

# The Carry Flag

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## 1 Based on

## 2 The Carry flag

- TOC: Carry flag
- Examples of signed and unsigned integer arithmetic
- The Carry flag in unsigned and signed computations
- Rules for the carry flag
- Method for computing the carry flag
- More examples of the carry flag

- The CARRY flag and OVERFLOW flag in binary arithmetic  
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[https://teaching.idallen.com/dat2343/10f/notes/  
040\\_overflow.ttx](https://teaching.idallen.com/dat2343/10f/notes/040_overflow.ttx)

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# Compiling 32-bit program on 64-bit gcc

- `gcc -v`
- `gcc -m32 t.c`
- `sudo apt-get install gcc-multilib`
- `sudo apt-get install g++-multilib`
- `gcc-multilib`
- `g++-multilib`
- `gcc -m32`
- `objdump -m i386`

- Examples of signed and unsigned integer arithmetic
- Carry flag in unsigned and signed computations
- Rules for the carry flag
- Method for computing the carry flag
- More examples of the carry flag

# TOC: Examples of signed and unsigned integer arithmetic

- Subtracting  $0x0000618D$  from  $0x0000195D$
- **Signed** and **unsigned** integer arithmetic examples
- 2's complement of the subtrahend  $0x0000618D$
- Interpreting  $0xFFFF9E73$  (2's complement of  $0x0000618D$ )
- **Unsigned** subtraction  $0x0000195D - 0x0000618D$
- **Signed** subtraction  $0x0000195D + (-0x0000618D)$
- 2's complement of the subtraction result  $0xFFFFB7D0$
- Interpreting the subtraction result  $0xFFFFB7D0$
- Summary of **signed** and **unsigned** subtractions
- Carry flag (CF) and **unsigned** integer overflows
- Overflow flag (OF) and **signed** integer overflows

# Subtracting 0x0000618D from 0x0000195D (1)

- a subtraction example

$$\begin{array}{r} \text{Minuend} \quad - \text{Subtrahend} \quad = \text{Difference} \\ 0x0000195D \quad - \quad 0x0000618D \quad = \quad 0xFFFFB7D0 \end{array}$$

- 2's complement

$$\begin{array}{r} 0x0000618D \quad \leftrightarrow \quad 0xFFFF9E73 \\ 0xFFFFB7D0 \quad \leftrightarrow \quad 0x00004830 \end{array}$$

- the results of **unsigned** and **signed** subtractions have the same bit pattern 0xFFFFB7D0

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Subtracting $0x0000618D$ from $0x0000195D$ (2)

- 2's complement

$$\begin{array}{l} \hline 0x0000618D \quad \leftrightarrow \quad 0xFFFF9E73 \quad * \\ 0xFFFFB7D0 \quad * \quad \leftrightarrow \quad 0x00004830 \\ \hline \end{array}$$

- interpreting  $0xFFFF9E73$

as an <b>unsigned</b> number	$+0xFFFF9E73$	$+4294942323_{10}$
as a <b>signed</b> number	$-0x0000618D$	$-24973_{10}$

- interpreting  $0xFFFFB7D0 = 0x0000195D - 0x0000618D$

as an <b>unsigned</b> number	$+0xFFFFB7D0$	$+4294948816_{10}$
as a <b>signed</b> number	$-0x00004830$	$-18480_{10}$

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>



# Signed and unsigned integer arithmetic examples

- $0x0000195D - 0x0000618D = 0xFFFFB7D0$

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unsigned subtraction	$0x0000195D - 0x0000618D$ 6493 - 24973	$= 0xFFFFB7D0$ $= 16^8 - 18480 = 4294948816$
	subtraction by hand	borrow (CF=1) $16^8 = 2^{32} = 4294967296$

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signed subtraction	$0x0000195D + (-0x0000618D)$	$= -0x00004830$
	$0x0000195D + 0xFFFF9E73$ 6493 - 24973	$= 0xFFFFB7D0$ $= -18480$
	the <i>transformed addition</i> using the 2's complement of <u>subtrahend</u>	CF = 1 ← Cn = 0 CF = !Cn for subtraction

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

## 2's complement of the subtrahend 0x0000618D

- the 2's complement of 0x0000618D  $\rightarrow$  0xFFFF9E73 ( $= -24973_{10}$ )

	F	F	F	F	9	E	7	3	
0xFFFF9E73	0x1111_1111_1111_1111_1001_1110_0111_0011								
0x0000618C	0x0000_0000_0000_0000_0110_0001_1000_1100								(1's complement)
0x0000618D	0x0000_0000_0000_0000_0110_0001_1000_1101								(2's complement)
	0	0	0	0	6	1	8	D	

- the 2's complement of 0xFFFF9E73  $\rightarrow$  0x0000618D ( $= +24973_{10}$ )

	0	0	0	0	6	1	8	D	
0x0000618D	0x0000_0000_0000_0000_0110_0001_1000_1101								
0xFFFF8E72	0x1111_1111_1111_1111_1001_1110_0111_0010								(1's complement)
0xFFFF8E73	0x1111_1111_1111_1111_1001_1110_0111_0011								(2's complement)
	F	F	F	F	9	E	7	3	

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Interpreting 0xFFFF9E73

- interpreting 0x0xFFFF9E73
  - as an **unsigned** Number :  $+0xFFFF9E73 (= +4294942323_{10})$

F	F	F	F	9	E	7	3
15	15	15	15	9	14	7	3
$16^7$	$16^6$	$16^5$	$16^4$	$16^3$	$16^2$	$16^1$	$16^0$

- as a **signed** number :  $-0x0000618D (= -24973_{10})$

0	0	0	0	6	1	8	D
0	0	0	0	6	1	8	13
$16^7$	$16^6$	$16^5$	$16^4$	$16^3$	$16^2$	$16^1$	$16^0$

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Unsigned subtraction 0x0000195D - 0x0000618D (1)

- 0x0000195D - 0x0000618D : unsigned subtraction

subtraction by hand

```

          0  0  0  0  1  9  5  D
0x0000195D  0x0000_0000_0000_0000_0001_1001_0101_1101
- 0x0000618D  0x0000_0000_0000_0000_0110_0001_1000_1101
-----
0xFFFFB7D0  1 0x1111_1111_1111_1111_1011_0111_1101_0000 (hand subtraction)
          1  F  F  F  F  B  7  D  0
          .
          V borrow (CF=1) : unsigned integer overflow
```

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

## Unsigned subtraction $0x0000195D - 0x0000618D$ (2)

- $0x0000195D - 0x0000618D$  : unsigned subtraction subtraction by hand
  - unsigned integer overflow is indicated by the carry flag (CF)
  - a borrow is indicated by the carry flag (CF=1)
    - whenever an unsigned integer overflow happened
    - $A - B$ , when  $A < B$ , for non-negative integers  $A, B$

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

# Signed subtraction $0x0000195D + (-0x0000618D)$ (1)

- $0x0000195D + (-0x0000618D)$  : signed subtraction

the *transformed addition* using the 2's complement of subtrahend

```

          0  0  0  0  1  9  5  D
0x0000195D  0x0000_0000_0000_0000_0001_1001_0101_1101 (+0x0000195D)
+ 0xFFFF9E73 0x1111_1111_1111_1111_1001_1110_0111_0011 (-0x0000618D)
          F  F  F  F  9  E  7  3
-----
0xFFFFB7D0 0 0x1111_1111_1111_1111_1011_0111_1101_0000 (hand addition)
0          F  F  F  F  B  7  D  0
-0x00004830 . 0x0000_0000_0000_0000_0100_1000_0011_0000 (2's complement)
.          0  0  0  0  4  8  3  0
V no carry in the transformed addition (Cn=0) --> (CF=1)
```

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

## Signed subtraction $0x0000195D + (-0x0000618D)$ (2)

- $0x0000195D + (-0x0000618D)$  : signed subtraction  
the *transformed addition* uses the 2's complement of a subtrahend
  - signed integer overflow is indicated by the **overflow** flag (OF)
  - when a subtraction is performed by a transformed addition, the **carry** flag (CF) is set by **inverting** the carry out ( $C_n$ ) of a transformed addition

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ADD instruction :  $CF \leftarrow c_n$     SUB instruction :  $CF \leftarrow \overline{c_n}$

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## 2's complement of the subtraction result 0xFFFFB7D0

- $0xFFFFB7D0 = 0x0000195D - 0x0000618D$
- the 2's complement of  $0xFFFFB7D0$  :  $0x00004830$  ( $= +18480_{10}$ )

	F	F	F	F	B	7	D	0
0xFFFFB7D0	0x1111_1111_1111_1111_1011_0111_1101_0000							
0x0000482F	0x0000_0000_0000_0000_0100_1000_0010_1111							(1's complement)
0x00004830	0x0000_0000_0000_0000_0100_1000_0011_0000							(2's complement)
	0	0	0	0	4	8	3	0

- the 2's complement of  $0x00004830$  :  $0xFFFFB7D0$  ( $= -18480_{10}$ )

	0	0	0	0	4	8	3	0
0x00004830	0x0000_0000_0000_0000_0100_1000_0011_0000							
0xFFFFB7CF	0x1111_1111_1111_1111_1011_0111_1100_1111							(1's complement)
0xFFFFB7D0	0x1111_1111_1111_1111_1011_0111_1101_0000							(2's complement)
	F	F	F	F	B	7	D	0

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>



# Interpreting the subtraction result 0xFFFFB7D0

- interpreting 0xFFFFB7D0 = 0x0000195D - 0x0000618D
  - as an **unsigned** Number : +0xFFFFB7D0 (= +4294948816<sub>10</sub> )

F	F	F	F	B	7	D	0
15	15	15	15	11	7	13	0
16 <sup>7</sup>	16 <sup>6</sup>	16 <sup>5</sup>	16 <sup>4</sup>	16 <sup>3</sup>	16 <sup>2</sup>	16 <sup>1</sup>	16 <sup>0</sup>

- as a **signed** number : -0x00004830 (= -18480<sub>10</sub> )

0	0	0	0	4	8	3	0
0	0	0	0	4	8	3	0
16 <sup>7</sup>	16 <sup>6</sup>	16 <sup>5</sup>	16 <sup>4</sup>	16 <sup>3</sup>	16 <sup>2</sup>	16 <sup>1</sup>	16 <sup>0</sup>

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# Summary of signed and unsigned subtractions (1)

- subtracting  $0x0000618D$  from  $0x0000195D$ 
  - $0x0000195D - 0x0000618D$  : unsigned integer subtraction  
hand subtraction
  - $0x0000195D + (-0x0000618D)$  : signed integer subtraction  
the *transformed addition* using the 2's complement of the subtrahend
  - the same result :  $0xFFFFB7D0$  (the same bit pattern)
    - interpreting as a unsigned integer  $4294948816_{10}$   
 $0xFFFFB7D0$  with a borrow (CF=1)  $2^{32} - 18480$
    - interpreting as a signed integer  $-18480_{10}$   
 $-0x00004830$  (meaningless CF=1)

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## Summary of **signed** and **unsigned** subtractions (2)

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**unsigned** subtraction     $0x0000195D - 0x0000618D$      $0xFFFFB7D0$   
4294948816<sub>10</sub>  
subtraction by hand    with a borrow (CF=1)

---

**signed** subtraction     $0x0000195D + (-0x0000618D)$      $-0x00004830$   
-18480<sub>10</sub>  
the *transformed addition* using  
the 2's complement of subtrahend

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# Carry flag (CF) and **unsigned** integer overflows

- $0x0000195D - 0x0000618D$  : **unsigned** subtraction
  - there is an **unsigned** integer overflow  
so the **carry** flag will be set ( $CF=1$ ) to indicate a **borrow**
  - $A - B$ , when  $A < B$ , for non-negative integers  $A, B$   
(unsigned integers can't be negative),

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# Overflow flag (OF) and signed integer overflows

- $0x0000195D + (-0x0000618D)$  : signed subtraction
  - there is no signed integer overflow  
the overflow flag won't be set (OF=0)
  - signed overflow occurs in the transformed addition,
    - two *positive* numbers are added and the result is a *negative*, ( $P + P \rightarrow N$ ), or
    - two *negative* numbers are added and the result is a *positive*, ( $N + N \rightarrow P$ )

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# TOC Carry flag in unsigned and signed computations

- 2's complement numbers : 4-bit
- Carrys in signed arithmetics
- Carrys in unsigned arithmetics
- Signed and unsigned arithmetics
- Using the carry flag as a borrow

## 2's complement numbers : 4-bit

	signed	unsigned
0111	(+7)	7
0110	(+6)	6
0101	(+5)	5
0100	(+4)	4
0011	(+3)	3
0010	(+2)	2
0001	(+1)	1
0000	(0)	0
1000	(-8)	8
1001	(-7)	9
1010	(-6)	10
1011	(-5)	11
1100	(-4)	12
1101	(-3)	13
1110	(-2)	14
1111	(-1)	15

# Carrys in unsigned arithmetics

- a carry in an **unsigned** addition can be obtained by performing a *hand* addition
  - **CF=1** represents a carry in an **unsigned** addition
- a borrow in an **unsigned** subtraction can be obtained by performing a *hand* subtraction
  - **CF=1** represents a borrow in an **unsigned** subtraction



# Carrys in signed arithmetics

- carry out in a **signed** addition / subtraction are carry out of an *n-bit* 2's complement adder
  - a **signed** subtraction  $A - B$  is performed by the transformed addtion  $A + \overline{B} + 1$
  - carry out is denoted by  $C_n$

# Signed and unsigned arithmetics (1)

	Addition	Subtraction
Unsigned	Hand Addition with a Carry ( <b>CF</b> )	Hand Subtraction with a Borrow ( <b>CF</b> )
Signed	2's complement addition with a carry out $C_n$ $A + B$	2's complement <b>transformed</b> addition with a carry out $C_n$ $A + \overline{B} + 1 \leftarrow A - B$
<b>CF</b> and $C_n$ relation	Carry <b>CF</b> = $C_n$ for <u>addition</u> instructions	Borrow <b>CF</b> = $!C_n$ for <u>subtraction</u> instructions

# Signed and unsigned arithmetics (2)

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**CF** (Carry Flag)

- a carry in an **unsigned** addition
- a borrow in an **unsigned** subtraction

---

**$C_n$**  (the carry out of an n-bit adder)

- a carry out in a **signed** addition
  - a carry out in a **signed** subtraction
- 

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Signed and unsigned arithmetics (3)

- **signed** and **unsigned** additions (or subtractions)
  - share the same *n-bit* result
  - a borrow (**CF**) in an **unsigned** subtraction is the inverted value of a carry out ( **$C_n$** ) in a **signed** subtraction

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# Signed and unsigned arithmetics (4)

- a borrow ( $CF$ ) in an unsigned subtraction is the inverted value of a carry out ( $C_n$ ) in a signed subtraction

unsigned add/sub		signed addition		signed subtraction
0011 (3)		0011 (+3)		0011 (+3)
-1110 -(14) SUB		+0010 +(2)		-1110 -(-2) SUB
-----		-----		-----
10101 (5) (-16)		00101 (+5)		00101 (+5)
CF=1		Cn=0		Cn=0
1101 (13)		1101 (-3)		1101 (-3)
-0010 -(2) SUB		+1110 +(-2)		-0010 -(+2) SUB
-----		-----		-----
01011 (11) (+11)		11011 (-5)		11011 (-5)
CF=0		Cn=1		Cn=1

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Using the carry flag as a borrow (1)

- a borrow ( $CF = 1$ ) occurs in an **unsigned** subtraction  $A - B$  when  $A < B$  as unsigned numbers
- computer hardware can detect a borrow ( $CF = 1$ ) in **unsigned** subtraction by looking at whether there is no carry out ( $C_n = 0$ ) in the transformed addition  $A + \overline{B} + 1$

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## Using the carry flag as a borrow (2)

- If there is no carry out ( $C_n = 0$ )  
in the transformed addition  $A + \overline{B} + 1$  of  $A - B$   
then there is a borrow ( $CF = 1$ )
- If there is a carry out ( $C_n = 1$ )  
in the transformed addition  $A + \overline{B} + 1$  of  $A - B$   
then there is no borrow ( $CF = 0$ )
- borrows  $CF = !C_n$  for subtraction instructions
- carries  $CF = C_n$  for addition instructions

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## Using the carry flag as a borrow (3)

- the same *addition* and *subtraction* instructions are used for both **unsigned** and **signed** integer arithmetic.
  - no special *addition* and *subtraction* instructions for **unsigned** and **signed** integer arithmetic

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# Using the carry flag as a borrow (4)

- the only difference is
  - which flags you *test* afterwards
    - **CF** for **unsigned** additions and subtractions
    - **OF** for **signed** additions and subtractions
  - how you *interpret* the result
    - Carry **CF** **unsigned** additions
    - Borrow **CF** **unsigned** subtractions
    - Overflow **OF** **signed** additions and subtractions

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# TOC Rules for the carry flag

- The 1st rule for setting the carry flag
- The 2nd rule for setting the carry flag
- Cases for clearing the carry flag
- Computing CF in unsigned additions and subtractions

## 2's complement numbers : 4-bit

	signed	unsigned
0111	(+7)	7
0110	(+6)	6
0101	(+5)	5
0100	(+4)	4
0011	(+3)	3
0010	(+2)	2
0001	(+1)	1
0000	(0)	0
1000	(-8)	8
1001	(-7)	9
1010	(-6)	10
1011	(-5)	11
1100	(-4)	12
1101	(-3)	13
1110	(-2)	14
1111	(-1)	15

# The 1st rule for setting the carry flag

- 1 **CF** = 1 : carry in **unsigned** addition
  - *hand addition rule*
  - the carry flag is set if the addition of two **unsigned** numbers causes a carry out of the most significant bits added.
  - **unsigned** integer overflow in **unsigned** addition

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# The 2nd rule for setting the carry flag

## ② $CF = 1$ : borrow in **unsigned** subtraction

- *hand subtraction rule*
- the carry flag is also set if the subtraction of two **unsigned** numbers requires a borrow into the most significant bits subtracted.
- **unsigned** integer overflow in **unsigned** subtraction

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# Cases for the 1st rule (1)

## ① CF = 1 : carry in **unsigned** addition

- all three interpretations have the same CF=1, the same S=0000

unsigned addition		signed addition	signed subtraction
0111 ( 7)		0111 (+7)	0111 (+7)
+1001 +( 9)		+1001 +(-7)	-0111 -(+7)
-----		-----	-----
10000 ( 0) +(16)		10000 ( 0)	10000 ( 0)
CF=1		Cn=1 -> CF=1	Cn=1 -> CF=1
CF means +16		CF meaningless	CF meaningless
S = 0000		S = 0000	S = 0000
* think hand		* think Cn of the corresponding addition	
addition		CF <- Cn	

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## Cases for the 1st rule (2)

	unsigned	signed
0111	7	+7
1001	9	-7

signed addition	0111 + 1001	(+7) + (-7)
	0111 + (-0111)	
signed subtraction	0111 - 0111	(+7) - (+7)

- **signed subtraction**  $0111 - 0111 = (+7) - (+7)$  is performed by its transformed addition  $0111 + 1001 = (+7) + (-7)$
- carry out  $C_4 = 1$  and  $CF = 1$

# Cases for the 2nd rule (1)

## 2 CF = 1 : borrow in **unsigned** subtraction

- all three interpretations have the same CF=1, the same S=0000

unsigned subtraction		signed addition		signed subtraction
0111 ( 7)		0111 (+7)		0111 (+7)
-1001 -( 9)		+0111 +(7)		-1001 -(-7)
-----		-----		-----
11110 (14) -(16)		01110 (14)		01110 (14)
CF=1		Cn=0 -> CF=1		Cn=0 -> CF=1
CF means -16		CF meaningless		CF meaningless
S = 1110		S = 1110		S = 1110
* think hand		* think Cn of the corresponding addition		
subtraction		CF <- Cn		

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## Cases for the 2nd rule (2)

	unsigned	signed
0111	7	+7
1001	9	-7

signed subtraction	0111 - 1001	(+7) - (-7)
	0111 - (-0111)	
signed addition	0111 + 0111	(+7) + (+7)

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Computing CF in unsigned additions and subtractions

- Computing CF in an **unsigned addition**
  - do the **signed** 2's complement addition
  - $C_n$  is the carry out
  - $CF \leftarrow C_n$  (CF means carry)
- Computing CF in an **unsigned subtraction**
  - do the transformed **signed** 2's complement addition
  - $C_n$  is the carry out
  - $CF \leftarrow !C_n$  (CF means borrow)

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# TOC: Method for computing the carry flag

- 2's complement numbers : 4-bit
- Addend and augend in a  $n$ -bit addition
- Full adder (hardware) operation
- Internal and external carry bits
- 2's complement signed addition and subtraction
- Carry flag computation

# Addend and augend in a $n$ -bit addition

$n$	bits	addended	$A$	$\{ a_{n-1}, a_{n-2}, \dots, a_1, a_0 \}$
$n$	bits	augend	$B$	$\{ b_{n-1}, b_{n-2}, \dots, b_1, b_0 \}$
$(n+1)$	bits	carry bits	$C$	$\{ c_n, c_{n-1}, c_{n-2}, \dots, c_1, c_0 \}$
$n$	bits	sum bits	$S$	$\{ s_{n-1}, s_{n-2}, \dots, s_1, s_0 \}$

external carry bits :  $c_n$  carry out,  $c_0$  carry in

$$\begin{array}{cccccc} a_{n-1} & a_{n-2} & \dots & a_1 & a_0 & \\ b_{n-1} & b_{n-2} & \dots & b_1 & b_0 & \\ \hline c_n & s_{n-1} & s_{n-2} & \dots & s_1 & s_0 \end{array}$$

# Full adder (hardware) operation

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full adder operation in the  $i^{\text{th}}$  bit position

$$\{c_{i+1}, s_i\} = a_i + b_i + c_i$$

---

$$\begin{array}{r} a_i \\ b_i \\ c_i \\ \hline c_{i+1} \quad s_i \end{array}$$

# Internal and external carry bits

external carries      $c_n$  output,  $c_0$  input  
 internal carries      $\{c_{n-1}, c_{n-2}, \dots, c_2, c_1\}$      output / input  
 sum bits                  $\{s_{n-1}, s_{n-2}, \dots, s_1, s_0\}$      output

	$a_{n-1}$	$a_{n-2}$	$\dots$	$a_1$	$a_0$
	$b_{n-1}$	$b_{n-2}$	$\dots$	$b_1$	$b_0$
$c_n$	$c_{n-1}$	$c_{n-2}$	$\dots$	$c_1$	$c_0$
	$s_{n-1}$	$s_{n-2}$	$\dots$	$s_1$	$s_0$

	$a_{n-1}$	$a_{n-2}$	$\dots$	$a_1$	$a_0$
	$b_{n-1}$	$b_{n-2}$	$\dots$	$b_1$	$b_0$
	$s_{n-1}$	$s_{n-2}$	$\dots$	$s_1$	$s_0$
$c_n$					$c_0$

# 2's complement signed addition and subtraction

- 2's complement signed addition

$$\{c_n, S\} = A + B = A + B + 0$$

	$a_{n-1}$	$a_{n-2}$	⋯	$a_1$	$a_0$
	$b_{n-1}$	$b_{n-2}$	⋯	$b_1$	$b_0$
	$c_{n-1}$	$c_{n-2}$	⋯	$c_1$	$0$
$c_n$	$s_{n-1}$	$s_{n-2}$	⋯	$s_1$	$s_0$

- 2's complement signed subtraction - via a transformed addition

$$\{c_n, S\} = A - B = A + \overline{B} + 1$$

	$a_{n-1}$	$a_{n-2}$	⋯	$a_1$	$a_0$
	$b_{n-1}$	$b_{n-2}$	⋯	$b_1$	$b_0$
	$c_{n-1}$	$c_{n-2}$	⋯	$c_1$	$1$
$c_n$	$s_{n-1}$	$s_{n-2}$	⋯	$s_1$	$s_0$

# Carry flag computation (1)

ADD (addition)	SUB (subtraction)
$CF = c_n$	$CF = \overline{c_n}$
normal carry of a 2's complement addition	<b>inverted</b> carry of a transformed addition
$A + B = A + B + 0$	$A - B = A + \overline{B} + 1$
$\{c_n, s_{n-1}\}$ $= a_{n-1} + b_{n-1} + c_{n-1}$	$\{c_n, s_{n-1}\}$ $= a_{n-1} + \overline{b_{n-1}} + c_{n-1}$

[https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14\\_1](https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14_1)



## Carry flag computation (2)

- In **unsigned** arithmetic,
  - the **carry flag** is used to detect *overflow*
  - the **carry flag** is used to extend *n-bit* result into *(n+1)-bit* result
  - for **addition**, the **carry flag** is a **carry out**
  - for **subtraction**, the **carry flag** is a **borrow in**
- In **signed** arithmetic,
  - the **carry flag** is useless
  - the **carry flag** neither detects overflow nor extends n-bit result

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Carry flag computation (3)

- In **unsigned** arithmetic,

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Addition	<b>CF = 1</b> means <b>carry out</b>	when <b>Cn = 1</b>
Subtraction	<b>CF = 1</b> means <b>borrow in</b>	when <b>Cn = 0</b>

---

- **CF** - Carry Flag in x86
- **Cn** - the normal carry out
  - the carry out of a 2's complement addition for **ADD**
  - the carry out of a *transformed* addition for **SUB**
- In **signed** arithmetic,
  - the **carry** flag is useless

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# TOC: More examples of the carry flag

- Summary I
- Summary II
- Cases for setting the carry flag
- Cases for clearing the carry flag

# Summary I

unsigned add/sub			signed addition			signed subtraction			CF	OF
1101	(13)		1101	(-3)		1101	(-3)			
+1110	+(14)	ADD	+1110	+(-2)	ADD	-0010	-(-2)			
-----	-----		-----	-----		-----	-----			
11011	(11)	(+16)	11011	(-5)		11011	(-5)		1	0
0011	( 3)		0011	(+3)		0011	(+3)			
-1110	-(-14)	SUB	+0010	+(+2)		-1110	-(-2)	SUB		
-----	-----		-----	-----		-----	-----			
10101	( 5)	(-16)	00101	(+5)		00101	(+5)		1	0
0011	( 3)		0011	(+3)		0011	(+3)			
+0010	+( 2)	ADD	+0010	+(+2)	ADD	-1110	-(-2)			
-----	-----		-----	-----		-----	-----			
00101	( 5)	(+ 0)	00101	(+5)		00101	(+5)		0	0
1101	(13)		1101	(-3)		1101	(-3)			
-0010	-(- 2)	SUB	+1110	+(-2)		-0010	-(-2)	SUB		
-----	-----		-----	-----		-----	-----			
01011	(11)	(+11)	11011	(-5)		11011	(-5)		0	0

# Summary II

unsigned add/sub			signed addition			signed subtraction			CF	OF
1011	(11)		1011	(-5)		1011	(-5)			
+1100	+(12)	ADD	+1100	+(-4)	ADD	-0100	-(+4)			
-----	-----		-----	-----		-----	-----			
10111	( 7) (+16)		10111	(+7)		10111	(+7)		1	1
0101	( 5)		0101	(+5)		0101	(+5)			
-1100	-(12)	SUB	+0100	+(+4)		-1100	-(-4)	SUB		
-----	-----		-----	-----		-----	-----			
11001	( 9) (-16)		01001	(-7)		01001	(-7)		1	1
0101	( 5)		0101	(+5)		0101	(+5)			
+0100	+( 4)	ADD	+0100	+(+4)	ADD	-1100	-(-4)			
-----	-----		-----	-----		-----	-----			
01001	( 9) (+ 0)		01001	(-7)		01001	(-7)		0	1
1011	(11)		1011	(-5)		1011	(-5)			
-0100	-( 4)	SUB	+1100	+(-4)		-0100	-(+4)	SUB		
-----	-----		-----	-----		-----	-----			
00111	( 7) ( 0)		10111	(+7)		10111	(+7)		0	1

# Cases for setting the carry flag (1) CF=1, OF=0

- unsigned integer overflow (CF=1 means +16)

* unsigned addition		* signed addition		signed subtraction
1101 (13)		1101 (-3)		1101 (-3)
+1110 +(14) ADD		+1110 +(-2) ADD		-0010 -(+2)
-----		-----		-----
11011 (11) (+16)		11011 (-5)		11011 (-5)
CF=1		Cn=1 -> CF=1		Cn=1 -> CF=1
CF means 16		CF meaningless		CF meaningless
S = 0000		S = 0000		S = 0000
* think hand		* think Cn of the corresponding addition		
addition		CF <- Cn (for unsigned addition)		

\* CF=1, S=1011, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for setting the carry flag (2) CF=1, OF=0

- unsigned integer overflow (CF=1 means -16)

* unsigned subtraction		signed addition		* signed subtraction
0011 (3)		0011 (+3)		0011 (+3)
-1110 -(14) SUB		+0010 +(2)		-1110 -(-2) SUB
-----		-----		-----
10101 (5) (-16)		00101 (+5)		00101 (+5)
CF=1		Cn=0 -> CF=1		Cn=0 -> CF=1
CF means -16		CF meaningless		CF meaningless
S = 0101		S = 0101		S = 0101
-----		-----		-----
* think hand subtraction		* think Cn of the transformed addition		
		CF <- !Cn (for unsigned subtraction)		
-----		-----		-----

\* CF=1, S=0101, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for setting the carry flag (3) CF=1, OF=1

- unsigned integer overflow (CF=1 means +16)

* unsigned addition		* signed addition		signed subtraction
1011 (11)		1011 (-5)		1011 (-5)
+1100 +(12) ADD		+1100 +(-4) ADD		-0100 -(+4)
-----		-----		-----
10111 ( 7) (+16)		10111 (+7)		10111 (+7)
CF=1		Cn=1 -> CF=1		Cn=1 -> CF=1
CF means +16		CF meaningless		CF meaningless
S = 0111		S = 0111		S = 0111
* think hand		* think Cn of the corresponding addition		
addition		CF <- Cn (for unsigned addition)		

\* CF=1, S=0111, OF=1 for all three interpretations



# Cases for setting the carry flag (4) CF=1, OF=1

- unsigned integer overflow (CF=1 means -16)

* unsigned subtraction		signed addition		* signed subtraction
0101 ( 5)		0101 (+5)		0101 (+5)
-1100 -(12) SUB		+0100 +(4)		-1100 -(-4) SUB
-----		-----		-----
11001 ( 9) (-16)		01001 (-7)		01001 (-7)
CF=1		Cn=0 -> CF=1		Cn=0 -> CF=1
CF means -16		CF meaningless		CF meaningless
S = 1001		S = 1001		S = 1001
-----		-----		-----
* think hand subtraction		* think Cn of the transformed addition		CF <- !Cn (for unsigned subtraction)
-----		-----		-----

\* CF=1, S=1001, OF=1 for all three interpretations

# Cases for clearing the carry flag (1) CF=0, OF=0

- no unsigned integer overflow (CF=0)

* unsigned addition		* signed addition	signed subtraction
0011 ( 3)		0011 (+3)	0011 (+3)
+0010 +( 2) ADD		+0010 +(2) ADD	-1110 -(-2)
-----		-----	-----
00101 ( 5) (+ 0)		00101 (+5)	00101 (+5)
CF=0		Cn=0 -> CF=0	Cn=0 -> CF=0
CF means 0		CF meaningless	CF meaningless
S = 0101		S = 0101	S = 0101
* think hand		* think Cn of the corresponding addition	
addition		CF <- Cn (for unsigned addition)	

\* CF=0, S=0101, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the carry flag (2) CF=0, OF=0

- no unsigned integer overflow (CF=0)

* unsigned addition	* signed addition	signed subtraction
1101 (13)	1101 (-3)	1101 (-3)
-0010 -( 2) SUB	+1110 +(-2)	-0010 -(+2) SUB
-----	-----	-----
11011 (11) (-16)	11011 (-5)	11011 (-5)
CF=0	Cn=0 -> CF=0	Cn=0 -> CF=0
CF means 0	CF meaningless	CF meaningless
S = 1011	S = 1011	S = 1011
-----	-----	-----
* think hand subtraction	* think Cn of the corresponding addition	
	CF <- Cn (for unsigned addition)	

\* CF=0, S=1011, OF=0 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)

# Cases for clearing the carry flag (3) CF=0, OF=1

- no unsigned integer overflow (CF=0)

* unsigned addition		* signed addition	signed subtraction
0101 ( 5)		0101 (+5)	0101 (+5)
+0100 +( 4) ADD		+0100 +( +4) ADD	-1100 -(-4)
-----		-----	-----
01001 ( 9) (+ 0)		01001 (-7)	01001 (-7)
CF=0		Cn=0 -> CF=0	Cn=0 -> CF=0
CF means +0		CF meaningless	CF meaningless
S = 1001		S = 1001	S = 1001
* think hand		* think Cn of the corresponding addition	
addition		CF <- Cn (for unsigned addition)	

\* CF=0, S=1001, OF=1 for all three interpretations

# Cases for clearing the carry flag (4) CF=0, OF=1

- no unsigned integer overflow (CF=0)

* unsigned subtraction		signed addition		* signed subtraction
1011 (11)		1011 (-5)		1011 (-5)
-0100 -( 4) SUB		+1100 +(-4)		-0100 -(+4) SUB
-----		-----		-----
00111 ( 7) ( 0)		10111 (+7)		10111 (+7)
CF=0		Cn=1 -> CF=0		Cn=1 -> CF=0
CF means 0		CF meaningless		CF meaningless
S = 0111		S = 0111		S = 0111
-----		-----		-----
* think hand subtraction		* think Cn of the transformed addition		
		CF <- !Cn (for unsigned subtraction)		
-----		-----		-----

\* CF=0, S=0111, OF=1 for all three interpretations

[http://teaching.idallen.com/dat2343/10f/notes/040\\_overflow.txt](http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt)