

## THE PATH-PAIRABILITY NUMBER OF PRODUCT OF STARS

ADAM S. JOBSON, ANDRÉ E. KÉZDY

*University of Louisville, Louisville, KY 40292*

**e-mail:** adam.jobson@louisville.edu  
andre.kezdy@louisville.edu

JENŐ LEHEL

*University of Louisville, Louisville, KY 40292*

and

*Alfréd Rényi Institute of Mathematics, Budapest, Hungary*

**e-mail:** lyency@gmail.com

AND

GÁBOR MÉSZÁROS

*University of Memphis, Memphis, TN 38152*

**e-mail:** gmeszaros@memphis.edu

### Abstract

The study of a graph theory model of certain telecommunications network problems lead to the concept of path-pairability, a variation of weak linkedness of graphs. A graph  $G$  is  $k$ -path-pairable if for any set of  $2k$  distinct vertices,  $s_i, t_i$ ,  $1 \leq i \leq k$ , there exist pairwise edge-disjoint  $s_i, t_i$ -paths in  $G$ , for  $1 \leq i \leq k$ . The *path-pairability number* is the largest  $k$  such that  $G$  is  $k$ -path-pairable. Cliques, stars, the Cartesian product of two cliques (of order at least three) are ‘fully pairable’; that is  $\lfloor n/2 \rfloor$ -pairable, where  $n$  is the order of the graph. Here we determine the path-pairability number of the Cartesian product of two stars.

**Keywords:** path-pairability, weak linkage, Cartesian product, star-like network, telecommunications network.

**2010 Mathematics Subject Classification:** 05C38, 05C40.

### REFERENCES

- [1] L. Csaba, R.J. Faudree, A. Gyárfás, J. Lehel and R.H. Schelp, *Networks communicating for each pairing of terminals*, *Networks* **22** (1992) 615–626.  
doi:10.1002/net.3230220702
- [2] R.J. Faudree, A. Gyárfás and J. Lehel, *Three-regular path pairable graphs*, *Graphs Combin.* **8** (1992) 45–52.  
doi:10.1007/BF01271707
- [3] R.J. Faudree, A. Gyárfás and J. Lehel, *Path-pairable graphs*, *J. Combin. Math. Combin. Comput.* **29** (1999) 145–157.
- [4] E. Gyóri, T.R. Mezei and G. Mészáros, *Note on terminal-pairability in complete grid graphs*, *Discrete Math.* **5** (2017) 988–990.  
doi:10.1016/j.disc.2017.01.014
- [5] A. Huck, *A sufficient condition for graphs to be weakly  $k$ -linked*, *Graphs Combin.* **7** (1991) 323–351.  
doi:10.1007/BF01787639
- [6] A.S. Jobson, A.E. Kézdy and J. Lehel, *The path-pairability of the products of paths* (2016), submitted.
- [7] E. Kubicka, G. Kubicki, and J. Lehel, *Path-pairable property for complete grids*, in: *Combinatorics, Graph Theory and Algorithms II* (1999) 577–586.
- [8] G. Mészáros, *Linkedness and Path-Pairability in the Cartesian Product of Graphs*, PhD Thesis (CEU, Budapest, 2015).
- [9] G. Mészáros, *On path-pairability in the Cartesian product of graphs*, *Discuss. Math. Graph Theory* **36** (2016) 743–758.  
doi:10.7151/dmgt.1888

Received 12 December 2016

Revised 27 December 2017

Accepted 27 December 2017