A Model for Decision Making Under the Influence of an Artificial Social Network

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Abstract—Decision making and the social processes that influence it are of great importance to many problems. The ways people make decisions can determine whether a new technology is successful, whether resources are allocated optimality, or how society responds to a crisis. In this paper, we propose a model for how these decisions can be influenced using modern information technology and social networks. Specifically, we study the use of artificial social networks to make influential users that are desirable to society as a whole. In real life, this would be achieved by the publication of "best users" by the authority behind the artificial network. Graphically, this creates a set of temporary edges from those users to other users. We first develop a general model for an abstract, general case. We then apply it to a specific case study in a smart grid. Our results suggest that the proposed methodology has the potential to move the equilibrium of a system to a more desirable state and that the degree of the improvement, as well as other, graph-theoretic characteristics, depends on the mathematics of the decision being made.

Index Terms—Decision making, dynamic networks, opinion dynamics, social networks.

I. INTRODUCTION

THE ability to predict and influence the decision making of individuals is of great interest in many fields. For example, the successful introduction of a new technology requires that the population of potential users willingly adopts and properly uses the technology. In addition, it is well-established that individuals acting in their own best interest can lead to collectively disastrous results. The ability to predict such results, as well as the ability to influence a population away from them, is therefore of great interest.

In this paper, we present a generalized framework for influencing the decision-making process in a social network. We combine techniques from past works on social influence and cascading behavior to create a robust model for simulating iterative decision making. We then demonstrate a method for manipulating the network in order to optimize the spread of positive traits throughout the network.

This paper is an expansion of the earlier work [1], wherein a basic framework for promoting influential users in a smart building setup is presented, and a continuation of [2], in which

the application of a social networking framework to user behavior in a smart grid is explored. In this paper, we extend the previous work in key ways. First and most importantly, we generalize the method of promoting influential users into a general framework and provide a model for simulating its effects. We additionally incorporate concepts such as homophily and noise into our model of edge creation and modification, in contrast with the idealized case that were used previously.

Section II covers the background information and literature reviews, Section III outlines our model, Section IV demonstrates abstract results, Section V demonstrates results from a specific implementation of this model, and Section VI concludes this paper.

II. BACKGROUND

Decision making is an important topic in psychology, economics, and social sciences. Research analyzing these behaviors in the psychology literature dates back decades [3]. In recent years, the rise of social networks as an important topic of research has led to research, combining these techniques with a network approach [4].

Researchers in many fields have worked to predict the decision-making behavior of groups of people for many years. Classical methods rely upon mathematical models, representing the populace of decision makers as a whole, often with differential equations to perform system modeling of the populace [3], [5]. The economic theory integrates into such a framework naturally, generating utility functions and treating people as rational agents trying to maximize their utilities [6]. Consequently, more and more work explored the insights that an agent-based setup could provide the analysis of decision making.

The rise of the social network analysis has added a whole new dimension to decision-making analysis. A social network provides a robust framework in which individuals can be modeled and their effects on each other can be analyzed. Early work modeling the spread of epidemics [7] was expanded to model the spread of ideas and actions throughout a network [8], [9]. One very prominent example of this is modeling the adoption of a new technology. Initially, simple thresholding models were often used in this area of research [10], but more sophisticated models emerged as research progressed [11]. This was aided in part by the production of real-world case studies, for example in agriculture [12], which provided an empirical mathematical foundation for modeling efforts.

Further aiding the analysis of decision making, how opinion dynamics progresses in a network was studied. Models were developed which simulate, over time, the ways that neighbors in a network influence each other's opinions and ideas [13]–[15]. Systems theory allowed the study of controllability of networks, that is, the ability to use "input nodes" to achieve a desired final state [16]. While many of these models result in all individuals converging toward the same opinion, other modes specifically seek to model how extreme opinions arise, as is sometimes seen in real life [17]. Techniques from other areas of science, such as Ising spin models from physics, can also be applied to simulate decision making and opinion spreading in networks [18].

The analysis of the progression of opinion dynamics in networks that change over time provides another interesting facet of this line of research. Many different models of opinion dynamics emerged from this paper. This paper showed that, even when individuals interacted only with similar neighbors, ultimate near-convergence was achieved [19]. The convergence patterns of networks with birth and death rates can also be analyzed [20]. In addition, techniques on phase transitions from physics were used to demonstrate "tipping points" in the parameters that govern network creation that distinguish the presence of a large, connected, homogeneous component [21]. Other papers study the effects that people self-sorting by traits have on the connectivity of the network [22] or the mechanisms by which multiple consensuses arise in a network through edge rewiring [23].

As the work of modeling the decision-making process progressed, so did research into how this process might be influenced. A great deal of economic research has studied how the use of incentives can affect the process by altering the utility functions or expected payoffs of the participants [24]. More recently, researchers have become interested in the use of influential nodes in a social network to provide a targeted intervention into the natural decision-making process [25], [26]. In addition, social network research has produced works studying the use of edge rewiring to influence certain decisions [27].

Our work in this paper expands on earlier work in a number of ways. First, we seek to provide a generalized, graph-theoretic framework for influencing the collective actions of a group. Moreover, we propose a novel extension of earlier works on edge rewiring and influential nodes. We will take advantage of modern communication technology to create a system in which the graph undergoes substantial changes at each time step with the intent of making certain users influential. This contrasts with existing work on opinion dynamics in time-variant or evolving networks, since here the evolution is deliberately controlled by a central authority.

III. METHODOLOGY

In this paper, we generalize our earlier model for decision making in a social network. We replace the smart-grid-specific equations with generalized formulas that can be tailored to a specific application. We additionally introduce more sophisticated and realistic methods for graph creation and evolution.

We use the following framework for modeling the users and their actions. Each user will have a set of characteristics that quantify their attitudes toward relevant features of the decision-making process in question. Broadly, we will categorize these characteristics as being either malleable or nonmalleable. The nonmalleable characteristics are those which are fixed for a given individual and will not change under social pressure. For example, a person's extraversion is not likely to change much over time, and users with high extraversion should be connected to more users in the social network. Conversely, the malleable characteristics are those which will evolve over time based on social influence from the user's neighbors in the social network. An example of a malleable characteristic is a user's willingness to enter into a specific program, which can change quickly as the user learns more about the program itself.

Each user in the network is assigned an individual utility function, which is based on their characteristics, as well as the actions of the other users in the network. As time progresses, each user plays a game to maximize their utility. In the case of our previous work, every user decided whether or not to join a smart-grid system and how to use their electricity to lower their electricity bill. In addition, we construct an overall utility function to measure the social optimality of the results across the social network. This function's output will always be negative, and we will seek to maximize it.

We will treat the individual agents as nodes in a directed, weighted social network. The graph construction will be based on the principles of homophily; that nodes are more likely to be connected to nodes similar to themselves [28], [29]. We therefore utilize the individuals' starting characteristics as personality indicators that will inform the graph construction. Specifically, we will generate a "similarity score" for each pair of users. This score will be defined as the norm of the difference of the users' characteristic vectors. The greater the similarity score, the more likely a pair of users are to have a unidirectional or bidirectional edge between them. These directed edges will provide the mechanism by which influence on the malleable characteristics will be spread throughout the network. All edges in this base network will have equal weight.

At each time interval, all directed links will propagate influence through the network. Our model will include multiple possible methods for performing this propagation. Let H_i^t denote individual i's characteristics at time t. Let $\bar{H}_i^t$ denote the average characteristic values in individual i's neighborhood at time t, weighted by edge weight. In this model, DeGroot's method [13] will be used to modify the malleable characteristics of each user based on the characteristics of the users with incoming edges. DeGroot's method updates an individual's characteristic by taking a weighted average over all user's characteristics and is widely used in social network analysis. Here, the weights are determined by the edge weight of the social network, and users with no connection are given a weight of zero

$$H_i^{t+1} = 0.5H_i^t + 0.5\bar{H}_i^t. \qquad (1)$$

Finally, we will apply our method for dynamically rewiring the network at each time interval. In a given implementation, we will establish a metric on the actions of users in order to determine which users are the "best" or most likely to
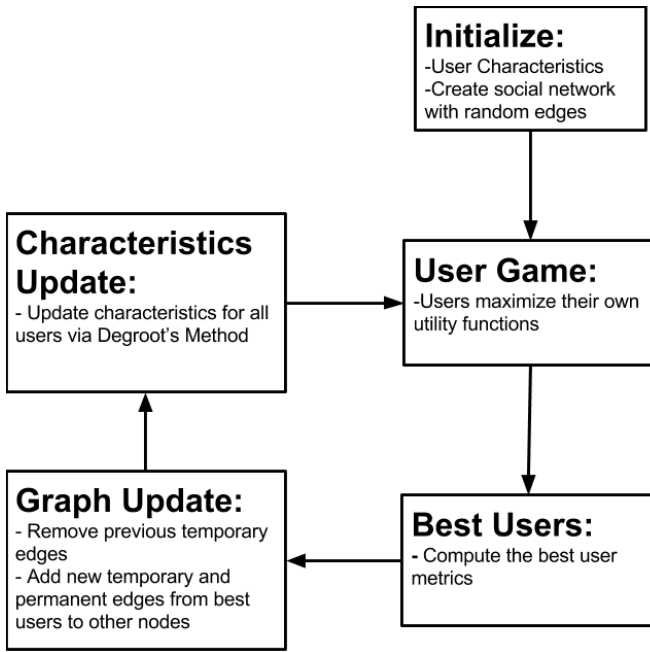
Fig. 1.   Block diagram representation of our method.

be a good influence on the other users. In our previous work, the best users (from the electricity company's point of view) had the smoothest load profile, and their habits were distributed across the social network [1]. The identities of these users will be widely disseminated to the userbase, thus adding potential outgoing edges from these best users to all other users in the system. These edges will be probabilistically added with likelihood based on their similarity scores. The weights on these edges are not necessarily the same as those for existing edges; rather, these weights will vary based upon the relative weight that the population will place on the artificial links as opposed to the basic links. At the end of each iteration, these temporary edges will be removed, returning to the base set of edges. Here, "iteration" refers to the cycles in which a new list of best users is generated. This would vary depending on the application, but would typically be at least one month.

Fig. 1 shows our method, which we will describe mathematically in Section IV.

IV. MODEL DESCRIPTION

We present a modular, generalized model for applying our techniques to any network decision-making problem where social influence is present.

The following notation will be used in the remainder of this paper. The graph is comprised of the set of vertices $\mathcal{V}$ and edges $\mathcal{E}$. $\mathcal{V}^*$ denotes the set of best users, and $\mathcal{E}^*$ denotes the set of temporary edges generated at each step. The total population size is denoted N. The action of user i is denoted a_i and is drawn from the action space A. We use $\mathbf{a}$ to denote the vector of actions of the entire userbase. User i's characteristics are represented by H_i, with $H_{i,m}$ and $H_{i,n}$ representing the malleable and nonmalleable characteristics, respectively.

Furthermore, $\overline{H_i}$ denotes the average characteristic values of the set of agents with outgiong edges to node i. Finally, the normalized characteristics, which are used in the edge generation step, are given by H_i^n. In the users' utility maximization game, $f_{a_i}(H_i, \mathbf{a})$ denotes the function that computes the action of user i as a function of user i's characteristics and the actions of the rest of the population. The social utility function of the set of actions is $U^s(\mathbf{a})$. The social function used to determine the best users is $B_i(\mathbf{a})$ for user i; in the cases where the optimality of each user is seperable, B_i will depend only on a_i. We will seek to maximize U^s; also, the users with the highest B values will be selected as the best users. H_{soc} denotes the net "social" characteristics of an agent, that is, a score from 0 to 1 representing the product of the agent's traits relating to social consciousness and openness to the kind of influence proposed in this paper. P_{edge} denotes the value the similarity score is multiplied by to obtain the edge probability for the initial social network. P_{temp} represents the probability of edge formation in an agent's temporary social network. M denotes the number of "best users" to generate in each iteration. W denotes the weight to give to these temporary edges.

The description detailed in Algorithm 1 specifies our generalized model.

V. GENERAL SIMULATIONS

We conducted experiments under several different specific implementations of this model. In each case, we defined a set of malleable and nonmalleable characteristics of the users, a decision-making function, and a measure of social optimality. We varied other parameters of the model and observed the behavior of several aspects of the system. First, we observed the maximum, minimum, and average values of both the malleable characteristics and the decisions made as the simulation progressed. Second, we studied the behavior of the systems modularity over time. We used the Louvain method [30] to find the community structure with the maximum modularity and recorded this maximum at each iteration. Modularity measures how strongly a network is divided into communities. A high modularity means several communities that are all densely collected together, but each community is not well connected to others. Low modularity indicates that the network connections are spread across users more evenly. Louvain's method is a greedy optimization algorithm that can be used to quickly and accurately calculate the modularity of a network [30]. Third, we studied lists of the users that were chosen as best users, to see if there was significant change over time or if the same users were chosen repeatedly. All simulations were performed using MATLAB and the optimization toolbox. For the Louvain method, the software package provided by the authors was used [31].

A. Test Case 1

We consider a generalized test case. In this case, each user will have two characteristics, denoted C and D. We assume that C is malleable and D is nonmalleable. C and D are both

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Algorithm 1 General Formulation for Influencing a Social Network

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Data: Action Space, Utilty Functions, Characteristics, Number of Agents, Number of Best Agents

Result: Social Utility Value With Influence of Best Agents

Initialization;

Generate characteristics of agents;

$\mathcal{E} \leftarrow \emptyset$;

for $i \leftarrow 1$ *to* N **do**
    for $j \leftarrow 1$ *to* $N, j \neq i$ **do**
        $\mathcal{E} \leftarrow \mathcal{E} \cup (i, j)$ with probability
        $1 - P_{\text{edge}} \| H_i^n - H_j^n \|$ and weight 1;
    end
end

Generate initial actions for each agent;

while *Stopping Condition Not Yet Met* **do**
    for $i \leftarrow 1$ *to* N **do**
        $a_i \leftarrow f_{a_i}(H_i, \mathbf{a})$
    end
    Select the M individuals with the smallest B values;
    $\mathcal{E}^* \leftarrow \emptyset$;
    for $i \in \mathcal{V}^*$ **do**
        for $j \leftarrow 1$ *to* N **do**
            $\mathcal{E}^* \leftarrow \mathcal{E}^* \cup (i, j)$ with probability $P_{\text{temp}} H_{\text{soc}}$ and weight W;
        end
    end
    for $i \leftarrow 1$ *to* N **do**
        $H_{i,m} \leftarrow 0.5 H_{i,m} + 0.5 \overline{H_{i,m}}$
    end
end

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on the interval $[0, 1]$. Initial values of C and D are drawn from $\mathcal{U}(0, 1)$. The "action" of each user is a number on the interval $[0, 2]$ and is obtained by adding C and D together. In this test case, we also use this value as the utility of the user. The social utility function is the average of all decisions. The product of C and D is used as the H_{soc} metric.

Mathematically

$$H_{i,m} = C_i \sim \mathcal{U}(0, 1) \tag{2}$$

$$H_{i,n} = D_i \sim \mathcal{U}(0, 1) \tag{3}$$

$$A = [0, 2] \tag{4}$$

$$f_{a_i}(H_i, \mathbf{a}) = D_i + C_i \tag{5}$$

$$B_i(\mathbf{a}) = a_i \tag{6}$$

$$U^s(\mathbf{a}) = \frac{1}{N} \sum_{i=1}^{N} a_i \tag{7}$$

$$H_{\text{soc}} = C_i D_i. \tag{8}$$

Unless otherwise specified, $P_{\text{edge}} = 5$, $P_{\text{temp}} = 1$, $W = 0.5$, $N = 200$, and $M = 5$.

B. Performance Goals

We are interested in examining the effect of increasing the number of best users on the network's modularity, the overall social utility, $U^s(\mathbf{a})$ present in the network, and the malleable characteristics C_i of the users. A high-modularity network means that most users are only talking to a few people, and it may be hard to influence a large set of users. Therefore, our method should aim to decrease the modularity of the social network and reach as many users as possible. For the sake of simplicity, we have chosen $U^s(\mathbf{a})$ as the mean action over all the users in the network. To maximize this social utility, our method should increase our user's action value over time. Since the action is defined as $D_i + C_i$, our method should increase the malleable characteristics to maximize each user's utility. This gives our test the simple goal of low values for modularity and high values for social utility and malleable characteristics. We implemented our method on each variation of our test case for zero (i.e., making no changes to the original network), one, and five best users to show that our method is robust to changes in parameter values and utility functions.
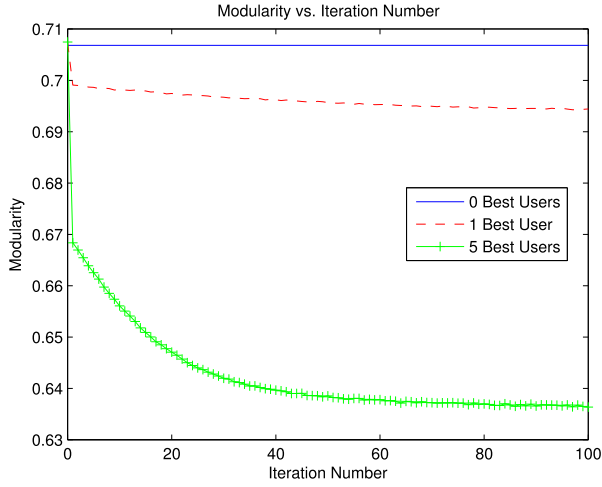
C. Results—Test Case 1

Fig. 2(a) shows the modularity result over time for the cases in which zero, one, and five best users are used. We can see that the modularity decreases over time under this framework, with greater decrease when the number of best users increases. Comparing these results with the C values over time as shown in Fig. 2(b) shows that the minimum value increases substantially as the number of best users increases. This in turn will cause the likelihood of edges forming between the best users and those at or near the minimum to increase; changing the number of edges in this manner will therefore decrease the modularity, as many disconnected pairs of nodes will have one or more common neighbors in the best users.

Further analysis of Fig. 2(b) shows that the addition of best users increases both the minimum and maximum values of C substantially. The addition is so dramatic that the minimum value when five best users are used is greater than the mean value when one best user is used. The difference, however, is not as substantial in the case of the maximum value. In fact, during the early iterations, having more best users actually worsens the maximum value. This can be explained by the fact that the best users are likely to be at or near the maximum value to begin with; the only influence they will have to be added will be from each other, which will tend to drop the C values of the very best users slightly. Furthermore, the difference between the mean and the extreme cases decreases with best users, with the mean being near the maximum when five best users exist.

Mirroring Fig. 2(b) and (c) shows the maximum, minimum, and mean user decision values when zero, one, and five best users are used. Recall from (7) that the mean of the users' decision values is the overall social utility. We can see that the results are generally similar, showing a steady increase in social utility and decrease between the distance between maximum and minimum when best users are added. This means that the other users are converging to the best users actions over time.
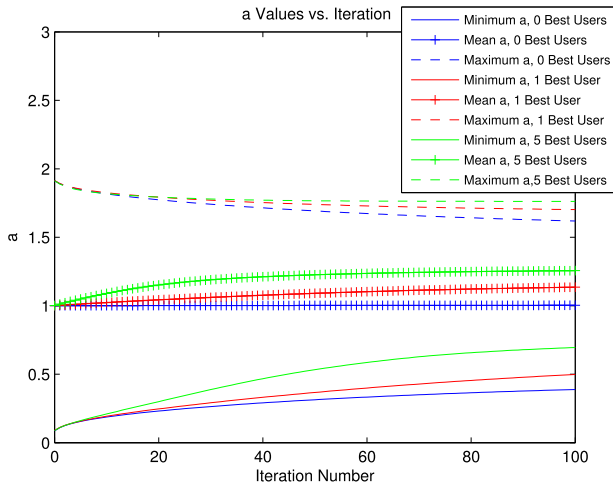
Fig. 3 shows the mean decision (social utility) values as the number of best users increases. We can clearly observe

(a)
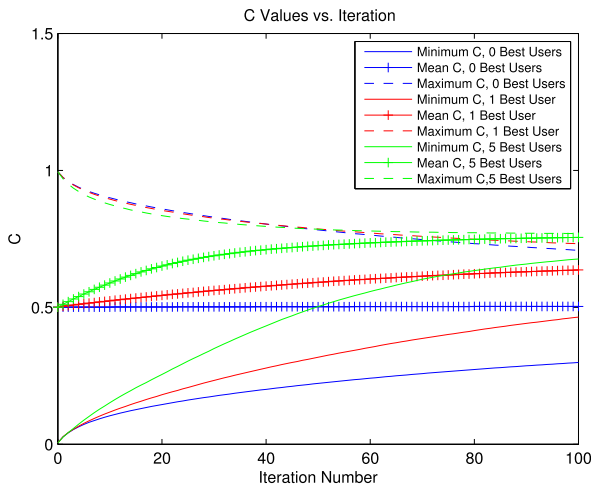
(b)

(c)

Fig. 2. Results of varying best users for Test Case 1. (a) Graph modularity. (b) Maximum, minimum, and mean C_i values. (c) Overall social utility as well as maximum and minimum individual decision values.

diminishing returns as the number increases, suggesting that the addition of more best users does not increase the maximum value of social utility and may not be worth the cost, depending on the application. Fig. 4 shows the effects as the artificial
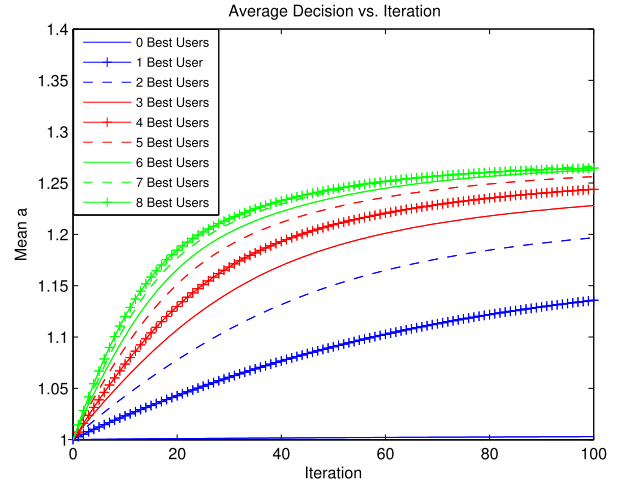

Fig. 3. Social utility values with varying numbers of best users for Test Case 1.
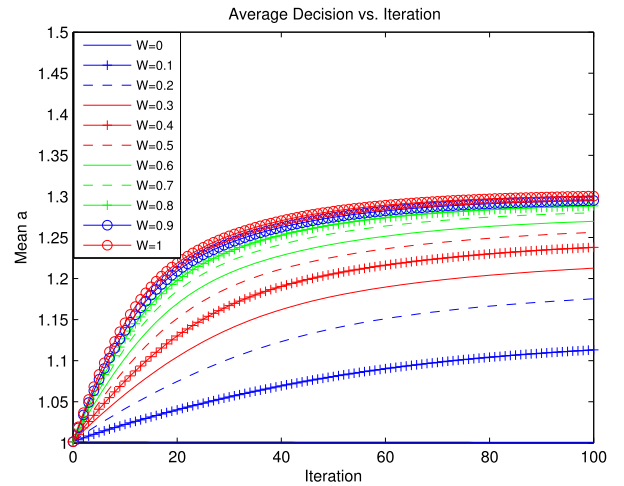

Fig. 4. Social utility values with varying W values.

edge weight varies from 0 to 1. Interestingly, increasing the weight of the edges is also subject to diminishing returns; the relationship between the weights and the final equilibrium is not linear. For small values, an increase in the weight produces a significant improvement in results, but for $W > 0.6$, the improvement becomes sufficiently marginal. This suggests that the methodology that we model could be successful even if the artificial links we create are valued significantly less than the base links, a likely scenario.

D. Test Cases 2 and 3

In this section, we consider two variants to the model presented earlier. In the first case, everything is the same except that C^2 is used in place of C when calculating the users' actions. In the second case, D^2 is used in place of D. These test cases decrease the impact of nonmalleable and malleable traits, respectively. Formally, for Test Case 2

$$f_{a_i}(H_i, \mathbf{a}) = D_i + C_i^2 \tag{9}$$

and for Test Case 3

$$f_{a_i}(H_i, \mathbf{a}) = D_i^2 + C_i. \tag{10}$$

with all other properties being the same.
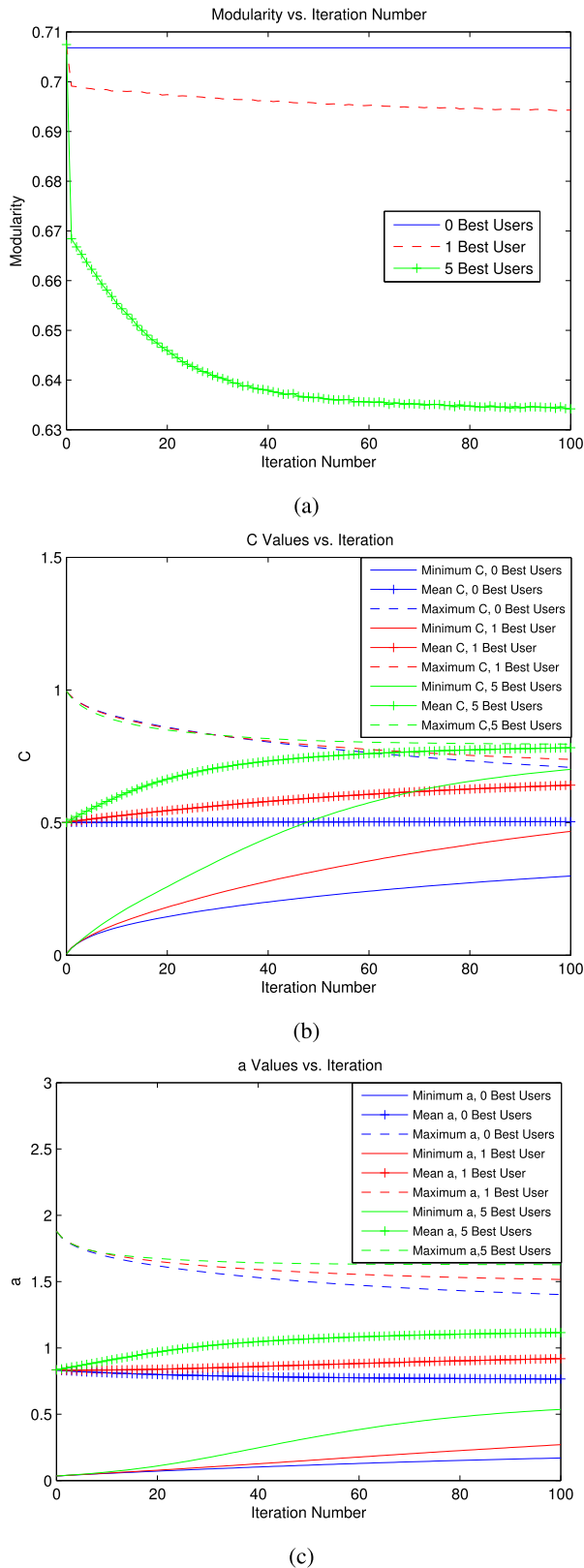
(a)

(b)

(c)

Fig. 5. Results of varying best users for Test Case 2 ($a_i = D_i + C_i^2$). (a) Graph modularity. (b) Maximum, minimum, and mean C_i values. (c) Overall social utility as well as maximum and minimum individual decision values.

E. Results—Test Cases 2 and 3

In Fig. 5(a), we see the results of the graph modularity when the C^2 term is used. This change does not significantly change the modularity of the graph.

Fig. 5(b) shows the C values when the C^2 term is used; again, we do not see any significant differences from Test Case 1.

Fig. 5(c) shows the decision values for the case in which the C^2 term is used. We see generally lower mean decision and social utility values; this is to be expected, since the mean C^2 term is lower than the mean C term. The overall trends are similar to Test Case 1, however, showing that the results remain similar, only shifted downward.

In Fig. 6(a), the modularity of the network is shown when a D^2 term is used. Unlike Test Case 2, we do see a small but noticeable increase in the modularity of the graph. This is likely caused by the effect the D^2 term has on the decision of which users are most suited to be the best users. At the high end of the spectrum, small variations in D will affect a user's decision more than small variations in C. This will cause the selection of best users with lower C values, thus providing worse influence and reducing the chances of edges being formed with lower users.

Fig. 6(b) shows the resultant C values from the use of a D^2 term, and Fig. 6(c) shows the decision values and social utility from the use of this term. We can observe that, in both the cases, the gains made in social utility are less than those in Test Case 2 or Test Case 1. This again suggests that the use of an exponent in the nonmalleable term has a detrimental effect on the ability of the users with the highest C values to be selected as best users.

Taken together, these results show how changes to the decision function can affect the quality of the influence from the best users.

F. Test Case 4

In this section, we compute a final variant of our test case. In this instance, the action space is an integer on $[0, 1]$, generated by taking the product of C and D.

Specifically

$$A = [0, 1] \tag{11}$$

$$f_{a_i}(H_i, \mathbf{a}) = D_i C_i \tag{12}$$

with all other aspects defined identical to before.
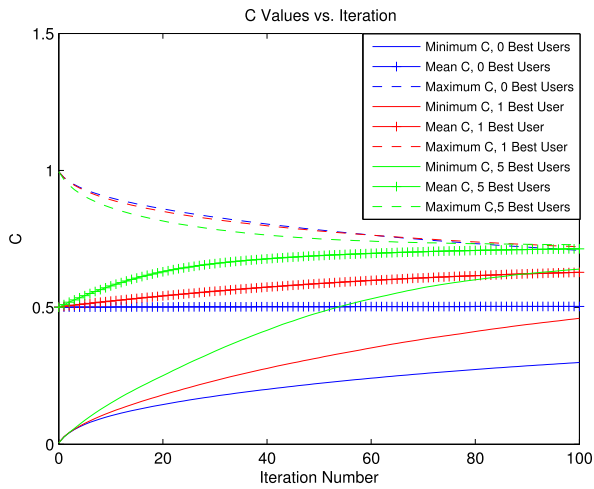
G. Results—Test Case 4

Fig. 7(a) shows the modularity results from the use of this decision function. We see modularities that are not significantly different from those we saw in the first two examples.

Fig. 7(b) shows the C values that result from this decision method. Again, there is no any substantial difference between these results and those for the first two examples.

In Fig. 7(c), we plot the decision results with zero, one, and five best users. In contrast to Fig. 7(a) and (b), Fig. 7(c) shows substantially different behavior than the previous examples. The curves showing the maximum values over time behave similar to those in the previous simulations, but the others differ significantly. The minimum values remain near zero, owing to the fact that a user with a very low D value cannot achieve an action that is not near zero. The mean value
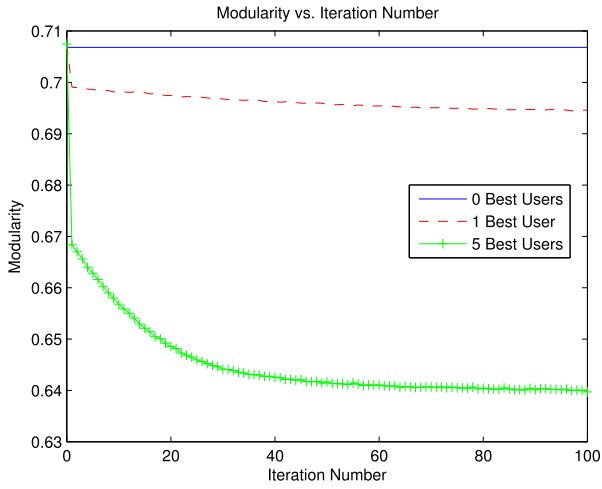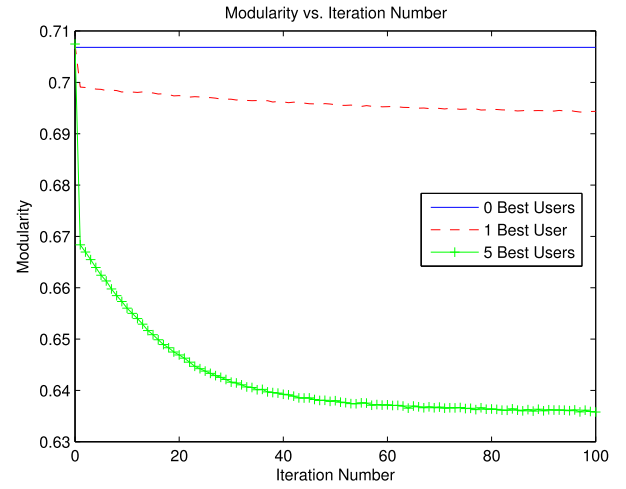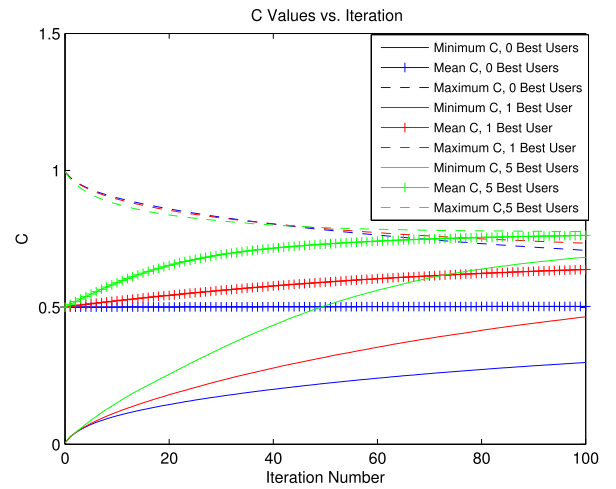
(a)

(b)
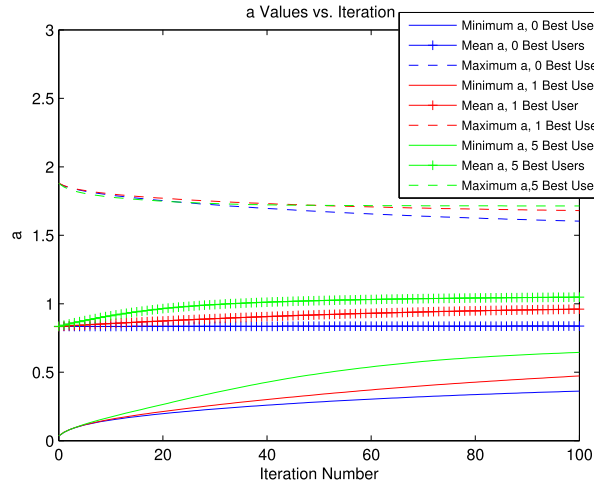
(c)

Fig. 6.   Results of varying best users for Test Case 3 ($a_i = D_i^2 + C_i$). (a) Graph modularity. (b) Maximum, minimum, and mean C_i values. (c) Overall social utility as well as maximum and minimum individual decision values.

(a)

(b)

(c)

Fig. 7.   Results of varying best users for Test Case 4 ($a_i = D_i C_i$). (a) Graph modularity. (b) Maximum, minimum, and mean C_i values. (c) Overall social utility as well as maximum and minimum individual decision values.
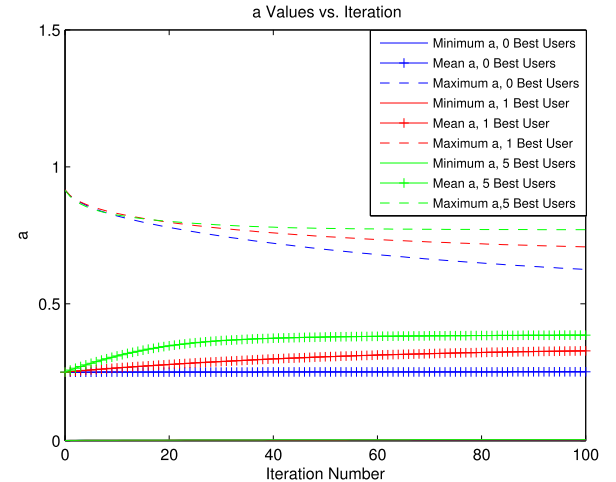
(social utility) curves begin at 25% of the maximum value and fail to reach half of the maximum value even when five best users are used. This contrasts with the previous test cases,

which all saw social utility above the halfway mark. The D value in this case sets a "cap" that is much more restrictive than that in the additive case.

These results, collectively, show how changes to the decision function can affect the quality of the final result, even when the behavior of the malleable characteristics is unchanged.

H. Choice of Best Users

In addition to all the tests discussed previously, we studied the choices of best users and their evolution over time. In all four choices of the decision function, the results were the same; the same five users were selected as the best at each iteration. Consequently, we can infer that for a simple, monotonic criterion like the one chosen here and the use of DeGroot's method for the updates, the same users will remain the best users.

VI. Conclusion

In this paper, we have presented preliminary work toward a generalized method for influencing decision making in social networks. We have presented theoretical results of manipulating a network in the way we proposed under a variety of circumstances. These ideas form a framework in which cutting-edge communication technology can be used to encourage users to behave in socially optimal ways. In the future, this framework could be elaborated upon or empirically tested.

Our results have compared the performance, both from a theoretic and a practical standpoint, of the model with different decision criteria. Specifically, our method reduces graph modularity, increases overall social utility, and changes the users malleable characteristics across a social network, and is robust to different types of utility functions. Our test cases demonstrated that the social utility of the users can be affected by the method for selecting best users, the choice of the utility function, and the ability of the nonmalleable characteristics to constrain the possible actions.

There is room for theoretical refinement of the ideas presented here. Ideas for future work include the possibility of different users having different utility functions or action spaces. More complex networks could also be used; weighted graphs or multimodal graphs could be introduced into the framework. One could also have the best users to receive assistance in optimizing their actions, to make them even more socially optimal.

From an empirical standpoint, this framework can be applied to other problems by changing the utility function, action space of user decisions, and relevant user characteristics. Example applications include smart-grid adoption and voluntary immunization programs. Implementing this framework in the real world requires some initial data gathering or extra computation. For example, when installing a smart grid in an apartment building, the utility company would need to know the initial characteristics and connections of the user population. This can be done through regular surveys or by formulating an inverse problem to deduce the characteristics and network connectivity from user actions and publicly available data such as Facebook friendship.

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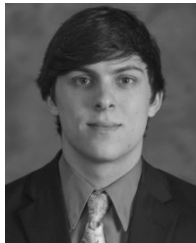
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