PHYSICAL CONSTRAINTS FOR THE CONTROL OF A PHYSICAL MODEL OF A TRUMPET

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ABSTRACT

In this paper, the control of a physical model of a trumpet is studied. Although this model clearly describes the mechanical and acoustical phenomena that are perceptually relevant, additional constraints must be imposed on the control parameters. In contrast with the model where the tube length can be varied continuously, only seven different tube lengths can be obtained with a real instrument. By studying the physical model and its implementation, different relationships between the control parameters and signal characteristics are identified. These relationships are then used to obtain the best set of tube lengths with respect to a given tuning frequency.

1. INTRODUCTION AND STATE OF THE ART

Physical modelling consists of describing the mechanical and acoustical phenomena that take place in a musical instrument in terms of a system of equations and in solving (numerically) these equations to obtain the sound output. The sound signal is computed from a set of time-varying control parameters that correspond with the gestures of the player. When playing the physical model in order to simulate an acoustic instrument, the way it is controlled is as important as the quality of the model itself. A physical model that is capable of simulating any sound of an acoustic instrument will still sound very unnatural if it is not controlled correctly. For synthesis techniques based on a signal model (for example additive synthesis), efficient estimation techniques are available leading to sound manipulations of a very high quality [1, 2]. For physical models, techniques that determine automatically the control parameters for a given sound are actively researched. We cite work on plucked strings [3, 4], bowed strings [5] and trumpet models [6, 7].

At the Analysis/Synthesis team of IRCAM a physical model of a trumpet was developed [8, 9, 10, 11, 12, 13]. Also, a real-time implementation was provided for the control of the model with an adapted instrument-like interface (sax MIDI, Yamaha WX7) [14]. Helie [7] proposed to invert the equations on which the model is based, resulting in an algorithm that automatically determines the lip frequency and damping factor from a synthesized sound for which the tube length and mouth pressure of the player are known. In [6], the author proposed a non parametric estimation technique based on nearest neighbor classification. The computation cost of the nearest neighbor search was reduced significantly using an efficient *branch and bound search algorithm* based on *Principal Component Analysis (PCA)* [15]. A disadvantage of this approach was that no constraints were imposed on the control parameters of the physical model resulting in control parameter sequences that were not physically acceptable. For example, when simulating a sound with vibrato the control parameters tried to simulate this by varying the tube length. This contradicts the control of a real instrument where the tube length is fixed for each note. By imposing this constraint, much better synthesis results were obtained.

2. DEFINING THE CONSTRAINTS

As stated in the introduction, the constraint that must be imposed is that a constant tube length must be used for each note. In contrast with the physical model where the tube length can be varied continuously, a real trumpet can only obtain seven different tube lengths. With the three valves, eight combinations can be obtained of which two result in the same tube length. In table 1 each column corresponds with a tube length and each row with a mode, resulting in a given note. As can be seen from the table, some notes can be obtained with different tube lengths. For example G4 can be obtained by exciting the sixth mode of tube length 1, the seventh mode of tube length 4 or the eighth mode of tube length 6. The following constraint is defined:

	tube length						
mode N	1	2	3	4	5	6	7
1	C2	B1	B♭1	A1	A♭1	G1	F♯1
2	C3	B2	B♭2	A2	A♭2	G1	F♯2
3	G3	F♯3	F3	E3	E♭3	D3	C♯3
4	C4	B3	B♭3	A3	A♭3	G3	F♯3
5	E4	E♭4	D4	C♯4	C4	B4	B♭4
6	G4	F♯4	F4	E4	E♭4	D4	C♯4
7	B♭4	A4	A♭4	G4	F♯4	F4	E4
8	C5	B4	B♭4	A4	A♭4	G4	F♯4

Table 1: *Notes obtained by exciting different modes of a given tube length*

> **Constraint 1:** "When a trumpet player interprets a score he chooses a correct finger position and excites the correct mode of the tube in order to obtain the desired note."

In other words, the player uses a mapping from the fundamental frequency to a mode and tube length couple. For simplicity, we will assume that the same combination is always used. In reality, other combinations can sometimes be preferred but this is considered beyond the scope of this article.

A second factor that influences the obtained fundamental frequency is the tuning of the instrument. The tuning valve adjusts all the tube lengths in order to correspond with a given reference frequency f_{ref}. Typically A3 corresponds with 440 Hz. This leads to a second constraint:

> **Constraint 2:** "Given a reference frequency, a set of seven tube lengths must be determined for the control of an entire trumpet performance"

3. THEORETICAL DERIVATION

3.1. The Physical Model

In this section, a (simplified) physical model of a trumpet and its implementation is described briefly. The upper lip of the trumpet player is modelled by a parallelepipedic mass attached to a damped spring. When the lips close, the characteristics of the spring change, introducing a non linearity to the oscillation. The body of the instrument is modelled by its impulse response function h_λ measured from a real instrument. The coupling between the lips and the instrument is obtained from the *Bernoulli equation* expressing the relation between volume flow u, lip position x, and pressure p. Assuming that the propagation of the wave is plane (linear propagation), p can be expressed as the sum of an incoming p_i and outgoing wave p_o [7, 11, 13].

In correspondence with the jMax implementation [14], the control parameters of the model are:

- P_M, pressure in the mouth
- P_T, tube length parameter, related to the number of times that the wave traverses the tube per second.
- P_D, lip damping
- P_L, lip resonance frequency

The algorithm for the sound synthesis from these parameters is given below:

$\lambda = \lfloor \frac{F_s}{2P_T} \rfloor$ `// Delay computation`
$p_{i,n} = \sum_{k=1}^{K} p_{o,n-k} h_{\lambda,k}$ `// Body Resonance`
`if` $x_n > 0$ `// Lips opened`
$$p_n = 2p_{i,n} - \tfrac{1}{2} A x_n (A x_n - \sqrt{(A x_n)^2 + 4|P_M - 2p_{i,n}|})$$
$$u_n = (p_n - p_{i,n})/Z_c$$
$$x_{n+1} = 2\sqrt{P_D}\cos(2\pi P_L/F_s)x_n - P_D x_{n-1} + \gamma(P_M - p_n)$$
`else // Lips closed`
$$p_n = 2p_{i,n}$$
$$u_n = 0$$
$$x_{n+1} = 2\sqrt{P_D}\cos(2\pi(2P_L)/F_s)x_n - P_D x_{n-1} + \gamma(P_M - p_n)$$
`end`
$$p_{o,n} = \tfrac{1}{2}(p_n + Z_c u_n)$$

Algorithm 1: *Trumpet synthesis*

where F_s is the sampling frequency, Z_c the acoustic impedance and A, γ being constants.

3.2. The Linear Response of the Body

The first two lines of the algorithm express the linear response of the body of the instrument h_λ. This reflection function was measured at the mouthpiece of a real instrument and was then simplified for computational reasons. It consists of the following parts

- h_1, the direct response at the mouthpiece
- a delay of λ zeros, corresponding with the cylindrical part of the tube where there is almost no reflection
- h_2, the reflection after travelling back and forth the instrument body

An example of the total reflection function h_λ is shown in figure 1. By changing the number of zeros between these two reflections, different tube lengths are simulated [8]. The number of zeros λ is computed from the tube length control parameter P_T using

$$\lambda = \lfloor \frac{F_s}{2P_T} \rfloor \tag{1}$$

The interval between the two maxima in the reflection function can be computed approximatively by adding the number of samples from the peaks to the zeros and is denoted λ_0. This implies that the total time for the wave to run back and forth the instrument body is given by

$$\tau = \frac{\lambda + \lambda_0}{F_s} \tag{2}$$

Therefore the resonance frequencies of the tube tube will be multiples of

$$f_\tau = \frac{F_s}{\lambda + \lambda_0} \tag{3}$$

As visualized in figure 1, these modes can be observed in the frequency domain. Expressing the total reflection function as

$$h_{\lambda,n} = h_{1,n} + h_{2,n-\tau} \tag{4}$$

its fourier transform yields

$$H_\lambda(f) = H_1(f) + H_2(f)e^{-2\pi i f \tau} \tag{5}$$

Maxima and minima of $|H_\lambda(f)|$ are obtained when $H_1(f)$ and $H_2(f)$ are in phase or antiphase respectively, implying that

$$|H_1(f)| - |H_2(f)| \le |H_\lambda(f)| \le |H_1(f)| + |H_2(f)| \tag{6}$$

This shows that bounds of $|H_\lambda(f)|$ are independent of λ. Therefore, the expression $|H_1(f)| + |H_2(f)|$ denotes the *spectral envelope* of $|H_\lambda(f)|$ as shown figure 1. Since $h_{1,n}$ and $h_{2,n}$ are sharply peaked, their spectrum is very smooth. Their phase difference is due to the modulator term $e^{-2\pi i f \tau}$ which determines the position of the resonances which are approximately equally spaced with an interval f_τ.

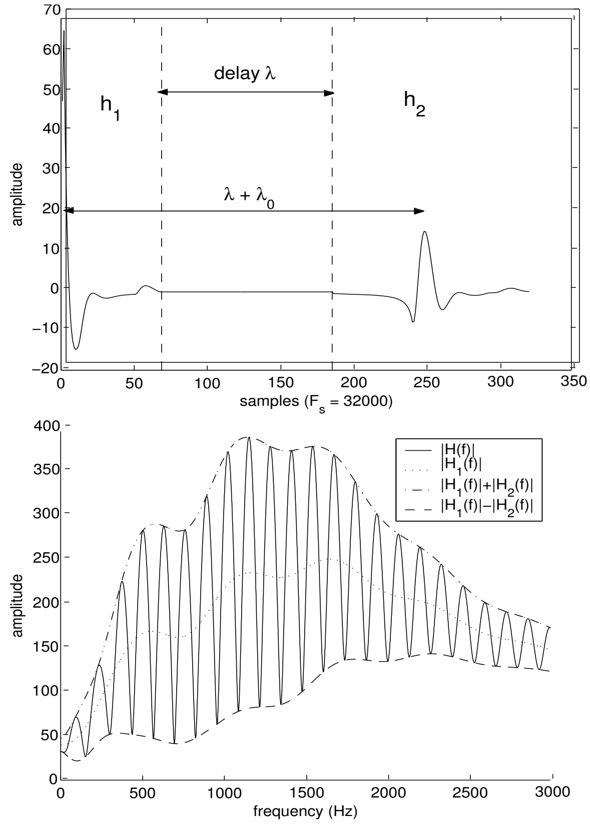

Figure 1: *Reflection function of the instrument body*

3.3. Resonance phenomena of the physical model

When the model produces a stable periodic sound, each period consists of an interval where the lips are opened and an interval where the lips are closed. When looking closely at the equation that calculates the pressure p_n when the lips are opened

$$p_n = 2p_{i,n} - \frac{1}{2}Ax_n\left(Ax_n - \sqrt{(Ax_n)^2 + 4|P_M - 2p_{i,n}|}\right) \tag{7}$$

one can derive by expressing the square root as a Taylor expansion that p_n approaches P_M when x_n is large. An example is given in figure 2. In this case the value of the mouth pressure P_M is 5000 Pa which is exactly the value that is obtained for p_n when the lips are opened ($x \gg 0$). For a large opening, the state variables take the values

$$\begin{aligned} p_n &= P_M \\ u_n &= (P_M - 2p_i)/Z_c \\ p_{o,n} &= P_M - p_{i,n} \\ P_M - p_n &= 0 \end{aligned} \tag{8}$$

Since the term $\gamma(P_M - p_n)$ expresses the external force that is exerted on the lips, it follows that the lips oscillate freely when they are largely opened. When $x < 0$, which means when the lips are closed, we obtain

$$\begin{aligned} p_n &= 2p_{i,n} \\ u_n &= 0 \\ p_{o,n} &= p_{i,n} \\ P_M - p_n &= P_M - 2p_{i,n} \end{aligned} \tag{9}$$

indicating that now an external force is exerted on the lips, expressed by

$$\gamma(P_M - 2p_{i,n}) \tag{10}$$

For small positive values of x_n, transition values for these state values are obtained. From this computation, it can be concluded that these state variables have the same period as the lip period. Therefore, when the fundamental frequency is measured from the sound signal produced by the physical model (this is the high pass filtered outgoing pressure $p_{0,n}$), the fundamental frequency of all the state variables is known. A second conclusion that is drawn, is that the lips are excited by an external force, essentially when they are closed. Since the strength of excitation force is dependent on $P_M - 2p_{i,n}$, the conditions are examined

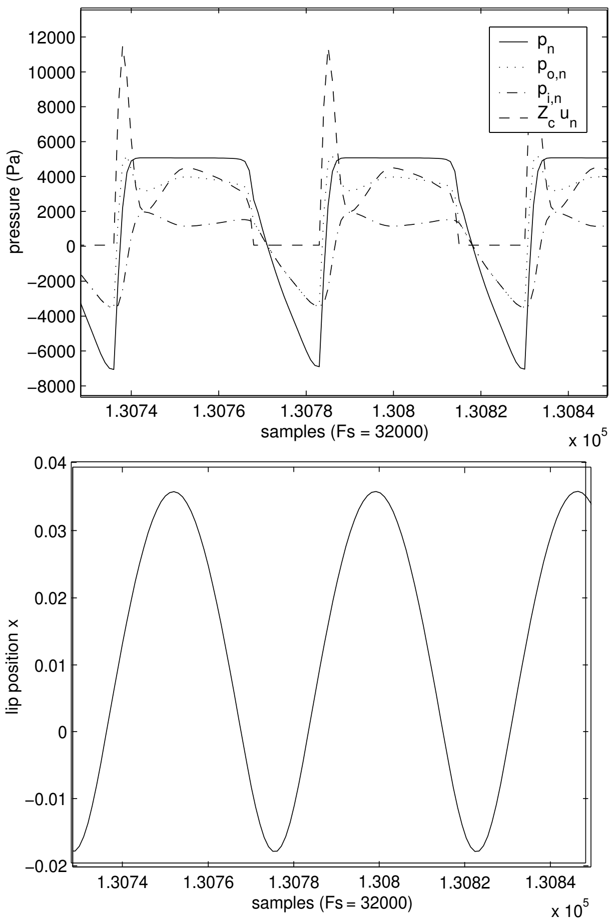

Figure 2: *Periodicity of lip position and state variables*

for which the excitation is the strongest, meaning that the resonance is maximal. For a fixed value for P_M this will be obtained for a negative value of $p_{i,n}$ with a maximal absolute value. Since $p_{i,n}$ is calculated from a convolution of the outgoing wave $p_{o,n}$ with the reflection function of the body $h_{\lambda,n}$ the maximal strength is obtained when the period of p_o is a multiple of the resonance frequency f_τ of the tube. In figure 3 an example is shown for the fourth mode of a fixed tube length. Both peaks of $h_{\lambda,n}$ coincide with negative values of p_o which results in a negative value for p_i with a maximal absolute value. In addition, the fact that four periods of p_o correspond with twice the tube length (the reflection function is measured at the mouthpiece) implies that the fourth mode is excited. Thus, for a fixed tube length, the trumpet model will resonate the strongest when the produced sound is a multiple of the first tube mode, f_τ. This implies that the amplitude of x_n is the highest when $f_0 = Nf_\tau$, where N is the mode index.

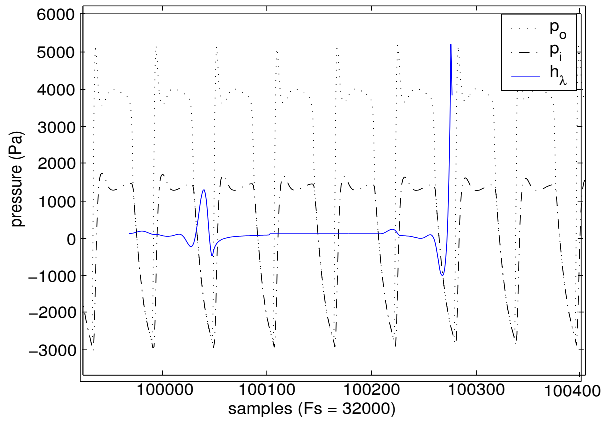

Figure 3: *pressure waves and reflection function*

3.4. Relationship with Lip Frequency P_L

It would be interesting to know which lip frequency is used in order to obtain this optimal resonance. This is difficult since the lip frequency does not correspond directly with the produced fundamental frequency. When the lips are opened, they oscillate freely, implying that the time interval that they are opened is $\frac{1}{2P_L}$. When the lips are closed, P_L is doubled, resulting in an interval of $\frac{1}{4P_L}$ when no external force is exerted. As a result, the total period has a length of of $\frac{3}{4P_L}$ and a corresponding frequency of $\frac{4P_L}{3}$. We will examine whether this is still a good approximation for the fundamental frequency even when the external force is exerted. At the bottom of figure 2 the lip position is shown in function of time.

4. TUBE LENGTH DETERMINATION

4.1. Validation Experiment

The computations in the previous sections can be validated by executing the following experiment. By varying the lip frequency from its lowest to its highest value, the maximal resonances of the lip position x_n can be observed. If the fundamental frequencies at these maximal resonances are multiples of f_τ, and the corresponding lip frequency is $\frac{3}{4}P_L$, then the previous reasoning is validated.

The experiment was performed for values of P_T being 150 and 70. F_s and λ_0 had the values 32000 and 128 respectively resulting in the f_τ-values of 136.7 and 89.9. From the results listed in table 2, the following conclusions can be drawn:

- For the high modes it is observed that f_τ is very close to f_0/N, meaning that the fundamental frequencies for the maximal resonances are very close to multiples of f_τ.

P_T	N	f_0	f_0/N	P_L	P_L/f_0
70	10	899	89.9	673.7	0.749
	9	809	89.8	605.4	0.748
	8	720	90.0	538.5	0.748
	7	630	90.1	471.3	0.748
	6	537	91.2	404	0.752
	5	463	92.6	344	0.743
	4	364	91.0	271	0.745
	3	276	92.0	205	0.743
	2	185	92.5	137	0.741
150	9	1231	136.7	922	0.749
	8	1094	136.7	819	0.749
	7	955	136.4	715	0.749
	6	818	136.3	614	0.751
	5	633	136.3	511	0.751
	4	551	137.7	412	0.748
	3	414	138	308	0.744
	2	277	138.5	207	0.747

Table 2: *Observations for different tube lengths*

- The ratio $\frac{P_L}{f_0}$ seems to be a constant very close to $\frac{3}{4}$
- These approximations are less accurate for the lower modes. In this case, the actual fundamental frequency is higher then the estimated value.

In the case of maximal resonance, the following relationships can be deduced for the physical model.

$$f_0 = N f_\tau \tag{11}$$

$$f_0 = \frac{4}{3} P_L \tag{12}$$

This corresponds with the first constraint that was expressed in section 2. When a player wishes to play a given note, a valve position must be chosen corresponding with a resonance frequency f_τ so that the desired fundamental frequency is a multiple of f_τ. In order to excite the correct mode N, the correct value of P_L must be used according to equation (12). This corresponds with the control of a real instrument and yields therefore an extra validation of the physical model.

4.2. Instrument Tuning

As indicated in table 1, seven different tube lengths are used in order to obtain all notes. Therefore, we wish to determine seven values for P_T that correspond with these tube lengths. The frequency f_0 of a note is calculated from a note index I and a reference frequency f_{ref} in the following manner

$$f_0 = f_{ref} 2^{\frac{I}{12}} \tag{13}$$

note	I	f_0	$f_\tau = \frac{f_0}{4}$	λ	P_T
C4	3	523.3	130.8	116.6	137.2
B3	2	493.9	123.5	131.2	122.0
B♭3	1	466.2	116.5	146.6	109.2
A3	0	440	110	162.9	98.2
A♭3	-1	415.3	103.8	180.2	88.8
G3	-2	392.0	98.0	198.5	80.6
F♯3	-3	370	92.5	218.0	73.4

Table 3: *Determination of P_T*

$\hat{\lambda}$	f_τ	f_0	I	$P_{T,min}$	$P_{T,max}$
117	130.6	522.4	2.97	135.6	136.7
131	123.6	494.2	2.01	121.3	122.1
147	116.4	465.5	0.97	108.2	108.8
163	110.0	439.9	-0.01	97.6	98.1
180	103.9	415.6	-0.99	88.4	88.9
199	97.6	391.4	-2.02	80.1	80.5
218	92.5	370	-3.00	73.1	73.3

Table 4: *Determination of P_T for $\hat{\lambda}$*

Taking 440 Hz as the reference frequency, meaning that $I = 0$ corresponds with the medium A, the fundamental frequencies of all notes are calculated. Knowing which notes are played using the fourth mode ($N = 4$), the values of f_τ and P_T can be deduced using equations (1) (neglecting the floor operator) and (3) as shown in table 3. This satisfies the second constraint that was imposed.

Furthermore, it must be taken into account that the value of λ is an integer value. Therefore, the integer value $\hat{\lambda}$ value closest to λ is used in order to recalculate the obtained fundamental frequencies to determine wether this is admissible. This results in an interval $P_L \in [P_{L,min}, P_{L,max}]$ for which all values result in the same value of $\hat{\lambda}$. The results are shown in table 4. The recalculation of I shows that the floor function for the computation of λ introduces a maximal deviation of three percent of a half tone.

Using appropriate combinations of tube length and lip frequency, as given in table 1, an ascending chromatic scale was synthesized. Figure 4 shows the note index computed from the fundamental frequency, according to equation (13). This figure confirms the previously drawn conclusion that the estimated f_0 is less accurate for lower modes.

5. FURTHER WORK AND CONCLUSIONS

This paper describes the physical constraints that must be imposed for the control of a physical model of a trumpet. A real trumpet only uses seven tube lengths while the tube

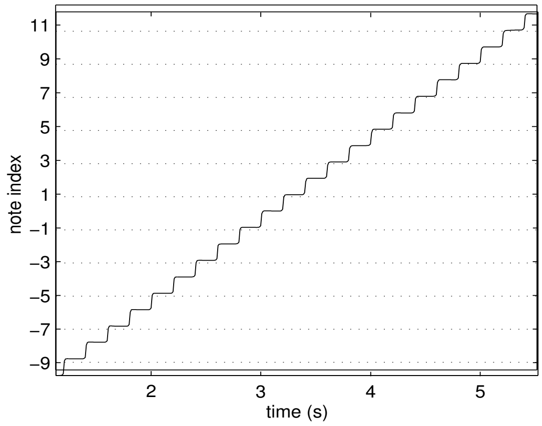

Figure 4: *Note index of a chromatic scale played by the physical model*

length parameter P_T of the physical model is continuous. Although the physical model used in this article is quite simplified, no significant changes in the synthesized sounds were observed. By a detailed study of the implementation of this physical model some very simple and approximative relationships between the fundamental frequency and the control parameters were identified. These relationships were then used in order to determine a set of seven tube lengths with respect to a given tuning frequency f_{ref}.

In addition, a number of parameters had to be determined manually. Due to the simplification of the reflection function h_λ additional filters were used to amplify the lower modes [8]. The resonance frequency of these filters was at the second and third mode of the tube and the amplitude of the filter was adapted manually in order to obtain a strength comparable with higher modes.

Simple note sequences were synthesized with respect to the physical constraints. Examples will be presented at the conference. Future research will increase the expressivity of the synthesis by integrating the physical constraints described in this paper with the automatic parameter estimation technique that is described in [6]. Also transient synthesis can further be improved.

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