnetism, gravitation in the form of structure laws, and finally by the equivalence of mass and energy. Our ultimate problem would be to modify our field laws in such a way that they would not break down for regions in which the energy is enormously concentrated.

But we have not so far succeeded in fulfilling this programme convincingly and consistently. The decision, as to whether it is possible to carry it out, belongs to the future. At present we must still assume in all our actual theoretical constructions two realities: field and matter.

25.1.14 Etc.

There are many other meanings associated with the field concept. But I stop here as the previous meanings are the most common and frequent ones in the literature.

25.2 These Different Field Definitions Contradict One Another

Several definitions and properties of the fields presented in Section 25.1 contradict one another. They are also against the basic definitions of Faraday and Maxwell. This Section presents some of these contradictions and the many problems introduced in physics after the advent of the field concept.

25.2.1 A Real Physical Entity Filling the Space Cannot be Identified with Space Itself

Faraday, Maxwell and the creators of the field concept defined it as a region of space in the neighborhood of source bodies like a gravitational mass, an electrified body, a magnet or a current carrying wire. That is, field was equated with space. Einstein and many modern scientists, on the other hand, maintained that the field is a real physical entity filling the space. That is, Einstein considered the field as something real in the space. This is obviously a contradiction. The basic concepts in physics are those of matter and empty space. Matter is something that has physical properties (it can be hard or soft, it can be hot or cold, it can be solid or liquid, it interacts with other matter, etc.) Space, on the other hand, has none of these properties. A body can occupy a region of space. It can also move from one region of space to another. But matter is not identical to space and should not be equated with it.

Therefore, if field is a region of space, as defined by Faraday and Maxwell, it cannot be a real physical entity. If, on the other hand, field is a real physical entity filling the space, then it cannot be identified with space itself. These two concepts exclude one another.

25.2.2 How Is it Possible for a Region of Space to Have Magnitude and Direction?

The usual conception of space is that it is a vacuum or empty region between material bodies. Faraday and Maxwell defined field as a region of space in the neighborhood of electrified bodies, magnets and current carrying wires. Maxwell, Einstein and most scientists argued that field is a vector quantity, with magnitude and direction. How is it possible for a region of empty space to have magnitude and direction?

25.2.3 How Is it Possible for a Region of Space to Propagate in Space?

The basic definition of field according to Faraday, Maxwell and many other scientists, is that it in general signifies a region of space around source bodies. An electric field, for instance, is a region of space around electrified particles. A magnetic field, on the other hand, is a region of space around magnets or around current carrying wires. An electromagnetic wave in Maxwell's theory is composed of electric and magnetic fields oscillating orthogonally to one another. Therefore, there should exist two regions of space orthogonal to one another, which is a very strange concept. This has never been explained by Maxwell nor by other authors.

In any event, according to Maxwell's original definition of a field as a region of space, an electromagnetic wave composed of electric and magnetic fields would also be a region of space. But this concept also creates other problems. According to Einstein, electromagnetic waves propagate in empty space. How is it possible for a region of space to propagate in space? This has never been explained by Einstein nor by any other person.

25.2.4 How Is it Possible for a Region of Space, Something Immaterial, to Interact with a Material Body?

Faraday and Maxwell defined field as a region of space around source bodies. Textbooks normally argue that a test body like a gravitational mass suffers a force when it is in the presence of a gravitational field. For instance, this test mass could be accelerated relative to the ground due to this gravitational field. An electrified test body, likewise, would feel the presence of an electric field, while a magnet and a current carrying wire would feel the presence of a magnetic field.

How is it possible for something immaterial, like a region of space, to act on a material body?

25.2.5 How Is it Possible for a Region of Space to Have Dimensions Different from Length, Area or Volume?

If a field is a region of space, as defined by Faraday, Maxwell and many other scientists, it should have dimension of "space." That is, the unit of measure of a field should coincide with that of length, area, or volume. In the International System of Units (SI or MKSA) the unit of length is the meter, represented by the symbol m, the unit of area is the square meter, represented by m^2, while the unit of volume is the cubic meter, represented by m^3.

But this is not what happens with the ordinary fields. The gravitational field $\vec{g}$ has the unit of acceleration, m/s^2, the electric field $\vec{E}$ has the unit $V/m = kgmC^{-1}s^{-2} = kgmA^{-1}s^{-3}$, while the magnetic field $\vec{B}$ has the unit Tesla, $T = Wm^{-2} = kgC^{-1}s^{-1} = kgA^{-1}s^{-2}$. All these units are different from the units of length, area and volume. Therefore, it is not possible to identify any of these "fields" with "space."

25.2.6 A Field Which Is Not a Region of Space Does Not Comply with Faraday and Maxwell's Theories

As shown before, several problems with the field concept are related with its definition as "a region of space." This has been the basic definition of field according to Faraday and Maxwell. If we drop this definition in order to avoid the previous problems, than we should no longer call it the "electromagnetic theory of Faraday and Maxwell," or the "Faraday-Maxwell field theory." After all, this will be a new model not compatible with the definitions, reasonings and concepts presented by Faraday and Maxwell. People advocating this new approach would not be following the ideas of Faraday and Maxwell. Therefore, these people should begin with a completely new conceptual framework. In particular, to avoid confusion, they should create another concept and give it a name different from "field". Moreover, these people should no longer associate the names of Faraday and Maxwell with their new theories.

In any event, even dropping the definition of field as a region of space, there are still many other problems and contradictions related with the other meanings associated with the field concept presented in Section 25.1. Some of these problems are discussed in the sequel.

25.2.7 Maxwell Argued that an Electromagnetic Wave Propagates in a Material Medium Filling All Space, the Ether. Einstein, On the Other Hand, Argued that an Electromagnetic Wave Propagates in Empty Space

A specific region of space can be empty of matter, like a vacuum. Or it can be filled with matter. In this second case we can have a region of space occupied by a material body, like a rock, for instance. We can also have a region of space occupied by a material medium, like water, air or another fluid. An empty region of space (a vacuum) and a region of space filled with matter are two completely different conceptions.

Faraday and Maxwell believed strongly in a material medium filling all space. Maxwell called it an ether.[758] As he said in the Preface of his *Treatise on Electricity and Magnetism*:[759]

> For instance, Faraday, in his mind's eye, saw lines of force traversing all space where the mathematicians saw centres of force attracting at a distance: Faraday saw a medium where they saw nothing but distance: Faraday sought the seat of the phenomena in real actions going on in the medium, they were satisfied that they had found it in a power of action at a distance impressed on the electric fluids.

In Maxwell's electromagnetic theory of light, the electromagnetic wave propagates in this material medium, the ether.[760] The relevant quotations were presented in Subsection 25.1.4.

He finished his book with the following statement:[761]

> [...] Hence all these theories lead to the conception of a medium in which the propagation takes place, and if we admit this medium as an hypothesis, I think it ought to occupy a prominent place in our investigations, and that we ought to endeavour to construct a mental representation of all the details of its action, and this has been my constant aim in this treatise.

Einstein, on the other hand, made the ether superfluous in his special theory of relativity and argued that the electromagnetic wave propagates in empty space:[762] The relevant quotations were presented in Subsection 25.1.5.

These are two completely opposite points of view. Therefore, it is wrong to say that Einstein's theories of relativity are compatible with Maxwell's electrodynamics. These are completely different conceptual frameworks. It does not make sense to keep Maxwell's equations, removing the material substance giving support to it, while at the same time arguing that the new theory agrees with Maxwell's points of view. To state the opposite, as has been done by Einstein, is to confuse everybody.

[758] [Max54, Vol. 1, Preface to the first edition, pp. v-xii, Vol. 2, §§ 641-646, pp. 278-283 and Chapter 23, §§ 846-866, pp. 480-493].
[759] [Max54, Vol. 1, p. ix].
[760] [Max54, Vol. 2, § 781, p. 431].
[761] [Max54, Vol. 2, § 866, p. 493].
[762] [Ein52, pp. 37-38] and [Ein78, p. 48].

Many textbooks quote approvingly the following statement by the German physicist Heinrich Hertz (1857-1894):[763]

> To the question, "What is Maxwell's theory?" I know of no shorter or more definite answer than the following: — Maxwell's theory is Maxwell's system of equations.

However, there is one person who certainly would reject it straight away, namely, Maxwell himself. He began and finished his most important book defending the idea of a material ether. His system of equations without the underlying material medium would make no sense to him.

For instance, whenever Maxwell dealt with the concept which he called "electric displacement", it was the displacement of a material medium. This material medium might be a dielectric or the ether:[764]

> The electric polarization of an elementary portion of a dielectric is a forced state into which the medium is thrown by the action of electromotive force, and which disappears when that force is removed. We may conceive it to consist in what we may call an electric displacement, produced by the electromotive intensity. [...]
>
> In this treatise, static electric induction is measured by what we have called the electric displacement, a directed quantity or vector which we have denoted by $\mathfrak{D}$, and its components by f, g, h. [...]
>
> The true current $\mathfrak{C}$ is made up of the conduction current $\mathfrak{K}$ and the variation of the electric displacement $\mathfrak{D}$, and since both of these depend on the electromotive intensity $\mathfrak{E}$, we find, as in Art. 611,
>
> $$\mathfrak{C} = (\mathsf{C} + \tfrac{1}{4\pi}\mathsf{K}\tfrac{d}{dt})\mathfrak{E} \ .$$

That is, according to Maxwell, there would be no electric displacement, no displacement current and no electromagnetic wave if there were no material medium to be displaced.

25.2.8 What is the Relation between the Electric and Magnetic Fields with the Ether?

Maxwell talked a lot about the electric and magnetic fields, usually emphasizing that they were regions of space around electrified bodies, around magnets and around current carrying wires. He also talked a lot about the ether, assuming that it was a material medium filling space. However, he never clearly distinguished between these concepts. Certainly they were not synonymous. However, what is the relation between an electromagnetic field and the ether?

The same can be said of Lorentz. He also talked a lot about electric and magnetic fields. He also was convinced of the existence of a material medium filling space, the ether. He assumed that this ether was at rest in relation to the frame of fixed stars. But he never discussed the relations between the field and ether concepts.

It is then difficult to understand their theories because we don't know the relations of these concepts which were essential for these two scientists.

[763] [Her00, p. 21].

[764] [Max54, § 60, p. 65; § 608, p. 252].

25.2.9 The Dimensions of $\vec{g}$, $\vec{E}$ and $\vec{B}$ are Different from One Another. This Means that They Are Not Magnitudes of the Same Kind. Therefore, They Could Not Receive the Same Denomination as that of "Field." They Are Magnitudes of Different Nature and Must be Classified in Different Categories

The Greek mathematician Euclid who flourished around 300 BC presented the classical theory of proportions in Books V and VI of his work *The Elements*:[765]

> Two magnitudes A and B are said to have a ratio to one another which are capable, when multiplied, of exceeding one another.

That is, two magnitudes A and B are of the same type if there are two natural numbers m and n such that $mA > B$ and $nB > A$. Examples of two magnitudes of the same type: two lengths, two areas, two volumes, two angles, two times intervals, two weights etc.

On the other hand, a length $d = 5\ m$ and a weight $W = 3\ N$ are magnitudes of different kinds. We cannot say which one is larger than the other. Likewise, there are no multiples of d which can be larger than W, nor multiples of W which can be larger than d. For this reason they received different names, length and weight, and also different units, meters (m) and Newtons (N).

In physics there are several kinds of energy. We have, for instance, kinetic energy, gravitational potential energy, elastic potential energy, electric potential energy, magnetic energy, nuclear energy, thermal energy, chemical energy etc. All of these magnitudes have the same unit, Joule, represented by J. These different kinds of energy can be compared with one another. We can, for instance, specify how many times a certain kinetic energy is greater or smaller than another specific electric energy. They can also be converted or modified into one another. For instance, when we release a rock at rest relative to the Earth, it falls to the ground. We say that its initial gravitational potential energy is being transformed into kinetic energy as the rock falls to the ground.

We also have several kinds of force: gravitational force, electric force, magnetic force, elastic force, nuclear force, frictional force, etc. All of these magnitudes have the same unit, Newton, represented by N. These forces, by acting simultaneously in the same body, can combine their effects. An example is the law of the parallelogram of forces which Newton presented in the first Corollary after his three laws of motion in the beginning of his book *Principia*. When a block of matter remains at rest in the ground, we say that the its downward weight is balanced by the upward normal force exerted by the floor. A force can be added or subtracted from another force, it is possible to say how many times a certain force is greater than another force etc.

The same behavior does not happen with $\vec{g}$, $\vec{E}$ and $\vec{B}$. In the International System of Units, for instance, the unit of measure of the gravitational field $\vec{g}$ is that of acceleration, m/s^2. The unit of the electric field $\vec{E}$ is that of $V/m = kgmC^{-1}s^{-2} = kgmA^{-1}s^{-3}$. The unit of the magnetic field $\vec{B}$ is that of Tesla, namely, $T = Wm^{-2} = kgC^{-1}s^{-1} = kgA^{-1}s^{-2}$. That is, these three magnitudes have different units of measure.

The magnitude $\vec{g}$ can be called a gravitational acceleration not only because it has the unit of acceleration, but also because it represents, for instance, the free fall acceleration of

[765] [Euc56, Book V, Definition 4, p. 114].

a test body released at rest relative to the ground. The magnitude $\vec{E}$, on the other hand, cannot be called an electrical acceleration. A particle of mass M and charge Q released at rest inside a capacitor in which there is an electric field $\vec{E}$ is accelerated relative to an inertial frame with an acceleration $\vec{a}$ given by $\vec{a} = Q\vec{E}/M$, and not by $\vec{a} = \vec{E}$. Likewise, the magnitude $\vec{B}$ cannot be called a magnetic acceleration.

As $\vec{g}$, $\vec{E}$ and $\vec{B}$ have different units, they represent different kinds of magnitude. Therefore, it does not make sense to classify them into the same category, namely, that of "field." They should receive different names. The magnitude $\vec{g}$ should receive the name acceleration or xxx. The magnitude $\vec{E}$ should receive the name yyy, but could not receive the name acceleration nor xxx. The magnitude $\vec{B}$ should receive the name zzz, but could not receive the names xxx nor yyy. We then would have a *gravitational acceleration* or a *gravitational xxx*, an *electric yyy* and a *magnetic zzz*.

By calling $\vec{g}$, $\vec{E}$ and $\vec{B}$ by the same generic name, *field*, creates only confusions and misunderstandings.

25.2.10 How Is it Possible to Have Action and Reaction Between a Field and a Material Body?

Usually Newton's third law of motion, the principle of action and reaction, is applied for the interaction between two material bodies. The force that a body A exerts on another body B is equal and opposite to the force exerted by B on A. This force can have several origins, namely, gravitational, elastic, electric, magnetic, etc. If bodies A and B are initially at rest relative to an inertial frame of reference, being free to move, their mutual interaction will cause them to be accelerated in this frame, moving in opposite directions.

Nowadays, on the other hand, this fact is described utilizing the field concept. Physicists then argue that the field produced by body A propagates in space, normally at light velocity. When this field reaches body B at a later time, the field interacts with B. How should we understand action and reaction in this field formulation? Body B, for instance, is accelerated relative to an inertial frame of reference by this interaction. What happens during this interaction with the field generated by A? Is there a force acting on this field and being generated by body B? Is this field accelerated relative to an inertial frame of reference? Does it accelerate as a whole (that is, the whole region occupied by this field) relative to an inertial frame of reference, or only the portion around body B is accelerated relative to an inertial frame of reference?

25.2.11 The Condensation of a Field Cannot Be an Elementary Particle of Matter

Einstein said that condensations of the electromagnetic field are elementary particles of matter.[766]

As O'Rahilly put it:[767]

> The field started as the humble offspring, the shadowy penumbra surrounding a charge; it ends by destroying not only electricity but matter!

[766] See Subsection 25.1.13.

[767] [O'R65, Vol. 2, p. 655].

This point of view presented by Einstein has been followed by many modern scientists.[768]

However, this point of view does not make sense. What does it mean a condensation of the electromagnetic field? Is it the electric field per unit volume, $d\vec{E}/dV$ or $d|\vec{E}|/dV$? Is it the magnetic field per unit volume, $d\vec{B}/dV$ or $|d\vec{B}|/dV$? And what did Einstein mean by an elementary particle of matter? Was he meaning its inertial mass m_i? Or its gravitational mass m_g? Or was he meaning its electric charge q? None of these choices make sense, after all these magnitudes ($d|\vec{E}|/dV$, $d|\vec{B}|/dV$, m and q) have different dimensions. They cannot be identified with one another. Therefore, it is incorrect to say that the elementary particles of matter are in their essence nothing else than condensations of the electromagnetic field.

25.2.12 Etc.

I could present many more examples showing the mutual contradictions between the several meanings associated with the field concept. But these are enough in order to indicate the logical problems associated with the present day field theories.

25.3 The Forces of Newton, Coulomb and Ampère

In 1687 Isaac Newton (1642-1727) proposed in the book *Principia* his law of universal gravitation.[769] According to Newton, the force of attraction between two particles is proportional to the product of their masses m and m', varies inversely as the inverse square of their distance r, acts along the straight line connecting the particles and follows the principle of action and reaction:

$$\frac{mm'}{r^2} \ . \tag{25.4}$$

In 1785 Charles Augustin de Coulomb (1736-1806) obtained an analogous expression describing the interaction between two electrified particles at rest relative to one another.[770] In this case one assumes the existence of two kinds of particles, namely, those electrified positively and negatively. Particles electrified with charges of the same sign repel one another, while particles electrified with charges of opposite sign attract one another. The force between two particles is proportional to the product of their electric charges e and e', varies inversely as the inverse square of their distance r, acts along the straight line connecting the particles and follows the principle of action and reaction:

$$\frac{ee'}{r^2} \ . \tag{25.5}$$

By working with long and thin artificial magnets with well-defined poles, Coulomb also obtained a similar expression describing the force between magnetic poles. In this case one assumes the existence of two magnetic fluids, namely, austral and boreal. They are also called North magnetic fluid and South magnetic fluid, respectively. The poles (or centers of action) of his uniformly magnetized bars were concentrated very close to their extremities. Poles of the same type repel one another, while poles of opposite type attract one another. Coulomb's

[768] [O'R65, Vol. 2, Chapter 13, Section 4: The 'Field', pp. 645-661] and [Hob13].
[769] [New34], [New52] and [New99]. Portuguese translation in [New90], [New08] and [New10].
[770] See footnote 288 on page 172.

force between two magnetic poles is proportional to the product of the pole strengths p and p', varies as the inverse square of their distance r, acts along the straight line connecting them and follows the principle of action and reaction:[771]

$$\frac{pp'}{r^2} \, . \tag{25.6}$$

I now discuss the work of André-Marie Ampère (1775-1836). His main book on electrodynamics was published in 1826, being fully available in French, Portuguese and English.[772]

Ampère's unification of the physics of his time has been discussed in detail in Section 22.2 (Ampère's Unification) of our book "Ampère's Electrodynamics — Analysis of the Meaning and Evolution of Ampère's Force between Current Elements, together with a Complete Translation of His Masterpiece: Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience",[773] and in Section 17.1 of our book "Wilhelm Weber's Main Works on Electrodynamics Translated into English", Volume 4: "Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation".[774]

Between 1820 and 1827 Ampère was able to unify three branches of physics, namely, magnetostatics, electrodynamics and electromagnetism. He was able to explain all of these electrodynamic phenomena utilizing his force between current elements. His force is proportional to the product of the current intensities i and i' of the two elements. It is also proportional to the product of the infinitesimal lengths ds and ds' of the elements. Moreover, it varies inversely as the square of their distance r. His force always complies with Newton's action and reaction law and points along the straight line connecting the elements, no matter the directions of the two interacting current elements. In these aspects his force is also similar to Newton's law of gravitation, just like Coulomb's laws for electrostatics and magnetostatics. However, Ampère's force depends also on the angle ε between the directions of the two current elements and on the angles θ and θ' between each current element and the straight line connecting their centers:

$$\frac{ii'dsds'}{r^2}\left(\cos\varepsilon - \frac{3}{2}\cos\theta\cos\theta'\right) = \frac{ii'dsds'}{r^2}\left(\sin\theta\sin\theta'\cos\omega - \frac{1}{2}\cos\theta\cos\theta'\right) \, . \tag{25.7}$$

Here ω is the angle between the planes drawn through each current element and the straight line joining them.

We discussed the meaning of these angles and presented several figures representing them in Section 2.8 (The Angles Appearing in Ampère's Force) of our book on Ampère's Electrodynamics. The angle ε between two elements of lengths ds and ds', with these elements oriented along the directions of the currents flowing in them, is represented in Figure 25.1 (a), with this angle taking its value between the following limits: 0 rad $\leq \varepsilon \leq$ π rad. This angle is not oriented, being always positive.

Ampère's three angles θ, θ' and ω are represented in Figure 25.1 (b). As happened with the angle ε between the two current elements, the following relations are valid for these three angles: 0 rad $\leq \theta \leq \pi$ rad, 0 rad $\leq \theta' \leq \pi$ rad and 0 rad $\leq \omega \leq \pi$ rad. These angles are also not oriented.

[771]See footnote 288 on page 172.

[772]See footnote 15 on page 18.

[773][AC11] with English translation in [AC15].

[774][Ass21m, Section 17.1: Ampère's Unification of Magnetism, Electrodynamics and Electromagnetism].

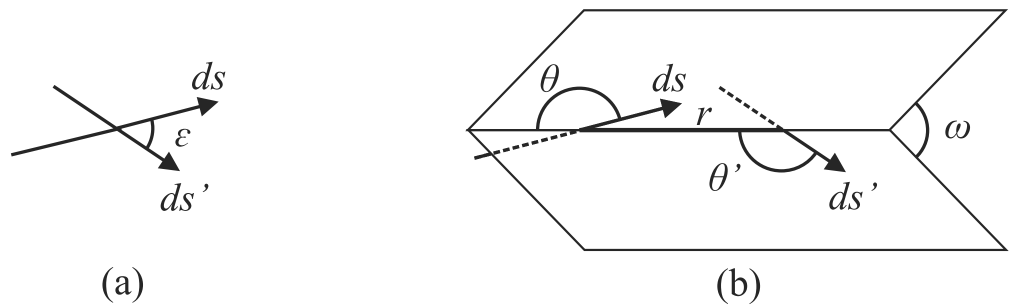


Figure 25.1: (a) Angle ε between two oriented current elements ds and ds'. (b) Ampère's three angles θ, θ' and ω.

By integrating this expression he showed that a closed circuit of arbitrary form exerts a force on a current element of another circuit which is always orthogonal to this element. Moreover, he showed that this force is also orthogonal to a certain straight line passing through the midpoint of this current element, being always contained in a plane perpendicular to this straight line. This plane was called the directing plane (*plan directeur* in French) and the straight line orthogonal to this plane was called directrix (*directrice* in French) or normal to the directing plane (*normale au plan directeur* in French).

Ampère's force $d^2\vec{F}_{21}$ exerted by a current element $i_2 d\vec{\ell}_2$ on another current element $i_1 d\vec{\ell}_1$ is expressed as follows in the International System of Units and in vector notation:[775]

$$d^2\vec{F}_{21} = -\frac{\mu_o}{4\pi} i_1 i_2 \frac{\hat{r}}{r^2} \left[2(d\vec{\ell}_1 \cdot d\vec{\ell}_2) - 3(\hat{r} \cdot d\vec{\ell}_1)(\hat{r} \cdot d\vec{\ell}_2) \right] = -d^2\vec{F}_{12} \ , \tag{25.8}$$

where $d^2\vec{F}_{12}$ is the force exerted by 1 on 2. Here $\mu_o \equiv 4\pi \times 10^{-7}\ kgm/(A^2 s^2)$ is a constant called vacuum permeability, or permeability of free space, r is the distance between the two current elements, while $\hat{r}$ is the unit vector pointing from the center of element 2 to the center of element 1.

By integrating this force over a closed circuit C_2 of arbitrary shape, Ampère obtained the force $d\vec{F}_{21}$ exerted by this circuit on a current element $i_1 d\vec{\ell}_1$ which did not belong to this closed circuit. Nowadays Ampère's result can be expressed mathematically in vector notation and in the International System of Units as follows:

$$d\vec{F}_{21} = \frac{\mu_o}{4\pi} i_1 i_2 \oint_{C_2} \frac{\hat{r}}{r^2} \left[3(\hat{r} \cdot d\vec{\ell}_1)(\hat{r} \cdot d\vec{\ell}_2) - 2(d\vec{\ell}_1 \cdot d\vec{\ell}_2) \right] = i_1 d\vec{\ell}_1 \times \oint_{C_2} \frac{\mu_o}{4\pi} \frac{i_2 d\vec{\ell}_2 \times \hat{r}}{r^2} \ . \tag{25.9}$$

That is, this force is always orthogonal to the current element $i_1 d\vec{\ell}_1$, no matter the shape of the closed circuit 2.

Integrating this force over a closed circuit C_1, we obtain the force $\vec{F}_{21}$ exerted by the closed circuit 2 on the closed circuit 1 as given by:

$$\vec{F}_{21} = \frac{\mu_o}{4\pi} i_1 i_2 \oint_{C_1} \oint_{C_2} \frac{\hat{r}}{r^2} \left[3(\hat{r} \cdot d\vec{\ell}_1)(\hat{r} \cdot d\vec{\ell}_2) - 2(d\vec{\ell}_1 \cdot d\vec{\ell}_2) \right]$$

[775][Ass92, Chapter 3], [Ass94, Chapter 4], [BA01, Chapter 5], [AH07, Chapter 1], [AH09, Chapter 1], [AH13, Chapter 1], [BA15, Chapter 5] and [Ass15a, Chapter 3].

$$= \frac{\mu_o}{4\pi} i_1 i_2 \oint_{C_1} \oint_{C_2} \frac{d\vec{\ell}_1 \times (d\vec{\ell}_2 \times \hat{r})}{r^2} = -\frac{\mu_o}{4\pi} i_1 i_2 \oint_{C_1} \oint_{C_2} \frac{(d\vec{\ell}_1 \cdot d\vec{\ell}_2)\hat{r}}{r^2} = -\vec{F}_{12} \ . \tag{25.10}$$

Here $\vec{F}_{12}$ is the force exerted by the closed circuit 1 on the closed circuit 2.

Ampère's force between current elements has been completely forgotten during the whole of the XXth century.

25.4 The So-called "Ampère's Circuital Law" is a Misnomer and should Not be Attributed to Ampère

Textbooks attribute to Ampère the so-called "Ampère's circuital law", that is, the line integral of a magnetic field around a closed loop is proportional to the electric current passing through the loop:

$$\oint \vec{B} \cdot d\vec{\ell} = \mu_0 I \ . \tag{25.11}$$

However:

- Ampère never derived such a law, he did not write it down in any format. He was not the author of "Ampère's circuital law".
- As a matter of fact, he never worked with the magnetic field concept.
- *Moreover, he did fight explicitly against anything circulating around a current carrying wire*!

A detailed discussion of this topic can be found in Chapter 16 (Ampère Against His Main Opponents) of our book on Ampère's Electrodynamics.[776]

The first person to write the circuital law, even without the displacement current, seems to have been Maxwell in his first paper dealing with electromagnetism of 1855, twenty years after Ampère's death.[777]

The so-called "Ampère's circuital law" is a misnomer and should not be attributed to Ampère.[778]

25.5 Ampère's Directrix Versus the Magnetic Field

In this Section I present a topic discussed at length in our book "Ampère's Electrodynamics — Analysis of the Meaning and Evolution of Ampère's Force between Current Elements, together with a Complete Translation of His Masterpiece: Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience".[779] Detailed references can be found in this book.

[776] [AC11] with English translation in [AC15].

[777] [Max58, p. 66 of Maxwell's original paper and p. 206 of Niven's book], [Whi73, pp. 242-245], [Ass94, Section 2.5: Maxwell's Equations] and [Erl99].

[778] [Erl99].

[779] See footnote 773.

25.5.1 Ampère's Directrix

In 1822 Ampère had obtained the final expression of his force between two current elements. In 1822, 1823 and 1824 he integrated this expression and calculated the force exerted by a current forming a closed circuit and acting on a current element $i'ds'$ not belonging to this closed circuit. He also calculated the force on this current element exerted by a current carrying circuit extended indefinitely in both directions to an infinite distance. Figure 25.2 (a) presents a closed circuit C of arbitrary shape carrying a current i interacting with an external current element $i'ds'$ located at an arbitrary point in space. Figure 25.2 (b) presents a circuit C of arbitrary shape, carrying a current i, extending indefinitely in both directions. In this last configuration the extremities of the circuit are located at infinite distances from $i'ds'$. This is the case, for instance, of a straight and filiform circuit of infinite length acting on $i'ds'$. By integrating his force between current elements he was able to obtain the magnitude of the force and its direction.

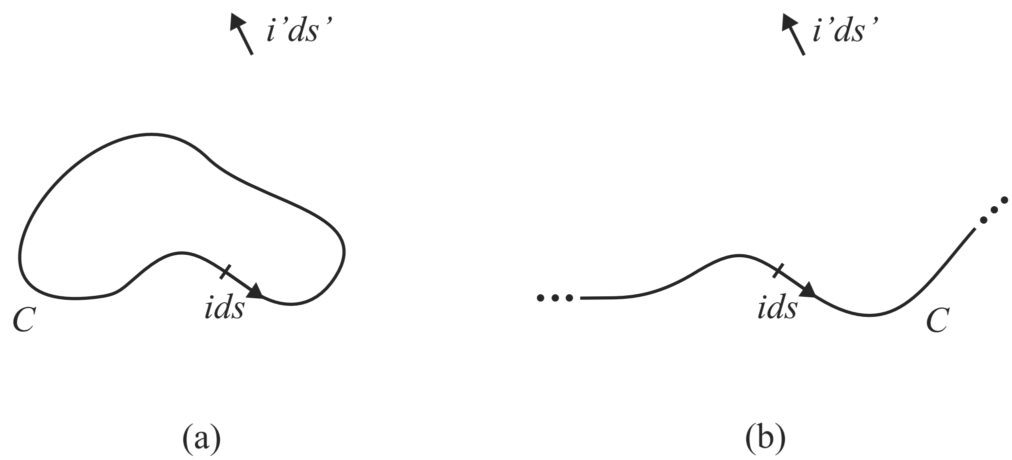


Figure 25.2: (a) Closed circuit C of arbitrary shape carrying a steady current i and an external current element $i'ds'$ located at an arbitrary point in space. (b) Circuit C of arbitrary shape extending indefinitely in both directions, carrying a steady current i, and acting on an external current element $i'ds'$.

Ampère calculated the net force acting on a current element $i'ds'$ which did not belong to the circuit C of arbitrary form represented by Figures 25.2 (a) and (b). He obtained that this net force was always orthogonal to the direction of the element $i'ds'$. Moreover, he concluded that this net force belonged to a certain plane passing through the center of $i'ds'$, no matter the direction of $i'ds'$. He called it the *directing plane* (*plan directeur* in French). The straight line orthogonal to this directing plane and passing through the midpoint of the current element $i'ds'$ was initially called *normal to the directing plane* (*normale au plan directeur* in French). In his main book, the *Théorie*, which was published in 1826, Ampère called it *directrix*, a term which will be adopted here.[780]

Let me give examples of two specific configurations.

As a first example, consider a constant current i flowing along the positive z axis through an infinite and straight conductor. The center of an external current element $i'ds'$ is at point A located in the xy plane. In this configuration the directing plane containing point A passes through the infinite current-carrying wire along the z axis, Figure 25.3 (a). The directrix

[780]See footnote 15 on page 18.

due to this wire at the location of the external current element is orthogonal to the directing plane, passing through point A. Figure 25.3 (b) presents this configuration with the z axis orthogonal to the plane of the paper.

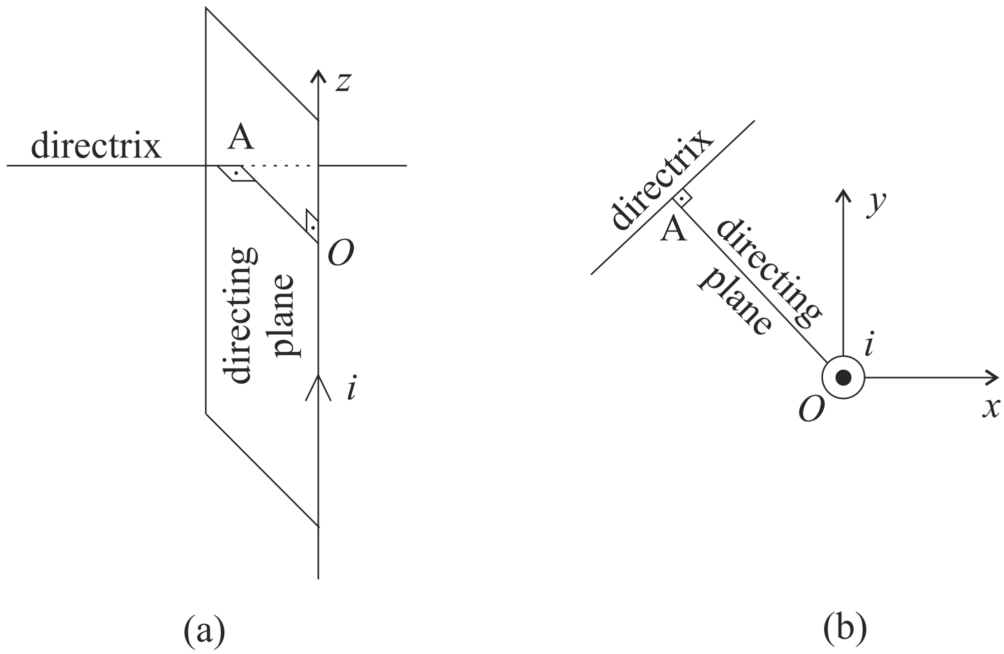


Figure 25.3: (a) Directing plane and directrix due to an infinite straight wire acting on a current element whose midpoint is at point A. (b) Configuration seen in the xy plane. The directrix is the straight line drawn in the xy plane, while the directing plane is orthogonal to the plane of the paper, passing along the straight line connecting A and the origin O of the coordinate system.

The force exerted by the infinite wire on the current element $i'ds'$ is always in the directing plane, no matter the orientation of $i'ds'$ relative to the infinite wire. It is orthogonal to the directrix and also orthogonal to $i'ds'$. Suppose that point A is located along the x axis. In this case the directrix will be a straight line parallel to the y axis. In particular, if this current element points along the x axis, the force will be parallel to the z axis; if this element points along the y axis, there will be no force acting on it; and if this element points along the z axis, the force will be parallel to the x axis.

As a second example, consider a circular loop of radius R located in the xy plane with its center at the origin O of the coordinate system. A constant current i flows anti-clockwise in this closed circuit. An external current element $i'ds'$ has its center at a point A located along the x axis. In this configuration the directing plane containing point A is the xy plane, Figure 25.4 (a). The directrix due to this circular loop at the location A of the external current element is a straight line parallel to the z axis, Figure 25.4 (b).

The force exerted by the circular wire on the current element $i'ds'$ is always in the xy directing plane, no matter the orientation of $i'ds'$. It is also orthogonal to the direction of $i'ds'$. In particular, if this current element points along the x axis, the force will be parallel to the y axis; if this element points along the y axis, the force will be parallel to the x axis; and if this element points along the z axis, there will be no force acting on it.

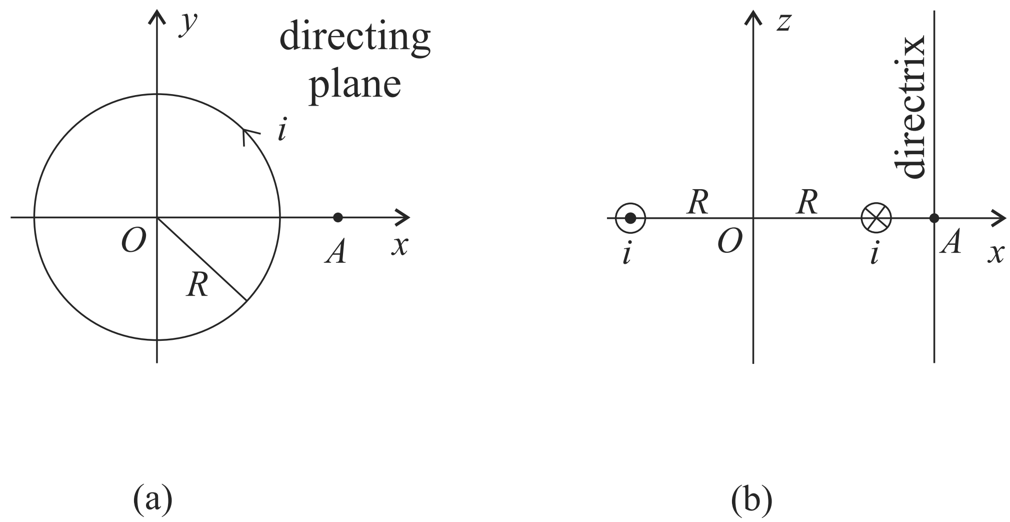


Figure 25.4: (a) A circular wire of radius R located in the xy plane, centered at the origin O of coordinate system and carrying a steady anti-clockwise current i. Point A is along the x axis. The directing plane coincides with the plane of the paper. (b) The same configuration as seen in the xz plane. The directrix passing through point A is the straight line parallel to the z axis.

25.5.2 The Directrix is Simply a Direction along which a Current Carrying Wire Exerts No Force on a Current Element. The Magnetic Field, on the other hand, Is "Something" that Is Produced by the Current Carrying Wire

I present here a very important conceptual distinction between Ampère's directrix and the magnetic field.

Consider a closed circuit C of arbitrary shape carrying a constant current i and a current element $i'ds'$ not belonging to the closed circuit C and located at a certain point A. According to Ampère, the directrix is simply a straight line passing through A and pointing in a certain direction in space. This specific direction will depend on the shape of the closed circuit and on the location of point A in relation to this circuit. Ampère showed how to calculate this direction. Two examples were given in the Subsection 25.5.1.

The direction of Ampère's directrix coincides with the direction of the magnetic field of the textbooks.[781] However, there is a sharp distinction between these two concepts.

According to Ampère, given the closed circuit C and the external current element $i'ds'$, the directrix simply indicates the direction along which no force acts on the current element, no matter the orientation of this current element in space.

According to Maxwell and to all textbooks on electromagnetism, on the other hand, this closed circuit C generates, creates or produces "something" in any external point A. No one has ever clearly explained what this "something" is. In any case, this "something" has been given a rather pompous name, namely, the "magnetic field". This "something" carries energy and linear momentum, exerts forces on current elements and on electrified particles, etc.

These are two completely different and opposite paradigms.

[781] See footnote 773.

25.5.3 According to Ampère, the Magnetic Force Is Not Mediated by a Magnetic Field

Consider a closed circuit C of arbitrary shape carrying a constant current i and a current element $i'ds'$ not belonging to the closed circuit C and located at a certain point A. Ampère obtained the force exerted by this closed circuit acting on $i'ds'$ by integrating his force between current elements. It should be emphasized that according to Ampère's original conceptions, this force resulted from a direct action between the closed conductor C carrying a current i and the current element $i'ds'$. That is, according to Ampère this force was not mediated through a "magnetic field". The net force was simply the result of a direct action at a distance between the current elements of the circuit and the external current element.

In the modern textbooks on electromagnetism, on the other hand, there is no direct action between the closed circuit C and the current element $i'ds'$. This closed circuit generates "something" in space called a magnetic field. This "something" propagates in space, typically at light velocity. When it reaches the location of the current element $i'ds'$, somehow it produces a force on it.

These are once more two completely different and opposite paradigms.

25.5.4 The Directrix Represents a Simple Straight Line and Not a Directed Straight Line. The Magnetic Field Vector, on the other hand, Represents a Directed Straight Line

Consider once more a given closed circuit of arbitrary shape carrying a steady current and a current element of another circuit located at an external point. Ampère's directrix will be a certain straight line passing through the center of the current element. The direction of this directrix coincides with the direction of the magnetic field vector passing through this point due to the closed current-carrying circuit. Despite the similarities of these concepts, there is a very important difference between them which must be emphasized, namely:

- Ampère's directrix represents a simple straight line. It is not a directed straight line.
- The magnetic field vector, on the other hand, represents a directed magnitude. As explained in every modern textbook on electromagnetism, it is obtained by the right-hand rule.

Consider, for instance, an infinite straight conductor carrying a constant current i along the positive z axis and an external point A, as shown in Figure 25.3. The directing plane containing point A passes through the z axis. The directrix due to the infinite wire will be a straight line orthogonal to the directing plane, Figure 25.5 (a). The magnetic field vector, on the other hand, will be an orientated line segment directed according to the right-hand rule, Figure 25.5 (b).

Consider now a constant current i flowing anti-clockwise along a circular loop of radius R located in the xy plane with its center at the origin O of the coordinate system, as shown in Figure 25.4. Figure 25.6 shows this system as seen on the xz plane which is orthogonal to the plane of the loop. I show three specific points where the center of an external current element $i'ds'$ might be located, namely, at A along the x axis, at the origin O and at a point D located somewhere in the xz plane. Figure 25.6 (a) shows the directrixes in these three

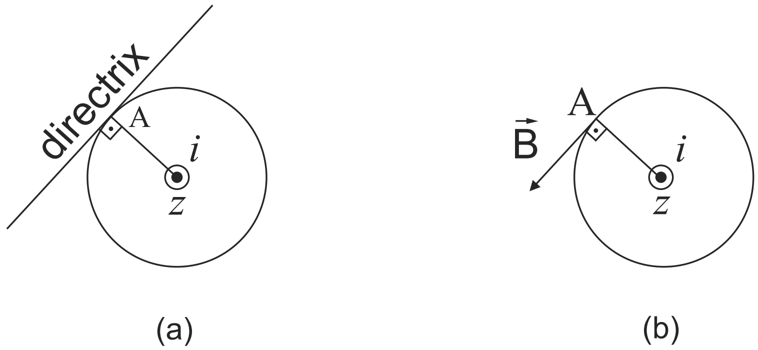


Figure 25.5: An infinite straight wire carrying a current i along the positive z axis, orthogonal to the plane of the paper. (a) Directrix at an external point A. (b) Magnetic field $\vec{B}$ produced by this current-carrying wire at an external point A.

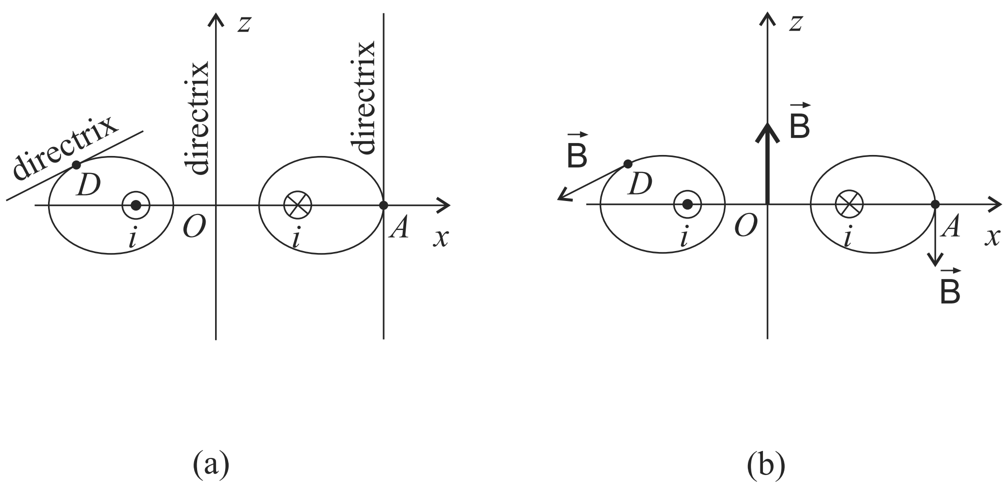


Figure 25.6: (a) The directrix at three points A, O and D located in the xz plane due to a circular loop in the xy plane carrying an anti-clockwise current i. (b) The magnetic field $\vec{B}$ at these three points.

points. Figure 25.6 (b) shows the magnetic fields produced by the circular loop in these three points.

Figures 25.5 and 25.6 show clearly the distinction between a directrix and a magnetic field. While the directrix is a straight, undirected line, the magnetic field is a directed magnitude.

25.5.5 The Magnetic Field Does Not Exist in Nature, But Only in the Minds of People Who Believe in It

Let me consider once again a constant current i flowing along the positive z axis through an infinite and straight conductor, as represented in Figure 25.3 (a). Consider a point A which does not belong to the z axis. It forms a plane with the z axis. This infinite and straight conductor will exert a force on a current element $i'ds'$ located at A. Ampère showed that the direction of this force will always be located in the directing plane, no matter the orientation of $i'ds'$ in space. This directing plane is exactly the plane containing the infinite straight wire and point A. This makes sense and complies with the principle of symmetry.

A straight line and a point A outside this straight line form a plane. Suppose there is an electric current flowing along this straight line. Suppose there is also an electrified particle or a current element located at point A. Any force acting on the electrified particle or on the current element must be contained in this plane, being directed along this plane.

The magnetic field due to this straight wire follows the right-hand rule, as represented in Figure 25.5 (b). However, this right-hand rule is an arbitrary choice made by man. Nothing obliges anyone to use the right-hand instead of the left-hand. This magnetic field violates the principle of symmetry.

A clear proof of this fact utilizes the principle of sufficient reason as formulated by Gottfried Wilhelm Leibniz (1646-1716):[782]

> 31. Our reasonings are grounded upon *two great principles, that of contradiction*, in virtue of which we judge false that which involves a contradiction, and true that which is opposed or contradictory to the false; (*Théod.* 44, 169.)
>
> 32. And *that of sufficient reason*, in virtue of which we hold that there can be no fact real or existing, no statement true, unless there be a sufficient reason, why it should be so and not otherwise, although these reasons usually cannot be known by us. (*Théod*. 44, 196.)

There is no sufficient reason why the magnetic field should follow the right-hand rule instead of the left-hand rule. Therefore, according to the principle of sufficient reason, the magnetic field cannot make this choice. The magnetic field defined by the right-hand rule is not real, it does not exist in nature.

As a second example, consider once more a constant current i flowing anti-clockwise along a circular loop of radius R located in the xy plane with its center at the origin O of the coordinate system, Figure 25.4 (a). Consider a point A in this plane. It can be located along the x axis, as represented in Figure 25.4 (a), or anywhere in the xy plane. This circular conductor carrying a steady current will exert a force on a current element $i'ds'$ located at A. The direction of this force will be located in the xy plane of the loop, no matter the orientation of $i'ds'$ in space. Ampère's directing plane coincides with the plane of the ring for any point A belonging to this plane. This makes sense and complies with the principle of symmetry. A force exerted on any electrified particle or on any current element located in the plane of the current carrying loop must be contained in the plane of the loop, being directed in this plane.

The magnetic field at the origin O of the loop points along the positive z axis, Figure 25.6 (b). There is no sufficient reason why the magnetic field at point O should point along the positive z axis, instead of pointing along the negative z axis. As the magnetic field can not make a choice between these opposite directions, it will not choose between these two possibilities. According to the principle of sufficient reason, the magnetic field at O pointing along the positive z axis, as given by the right-hand rule, is fictitious, it does not exist in nature.

Consider now a point A outside the ring, located along the x axis, Figure 25.6 (b). The magnetic field given by the right-hand rule points along the negative z axis. There is no sufficient reason why the magnetic field at point A should point along the negative z axis, instead of pointing along the positive z axis. As the magnetic field can not make a choice between these opposite directions, it will not choose between these two possibilities.

[782][Lei48, §§ 31 and 32, p. 235].

According to the principle of sufficient reason, the magnetic field at A pointing along the negative z axis, as given by the right-hand rule, is fictitious, it does not exist in nature.

According to the principle of sufficient reason, examples like these prove that the magnetic field given by the right-hand rule does not exist in nature. Despite what we read in all textbooks on electromagnetism, this magnetic field is a fiction, an imaginary concept, which only exists in the minds of people who believe in it.[783]

A closed circuit of arbitrary shape carrying a steady current i exerts a force on another piece of wire carrying a steady current i'. This force can be measured. Ampère taught how to calculate its magnitude and direction by integrating his force between current elements. He did not utilize the magnetic field concept. This concept is superfluous and against the basic principles of symmetry. Ampère's formula complies with the principle of symmetry and agrees with all measurements. This is the force which should be taught in the physics textbooks.[784]

25.6 Weber Never Worked with the Field Concept

Weber's electrodynamics is a completely different paradigm from the field theories of Faraday, Maxwell, Lorentz and Einstein. Wilhelm Weber (1804-1891) was a contemporary of Michael Faraday (1791-1867) and James Clerk Maxwell (1831-1879). In his treatises he quoted many works by Faraday as we can see in his Collected Works.[785] He also quoted a few times some of Maxwell's papers.[786] The 6 volumes of his Collected Works amount to more than 3000 pages. We can read all of them and we will not find the electric field and magnetic field concepts. The only exception is a single place where he quoted Faraday's words related to diamagnetism.[787]

The main result of Ampère's researches was his force between two current elements. It is fascinating to read Weber's first major Memoir on *Electrodynamic Measurements* and to see how he expressed Ampère's force between current elements based on the motion of the positive and negative electrified particles composing each current element.

Faraday expressed the law of electromagnetic induction as being due to a wire cutting the magnetic curves, as can be seen in Article 114 of his original paper read in 1831:[788]

> 114. The relation which holds between the magnetic pole, the moving wire or metal, and the direction of the current evolved, i.e., *the law* which governs the evolution of electricity by magneto-electric induction, is very simple, although rather difficult to express. If in *Fig. 12*, PN represent a horizontal wire passing by a marked

[783] This is my point of view (Andre Koch Torres Assis), [Ass99] with German translation in [Ass14a] and Portuguese translation in [Ass19a].

[784] See footnote 15 on page 18.

[785] [Web92e], [Web92d], [Web93c], [Web94g], [WW93] and [WW94]. Most of his papers have already been translated into English in the 4 Volumes of the book *Wilhelm Weber's Main Works on Electrodynamics Translated into English*: [Ass21j], [Ass21k], [Ass21l] and [Ass21m].

[786] [AW03].

[787] [Web48b, p. 257 of Weber's *Werke*] and [Web48c, p. 257 of Weber's *Werke*] with English translation in [Web21n, p. 251]. See also [Far46c, p. 73, Article 2429] with German translation in [Far47, p. 48, Article 2429].

[788] [Far32a, p. 154 of the Philosophical Transactions and p. 281 of the Great Books of the Western World] with German translation in [Far32b] and [Far89], and Portuguese translation in [Far11, p. 193].

magnetic pole,[789] so that the direction of its motion shall coincide with the curved line proceeding from below upwards; or if its motion parallel to itself be in a line tangential to the curved line, but in the general direction of the arrows; or if it pass the pole in other directions, but so as to cut the magnetic curves[790] in the same general direction, or on the same side as they would be cut by the wire if moving along the dotted curved line—then the current of electricity in the wire is from P to N. [...]

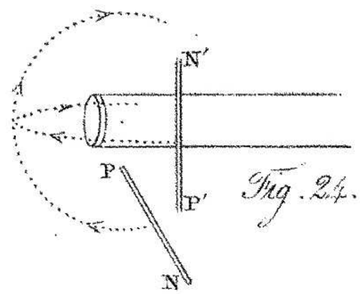


Figure 25.7: Figure 12 of the *Great Books of the Western World* coincides with Figure 24 of Faraday's original paper of 1831 in the *Philosophical Transactions*.

Nowadays Faraday's law of induction is expressed in terms of the electromotive force (or emf) induced in a closed circuit by a temporal change of the flux of the magnetic field that passes through the circuit. A typical statement of this flux rule has been given by Feynman, Leighton and Sands:[791]

> The amount of emf is given by a simple rule discovered by Faraday. (We will just state the rule now and wait until later to examine it in detail.) The rule is that when the magnetic flux that passes through the loop (this flux is the normal component of $\mathbf{B}$ integrated over the area of the loop) is changing with time, the emf is equal to the rate of change of the flux. We will refer to this as "the flux rule."

In 1846 Weber also utilized Faraday's law of induction in order to deduce his own force between electrified particles. However, Weber did not utilized these concepts of magnetic curve, lines of force, a wire cutting the magnetic curves or cutting the lines of magnetic force, rate of change of the magnetic flux, not even the basic concept of a magnetic field![792]

The only concepts which Weber utilized were those that he could observe and measure. In Part III, Sections 10 and 11 of his paper, he mentions that Faraday obtained induction in three main configurations. In the first configuration two closed circuits remained at rest relative to one another, with a current in the primary circuit and none in the secondary circuit. However, a current would be induced in the secondary circuit whenever there was a change in the current intensity in the primary circuit. The second configuration was that in

[789][Note by Faraday presented in Article 44 of this paper:] To avoid any confusion as to the poles of the magnet, I shall designate the pole pointing to the north as the marked pole; [...]

[790][Note by Faraday:] By magnetic curves, I mean the lines of magnetic forces, however modified by the juxtaposition of poles, which would be depicted by iron filings; or those to which a very small magnetic needle would form a tangent.

[791][FLS64, p. 16-1].

[792][Web46] with a partial French translation in [Web87] and a complete English translation in [Web21d].

which the current in the primary circuit was steady. A current might also be induced in the secondary circuit when there was a relative motion between the two circuits. In the third configuration the primary circuit with steady current was replaced by a permanent magnet. A current might also be induced in the secondary circuit whenever there was a relative motion between the magnet and the secondary circuit. Utilizing his electrodynamometer Weber could measure the current in the primary circuit, the induced current in the secondary circuit and also the relative motion between its two coils.

Weber presented the deduction of his force in Sections 18-21, of his 1846 treatise.[793] He assumed three facts which were in part based indirectly on observation. He also decomposed each current element into oppositely electrified particles moving relative to the conduction wire with certain drift velocities. In particular, Faraday's experimental discoveries were represented in his third special fact, namely:

> In order to obtain for this investigation the most reliable possible guideline based on experience, the foundation will be *three special facts*, which are in part based indirectly on observation, in part contained directly in Ampère's law, which is confirmed by all measurements. [...]
>
> The *third fact* is, that a current element, which lies together with a wire element in a straight line coinciding with the directions of both elements, induces a *like-* or *opposite-directed* current in the wire element, according to whether the intensity of its own current *decreases* or *increases*.

It is fascinating to see these two completely different paradigms into action. In one of them we have magnetic curves, lines of force, magnetic field, flux of magnetic field etc. In the other we have electrified particles, their distances, relative velocities and relative accelerations.

25.7 Weber's Unification of the Laws of Coulomb, Ampère and Faraday

Weber obtained the unification of all branches of electrodynamics known during the XIXth century. Beginning with Ampère's force between current elements and supposing each current element as being composed of positive and negative electrified particles moving in opposite directions relative to the body of the conductor with certain drift velocities, he proposed in 1846 a *fundamental law of electric action*.[794] It was a force combining Coulomb's law for the interaction between electrified particles separated by a distance r, together with a component depending on the relative velocity dr/dt between the particles and another component depending on their relative acceleration d^2r/dt^2.

In 1852 he presented his law utilizing a constant c representing the uniform relative velocity at which the force between the particles would fall to zero.[795] Weber's c (known throughout the 19th century as the *Weber constant*) was first measured by Weber and Kohlrausch in 1854-1856. They obtained $c = 4.39 \times 10^8\ m/s$.[796]

[793] [Web46, pp. 132-157 of Weber's *Werke*] with English translation in [Web21d, pp. 129-149].

[794] See footnote 792.

[795] [Web52b, p. 366 of Weber's *Werke*] with English translation in [Web21e, p. 346].

[796] [Web55] with English translations at [Web21i]; [WK56] with English translation in [WK21] and Portuguese translation in [WK08]; and [KW57] with English translation in [KW21]. See also [Ass21h].

In 1857 Weber and Kirchhoff were the first to derive theoretically the complete telegraph equation working independently from one another and arriving simultaneously at the same result. Utilizing the modern concepts and usual terminology of circuit theory, we can say that they were the first to take into account not only the capacitance and resistance of the wire, but also its self-inductance. For a circuit of negligible resistance, they concluded that the velocity of propagation of an electric wave along the wire would be given by $c/\sqrt{2} = 3 \times 10^8\ m/s$. This value coincided with the known light velocity in vacuum, v_L, as deduced from astronomical observations and from terrestrial optical experiments. That is,

$$\frac{c}{\sqrt{2}} = v_L \qquad \text{or} \qquad c = \sqrt{2} \cdot v_L \ . \tag{25.12}$$

This result was independent of the cross section and conductivity of the wire, and also independent of its surface density of electricity. Kohlrausch, who was collaborating with Weber on some experiments related with the propagation of electromagnetic waves, died in 1858. Weber's work has been delayed in publication and appeared only in 1864.[797]

Weber's 1846 fundamental force can then be expressed in terms of his 1852 constant c and also in terms of light velocity in vacuum, v_L, as follows:

$$\frac{ee'}{r^2}\left[1 - \frac{1}{c^2}\left(\frac{dr}{dt}\right)^2 + \frac{2}{c^2}r\frac{d^2r}{dt^2}\right] = \frac{ee'}{r^2}\left[1 - \frac{1}{2v_L^2}\left(\frac{dr}{dt}\right)^2 + \frac{1}{v_L^2}r\frac{d^2r}{dt^2}\right] \ . \tag{25.13}$$

The force between two particles was thus dependent upon their *relative velocity*, dr/dt, and *relative acceleration*, d^2r/dt^2. Velocities of the particles relative to the observer or frame of reference are not relevant here. Weber's force points along the straight line connecting the interacting particles and follows the principle of action and reaction.

Weber had also introduced in 1848 a potential energy from which he could deduce his force law.[798] Weber's potential energy can be written in terms of Weber's constant c and light velocity v_L as follows:

$$\frac{ee'}{r}\left[1 - \frac{1}{c^2}\left(\frac{dr}{dt}\right)^2\right] = \frac{ee'}{r}\left[1 - \frac{1}{2v_L^2}\left(\frac{dr}{dt}\right)^2\right] \ . \tag{25.14}$$

When there is no motion between the particles, $dr/dt = 0$ and $d^2r/dt^2 = 0$, Weber's force reduces to Coulomb's law. Therefore, the whole of electrostatics is contained in Weber's electrodynamics, including Gauss' flux law.

Consider now two conductors carrying steady currents i and i'. Two current elements of lengths ds and ds' are represented by ids and $i'ds'$. We can consider a neutral current element ids as composed of equal and opposite charges, e_+ and $e_- = -e_+$, moving relative to the matter of this conductor with velocities v_+ and v_-. Likewise, we can consider a neutral current element $i'ds'$ as composed of equal and opposite charges, e'_+ and $e'_- = -e'_+$, moving relative to the body of this conductor with velocities v'_+ and v'_-. The interaction of ids with $i'ds'$ is then composed of four terms, namely, the force between e_+ and e'_+, the force between e_+ and e'_-, the force between e_- and e'_+, and the force between e_- and e'_-. By adding these four expressions, Weber was able to deduce in 1846 Ampère's force between current elements from his fundamental force between electrified particles. Therefore, the whole of Ampère's

[797] [Kir57b] with English translation in [Kir57a] and [Kir21c], [Pog57] with English translation in [Pog21], and [Web64] with English translation in [Web21c]. See also [Ass21h].

[798] [Web48a] with English translations in [Web52c], [Web66c], [Web19] and [Web21p].

electrodynamics is contained in Weber's force law. As Ampère had unified magnetostatics, electrodynamics and electromagnetism with his force law, all of these three branches of physics are contained in Weber's electrodynamics (including the so-called magnetic circuital law).

In 1831 Michael Faraday (1791-1867) discovered electromagnetic induction.[799] That is, the induction of a current in a secondary circuit by varying the current intensity in a nearby primary circuit. He also showed that it was possible to induce a current in a secondary circuit when there was a relative motion between this secondary circuit and a primary circuit carrying a steady current (or when there was a relative motion between this secondary circuit and a nearby magnet). These phenomena became known as Faraday's law of induction. In 1846 Weber was able to deduce quantitatively Faraday's law of induction from his fundamental force law. James Clerk Maxwell (1831-1879) could only obtain a mathematical formulation of Faraday's law many years after Weber.

Therefore, essentially all electrodynamic phenomena known during the working periods of Maxwell and Weber could be explained quantitatively from Weber's law given by Equations (25.13) and (25.14).

Newton and Coulomb's force laws depend on the distance r between the interacting particles. As it deals with a much broader class of phenomena, Weber's force depends also on the *relative* velocity, dr/dt, and *relative* acceleration, d^2r/dt^2, between the interacting particles. These are *relational magnitudes* which have the same values for all observers and for all frames of reference. These magnitudes have the same value even when comparing an inertial frame of reference with a non inertial frame of reference. They are intrinsic magnitudes related only to the interacting bodies.[800]

[799] See footnote 788.

[800] [Ass13] and [Ass14b].

Bibliography

[AB23] A. K. T. Assis and L. L. Bucciarelli. *Coulomb's Memoirs on Torsion, Electricity, and Magnetism Translated into English*. Apeiron, Montreal, 2023. Available at www.ifi.unicamp.br/~assis.

[AC06] A. K. T. Assis and J. P. M. C. Chaib. Nota sobre o magnetismo da pilha de Volta — tradução comentada do primeiro artigo de Biot e Savart sobre eletromagnetismo. *Cadernos de História e Filosofia da Ciência*, 16:303–309, 2006.

[AC11] A. K. T. Assis and J. P. M. d. C. Chaib. *Eletrodinâmica de Ampère: Análise do Significado e da Evolução da Força de Ampère, Juntamente com a Tradução Comentada de Sua Principal Obra sobre Eletrodinâmica*. Editora da Unicamp, Campinas, 2011.

[AC12] A. K. T. Assis and J. P. M. C. Chaib. Ampère's motor: Its history and the controversies surrounding its working mechanism. *American Journal of Physics*, 80:990–995, 2012. Doi: 10.1119/1.4746698.

[AC15] A. K. T. Assis and J. P. M. C. Chaib. *Ampère's Electrodynamics — Analysis of the Meaning and Evolution of Ampère's Force between Current Elements, together with a Complete Translation of His Masterpiece: Theory of Electrodynamic Phenomena, Uniquely Deduced from Experience*. Apeiron, Montreal, 2015. Available at www.ifi.unicamp.br/~assis.

[AGV07] R. Achilles and J. Guala-Valverde. Action at a distance: a key to homopolar induction. *Apeiron*, 14:169–183, 2007.

[AH07] A. K. T. Assis and J. A. Hernandes. *The Electric Force of a Current: Weber and the Surface Charges of Resistive Conductors Carrying Steady Currents*. Apeiron, Montreal, 2007. Available at www.ifi.unicamp.br/~assis.

[AH09] A. K. T. Assis and J. A. Hernandes. *A Força Elétrica de uma Corrente: Weber e as Cargas Superficiais de Condutores Resistivos com Correntes Constantes*, volume 73 of *Coleção Acadêmica*. Edusp and Edufal, São Paulo and Maceió, 2009.

[AH13] A. K. T. Assis and J. A. Hernandes. *Elektrischer Strom und Oberflächenladungen: was Wilhelm Weber schon vor mehr als 150 Jahre wußte*. Apeiron, Montreal, 2013. German translation by H. Härtel. Available at www.ifi.unicamp.br/~assis.

[Amp20a] A.-M. Ampère. Analyse des mémoires lus par M. Ampère a l'Académie des Sciences, dans les séances des 18 et 25 septembre, des 9 et 30 octobre 1820. *Annales Générales des Sciences Physiques*, 6:238–257, 1820.

[Amp20b] A.-M. Ampère. Analyse des mémoires lus par M. Ampère a l'Académie des Sciences, dans les séances des 18 et 25 septembre, des 9 et 30 octobre 1820. 20 pages. Reprint of Annales Générales des Sciences Physiques, Vol. 6, pp. 238-257 (1820), 1820.

[Amp20c] A.-M. Ampère. Suite du Mémoire sur l'Action mutuelle entre deux courans électriques, entre un courant électrique et un aimant ou le globe terrestre, et entre deux aimants. *Annales de Chimie et de Physique*, 15:170–218, 1820.

[Amp21] A.-M. Ampère. Suite de la Note sur un Appareil à l'aide duquel on peut vérifier toutes les propriétés des conducteurs de l'électricité voltaïque, découvertes par M. Ampère. *Annales de Chimie et de Physique*, 18:313–333, 1821.

[Amp22a] A.-M. Ampère. Expériences relatives à de nouveaux phénomènes électrodynamiques. *Annales de Chimie et de Physique*, 20:60–74, 1822.

[Amp22b] A.-M. Ampère. Expériences relatives aux nouveaux phénomènes électrodynamiques que j'ai obtenus au mois de décembre 1821. In A.-M. Ampère, editor, *Recueil d'Observations Électro-dynamiques*, pages 237–250. Crochard, Paris, 1822. Despite this date, the volume of the Recueil was only published in 1823.

[Amp23] A.-M. Ampère. Mémoire sur la théorie mathématique des phénomènes électrodynamiques uniquement déduite de l'expérience, dans lequel se trouvent réunis les Mémoires que M. Ampère a communiqués à l'Académie royale des Sciences, dans les séances des 4 et 26 décembre 1820, 10 juin 1822, 22 décembre 1823, 12 septembre et 21 novembre 1825. *Mémoires de l'Académie Royale des Sciences de l'Institut de France*, 6:175–387, 1823. Despite this date, the work was only published in 1827.

[Amp26] A.-M. Ampère. *Théorie des Phénomènes Électro-dynamiques, Uniquement Déduite de l'Expérience*. Méquignon-Marvis, Paris, 1826.

[Amp28] A.-M. Ampère. Note sur l'action mutuelle d'un aimant et d'un conducteur voltaïque. *Annales de Chimie et de Physique*, 37:113–139, 1828.

[Amp65] A.-M. Ampère. On the Mathematical Theory of Electrodynamic Phenomena, Experimentally Deduced. In R. A. R. Tricker*, Early Electrodynamics — The First Law of Circulation*, pages 155–200, New York, 1965. Pergamon. Partial English translation by O. M. Blunn of Ampère's work "Mémoire sur la théorie mathématique des phénomènes électro-dynamiques uniquement déduite de l'expérience", *Mémoires de l'Académie royale des Sciences de l'Institut de France*, Vol. 6, pp. 175-387 (1823), issued 1827.

[Amp69a] A. M. Ampère. New names. In W. F. Magie, editor, *A Source Book in Physics*, page 447, New York, 1969. McGraw-Hill. Extract from a paper entitled "Experiments on the new electrodynamical phenomena," in the *Annales de Chimie et de Physique*, Series II, Vol. 20, p. 60, 1822.

[Amp69b] A. M. Ampère. The solenoid. Circuits and magnetic shells. In W. F. Magie, editor, *A Source Book in Physics*, pages 456–460, New York, 1969. McGraw-Hill.

Extracts from "Théorie des phénomènes électrodynamiques uniquement déduite de l'expérience," Paris, 1826.

[Amp12] A.-M. Ampère. Mathematical Theory of Electrodynamic Phenomena, Uniquely Derived from Experiments. Translated by M. D. Godfrey. Available at https://archive.org/details/AmpereTheorieEn and https://sites.google.com/site/michaeldgodfrey/physics-information-and-communication, 2012.

[And07] R. J. Anderton. Einstein, ether and unified field. *The General Science Journal*, 2007. Available at www.gsjournal.net/Science-Journals/Essays/View/408.

[Ano89] Anonymous. Gauss and the electric telegraph. *Popular Science Monthly*, 34:396–399, 1889. Available at https://en.wikisource.org/wiki/Popular_Science_Monthly/Volume_34/January_1889/Gauss_and_the_Electric_Telegraph.

[AP92] A. K. T. Assis and F. M. Peixoto. On the velocity in the Lorentz force law. *The Physics Teacher*, 30:480–483, 1992.

[Ara24] F. J. D. Arago. Untitled. *Annales de Chimie et de Physique*, 27:363, 1824.

[Ara25] F. J. D. Arago. Untitled. *Annales de Chimie et de Physique*, 28:325–326, 1825.

[Ara26] F. J. D. Arago. Note concernant les phénomènes magnétiques auxquels le mouvement donne naissance. *Annales de Chimie et de Physique*, 32:213–223, 1826.

[Ara54] F. J. D. Arago. Du magnétisme de rotation. In J.-A. Barral, editor, *Oeuvres Complètes de François Arago*, volume 4, pages 424–448. Gide et J. Baudry, Paris, 1854.

[ARV09] A. K. T. Assis, J. E. A. Ribeiro, and A. Vannucci. The field concepts of Faraday and Maxwell. In M. S. D. Cattani, L. C. B. Crispino, M. O. C. Gomes, and A. F. S. Santoro, editors, *Trends in Physics: Festscrift in Homage to Prof. José Maria Filardo Bassalo*, pages 31–38. Editora Livraria da Física, São Paulo, 2009.

[ARW04] A. K. T. Assis, K. Reich, and K. H. Wiederkehr. On the electromagnetic and electrostatic units of current and the meaning of the absolute system of units — For the 200th anniversary of Wilhelm Weber's birth. *Sudhoffs Archiv*, 88:10–31, 2004.

[Ass90] A. K. T. Assis. Deriving Ampère's law from Weber's law. *Hadronic Journal*, 13:441–451, 1990.

[Ass92] A. K. T. Assis. *Curso de Eletrodinâmica de Weber*. Setor de Publicações do Instituto de Física da Universidade Estadual de Campinas — UNICAMP, Campinas, 1992. Notas de Física IFGW Número 5. Available at www.ifi.unicamp.br/~assis and www.bibliotecadigital.unicamp.br/document/?down=60362.

[Ass94] A. K. T. Assis. *Weber's Electrodynamics*. Kluwer, Dordrecht, 1994. Nowadays this book is available through Springer. Doi: 10.1007/978-94-017-3670-1 and www.springer.com/gp/book/9780792331377.

[Ass95] A. K. T. Assis. Weber's force versus Lorentz's force. *Physics Essays*, 8:335–341, 1995.

[Ass98] A. K. T. Assis. Newton e suas grandes obras: o Principia e o Óptica. In M. J. P. M. de Almeida and H. C. da Silva, editors, *Linguagens, Leituras e Ensino da Ciência*, pages 37–52. Mercado de Letras/Associação de Leitura do Brasil, Campinas, 1998.

[Ass99] A. K. T. Assis. Arguments in favour of action at a distance. In A. E. Chubykalo, V. Pope, and R. Smirnov-Rueda, editors, *Instantaneous Action at a Distance in Modern Physics — "Pro" and "Contra"*, pages 45–56, Commack, 1999. Nova Science Publishers.

[Ass03] A. K. T. Assis. Tradução de uma obra de Gauss. *Revista Brasileira de Ensino de Física*, 25:226–249, 2003. Paper in Portuguese: "Translation of a work by Gauss".

[Ass10a] A. K. T. Assis. *The Experimental and Historical Foundations of Electricity*. Apeiron, Montreal, 2010. Available at www.ifi.unicamp.br/~assis.

[Ass10b] A. K. T. Assis. *Os Fundamentos Experimentais e Históricos da Eletricidade*. Apeiron, Montreal, 2010. Available at www.ifi.unicamp.br/~assis.

[Ass11] A. K. T. Assis. Newton and inverse problems. In D. Krause and A. Videira, editors, *Brazilian Studies in Philosophy and History of Science: An Account of Recent Works*, volume 290 of Boston Studies in the Philosophy of Science, chapter 3, pages 71–76. Springer, Dordrecht, 2011. Doi: 10.1007/978-90-481-9422-3_3.

[Ass13] A. K. T. Assis. *Mecânica Relacional e Implementação do Princípio de Mach com a Força de Weber Gravitacional*. Apeiron, Montreal, 2013. Available at www.ifi.unicamp.br/~assis.

[Ass14a] A. K. T. Assis. Argumente zugunsten der Fernwirkung. Translated (2014) from the English by M. Pohl. Original paper: A. K. T. Assis, "Arguments in favour of action at a distance," in: Instantaneous Action at a Distance in Modern Physics - "Pro" and "Contra", Edited by A. E. Chubykalo, V. Pope and R. Smirnov-Rueda (Nova Science Publishers, Commak, 1999), pp. 45-56. Available at www.ifi.unicamp.br/~assis, 2014.

[Ass14b] A. K. T. Assis. *Relational Mechanics and Implementation of Mach's Principle with Weber's Gravitational Force*. Apeiron, Montreal, 2014. Available at www.ifi.unicamp.br/~assis.

[Ass15a] A. K. T. Assis. *Eletrodinâmica de Weber: Teoria, Aplicações e Exercícios*. Editora da Unicamp, Campinas, 2nd edition, 2015. Available at www.ifi.unicamp.br/~assis and https://editoraunicamp.com.br/catalogo/?id=1718.

[Ass15b] A. K. T. Assis. *The Experimental and Historical Foundations of Electricity*. Apeiron, Montreal, 2015. Book in Russian translated from the English version by A. Baraov. Available at www.ifi.unicamp.br/~assis.

[Ass17a] A. K. T. Assis. *I Fondamenti Sperimentali e Storici dell'Elettricità*. Associazione per l'Insegnamento della Fisica, Parma, 2017. La Fisica nella Scuola, Anno L, n. 2 Supplemento, Quaderno 26. Translated by P. Cerreta, A. Cerreta and R. Cerreta. Edited by P. Cerreta, R. Serafini and R. Urigu. Available at www.ifi.unicamp.br/~assis.

[Ass17b] A. K. T. Assis. Newton e os problemas inversos. In E. T. César, T. C. Soares, and E. E. Reinehr, editors, *Ciência em Dia: Jornadas de Divulgação Científica — Ano Internacional da Luz*, pages 9–24. Editora Livraria da Física, São Paulo, 2017.

[Ass18a] A. K. T. Assis. *The Experimental and Historical Foundations of Electricity*, volume 2. Apeiron, Montreal, 2018. Available at www.ifi.unicamp.br/~assis.

[Ass18b] A. K. T. Assis. *Os Fundamentos Experimentais e Históricos da Eletricidade*, volume 2. Apeiron, Montreal, 2018. Available at www.ifi.unicamp.br/~assis.

[Ass19a] A. K. T. Assis. Argumentos a favor da ação a distância. *Revista Educar Mais*, 3:202–218, 2019. Doi: 10.15536/reducarmais.3.2019.202-218.1505.

[Ass19b] A. K. T. Assis. *The Experimental and Historical Foundations of Electricity*, volume 2. Apeiron, Montreal, 2019. Book in Russian translated from the volume 2 of the English version by A. Baraov. Available at www.ifi.unicamp.br/~assis.

[Ass21a] A. K. T. Assis. Editor's Comments on Kirchhoff's 1857 Paper on the Motion of Electricity in Wires. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume III: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity, pages 221–224, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Ass21b] A. K. T. Assis. Editor's Comments on Poggendorff's 1857 Paper. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume III: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity, pages 227–228, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Ass21c] A. K. T. Assis. Editor's Introduction to Carl Neumann's 1868 Paper. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume IV: Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation, pages 13–15, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Ass21d] A. K. T. Assis. Editor's Introduction to Gauss' Work on the Fundamental Law for All Interactions of Galvanic Currents. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume IV: Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation, pages 9–10, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Ass21e] A. K. T. Assis. Editor's Introduction to Weber's Second Memoir on Electrodynamic Measurements. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume II: Weber's Fundamental Force and the Unification of the Laws of Coulomb, Ampère and Faraday, pages 287–290, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Ass21f] A. K. T. Assis. Editor's Introduction to Weber's Sixth Memoir on Electrodynamic Measurements. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume IV: Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation, pages 59–65, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Ass21g] A. K. T. Assis. Gauss and Weber's absolute system of units and its difference to the modern "Gaussian" system of units. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume I: Gauss and Weber's Absolute System of Units, pages 207–212, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Ass21h] A. K. T. Assis. The origins and meanings of Weber's constant c, of Maxwell's constant c and the relation of these two different constants with light velocity in vacuum. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume III: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity, pages 385–396, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Ass21i] A. K. T. Assis. Why did I decide to work in this project of the English translation of Weber's main works on electrodynamics. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume I: Gauss and Weber's Absolute System of Units, pages 15–19, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Ass21j] A. K. T. Assis (editor). *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume 1: Gauss and Weber's Absolute System of Units. Apeiron, Montreal, 2021. Available at www.ifi.unicamp.br/~assis.

[Ass21k] A. K. T. Assis (editor). *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume 2: Weber's Fundamental Force and the Unification of the Laws of Coulomb, Ampère and Faraday. Apeiron, Montreal, 2021. Available at www.ifi.unicamp.br/~assis.

[Ass21l] A. K. T. Assis (editor). *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume 3: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity. Apeiron, Montreal, 2021. Available at www.ifi.unicamp.br/~assis.

[Ass21m] A. K. T. Assis (editor). *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume 4: Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation. Apeiron, Montreal, 2021. Available at www.ifi.unicamp.br/~assis.

[Ass22] A. K. T. Assis. *Tradução Comentada das Principais Obras de Coulomb sobre Eletricidade e Magnetismo*. Apeiron, Montreal, 2022. Available at www.ifi.unicamp.br/~assis.

[AT94] A. K. T. Assis and D. S. Thober. Unipolar induction and Weber's electrodynamics. In M. Barone and F. Selleri, editors, *Frontiers of Fundamental Physics*, pages 409–414, New York, 1994. Plenum Press.

[AW03] A. K. T. Assis and K. H. Wiederkehr. Weber quoting Maxwell. *Mitteilungen der Gauss-Gesellschaft*, 40:53–74, 2003.

[AWW11] A. K. T. Assis, K. H. Wiederkehr, and G. Wolfschmidt. *Weber's Planetary Model of the Atom*, volume 19 of *Nuncius Hamburgensis — Beiträge zur Geschichte der Naturwissenschaften*. Tredition Science, Hamburg, 2011. Edited by G. Wolfschmidt.

[AWW14] A. K. T. Assis, K. H. Wiederkehr, and G. Wolfschmidt. *O Modelo Planetário de Weber para o Átomo*. Apeiron, Montreal, 2014. Available at www.ifi.unicamp.br/~assis.

[AWW18] A. K. T. Assis, K. H. Wiederkehr, and G. Wolfschmidt. *Webers Planeten-Modell des Atoms*. Apeiron, Montreal, 2018. Translated by H. Härtel. Available at www.ifi.unicamp.br/~assis.

[BA01] M. d. A. Bueno and A. K. T. Assis. *Inductance and Force Calculations in Electrical Circuits*. Nova Science Publishers, Huntington, New York, 2001.

[BA15] M. Bueno and A. K. T. Assis. *Cálculo de Indutância e de Força em Circuitos Elétricos*. Apeiron, Montreal, 2nd edition, 2015. Available at www.ifi.unicamp.br/~assis.

[Bau22] C. Baumgärtel. *Aspects of Weber Electrodynamics*. PhD thesis, Faculty of Science and Engineering, The University of Liverpool, UK, Liverpool, 2022. Available at https://livrepository.liverpool.ac.uk/3165052.

[Bec43] E. Becquerel. Des lois du dégagement de la chaleur pendant le passage des courants électriques à travers les corps solides et liquides. *Annales de Chimie et de Physique*, 9:21–70, 1843.

[Bec79] H. G. Beckmann. Wilhelm Webers Wissenschaftliche Apparate in der Historischen Sammlung des I. Physikalischen Instituts der Universität Göttingen. Schriftliche Hausarbeit im Rahmen der fachwissenschaftlichen Prüfung für das Lehramt an Gymnasien, 1979.

[Ber36a] Berzelius. Considerations respecting a new power which acts in the formation of organic bodies. *The Edinburgh New Philosophical Journal*, 21:223–228, 1836.

[Ber36b] Berzélius. Quelques idées sur une nouvelle force agissant dans les combinaisons des corps organiques. *Annales de Chimie et de Physique*, 61:146–151, 1836.

[Ber36c] J. Berzelius. Einige Ideen über eine bei der Bildung organischer Verbindungen in der lebenden Natur wirksame, aber bisher nich bemerkte Kraft. *Jahres-Bericht über die Fortschritte der physischen Wissenschaften von Jacob Berzelius*, 15:237–245, 1836.

[Ber56] W. Bernhardt. *Dr. Ernst Chladni, der Akustiker. Eine Biographie und geschichtliche Darstellung seiner Entdeckungen zur Erinnerung an seinen hundertjährigen Geburtstag den 30. November 1856.* Franz Mohr's Buchhandlung, Wittenberg, 1856.

[Bio21] J. B. Biot. *Précis élémentaire de Physique expérimentale*, volume II. Deterville, Paris, second edition, 1821.

[Blo82] C. Blondel. *A.-M. Ampère et la Création de l'Électrodynamique (1820-1827).* Bibliothèque Nationale, Paris, 1982.

[Blo05] C. Blondel, 2005. Ampère et l'Histoire de l'Électricité. Site of the CNRS which was coordinated by Christine Blondel. Available at www.ampere.cnrs.fr.

[BM22] C. Baumgärtel and S. Maher. Resolving the paradox of unipolar induction: new experimental evidence on the influence of the test circuit. *Scientific Reports*, 12:16791, 2022. Doi: 10.1038/s41598-022-21155-x. Available at https://www.nature.com/articles/s41598-022-21155-x.

[Bor08] G. Borvon. Dans les coulisses du congrès international des électriciens de 1881. Available at http://www.ampere.cnrs.fr/parcourspedagogique, 2008.

[Bor09] G. Borvon. Histoire de l'électricité: l'histoire des unités électriques. Available at http://seaus.free.fr/spip.php?article324, 2009.

[Bor12] G. Borvon. History of the electrical units. Available at http://seaus.free.fr/spip.php?article964, 2012.

[Bor13] G. Borvon. Histoire des unités électriques. Le premier congrès international des électricienn à Paris en 1881. Available at http://histoires-de-sciences.over-blog.fr/2013/11/histoire-des-unit%C3%A9s-%C3%A9lectriques.html, 2013.

[Bri69] C. Briot. *Théorie Mécanique de la Chaleur.* Gauthier-Villars, Paris, 1869.

[BS20] J. B. Biot and F. Savart. Note sur le magnétisme de la pile de Volta. *Annales de Chimie et de Physique*, 15:222–223, 1820.

[BS24] J. B. Biot and F. Savart. Sur l'aimantation imposée aux métaux par l'électricité en mouvement. In J. B. Biot, *Précis Élémentaire de Physique Expérimentale,* vol. II, troisième édition, pages 704–774. Gauthier-Villars, Paris, 1824.

[BS85] J. B. Biot and F. Savart. Sur l'aimantation imprimée aux métaux par l'électricitè en mouvement, in: *Précis élémentaire de Physique expérimentale*, troisième édition (Paris: Deterville, 1824), Vol. II, pp. 704-774. In J. Joubert, editor, *Collection de Mémoires relatifs a la Physique — Tome II: Mémoires sur l'Électrodynamique*, pages 80–127, Paris, 1885. Gauthier-Villars.

[BS65a] J. B. Biot and F. Savart. Magnetization of metals by electricity in motion. In R. A. R. Tricker, *Early Electrodynamics — The First Law of Circulation*, pages 119–139, New York, 1965. Pergamon. Extracts from Biot's *Précis elémentaire de Physique expérimentale*, Vol. II, 3rd edition, pp. 707-723 and 740-746, 1824 (Translated by O. M. Blunn).

[BS65b] J. B. Biot and F. Savart. Note on the magnetism of Volta's battery. In R. A. R. Tricker, *Early Electrodynamics — The First Law of Circulation*, pages 118–119, New York, 1965. Pergamon. Translated by O. M. Blunn.

[CA09] J. P. M. d. C. Chaib and A. K. T. Assis. Sobre os efeitos das correntes elétricas (segunda parte) — Tradução da primeira obra de Ampère sobre eletrodinâmica. *Revista Brasileira de História da Ciência*, 2:118–145, 2009.

[CA13] J. P. M. d. C. Chaib and A. K. T. Assis. Motor de Ampère: elementos para um ensino crítico de física. In C. C. Silva and M. E. B. Prestes, editors, *Aprendendo Ciência e sobre Sua Natureza: abordagens históricas e filosóficas*, pages 55–70. Tipographia Editora Expressa, São Carlos, 2013.

[Cha09] J. P. M. d. C. Chaib. *Análise do Significado e da Evolução do Conceito de Força de Ampère, juntamente com a Tradução Comentada de sua Principal Obra sobre Eletrodinâmica*. PhD thesis, University of Campinas — UNICAMP, Campinas, Brazil, 2009. Supervisor: A. K. T. Assis. Available at www.ifi.unicamp.br/~assis and http://repositorio.unicamp.br/Acervo/Detalhe/435449.

[Chi64] R. A. Chipman. Contributions from the museum of history and tecnology: Science and technology paper 38: The earliest electromagnetic instruments. *Bulletin of the United States National Museum*, pages 121–136, 1964. Available at https://www.gutenberg.org/ebooks/34061.

[Chr33] S. H. Christie. Experimental determination of the laws of magneto-electric induction in different masses of the same metal, and its intensity in different metals. *Philosophical Transactions*, 123:95–142, 1833.

[Cla52] R. Clausius. Ueber die Anordnung der Elektricität auf einer einzelen sehr dünnen Platte und auf den beiden Belegungen einer Franklin'schen Tafel. *Annalen der Physik und Chemie*, 86:161–205, 1852.

[Col00] E. S. d. Colombo. Traducción de la carta de Alessandro Volta: sobre la electricidad excitada por el simple contacto de substancias conductoras de distintas especies. *Llull: Revista de la Sociedad Española de Historia de las Ciencias y de las Técnicas*, 23:763–784, 2000. Available at Dialnet.unirioja.es/descarga/articulo/2961105.pdf.

[Con82] Congrès International des Électriciens, 1882. G. Masson. Paris. Available at https://archive.org/details/congrsinternati38unkngoog/page/n6/mode/2up.

[Cou88a] C. A. Coulomb. Premier mémoire sur l'électricité et le magnétisme. Construction et usage d'une balance électrique, fondée sur la propriété qu'ont les fils de métal, d'avoir une force de réaction de torsion proportionelle à l'angle de torsion. Détermination expérimentale de la loi suivant laquelle les élémens des corps électrisés du même genre d'électricité, se repoussent mutuellement. *Mémoires de l'Académie royale des Sciences. Année 1785*, 88:569–577, 1788. Published in 1788. Reprinted in A. Potier (ed.), *Collection de Mémoires relatifs a la Physique*, Volume 1: *Mémoires de Coulomb*, pp. 107-115 (Gauthiers-Villars, Paris, 1884).

[Cou88b] C. A. Coulomb. Second mémoire sur l'électricité et le magnétisme. Où l'on détermine, suivant quelles loix le fluide magnétique, ainsi que le fluide électrique, agissent, soit par répulsion, soit par attraction. *Mémoires de l'Académie royale des Sciences. Année 1785*, 88:578–611, 1788. Published in 1788. Reprinted in A. Potier (ed.), *Collection de Mémoires relatifs a la Physique*, Volume 1: *Mémoires de Coulomb*, pp. 116-146 (Gauthiers-Villars, Paris, 1884).

[Cou90a] C. A. Coulomb. Erste Abhandlung über die Elektricität und den Magnetismus. In W. König, editor, *Vier Abhandlungen über die Elektricität und den Magnetismus von Coulomb*, pages 3–11. Wilhelm Engelmann, Leipzig, 1890. Uebersetzt und herausgegeben von W. König.

[Cou90b] C. A. Coulomb. Zweite Abhandlung über die Elektricität und den Magnetismus. In W. König, editor, *Vier Abhandlungen über die Elektricität und den Magnetismus von Coulomb*, pages 12–42. Wilhelm Engelmann, Leipzig, 1890. Uebersetzt und herausgegeben von W. König.

[CS02] R. W. Chabay and B. A. Sherwood. *Matter & Interactions*, volume 2: Electric and Magnetic Interactions. Wiley, New York, 2002.

[Cur09] J. C. Curé. *Einstein on Trial or Metaphysical Principles of Natural Philosophy*. Shirley Ramsey, Caracas, 3rd edition, 2009.

[Dör91] M. Dörries. Prior history and aftereffects: hysteresis and "Nachwirkung" in 19th-century physics. *Historical Studies in the Physical and Biological Sciences*, 22:25–55, 1991.

[Edl67] E. Edlund. Untersuchung über den galvanischen Lichtbogen. *Annalen der Physik und Chemie*, 131:586–607, 1867.

[Edl86] E. Edlund. On the resistance in the voltaic arc. *Philosophical Magazine*, 21:131:366–368, 1886.

[EI38] A. Einstein and L. Infeld. *The Evolution of Physics*. Cambridge University Press, London, 1938.

[Ein05] A. Einstein. Zur Elektrodynamik bewegter Körper. *Annalen der Physik*, 17:891–921, 1905.

[Ein20a] A. Einstein. *Relativity: The Special and General Theory*. Henry Holt and Company, New York, 1920. Translated by R. W. Lawson. Available at www.bartleby.com/173/.

[Ein20b] A. Einstein. *Über die spezielle und die allgemeine Relativitätstheorie (Gemeinverständlich)*. F. Vieweg, Braunschweig, 5th edition, 1920.

[Ein22] A. Einstein. *Sidelights on Relativity*. Methuen, London, 1922. Translated by G. B. Jeffery.

[Ein24] A. Einstein. Über den Äther. *Verhandlungen der Schweizerischen Naturforschenden Gesellschaft*, 105:85–93, 1924.

[Ein52] A. Einstein. On the electrodynamics of moving bodies. In A. Einstein, H. A. Lorentz, H. Weyl and H. Minkowski, *The Principle of Relativity*, pages 35–65, New York, 1952. Dover. Translated by H. Perrett and G. B. Jeffery.

[Ein78] A. Einstein. Sobre a electrodinâmica dos corpos em movimento. In A. Einstein, H. Lorentz, H. Weyl and H. Minkowski, *O Princípio da Relatividade*, Lisboa, 2nd edition, 1978. Fundação Calouste Gulbenkian. Pages 47-86. Translated by M. J. Saraiva.

[Ein91] A. Einstein. On the ether. In S. Saunders and H. R. Brown, editors, *The Philosophy of Vacuum*, pages 13–20. Clarendon, Oxford, 1991. Translated by S. W. Saunders.

[Eke01] S. Ekelöf. The genesis of the Wheatstone bridge. *Engineering Science and Education Journal*, 10:37–40, 2001.

[Erl99] H. Erlichson. Ampère was not the author of "Ampère's circuital law". *American Journal of Physics*, 67:448–450, 1999.

[Euc56] Euclid. *The Thirteen Books of The Elements*, volume 2, Books III-IX. Dover, New York, 1956. Translated with introduction and commentary by Sir Thomas L. Heath.

[Far21] M. Faraday. Sur les Mouvemens électro-magnétiques et la théorie du magnétisme. *Annales de Chimie et de Physique*, 18:337–370, 1821.

[Far22a] M. Faraday. Description of an electro-magnetical apparatus for the exhibition of rotatory motion. *The Quarterly Journal of Science, Literature, and the Arts*, 12:283–285, 1822.

[Far22b] M. Faraday. On some new electro-magnetical motions, and on the theory of magnetism. *The Quarterly Journal of Science, Literature, and the Arts*, 12:74–96, 1822.

[Far26] J. Farrar. *Elements of Electricity, Magnetism, and Electro-magnetism, embracing the Late Discoveries and Improvements, digested into the Form of a Treatise; being the Second Part of a Course of Natural Philosophy.* Cambridge University Press, Cambridge, 1826. This volume, with the exception of the notes, is selected from Biot's *Précis Elémentaire de Physique*, third edition, printed at Paris in 1824, and translated with such alterations as were found necessary in order to adapt it to the English reader.

[Far32a] M. Faraday. Experimental researches in electricity. *Philosophical Transactions*, 122:125–162, 1832. Read November 24, 1831. Reprinted in *Great Books of the Western World*, R. M. Hutchins (editor), (Encyclopaedia Britannica, Chicago, 1952), Vol. 45: Lavoisier, Fourier, Faraday. Pp. 265-285, § 1-139.

[Far32b] M. Faraday. Experimental-Untersuchungen über Elektricität. *Annalen der Physik und Chemie*, 25:91–142, 1832.

[Far46a] M. Faraday. Experimental researches in electricity. Nineteenth series. *Philosophical Transactions*, 136:1–20, 1846. Read November 20, 1845. Reprinted in *Great Books of the Western World*, R. M. Hutchins (editor), (Encyclopaedia Britannica, Chicago, 1952), Vol. 45: Lavoisier, Fourier, Faraday. Pp. 595-607, § 2146-2242.

[Far46b] M. Faraday. Experimental researches in electricity. Twentieth series. *Philosophical Transactions*, 136:21–40, 1846. Read December 18, 1845. Reprinted in *Great Books of the Western World*, R. M. Hutchins (editor), (Encyclopaedia Britannica, Chicago, 1952), Vol. 45: Lavoisier, Fourier, Faraday. Pp. 607-619, § 2243-2342.

[Far46c] M. Faraday. Experimental researches in electricity. Twenty-first series. *Philosophical Transactions*, 136:41–62, 1846. Read January 8, 1846. Reprinted in *Great Books of the Western World*, R. M. Hutchins (editor), (Encyclopaedia Britannica, Chicago, 1952), Vol. 45: Lavoisier, Fourier, Faraday. Pp. 619-632, § 2343-2453.

[Far47] M. Faraday. Ein und zwanzigste Reihe von Experimental-Untersuchungen über Elektricität. *Annalen der Physik und Chemie*, 70:24–59, 1847.

[Far89] M. Faraday. *Experimental-Untersuchungen über Elektricität*, volume I. Springer, Berlin, 1889. Deutsche Uebersetzung von S. Kalischer.

[Far52a] M. Faraday. Description of an electro-magnetical apparatus for the exhibition of rotary motion. In R. M. Hutchins, editor, *Great Books of the Western World,* Vol. 45: *Lavoisier, Fourier, Faraday*, pages 807–809. Encyclopaedia Britannica, Chicago, 1952. Reprint of the Quarterly Journal of Science, Vol. 12, pp. 283-285 (1822).

[Far52b] M. Faraday. *Experimental Researches in Electricity*, volume 45, pp. 253-898 of *Great Books of the Western World*. Encyclopaedia Britannica, Chicago, 1952.

[Far52c] M. Faraday. On some new electro-magnetical motions and on the theory of magnetism. In R. M. Hutchins, editor, *Great Books of the Western World,* Vol. 45: *Lavoisier, Fourier, Faraday*, pages 795–807. Encyclopaedia Britannica, Chicago, 1952. Reprint of the Quarterly Journal of Science, Vol. 12, pp. 74-96 (1822).

[Far11] M. Faraday. Pesquisas experimentais em eletricidade. *Caderno Brasileiro de Ensino de Física*, 28:152–204, 2011. Portuguese translation by A. K. T. Assis and L. F. Haruna. Doi: 10.5007/2175-7941.2011v28n1p152.

[Fec29] G. T. Fechner. *Lehrbuch des Galvanismus und der Elektrochemie*, volume 3. Leopold Voß, Leipzig, 2nd edition, 1829.

[Fec45] G. T. Fechner. Ueber die Verknüpfung der Faraday'schen Inductions-Erscheinungen mit den Ampère'schen elektro-dynamischen Erscheinungen. *Annalen der Physik und Chemie*, 64:337–345, 1845.

[Fec21] G. T. Fechner. On the connection between Faraday's induction phenomena and Ampère's electrodynamic phenomena. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume II: Weber's Fundamental Force and the Unification of the Laws of Coulomb, Ampère and Faraday, pages 19–26, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Fey33a] E. Feyerabend. Der erste praktisch verwendete elektrische Telegraph. *Technikgeschichte*, 22:83–87, 1933.

[Fey33b] E. Feyerabend. *Der Telegraph von Gauss und Weber im Werden der elektrischen Telegraphie*. Reichsportminsterium, Berlin, 1933.

[FLS63] R. P. Feynman, R. B. Leighton, and M. Sands. *The Feynman Lectures on Physics*. Addison-Wesley, Reading, 1963. Volume 1: Mainly Mechanics, Radiation, and Heat.

[FLS64] R. P. Feynman, R. B. Leighton, and M. Sands. *The Feynman Lectures on Physics*. Addison-Wesley, Reading, 1964. Volume 2: Mainly Electromagnetism and Matter.

[Fra81] O. I. Franksen. *H. C. Ørsted — A Man of the Two Cultures*. Strandbergs Forlag, Birkerød, 1981.

[Fre25] A. Fresnel. Ueber das Licht. *Annalen der Physik und Chemie*, 3:89–128, 1825. Doi: 10.1002/andp.18250790107.

[Fri82] H. Fricke. *Two rival programmes in 19th. century classical electrodynamics: action-at-a-distance versus field theories*. PhD thesis, The London School of Economics and Political Science, London, 1982. Available at http://etheses.lse.ac.uk/3430/.

[FS16] É. J. B. Ferreira and A. P. B. Silva. Termomagnetismo ou termoeletricidade? Um estudo do trabalho de Thomas Johan Seebeck. *Caderno Brasileiro de Ensino de Física*, 33:861–878, 2016. Doi: 10.5007/2175-7941.2016v33n3p861.

[GA94] P. Graneau and A. K. T. Assis. Kirchhoff on the motion of electricity in conductors. *Apeiron*, 19:19–25, 1994.

[Gal14] Galileo Galilei. *Dialogues Concerning Two New Sciences*. Macmillan, New York, 1914. Translated by H. Crew and A. d. Salvio.

[Gal85] Galileu Galilei. *Duas Novas Ciências*. Ched Editorial, Istituto Italiano di Cultura and Nova Stella, São Paulo, 1985. Translation and notes by L. Mariconda and P. R. Mariconda.

[Gar04] D. Gardelli. Concepções de interação física: Subsídios para uma abordagem histórica do assunto no ensino médio. Master's thesis, Instituto de Física, Universidade de São Paulo, Brasil, 2004.

[Gau09] C. F. Gauss. *Theoria Motus Corporum Coelestium in Sectionibus Conicis solem Ambientium*. Friedrich Perthes & Johann Heinrich Besser, Hamburg, 1809.

[Gau32] C. F. Gauss. Anzeige der "Intensitas Vis Magneticae Terrestris ad Mensuram Absolutam Revocata". *Göttingische gelehrte Anzeigen*, 205-207:2041–2058, 1832. Vol. 205 from December 24, 1832, pp. 2041-2048; Vols. 206 and 207 from December 27, 1832, pp. 2049-2058. Reprinted in C. F. Gauss's *Werke*, Vol. 5, pp. 293-304 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1867). Corrected version in: Astronomische Nachrichten 10, 1833, Nr. 238, pp. 349-360: Anzeige der Abhandlung des Herrn Hofraths Gauß: Intensitas vis magneticae terrestris ad mensuram absolutam revocata.

[Gau33a] C. F. Gauss. [abstract of the paper:] Intensitas vis magneticae terrestris ad mensuram absolutam revocata. *Philosophical Magazine*, 2:291–299, 1833.

[Gau33b] C. F. Gauss. Die Intensität der erdmagnetischen Kraft, zurückgeführt auf absolutes Maass. *Annalen der Physik und Chemie*, 28:241–273 and 591–615, 1833. Translated by J. C. Poggendorff.

[Gau34a] C. F. Gauss. 128. Stuck. Den 9. August 1834. *Göttingische gelehrte Anzeigen*, 2:1265–1274, 1834. Reprinted in C. F. Gauss's *Werke*, Vol. 5, pp. 519-525 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1867).

[Gau34b] C. F. Gauss. Beobachtungen der magnetischen Variation in Göttingen und Leipzig, am 1. und 2. October 1834. *Annalen der Physik und Chemie*, 33:426–433, 1834. Reprinted in C. F. Gauss's *Werke*, Vol. 5, pp. 525-528 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1867).

[Gau34c] C. F. Gauss. Mesure absolue de l'Intensité du Magnétisme terrestre. *Annales de Chimie et de Physique*, 57:5–69, 1834.

[Gau35] C. F. Gauss. Berichte über die in dem magnetischen Observatorium, und in Verbindung damit anderwärts gemachten Beobachtungen. *Göttingische gelehrte Anzeigen*, 1:345–357, 1835. 36. Stuck der *Göttingische gelehrte Anzeigen*, den 7. Merz 1835. Reprinted in C. F. Gauss's *Werke*, Vol. 5, pp. 528-536 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1867).

[Gau36a] C. F. Gauss, 1836. Ob isměrenii zemnago magnitizma. (Soč[inenie] Karl[a] Frid[richa] Gaussa). Per[evël] A. Drašusov. Učenyja zapiski Imperatorskago universiteta čast 11, 1836, Nr. 7 (Januar), str. 3-22; Nr. 8 (Februar), str. 246-271; Nr. 9 (März), str. 341-381.

[Gau36b] C. F. Gauss. Erdmagnetismus und Magnetometer. In H. C. Schumacher, editor, *Jahrbuch für 1836*, pages 1–47. J. G. Cotta'schen, Stuttgart, 1836. Reprinted in C. F. Gauss's *Werke*, Vol. 5, pp. 315-344 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1867).

[Gau37a] C. F. Gauss. [abstract of the paper:] Intensitas vis magneticae terrestris ad mensuram absolutam revocata. *Abstracts of the Papers Printed in the Philosophical Transactions of the Royal Society of London, from 1830 to 1837 inclusive*, 3:166–174, 1837.

[Gau37b] C. F. Gauss. Ein neues Hülfsmittel für die magnetischen Beobachtungen. *Göttingische gelehrte Anzeigen*, 3:1721–1728, 1837. 173. Stuck. Den 30. October 1837. Reprinted in C. F. Gauss's *Werke*, Vol. 5, pp. 352-356 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1867).

[Gau37c] C. F. Gauss. Misura assoluta dell' intensità della forza magnetica terrestre. *Effemeridi astronomiche di Milano*, primo supplemento:3–132, 1837. Tradotta e commentata da P. Frisiani.

[Gau38a] C. F. Gauss. Anleitung zur Bestimmung der Schwingungsdauer einer Magnetnadel. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1837*, volume II, chapter IV, pages 58–80. Dieterichschen Buchhandlung, Göttingen, 1838. Reprinted in Carl Friedrich Gauss *Werke*, Vol. 5, pp. 374-394 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1867).

[Gau38b] C. F. Gauss. Ueber ein neues, zunächst zur unmittelbaren Beobachtung der Veränderungen in der Intensität des horizontalen Theils des Erdmagnetismus bestimmtes Instrument. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1837*, volume II, chapter I, pages 1–19. Dieterichschen Buchhandlung, Göttingen, 1838. Reprinted in Carl Friedrich Gauss *Werke*, Vol. 5, pp. 357-373 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1867).

[Gau39] C. F. Gauss. Allgemeine Theorie des Erdmagnetismus. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1838*, volume III, chapter I, pages 1–57. Weidmannschen Buchhandlung, Leipzig, 1839. Reprinted in Carl Friedrich Gauss *Werke*, Vol. 5, pp. 119-194 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1867).

[Gau40] C. F. Gauss. Allgemeine Lehrsätze in Beziehung auf die im Verkehrten Verhältnisse des Quadrats der Entfernung wirkenden Anziehungs- und Abstossungs-kräfte. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1839*, volume IV, chapter I, pages 1–51. Weidmannschen Buchhandlung, Leipzig, 1840. Reprinted in Carl Friedrich Gauss *Werke*, Vol. 5, pp. 195-242 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1867).

[Gau41a] C. F. Gauss. General theory of terrestrial magnetism. In R. Taylor, editor, *Scientific Memoirs,* Vol. 2, pages 184–251, London, 1841. Richard and John E. Taylor. Translated by Mrs. Sabine, and revised by Sir John Herschel, Bart.

[Gau41b] C. F. Gauss. Intensitas vis magneticae terrestris ad mensuram absolutam revocata. *Commentationes Societatis Regiae Scientiarum Goettingensis Recentiores*, 8:3–44, 1841. Delivered before the Society in 15 December 1832. Reprinted in C. F. Gauss's *Werke*, Vol. 5, pp. 79-118 (Königliche Gesellschaft der Wissenschaften, Göttingen, 1867).

[Gau41c] C. F. Gauss. On a new instrument for the direct observation of the changes in the intensity of the horizontal portion of the terrestrial magnetic force. In R. Taylor, editor, *Scientific Memoirs,* Vol. 2, pages 252–267, London, 1841. Richard and John E. Taylor. From the *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1837*, Volume II, Chapter I, pp. 1-19. — Herausgegeben von Carl Friederich Gauss und Wilhelm Weber. Göttingen, 1838. Translated by Mr. William Francis, and revised by Major Sabine and Professor Lloyd.

[Gau43] C. F. Gauss. General propositions relating to attractive and repulsive forces acting in the inverse ratio of the square of the distance. In R. Taylor, editor, *Scientific Memoirs,* Vol. 3, pages 153–196, London, 1843. Richard and John E. Taylor.

[Gau57] C. F. Gauss. *Theory of the Motion of Heavenly Bodies Moving about the Sun in Conic Sections.* Little, Brown & Co., Boston, 1857. Translated by C. H. Davis.

[Gau67] C. F. Gauss. Grundgesetz für alle Wechselwirkungen galvanischer Ströme. In *Carl Friedrich Gauss's Werke*, volume 5, pages 616–617. Königlichen Gesellschaft der Wissenschaften zu Göttingen, Göttingen, 1867. Available at https://gdz.sub.uni-goettingen.de/volumes/id/PPN235957348.

[Gau94] C. F. Gauss. Die Intensität der erdmagnetischen Kraft auf absolutes Maass zurückgeführt. In E. Dorn, editor, *Ostwald's Klassiker der exakten Wissenschaften, Vol. 53.* Wilhelm Engelmann Verlag, Leipzig, 1894. Translation by A. Kiel, notes by E. Dorn. Reprinted in: *Carl Friedrich Gauss — Werke — Supplement* — Band 3: Varia: 15 Abhandlungen in deutscher Übersetzung. Mit einer Einleitung, einer Bibliographie und Registern herausgegeben von Karin Reich. Hrsg. von der Akademie der Wissenschaften zu Göttingen, Universitätsverlag Göttingen 2019, S. 533-594.

[Gau35] C. F. Gauss. The absolute measure of magnetic force. In W. F. Magie, editor, *A Source Book in Physics*, pages 519–524, New York, 1935. McGraw-Hill.

[Gau52] C. F. Gauss, 1952. Intensivnost semnoj magnitnoj sily, privedennaja k absoljutnoj mere. In: B. M. Janovskij (Hrsg): Karl Fridrich Gauss. Izbrannye trudy po zemnomu magnetizmu. Perevod A. N. Krylova. Moskau 1952, str. 23-75.

[Gau75] C. F. Gauss. Letter from Gauss to Schumacher, April 29, 1845. In *Carl Friedrisch Gauss, Werke, Ergänzungsreihe, Vol. V, Briefwechsel C. F. Gauss — H. C. Schumacher, Part 2*, pages 436–440. Georg Olms Verlag, Hildesheim, 1975.

[Gau03] C. F. Gauss. The intensity of the earth's magnetic force reduced to absolute measurement. Translated in 1995 from the German by Susan P. Johnson, edited by L. Hecht. Available since 2003 at http://21sci-tech.com/translation.html, 2003.

[Gau21a] C. F. Gauss. Fundamental law for all interactions of galvanic currents. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume IV: Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation, page 11, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Gau21b] C. F. Gauss. [Gauss, 1832, Abstract of the paper:] *intensitas vis magneticae terrestris ad mensuram absolutam revocata*. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume I: Gauss and Weber's Absolute System of Units, pages 37–45, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Gau21c] C. F. Gauss. *Teoría General del Magnetismo Terrestre*. Catarata, Madrid, 2021. Introducción, traducción y notas de J. M. Vaquero.

[Gau21d] C. F. Gauss. The absolute measure of the earth's magnetic force. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume I: Gauss and Weber's Absolute System of Units, pages 49–80, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Gau d] C. F. Gauss, [s. d.]. The complete correspondence of Carl Friedrich Gauß. Site of the Department "Digital Library" of the State and University Library in Göttingen on behalf of the Academy of Sciences in Göttingen. The metadata of the letters have been made available by Prof. Dr. Menso Folkerts. Available at https://gauss.adw-goe.de.

[Gil71a] C. S. Gillmor. *Coulomb and the Evolution of Physics and Engineering in Eighteenth-Century France*. Princeton University Press, Princeton, 1971.

[Gil71b] C. S. Gillmor. Coulomb, Charles Augustin. In C. C. Gillispie, editor, *Dictionary of Scientific Biography,* Vol. 3, pages 439–447. Charles Scribner's Sons, New York, 1971.

[Gri89] D. J. Griffiths. *Introduction to Electrodynamics*. Prentice Hall, Englewood Cliffs, 2nd edition, 1989.

[Gro39] W. R. Grove. Volta'sche Säule von großser elektro-chemischer Kraft. *Annalen der Physik und Chemie*, 48:300–304, 1839.

[GT14] K. H. Glassmeier and B. T. Tsurutani. Carl Friedrich Gauss — General Theory of Terrestrial Magnetism — a revised translation of the German text. *History of Geo- and Space Sciences*, 5:11–62, 2014. Doi: 10.5194/hgss-5-11-2014.

[GT19] P. M. Grant and J. S. Thompson. Standardization of the Ohm as a unit of electrical resistance, 1861-1867. *Proceedings of the IEEE*, 107:2281–2289, 2019. Doi: 10.1109/JPROC.2019.2945495.

[GV02] J. Guala-Valverde. Why homopolar devices cannot be additive? *Spacetime & Substance*, 3:186–187, 2002.

[GVM01] J. Guala-Valverde and P. Mazzoni. The unipolar dynamotor: a genuine relational engine. *Apeiron*, 8:41–52, 2001.

[GVMA02] J. Guala-Valverde, P. Mazzoni, and R. Achilles. The homopolar motor: A true relativistic engine. *American Journal of Physics*, 70:1052–1055, 2002.

[GW37] C. F. Gauss and W. Weber. *Resultate aus den Beobachtungen des magnetisches Vereins im Jahre 1836*, volume I. Dieterichschen Buchhandlung, Göttingen, 1837.

[GW38] C. F. Gauss and W. Weber. *Resultate aus den Beobachtungen des magnetisches Vereins im Jahre 1837*, volume II. Dieterichschen Buchhandlung, Göttingen, 1838.

[GW39] C. F. Gauss and W. Weber. *Resultate aus den Beobachtungen des magnetisches Vereins im Jahre 1838*, volume III. Dieterichschen Buchhandlung, Göttingen, 1839.

[GW40a] C. F. Gauss and W. Weber. *Atlas des Erdmagnetismus nach den Elementen der Theorie Entworfen — Supplement zu den Resultaten aus den Beobachtungen des magnetisches Vereins unter Mitwirkung von C. W. B. Goldschmidt*. Weidmann'sche Buchhandlung, Leipzig, 1840. Reprinted in C. F. Gauss's *Werke*, Vol. 12, pp. 335-408 (Königlichen Gesellschaft der Wissenschaften, Göttingen, 1929).

[GW40b] C. F. Gauss and W. Weber. *Resultate aus den Beobachtungen des magnetisches Vereins im Jahre 1839*, volume IV. Weidmannschen Buchhandlung, Leipzig, 1840.

[GW41] C. F. Gauss and W. Weber. *Resultate aus den Beobachtungen des magnetisches Vereins im Jahre 1840*, volume V. Weidmannschen Buchhandlung, Leipzig, 1841.

[GW43] C. F. Gauss and W. Weber. *Resultate aus den Beobachtungen des magnetisches Vereins im Jahre 1841*, volume VI. Weidmannschen Buchhandlung, Leipzig, 1843.

[Han74] W. G. Hankel. *Elektrische Untersuchungen*, volume 9: Über die thermoelektrischen Eigenschaften des Schwerspathes. Hirzel, Leipzig, 1874.

[Hea89] O. Heaviside. On the electromagnetic effects due to the motion of electrification through a dielectric. *Philosophical Magazine*, 27:324–339, 1889.

[Hei81] J. L. Heilbron. The electrical field before Faraday. In G. N. Cantor and M. J. S. Hodge, editors, *Conceptions of Ether: Studies in the History of Ether Theories 1740-1900*, pages 187–213. Cambridge University Press, Cambridge, 1981.

[Hel47] H. von Helmholtz. *Über die Erhaltung der Kraft*. Engelmann, Leipzig, 1847. Reprinted in H. Helmholtz, Wissenschaftliche Abhandlungen (Johann Ambrosius Barth, Leipzig, 1882), Vol. 1, Article 2, pp. 12-75.

[Hel53] H. Helmholtz. On the conservation of force; a physical memoir. In J. Tyndall and W. Francis, editors, *Scientific Memoirs,* Vol. on Natural Philosophy, pages 114–162, London, 1853. Taylor and Francis. Translated by J. Tyndall. Available at https://books.google.com.br/books?id=C1i4AAAAIAAJ&hl=pt-BR&source=gbs_navlinks_s.

[Hel72a] H. Helmholtz. Ueber die Theorie der Elektrodynamik. *Monatsberichte der Berliner Akademie der Wissenschaften*, pages 247–256, 1872. Reprinted in H. Helmholtz, Wissenschaftliche Abhandlungen (Johann Ambrosius Barth, Leipzig, 1882), Vol. 1, Article 34, pp. 636-646.

[Hel72b] H. von Helmholtz. On the theory of electrodynamics. *Philosophical Magazine*, 44:530–537, 1872.

[Hel73] H. v. Helmholtz. Ueber die Theorie der Elektrodynamik. Zweite Abhandlung. Kritisches. *Journal für die reine und angewandte Mathematik*, 75:35–66, 1873. Reprinted in H. Helmholtz, Wissenschaftliche Abhandlungen (Johann Ambrosius Barth, Leipzig, 1882), Vol. 1, Article 35, pp. 647-683; with additional material from 1881 on pp. 684-687.

[Hel82] H. Helmholtz. Ueber die Theorie der Elektrodynamik. In H. Helmholtz, editor, *Wissenschaftliche Abhandlungen*, volume 1, pages 636–646. Johann Ambrosius Barth, Leipzig, 1882.

[Hel66] H. von Helmholtz. On the conservation of force; a physical memoir. In J. Tyndall and W. Francis, editors, *Scientific Memoirs,* Vol. on Natural Philosophy, pages 114–162, New York, 1966. Johnson Reprint Corporation. Translated by J. Tyndall.

[Hen11] J. Henry. Gravity and *de gravitatione*: the development of Newton's ideas on action at a distance. *Studies in History and Philosophy of Science*, 42:11–27, 2011. Doi: 10.1016/j.shpsa.2010.11.025.

[Hen14] J. Henry. Newton and action at a distance between bodies—A response to Andrew Janiak's "Three concepts of causation in Newton". *Studies in History and Philosophy of Science*, 47:91–97, 2014. Doi: 10.1016/j.shpsa.2014.03.001.

[Hen19] J. Henry. Newton and action at a distance. In E. Schliesser and C. Smeenk, editors, *The Oxford Handbook of Newton*, pages 1–34, Oxford, 2019. Oxford University Press. Doi: 10.1093/oxfordhb/9780199930418.013.17.

[Hen20] J. Henry. Primary and secondary causation in Samuel Clarke's and Isaac Newton's theories of gravity. *Isis*, 111:542–561, 2020.

[Her00] H. Hertz. *Electric Waves: Being Researches on the Propagation of Electric Action With Finite Velocity Through Space*. Macmillan, London, 2nd edition, 1900. Translated by D. E. Jones.

[HM95] M. A. Heald and J. B. Marion. *Classical Electromagnetic Radiation*. Saunders, Fort Worth, 3rd edition, 1995.

[Hob13] A. Hobson. There are no particles, there are only fields. *American Journal of Physics*, 81:211–223, 2013. Doi: 10.1119/1.4789885.

[Hur10] P. G. Huray. *Maxwell's Equations*. Wiley: IEEE, Hoboken, 2010.

[Jac51] Jacobi. Sur quelques points de la galvanométrie. *Comptes Rendues de l'Académie des Sciences de Paris*, 33:277–282, 1851.

[Jac66] C. G. J. Jacobi. *Vorlesungen über Dynamik*. Reimer, Berlin, 1866. Herausgegeben von A. Clebsch.

[Jac84] C. G. J. Jacobi. *Vorlesungen über Dynamik*. Reimer, Berlin, 1884. 2. Auflage 1884 als Supplementband der Gesammelten Werke von C. G. J. Jacobi (Herausgegeben von E. Lottner).

[Jac75] J. D. Jackson. *Classical Electrodynamics*. John Wiley & Sons, New York, second edition, 1975.

[Jac09] C. G. J. Jacobi. Lectures on dynamics. In A. Clebsch, editor, *Jacobi's Lectures on Dynamics*. Hindustan Book Agency, New Delhi, 2nd edition, 2009. Texts and Readings in Mathematics, Volume 51. Delivered at the University of Königsberg in the winter semester 1842-1843 and according to the notes prepared by C. W. Brockardt. Translated by K. Balagangadharan. Translation edited by B. Banerjee. Available at https://link.springer.com/book/10.1007/978-93-86279-62-0.

[Jan09] P. Janet. La vie et les oeuvres d'Eleuthère Mascart. *Revue Générale des Sciences Pures et Appliquées*, 20:574–593, 1909.

[Jea41] J. Jeans. *The Mathematical Theory of Electricity and Magnetism*. Cambridge University Press, Cambridge, 1941. Reprint of the 5th edition of 1925.

[Jou41a] J. P. Joule. On the heat evolved by metallic conductors of electricity, and in the cells of a battery during electrolysis. *Philosophical Magazine*, 19:260–277, 1841. Reprinted in The Annals of Electricity, Magnetism, and Chemistry; and Guardian of Experimental Science, Vol. 8, pp. 287-301 (1842).

[Jou41b] J. P. Joule. On the production of heat by voltaic electricity. *Proceedings of the Royal Society of London*, 4:280–281, 1841. Reprinted in Athenaeum, Vol. 697, 192 (1841) and Philosophical Magazine, Vol. 18, pp. 308-309 (1841).

[Jou42] J. P. Joule. Sur la chaleur développée dans les conducteurs métalliques et dans les auges d'une pile sous l'influence de lélectricité. *Archives de l'Électricité. Supplément a la Bibliothèque Universelle de Genève*, 2:54–79, 1842.

[Jou43] J. P. Joule. On the caloric effects of magneto-electricity and on the mechanical value of heat. *Philosophical Magazine*, 23:263–276, 347–355 and 435–443, 1843.

[Ker13] M. Kersting, 2013. Der Gauss-Weber-Telegraph. In: Sammlung und Physikalisches Museum. Universität Goettingen - Fakultät für Physik, p. 7. Available at https://www.uni-goettingen.de/de/document/download/0188fd8f56a96739a307f28545ed41d2.pdf/Gauss_Weber_Telegraf_Magdalena%20Kersting.pdf.

[Kir49] G. Kirchhoff. Ueber eine Ableitung der Ohm'schen Gesetze, welche sich an die Theorie der Elektrostatik anschliesst. *Annalen der Physik und Chemie*, 78:506–513, 1849. Reprinted in G. Kirchhoff's *Gesammelte Abhandlungen* (Barth, Leipzig, 1882), pp. 49-55.

[Kir50] G. Kirchhoff. On a deduction of Ohm's law, in connexion with the theory of electrostatics. *Philosophical Magazine*, 37:463–468, 1850.

[Kir54] G. Kirchhoff. Démonstration des lois de Ohm fondée sur les principes ordinaires de l'Électricité statique. *Annales de Chimie et de Physique*, 41:496–500, 1854.

[Kir57a] G. Kirchhoff. On the motion of electricity in wires. *Philosophical Magazine*, 13:393–412, 1857. Available at https://archive.org/stream/londonedinburghp13maga#page/392/mode/2up.

[Kir57b] G. Kirchhoff. Ueber die Bewegung der Elektricität in Drähten. *Annalen der Physik und Chemie*, 100:193–217, 1857. Reprinted in G. Kirchhoff's *Gesammelte Abhandlungen* (Barth, Leipzig, 1882), pp. 131-154.

[Kir57c] G. Kirchhoff. Ueber die Bewegung der Elektricität in Leitern. *Annalen der Physik und Chemie*, 102:529–544, 1857. Reprinted in G. Kirchhoff's *Gesammelte Abhandlungen* (Barth, Leipzig, 1882), pp. 154-168.

[Kir21a] G. Kirchhoff. On a deduction of Ohm's laws, in connexion with the theory of electrostatics. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume II: Weber's Fundamental Force and the Unification of the Laws of Coulomb, Ampère and Faraday, pages 261–265, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Kir21b] G. Kirchhoff. On the motion of electricity in conductors. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume III: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity, pages 229–241, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Kir21c] G. Kirchhoff. On the motion of electricity in wires. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume III: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity, pages 201–220, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Koh73] F. Kohlrausch. Zurückführung der Siemens'schen Widerstandseinheit auf absolutes Maass. *Annalen der Physik und Chemie*, 6 (Supplementary Volume):1–35, 1873.

[Koh74] F. Kohlrausch. Ueber Thermoelektricität, Wärme- und Elektricitätsleitung. *Nachrichten der Göttinger Gesellschaft der Wissenschaften*, 4:65–86, 1874.

[Koh83] F. Kohlrausch. *An Introduction to Physical Measurements — with appendices on absolute electrical measurement, etc.* J. & A. Churchill, London, 2nd edition, 1883. Translated from the fourth German edition by T. H. Waller and H. R. Procter.

[KS05] S. Krapas and M. C. d. Silva. Forças que atuam a distância: representações em livros didáticos do ensino médio e a história da ciência. In M. A. Moreira, C. C. Sahelices, and J. M. Villagrá, editors, *Actas do II Encuentro Iberoamericano sobre Investigación Básica em Educación em Ciências*, pages 351–366. Servicios de Publicaciones de la Universidad de Burgos, Burgos, 2005.

[KS08] S. Krapas and M. C. d. Silva. O conceito de campo: polissemia nos manuais, significados na física do passado e da atualidade. *Ciência & Educação*, 14:15–33, 2008.

[KW57] R. Kohlrausch and W. Weber. Elektrodynamische Maassbestimmungen insbesondere Zurückführung der Stromintensitäts-Messungen auf mechanisches Maass. *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig, mathematisch-physischen Classe*, 3:221–292, 1857. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 609-676.

[KW21] R. Kohlrausch and W. Weber. Electrodynamic measurements, fourth memoir, specially attributing mechanical units to measures of current intensity. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume III: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity, pages 141–199, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[LA98] E. K. Lauridsen and N. Abrahamsen. The history of astatic magnet systems and suspensions. *Centaurus*, 40:135–169, 1998.

[Lan09] P. Langevin. L'Oeuvre de E. Mascart. *La Revue du Mois*, pages 385–406, 1909. Available at https://www.annales.org/archives/x/mascart.html.

[Lar82] D. B. Larson. *The Neglected Facts of Science*. North Pacific Publishers, Portland, 1982. Available at https://www.calameo.com/books/0072567018dc66898732d.

[LB67] D. Lardner and E. B. Bright. *The Electric Telegraph*. James Walton, London, 3rd edition, 1867. A new edition revised and re-written by E. B. Bright.

[Lei48] Leibniz. *The Monadology and Other Philosophical Writings*. Oxford University Press, Oxforc, 1948. Translated with introduction and notes by R. Latta. Available at https://ia803402.us.archive.org/22/items/monadologyotherp00gott/monadologyotherp00gott.pdf.

[Len34] E. Lenz. Ueber die Bestimmung der Richtung der durch elektrodynamische Vertheilung erregten galvanischen Ströme. *Annalen der Physik und Chemie*, 31:483–494, 1834.

[Len43a] E. Lenz. Ueber die Gesetze der Wärme-Entwicklung durch den galvanischen Strom. *Annalen der Physik und Chemie*, 59:203–240, 1843.

[Len43b] E. Lenz. Ueber die Gesetze der Wärme-Entwicklung durch den galvanischen Strom (fortsetzung). *Annalen der Physik und Chemie*, 59:407–420, 1843.

[Len43c] H. F. E. Lenz. Ueber die Gesetze der Wärme-Entwicklung durch den galvanischen Strom. *Bulletin de la Classe Physico-mathématique de l'Académie Impériale des Sciences de Saint-Pétersbourg*, 1:column 209 – column 253, 1843.

[Len44a] E. Lenz. Ueber die Gesetze der Wärme-Entwicklung durch den galvanischen Strom. *Annalen der Physik und Chemie*, 61:18–49, 1844.

[Len44b] H. F. E. Lenz. Ueber die Gesetze der Wärme-Entwicklung durch den galvanischen Strom. *Bulletin de la Classe Physico-mathématique de l'Académie Impériale des Sciences de Saint-Pétersbourg*, 2:column 161 – column 188, 1844.

[Len69] H. F. E. Lenz. Lenz' law. In W. F. Magie, editor, *A Source Book in Physics*, pages 511–513, New York, 1969. McGraw-Hill. Extract from a paper published in the *Annalen der Physik und Chemie*, vol. 31, p. 483, 1834, entitled "Ueber die Bestimmung der Richtung der durch elektrodynamische Vertheilung erregten galvanischen Ströme.".

[LL75] L. D. Landau and E. M. Lifshitz. *The Classical Theory of Fields*. Butterworth-Heinemann, Burlington, 1975. Fourth revised English edition. Course of Theoretical Physics, Volume 2.

[Llo80] J. T. Lloyd. Lord Kelvin demonstrated. *The Physics Teacher*, 18:16–24, 1980.

[Llo07] J. T. Lloyd. Lorde Kelvin demonstrado. *Revista Brasileira de Ensino de Física*, 29:499–508, 2007. Translation by A. K. T. Assis of J. T. Lloyd, "Lord Kelvin demonstrated," The Physics Teacher, Vol. 18, pp. 16-24 (1980).

[Lor92] H. A. Lorentz. *La Théorie Électromagnétique de Maxwell et Son Application aux Corps Mouvants*. E. J. Brill, Leiden, 1892. Extrait des *Archives néerlandaises des Sciences exactes et naturelles*, T. XXV.

[Lor95] H. A. Lorentz. *Versuch einer Theorie der Electrischen und Optischen Erscheinungen in Bewegten Körpern*. E. J. Brill, Leiden, 1895. Reprinted in H. A. Lorentz, Collected Papers (Springer, Dordrecht, 1937), Vol. V, pp. 1-138. English Wikisource translation available at https://en.wikisource.org/wiki/Translation:Attempt_of_a_Theory_of_Electrical_and_Optical_Phenomena_in_Moving_Bodies.

[Lor09] H. A. Lorentz. *The Theory of Electrons and Its Applications to the Phenomena of Light and Radiant Heat*. Macmillan, New York, 1909. A course of lectures delivered in Columbia University, New York, in March and April, 1906.

[Lor15] H. A. Lorentz. *The Theory of Electrons*. Teubner, Leipzig, second edition, 1915. Reprinted in *Selected Works of H. A. Lorentz*, vol. 5, N. J. Nersessian (ed.), (Palm Publications, Nieuwerkerk, 1987).

[Lor31] H. A. Lorentz. *Lectures on Theoretical Physics*, volume 3. MacMilan, London, 1931.

[LSN21] A. R. d. S. Lima, A. P. B. d. Silva, and L. F. d. Nascimento. Uma proposta histórica e experimental para o estudo dos multiplicadores do efeito magnético. *Experiências em Ensino de Ciências*, 16:185–206, 2021.

[MA08] C. P. Magnaghi and A. K. T. Assis. Sobre a eletricidade excitada pelo simples contato entre substâncias condutoras de tipos diferentes — Uma tradução comentada do artigo de Volta de 1800 descrevendo sua invenção da pilha elétrica. *Caderno Brasileiro de Ensino de Física*, 25:118–140, 2008.

[Mal82] S. R. C. Malin. Sesquicentenary of Gauss's first measurement of the absolute value of magnetic intensity. *Philosophical Transactions*, 306:5–8, 1982.

[Mar84] R. d. A. Martins. Mayer e a conservação da energia. *Cadernos de História e Filosofia da Ciência*, 6:63–84, 1984.

[Mar86] R. d. A. Martins. Ørsted e a descoberta do eletromagnetismo. *Cadernos de História e Filosofia da Ciência*, 10:89–114, 1986.

[Mar90] S. Marinov. *The Thorny Way of Truth*, volume VIII. International Publishers East-West, Graz, 1990.

[Mar22] R. d. A. Martins. Joule's 1840 manuscript on the production of heat by voltaic electricity. *Notes and Records*, 76:117–153, 2022. Doi: 10.1098/rsnr.2020.0027.

[Max62] J. C. Maxwell, 1861-62. On physical lines of force. Philosophical Magazine, Vol. 21, pp. 161-175, 281-291 and 338-348 (1861); Vol. 23, pp. 12-24 and 85-95 (1862). Reprinted in W. D. Niven (ed.), The Scientific Papers of James Clerk Maxwell (Cambridge University Press, Cambridge, 1890), Vol. 1, pp. 451-513.

[Max58] J. C. Maxwell. On Faraday's lines of force. *Transactions of the Cambridge Philosophical Society*, 10:27–83, 1858. Read Dec. 10, 1855, and Feb. 11, 1856. Reprinted in W. D. Niven (ed.), The Scientific Papers of James Clerk Maxwell (Cambridge University Press, Cambridge, 1890), Vol. 1, pp. 155-229.

[Max65] J. C. Maxwell. A dynamical theory of the electromagnetic field. *Philosophical Transactions*, 155:459–512, 1865. Read December 8, 1864. Reprinted in W. D. Niven (ed.), The Scientific Papers of James Clerk Maxwell (Cambridge University Press, Cambridge, 1890), Vol. 1, pp. 526-597.

[Max67] J. C. Maxwell. On the dynamical theory of gases. *Philosophical Transactions*, 157:49–88, 1867. Reprinted in W. D. Niven (ed.), The Scientific Papers of James Clerk Maxwell (Cambridge University Press, Cambridge, 1890), Vol. 2, pp. 26-78.

[Max73a] J. C. Maxwell. *A Treatise on Electricity and Magnetism*, volume I. Clarendon Press, Oxford, 1873.

[Max73b] J. C. Maxwell. *A Treatise on Electricity and Magnetism*, volume II. Clarendon Press, Oxford, 1873.

[Max54] J. C. Maxwell. *A Treatise on Electricity and Magnetism*. Dover, New York, 1954. Two volumes. Unabridged republication of the third edition of 1891.

[Max65a] J. C. Maxwell. A dynamical theory of the electromagnetic field. In W. D. Niven, editor, *The Scientific Papers of James Clerk Maxwell*, pages 526–597 (vol. 1), New York, 1965. Dover. Article originally published in 1865.

[Max65b] J. C. Maxwell. On Faraday's lines of force. In W. D. Niven, editor, *The Scientific Papers of James Clerk Maxwell*, pages 155–229 (vol. 1), New York, 1965. Dover. Article originally published in 1855.

[Max65c] J. C. Maxwell. On physical lines of force. In W. D. Niven, editor, *The Scientific Papers of James Clerk Maxwell*, pages 451–513 (vol. 1), New York, 1965. Dover. Article originally published in 1861/2.

[May42] J. R. Mayer. Bermerkungen über die Kräfte der unbelebten Natur. *Annalen der Chemie und Pharmacie*, 42:233–240, 1842.

[May62] J. R. Mayer. Remarks on the forces of inorganic nature. *Philosophical Magazine*, 24:371–377, 1862. Translated by G. C. Foster.

[May84] J. R. Mayer. Observações sobre as forças da natureza inanimada. *Cadernos de História e Filosofia da Ciência*, 6:85–95, 1984. Translated by R. d. A. Martins.

[MB82] S. R. C. Malin and D. R. Barraclough. 150th anniversary of Gauss's first absolute magnetic measurement. *Nature*, 297:285, 1982.

[Mey54] A. M. Meyner. *Geschichte der Stadt Wittenberg: aus archivalischen und andern zuverlässigen Quellen geschöpft und bearbeitet*. Hermann Reubürger, Dessau, 1854.

[MF94] F. Merkel and O. Fischer. Untitled. In *Wilhelm Weber's Werke*, volume 6: Mechanik der menschlichen Gehwerkzeuge, page iii, Berlin, 1894. Springer.

[MJ64] J. C. Maxwell and F. Jenkin. On the elementary relations between electrical measurements. *Report of the Thirty-Third Meeting of the British Association for the Advancement of Science; held at Newcastle-upon-Tyne in August and September 1863*, pages 130–163, 1864.

[MJ65] J. C. Maxwell and F. Jenkin. On the elementary relations between electrical measurements. *Philosophical Magazine*, 29:507–525, 1865. Doi: 10.1080/14786446508643913.

[MRGL10] F. Martin-Rodriguez, G. B. Garcia, and M. A. Lires. Technological archaeology: technical description of the Gauss-Weber telegraph. In IEEE, editor, *2010 Second Region 8 IEEE Conference on the History of Communications*, Madrid, 2010. Histelcon. Doi: 10.1109/HISTELCON.2010.5735309.

[MS20] R. A. Martins and A. P. B. Silva. Joule's experiments on the heat evolved by metallic conductors of electricity. *Foundations of Science*, pages 1–77, 2020. Doi: 10.1007/s10699-020-09681-1.

[Ner33] J. J. Nervander. Mémoire sur un galvanomètre à châssis cylindrique par lequel on obtient immédiatement et sans calcul la mesure de l'intensité du courant électrique qui produit la déviation de l'aiguille aimantée. *Annales de Chimie et de Physique*, 55:156–184, 1833.

[Ner89] N. J. Nersessian. Faraday's field concept. In D. Gooding and F. A. J. L. James, editors, *Faraday Rediscovered*, pages 175–187. Macmillan Press, Basingstoke, 1989.

[Neu46] F. E. Neumann. Allgemeine Gesetze der inducirten elektrischen Ströme. *Annalen der Physik und Chemie*, 67:31–43, 1846.

[Neu47] F. E. Neumann. Allgemeine Gesetze der inducirten elektrischen Ströme. *Abhandlungen der Königlichen Akademie der Wissenschaften zu Berlin*, pages 1–87, 1847. Gelesen in der Akademie der Wissenschaften am 27. October 1845.

[Neu48] F.-E. Neumann. Essai d'une théorie mathématique de l'induction. *Journal de mathématiques pures et appliquées*, 13:113–178, 1848. Traduit par M. A. Bravais.

[Neu58] C. Neumann. *Explicare tentatur quomodo fiat ut lucis planum polarisationis per vires electricas vel magneticas declinetur*. Halis Saxonum, 1858.

[Neu63] C. Neumann. *Die magnetische Drehung der Polarisationsebene des Lichts — Versuch einer mathematischen Theorie*. Verlag des Buchhandlung des Waisenhauses, Halle, 1863.

[Neu68a] C. Neumann. *Die Principien der Elektrodynamik. Eine mathematische Untersuchung*. H. Laupp, Tübingen, 1868. Reprinted in *Mathematische Annalen*, Vol. 17, pp. 400-434 (1880).

[Neu68b] C. Neumann. Resultate einer Untersuchung über die Principien der Elektrodynamik. *Nachrichten der Göttinger Gesellschaft der Wissenschaften*, 10:223–234, 1868.

[Neu69] C. Neumann. Notizen zu einer kürzlich erschienenen Schrift über die Principien der Elektrodynamik. *Mathematische Annalen*, 1:317–324, 1869.

[Neu71] C. Neumann. Elektrodynamische Untersuchungen mit besonderer Rücksicht auf das Princip der Energie. *Berichte über die Verhandlungen der Königlich Sächsischen Gesellshaft der Wissenschaften zu Leipzig. Mathemathisch-Physische Classe*, 23:386–449, 1871.

[Neu72] C. Neumann. Ueber die Elementargesetze der Kräfte elektrodynamischen Ursprungs. *Mathematische Annalen*, 5:602–624, 1872.

[Neu74] C. Neumann. Ueber das von Weber für die elektrischen Kräfte aufgestellte Gesetz. *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig, mathematisch-physischen Classe*, 11:77–200, 1874.

[Neu21a] C. Neumann. Notes on a recently published essay on the principles of electrodynamics. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume IV: Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation, pages 49–54, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Neu21b] C. Neumann. Principles of electrodynamics. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume IV: Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation, pages 17–47, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[New34] I. Newton. *Mathematical Principles of Natural Philosophy*. University of California Press, Berkeley, 1934. Cajori edition.

[New52] I. Newton. *Mathematical Principles of Natural Philosophy*, volume 34, pp. 1-372 of *Great Books of the Western World*. Encyclopaedia Britannica, Chicago, 1952. Translated by A. Motte and revised by F. Cajori.

[New79] I. Newton. *Opticks*. Dover, New York, 1979.

[New90] I. Newton. *Principia — Princípios Matemáticos de Filosofia Natural*. Nova Stella/Edusp, São Paulo, 1990. Livro I: O Movimento dos Corpos. Portuguese translation by T. Ricci, L. G. Brunet, S. T. Gehring and M. H. C. Célia.

[New96] I. Newton. *Óptica*. Edusp, São Paulo, 1996. Portuguese translation, introduction and notes by A. K. T. Assis.

[New99] I. Newton. *The Principia: Mathematical Principles of Natural Philosophy*. University of California Press, Berkeley, 1999. A new translation by I. B. Cohen and A. Whitman, assisted by J. Budenz.

[New08] I. Newton. *Principia — Princípios Matemáticos de Filosofia Natural*. Edusp, São Paulo, 2008. Livro II: O Movimento dos Corpos (em Meios com Resistência). Livro III: O Sistema do Mundo (Tratado Matematicamente). Portuguese translation by A. K. T. Assis.

[New10] I. Newton. *Principia — Princípios Matemáticos de Filosofia Natural*. Folha de São Paulo, São Paulo, 2010. Livro III: O Sistema do Mundo (Tratado Matematicamente). Coleção Folha de São Paulo: Livros que Mudaram o Mundo, Volume 9. Portuguese translation by A. K. T. Assis.

[Nor96] H. S. Norrie. *Ruhmkorff Induction-Coils, Their Construction, Operation and Application*. Spon & Chamberlain, New York, 1896.

[Oer20a] H. C. Oersted. Expériences sur l'effet du conflict électrique sur l'aiguille aimantée. *Annales de Chimie et de Physique*, 14:417–425, 1820.

[Oer20b] H. C. Oersted. Experimenta circa effectum conflictus electrici in acum magneticam. Copenhagen, Denmark, self-published. Reprinted in O. I. Frankson, H. C. Ørsted: A Man of the Two Cultures (Birkerød, Strandbergs Forlag, 1981), pp. 19-22, 1820.

[Oer20c] H. C. Oersted. Experiments on the effect of a current of electricity on the magnetic needle. *Annals of Philosophy*, 16:273–277, 1820. Translated from a printed account drawn up in Latin by the author and transmitted by him to the Editor of the *Annals of Philosophy*. Reprinted in Selected Works of Hans Christian Ørsted (Princeton, Princeton University Press, 1998), translated and edited by K. Jelved, A. D. Jackson and O. Knudsen.

[Oer20d] J. C. Oersted. Versuche über die Wirkung des electrischen Conflicts auf die Magnetnadel. *Annalen der Physik und Chemie*, 6:295–304, 1820. Translated by Gilbert.

[Oer65] H. C. Oersted. Experiments on the effect of a current of electricity on the magnetic needle. In R. A. R. Tricker, *Early Electrodynamics — The First Law of Circulation*, pages 113–117, New York, 1965. Pergamon. Translation from Thomson's *Annals of Philosophy*, October 1820. Translated from a printed account drawn up in Latin by the author and transmitted by him to the Editor of the *Annals of Philosophy*.

[Ohm26a] G. S. Ohm. Bestimmung des Gesetzes, nach welchem Metalle die Kontakt-Elektrizität leiten, nebst einem Entwurfe zu einer Theorie des Voltaschen Apparates und des Schweiggerschen Multiplikators. *Journal für Chemie und Physik*, 46:137–166, 1826. Reprinted in *Ostwald's Klassiker der exakten Wissenschaften*, Nr. 244, C. Piel (ed.), (Akademische Verlagsgesellschaft, Leipzig, 1938), pp. 8-29.

[Ohm26b] G. S. Ohm. Ein Nachtrag zu dem vorstehenden Aufsatz. *Annalen der Physik und Chemie*, 7:117–118, 1826.

[Ohm26c] G. S. Ohm. Versuch einer Theorie der durch galvanische Kräfte hervorgebrachten elektroskopischen Erscheinungen. *Annalen der Physik und Chemie*, 6:459–469, 1826.

[Ohm26d] G. S. Ohm. Versuch einer Theorie der durch galvanische Kräfte hervorgebrachten elektroskopischen Erscheinungen (Beschluss). *Annalen der Physik und Chemie*, 7:45–54, 1826.

[Ohm27] G. S. Ohm. *Die Galvanische Kette, mathematisch bearbeitet*. T. H. Riemann, Berlin, 1827.

[Ohm60] G. S. Ohm. *Théorie Mathématique des Courants Électriques*. L. Hachette et Co., Paris, 1860. Traduction, préface et notes de J.-M. Gaugain.

[Ohm66] G. S. Ohm. The galvanic circuit investigated mathematically. In R. Taylor, editor, *Scientific Memoirs,* Vol. 2, pages 401–506, New York, 1966. Johnson Reprint Corporation. English translation by W. Francis.

[O'R65] A. O'Rahilly. *Electromagnetic Theory — A Critical Examination of Fundamentals*. Dover, New York, 1965.

[Ørs86] H. C. Ørsted. Experiências sobre o efeito do conflito elétrico sobre a agulha magnética. *Cadernos de História e Filosofia da Ciência*, 10:115–122, 1986. Translated by R. d. A. Martins.

[Ørs98] H. C. Ørsted. Experiments on the effect of the electric conflict on the magnetic needle. In K. Jelved, A. D. Jackson, and O. Knudsen, editors, *Selected Scientific Works of Hans Christian Ørsted*, pages 413–416. Princeton University Press, Princeton, 1998. Article originally written in Latin in 1820.

[Pai82] A. Pais. *'Subtle is the Lord...' — The Science and the Life of Albert Einstein*. Oxford University Press, Oxford, 1982.

[Pai86] A. Pais. *Inward Bound — Of Matter and Forces in the Physical World*. Clarendon Press, Oxford, 1986.

[Pel34] Peltier. Nouvelles expériences sur la caloricité des courans électriques. *Annales de Chimie et de Physique*, 56:371–386, 1834.

[Pog26] J. C. Poggendorff. Ein Vorschlag zum Messen der magnetischen Abweichung. *Annalen der Physik und Chemie*, 7:121–130, 1826.

[Pog57] J. C. Poggendorff. Bemerkung zu dem Aufsatz des Herrn Prof. Kirchhoff. *Annalen der Physik und Chemie*, 100:351–352, 1857. Reprinted in Wilhelm Weber's *Werke*, Vol. 4, p. 242, H. Weber (ed.), (Springer, Berlin, 1894).

[Pog21] J. C. Poggendorff. Comment on the paper by Prof. Kirchhoff. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume III: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity, page 225, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Poi12a] Poisson. Extrait d'un mémoire sur la distribution de l'électricité à la surface des corps conducteurs. *Journal de Physique, de Chimie, d'Histoire Naturelle et des Arts*, 75:229–237, 1812.

[Poi12b] Poisson. Mémoire sur la distribution de l'électricité à la surface des corps conducteurs. *Mémoires de la Classe des Sciences Mathématiques et Physiques*, pages 1–92, 1812. Année 1811. Première partie. Lu les 9 mai et 3 août 1812.

[Poi13] Poisson. Second mémoire sur la distribution de l'électricité a la surface des corps conducteurs. *Journal de Physique, de Chimie, d'Histoire Naturelle et des Arts*, 77:380–386, 1813. Lu à l'Institut, le 6 septembre 1813.

[Poi14] Poisson. Second mémoire sur la distribution de l'électricité à la surface des corps conducteurs. *Mémoires de la Classe des Sciences Mathématiques et Physiques*, pages 163–274, 1814. Année 1811. Seconde partie. Lu le 6 septembre 1813.

[Poi19] Poisson. Essay on the distribution of electricity on the surface of conducting bodies. Translated by S. Gallagher. Available at https://histomathsci.blogspot.com, 2019.

[Pot84] A. Potier. *Collection de Mémoires relatifs a la Physique*, volume 1: *Mémoires de Coulomb*. Gauthiers-Villars, Paris, 1884.

[Pou37] Pouillet. Mémoire sur la pile de Volta et sur la loi générale de l'intensité que prennent les courrants, soit qu'ils proviennent d'un seul élément, soit qu'ils proviennnent d'une pile à grande ou à petite tension. *Comptes Rendues de l'Académie des Sciences de Paris*, 4:267–279, 1837.

[Pri83] W. Pricha. *Von Ampère zu Maxwell. Wilhelm Webers Briefe an Carl August v. Steinheil und die Elektrodynamik Ihres Zeit*. PhD thesis, Universität Bremen, 1983.

[RA11] S. Reif-Acherman. Studies on the temperature dependence of electric conductivity for metals in the nineteenth century: a neglected chapter in the history of superconductivity. *Revista Brasileira de Ensino de Física*, 33:4602, 2011. Doi: 10.1590/S1806-11172011000400020.

[Rei02] K. Reich. *Gauß' Werke in Kurzfassung*. Erwin Rauner, Augsburg, 2002. Algorismus, Studien zur Geschichte der Mathematik und der Naturwissenschaften, Vol. 39.

[Rei13] K. Reich. Die Beziehungen zwischen Kopenhagen und Göttingen auf dem Gebiet des Erdmagnetismus: Ergebnisse einer Analyse der Briefe, die "Hans Christian Oersted" mit "Carl Friedrich Gauß" und "Wilhelm Weber" wechselte. *Sudhoffs Archiv*, 97:21–38, 2013.

[Rei19] K. Reich, 2019. Die Intensität der erdmagnetischen Kraft: Nr. 15, pp. xxxix-xlii. In *Carl Friedrich Gauss — Werke — Supplement* — Band 3: Varia: 15 Abhandlungen in deutscher Übersetzung. Mit einer Einleitung, einer Bibliographie und Registern herausgegeben von Karin Reich. Hrsg. von der Akademie der Wissenschaften zu Göttingen, Universitätsverlag Göttingen 2019, S. 533-594.

[Rib08] J. E. A. Ribeiro. Sobre a Força de Lorentz, os Conceitos de Campo e a "Essência" do Eletromagnetismo Clássico. Master's thesis, Instituto de Física, Universidade de São Paulo, Brasil, 2008.

[Rie67a] B. Riemann. A contribution to electrodynamics. *Philosophical Magazine*, 34:368–372, 1867. This text was written in 1858.

[Rie67b] B. Riemann. Ein Beitrag zur Elektrodynamik. *Annalen der Physik und Chemie*, 131:237–243, 1867. This text was written in 1858.

[Rie92a] E. Riecke. Vorwort zum zweiten Bande. In *Wilhelm Weber's Werke*, volume 2: Magnetismus, pages iii–vi, Berlin, 1892. Springer.

[Rie92b] E. Riecke. *Wilhelm Weber (geb. 24. October 1804, gest. 23. Juni 1891).* Dieterichsche Verlags-Buchhandlung, Göttingen, 1892. Rede gehalten in der öffentlichen Sitzung der K. Gesellschaft der Wissenschaften am 5. December 1891. Reprinted in Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen, Vol. 38, pp. 1-44 (1892).

[Rie93] E. Riecke. Vorwort zum fünften Bande. In *Wilhelm Weber's Werke*, volume 5: Wellenlehre, pages iii–v, Berlin, 1893. Springer.

[Rie77] B. Riemann. A contribution to electrodynamics. In C. White*, Energy Potential: Toward a New Electromagnetic Field Theory*, pages 295–300, New York, 1977. Campaigner. Translated by J. J. Cleary Jr.

[RVA08] J. E. A. Ribeiro, A. Vannucci, and A. K. T. Assis. The multiple definitions of 'field' in the context of electromagnetism. In M. G. P. Batista, editor, *Proceedings of the VI Taller Internacional "ENFIQUI 2008" — La Enseñanza de la Física y la Química*, pages 1–4, Matanzas, Cuba, 2008. Universidad Pedagógica "Juan Marinello".

[Sch20] J. S. Schweigger. Zusätze zu Oersted's elektro-magnetische Versuchen. *Allgemeine Literatur-Zeitung*, 296:cols. 622–624, 1820.

[Sch21a] Schweiger. Sur l'electro-magnétisme. *Bibliotheque Universelle des Sciences, Belles-Lettres, et Arts*, 16:197–200, 1821.

[Sch21b] J. S. Schweigger. Noch einige Worte über diese neuen elektromagnetischen Phänomene. *Journal für Chemie und Physik*, 31:35–41, 1821.

[Sch21c] J. S. Schweigger. Über Elektromagnetismus. *Journal für Chemie und Physik*, 31:7–17, 1821.

[Sch21d] J. S. Schweigger. Zusätze zu Oersted's elektromagnetische Versuchen. *Journal für Chemie und Physik*, 31:1–6, 1821.

[Sch88] U. Schmucker. The Wingst geomagnetic observatory and the development of geomagnetism during the past fifty years. *Deutsche Hydrographische Zeitschrift*, 41:93–107, 1988. Doi: 10.1007/BF02225920.

[See25] Seebeck. Magnetische Polarisation der Metalle und Erze durch Temperatur-Differenz. *Abhanglungen der Königlichen Akademie der Wissenschaften zu Berlin — Aus den Jahren 1822 und 1823*, pages 265–373, 1825. Auszug aus vier Vorlesungen, welche in der Akademie der Wissenschaften am 16. August, am 18. und 25. Oktober 1821 und am 11. Februar 1822 gehalten worden.

[See26] T. J. Seebeck. Ueber die Magnetische Polarisation der Metalle und Erze durch Temperatur-Differenz. *Annalen der Physik und Chemie*, 6:1–10, 133–160 and 253–286, 1826. Ausgezogen aus den so eben erschienen Denkschriften für 1822 und 1823.

[See69] T. J. Seebeck. Thermoelectric currents. In W. F. Magie, editor, *A Source Book in Physics*, pages 461–464, New York, 1969. McGraw-Hill.

[Sie60] W. Siemens. Vorschlag eines reproducirbaren Widerstandsmaaßes. *Annalen der Physik und Chemie*, 186:1–20, 1860. Doi: 10.1002/andp.18601860502.

[Sie61] W. Siemens. Proposal for a new reproducible standard measure of resistance to galvanic currents. *Philosophical Magazine*, 21:25–38, 1861. Translated by F. Guthrie.

[Sih21] A. Sihvola. Johan Jacob Nervander and the quantification of electric current. *IEEE Antennas & Propagation Magazine*, 63:123–128, 2021. Doi: 10.1109/MAP.2020.3039803.

[SK07] M. C. d. Silva and S. Krapas. Controvérsia ação a distância/ação mediada: abordagens didáticas para o ensino das interações físicas. *Revista Brasileira de Ensino de Física*, 29:471–479, 2007.

[SP13] K. W. Scheller and T. J. Pickett. F = qv x b: v is with respect to what? *The Physics Teacher*, 51:169–170, 2013.

[Ten97] J. Tennenbaum. Wilhelm Weber's 'Aphorisms' — On the hypotheses of temporal ordering. *21st Century Science & Technology*, 10(2):50–51, 1997.

[Tho81] J. J. Thomson. On the electric and magnetic effects produced by the motion of electrified bodies. *Philosophical Magazine*, 11:229–249, 1881.

[Tho04] J. J. Thomson. *Electricy and Matter*. Yale University Press, New Haven, 1904.

[Tho21] J. J. Thomson. *Elements of the Mathematical Theory of Electricity and Magnetism*. Cambridge University Press, Cambridge, 5th edition, 1921.

[Tho29] J. J. Thomson. *Beyond the Electron*. Cambridge University Press, Cambridge, 1929.

[Tim05] A. Timm. Der elektromagnetische Telegraph von Gauß und Weber. *Göttinger Bibliothekschriften*, 30:169–187, 2005.

[Tom12a] F. D. Tombe. Maxwell's original equations. *The General Science Journal*, 2012. Available at www.gsjournal.net/Science-Journals/Essays/View/3889.

[Tom12b] F. D. Tombe. The significance of Maxwell's equations. *The General Science Journal*, 2012. Available at www.gsjournal.net/Science-Journals/Essays/View/4258.

[Tun92] P. Tunbridge. *Lord Kelvin: His Influence on Electrical Measurements and Units*. Peter Peregrinus Ltd., London, 1992. IEE History of Technology Series 18.

[Voi92] W. Voigt. Vorwort zum ersten Bande. In *Wilhelm Weber's Werke*, volume 1: Akustik, Mechanik, Optik und Wärmelehre, pages iii–iv, Berlin, 1892. Springer.

[Voi99] W. Voigt. Untitled. *Zeitschrift des Vereins deutscher Ingenieure*, 43:824–825, 1899.

[Vol00a] A. Volta. On the electricity excited by the mere contact of conducting substances of different kinds. *Philosophical Transactions*, 90:403–431, 1800. Letter in French from A. Volta to J. Banks dated March 20, 1800. It was read before the Royal Society in June 26, 1800.

[Vol00b] A. Volta. On the electricity excited by the mere contact of conducting substances of different kinds. *Philosophical Magazine*, 7:289–311, 1800.

[Vol23] A. Volta. Sull'elettricità eccitata dal semplice contatto di sostanze conduttrici di diversa natura in una lettera di Alessandro Volta a Sir Joseph Banks. In *Le Opere di Alessandro Volta*, Vol. 1. Ulrico Hoepli, Milano, 1923. Available at echo.mpiwg-berlin.mpg.de/content/electricity.

[Vol64] A. Volta. On the electricity excited by the mere contact of conducting substances of different kinds. In B. Dibner, *Alessandro Volta and the Electric Battery*, pages 111–131. Franklin Watts, New York, 1964. Translated from the author's paper published in French in the Philosophical Transactions for 1800, Part 2.

[Web25a] W. Weber. Auszug aus den die Theorie des Schalles und Klanges betreffenden Aufsätzen von Felix Savart. Mit einigen Bemerkungen über scheinbare Widersprüche zwischen Savart's Entdeckungen und Chladni's früheren Arbeiten und andere Zusätzen. *Journal für Chemie und Physik*, 14:385–428, 1825. Reprinted in Wilhelm Weber's *Werke*, Vol. 1, pp. 3-28, W. Vogt (ed.), (Springer, Berlin, 1892).

[Web25b] W. Weber. Ueber Savart's Klangversuche. *Journal für Chemie und Physik*, 15:257–310, 1825. Reprinted in Wilhelm Weber's *Werke*, Vol. 1, pp. 29-50, W. Vogt (ed.), (Springer, Berlin, 1892).

[Web27] W. Weber, 1827. Leges oscillationis oriundae si duo corpora diversa celeritate oscillantia ita conjungutur ut oscillare non possint nisi simul et synchronice exemplo illustratae tuborum linguatorum. Reprinted in Wilhelm Weber's *Werke*, Vol. 1, pp. 207-256, W. Vogt (ed.), (Springer, Berlin, 1892).

[Web28] W. Weber. Kompensation der Orgelpfeifen. *Annalen der Physik und Chemie*, 14:397–408, 1828. Reprinted in *Wilhelm Weber's Werke*, Vol. 1, W. Voigt (ed.), (Springer, Berlin, 1892), pp. 257-265.

[Web35] W. Weber. Ueber die Elasticität der Seidenfäden. *Annalen der Physik und Chemie*, 34:247–257, 1835. Reprinted in *Wilhelm Weber's Werke*, Vol. 1, W. Voigt (ed.), (Springer, Berlin, 1892), pp. 438-444.

[Web38] W. Weber. Das Induktions-Inklinatorium. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1837*, volume II, chapter V, pages 81–96. Dieterichschen Buchhandlung, Göttingen, 1838. Reprinted in *Wilhelm Weber's Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 75-88.

[Web39a] W. Weber. Das transportable Magnetometer. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1838*, volume III, chapter III, pages 68–85. Weidmannschen Buchhandlung, Leipzig, 1839. Reprinted in *Wilhelm Weber's Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 89-104.

[Web39b] W. Weber. Der Induktor zum Magnetometer. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1838*, volume III, chapter IV, pages 86–101. Weidmannschen Buchhandlung, Leipzig, 1839. Reprinted in *Wilhelm Weber's Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 105-118.

[Web39c] W. Weber. Der Rotationsinductor. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1838*, volume III, chapter V, pages 102–117. Weidmannschen Buchhandlung, Leipzig, 1839. Reprinted in *Wilhelm Weber's Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 119-131.

[Web40] W. Weber. Unipolare Induction. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1839*, volume IV, chapter III, pages 63–90. Weidmannschen Buchhandlung, Leipzig, 1840. Reprinted in Wilhelm Weber's *Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 153-175, extract with some amendments and modifications by Weber in Wilhelm Weber's *Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 176-179.

[Web41a] W. Weber. Messung starker galvanischer Ströme bei geringem Widerstande nach absolutem Maasse. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1840*, volume V, chapter VIII, pages 83–90. Weidmannschen Buchhandlung, Leipzig, 1841. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 6-12.

[Web41b] W. Weber. On a transportable magnetometer. In R. Taylor, editor, *Scientific Memoirs,* Vol. 2, pages 565–586, London, 1841. Taylor and Francis. This article is translated partly from the *Resultate aus den Beobachtungen des magnetischen Vereins im Jare 1838*, Volume III, Chapter III, pp. 68-85, and partly from manuscript communications from M. Weber to Major Sabine. Translation presented by Major Sabine. Available at https://www.biodiversitylibrary.org/bibliography/2501#/summary.

[Web41c] W. Weber. Über magnetische Friction. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1840*, volume V, chapter IV, pages 46–58. Weidmannschen Buchhandlung, Leipzig, 1841. Reprinted in *Wilhelm Weber's Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 200-211.

[Web41d] W. Weber. Ueber das elektrochemische Aequivalent des Wassers. In C. F. Gauss and W. Weber, editors, *Resultate aus den Beobachtungen des magnetischen Vereins im Jahre 1840*, volume V, chapter IX, pages 91–98. Weidmannschen Buchhandlung, Leipzig, 1841. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 13-18.

[Web41e] W. Weber. Unipolare Induction. *Annalen der Physik und Chemie*, 52:353–386, 1841. Reprinted in Wilhelm Weber's *Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 153-175, abstract in Wilhelm Weber's *Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 176-179.

[Web42] W. Weber. Ueber das elektrochemische Aequivalent des Wassers. *Annalen der Physik und Chemie*, 55:181–189, 1842. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 13-18.

[Web46] W. Weber. Elektrodynamische Maassbestimmungen — Über ein allgemeines Grundgesetz der elektrischen Wirkung. *Abhandlungen bei Begründung der Königlich Sächsischen Gesellschaft der Wissenschaften am Tage der zweihundertjährigen Geburtstagfeier Leibnizen's herausgegeben von der Fürstlich Jablonowskischen Gesellschaft (Leipzig)*, pages 211–378, 1846. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 25-214.

[Web48a] W. Weber. Elektrodynamische Maassbestimmungen. *Annalen der Physik und Chemie*, 73:193–240, 1848. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 215-254.

[Web48b] W. Weber. Über die Erregung und Wirkung des Diamagnetismus nach den Gesetzen inducirter Ströme. *Annalen der Physik und Chemie*, 73:241–256, 1848. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 255-268.

[Web48c] W. Weber. Über die Erregung und Wirkung des Diamagnetismus nach den Gesetzen inducirter Ströme. *Berichte über die Verhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig, mathematisch-physische Klasse*, I aus den Jahren 1846 und 1847:346–358, 1848. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 255-265.

[Web51] W. Weber. Messungen galvanischer Leitungswiderstände nach einem absolutem Maasse. *Annalen der Physik und Chemie*, 82:337–369, 1851. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 276-300.

[Web52a] W. Weber. Elektrodynamische Maassbestimmungen insbesondere über Diamagnetismus. *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig, mathematisch-physischen Classe*, 1:485–577, 1852. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 473-554.

[Web52b] W. Weber. Elektrodynamische Maassbestimmungen insbesondere Widerstandsmessungen. *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig, mathematisch-physischen Classe*, 1:199–381, 1852. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 301-471.

[Web52c] W. Weber. On the measurement of electro-dynamic forces. In R. Taylor, editor, *Scientific Memoirs,* Vol. 5, pages 489–529, London, 1852. Taylor and Francis. Available at https://archive.org/details/in.ernet.dli.2015.212784 and https://www.biodiversitylibrary.org/bibliography/2501#/summary.

[Web52d] W. Weber. Ueber den Zusammenhang der Lehre vom Diamagnetismus mit der Lehre von dem Magnetismus und der Elektricität. *Annalen der Physik und Chemie*, 87:145–189, 1852. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 555-590.

[Web53a] W. Weber. Die Anwendung der magnetischen Induction auf Messung der Inclination mit dem Magnetometer. *Annalen der Physik und Chemie*, 90:209–247, 1853.

[Web53b] W. Weber. On the connexion of diamagnetism with magnetism and electricity. In J. Tyndall and W. Francis, editors, *Scientific Memoirs,* Vol. on Natural Philosophy, pages 163–199, London, 1853. Taylor and Francis. Translated by J. Tyndall. Available at https://books.google.com.br/books?id=C1i4AAAAIAAJ&hl=pt-BR&source=gbs_navlinks_s.

[Web53c] W. Weber. Ueber die Anwendung der magnetischen Induktion zur Messung der Inklination mit dem Magnetometer. *Göttinger Nachrichten*, 2:17–24, 1853. Reprinted in Wilhelm Weber's *Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 328-332.

[Web53d] W. E. Weber. Ueber die Anwendung der magnetischen Induktion auf Messung der Inklination mit dem Magnetometer. *Abhandlungen der Königlichen Gesellschaft der Wissenschaften zu Göttingen*, 5:3–58, 1853. Reprinted in *Wilhelm Weber's Werke*, Vol. 2, E. Riecke (ed.), (Springer, Berlin, 1892), pp. 277-327.

[Web55] W. Weber. Vorwort bei der Übergabe der Abhandlung: *Elektrodynamische Maassbestimmungen, insbesondere Zurückführung der Stromintensitäts-Messungen auf mechanisches Maass. Berichte über die Verhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig, mathematisch-physische Klasse*, 17:55–61, 1855. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 591-596.

[Web61] W. Weber. On the measurement of electric resistance according to an absolute standard. *Philosophical Magazine*, 22:226–240 and 261–269, 1861. Translated by Dr. E. Atkinson.

[Web62] W. Weber. Zur Galvanometrie. *Abhandlungen der Königlich Gesellschaft der Wissenschaften zu Göttingen, mathematische Klasse*, 10:3–96, 1862. Reprinted in *Wilhelm Weber's Werke*, Vol. 4, H. Weber (ed.), (Springer, Berlin, 1894), pp. 17-96.

[Web64] W. Weber. Elektrodynamische Maassbestimmungen insbesondere über elektrische Schwingungen. *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig, mathematisch-physischen Classe*, 6:571–716, 1864. Reprinted in *Wilhelm Weber's Werke*, Vol. 4, H. Weber (ed.), (Springer, Berlin, 1894), pp. 105-241.

[Web71] W. Weber. Elektrodynamische Maassbestimmungen insbesondere über das Princip der Erhaltung der Energie. *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig, mathematisch-physischen Classe*, 10:1–61, 1871. Reprinted in *Wilhelm Weber's Werke*, Vol. 4, H. Weber (ed.), (Springer, Berlin, 1894), pp. 247-299.

[Web72] W. Weber. Electrodynamic measurements — Sixth memoir, relating specially to the principle of the conservation of energy. *Philosophical Magazine*, 43:1–20 and 119–149, 1872. Translated by Professor G. C. Foster, F.R.S., from the *Abhandlungen der mathem.-phys. Classe der Königlich Sächsischen Gesellschaft der Wissenschaften*, vol. x (January 1871).

[Web74] W. Weber. Ueber das Aequivalent lebendiger Kräfte. *Annalen der Physik und Chemie*, Jubelband:199–213, 1874. Reprinted in *Wilhelm Weber's Werke*, Vol. 4, H. Weber (ed.), (Springer, Berlin, 1894), pp. 300-311.

[Web75] W. Weber. Ueber die Bewegung der Elektricität in Körpern von molekularer Konstitution. *Annalen der Physik und Chemie*, 156:1–61, 1875. Reprinted in *Wilhelm Weber's Werke*, Vol. 4, H. Weber (ed.), (Springer, Berlin, 1894), pp. 312-357.

[Web78] W. Weber. Elektrodynamische Maassbestimmungen insbesondere über die Energie der Wechselwirkung. *Abhandlungen der Königlich Sächsischen Gesellschaft der Wissenschaften zu Leipzig, mathematisch-physischen Classe*, 11:641–696, 1878. Reprinted in *Wilhelm Weber's Werke*, Vol. 4, H. Weber (ed.), (Springer, Berlin, 1894), pp. 361-412.

[Web87] W. Weber. Mesures électrodynamiques. In J. Joubert, editor, *Collection de Mémoires relatifs a la Physique,* Vol. III: *Mémoires sur l'Électrodynamique*, pages 289–402. Gauthier-Villars, Paris, 1887.

[Web92a] H. Weber. Wilhelm Weber — Eine Lebensskizze. *Deutsche Revue*, 3:183–202, 1892.

[Web92b] H. Weber. Wilhelm Weber — Eine Lebensskizze (Fortsetzung). *Deutsche Revue*, 3:324–345, 1892.

[Web92c] H. Weber. Wilhelm Weber — Eine Lebensskizze (Schluß). *Deutsche Revue*, 4:99–118, 1892.

[Web92d] W. Weber. *Wilhelm Weber's Werke,* E. Riecke (ed.), volume 2, *Magnetismus*. Springer, Berlin, 1892. Available at http://archive.org/details/wilhelmweberswe01fiscgoog.

[Web92e] W. Weber. *Wilhelm Weber's Werke,* W. Voigt, (ed.), volume 1, *Akustik, Mechanik, Optik und Wärmelehre*. Springer, Berlin, 1892.

[Web93a] H. Weber. Vorwort zum dritten Bande. In *Wilhelm Weber's Werke*, volume 3: Galvanismus und Elektrodynamik. Erster Theil, pages iii–v, Berlin, 1893. Springer.

[Web93b] H. Weber. *Wilhelm Weber — Eine Lebensskizze*. Eduard Trewendt, Breslau, 1893. Available at https://books.google.de/books/about/Wilhelm_Weber.html?id=pu3mpnrYpuUC&redir_esc=y.

[Web93c] W. Weber. *Wilhelm Weber's Werke,* H. Weber (ed.), volume 3, *Galvanismus und Elektrodynamik*, first part. Springer, Berlin, 1893. Available at http://archive.org/details/wilhelmweberswe02fiscgoog.

[Web94a] H. Weber. Vorwort zum vierten Bande. In *Wilhelm Weber's Werke*, volume 4: Galvanismus und Elektrodynamik. Zweiter Theil, pages iii–viii, Berlin, 1894. Springer.

[Web94b] W. Weber. Aphorismen. In H. Weber, editor, Wilhelm Weber's *Werke,* Vol. 4, pages 630–632, Berlin, 1894. Springer.

[Web94c] W. Weber. Bemerkungen zu der Abhandlung: "Untersuchung über den galvanischen Lichtbogen" von Prof. E. Edlund. In H. Weber, editor, Wilhelm Weber's *Werke,* Vol. 4, pages 578–583, Berlin, 1894. Springer.

[Web94d] W. Weber. Elektrodynamische Maassbestimmungen insbesondere über den Zusammenhang des elektrischen Grundgesetzes mit dem Gravitationsgesetze. In H. Weber, editor, Wilhelm Weber's *Werke,* Vol. 4, pages 479–525, Berlin, 1894. Springer.

[Web94e] W. Weber. Elektroskopische und elektrodynamische Wirkungen der freien Elektricität geschlossener Ketten. In H. Weber, editor, Wilhelm Weber's *Werke,* Vol. 4, pages 616–621, Berlin, 1894. Springer.

[Web94f] W. Weber. Ueber Elektrothermismus. (Ueber Elektricität und Wärme). In H. Weber, editor, Wilhelm Weber's *Werke,* Vol. 4, pages 622–629, Berlin, 1894. Springer.

[Web94g] W. Weber. *Wilhelm Weber's Werke,* H. Weber, (ed.), volume 4, *Galvanismus und Elektrodynamik*, second part. Springer, Berlin, 1894.

[Web94h] W. Weber. Zur Galvanometrie (Auszug). In H. Weber, editor, Wilhelm Weber's *Werke,* Vol. 4, pages 526–538, Berlin, 1894. Springer.

[Web66a] W. Weber. On a transportable magnetometer. In R. Taylor, editor, *Scientific Memoirs,* Vol. 2, pages 565–586, New York, 1966. Johnson Reprint Corporation.

[Web66b] W. Weber. On the connexion of diamagnetism with magnetism and electricity. In J. Tyndall and W. Francis, editors, *Scientific Memoirs,* Vol. on Natural Philosophy, pages 163–199, New York, 1966. Johnson Reprint Corporation. Translated by J. Tyndall.

[Web66c] W. Weber. On the measurement of electro-dynamic forces. In R. Taylor, editor, *Scientific Memoirs,* Vol. 5, pages 489–529, New York, 1966. Johnson Reprint Corporation.

[Web97] W. Weber. Aphorisms. *21st Century Science & Technology*, 10(2):52–53, 1997. English translation by J. Tennenbaum.

[Web07] W. Weber, 2007. Determinations of electrodynamic measure: concerning a universal law of electrical action, 21st Century Science & Technology, posted March 2007, translated by S. P. Johnson, edited by L. Hecht and A. K. T. Assis. Available at http://21sci-tech.com/translation.html and www.ifi.unicamp.br/~assis.

[Web19] W. Weber, 2019. On the measurement of electro-dynamic forces. Classics of Measure no. 1. Second edition. Version of 21 February 2019. This English translation appeared originally in R. Taylor (editor), *Scientific Memoirs, selected from the Transactions of Foreign Academies of Science and Learned Societies*, Volume V, part 20, article 14 (London: Taylor and Francis, 1852). Available at www.sizes.com/library/classics/Weber1.pdf.

[Web21a] W. Weber. Aphorisms. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume IV: Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation, pages 211–213, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21b] W. Weber. Electrodynamic measurements, eighth memoir, relating specially to the connection of the fundamental law of electricity with the law of gravitation. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume IV: Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation, pages 173–210, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21c] W. Weber. Electrodynamic measurements, fifth memoir, relating specially to electric oscillations. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume III: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity, pages 267–383, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21d] W. Weber. Electrodynamic measurements, first memoir, relating specially to a general fundamental law of electric action. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume II: Weber's Fundamental Force and the Unification of the Laws of Coulomb, Ampère and Faraday, pages 33–203, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21e] W. Weber. Electrodynamic measurements, second memoir, relating specially to measures of resistance. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume II: Weber's Fundamental Force and the Unification of the Laws of Coulomb, Ampère and Faraday, pages 291–441, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21f] W. Weber. Electrodynamic measurements, seventh memoir, relating specially to the energy of interaction. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume IV: Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation, pages 123–164, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21g] W. Weber. Electrodynamic measurements, sixth memoir, relating specially to the principle of the conservation of energy. In A. K. T. Assis, editor, *Wilhelm Weber's

Main Works on Electrodynamics Translated into English, volume IV: Conservation of Energy, Weber's Planetary Model of the Atom and the Unification of Electromagnetism and Gravitation, pages 67–111, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21h] W. Weber. Electrodynamic measurements, third memoir, relating specially to diamagnetism. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume III: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity, pages 11–83, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21i] W. Weber. Foreword to the submission of the treatise: electrodynamic measurements, specially attributing mechanical units to measures of current intensity. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume III: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity, pages 125–129, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21j] W. Weber. Measurement of strong galvanic currents with low resistance according to absolute measure. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume I: Gauss and Weber's Absolute System of Units, pages 187–193, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21k] W. Weber. On a transportable magnetometer. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume I: Gauss and Weber's Absolute System of Units, pages 151–182, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21l] W. Weber. On the connexion of diamagnetism with magnetism and electricity. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume III: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity, pages 85–120, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21m] W. Weber. On the electrochemical equivalent of water. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume I: Gauss and Weber's Absolute System of Units, pages 195–200, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21n] W. Weber. On the excitation and action of diamagnetism according to the laws of induced currents. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume II: Weber's Fundamental Force and the Unification of the Laws of Coulomb, Ampère and Faraday, pages 249–257, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21o] W. Weber. On the measurement of electric resistance according to an absolute standard. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume II: Weber's Fundamental Force and the Unification of the Laws of Coulomb, Ampère and Faraday, pages 267–286, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Web21p] W. Weber. On the measurement of electro-dynamic forces. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume II: Weber's Fundamental Force and the Unification of the Laws of Coulomb, Ampère and Faraday, pages 207–247, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Wes90] J. P. Wesley. Weber electrodynamics, Part I. General theory, steady current effects. *Foundations of Physics Letters*, 3:443–469, 1990.

[Whe43] C. Wheatstone. An account of several new instruments and processes for determining the constants of a voltaic circuit. *Philosophical Transactions*, 133:303–327, 1843.

[Whe44a] C. Wheatstone. Beschreibung verschiedener neuen Instrumente und Methoden zur Bestimmung der Constanten einer Volta'schen Kette. *Annalen der Physik und Chemie*, 62:499–543, 1844.

[Whe44b] C. Wheatstone. Descritption de plusieurs instruments et procédés nouveaux pour déterminer les constantes d'un circuit voltaique. *Annales de Chimie et de Physique*, 10:257–293, 1844. Translated by M. J.-V. Fayeux.

[Whi73] E. T. Whittaker. *A History of the Theories of Aether and Electricity*, volume 1: *The Classical Theories*. Humanities Press, New York, 1973.

[Wie60] K. H. Wiederkehr. Wilhelm Webers Stellung in der Entwicklung der Elektrizitätslehre. Dissertation, Universität Hamburg, 1960.

[Wie67] K. H. Wiederkehr. *Wilhelm Eduard Weber — Erforscher der Wellenbewegung und der Elektrizität (1804-1891)*, volume 32 of *Grosse Naturforscher,* H. Degen (ed.). Wissenschaftliche Verlagsgesellschaft, Stuttgart, 1967.

[WK56] W. Weber and R. Kohlrausch. Über die Elektricitätsmenge, welche bei galvanischen Strömen durch den Querschnitt der Kette fliesst. *Annalen der Physik und Chemie,* J. C. Poggendoff (ed.), 99:10–25, 1856. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 597-608.

[WK68] W. Weber and R. Kohlrausch. Über die Einführung absoluter elektrischer Maße. In S. Balke, H. Gericke, W. Hartner, G. Kerstein, F. Klemm, A. Portmann, H. Schimank, and K. Vogel, editors, *Ostwalds Klassiker der exakten Wissenschaften,* new series, Vol. 5. Friedrich-Vieweg & Sohn, Braunschweig, 1968. Commented by F. Kohlrausch and K. H. Wiederkehr.

[WK03] W. Weber and R. Kohlrausch. On the amount of electricity which flows through the cross-section of the circuit in galvanic currents. In F. Bevilacqua and E. A. Giannetto, editors, *Volta and the History of Electricity*, pages 287–297. Università degli Studi di Pavia and Editore Ulrico Hoepli, Milano, 2003. Translated by S. P. Johnson and edited by L. Hecht. Available at http://ppp.unipv.it/PagesIT/VoltaHistElecSec3Frame.htm and www.ifi.unicamp.br/~assis.

[WK08] W. Weber and R. Kohlrausch. Sobre a quantidade de eletricidade que flui através da seção reta do circuito em correntes galvânicas. *Revista Brasileira de História da Ciência*, 1:94–102, 2008. Portuguese translation by A. K. T. Assis.

[WK21] W. Weber and R. Kohlrausch. On the amount of electricity which flows through the cross-section of the circuit in galvanic currents. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume III: Measurement of Weber's Constant c, Diamagnetism, the Telegraph Equation and the Propagation of Electric Waves at Light Velocity, pages 131–140, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[Wol05] G. Wolfschmidt (ed.). *Vom Magnetismus zur Elektrodynamik: anläßlich des 200. Geburtstags von Wilhelm Weber (1804-1891) und des 150. Todestages von Carl Friedrich Gauss (1777-1855).* Schwerpunkt Geschichte der Naturwissenschaften, Mathematik und Technik, Hamburg, 2005.

[Woo68] A. E. Woodruff. The contributions of Hermann von Helmholtz to electrodynamics. *Isis*, 59:300–311, 1968.

[Woo81] A. E. Woodruff. Weber, Wilhelm Eduard. In C. C. Gillispie, editor, *Dictionary of Scientific Biography,* Vol. 14, pages 203–209, New York, 1981. Charles Scribner's Sons.

[WSH03] D. Warner, R. Sherman, and K. Hentschel. The several faces of earth induction. *Bulletin of the Scientific Instrument Society*, 76:30–34, 2003.

[WW25] E. H. Weber and W. Weber. *Wellenlehre auf Experimente gegründet oder über die Wellen tropfbarer Flüssigkeiten mit Anwendung auf die Schall- und Lichtwellen.* Gerhard Fleischer, Leipzig, 1825. Reprinted in *Wilhelm Weber's Werke*, Vol. 5, E. Riecke (ed.), (Springer, Berlin, 1893). Available at http://archive.org/details/wilhelmweberswe00fiscgoog.

[WW36] W. Weber and E. Weber. *Mechanik der menschlichen Gehwerkzeuge. Eine anatomisch-physiologische Untersuchung.* Dietrich, Göttingen, 1836. Reprinted in *Wilhelm Weber's Werke*, Vol. 6, F. Merkel and O. Fischer (eds.), (Springer, Berlin, 1894). Available at http://archive.org/details/wilhelmweberswe03fiscgoog.

[WW41a] F. Wöhler and W. Weber. Neue galvanische Säule. *Journal für praktische Chemie*, 23:313–316, 1841.

[WW41b] F. Wöhler and W. Weber. Ueber eine neue Construction der Grove'schen Säulen. *Annalen der Chemie und Pharmacie*, 38:307–311, 1841. Reprinted in *Polytechnisches Journal*, Vol. 81, pp. 273-275 (1841).

[WW41c] F. Wöhler and W. Weber. Zusammensetzung galvanischer Säulen. *Göttingische gelehrte Anzeigen*, 1:801–804, 1841. 81. Stuck der *Göttingische gelehrte Anzeigen*, den 24. May 1841. Reprinted in *Wilhelm Weber's Werke*, Vol. 3, H. Weber (ed.), (Springer, Berlin, 1893), pp. 3-5.

[WW93] E. H. Weber and W. Weber. *Wilhelm Weber's Werke,* E. Riecke (ed.), volume 5, *Wellenlehre auf Experimente gegründet oder über die Wellen tropfbarer Flüssigkeiten mit Anwendung auf die Schall- und Lichtwellen.* Springer, Berlin, 1893. Originally published in 1825. Available at http://archive.org/details/wilhelmweberswe00fiscgoog.

[WW94] W. Weber and E. Weber. *Wilhelm Weber's Werke,* F. Merkel and O. Fischer (editors), volume 6, *Mechanik der menschlichen Gehwerkzeuge. Eine anatomisch-physiologische Untersuchung.* Springer, Berlin, 1894. Originally published in 1836. Available at http://archive.org/details/wilhelmweberswe03fiscgoog.

[WW92] W. Weber and E. Weber. *Mechanics of the Human Walking Apparatus.* Springer, Berlin, 1992. Translated by P. Maquet and R. Furlong.

[WW21] F. Wöhler and W. Weber. Composition of galvanic piles. In A. K. T. Assis, editor, *Wilhelm Weber's Main Works on Electrodynamics Translated into English*, volume I: Gauss and Weber's Absolute System of Units, pages 183–185, Montreal, 2021. Apeiron. Available at www.ifi.unicamp.br/~assis.

[WZ80] W. Weber and F. Zöllner. Ueber Einrichtungen zum Gebrauch absoluter Maasse in der Elektrodynamik mit praktischer Anwendung. *Berichte über die Verhandlungen der Königlich Sächsischen Geselschaft der Wissenschaften zu Leipzig, mathematisch-physische Klasse*, 32:77–143, 1880. Reprinted in *Wilhelm Weber's Werke*, Vol. 4, H. Weber (ed.), (Springer, Berlin, 1894), pp. 420-476.

[Yag20] A. D. Yaghjian. Maxwell's derivation of the Lorentz force from Faraday's law. Available at https://arxiv.org/abs/1911.04605, 2020.

[Zet77] K. E. Zetzsche. *Handbuch der elektrischen Telegraphie*, volume 1: Geschichte der elektrischen Telegraphie. Julius Springer, Berlin, 1877.

This is the fifth of 5 volumes of the book "Wilhelm Weber's Main Works on Electrodynamics Translated into English".

It contains the following translations: (a) The introduction of Gauss' 1840 paper on the general propositions relating to attractive and repulsive forces acting in the inverse ratio of the square of the distance. (b) Weber's two papers on unipolar induction, published in 1840 and 1841, preceded with introductory comments by the editor. (c) Weber's large paper on galvanometry published in 1862. (d) Weber's 1874 paper on the equivalent of vis viva, preceded by the translator's introduction. (e) Weber's 1875 treatise on the motion of electricity in bodies of molecular constitution.

Articles with biographical content were subsequently included: (a) Eduard Riecke's memorial speech given at the public meeting of the Royal Society of Sciences of Göttingen on December 5, 1891. (b) Heinrich Weber's 1893 biographical sketch of Wilhelm Weber. (c) W. Voigt's 1899 speech on the unveiling of the monument to Gauss and Weber in Göttingen. (d) The prefaces to the six volumes of the Collected Works of Wilhelm Weber published between 1892 and 1894.

It also contains the following treatises by Wilhelm Weber only published posthumously in his Collected Works: (a) on galvanometry (excerpt). (b) Comments on the paper: "investigation into the electric arc" by Prof. E. Edlund. (c) Electroscopic and electrodynamic actions of free electricity in closed conductors. (d) On electrothermism (on electricity and heat).

This fifth volume also contains three chapters discussing additional topics: (a) The *Weber* as an electrical unit of measure. (b) The velocity in Weber's electrodynamics versus the velocities in different field theories. (c) Weber's electrodynamics versus different field theories.

A large bibliography is included at the end of this volume 5.

About the Editor: Prof. Andre Koch Torres Assis has been working on Weber's law applied to electromagnetism and gravitation for more than 30 years: https://www.ifi.unicamp.br/~assis

978-1-987980-35-6