

## Research Article

# Consensus Model with Double Feedback Mechanism Based on Dynamic Trust Relationship in Social Network Group Decision-Making

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## ABSTRACT

In social network group decision-making, adjusting the opinions of decision makers is often used to promote consensus. But decision makers are not always willing to accept the feedback mechanism, and decision-makers' trust relationships are not constant during the consensus process. This paper constructs a novel consensus model with double feedback mechanism based on dynamic relationship to solve social network group decision-making problems. It overcomes the deficiency of single feedback mechanism and improves consensus degrees. This double feedback mechanism comprises a weight feedback mechanism and an alternative-contents feedback mechanism. In each round of this double feedback mechanism, the trust relationships between decision makers are dynamic, the trust scores are adjusted to update the weights of decision makers on the basis of the weight feedback mechanism. Synchronously, adjust the contents of alternatives with low dominance degree according to the alternative-contents feedback mechanism, which can change the evaluation opinions of the decision makers. Moreover, an example of a decision-making problem about a Public-Private-Partnership construction project illustrates the feasibility and validity of the proposed method. Finally, the comparisons between this paper and existing studies help point out the advantages of the proposed methods which can improve decision makers' consensus degrees and satisfaction with alternatives.

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## 1. INTRODUCTION

With the rapid development of social media such as WeChat and Weibo, interactions between decision makers (DMs) have become more convenient and frequent. Thus, it is necessary to consider the social network relationship among DMs in group decision-making (GDM). Consequently, a new type of social network group decision-making (SNGDM) problem has become a hot issue [1–12]. SNGDM is a process to obtain an optimal solution from a set of alternatives by several DMs who may be known or connected through certain social relations such as trust. The consensus reaching process is usually crucial for the resolution of SNGDM problems. In this process, disagreement among DMs blocks consensus reaching, which may lead to incompatibility of choosing the optimal alternative between members of the group [13,14]. The consensus model is an effective method to reduce or eliminate incompatibility. The model usually involves a feedback mechanism, which is the key for DMs to reach an agreement on finite alternatives, to guide incompatible DMs with advice on how to revise their judgments [15–18].

According to recent papers about SNGDM, relevant studies can be mainly classified into four categories [10]: (1) estimate missing DMs' preference information using social network, (2) preference aggregation using social network, (3) feedback mechanism design for consensus reaching process in SNGDM which is the focus of this paper, and (4) consensus reaching process based on opinion evolution in SNGDM. Regarding research on the feedback mechanism of consensus reaching process in SNGDM, Zhang *et al.* [10] constructed a new feedback mechanism which can provide advice of adjustment to DMs by considering the leadership and the bounded confidence of DMs in SNGDM. Wu *et al.* [11] calculated the trust scores of DMs based on self-confidence in social network, to determine the weights of DMs, and adjust the opinions of those with low consensus through feedback mechanism. To minimize the total compensation cost in the worst case, Lu *et al.* [12] constructed a consensus model based on robust optimization, in which DMs' opinions can be adjusted based on the unit adjustment cost of DMs in feedback mechanism.

The existing literature mostly studies the consensus model with single feedback mechanism [19–29]. Most of the single feedback mechanisms proposed in these literatures produce recommendation advices by implementing a linear weighted approach with a

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fixed or static feedback parameter. As pointed out by Wu and Chiclana in [30], these feedback mechanisms share the common limitation of forcing the incompatible DMs to implement the recommendation advices without considering whether they like them or not. In contrast, double feedback mechanism can make up for this deficiency and improve the efficiency of the consensus reaching process [31].

However, most of the current researches on consensus models with double feedback mechanism assumed that DMs were independent. Few studies considered trust relationships between DMs in double feedback mechanism, and none considered the dynamic changes of trust relationships between DMs. In the real SNGDM context, information asymmetry and DMs' private information make the establishment of trust relationship a process. And it is interaction and communication between DMs that makes the trust relationship dynamic change. Dong *et al.* [32] established a consensus model with double feedback mechanism including a judgment feedback mechanism (JFM) and a weight feedback mechanism (WFM), which adjusted opinions and weights of DMs, but assumed that DMs are independent. As regards social network, Campion and Lord [18] believed that trust relationships between DMs affect decision results, and DMs will vary their evaluations with the interaction feedback. Wu *et al.* [4] proposed a visual social network consensus model based on trust relationship, using recommendation mechanism to provide adjustment suggestions to DMs with less contribution in the consensus process. Bodily and Samuel [33] also assumed that each individual is willing to revise his or her own social preferences to adapt to other's social preferences. Subsequent scholars considered the trust relationship between DMs and studied the DFM consensus model. Tian *et al.* [34] determined initial weights of DMs through trust relationships between DMs and built a consensus model with double feedback mechanism. However, they ignored the influence of dynamic changes of trust relationships between DMs on decision results in social network. When DMs interacted and communicated during the feedback process, their trust relationship had not changed in [34]. In fact, social relations such as trust relationships between DMs in social network not only exist, but also change dynamically with the interaction between DMs in social network, which will have a profound impact on DMs' cognition and subsequent decisions [35]. Hence, in this paper, we consider dynamic trust relationship during the consensus process.

Meanwhile, difference of DMs' opinions mainly stems from the difference in their judgment on contents of alternatives [36]. So, appropriately adjustment of the contents of alternatives can lead to the change of DMs' opinions and promote consensus reaching. Take alternatives of Public-Private-Partnership (PPP) construction project as an example, different DMs have different opinions on its investment of alternative. Consensus failure is often caused by the fact that most DMs have widely divergent opinions on a particular alternative, rather than by the low consensus degree of one DM. And moderate adjustment of the contents of alternatives, such as the amount of investment, may more easily affect DMs' opinions. Therefore, from the perspective of dynamic change of trust relationships and the appropriate adjustment of contents of alternatives in social network, it is necessary to deeply explore the consensus model with double feedback mechanism in SNGDM.

Although the change of DMs' opinions caused by the adjustment of alternatives has attracted scholars' attention, there are few studies

on the consensus model with double feedback mechanism involving the adjustment of the contents of alternatives. By introducing new alternatives randomly, Pérez *et al.* [37] eliminated alternatives with low alternative dominance degree to promote consensus reaching, but they only studied consensus model with single feedback mechanism. In fact, eliminating and updating alternatives to change DMs' opinions may lead to group thinking of DMs [38]. Adjusting contents of alternatives appropriately can not only alleviate conflict of DMs' opinion, but also take minority opinions into account, which can improve their satisfaction. Therefore, in order to make DMs more willing to modify evaluation opinions, this paper needs to consider how to identify and adjust controversial alternatives, so as to improve DMs' satisfaction degree. Considering that the DMs' evaluation of alternatives has cognitive limitations and uncertainties, the 2-tuple linguistic preference relation (2-tuple LPR) is widely used to represent DMs' evaluation of alternatives in GDM problems [39–45]. So this paper also adopts 2-tuple LPR to represent DMs' opinions.

The primary objective of this work is to build a consensus model with double feedback mechanism considering the dynamic change of trust relationship between DMs and the adjustment of contents of alternatives. Main contributions of the proposed model, compared with existing studies, are as follows: (1) A consensus framework with double feedback mechanism for SNGDM is constructed based on the WFM and the alternative-contents feedback mechanism (AFM), which is an extension of the existing research. (2) This paper considers the influence of the change of trust relationship on the DMs' weights. Although trust relationships between DMs are considered in [34], it assumes that trust relationships between DMs remain unchanged. In contrast, this paper makes the adjustment of DMs' weights more flexible through the dynamic change of trust relationships between DMs, so that the consensus degrees in each round of consensus process can be significantly improved. (3) Alternative classification method is proposed, which moderately adjusts the contents of low dominance degree alternative to guide the DMs to change their opinions, so as to improve the final consensus level and decision quality. (4) This paper improves DMs' consensus degrees and makes DMs more satisfied with the alternative by comparison and analysis. This provides an effective method for PPP construction project selection.

The remainder of this paper is organized as follows: The basic concepts of the 2-tuple preference relation and trust matrix are introduced in Section 2. Then, the consensus framework for a double feedback mechanism including the WFM and the AFM is constructed in Section 3. Based on the proposed consensus framework, the consensus model with double feedback mechanism is developed in Section 4. In Section 5, a numerical example is provided to illustrate the applicability and efficiency of the consensus model with double feedback mechanism. Some comparison analysis is given in Section 6. Finally, some concluding remarks are presented in Section 7.

## 2. PRELIMINARIES

To facilitate the consensus modeling, this section first introduces some basic concepts and definitions related to this paper. In a SNGDM problem, we denote the set of alternatives as  $A = \{a_1, a_2, \dots, a_n\}$  ( $n \geq 2$ ), DMs as  $E = \{e^1, e^2, \dots, e^m\}$  ( $m \geq 2$ ), and

their weight vector as  $W = (w^1, w^2, \dots, w^m)$ , where  $w^h \geq 0$ ,  $h = 1, 2, \dots, m$ ,  $\sum_{h=1}^m w^h = 1$ . Based on this, this section introduces the concepts of 2-tuple LPR, social network, and trust matrix.

### 2.1. The 2-Tuple LPR

In this study, 2-tuple LPR is used to represent DMs' evaluation of alternatives. In the 2-tuple LPR  $S = ((s_k, \alpha_k))_{n \times n}$ ,  $(s_k, \alpha_k)$  represents the DM's 2-tuple linguistic evaluation information, where  $s_k$  is an element in the predefined linguistic term set  $S = \{s_0, s_1, \dots, s_g\}$  with odd granularity  $g + 1$ , and  $\alpha_k$  is the symbolic translation value that represents the deviation between  $s_k$  and the evaluation value. According to Herrera and Martinez [46,47],  $\beta \in [0, g]$  is a symbolic aggregation value of a 2-tuple linguistic evaluation information, which can be represented by the function  $\Delta$  as the 2-tuple linguistic evaluation information:

$$\Delta(\beta) = (s_k, \alpha_k) = \begin{cases} s_k & k = \text{round}(\beta) \\ \alpha_k = \beta - k & \alpha \in [-0.5, 0.5) \end{cases} \quad (1)$$

where *round* is the rounding operator. Accordingly, the inverse function  $\Delta^{-1}$  can be defined as

$$\Delta^{-1}(s_k, \alpha_k) = k + \alpha_k = \beta \quad (2)$$

### 2.2. Social Network

A social network is a platform where DMs communicate with each other and form a relatively stable system of social relations, which in turn can be used to study relationships among DMs using what is known as social network analysis [2]. The strength of the relationship between DMs and the structural characteristics of the social network are the focus of social network analysis. DMs in a social network and the connections between them are usually expressed in three ways: (1) graph, (2) the adjacency matrix, and (3) algebraic. In this paper, a social network is defined by a directed graph with nodes representing DMs and edges corresponding to the trust relationship between DMs.  $e^k \rightarrow e^h$  signifies a direct trust relationship between  $e^k$  and  $e^h$ . And the specific trust degree between DMs is represented by the trust matrix.

### 2.3. Trust Matrix

Trust matrix is used in this paper to represent the specific trust relationship between DMs. Unknown trust evaluation values exist in the trust matrix due to differences in DMs' occupations and locations [48]. Therefore, referring to [49], this paper proposes the dual trust propagation operator to complete the trust matrix, and defines the trust score of DM through the trust function of DM.

Let  $\Lambda_0 = (\lambda^{hl})_{m \times m}$  be the initial trust matrix, where  $\lambda^{hl}$  is the evaluation value of  $e^l$ 's trust on  $e^h$ , satisfying  $h, l \in \{1, 2, \dots, m\}$  and  $h \neq l$ .

**Definition 1.** [48] Let  $\lambda = (t, d)$  be the trust evaluation value, and  $t, d \in [0, 1]$ , where  $t$  represents the trust value and  $d$  represents the distrust value.

#### 2.3.1. Dual trust propagation operator

In order to complete the trust matrix, a dual trust propagation operator is proposed based on the concepts of t-norms (triangular norms) and t-conorms (triangular conorms). T-norms and t-conorms operators are expressed by Einstein product and Einstein sum operators, respectively.

**Definition 2.** [49] The dual trust propagation operator  $P_D$  can be defined as considering the case of decrease in trust value and increase in distrust value in trust propagation:

$$P_D(\lambda_1, \dots, \lambda_n) = (E_{\otimes}(t_1, \dots, t_n), E_{\oplus}(d_1, \dots, d_n)) \quad (3)$$

where  $E_{\otimes}$  and  $E_{\oplus}$  are the Einstein product operator and the Einstein sum operator, respectively.

**Definition 3.** [49] Einstein product operator is expressed as

$$E_{\otimes}(t_1, \dots, t_n) = \frac{2 \prod_{i=1}^n t_i}{\prod_{i=1}^n (2 - t_i) + \prod_{i=1}^n t_i} \quad (4)$$

where  $t_1, \dots, t_n$  are  $n$  number trust values.

**Definition 4.** [49] Einstein sum operator is expressed as

$$E_{\oplus}(d_1, \dots, d_n) = \frac{\prod_{i=1}^n (1 + d_i) - \prod_{i=1}^n (1 - d_i)}{\prod_{i=1}^n (1 + d_i) + \prod_{i=1}^n (1 - d_i)} \quad (5)$$

where  $d_1, \dots, d_n$  are  $n$  number distrust values.

According to Eqs. (4) and (5), the properties of t-norms and t-conorms operators are as follows:

**Property 1.**  $E_{\otimes}(t_1, \dots, t_n) \leq \min(t_1, \dots, t_n)$

**Property 2.**  $\max(d_1, \dots, d_n) \leq E_{\oplus}(d_1, \dots, d_n)$

#### 2.3.2. Trust score of DM

After completing the trust matrix with dual trust propagation operator, the trust function of DMs can be represented as follows:

$$\lambda^h = (t^h, d^h) = \frac{1}{m-1} \sum_{l=1, l \neq h}^m \lambda^{hl} \quad (6)$$

Based on the corresponding trust function, the trust score of  $e^h$  ( $TS^h$ ) can be calculated as follows:

$$TS^h = \frac{t^h - d^h + 1}{2} \quad (7)$$

## 3. CONSTRUCTION OF CONSENSUS FRAMEWORK WITH DOUBLE FEEDBACK MECHANISM

For the SNGDM problem, a directed graph represents the trust relationship between DMs, whereas the edge between two DMs means direct trust relationship between them, and DMs provide their 2-tuple LPRs  $S = ((s_k, \alpha_k))_{n \times n}$  for finite alternatives. Once DMs' preferences reach a certain degree of consensus, the alternatives can be ranked and selected. Otherwise, the double feedback mechanism needs to be activated to promote the consensus.

The breakthrough points of this research on SNGDM consensus model are how to reset DMs' weights in line of the dynamic change of trust relationships between DMs, and how to guide DMs to change their evaluations through appropriate adjustment of contents of alternatives as well. Therefore, the consensus framework with double feedback mechanism for SNGDM is proposed in Figure 1. If DM's consensus degree  $CI^h$  is less than the threshold  $\gamma$ , the WFM and the AFM are started synchronously.

The framework is specifically described as: (1) Considering the influence of the change of trust relationships on DMs' weights, the adjustment direction of trust relationship is determined by comparing the ranking values of consensus degrees and trust scores. The WFM is built by adjusting trust scores to reset a new round of DMs' weights. (2) The AFM is constructed. The alternatives are divided into high and low dominance degree alternatives by alternative dominance degree classification method, and the contents of low dominance degree alternatives are appropriately adjusted to change the evaluation of DMs.

Finally, group preference relation and DMs' consensus degrees are calculated based on the new round of DMs' weights and 2-tuple linguistic evaluation matrices after the operation of the abovementioned double feedback mechanism. Once termination conditions are met, the consensus process with double feedback mechanism ends and transfers into alternative-selection process. Otherwise, next round of double feedback mechanism is carried out until the termination conditions are met.

#### 4. CONSTRUCTION OF CONSENSUS MODEL WITH DOUBLE FEEDBACK MECHANISM

In order to build a consensus model with double feedback mechanism for SNGDM, we put forward relevant assumptions based on Figure 1: (1) Every DM uses the 2-tuple LPR to evaluate alternatives. There is no difference in the understanding of linguistic granularity among DMs. The initial alternative set and the selection

of contents of alternatives are predetermined. (2) DMs are serious and responsible, and there is no such behavior as private alliance. (3) The maximum round for reaching consensus is  $T_{max}$ .

In the case of  $CI^h < \gamma$ , we build a consensus model with WFM and AFM, the first task is to determine the consensus degrees of DMs.

#### 4.1. Calculation of DMs' Consensus Degrees

In this section, the distance between the 2-tuple LPR and the group preference relation is used to calculate the consensus degree of DM.

Firstly, the DM's weight is determined by the DM's trust-score set  $TS = \{TS^1, \dots, TS^m\}$ . The weight of DM can be obtained, according to [49] and [50], as

$$w^{\sigma(h)} = Q \left( \frac{T(\sigma(h))}{T(\sigma(m))} \right) - Q \left( \frac{T(\sigma(h-1))}{T(\sigma(m))} \right) \quad (8)$$

where  $T(\sigma(h)) = \sum_{f=1}^h TS^{\sigma(f)}$ , and  $\sigma(f)$  represents that  $TS^{\sigma(f)}$  is the  $f^h$  largest value in the DM's trust-score set  $\{TS^1, \dots, TS^m\}$ , and  $Q$  is the nondecreasing proportional quantifier.

Secondly, the group preference relation is denoted as  $\tilde{S} = ((\tilde{s}, \tilde{\alpha}))_{n \times n}$ , wherein the element  $(\tilde{s}, \tilde{\alpha})$  is expressed as

$$(\tilde{s}, \tilde{\alpha}) = \Delta \left( \sum_{h=1}^m w^h \Delta^{-1}(s^h, \alpha^h) \right) = \Delta \left( \sum_{h=1}^m w^h \beta_{ij}^h \right) \quad (9)$$

Finally, the consensus degree of each DM is calculated as follows:

$$CI^h = 1 - \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n d(S_{ij}^h, \tilde{S}_{ij}) \quad (10)$$

$$= 1 - \frac{1}{n^2} \sum_{i=1}^n \sum_{j=1}^n \frac{|\Delta^{-1}(s_{ij}, \alpha_{ij}) - \Delta^{-1}(\tilde{s}_{ij}, \tilde{\alpha}_{ij})|}{g + 1}$$

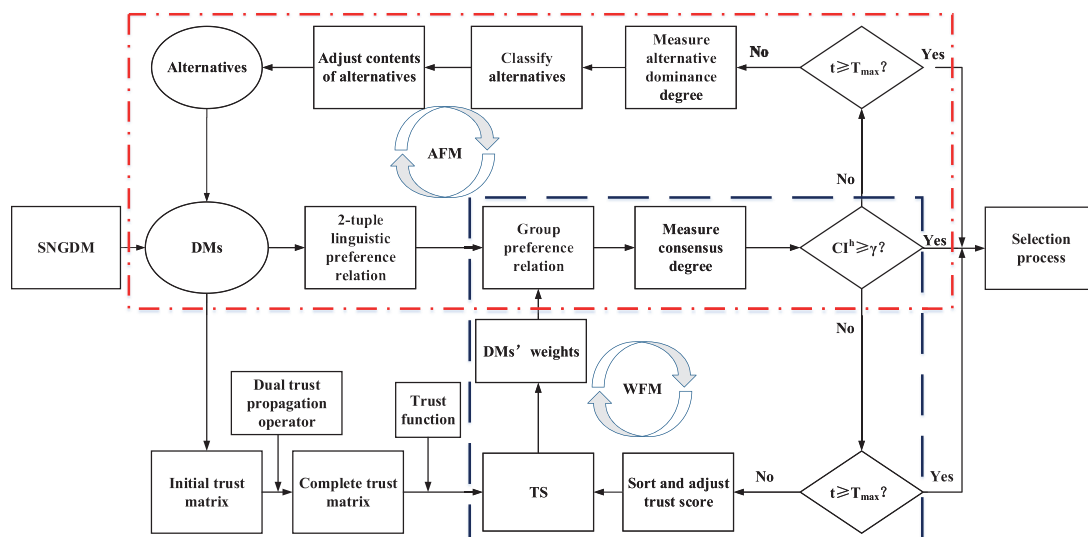


Figure 1 | Consensus framework with double feedback mechanism for social network group decision-making (SNGDM).

## 4.2. Construction of Double Feedback Mechanism

A consensus model with double feedback mechanism is developed to improve the consensus degrees of DMs. This double feedback mechanism includes the WFM and the AFM. The details are introduced in the following context.

### 4.2.1. Construction of WFM

The weights of DM are reset according to the influence of the dynamic change of the DMs' trust scores, Firstly, the adjustment direction of DMs' trust scores is determined by sorting and comparing the consensus-degree set and trust-score set of DMs. Secondly, adjust trust scores and obtain the next round of DMs' trust scores. Finally, the weights of DMs are reset according to the next round of DMs' trust scores.

- i. Determine the adjustment direction of trust scores  
First of all, rank the consensus-degree set  $CI$  and the trust-score set  $TS$  of each DM, respectively. According to [51], let  $o^k(\cdot)$  be the ranking function, then rank the consensus-degree set  $CI = \{CI^1, \dots, CI^m\}$  of each DM as follows:

$$O_{CI}^K = \{o^k(CI^1), \dots, o^k(CI^m)\} \quad (11)$$

Then the order of the trust-score set  $TS$  is

$$O_{TS}^K = \{o^k(TS^1), \dots, o^k(TS^m)\} \quad (12)$$

Secondly, the adjustment direction of DMs' trust scores is determined by comparing  $o^k(CI^h)$  and  $o^k(TS^h)$  which are ranking values of  $CI$  and  $TS$ , respectively. The indicator function  $\Delta = \{\delta^1, \dots, \delta^m\}$  is introduced in this paper to compare two ranking values. Referring to [52], the indicator function  $\Delta$  is expressed as

$$\delta^h = \begin{cases} 1 & \text{if } o^k(CI^h) < o^k(TS^h) \\ 0 & \text{if } o^k(CI^h) = o^k(TS^h) \\ -1 & \text{if } o^k(CI^h) > o^k(TS^h) \end{cases} \quad (13)$$

- ii. The next round of trust scores adjustment  
The adjustment direction of DMs' trust scores is determined according to the indicator function  $\Delta$ , the trust score of each DM in the  $t + 1$  round of consensus process is

$$TS_{t+1}^h = (1 + \delta^h \times \theta) TS_t^h \quad (14)$$

where  $\theta \in (0, 1)$  represents the control parameter of weight adjustment, and the smaller  $\theta$  is, the greater the weight adjustment.

- iii. Reset the weight of DM  
The weight of DM is reset according to the trust score of DM in  $t + 1$  round consensus process. According to Eqs. (8) and (14), the weight of DM is determined as follows:

$$w_{t+1}^{\sigma(h)} = Q\left(\frac{T(\sigma(h))_{t+1}}{T(\sigma(m))_{t+1}}\right) - Q\left(\frac{T(\sigma(h-1))_{t+1}}{T(\sigma(m))_{t+1}}\right) \quad (15)$$

where  $T(\sigma(h))_{t+1} = \sum_{j=1}^h TS_{t+1}^{\sigma(j)}$ ,  $TS_{t+1}^{\sigma(j)}$  represents the  $j^{\text{th}}$  largest value of the trust-score set  $\{TS^1, \dots, TS^m\}$  in the  $(t + 1)$  round of consensus process.

### 4.2.2. Construction of AFM

In order to construct the AFM, this paper refers to [36] to calculate the dominance degree of each alternative. Then, the alternative classification method is put forward, and we divide the alternatives into high and low dominance degree alternatives according to their respective dominance degrees. Finally, the adjustment range of contents is determined for low dominance degree alternatives, and then the 2-tuple LPR of DM  $e^h$  is also determined.

- i. Determination of the dominance degree of alternatives  
The set of 2-tuple linguistic evaluation of alternative  $a_i$  over  $a_j$  by DM  $e^h$  is  $\{S_{ij}^1, S_{ij}^2, \dots, S_{ij}^m\}$ . It is aggregated into the group 2-tuple linguistic evaluation value  $S_{ij}^c$  by the weighted average (WA) operator:

$$S_{ij}^c = WA(S_{ij}^1, S_{ij}^2, \dots, S_{ij}^m) = \sum_{h=1}^t w^h S_{ij}^h \quad (16)$$

where  $w^h \in [0, 1]$  is the weight of DM  $e^h$ . According to  $S_{ij}^c$ , the dominance degree of alternative can be calculated as

$$QGDD_i = \emptyset_Q(S_{i1}^c, S_{i2}^c, \dots, S_{i(i-1)}^c, S_{i(i+1)}^c, \dots, S_{in}^c) \quad (17)$$

where  $QGDD_i$  is the dominance degree of the alternative  $a_i$  ( $i = 1, 2, \dots, n$ ) on "fuzzy majority," and  $Q$  is the nondecreasing proportional quantifier.

- ii. Classification of alternatives  
It can be seen from Eq. (17) that the dominance-degree set of all alternatives is  $\{QGDD_1, \dots, QGDD_n\}$ , then the order of the dominance degree of alternative is  $O_{QGDD}^K = \{o^k(QGDD_1), \dots, o^k(QGDD_n)\}$ . And then the alternatives are divided into high dominance degree and low dominance degree alternatives.

  - (a) Identify the high dominance degree alternative  $a_i$  by smaller order values of alternatives' dominance degree:

$$A_{high}^t = \{a_i | a_i \in A, o^k(QGDD_i) \leq \text{round}(\alpha \times n)\} \quad (18)$$

- (b) Identify the low dominance degree alternative  $a_i$  by larger order values of alternatives' dominance degree:

$$A_{low}^t = \{a_i | a_i \in A, o^k(QGDD_i) > \text{round}(\alpha \times n)\} \quad (19)$$

In Eqs. (18) and (19),  $\alpha \in (0, 1)$  is the classification parameters of alternatives.

- (c) Adjustment range determination of low dominance alternative's contents  
According to the classification of alternatives, contents

adjustment-range of low dominance alternative  $a_i$  is as follows:

$$a_i^{t+1} \begin{cases} a_i^t & \text{if } a_i^t \in A_{high}^t \\ (1 - \eta) a_i^t + \eta a_{high}^t & \text{if } a_i^t \in A_{low}^t \end{cases} \quad (20)$$

where  $t$  represents the round of the consensus process,  $\eta \in (0, 1)$  is the adjustment parameter of alternative's contents. The larger the  $\eta$ , the larger the adjustment range is of low dominance alternative's contents.

### 4.3. The Termination of Consensus Process in SNGDM

For termination of consensus process in SNGDM, one of two conditions should be met:

- i. After the  $t^{th}$  round of consensus process, the consensus process ends if all  $CI^h \geq \gamma$ ,  $h \in \{1, \dots, m\}$ , and output  $t = t^*$ ;
- ii. If the predefined maximum round of consensus process  $T_{max}$  is reached, the consensus process ends and let  $t = T_{max}$ .

### 4.4. Steps of the Consensus Model with Double Feedback Mechanism

According to the above analysis, steps of the consensus model with double feedback mechanism are as follows:

**Step1.** Input the initial 2-tuple LPR  $S_0^h = \left( (s_{ij,0}^h, \alpha_{ij,0}^h) \right)_{n \times n}$  and the initial trust matrix  $\Lambda_0 = (\lambda^{lh})_{m \times m}$ , and use Eq. (3) to complete the trust matrix.

**Step2.** The DM's trust function is calculated according to Eq. (6), and transformed into trust score by Eq. (7). Then either the initial DM weight is calculated by Eq. (8) or the DM weight is reset by Eq. (15).

**Step3.** The 2-tuple LPRs  $S_0^h = \left( (s_{ij,0}^h, \alpha_{ij,0}^h) \right)_{n \times n}$  of DMs are aggregated by Eq. (11) & DMs' weights. Then the group preference relation  $\bar{S}^h = \left( (\bar{s}_{ij}^h, \bar{\alpha}_{ij}^h) \right)_{n \times n}$  is obtained.

**Step4.** Obtain the consensus degree  $CI^h$  of each DM according to Eq. (10), and judge whether terminate the consensus process or not. If the conditions are met, turn to step 6, otherwise turn to step 5.

**Step5.** Let  $t = t + 1$  if  $CI^h < \gamma$ , and run the synchronous double feedback mechanism. For the WFM, first of all, obtain the ranking of consensus-degree set and trust-score set of DMs by Eqs. (11) and (12). Then determine the adjustment direction of trust score by Eq. (13). Finally, adjust trust score by Eq. (14) so as to reset DMs' weights, and obtain  $W_t = W_{t+1}$ . For the AFM, first, calculate the dominance degree of alternatives according to Eq. (17). Then, identify high and low dominance degree alternatives by Eqs. (18) and (19). Finally, determine the adjustment range of low dominance degree alternatives' contents through Eq. (20), and obtain  $a_i^t = a_i^{t+1}$ . At the end of step 5, go to step 2.

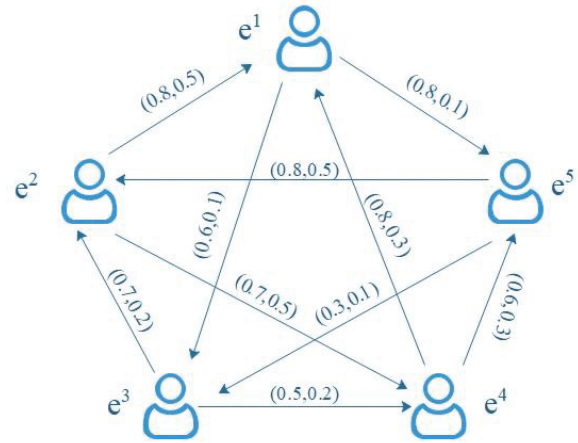


Figure 2 | A network of trust among the five decision makers (DMs).

**Step6.** Let  $t \leftarrow t^*$ , the consensus process is over and group opinions have reached a stable and consistent state. Then output the final group preference relation  $\bar{S}^h = \left( (\bar{s}_{ij}^h, \bar{\alpha}_{ij}^h) \right)_{n \times n}$  and final trust matrix  $\bar{\Lambda} = \left( \bar{\lambda}^{lh} \right)_{m \times m}$ .

**Step7.** According to the final group preference relation, the alternatives are sorted and the best alternative is selected.

## 5. ILLUSTRATIVE EXAMPLE

In this section, an example of a provincial finance department's PPP pilot-projects selection is given to verify the feasibility and effectiveness of the proposed consensus model with double feedback mechanism.

One of the five alternatives  $A = \{a_1, a_2, a_3, a_4, a_5\}$  to be included in the PPP database is selected as the pilot project. The ratios of construction cost of these five alternatives to performance appraisal are  $a_1 = 21\%$ ,  $a_2 = 31\%$ ,  $a_3 = 36\%$ ,  $a_4 = 30\%$ , and  $a_5 = 27\%$ , respectively. Five DMs  $E = \{e^1, e^2, e^3, e^4, e^5\}$  from budget, the industry, engineering technology, financial service, as well as legal consulting fields are invited. After discussion among the DMs, the alternatives are evaluated using a seven-granularity linguistic term set:

$$S = \{s_0 = \text{extremely poor}, s_1 = \text{very poor}, s_2 = \text{poor}, s_3 = \text{medium}, s_4 = \text{good}, s_5 = \text{very good}, s_6 = \text{extremely good}\}$$

The DM evaluates trust relationships with other DMs, and the trust relationship between five DMs is shown in Figure 2, and the initial trust matrix  $\Lambda_0$  of the DMs can be obtained as

$$\Lambda_0 = \begin{bmatrix} - & - & (0.6, 0.1) & - & (0.8, 0.1) \\ (0.8, 0.5) & - & - & (0.7, 0.5) & - \\ - & (0.7, 0.2) & - & (0.5, 0.2) & - \\ (0.8, 0.3) & - & - & - & (0.6, 0.3) \\ - & (0.8, 0.5) & (0.3, 0.1) & - & - \end{bmatrix}$$

The complete trust network as shown in Figure 3 is obtained by completing the trust matrix with dual trust propagation operator.

The complete trust matrix can be obtained through Eq. (3) as follows:

$$\Lambda_0 = \begin{bmatrix} - & (0.50, 0.43) & (0.60, 0.10) & (0.25, 0.29) & (0.80, 0.10) \\ (0.80, 0.50) & - & (0.44, 0.57) & (0.70, 0.50) & (0.50, 0.63) \\ (0.53, 0.64) & (0.70, 0.20) & - & (0.50, 0.20) & (0.25, 0.47) \\ (0.80, 0.30) & (0.44, 0.70) & (0.29, 0.39) & - & (0.60, 0.30) \\ (0.62, 0.80) & (0.80, 0.50) & (0.30, 0.10) & (0.32, 0.55) & - \end{bmatrix}$$

The complete trust matrix is used to calculate the values of DMs' trust function, and the trust scores of the five DMs are

$$TS^1=0.565, TS^2=0.575, TS^3=0.560, TS^4=0.525, TS^5=0.580.$$

According to the ranking function, the trust-score set of five DMs is sorted as follows:

$$TS^{\sigma(1)}=TS^5, TS^{\sigma(2)}=TS^2, TS^{\sigma(3)}=TS^1, TS^{\sigma(4)}=TS^3, TS^{\sigma(5)}=TS^4.$$

The initial weights of DMs are calculated by DMs' trust scores, and the weights of DMs are inferred as  $w^{\sigma(1)}=0.35$ ;  $w^{\sigma(2)}=0.2$ ;  $w^{\sigma(3)}=0.17$ ;  $w^{\sigma(4)}=0.15$ ;  $w^{\sigma(5)}=0.13$ , using the Basic Unit-interval Monotone (BUM) membership function of the fuzzy linguistic quantifier  $Q(r) = r^{2/3}$  in Eq. (8).

The corresponding DMs' weights are

$$w^1=0.17, w^2=0.2, w^3=0.15, w^4=0.13, w^5=0.35.$$

The 2- tuple linguistic evaluation matrices given by five DMs are as follows:

$$S^1 = \begin{bmatrix} (s_3, 0) & (s_2, 0) & (s_1, 0) & (s_5, 0) & (s_4, 0) \\ (s_4, 0) & (s_3, 0) & (s_2, 0) & (s_6, 0) & (s_5, 0) \\ (s_5, 0) & (s_4, 0) & (s_3, 0) & (s_6, 0) & (s_6, 0) \\ (s_1, 0) & (s_0, 0) & (s_0, 0) & (s_3, 0) & (s_2, 0) \\ (s_2, 0) & (s_1, 0) & (s_0, 0) & (s_4, 0) & (s_3, 0) \end{bmatrix}$$

$$S^2 = \begin{bmatrix} (s_3, 0) & (s_2, 0) & (s_1, 0) & (s_0, 0) & (s_4, 0) \\ (s_4, 0) & (s_3, 0) & (s_2, 0) & (s_1, 0) & (s_5, 0) \\ (s_5, 0) & (s_4, 0) & (s_3, 0) & (s_2, 0) & (s_6, 0) \\ (s_6, 0) & (s_5, 0) & (s_4, 0) & (s_3, 0) & (s_6, 0) \\ (s_2, 0) & (s_1, 0) & (s_0, 0) & (s_0, 0) & (s_3, 0) \end{bmatrix}$$

$$S^3 = \begin{bmatrix} (s_3, 0) & (s_0, 0) & (s_2, 0) & (s_1, 0) & (s_4, 0) \\ (s_6, 0) & (s_3, 0) & (s_5, 0) & (s_4, 0) & (s_6, 0) \\ (s_4, 0) & (s_1, 0) & (s_3, 0) & (s_2, 0) & (s_5, 0) \\ (s_5, 0) & (s_2, 0) & (s_4, 0) & (s_3, 0) & (s_6, 0) \\ (s_2, 0) & (s_0, 0) & (s_1, 0) & (s_0, 0) & (s_3, 0) \end{bmatrix}$$

$$S^4 = \begin{bmatrix} (s_3, 0) & (s_4, 0) & (s_1, 0) & (s_5, 0) & (s_2, 0) \\ (s_2, 0) & (s_3, 0) & (s_0, 0) & (s_4, 0) & (s_1, 0) \\ (s_5, 0) & (s_6, 0) & (s_3, 0) & (s_6, 0) & (s_4, 0) \\ (s_1, 0) & (s_2, 0) & (s_0, 0) & (s_3, 0) & (s_0, 0) \\ (s_4, 0) & (s_5, 0) & (s_2, 0) & (s_6, 0) & (s_3, 0) \end{bmatrix}$$

$$S^5 = \begin{bmatrix} (s_3, 0) & (s_1, 0) & (s_0, 0) & (s_2, 0) & (s_0, 0) \\ (s_5, 0) & (s_3, 0) & (s_1, 0) & (s_4, 0) & (s_2, 0) \\ (s_6, 0) & (s_5, 0) & (s_3, 0) & (s_6, 0) & (s_4, 0) \\ (s_4, 0) & (s_2, 0) & (s_0, 0) & (s_3, 0) & (s_1, 0) \\ (s_6, 0) & (s_4, 0) & (s_2, 0) & (s_5, 0) & (s_3, 0) \end{bmatrix}$$

The group preference relation is calculated by DMs' weights and 2-tuple LPRs:

$$\tilde{S} = \begin{bmatrix} (s_3, 0) & (s_2, -0.39) & (s_1, -0.2) & (s_2, 0.35) & (s_2, 0.34) \\ (s_4, 0.39) & (s_3, 0) & (s_2, -0.16) & (s_4, 0.26) & (s_4, -0.42) \\ (s_5, 0.2) & (s_4, 0.16) & (s_3, 0) & (s_5, -0.4) & (s_5, -0.11) \\ (s_4, -0.35) & (s_2, 0.26) & (s_1, 0.4) & (s_3, 0) & (s_3, -0.21) \\ (s_4, -0.34) & (s_2, 0.42) & (s_1, 0.11) & (s_3, 0.21) & (s_3, 0) \end{bmatrix}$$

Then, the consensus degrees  $CI^h$  are calculated using Eq. (10) as

$$CI^1 = 0.8624, CI^2 = 0.8190, CI^3 = 0.7991, CI^4 = 0.8247, CI^5 = 0.8759.$$

This example sets the threshold value  $\gamma$  to 0.9, and each DM's  $CI^h$  fails to meet the threshold. Thus, the double feedback mechanism is activated in the first round of consensus process.

- i. The operation of double feedback mechanism in the first round of consensus process

Through the WFM and the AFM in the first round, consensus of DMs is promoted.

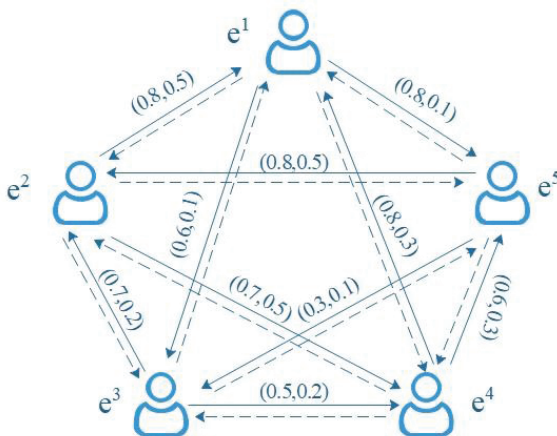


Figure 3 | The trust network after completing the trust matrix.

(a) The process of WFM

The ranking of DMs' consensus-degrees set  $CI$  is  $O_{CI}^k = \{o^k(CI^1), \dots, o^k(CI^5)\} = \{2, 4, 5, 3, 1\}$ , and the ranking of DMs' trust-scores set  $TS$  is  $O_{TS}^k = \{o^k(TS^1), \dots, o^k(TS^5)\} = \{3, 2, 4, 5, 1\}$ . Then the indicator function is  $\Delta = \{\delta^1, \dots, \delta^5\} = \{1, -1, -1, 1, 0\}$ , which determines the direction of DMs' trust score adjustment. By Eq. (14), adjust five DMs' trust scores to  $TS_{t=1}^1=0.678$ ,  $TS_{t=1}^2=0.46$ ,  $TS_{t=1}^3=0.448$ ,  $TS_{t=1}^4=0.630$ , and  $TS_{t=1}^5=0.580$ . Then reset the DMs' weights to  $w_{t=1}^1=0.389$ ,  $w_{t=1}^2=0.121$ ,  $w_{t=1}^3=0.110$ ,  $w_{t=1}^4=0.214$ , and  $w_{t=1}^5=0.166$ .

$$S_{t=1}^2 = \begin{bmatrix} (s_3, 0) & (s_2, 0) & (s_1, 0) & (s_4, 0) & (s_5, 0) \\ (s_4, 0) & (s_3, 0) & (s_2, 0) & (s_5, 0) & (s_6, 0) \\ (s_5, 0) & (s_4, 0) & (s_3, 0) & (s_6, 0) & (s_6, 0) \\ (s_2, 0) & (s_1, 0) & (s_0, 0) & (s_3, 0) & (s_4, 0) \\ (s_1, 0) & (s_0, 0) & (s_0, 0) & (s_2, 0) & (s_3, 0) \end{bmatrix}$$

$$S_{t=1}^3 = \begin{bmatrix} (s_3, 0) & (s_2, 0) & (s_1, 0) & (s_4, 0) & (s_5, 0) \\ (s_4, 0) & (s_3, 0) & (s_2, 0) & (s_5, 0) & (s_6, 0) \\ (s_5, 0) & (s_4, 0) & (s_3, 0) & (s_6, 0) & (s_6, 0) \\ (s_2, 0) & (s_1, 0) & (s_0, 0) & (s_3, 0) & (s_4, 0) \\ (s_1, 0) & (s_0, 0) & (s_0, 0) & (s_2, 0) & (s_3, 0) \end{bmatrix}$$

(b) The process of AFM

In addition to the WFM, the contents of alternatives need to be adjusted.

Firstly, the dominance degrees are calculated as  $QGDD_1 = 1.821$ ,  $QGDD_2 = 3.47$ ,  $QGDD_3 = 4.672$ ,  $QGDD_4 = 2.386$ ,  $QGDD_5 = 2.605$ . We set  $\alpha = 0.7, \eta = 0.2$ . Then, the high and low dominance degrees alternative-sets are  $A_{high} = \{a_3, a_2, a_5, a_4\}$  and  $A_{low} = \{a_1\}$ . Adjust the contents of the low dominance degree alternative to  $a_1^{t=1} = 24\%$ . The 2-tuple LPRs given by five DMs are adjusted to

$$S_{t=1}^4 = \begin{bmatrix} (s_3, 0) & (s_4, 0) & (s_1, 0) & (s_5, 0) & (s_2, 0) \\ (s_2, 0) & (s_3, 0) & (s_0, 0) & (s_4, 0) & (s_1, 0) \\ (s_5, 0) & (s_6, 0) & (s_3, 0) & (s_6, 0) & (s_4, 0) \\ (s_1, 0) & (s_2, 0) & (s_0, 0) & (s_3, 0) & (s_0, 0) \\ (s_4, 0) & (s_5, 0) & (s_2, 0) & (s_6, 0) & (s_3, 0) \end{bmatrix}$$

$$S_{t=1}^5 = \begin{bmatrix} (s_3, 0) & (s_2, 0) & (s_0, 0) & (s_4, 0) & (s_1, 0) \\ (s_4, 0) & (s_3, 0) & (s_1, 0) & (s_5, 0) & (s_2, 0) \\ (s_6, 0) & (s_5, 0) & (s_3, 0) & (s_6, 0) & (s_4, 0) \\ (s_2, 0) & (s_1, 0) & (s_0, 0) & (s_3, 0) & (s_0, 0) \\ (s_5, 0) & (s_4, 0) & (s_2, 0) & (s_6, 0) & (s_3, 0) \end{bmatrix}$$

$$S_{t=1}^1 = \begin{bmatrix} (s_3, 0) & (s_2, 0) & (s_1, 0) & (s_5, 0) & (s_4, 0) \\ (s_4, 0) & (s_3, 0) & (s_2, 0) & (s_6, 0) & (s_5, 0) \\ (s_5, 0) & (s_4, 0) & (s_3, 0) & (s_6, 0) & (s_6, 0) \\ (s_1, 0) & (s_0, 0) & (s_0, 0) & (s_3, 0) & (s_2, 0) \\ (s_2, 0) & (s_1, 0) & (s_0, 0) & (s_4, 0) & (s_3, 0) \end{bmatrix}$$

Secondly, the group preference relation is determined to be

$$\tilde{S}_{t=1} = \begin{bmatrix} (s_3, 0) & (s_2, 0.428) & (s_1, -0.166) & (s_5, -0.397) & (s_3, 0.305) \\ (s_4, -0.428) & (s_3, 0) & (s_1, 0.406) & (s_5, 0.175) & (s_4, -0.123) \\ (s_5, 0.166) & (s_5, -0.406) & (s_3, 0) & (s_6, 0) & (s_5, 0.24) \\ (s_1, 0.397) & (s_1, -0.175) & (s_0, 0) & (s_3, 0) & (s_2, -0.298) \\ (s_3, -0.305) & (s_2, 0.123) & (s_1, -0.24) & (s_4, 0.298) & (s_3, 0) \end{bmatrix}$$

Finally, DMs' consensus degrees are recalculated as  $CI_{t=1}^1 = 0.9396$ ,  $CI_{t=1}^2 = 0.8989$ ,  $CI_{t=1}^3 = 0.8989$ ,  $CI_{t=1}^4 = 0.8647$ , and  $CI_{t=1}^5 = 0.8906$ , respectively.

At this point,  $CI_{t=1}^2$ ,  $CI_{t=1}^3$ ,  $CI_{t=1}^4$ , and  $CI_{t=1}^5$  do not meet the threshold, and the double feedback mechanism needs to be activated for a second round of consensus process.

LPRs given by five DMs are adjusted to

$$S_{t=2}^1 = \begin{bmatrix} (s_3, 0) & (s_2, 0) & (s_1, 0) & (s_5, 0) & (s_4, 0) \\ (s_4, 0) & (s_3, 0) & (s_2, 0) & (s_6, 0) & (s_5, 0) \\ (s_5, 0) & (s_4, 0) & (s_3, 0) & (s_6, 0) & (s_6, 0) \\ (s_1, 0) & (s_0, 0) & (s_0, 0) & (s_3, 0) & (s_2, 0) \\ (s_2, 0) & (s_1, 0) & (s_0, 0) & (s_4, 0) & (s_3, 0) \end{bmatrix}$$

ii. The operation of double feedback mechanism in the second round of consensus process

Through the WFM in the second round of consensus process, the weights of DMs are reset to  $w_{t=2}^1=0.395$ ,  $w_{t=2}^2=0.192$ ,  $w_{t=2}^3=0.160$ ,  $w_{t=2}^4=0.136$ , and  $w_{t=2}^5=0.117$ .

Adjust the contents of alternatives in the second round of consensus reaching process, and the high and low dominance degree alternative set are identified as  $A_{high} = \{a_3, a_1, a_2, a_5\}$  and  $A_{low} = \{a_4\}$ , respectively. Adjust the content of the low dominance degree alternative to  $a_4^{t=2} = 31.2\%$ . Then the 2-tuple

$$S_{t=2}^2 = \begin{bmatrix} (s_3, 0) & (s_2, 0) & (s_1, 0) & (s_4, 0) & (s_5, 0) \\ (s_4, 0) & (s_3, 0) & (s_2, 0) & (s_5, 0) & (s_6, 0) \\ (s_5, 0) & (s_4, 0) & (s_3, 0) & (s_6, 0) & (s_6, 0) \\ (s_2, 0) & (s_1, 0) & (s_0, 0) & (s_3, 0) & (s_4, 0) \\ (s_1, 0) & (s_0, 0) & (s_0, 0) & (s_2, 0) & (s_3, 0) \end{bmatrix}$$

$$S_{t=2}^3 = \begin{bmatrix} (s_3, 0) & (s_2, 0) & (s_1, 0) & (s_4, 0) & (s_5, 0) \\ (s_4, 0) & (s_3, 0) & (s_2, 0) & (s_5, 0) & (s_6, 0) \\ (s_5, 0) & (s_4, 0) & (s_3, 0) & (s_6, 0) & (s_6, 0) \\ (s_2, 0) & (s_1, 0) & (s_0, 0) & (s_3, 0) & (s_4, 0) \\ (s_1, 0) & (s_0, 0) & (s_0, 0) & (s_2, 0) & (s_3, 0) \end{bmatrix}$$



$$S^4_{t=2} = \begin{bmatrix} (s_3, 0) & (s_4, 0) & (s_2, 0) & (s_6, 0) & (s_5, 0) \\ (s_2, 0) & (s_3, 0) & (s_1, 0) & (s_5, 0) & (s_4, 0) \\ (s_4, 0) & (s_5, 0) & (s_3, 0) & (s_6, 0) & (s_6, 0) \\ (s_0, 0) & (s_1, 0) & (s_0, 0) & (s_3, 0) & (s_2, 0) \\ (s_1, 0) & (s_2, 0) & (s_0, 0) & (s_4, 0) & (s_3, 0) \end{bmatrix}$$

$$S^5_{t=2} = \begin{bmatrix} (s_3, 0) & (s_2, 0) & (s_1, 0) & (s_5, 0) & (s_4, 0) \\ (s_4, 0) & (s_3, 0) & (s_2, 0) & (s_6, 0) & (s_5, 0) \\ (s_5, 0) & (s_4, 0) & (s_3, 0) & (s_6, 0) & (s_5, 0) \\ (s_1, 0) & (s_0, 0) & (s_0, 0) & (s_3, 0) & (s_2, 0) \\ (s_2, 0) & (s_1, 0) & (s_0, 0) & (s_4, 0) & (s_3, 0) \end{bmatrix}$$

Secondly, the group preference relation is determined to be

$$\tilde{S}_{t=2} = \begin{bmatrix} (s_3, 0) & (s_2, 0.272) & (s_1, 0.136) & (s_5, -0.216) & (s_4, 0.488) \\ (s_4, -0.272) & (s_3, 0) & (s_2, -0.136) & (s_6, 0.488) & (s_5, 0.216) \\ (s_5, -0.136) & (s_4, 0.136) & (s_3, 0) & (s_6, 0) & (s_6, -0.117) \\ (s_1, 0.216) & (s_0, 0.488) & (s_0, 0) & (s_3, 0) & (s_3, -0.296) \\ (s_2, -0.488) & (s_1, -0.216) & (s_0, 0) & (s_3, 0.296) & (s_3, 0) \end{bmatrix}$$

Finally, DMs' consensus degrees are recalculated as  $CI^1_{t=2} = 0.9690$ ,  $CI^2_{t=2} = 0.9487$ ,  $CI^3_{t=2} = 0.9487$ ,  $CI^4_{t=2} = 0.9123$ , and  $CI^5_{t=2} = 0.9646$ , respectively.

Now that the consensus degrees  $CI^h$  of all DMs completely meet the threshold value and meet the termination judgment conditions, the consensus process ends and get the final ranking of alternatives  $a_3 > a_2 > a_1 > a_5 > a_4$ . Hence,  $a_3$  is selected as the PPP pilot project.

**Table 1** | Results comparison with different methods.

Literature	Consensus Rounds	Alternatives Ranking
Wu et al. [49]	7	$a_3 > a_2 > a_5 > a_4 > a_1$
Tian et al. [34]	7	$a_3 > a_2 > a_5 > a_4 > a_1$
This paper	2	$a_3 > a_2 > a_1 > a_5 > a_4$

**Table 2** |  $CI^h$  of DMs at each round with the method in [49].

Round	$CI^1$	$CI^2$	$CI^3$	$CI^4$	$CI^5$	Revised	Feedback Mechanism
1	0.8851	0.8525	0.8385	0.8590	0.8967	All	JFM
2	0.9018	0.8718	0.8668	0.8828	0.9069	All	JFM
3	0.9027	0.8827	0.8896	0.9047	0.9119	$e^2 e^3 e^4$	JFM
4	0.9045	0.8921	0.9014	0.9100	0.9164	$e^2 e^3$	JFM
5	0.9258	0.8990	0.9003	0.9118	0.9117	$e^2$	JFM
6	0.9068	0.9030	0.8995	0.9124	0.9179	$e^2$	JFM
7	0.9074	0.9017	0.9067	0.9137	0.9189	$e^3$	JFM

DM, decision maker; JFM, judgment feedback mechanism.

## 6. COMPARATIVE ANALYSIS AND DISCUSSION

Comparative analysis is conducted between the proposed approach and existing methods in [34,49], which inspired this paper. Although Dong et al. [32] studied the consensus model with double feedback mechanism, it assumed that DMs were independent, so this paper does not make a comparison with the method in [32]. Different from the cost consensus metric established by Álvaro et al. [53] for the minimum cost models, this paper illustrates the effectiveness of the proposed model by improving the consensus degree of DMs in each round and reducing the consensus rounds. Methods from [34,49] are applied to illustrative example of this paper, and the results obtained by the proposed approach are compared with those from the literature. For convenience, the dual trust propagation operator is used, with the weights of DMs as  $w^1=0.17$ ;  $w^2=0.2$ ;  $w^3=0.15$ ;  $w^4=0.13$ ;  $w^5=0.35$ .

### i. Comparison with single feedback SNGDM method in [49]

In [49], a multi-criteria GDM (MCGDM) problem with trust function was addressed, and a trust-based recommendation mechanism was developed for reaching consensus. Then the method of [49] is applied to the illustrative example of this paper, with predefined consensus degree 0.9. The ranking of the alternatives after consensus reaching and  $CI^h$  of DMs at each round with the method in [49] are given in Tables 1 and 2. It can be seen from Table 1, that both the results of the optimal alternative obtained by [49] and the proposed approach are  $a_3$ . Nevertheless, consensus is reached after only two rounds in the proposed approach in contrast with seven rounds using the trust-based recommendation mechanism of [49].

### ii. Comparison with double feedback SNGDM method in [34]

In [34], a SNGDM problem with trust function was addressed, and a double feedback mechanism including JFM and WFM is developed for reaching consensus. Whereas in illustrative example of this paper, consensus degree is predefined as 0.9 and control parameters are predefined as  $\mu = \nu = 0.5$ . Then the method of [34] is applied to the illustrative example of this paper. The consensus is reached after seven rounds by method of [34] using the JFM and WFM. The ranking of the alternatives after consensus reaching and  $CI^h$  of DMs at each round with the method in [34] are given in Tables 1 and 3. It can be seen from Tables 1 and 3 that in [34] or the proposed approach, the optimal alternative is obtained as  $a_3$ . However, consensus

**Table 3** |  $CI^h$  of DMs at each round with the method in [34].

Round	$CI^1$	$CI^2$	$CI^3$	$CI^4$	$CI^5$	Feedback Mechanism	
						JFM	WFM
1	0.8647	0.8094	0.8559	0.8332	0.8832	√	
2	0.8747	0.8735	0.8433	0.8489	0.8936	√	
3	0.8745	0.8645	0.8315	0.8584	0.9008		√
4	0.8730	0.8590	0.9070	0.8639	0.9051	√	
5	0.8745	0.8994	0.8960	0.8692	0.9082	√	
6	0.8750	0.9031	0.9032	0.9135	0.9077	√	
7	0.9140	0.9027	0.9027	0.9151	0.9165	√	

DM, decision maker; JFM, judgment feedback mechanism; WFM, weight feedback mechanism.

is reached after two rounds in the proposed approach, in contrast with seven rounds by method of [34].

Although both the proposed method and method in [34] depend on trust relationship to obtain DMs' weights, the double feedback mechanisms are different. In contrast with method of [34] which assumes invariant weights of DMs, the proposed approach depends on dynamic trust relationship between DMs to change the weight of each DM. In addition, the proposed approach adopts AFM and adjusts contents of alternatives with low dominance degree to promote consensus-reaching. Overall, the proposed approach is an improved approach to provide a new and effective way to address SNGDM problems with dynamic trust relationship between DMs.

According to the aforementioned comparison analysis, the advantages of using the proposed approach to solve the SNGDM can be summarized as follows:

1. The optimal alternative obtained in this paper is the same as that obtained in [34,49], which reflects the feasibility of the proposed model in this paper. At the same time, DMs' consensus degrees greatly improve after two rounds. It indicated that the proposed model in this paper can effectively improve the consensus degrees of DMs.
2. The ranking results of alternatives in this paper are slightly different from those in [34,49], because the contents of  $a_1$  and  $a_4$  are respectively adjusted through the AFM in consensus reaching process. It indicated that AFM can change DMs' opinions on the evaluation of alternatives and improve their satisfaction with the alternatives.
3. This paper has guided significance for the dispute settlement of PPP construction projects. PPP projects involve many subjects, so it is more necessary to consider the trust relationship between DMs to promote consensus reaching. It should adjust the contents of alternatives timely, so as to improve DMs' satisfaction on alternatives and solve the project disputes effectively.

## 7. CONCLUSIONS

This paper studies the SNGDM problem under the condition of the dynamic change of trust relationship between DMs and appropriate adjustment of alternative contents. On the basis of constructing

a consensus framework with double feedback mechanism for SNGDM, a consensus model with double feedback mechanism was proposed. The main conclusions of the proposed model are as follows:

- i. Interaction in the consensus process is the basis for the change in trust relationships between DMs. Trust scores among DMs change dynamically with the WFM, so that DMs can adjust the trust relationship network more flexibly. Accordingly, weights of DMs change so that the degree of consensus in each round of consensus process can be significantly improved.
- ii. Difference in DMs' judgment of alternative contents is the main reason for opinion conflict. This paper builds the AFM to appropriate adjustment of the contents of low dominance alternatives, change DMs' evaluations and reduce conflict of DMs' opinions. It not only took into account evaluations of the minority DMs and avoided group thinking, but also improved DMs' satisfaction of alternatives, so as to improve the quality and efficiency of consensus.
- iii. The optimal alternative obtained by three methods is the same, which shows that the proposed consensus model is feasible. The application of AFM makes the alternative ranking slightly changed in this paper, but DM's satisfaction to the alternatives is improved. Furthermore, consensus degrees in this model have been significantly improved in each round, compared with [34,49], indicating its high efficiency in consensus reaching. This has guiding significance for the dispute settlement of PPP construction projects in reality.

However, the consensus model proposed in this paper also has limitations. Further study in the future is to determine the reasonable range of control parameters  $\theta$  and  $\eta$  through sensitivity analysis. Moreover, some scholars have focused on GDM based on hesitant fuzzy linguistic information [54–57]. Considering the cognitive limitations and fuzziness of DMs, we will use hesitant fuzzy linguistic sets to represent DMs' preferences in the future research on consensus models. Existing research has also been extended to the field of large-scale GDM [12,54,58], so it will also be interesting to explore more challenging consensus model for large-scale SNGDM.

## CONFLICTS OF INTEREST

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

## AUTHORS' CONTRIBUTIONS

Yueqin Gu: contributed significantly to the model construction of the study, manuscript writing and revision; Tiantian Hao contributed significantly to the manuscript preparation and manuscript revision; Dong Cheng: contributed significantly to providing guidance for writing and helped perform the analysis with constructive discussions; Juan Wang: contributed significantly to the polishing of the sentences in the paper; Faxin Cheng: contributed significantly to the research ideas and the direction of

future research, and helped perform the analysis with constructive discussions.

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