A CLASSICAL MORPHOLOGICAL APPROACH TO COLOR IMAGE FILTERING BASED ON SPACE FILLING CURVES

E.Stringa, A. Teschioni, C.S. Regazzoni

Department of Biophysical and Electronic Engineering
University of Genoa
Via Opera Pia 11A, 16145 Genova, ITALY

ABSTRACT

In the last decade, nonlinear image processing algorithms have been developed for noise removing. With respect to linear methods, they present the advantage of minimizing distortions of informative characteristics. Morphological filters are a class of nonlinear techniques that have been successfully applied in filtering monochromatic images, but their extension to vectorial -valued images encountered great difficulties, cause of the value-ordering step necessary for implementing them. This paper presents a new class of morphological operators for multivalued images. The problem of ordering the vectorial values is solved by the introduction of a vectorial-into-scalar transformation, based on the concept of Space Filling Curves: such a transformation allows one to preserve module information and also to take into account angle information.

1. INTRODUCTION

The development of efficient algorithms for restoration of grey-level or multivariate images starting from their noisy versions is an important task in image processing. Traditionally, linear filtering techniques have been used to solve such problem, but, even though this is a mathematically straightforward approach, it might cause some distortions of most informative image characteristics.

An alternative solution that has been studied in the last years is to employ non-linear filtering techniques, in particular, to use a class of non-linear operators known as morphological operators.

Mathematical Morphology (MM), which represents the framework for the development of this kind of operators, has originally been introduced by Matheron [1] and Serra [2] for application to binary images. MM provides a conceptual basis for facing image processing with non-linear tools; In particular, MM is based on representing images as sets, thus making it easier to evaluate the information about morphological structures present in an image. MM is characterized by the definition of a set of elementary operators (i.e., dilation and erosion) which can be combined in order to form more complex operators (e.g., opening and closing).

Even though this kind of operators has been extensively employed to design robust and efficient binary or grey-level image processing techniques, the large availability of multivalued images (e.g., colour images, motion fields) makes it useful to generalize them to the processing of multivalued information. However, in this case, one has to take into account that sorting operations over sets of scalar MM operators cannot be directly performed to consider both the module and angle information of each vector. New approaches [3,4] to multivalued image filtering have been developed in the past few years to cope with this problem.

The main drawbacks of such approaches lie in requiring a high computational load and in not taking into appropriate consideration the correlation between different vector components. This letter presents a new approach to the definition of a new class of morphological operators able to overcome the sorting problem for multivalued images. In particular, a transformation, based on the concept of space filling curves [5] (Fig.1), from multivalued sets into scalar sets is performed before applying scalar filtering operator: such a transformation allows one to preserve module information, represented by an appropriate norm, and also to take into account angle information.

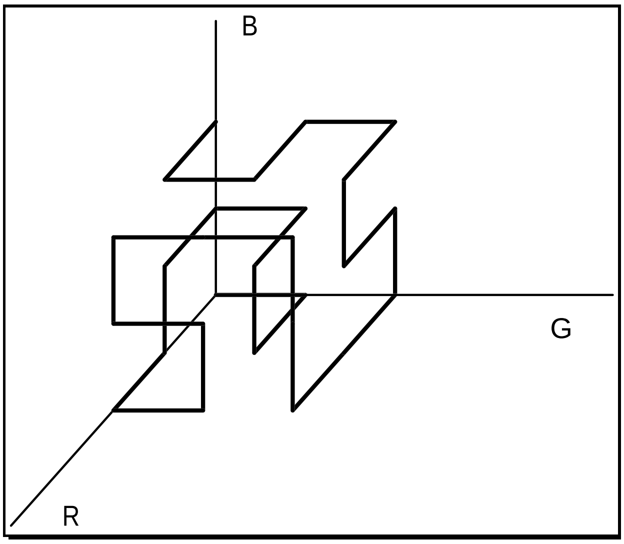

Figure 1. 3-D Space Filling Curve used for the Vectorial-to-Scalar Transformation.

2. MATHEMATICAL MORPHOLOGY AND SPACE FILLING CURVES

The generalization of fundamental morphological operations to multivariate signals requires the definition of an ordering relation in the signal value space, i.e., in a vector space.

A functional definition of morphological operators has been proposed by Cheng et al. [6] for grey-scale images. According to it, the basic morphological operations (i.e., dilation and erosion) can be obtained by replacing the value at a certain pixel with the largest (dilation) or smallest (erosion), value in a specific neighbourhood of that pixel. Let Z denote the set of integers and let $x(i,j)$ denote a discrete grey-level image signal, where $\{i,j\} \subset Z^2$ and let set $\{x\} \subset$ Z. A structuring element (SE) B can be expressed by its support domain $B \subset Z^2$; throughout this letter, only flat SEs will be considered. Let us denote as $B^S = \{ -b : b \in B \}$ the symmetric set of B and let us define $B_{u,v}$ as the translation of B by (u,v), where $(u,v) \in Z^2$. The result of the dilation of an image by means of B is defined as:

$$\left(x \oplus B^S\right)(i, j) = \max_{(u,v) \in B_{i,j}} \left(x(u,v) \right) \qquad (1)$$

and the result of image erosion by means of B is defined as:

$$\left(x \,\Theta\, B^S\right)(i, j) = \min_{(u,v) \in B_{i,j}} \left(x(u,v) \right) \qquad (2)$$

When such operations have to be generalized to multivalued images, a criterion for ranking multivalued sets is necessary.

In this letter, we propose the use of the approach to multivalued-set ranking described in [7] for median filtering of colour images. This method involves two steps. The first step is a vectorial-into-scalar transformation whose aim is reducing the dimensionality of an input multivalued field to that of a scalar field by preserving the module and angle information of each vector. The largest (smallest) value of the transformed set is computed as the output of dilation (erosion).

As a result, multivalued ordering is reduced to scalar ordering, and computational complexity is decreased by obtaining, at the same time, quite good filtering performances.

From a mathematical point of view, the problem can be stated as the search for a function $\gamma : \mathrm{Z} \rightarrow \mathrm{Z}^{\mathrm{N}}$, $\gamma = \gamma(\mathrm{t})$, $\mathrm{t} \in \mathrm{Z}$, $\gamma : \mathrm{Z} \rightarrow \mathrm{Z}^{\mathrm{N}}$ that allows one to map, in a continuous and unique way, N-D values into a one-dimensional space. Space Filling Curves (SFC) [5] satisfy this requirement. In [7], an approach based on SFC has been proposed, that allows one to perform another operation requiring multivalued ordering, i.e. vectorial median filtering. This operation involves two steps: 1) absolute sorting of a multivalued space based on Peano SFC to map multivalued data into an appropriate 1D space, and 2) median filtering operation in the scalar space. The main advantage of this approach is computational efficiency, as compared with those of other vectorial median filtering techniques with comparable performances. In this letter, we suggest using the same SFC as described in [7] (Fig. 1) as a basis for the selection of the minimum and maximum vectors within an SE B. According to the proposed approach, the following operators can be used for Reduced Vectorial Morphology (RVM): *Reduced Vectorial Morphological Dilation* (RVMD) of a multivalued signal $\boldsymbol{x}(i,j)$, $\boldsymbol{x} \in Z^N$, $\boldsymbol{x}(i,j) = (x_1(i,j), \ldots, x_N(i,j))$, is defined as:

$$\delta_B[\boldsymbol{x}(i,j)] = (\boldsymbol{x} \oplus B^S)(i,j) = T^{-1}\left\{ \max_{(u,v) \in B_{i,j}} [T(\boldsymbol{x}(u,v))] \right\} \quad (3)$$

and *Reduced Vectorial Morphological Erosion* (RMVE) can be introduced as:

$$\varepsilon_B[\boldsymbol{x}(i,j)] = (\boldsymbol{x} \Theta B^S)(i,j) = T^{-1}\left\{ \min_{(u,v) \in B_{i,j}} [T(\boldsymbol{x}(u,v))] \right\} \quad (4)$$

The vectorial-into-scalar transformation T(•) is defined as $T(\boldsymbol{x}(u,v)) = \gamma^{-1}(\boldsymbol{x}(u,v))$, as in [7].

Thanks to the reversibility of the SFC-based transformation, it can be shown that a set of properties of MM still hold.

3. RESULTS ON COLOR IMAGES

The new presented operators were tested by using a 256x256 color image of a squirrel: this image was previously employed during the tests described in [7]. The image was corrupted by salt-and-pepper impulsive noise at different corruption levels. The original color squirrel image corrupted by 4% impulsive noise is shown if Fig.2.

Square 3x3 SEs were used for RVMO (Fig.3) and RVMC (Fig.4) and quantitative evaluations (Table 1) were obtained by comparing their outputs and the output of a cascade of RVMO and RVMC (Reduced Vectorial Morphological Open-Close – RVMOC) (Fig.5) with the original image and with the RVMF [7] (Fig.6) output in terms of signal-to-noise ratio (SNR).

Concerning the RVM operators, the obtained results show similar behaviors to those of morphological counterparts on scalar images. These results lead us to conjecture that the proposed approach should make it possible to generalize scalar MM processing methods to multivalued data. Moreover, composing RVM operators allows one to increase SNR values. It can be seen that RVMOC output is comparable to the output resulting from the fifth iteration of RVMF, as it is forecasted from the theory.

Figure 2. Original color squirrel image corrupted by 4% impulsive noise.

	4% impulsive	6% impulsive	8% impulsive
ORIGINAL NOISY IMAGE SNR	10.68	8.96	7.68
3X3 RVMO IMAGE SNR	16.87	14.70	12.64
3X3 RVMC IMAGE SNR	15.86	12.17	10.32
3X3 RVMF OUTPUT SNR	20.89	20.57	19.58
3X3 RVMOC OUTPUT SNR	17.50	16.13	14.09
3X3 5^{TH} RVMF OUTPUT SNR	17.80	16.87	15.65

Table 1. Quantitative results for noisy squirrel images.

4. CONCLUSIONS

In this paper a new morphological approach for color image restoration has been presented. The question of ordering vectorial values assumed by the image intensity, necessary for the application of classical morphological operations, has been solved by introducing a reversible transformation from the vectorial color space into a scalar space before the filtering operation. The reversibility property of the transformation based on the space filling curves is used in order to derive the color image after the morphological filtering performed in the scalar space. Experimental results show that the proposed morphological approach to color image restoration should make it possible to generalize scalar MM processing methods to multivalued data.

Figure 3. Color squirrel image corrupted by 4% impulsive noise and filtered by RVMO.

Figure 4. Color squirrel image corrupted by 4% impulsive noise and filtered by RVMC.

REFERENCES

[1] G. Matheron, *Random Sets and Image Geometry*, New York, Wiley, 1975.

[2] J. Serra, *Image Analysis and Mathematical Morphology*, San Diego, CA, Academic, 1982.

[3] C. Vertan, V. Popescu and V. Buzuloiu, *Morphological Like Operators for Color Images,* Signal Processing VIII: Theories and Applications, 1996, pp. 165-168.

[4] K. Tang, J. Astola and Y. Neuvo, *Nonlinear Multivariate Image Filtering Techniques,* IEEE Trans. on Image Processing, vol.4, pp. 788-798, June 1995.

[5] H. Sagan, *Space Filling Curves*, Springer-Verlag, 1994.

[6] F. Cheng and A.N. Venetsanopoulos, *An Adaptive Morphological Filter for Image Processing*, IEEE Trans. on Image Processing, vol. 1, pp. 533-539, October 1992.

[7] C.S. Regazzoni and A. Teschioni, *A New Approach to Vector Median Filtering based on Space Filling Curves,* IEEE Trans. on Image Processing, vol. 6, pp. 1025-1037, July 1997.

[8] M. Charif-Chefchaouni and D. Schonfeld, *Morphological Representation of Order Statistics Filters,* IEEE Trans. on Image Processing, vol. 4, pp. 838-845, June 1995.

Figure 5. Color squirrel image corrupted by 4% impulsive noise and filtered by RVMOC.

Figure 6. Color squirrel image corrupted by 4% impulsive noise and filtered by RVMF.