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Heterogeneity Enhanced Order in a Chaotic Neural Network

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Abstract. Order of the mean field by heterogeneity is studied in the turbulent phase of a chaotic neural network. Heterogeneity means the distributed randomness of the input in each neuron or the weight in the network. The average power spectrum of the mean field is used to observe the order and to focus on its peak sharpness. The sharpness of the power peak grows remarkably in the turbulent phase, except around the phase, due to the input disorder. One can find the maximum of the power sharpness as the weight disorder increases in the turbulent phase. We suppose that this ordering effect is important for processing information for actual neural networks because of the general existence of such heterogeneity.

1. Introduction

The effects of noise and disorder in nonlinear systems have received considerable attention. Stochastic resonance is one of famous examples for noise-induced order [9]. This is a cooperative effect between noise and a weak sinusoid. In N globally coupled integrated and fire neurons, noise creates synchronized state in the mean field and the individual neuron, which is not found in the isolated neuron [6]. Several studies have focused on a stabilization in chaotic systems. The flip-flop process in the Lorentz system is stabilized by noise [8]. The distribution in a power spectrum is ordered by noise in the Belousov-Zhabotinskii (B-Z) map [4].

The effects of disorder in nonlinear systems have also been reported to produce order as the effect of noise. In a spatially extended system of forced, damped, and nonlinear pendula, disorder makes chaotic patterns into complex but regular periodic patterns [3]. A globally coupled map is also reported to create order in the mean field by disorder in each map [7]. Most of the studies reported that the noise or disorder increases the coherence of the system.

In this paper, we focus on a chaotic neural network that has been proposed in studies of the giant axons of squid and the Hodgkin-Huxley equation [2]. The neuron model in this network was the first to provide a chaotic response; some parameter combinations trigger a chaotic output. These associative chaotic

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dynamics of the network are used for information processing, especially as a means of escaping from local minima in optimization and retrieving associative memories [10, 1].

Heterogeneity means the distributed randomness of the input in each neuron or the weight in the network. The average power spectrum of the mean field is used to observe the order and to focus on the sharpness of its peak. To measure the sharpness of the power spectrum peak, we introduce the modified autocorrelation function of an average spectrum. We guess that this ordering effect is important for processing information for actual neural networks because of the general existence of such heterogeneity.

2. Ordering in the Mean Field of a Chaotic Neural Network with Heterogeneity

2.1. Chaotic neural network

The original chaotic neural network is represented as the dynamics of the difference equations of three variables [2]. A certain parameter combination can reduce variables into one. We study a global coupling network with a heterogeneous coupling or a heterogeneous input. The networks can be represented by the following equation.

$$\begin{aligned} y_i(t+1) &= ky_i(t) - \alpha f(y_i(t)) + \sum_{j=1}^{N} w_{i,j}\{f(y_j(t)) - f(y_i(t))\} + a_i, \\ x_i(t+1) &= f(y_i(t+1)), \\ f(z) &= \frac{1}{1+\exp(-\frac{z}{\epsilon})}, \end{aligned} \tag{1}$$

where
$y_i(t)$: internal state of ith neuron at time t,
$x_i(t)$: output of ith neuron at time t,
k : damping constant,
α : refractory parameter ($\alpha \geq 0$),
$w_{i,j}$: weight from jth neuron to ith neuron,
a_i : parameter of ith neuron based on the input and the threshold,
ϵ : steepness parameter of the sigmoid function.
We call parameter a_i the input for a bifurcation parameter. The bifurcation diagram and the Lyapunov exponent of the chaotic neuron model appeared in Ref. [2, 1]. We fix the parameter set at $k = 0.7$, $\alpha = 1.0$, and $\epsilon = 0.02$ throughout this paper. We normally consider $N = 100$ networks and set the initial values $y_i(0)$ as uniform random numbers distributed over $[-1, 1]$. We note that the equation has interactions for the difference between one neuron and the other neurons. This formulation corresponds to the original chaotic neural network [2] with α_i depending on each neuron. Disorder of the weight

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means that $w_{i,j}$ is distributed over $[w - \Delta w, w + \Delta w]$. We define the weight disorder ratio D_w as $\frac{\Delta w}{w}$. Disorder of the input also means that a_i is distributed over $[a - \Delta a, a + \Delta a]$. We also define the input disorder ratio D_a as $\frac{\Delta a}{a}$. We use the identical weight or input configuration for one disorder ratio.

The mean field $m(t)(= \frac{1}{N}\sum_{i=1}^{N} y_i(t))$ of neuron internal state $y_i(t)$ is used to calculate the power spectrum. We consider 4096 time steps for calculating the power spectrum by discarding 8192 transient time steps. The average power spectrum is calculated from 10 mean fields from 10 initial values. To measure the sharpness of the power spectrum peak, we employed the modified autocorrelation function as in Ref. [5], which is defined by

$$C = \frac{1}{M}\sum_{i=1}^{M} \frac{\sum_{j=1}^{M} P_{j+i \bmod M} P_j}{\sum_{j=1}^{M} P_j P_j}, \tag{2}$$

where P_i is the value of the power at the ith frequency index, and M is the number of discrete points in the spectrum. This provides a good measure of the flatness of a spectrum. C takes the value 1 when the spectrum is completely flat, and 0 when there are just δ-peaks. A better indicator of the sharpness of the peaks is given by

$$S = -\log_{10} C. \tag{3}$$

$S = 0$ is the signature of a completely flat spectrum and $S \to \infty$ is the signature of (very sharp) δ-peaks. We also use the KS entropy, which represents the amplification ratio of the tangential volume.

2.2. Results of numerical calculations

First, we show the average power spectrum at a certain ratio of each disorder. Figure 1 represents the average power spectrum at the input disorder ratio $D_a = 0.2$. The peak grows when the disorder ratio has a nonzero value. Figure 2 shows the average power spectrum at the weight disorder ratio $D_w = 1.0$. The result is similar to the input case.

Next, we show the sharpness of the power spectrum peak when the disorder ratio increases. Figure 3 represents the sharpness of the power spectrum peak and the corresponding KS entropy when the input disorder ratio changes. We found the parameter regions where the sharpness of the power peak increases as the input disorder ratio increases. From the Lyapunov spectrum calculations, we notice that the significantly increased regions of sharpness have smooth curves in the Lyapunov spectrum. This means that the input disorder can make order of the power spectrum in the turbulent phase. However, this ordering is only found in a certain range of weights except for small and large weight values in the turbulent phase. We are not sure of the ordering around the border of the turbulent phase. We could not observe a clear relation between the sharpness of the power spectrum peak and the KS entropy, but there is a decreasing trend of the KS entropy with the increasing disorder ratio.

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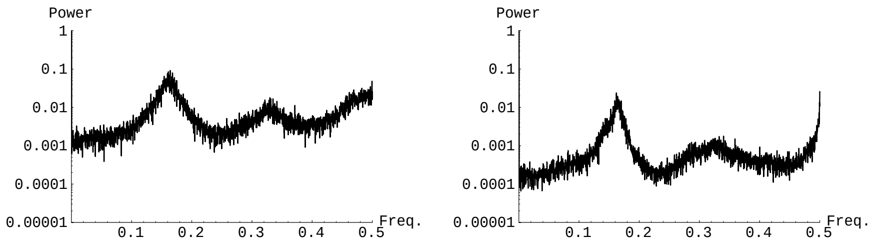

(a) $D_w = 0.0$, $D_a = 0.0$ (b) $D_w = 0.0$, $D_a = 0.2$

Figure 1: Average power spectrum at input disorder $D_a = 0.2$ (network has $w = 0.005$ and $a = 0.35$).

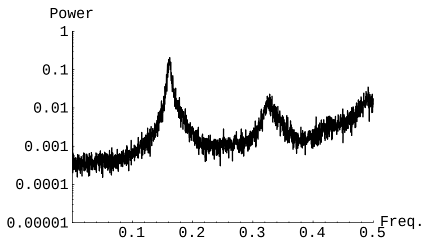

Figure 2: Average power spectrum at weight disorder $D_w = 1.0$ (network has $w = 0.005$ and $a = 0.35$).

Figure 4 shows the sharpness of the power spectrum peak and the corresponding KS entropy when the weight disorder ratio changes. We also discovered parameter regions where the sharpness of the power peak has an increasing trend as the weight disorder ratio increases. Here, the phase within the calculated range of the disorder ratio is turbulent. This situation is different from the result in the input disorder. We found the maximum power sharpness as the weight disorder increases in the turbulent phase.

3. Conclusion and Discussion

We studied the order of the mean field by heterogeneity in the turbulent phase of a chaotic neural network. The distributed randomness of the input in each neuron or the weight in the network was introduced as heterogeneity. The average power spectrum of the mean field had sharper peaks at nonzero values of the disorder ratio in the turbulent phase. We employed the modified auto-

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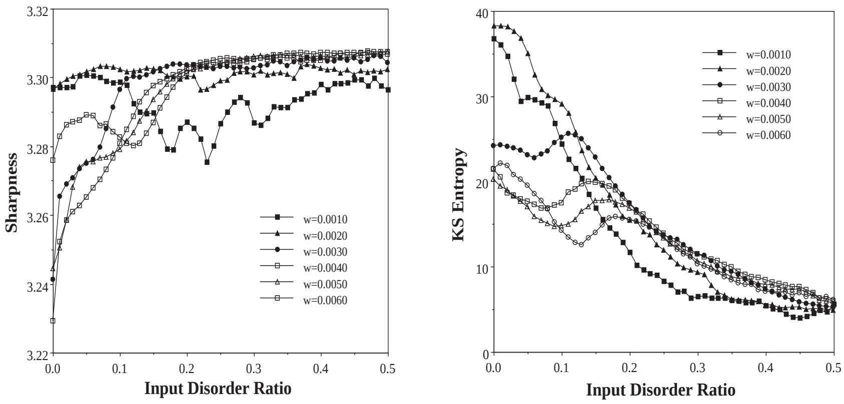

(a) Power peak sharpness (b) KS entropy

Figure 3: Power spectrum peak sharpness and corresponding KS entropy plotted as a function of the input disorder ratio D_a (network has $w = 0.005$ and $a = 0.35$).

correlation function of an average power spectrum to measure the sharpness of the power peak. We found the parameter regions where the sharpness of the power peak increases as the input disorder ratio increases. The sharpness of the power peak grew remarkably in the turbulent phase, except around the phase, due to the input disorder. One can find the maximum power sharpness as the weight disorder increases in the turbulent phase. We discovered differences in the increasing regions of the power sharpness according to each disorder ratio. However, we notice that these two disorders have a similar effect at the right-hand side of Eq. (1) from the viewpoint of the mathematical formulation. As a result of this similarity, there is a possibility of finding the same mechanism for ordering by any parameter disorder. We also notice the weight disorder makes the order weaker than the input disorder. This may be due to a different degree of ordering effect by the same ratio in the input and weight disorder. We had a decreasing trend of KS entropy as the increasing of the disorder ratio; however, we could not observed a clear relation between them. To analyze these phenomena in detail, we need a smoothness measure of the Lyapunov spectrum to decide the turbulent degree in the turbulence. This may provide a precise condition for ordering in the turbulent phase.

Ordering may be found in other phases since it is important for actual biological nerve systems. This effect may be used to process information in actual neural networks because we can easily find the heterogeneity of each element and the randomness of the interaction among elements in biological systems.

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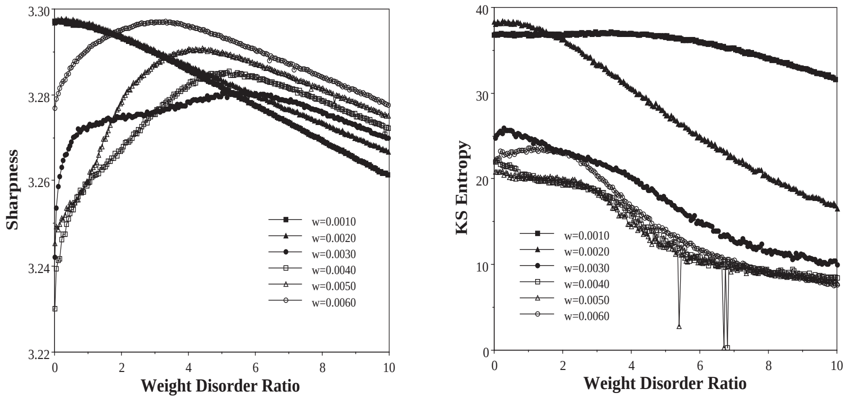

(a) Power peak sharpness (b) KS entropy

Figure 4: Power spectrum peak sharpness and corresponding KS entropy plotted as a function of the weight disorder ratio D_w (network has $w = 0.005$ and $a = 0.35$).

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