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Patch-Based Bilateral Filter and Local M-Smoother for Image Denoising

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Abstract. In the field of image analysis, denoising is an important preprocessing task. The design of an efficient, robust, and computationally effective edge-preserving denoising algorithm is a widely studied, and yet unsolved problem. One of the most efficient edge-preserving denoising algorithms is the bilateral filter, which is an intuitive generalization of the local M-smoother. In this paper, we propose to modify both the bilateral filter and the local M-smoother to use patches of the image instead of single pixels in the denoising process. With this modification, the filtering effet becomes more sensitive to the different areas of the image and the filtering results improve. The denoising quality of these patch-based filters is evaluated on test images and compared to the classical bilateral filtering and local M-smoother.

1 Introduction

Nowadays, digital images are used in many fields of application (ranging from multimedia home applications to professional ones in medicine, security, or geography, for instance). However, these images often come from an acquisition process that produces noise and blur on the resulting image. These artifacts can lead to misinterpretations, lower the confidence in the images, or even lead to a suboptimal decision making. Therefore, attenuating noise and blur with appropriate tools proves to be mandatory before any further image analysis.

In denoising algorithms, the challenge is to identify as accurately as possible the noisefree signal in the image. In particular, imperfect noise removal could possibly damage fine textures and region edges. Most denoising tools are indeed filters that trim high frequencies of the measured signal; suboptimal parameter tuning might thus blur the signal, which is obviously a highly undesired adverse effect. Denoising tools that can attenuate noise with minimal blurring are often said to be *edge-preserving*.

Unsupervised edge-preserving denoising of images can be achieved with various paradigms: partial differential equations [1], Bayesian denoising [2], kernel regression [3], gradient approximation [4], total variation [5], wavelet transform [6], density approximation, and robust statistics. As they basically achieve some sort of mode identification, the local M-smoother [7] (LMS) and bilateral filtering [8] (BF) can easily be cast within the two last paradigms. Because of their intuitive formulation, low computational cost, and global efficiency, BF and LMS have been quite popular options among

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researchers and practitioners. Still, their intrinsic design makes them optimal only for piecewise constant signals. In any other case, such as ramps or even slight edge blur, their performance significantly drop, because modes of the signal tend to be less clearly separated.

This paper aims at adapting BF and LMS in order to use groups of adjacent pixels in the image, called *patches*, instead of single pixels. Replacing scalar values with multidimensional information such as patches in the mode identification process hopefully digs the gaps between modes. Hence, the denoising process should also be more accurate. Patch-based denoising has already been investigated by Kervrann [9], [10]. His approach produces good results but relies on empirical arguments and does not take into account the distance between the pixels of a patch. The consequence is that results around edges critically depend of the size of the selected patches which can lead to lower performances in these areas.

The remainder of this paper is organized as follows. Section 2 will briefly introduce the local M-smoother and bilateral filtering, before moving to their extension to patches. The benchmark images used to assess the denoising quality are introduced in Section 3, along with other aspects the experimental setup. Section 4 presents and comments the results. Finally, Section 5 draws the conclusions and sketches some directions for future work.

2 Image Model and Patch-Based Filtering

2.1 Image Model

Let us define a D-dimensional image as a vector of pixels in which the ith pixel position can be uniquely identified by vector $\mathbf{x}_i = [x_{i1}, \ldots, x_{iD}]^T$. The pixel at coordinate $\mathbf{x}_i$ has an observed intensity $f_i = u_i + \varepsilon_i$ which consists of the noisefree signal u_i and the noise component ε_i. In the following developments, we assume that all noise components ε_i are Gaussian and i.i.d.

2.2 Local M-Smoother and Bilateral Filter

The derivations of the LMS and BF within the framework of mode identification are detailed in [7] and [11], for instance. The objective function is

$$E(\mathbf{u}) = \sum_{i=1}^{P} \sum_{j \in N_i} w_{ij} \Psi\left(-\frac{1}{2}(u_i - f_j)^2\right) \quad , \tag{1}$$

where P is the number of pixels, N_i is the neighborhood of $\mathbf{x}_i$ and $w_{ij} = \exp(-\frac{(\mathbf{x}_i - \mathbf{x}_j)^2}{2\rho^2})$ is a spatial weight. The radiometric kernel $\Psi(v) = \exp(-\frac{u}{2\sigma^2})$ plays the same role as in a Parzen window estimator. Maximizing $E(\mathbf{u})$ can be achieved with a fixed-point strategy and leads to the update rule

$$\hat{u}_i^{k+1} = \frac{\sum_{j \in N_i} w_{ij} \Psi'\left(\frac{1}{2}(\hat{u}_i^k - f_j)^2\right) f_j}{\sum_{j \in N_i} w_{ij} \Psi'\left(\frac{1}{2}(\hat{u}_i^k - f_j)^2\right)} \quad , \tag{2}$$

which is initialized with $\hat{u}_i^0 = f_i$. As can be seen, the local M-smoother performs a weighted average of the surrounding pixel values, in order to obtain the filtered value of a given pixel. The global weights are thus the products of both spatial and radiometric kernels, which makes them adaptive and anisotropic.

Bilateral filtering is an intuitive generalization of the LMS. In bilateral filtering, the noisy values f_j are replaced in the update rule with the filtered values u_j^k, which are assumed to be better estimates of the noisefree data. This leads to

$$\hat{u}_i^{k+1} = \frac{\sum_{j \in N_i} w_{ij} \Psi' \left(\frac{1}{2} (\hat{u}_i^k - \hat{u}_j^k)^2 \right) \hat{u}_j^k}{\sum_{j \in N_i} w_{ij} \Psi' \left(\frac{1}{2} (\hat{u}_i^k - \hat{u}_j^k)^2 \right)} \quad , \tag{3}$$

with the same initialization as for the LMS. This modification also means that the filter becomes nonlocal after the first update, as the intensities that were out of the neighborhood of $\hat{u}_i^k$ have been involved in the determination of $\hat{u}_j^k$ at a previous step. Because of this diffusion process, BF can converge to a constant image and has to be stopped after a few iterations.

2.3 Patch-Based Bilateral Filter

Each pixel of an image can be interpreted as the realization of a random variable with some expected value. The LMS and BF rely on the idea that the pixel realizations are locally distributed among one or several clearly separated modes. The use of patches is motivated by the observation that it is usually easier to identify modes in a multidimensional space, as the risk of overlapping is lower. For this purpose, if D is the image dimensionality, let us define patch P_i^k as the subset of p^D pixel values at iteration k, centered on $\mathbf{x}_i$. Patch-based bilateral filtering (PBBF) calculates the weights between the pixels in $\mathbf{x}_i$ and $\mathbf{x}_j$ using $d(P_i, P_j)$, the distance between patches P_i and P_j. The PBBF update rule is then written as

$$\hat{u}_i^{k+1} = \frac{\sum_{j \in N_i} w_{ij} \Psi' \left((d(P_j^k, P_i^k) \right) \hat{u}^k}{\sum_{j \in N_i} w_{ij} \Psi' \left((d(P_j^k, P_i^k) \right)} \quad . \tag{4}$$

The choice of a particular distance function can be guided by the noise model or other properties of the image. A classical option is the L_2 distance

$$d(P_j^k, P_i^k) = \sqrt{\sum_{n=1}^{p^D} (P_{j_n}^k - P_{i_n}^k)^2} \quad , \tag{5}$$

where $P_{j_n}^k$ is the nth pixel in patch P_j^k.

Patches can be introduced in the LMS as well. In this case, the update rule for the patch-based local M-smoother (PBLMS) is written as

$$\hat{u}_i^{k+1} = \frac{\sum_{j \in N_i} w_{ij} \Psi' \left((d(P_j^k, P_i^0) \right) f_j}{\sum_{j \in N_i} w_{ij} \Psi' \left((d(P_j^k, P_i^0) \right)} \quad . \tag{6}$$

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Here the patch centered on $\mathbf{x}_i$ contains values from the original image f. If the patch size is set to 1 and the chosen distance is L_2, the PBBF and PBLMS algorithms obviously reduce to the classical BF and LMS, respectively.

3 Databases and Experimental Design

3.1 Databases

Performance assessement is achieved using a set of digital pictures[1] that is widely used in the literature ([12], [13], [14] and [15]). The images contain 512^2 pixels with values between 0 and 255. All images have been polluted by i.i.d. additive Gaussian noise with standard deviations of 5, 10, and 20.

3.2 Experimental design

For each filter, the filter parameters ρ and σ have been optimized with a training image. Next, denoising performances have been evaluated 50 times on the same image polluted with noise following the same noise model that the one used to generate the training image. This methodology has been used for the PBBF, PBLMS, BF, and LMS. The parameters ρ and σ have been optimized for 1 to 15 iterations, which means 15 experiments for each algorithm, image, and noise model. For each image and noise model, the best performances have been selected in each set of 15 experiments. The patch-to-patch distance is the L_2 distance.

The denoising performances are evaluated with the root mean square error, defined as

$$\mathrm{RMSE} = \sqrt{\frac{1}{M}\sum_{m=1}^{M}\frac{1}{I}\sum_{i=1}^{I}(\hat{u}_i^k - u_i)^2} \ , \tag{7}$$

where M is the number of denoising trials, m is the trial index, I is the total number of pixels in the image and i the pixel index.

4 Results

The denoising results are given in Table 1. In terms of RMSE, the patch-based filters outperform the classical LMS and BF algorithms. For all noise models, the best results are given either by the PBBF or PBLMS depending of the image. The variance of the RMSE over the trials is of the same order of magnitude for all filters and proves to be small very small with respect to the corresponding RMSE values. Typical results are illustrated in Fig. 1. These images are samples of the original Lena image which was analyzed. Although all filters correctly preserve edges, the images obtained with patch-based filters look less noisy than those computed by classical filters. Moreover, fine details appear to be more salient.

[1] Available at http://decsai.ugr.es/~javier/denoise/test_images/index.htm.

noise std	5		10		20	
Lena	mRMSE	vRMSE	mRMSE	vRMSE	mRMSE	vRMSE
LMS	3.3728	$3.7430.10^{-5}$	5.2371	$8.3978.10^{-5}$	8.2675	$3.2041.10^{-4}$
BF	3.3724	$3.2498.10^{-5}$	5.1428	$1.0774.10^{-4}$	7.4586	$3.6691.10^{-4}$
PBLMS	3.1642	$3.0059.10^{-5}$	4.6202	$8.0814.10^{-5}$	6.7656	$4.3030.10^{-4}$
PBBF	3.1642	$2.0347.10^{-5}$	4.6055	$1.1671.10^{-4}$	6.6330	$5.3355.10^{-4}$
Barbara	mRMSE	vRMSE	mRMSE	vRMSE	mRMSE	vRMSE
LMS	4.0035	$4.3394.10^{-5}$	6.9220	$1.7707.10^{-4}$	11.731	$5.1862.10^{-4}$
BF	4.0047	$5.6820.10^{-5}$	6.9176	$1.8908.10^{-4}$	11.526	$4.8474.10^{-4}$
PBLMS	3.5672	$5.3594.10^{-5}$	5.8781	$1.2557.10^{-4}$	9.2186	$5.9284.10^{-4}$
PBBF	3.5673	$2.7393.10^{-5}$	5.8776	$1.3783.10^{-4}$	9.2703	$6.6194.10^{-4}$
Boat	mRMSE	vRMSE	mRMSE	vRMSE	mRMSE	vRMSE
LMS	3.9298	$3.0525.10^{-5}$	6.1785	$1.1108.10^{-4}$	9.5849	$3.3252.10^{-4}$
BF	3.8959	$3.8741.10^{-5}$	6.0238	$1.1891.10^{-4}$	8.8940	$3.4878.10^{-4}$
PBLMS	3.8026	$3.2754.10^{-5}$	5.6561	$1.1671.10^{-4}$	8.3808	$4.1329.10^{-4}$
PBBF	3.8011	$2.2535.10^{-5}$	5.6688	$1.2218.10^{-4}$	8.2600	$3.8027.10^{-4}$
House	mRMSE	vRMSE	mRMSE	vRMSE	mRMSE	vRMSE
LMS	3.3561	$7.7022.10^{-5}$	5.1959	$6.8212.10^{-4}$	8.5771	$1.2275.10^{-3}$
BF	3.2924	$8.9099.10^{-5}$	5.0704	$5.0182.10^{-4}$	7.5748	$1.5632.10^{-3}$
PBLMS	3.1979	$1.3145.10^{-4}$	4.6722	$6.7429.10^{-4}$	6.8100	$2.4467.10^{-3}$
PBBF	3.1555	$1.0939.10^{-4}$	4.5479	$4.7471.10^{-4}$	6.4854	$2.8182.10^{-3}$

Table 1: Averaged RMSE results for 50 images polluted with the same noise model (additive i.i.d. Gaussian noise with standard deviation 5,10 or 20) 'mRMSE' is the mean RMSE for the 50 images, and 'vRMSE' is the variance of the RMSE over those images.

Fig. 1: Example of the Lena image. Top row, left: original image, middle: bilateral filtering, right: patch-based bilateral filtering. Bottom row, left: noisy image, middle: local M-smoother, right: patch-based local M-smoother.

5 Conclusions

The introduction of patches in the classical bilateral filter and local M-smoother algorithms relies on the idea that a filter whose weights are based on vector comparison rather than scalar ones is able to better separate signal modes. Our experiments show that the patch-based filters outperforms bilateral filtering and the local M-smoother both in terms of RMSE and of visual inspection. Future work will focus on more complex noise models and will investigate other patch-to-patch distance functions.

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