

Tapped Delay

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Based on

Introduction to Signal Processing

S. J. Ofranidis

The necessities in DSP C Programming

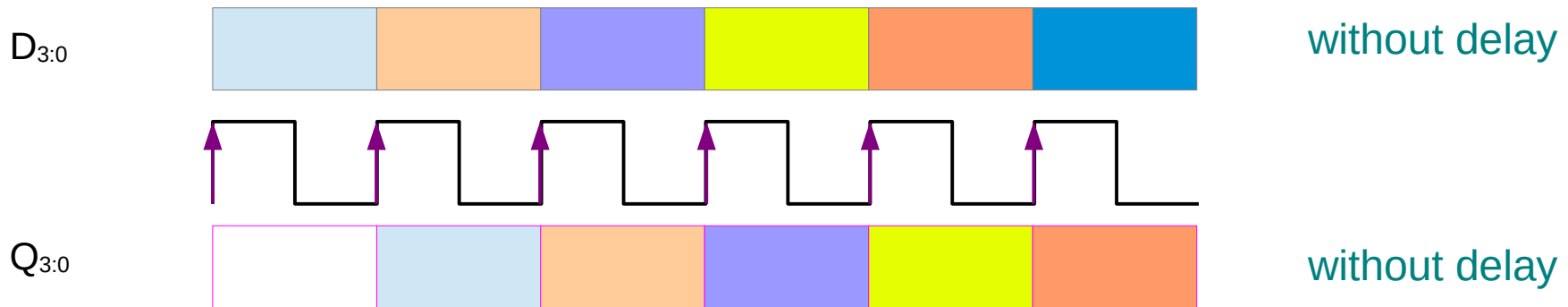
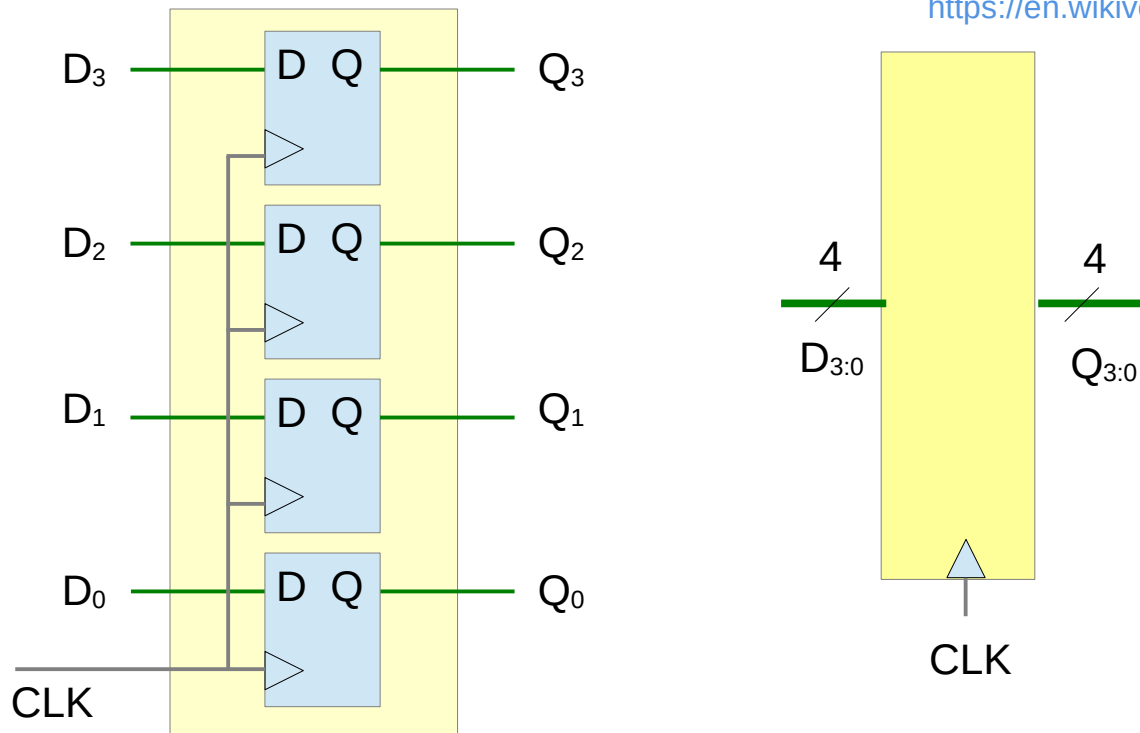
FIR Filter (A.pdf) 20191114

D Flip Flop

Considering the widely used
Edge triggered
D-type Flip Flops

Register

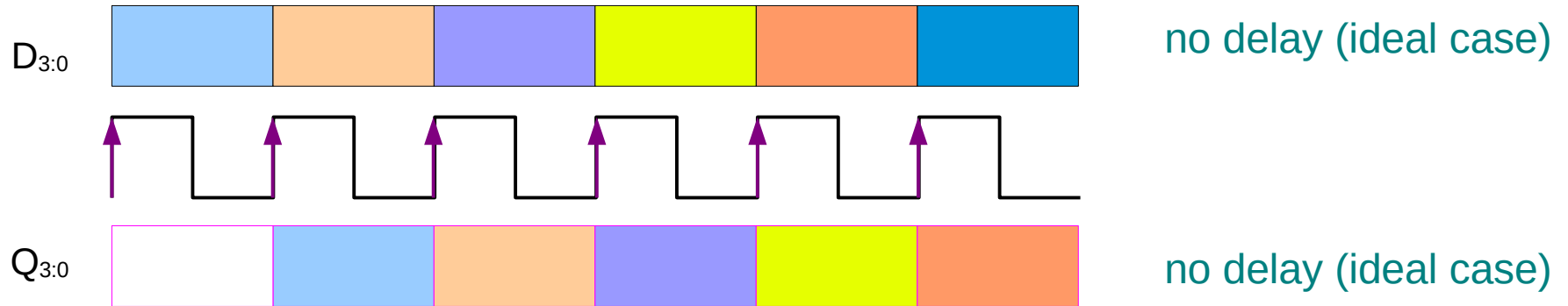
https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



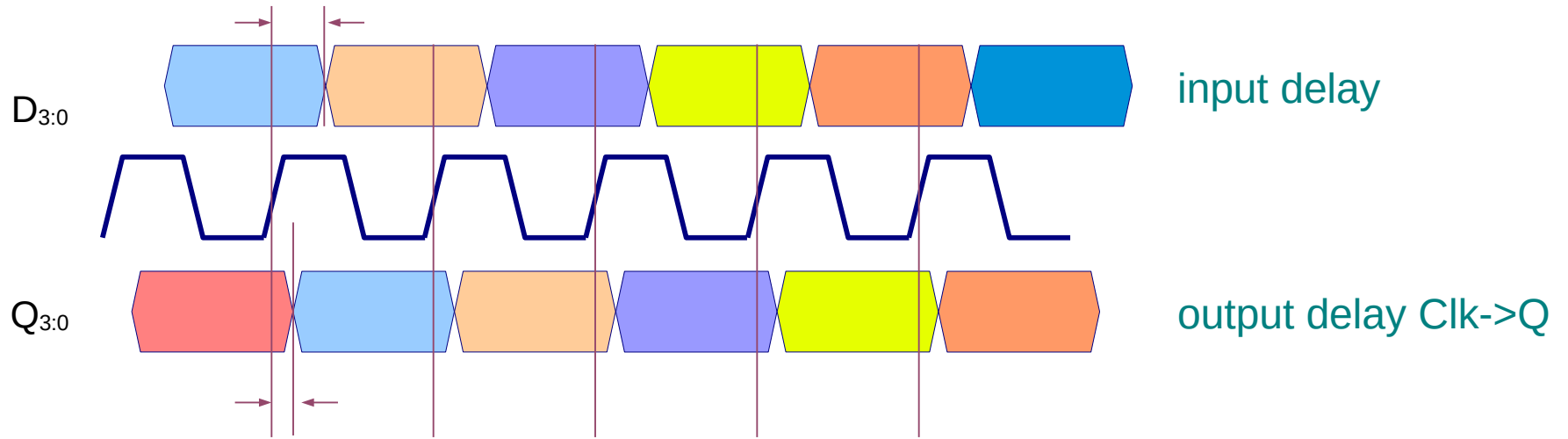
Types of Timing Diagrams

a timing diagram without delays

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

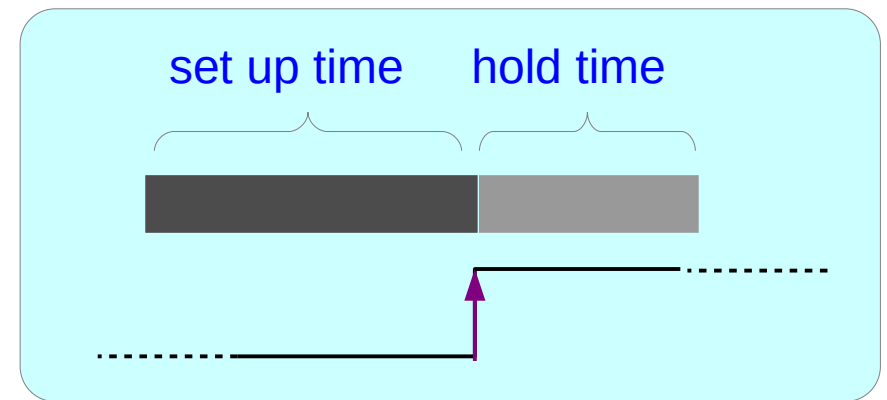
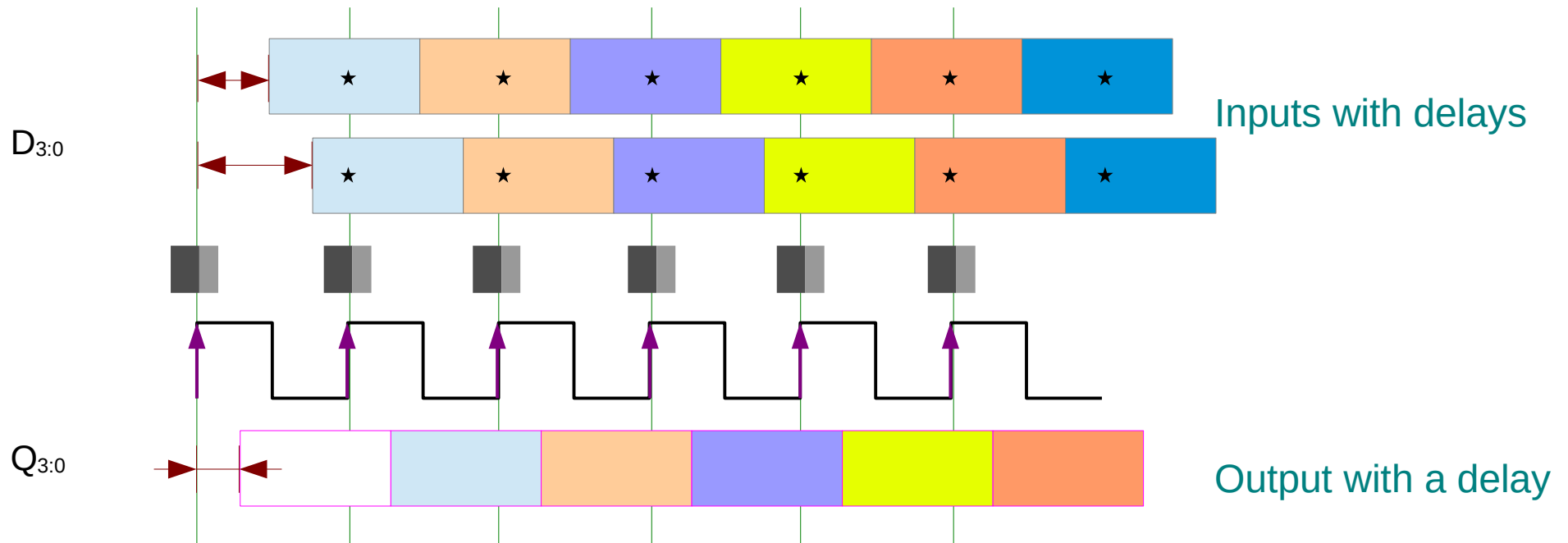


a timing diagram with delays



Setup & Hold Time (1)

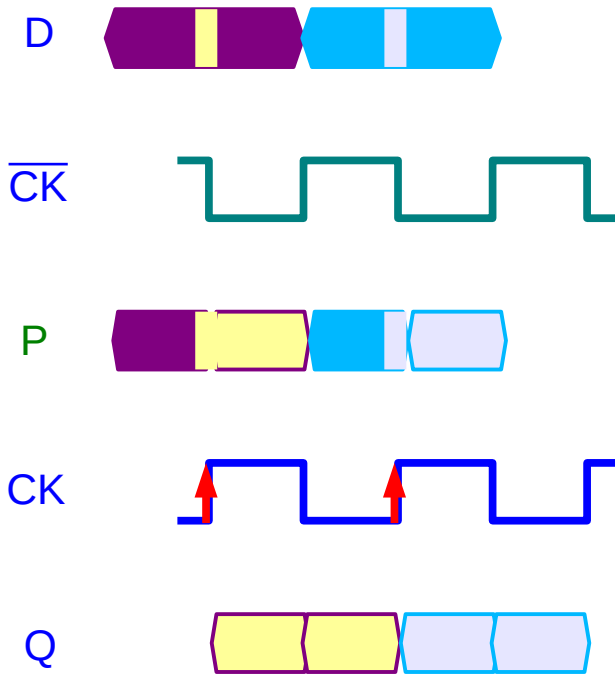
https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



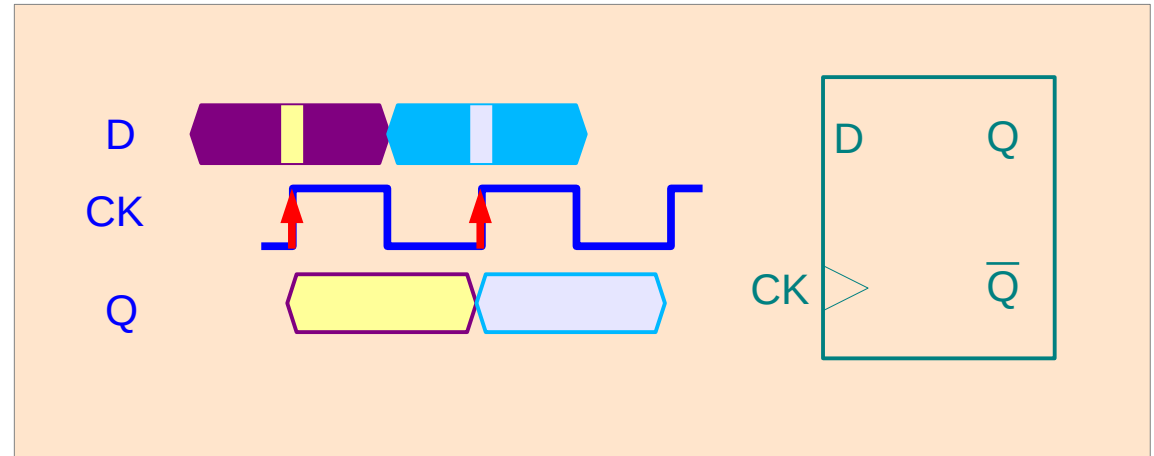
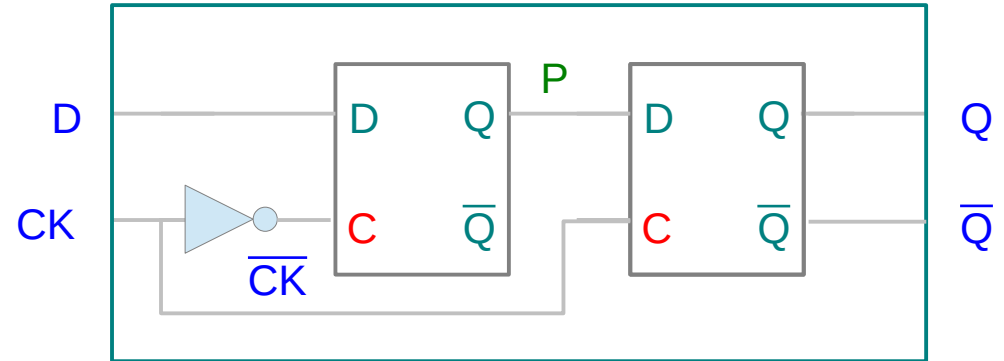
Master-Slave D FlipFlop – Rising Edge

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

Master D Latch



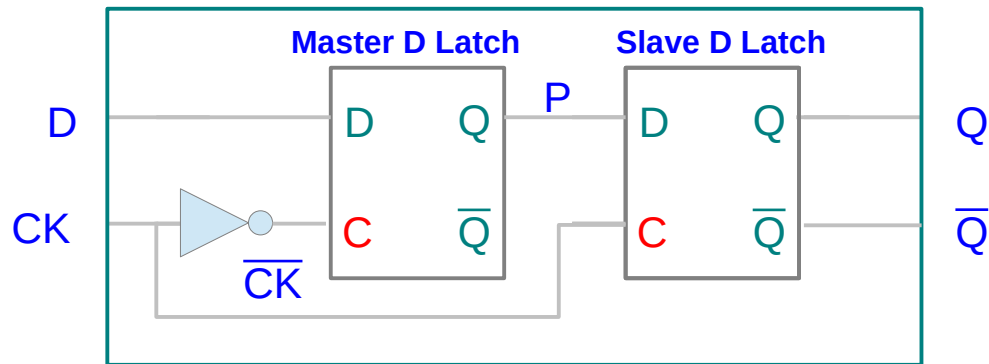
most of D data in a time slot (dark shaded area) is not used but only the data around the rising edge of the clock CK



Slave D Latch

Master-Slave D FlipFlop – Rising Edge

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design

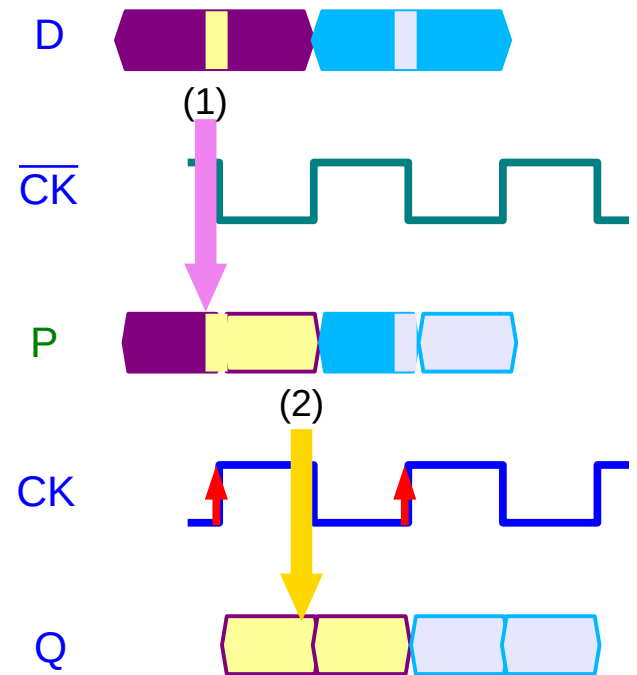


(1) the current input D gets stored in the master latch

(2) the current content P gets stored in the slave latch

Using **inverted clocks enable** (1) and (2) to be executed sequentially

Master D Latch

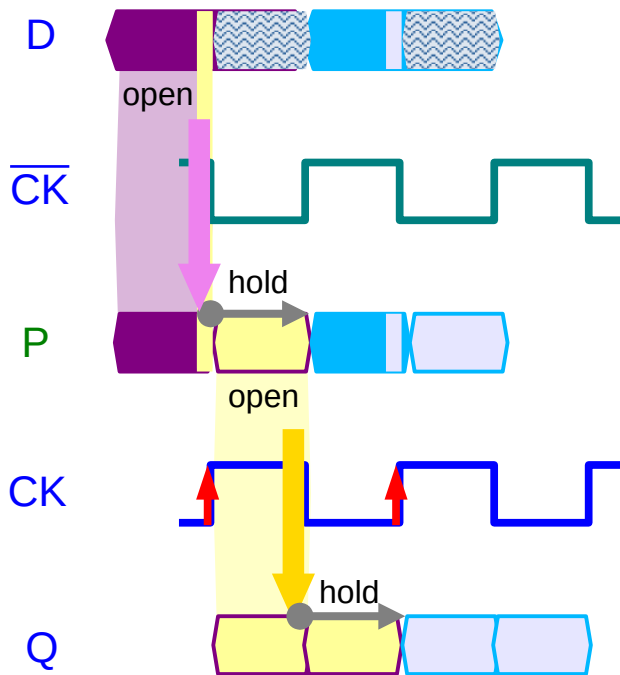


Slave D Latch


Master-Slave D FlipFlop – open and hold

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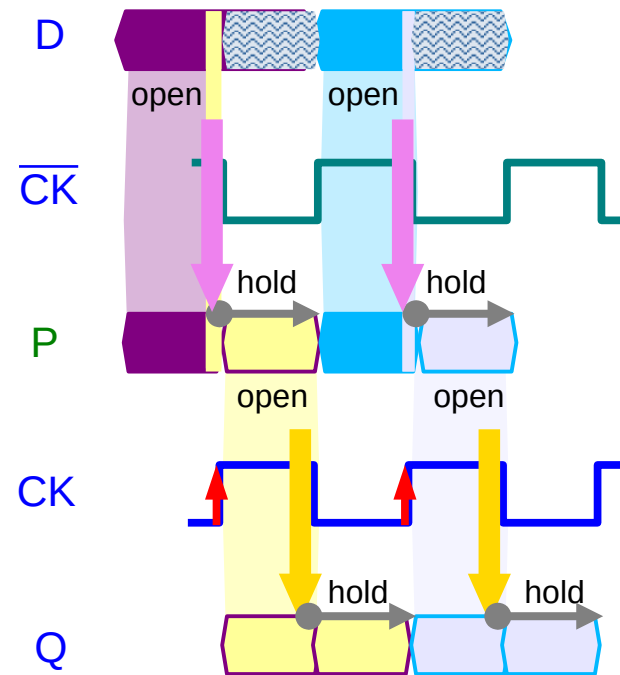
Master D Latch




Slave D Latch

 shaded area has no effect in operation

Master D Latch

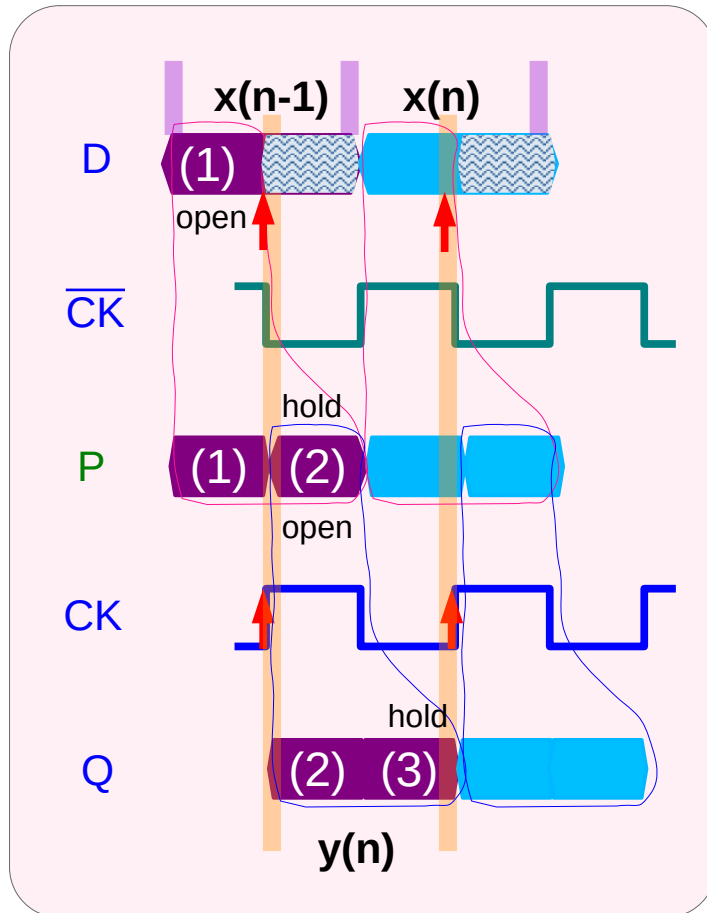


Slave D Latch

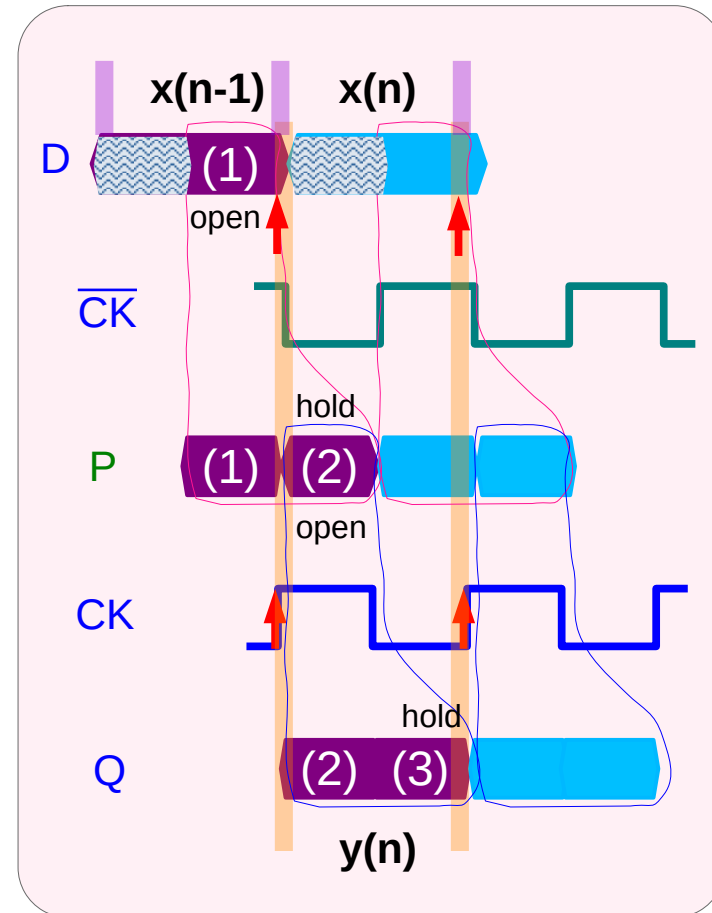
 shaded area has no effect in operation

Master-Slave D FlipFlop – typical vs. ideal timing (1)

Typical Timing

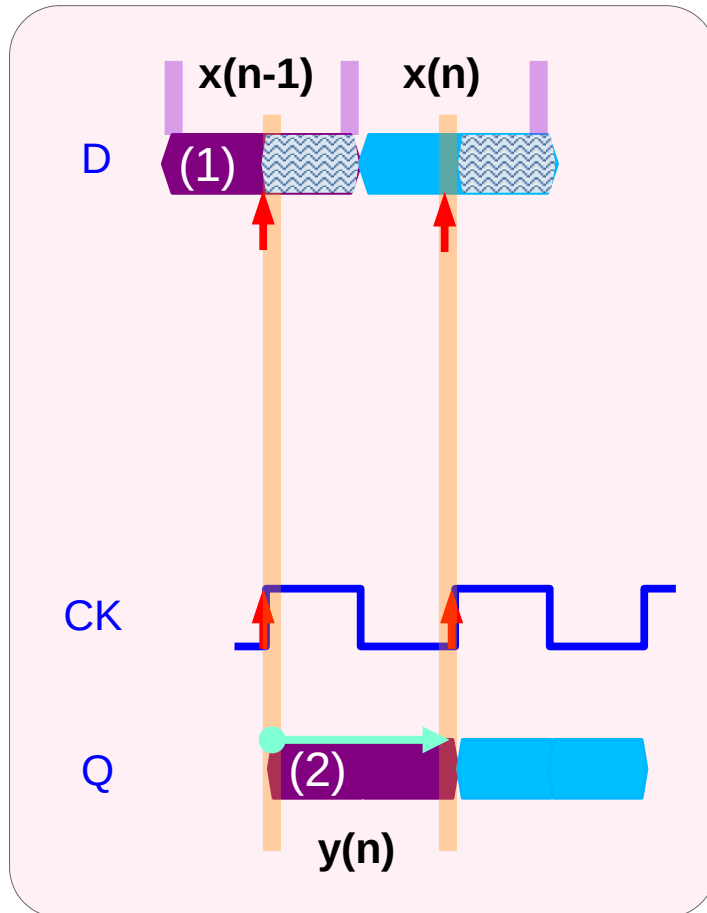


Ideal Timing

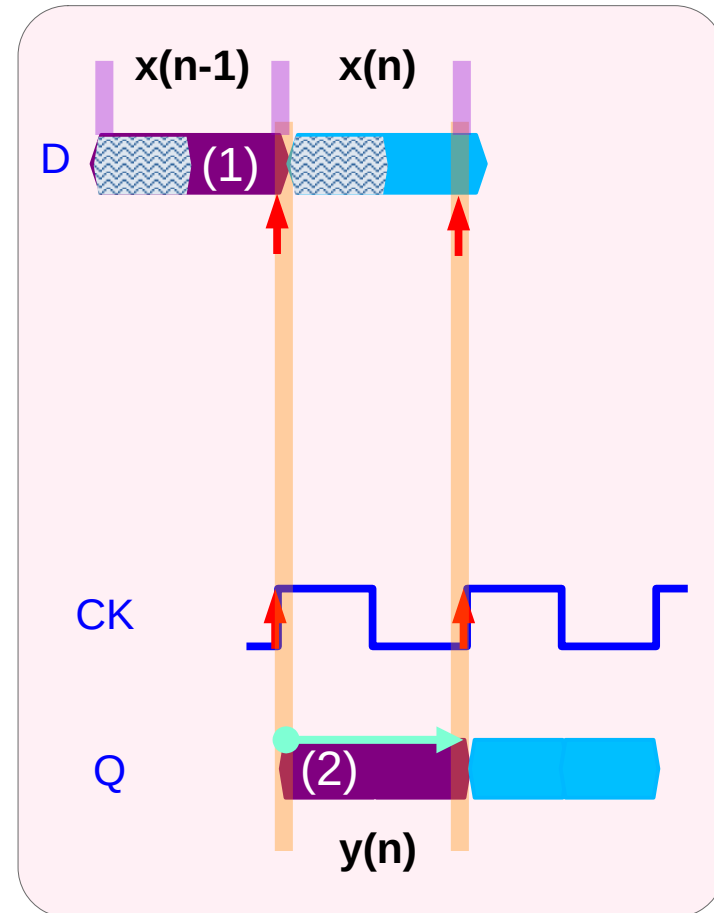


Master-Slave D FlipFlop – typical vs. ideal timing (2)

Typical Timing

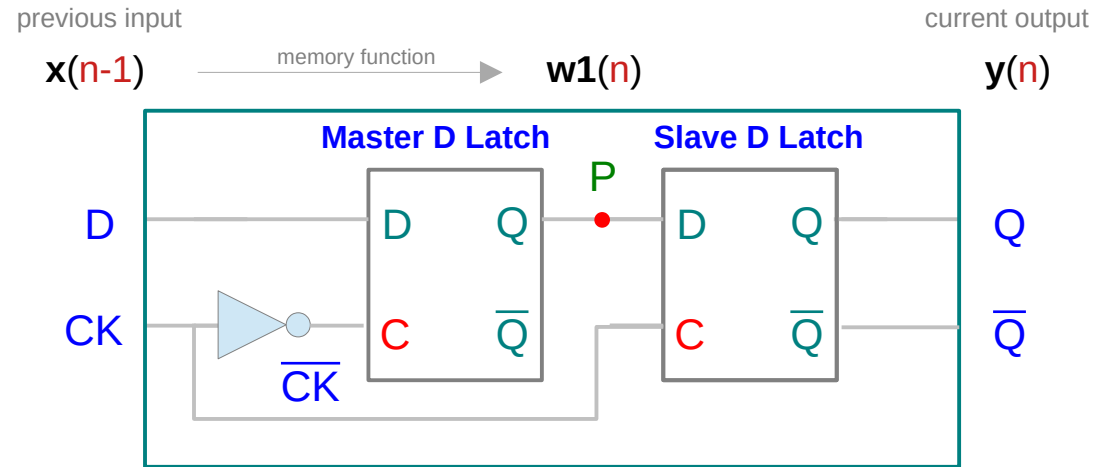
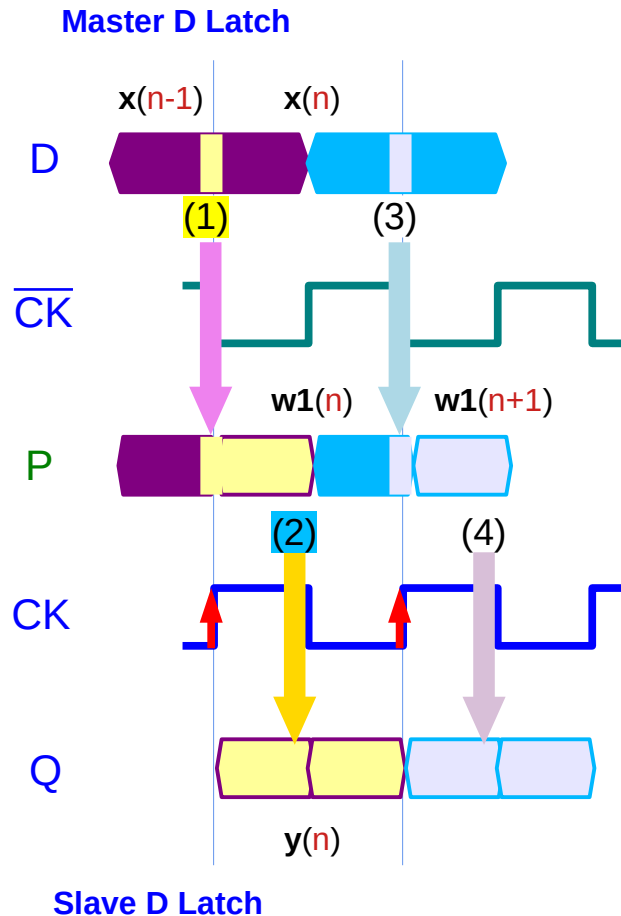


Ideal Timing



Master-Slave D FlipFlop – Rising Edge Sampling

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



(1) the input $x(n-1)$ gets stored in the master latch

(2) the content of $w1(n)$ gets stored in the slave latch

$$w1(n) = x(n-1)$$

$$y(n) = w1(n)$$

WR $w1(n)$

RD $w1(n)$

first, write

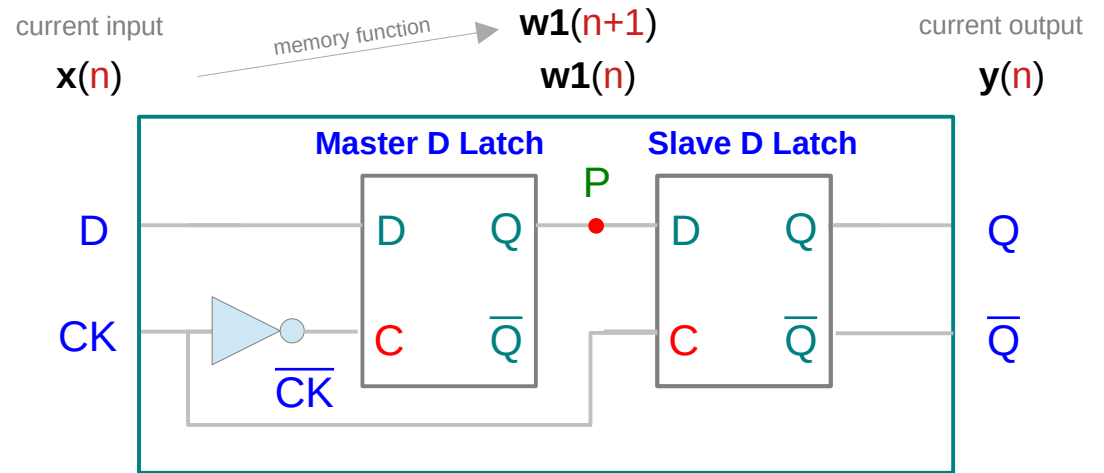
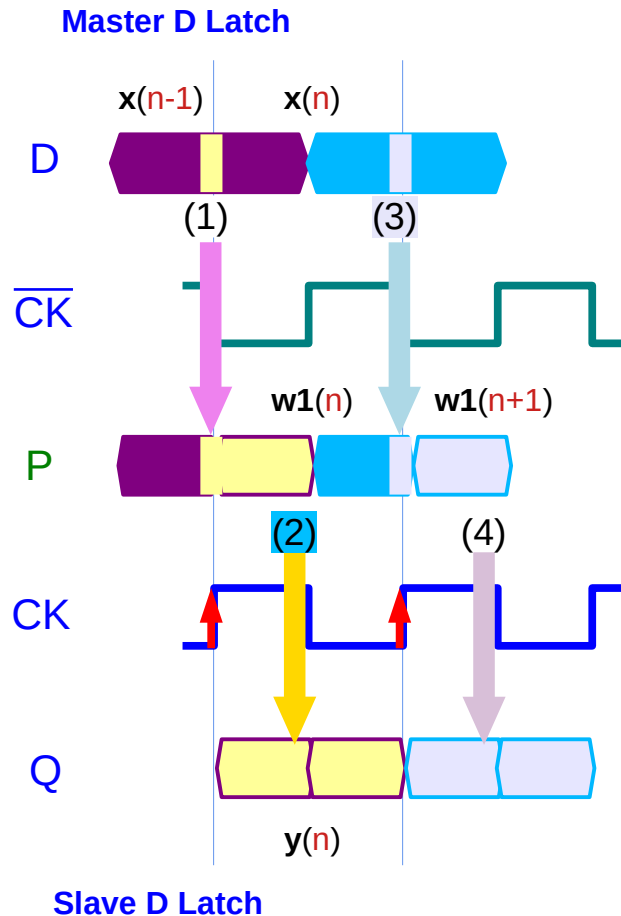
then, read

at the end of the previous time slot $n-1$, $x(n-1)$ gets stored in $w1(n)$ for the current time slot n

during the current time slot n , $y(n)$ outputs $w1(n)$ which holds the value of $x(n-1)$ at the end of the previous time slot $n-1$

Master-Slave D FlipFlop – Rising Edge Sampling

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



(2) the content of $w1(n)$ gets stored in the slave latch

$$y(n) = w1(n)$$

RD $w1(n)$

first, read

during the current time slot n , $y(n)$ outputs $w1(n)$ which holds the value of $x(n-1)$ at the end of the previous time slot $n-1$

(3) the input $x(n)$ gets stored in the master latch

$$w1(n+1) = x(n)$$

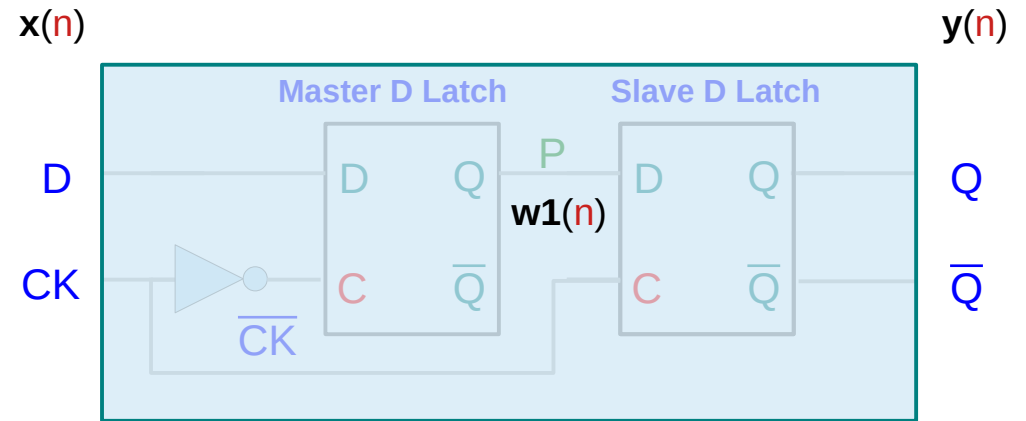
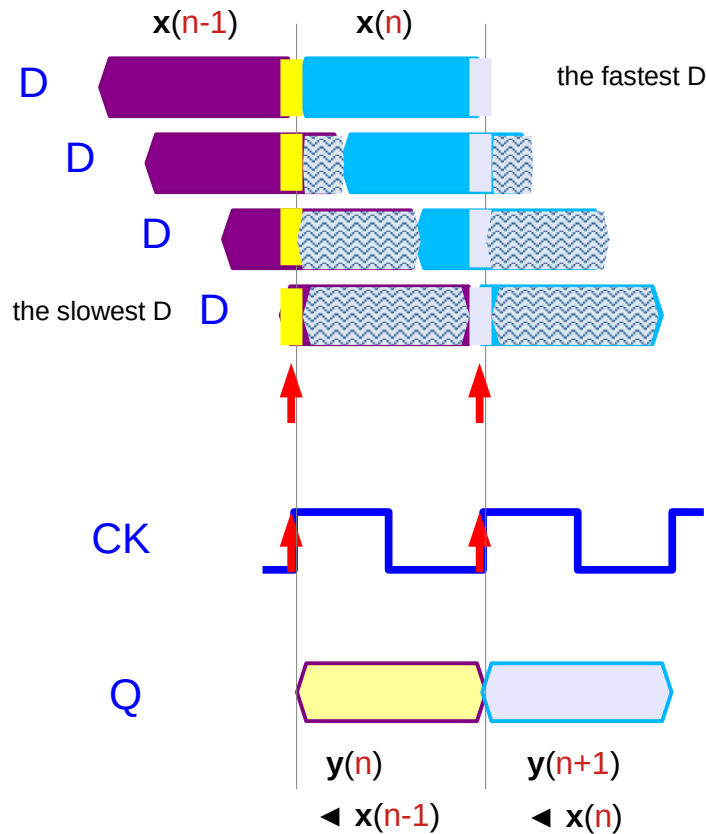
WR $w1(n+1)$

then, write

at the end of the current time slot n , $x(n)$ gets stored in $w1(n+1)$ for the next time slot $n+1$

Master-Slave D FlipFlop – Rising Edge Sampling

https://en.wikiversity.org/wiki/The_necessities_in_Digital_Design



$$y(n) = w1(n)$$

RD $w1(n)$

first, read

during the current time slot n ,
 $y(n)$ outputs $w1(n)$ which holds
the value of $x(n-1)$ at the end
of the previous time slot $n-1$

$$w1(n+1) = x(n)$$

WR $w1(n+1)$

then, write

at the end of
the current time slot n ,
 $x(n)$ gets stored in $w1(n+1)$
for the next time slot $n+1$

Fixed point representation

fractional numbers

Floating Point Representation

Fixed Point Representation

integer + implicit fixed scaling factor

a 2's complement number $(11110101)_2 = -11$

could represent $-11 \cdot 2^{-3} = -88$,
 $-11 \cdot 2^{-5} = -0.34375$
 $-11 \cdot 2^{-12} = -0.002685546875$

with implied scaling factors -3, -5, -12

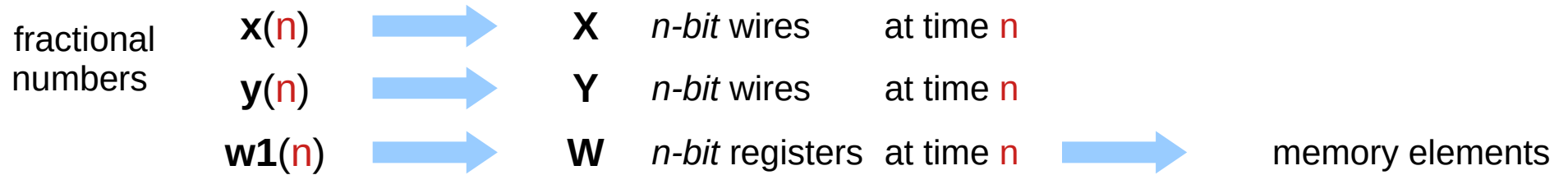
fractional numbers

$x(n)$  **X** *n-bit* wires at time *n*

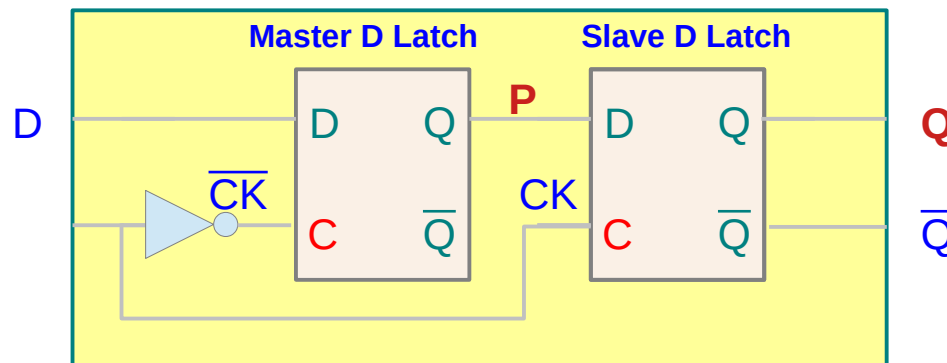
$y(n)$  **Y** *n-bit* wires at time *n*

$w1(n)$  **W** *n-bit* registers at time *n*

Memory Elements

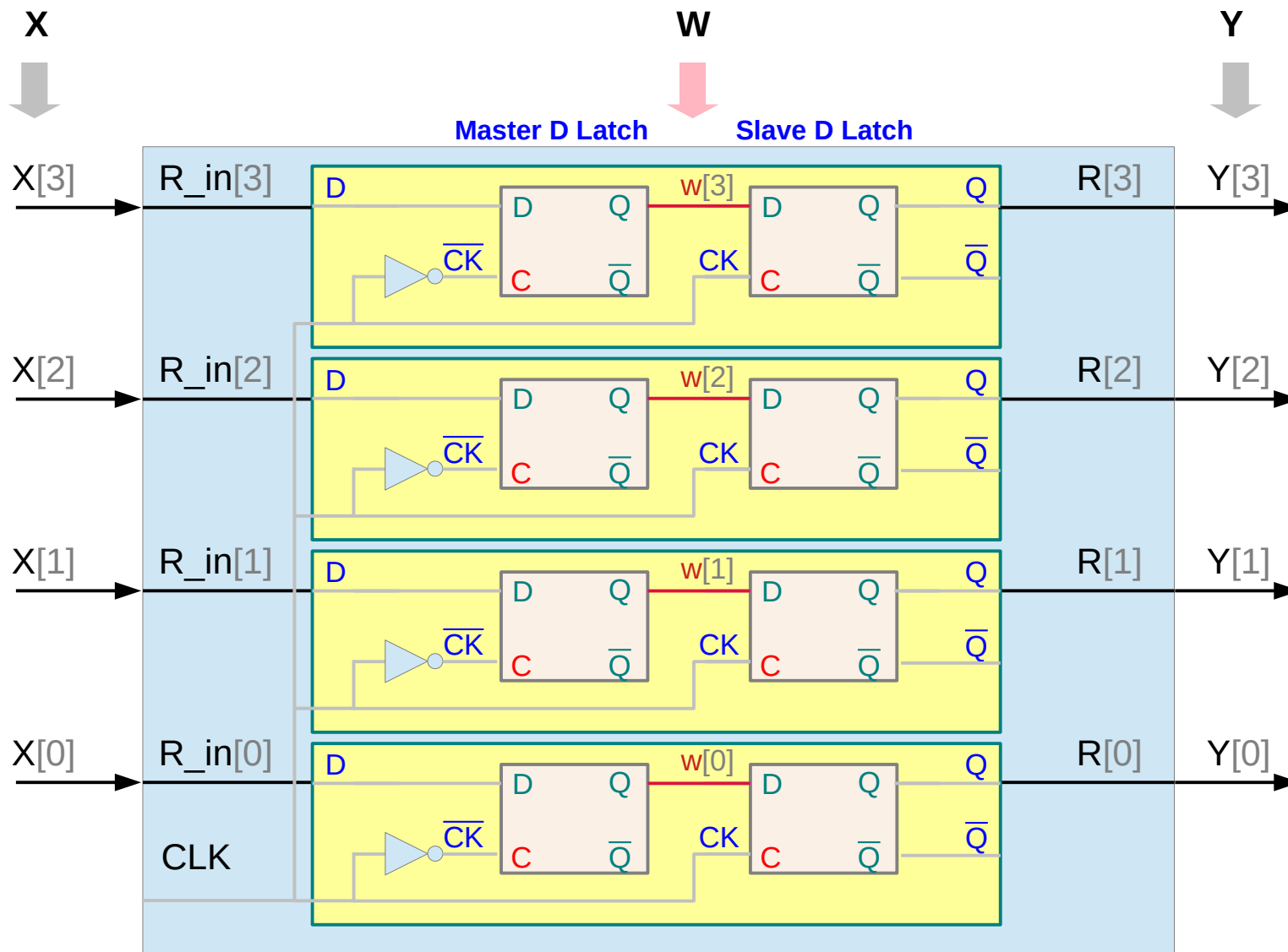


1. using the **P** outputs of **master latches** as the name for **memory elements**
2. using the **Q** outputs of **flipflops** as the name for **memory elements**

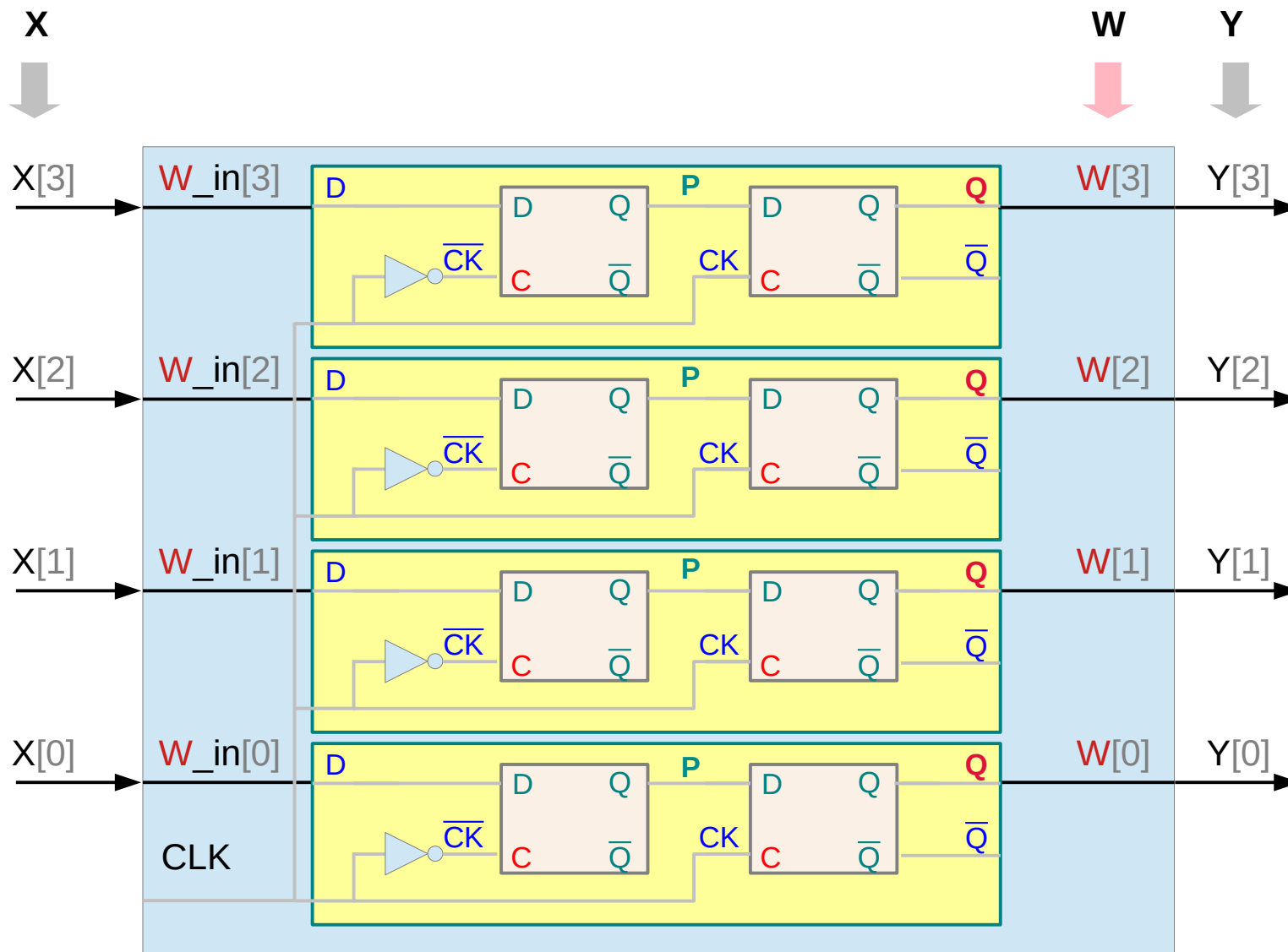


D FlipFlop using Master and Slave D Latches

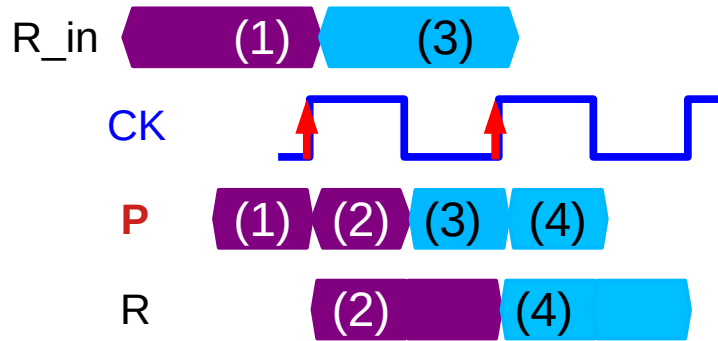
(1) Using master latch outputs P



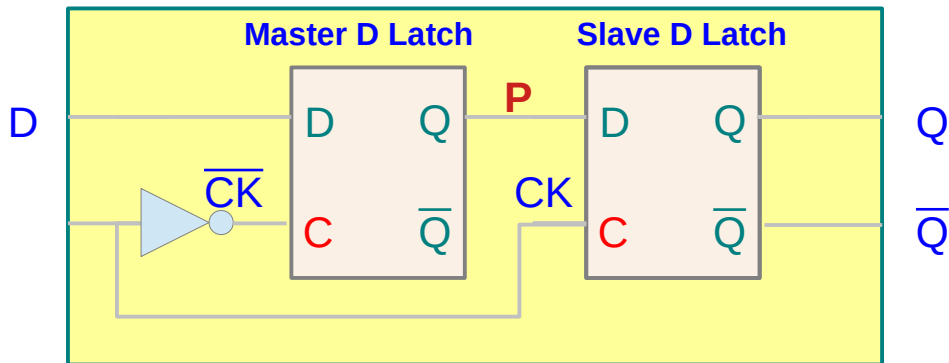
(2) Using flipflop outputs Q



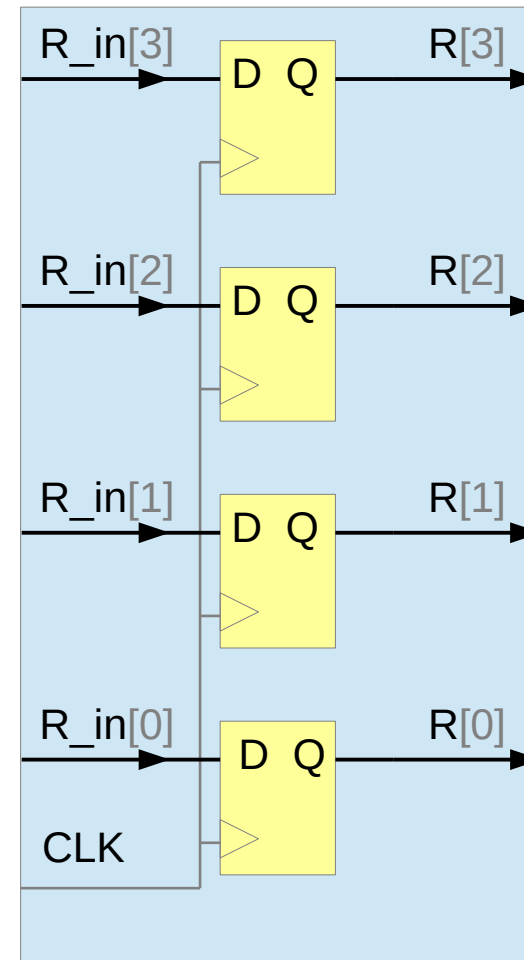
(1) 4-bit Register Using master latch outputs **P**



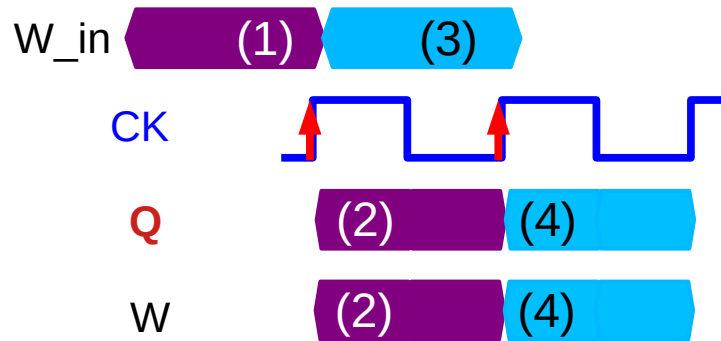
D FlipFlop using Master and Slave D Latches



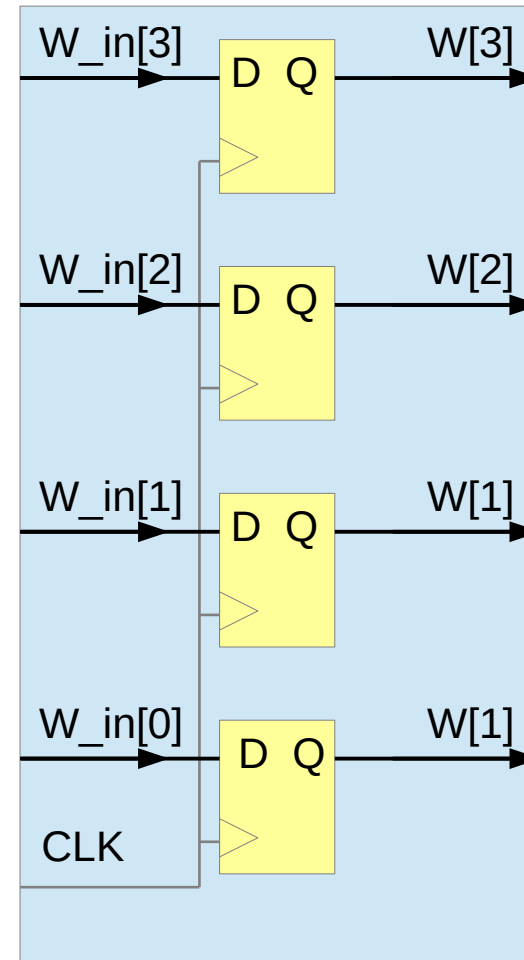
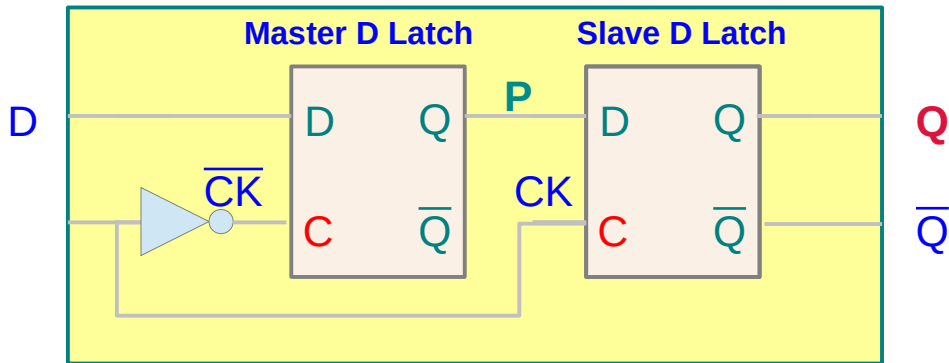
4-bit Register using 4 D flipflops



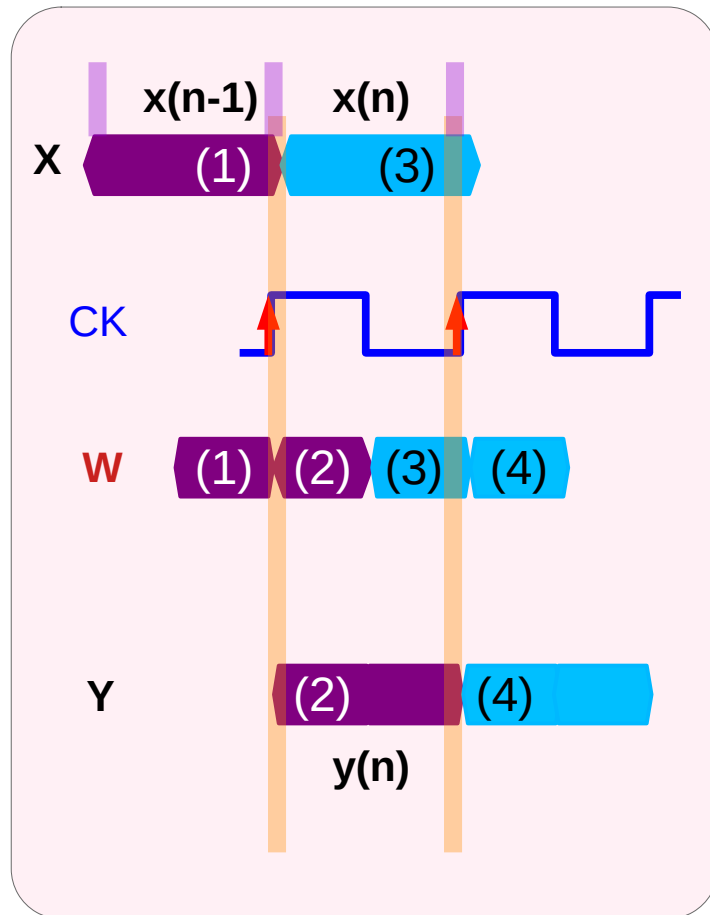
(2) 4-bit Register Using flipflop outputs Q



D FlipFlop using Master and Slave D Latches



(1) Timing diagrams with master latch outputs **P**



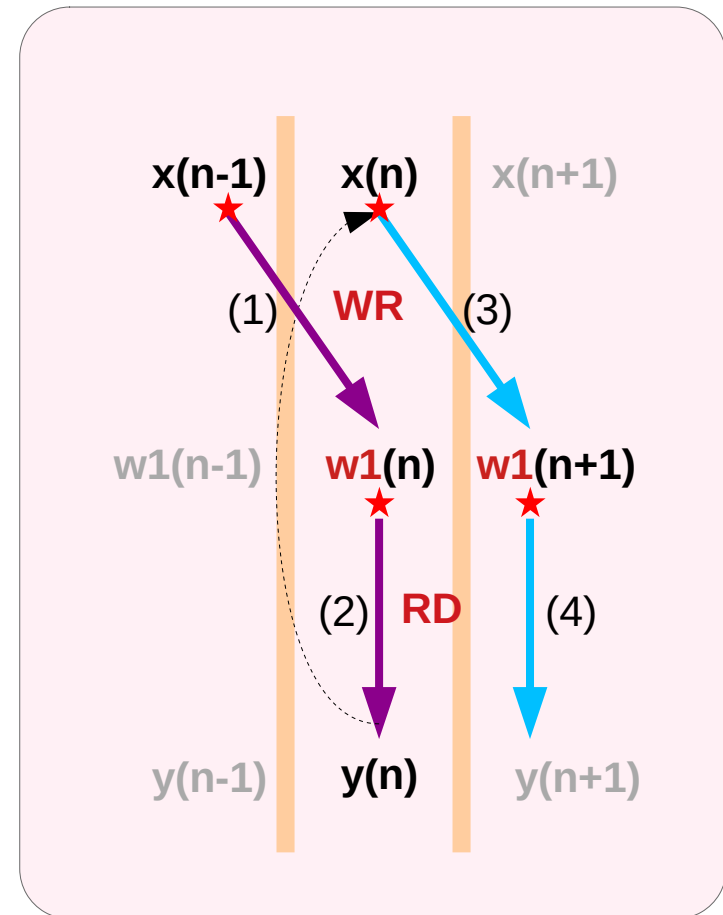
Hardware model

data centered view

Input

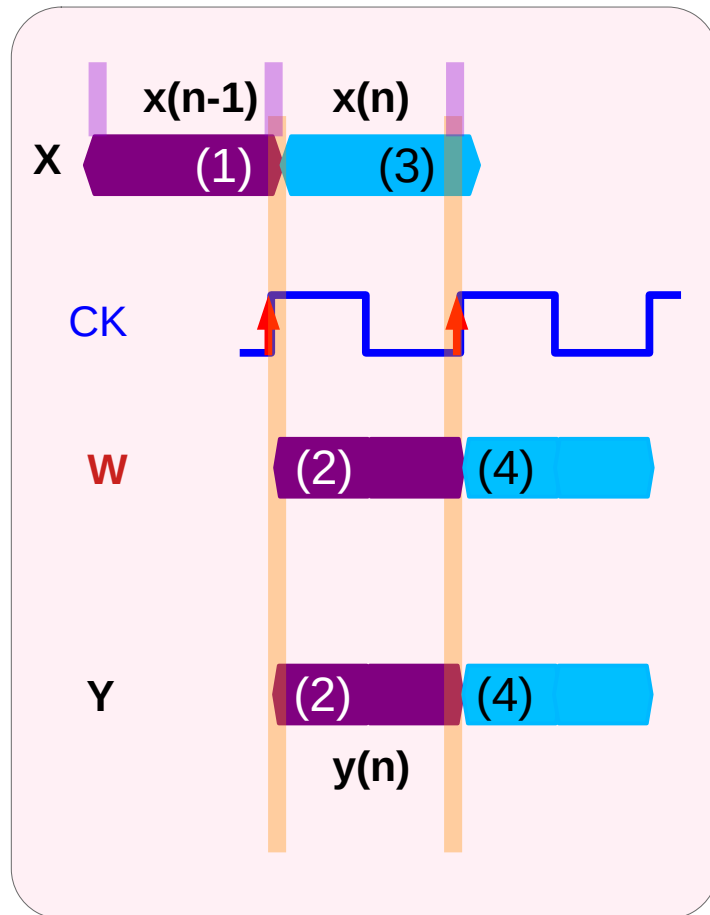
Internal State

Output



The same **DSP C** model

(2) Timing diagrams with flipflop outputs Q



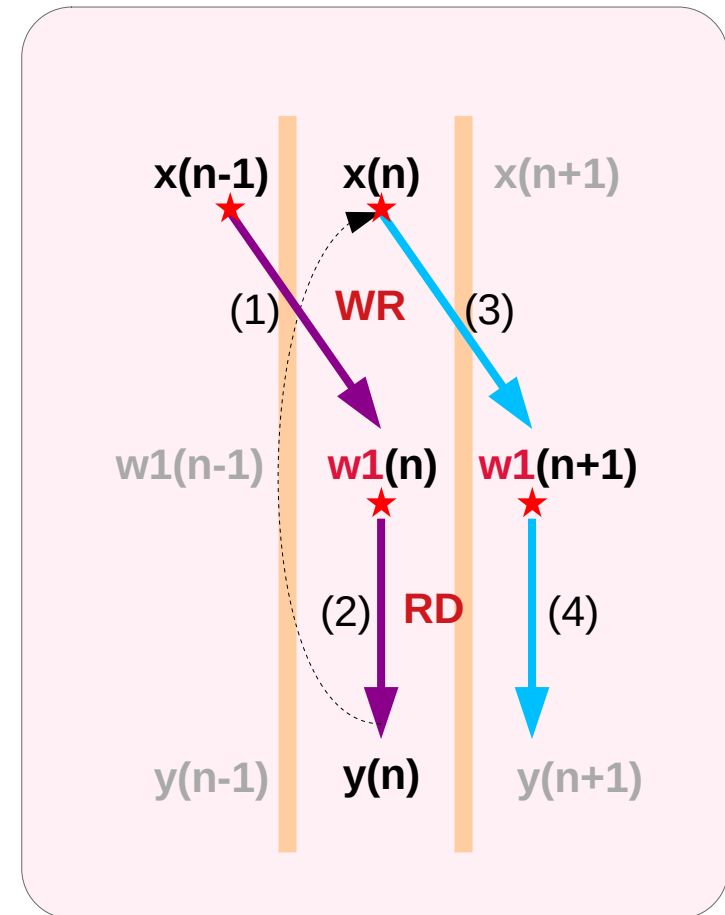
Hardware model

data centered view

Input

Simplified Internal State

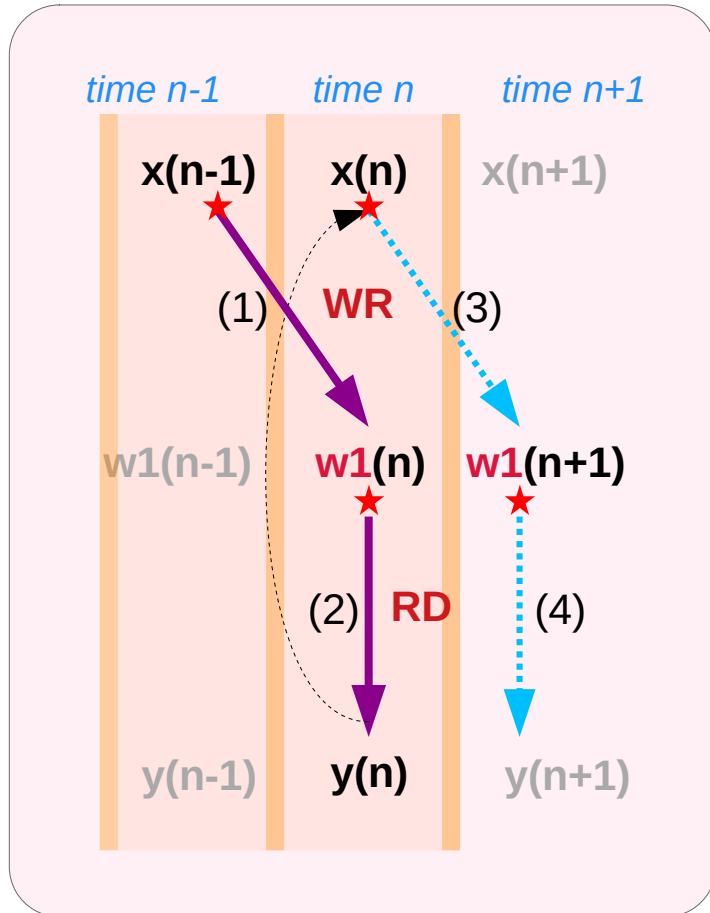
Output



The same DSP C model

Data centered operations

data centered operations



DSP C model

operation spans two time steps ($n-1, n$)

- (1) $w1(n) = x(n-1)$ *WR* $w1(n)$
 (2) $y(n) = w1(n)$ *RD* $w1(n)$

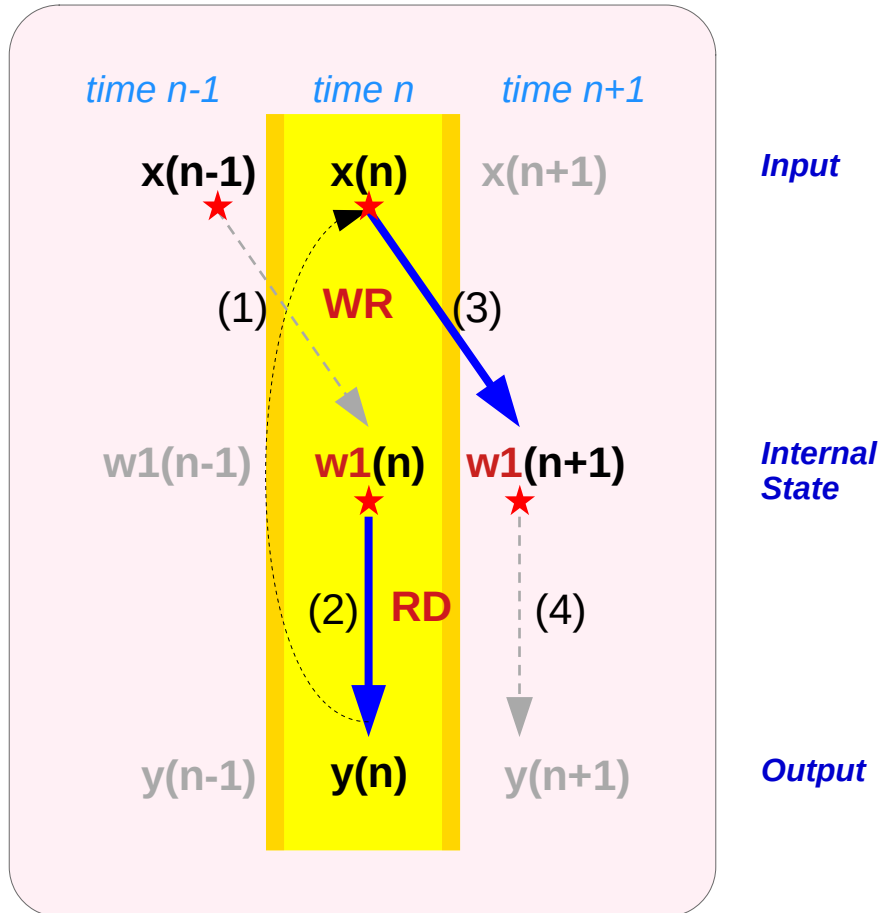
first **WR**, then **RD**
 must avoid **RAW (read after write)** hazards

here, because the written data is read out,
 there is no **RAW (read after write)** hazards

$$\begin{array}{l}
 \underline{y(n-1) = w1(n-1)} \quad \underline{w1(n) = w1(n-1)} \\
 \underline{w1(n) = x(n-1)} \\
 \underline{y(n) = w1(n)} \quad \underline{w1(n+1) = w1(n)} \\
 \underline{w1(n+1) = x(n)} \\
 \underline{y(n+1) = w1(n+1)} \quad \underline{w1(n+2) = w1(n+1)} \\
 \underline{w1(n+2) = x(n+1)} \\
 \underline{y(n+2) = w1(n+2)}
 \end{array}$$

Time centered operations

time centered operations



DSP C model

operation spans only one time step (n)

- (2) $y(n) = w1(n)$ *RD* $w1(n)$
 (3) $w1(n+1) = x(n)$ *WR* $w1(n)$

first **RD**, then **WR**
 no **RAW (read after write)** hazards

| | |
|--------------------|---------------------|
| $y(n-1) = w1(n-1)$ | $w1(n) = w1(n-1)$ |
| $w1(n) = x(n-1)$ | |
| $y(n) = w1(n)$ | $w1(n+1) = w1(n)$ |
| $w1(n+1) = x(n)$ | |
| $y(n+1) = w1(n+1)$ | $w1(n+2) = w1(n+1)$ |
| $w1(n+2) = x(n+1)$ | |
| $y(n+2) = w1(n+2)$ | |

Data vs. time centered operations (1)

data centered operations

time centered operations

| | | | |
|---|---------------------|-------------------|-------------------|
| $y(n-1) = w1(n-1)$ <i>output from $w1(n-1)$</i> <hr/> $w1(n) = x(n-1)$ <i>input to $w1(n)$</i> | $w1(n) = w1(n-1)$ | } <i>time n-1</i> | |
| $y(n) = w1(n)$ <i>output from $w1(n)$</i> <hr/> $w1(n+1) = x(n)$ <i>input to $w1(n+1)$</i> | $w1(n+1) = w1(n)$ | | } <i>time n</i> |
| $y(n+1) = w1(n+1)$ <i>output from $w1(n+1)$</i> <hr/> $w1(n+2) = x(n+1)$ <i>input to $w1(n+2)$</i> | $w1(n+2) = w1(n+1)$ | | |
| $y(n+2) = w1(n+2)$ <i>output from $w1(n+2)$</i> | | | } <i>time n+2</i> |

- (1) $w1(n) = x(n-1)$ *WR $w1(n)$*
 (2) $y(n) = w1(n)$ *RD $w1(n)$*

- (2) $y(n) = w1(n)$ *RD $w1(n)$*
 (1) $w1(n) = x(n)$ *WR $w1(n)$*

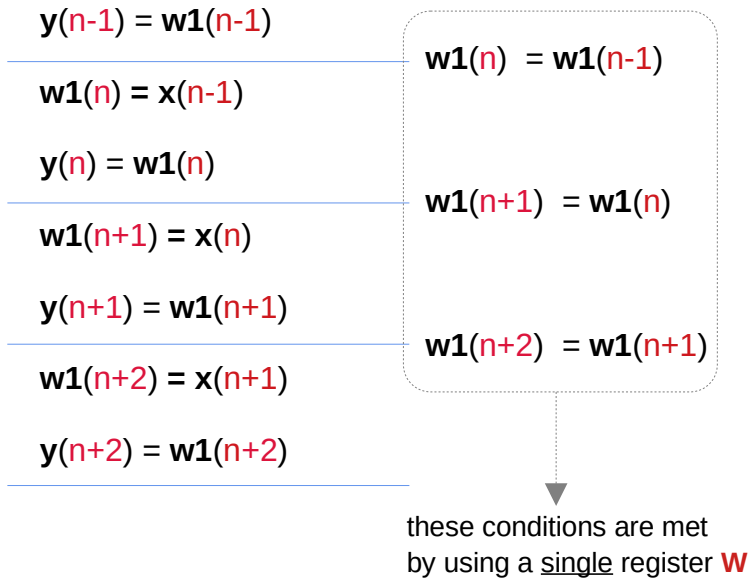
| | |
|----------|---------------------------|
| $W1 = X$ | <i>WR $W1$</i> |
| $Y = W1$ | <i>RD $W1$</i> |

| | |
|----------|---------------------------|
| $Y = W1$ | <i>RD $W1$</i> |
| $W1 = X$ | <i>WR $W1$</i> |

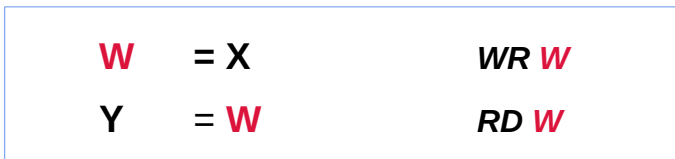
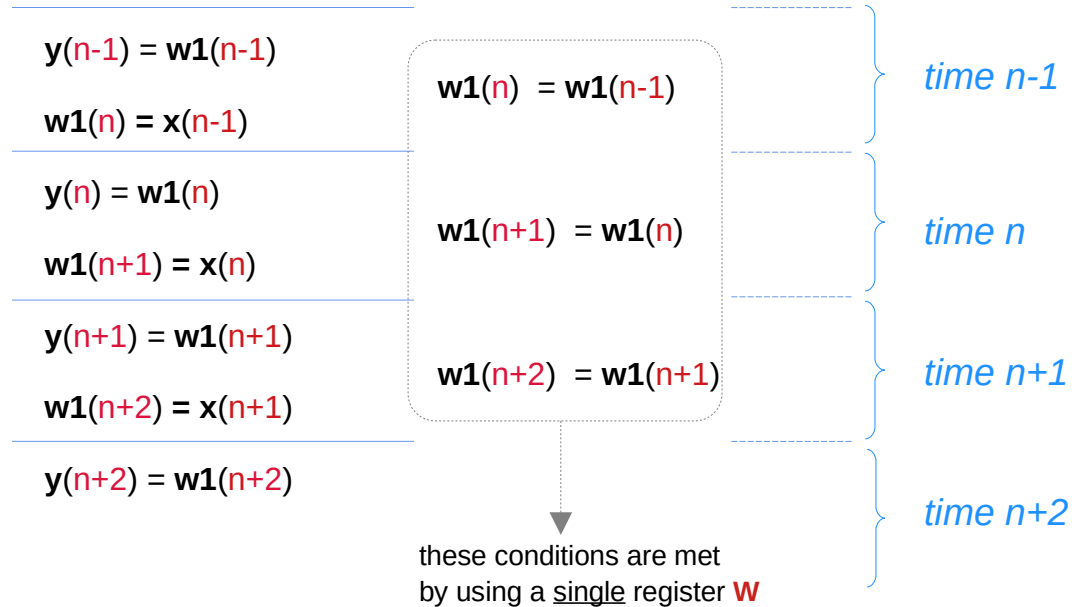
← *Register Transfer Level*

Data vs. time centered operations (2)

data centered operations



time centered operations



Data centered operations

data centered operations

in the Register Transfer Level

| | | | |
|---|---------------------|---|------------|
| $y(n-1) = w1(n-1)$ <small>output $y(n-1)$ from $w1(n-1)$</small> $w1(n) = x(n-1)$ <small>input $x(n-1)$ to $w1(n)$</small> | $w1(n) = w1(n-1)$ | $Y = W1$ <small>output $y(n-1)$ from $W1$</small> $W1 = X$ <small>input $x(n-1)$ to $W1$</small> | } time n-1 |
| $y(n) = w1(n)$ <small>output $y(n)$ from $w1(n)$</small> $w1(n+1) = x(n)$ <small>input $x(n)$ to $w1(n+1)$</small> | $w1(n+1) = w1(n)$ | $Y = W1$ <small>output $y(n)$ from $W1$</small> $W1 = X$ <small>input $x(n)$ to $W1$</small> | |
| $y(n+1) = w1(n+1)$ <small>output $y(n+1)$ from $w1(n+1)$</small> $w1(n+2) = x(n+1)$ <small>input $x(n+1)$ to $w1(n+2)$</small> | $w1(n+2) = w1(n+1)$ | $Y = W1$ <small>output $y(n+1)$ from $W1$</small> $W1 = X$ <small>input $x(n+1)$ to $W1$</small> | } time n+1 |
| $y(n+2) = w1(n+2)$ <small>output $y(n+2)$ from $w1(n+2)$</small> | | $Y = W1$ <small>output $y(n+2)$ from $W1$</small> | |
| (1) $w1(n) = x(n-1)$ $WR\ w1(n)$ (2) $y(n) = w1(n)$ $RD\ w1(n)$ | | $W1 = X$ $WR\ W1$ $Y = W1$ $RD\ W1$ | |

Time centered operations

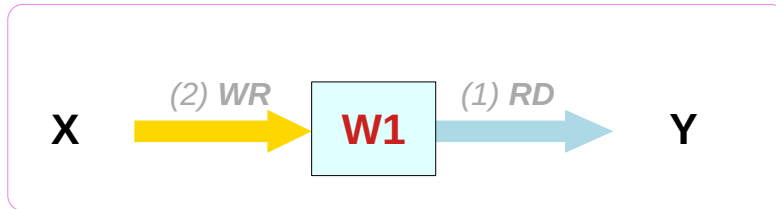
time centered operations

in the Register Transfer Level

| | | |
|--|---|--|
| $y(n-1) = w1(n-1)$ <small>output from $w1(n-1)$</small> $w1(n) = x(n-1)$ <small>input to $w1(n)$</small> $w1(n) = w1(n-1)$ | $Y = W1$ <small>output $y(n-1)$ from $W1$</small> $W1 = X$ <small>input $x(n-1)$ to $W1$</small> | } time n-1 } time n } time n+1 } time n+2 |
| $y(n) = w1(n)$ <small>output from $w1(n)$</small> $w1(n+1) = x(n)$ <small>input to $w1(n+1)$</small> $w1(n+1) = w1(n)$ | $Y = W1$ <small>output $y(n)$ from $W1$</small> $W1 = X$ <small>input $x(n)$ to $W1$</small> | |
| $y(n+1) = w1(n+1)$ <small>output from $w1(n+1)$</small> $w1(n+2) = x(n+1)$ <small>input to $w1(n+2)$</small> $w1(n+2) = w1(n+1)$ | $Y = W1$ <small>output $y(n+1)$ from $W1$</small> $W1 = X$ <small>input $x(n+1)$ to $W1$</small> | |
| $y(n+2) = w1(n+2)$ <small>output from $w1(n+2)$</small> | $Y = W1$ <small>output $y(n+2)$ from $W1$</small> | |
| (2) $y(n) = w1(n)$ $RD\ w1(n)$ | $Y = W1$ $RD\ W1$ | |
| (1) $w1(n) = x(n)$ $WR\ w1(n)$ | $W1 = X$ $WR\ W1$ | |

Simultaneous RD and WR actions

Register Transfer Level



at time n

at time $n+1$

$$Y = W1$$

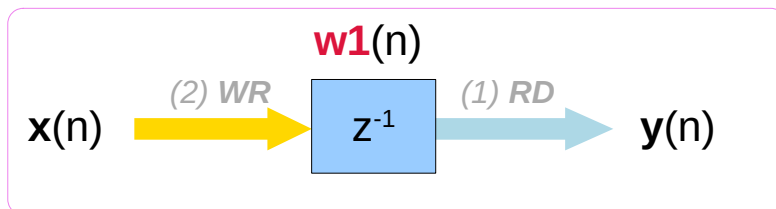
$$W1 = X$$

$$Y = W1$$

$$W1 = X$$

(1) RD
(2) WR

DSP C Model



at time n

at time $n+1$

$$y(n) = w1(n)$$

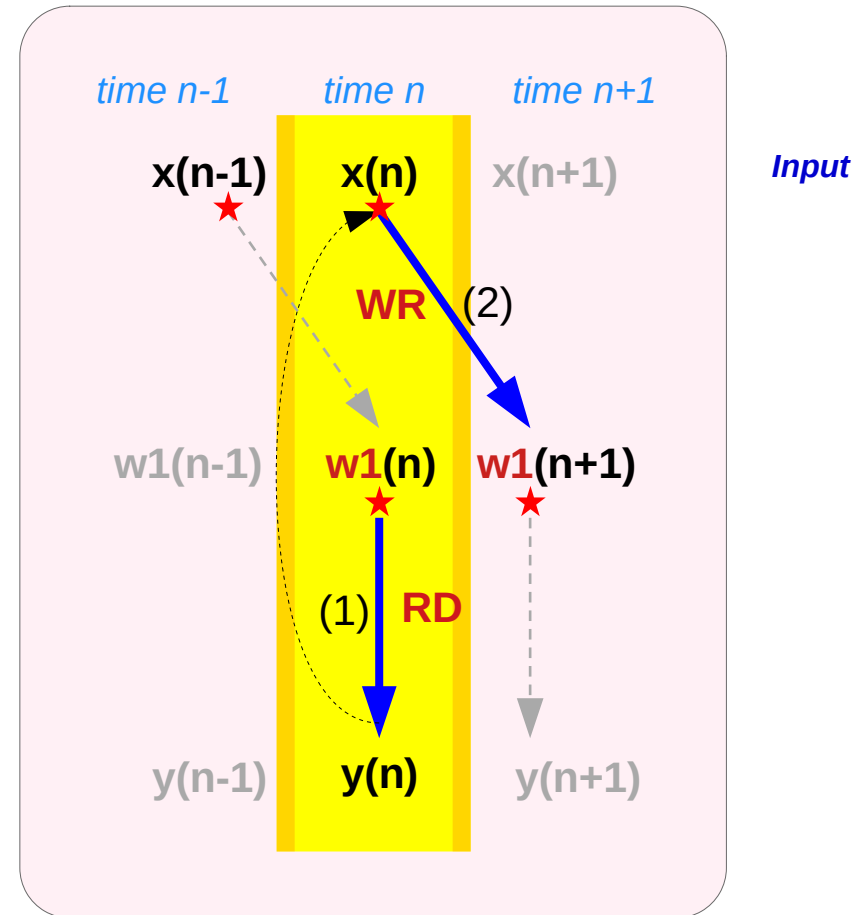
$$w1(n+1) = x(n)$$

$$y(n+1) = w1(n+1)$$

$$w1(n+2) = x(n+1)$$

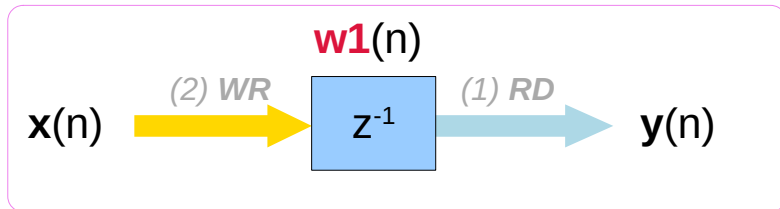
(1) RD
(2) WR

time centered operations



DSP C model

Simultaneous **RD** and **WR** actions



| | | |
|-----------------|----------------|-----------------|
| current content | w1(n) | = x(n-1) |
| next content | w1(n+1) | = x(n) |

at time n

$$y(n) = w1(n)$$

$$w1(n+1) = x(n)$$

at time $n+1$

$$y(n+1) = w1(n+1)$$

$$w1(n+2) = x(n+1)$$

- (1) **RD** *read internal state*
 (2) **WR** *update internal state*

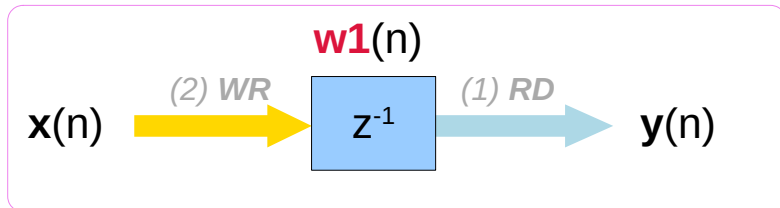
at time n ,

- 1) the content of the register **w1(n)**
becomes the output **y(n)**
- 2) the input **x(n)** is saved and
becomes the new content **w1(n+1)**

RD access of **w1(n) = x(n-1)**

WR access of **w1(n+1) = x(n)**

Current content $w1(n)$ and current input $x(n)$

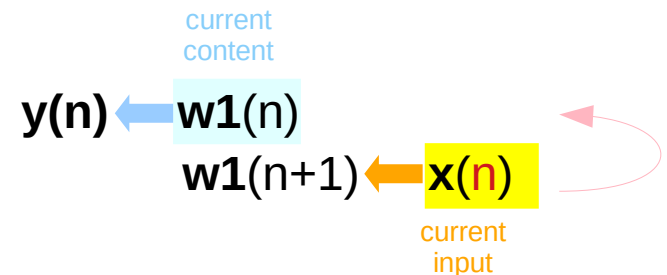


| | | |
|-----------------|-----------|------------|
| current content | $w1(n)$ | $= x(n-1)$ |
| next content | $w1(n+1)$ | $= x(n)$ |

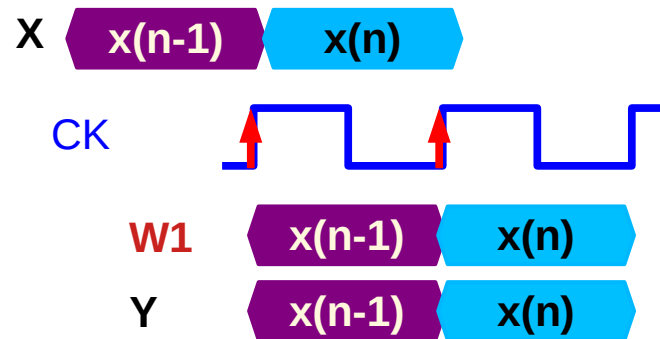
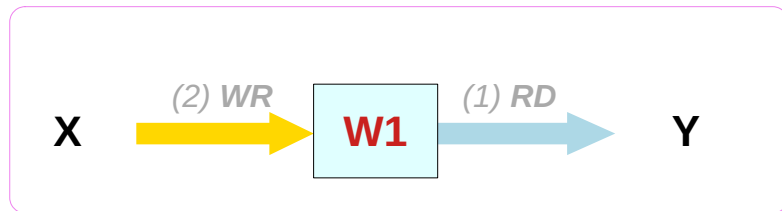
a register holding the previous input sample $x(n-1)$

- (1) the **current content** $x(n-1)$ is clocked out to the output
- (2) the **current input** $x(n)$ gets stored in the register

It will be held for one sampling instant and become the output at the next time $n+1$



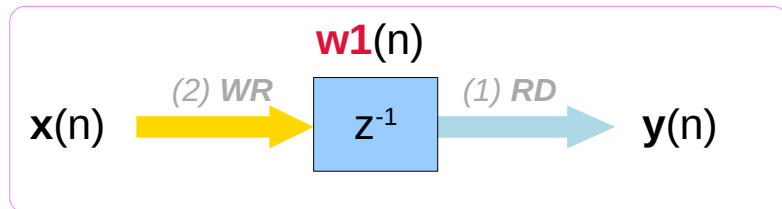
Current content $w1(n)$ and current input $x(n)$



*simulate a clocked hardware
ignoring delay constraints in hardware*

zero-delay simulation

Delay element modeling



| | |
|--------------------------------|-----------------------------------|
| $w1(n) \longrightarrow y(n)$ | $y(n) = w1(n)$ (1) RD old w1 |
| $x(n) \longrightarrow w1(n+1)$ | $w1(n+1) = x(n)$ (2) WR new w1 |

The content of the delay register at time n as the **internal state** of the filter by

internal state at time n

$$w1(n) = x(n-1)$$

internal state at time $n+1$

$$w1(n+1) = x(n)$$

output at time n

$$y(n) = w1(n)$$

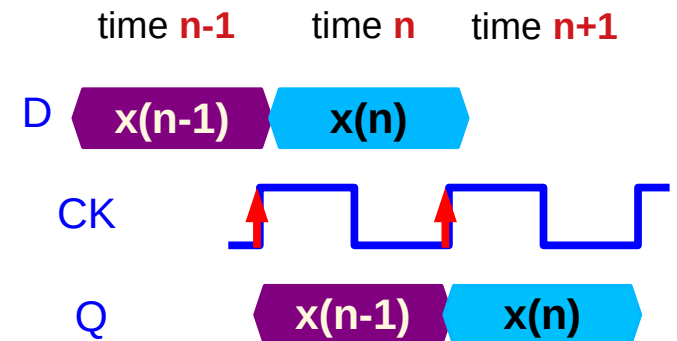
RD before WR

WAR (Write after Read) Access

WR at time $n-1$

WR at time n

RD at time n



*simulate a clocked hardware
ignoring delay constraints in hardware*

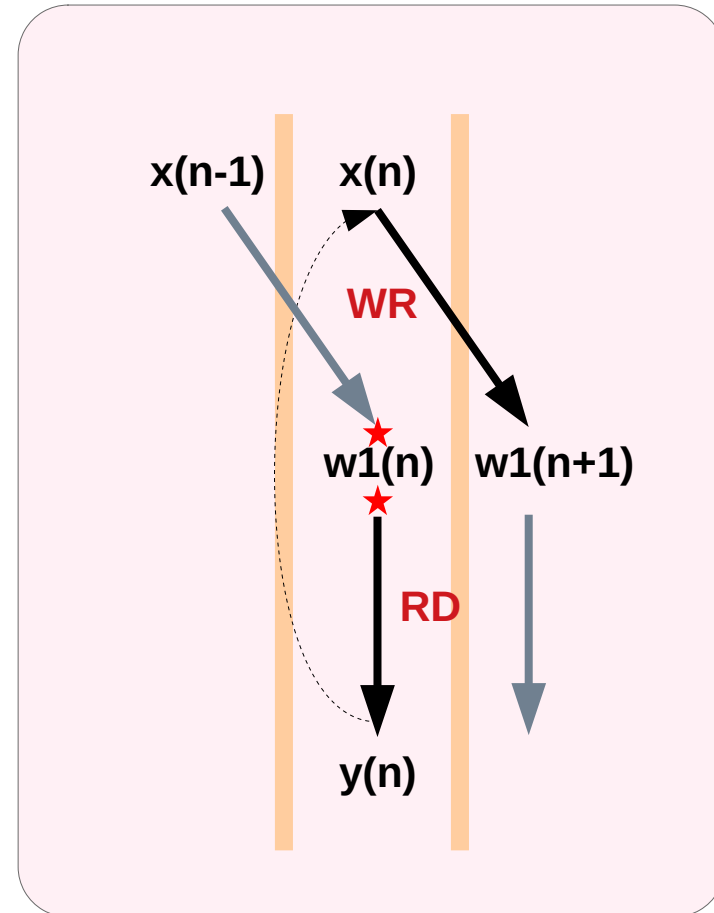
zero-delay simulation

WAR (Write after Read)

| | | |
|--------------------|--------|--------|
| $y(n) = w1(n)$ | (1) RD | old w1 |
| $w1(n+1) = x(n)$ | (2) WR | new w1 |
| $y(n+1) = w1(n+1)$ | (1) RD | old w1 |
| $w1(n+2) = x(n+1)$ | (2) WR | new w1 |

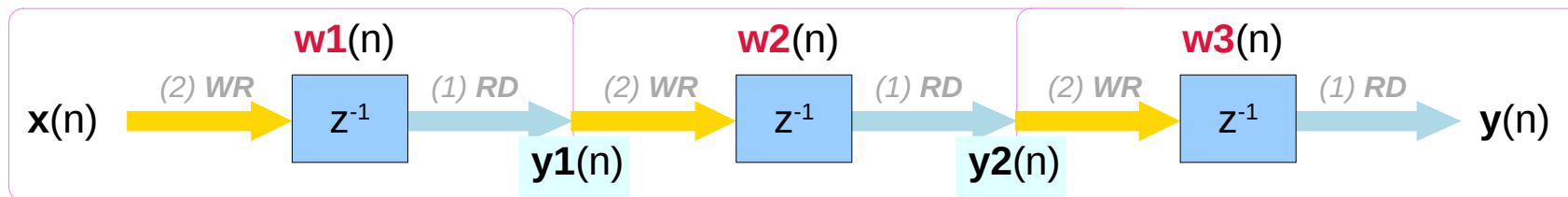
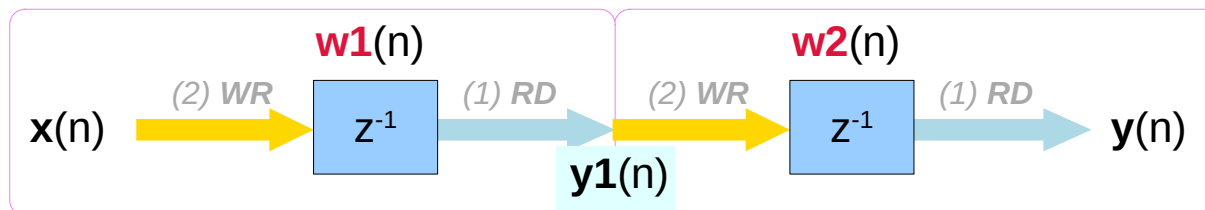
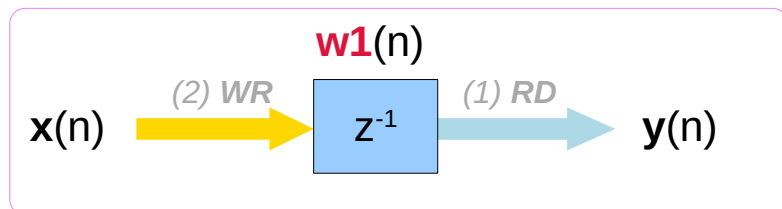
| | | |
|--------------------|--------|--------|
| $w1(n+1) = x(n)$ | (2) WR | new w1 |
| $y(n) = w1(n)$ | (1) RD | old w1 |
| $w1(n+2) = x(n+1)$ | (2) WR | new w1 |
| $y(n+1) = w1(n+1)$ | (1) RD | old w1 |

WAR (Write after Read) Violation



DSP C model

Single, Double, Triple Delay – Summary



single delay

$$\begin{aligned} y(n) &= w1(n) && \text{output} \\ w1(n+1) &= x(n) && \text{input} \end{aligned}$$

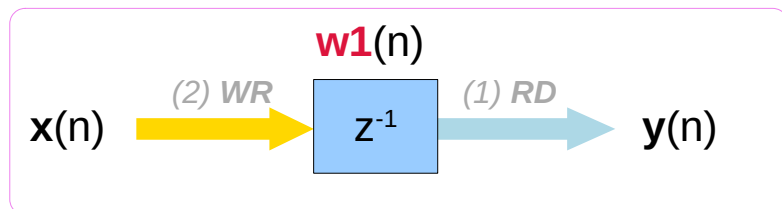
double delay

$$\begin{aligned} y(n) &= w2(n) && \text{output} \\ w2(n+1) &= w1(n) && \\ w1(n+1) &= x(n) && \text{input} \end{aligned}$$

triple delay

$$\begin{aligned} y(n) &= w3(n) && \text{output} \\ w3(n+1) &= w2(n) && \\ w2(n+1) &= w1(n) && \\ w1(n+1) &= x(n) && \text{input} \end{aligned}$$

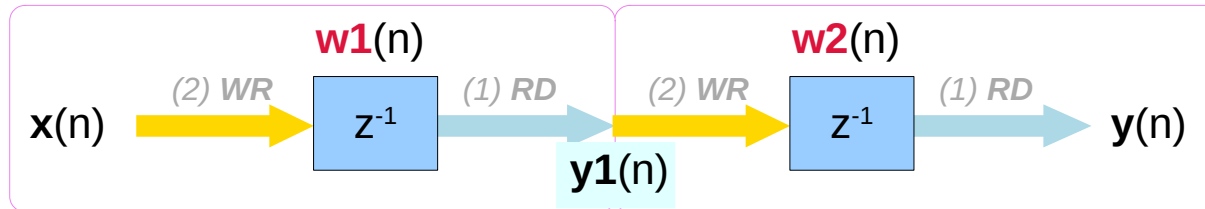
Single Delay – IO Equations



single delay

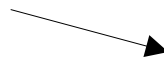
| | |
|------------------|--------|
| $y(n) = w1(n)$ | output |
| $w1(n+1) = x(n)$ | input |

Double Delay – IO Equations



$$\begin{aligned} \mathbf{y1}(n) &= \mathbf{w1}(n) \\ \mathbf{w1}(n+1) &= \mathbf{x}(n) \end{aligned}$$

$$\mathbf{w1}(n+1) = \mathbf{x}(n)$$



$$\begin{aligned} \mathbf{y}(n) &= \mathbf{w2}(n) \\ \mathbf{w2}(n+1) &= \mathbf{y1}(n) \end{aligned}$$

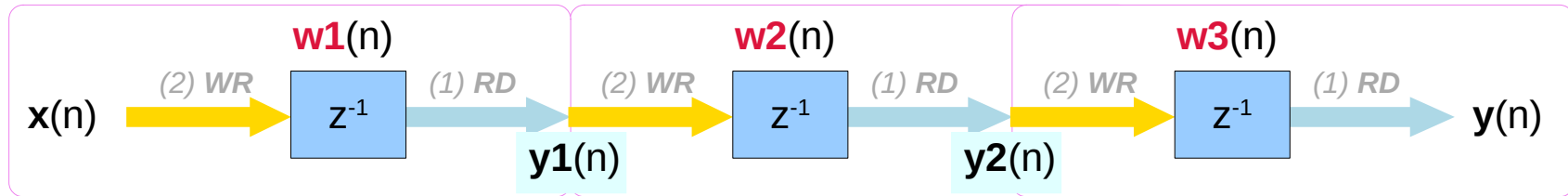
$$\begin{aligned} \mathbf{y}(n) &= \mathbf{w2}(n) \\ \mathbf{w2}(n+1) &= \mathbf{w1}(n) \end{aligned}$$

$$\begin{aligned} \mathbf{y}(n) &= \mathbf{w2}(n) \\ \mathbf{w2}(n+1) &= \mathbf{w1}(n) \\ \mathbf{w1}(n+1) &= \mathbf{x}(n) \end{aligned}$$

output

input

Triple Delay – IO Equations



$$\begin{aligned} y_1(n) &= w_1(n) \\ w_1(n+1) &= x(n) \end{aligned}$$

$$\begin{aligned} y_2(n) &= w_2(n) \\ w_2(n+1) &= y_1(n) \end{aligned}$$

$$\begin{aligned} y(n) &= w_3(n) \\ w_3(n+1) &= y_2(n) \end{aligned}$$

$$w_1(n+1) = x(n)$$

$$w_2(n+1) = w_1(n)$$

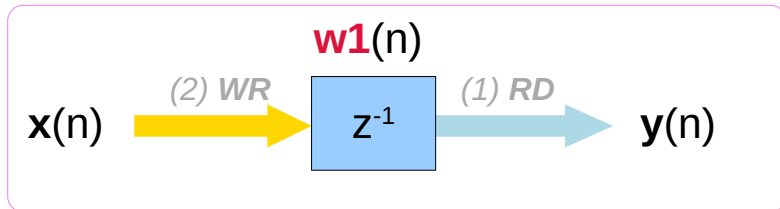
$$\begin{aligned} y(n) &= w_3(n) \\ w_3(n+1) &= w_2(n) \end{aligned}$$

$$\begin{aligned} y(n) &= w_3(n) \\ w_3(n+1) &= w_2(n) \\ w_2(n+1) &= w_1(n) \\ w_1(n+1) &= x(n) \end{aligned}$$

output

input

Single Delay

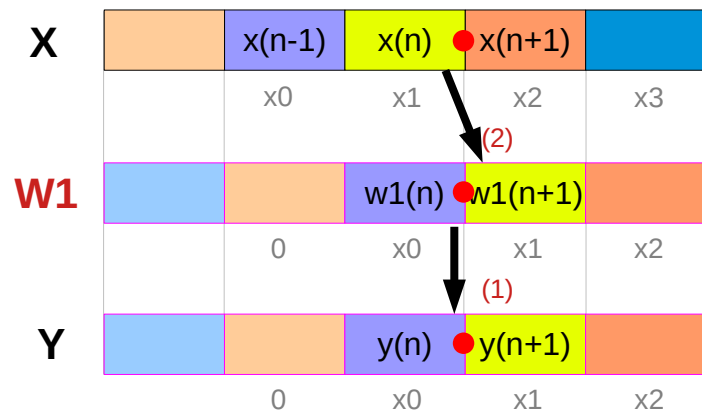


single delay

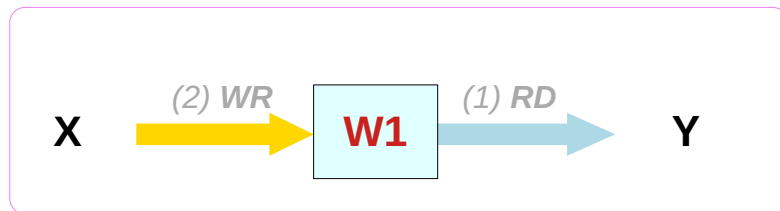
$$y(n) = w1(n)$$

$$w1(n+1) = x(n)$$

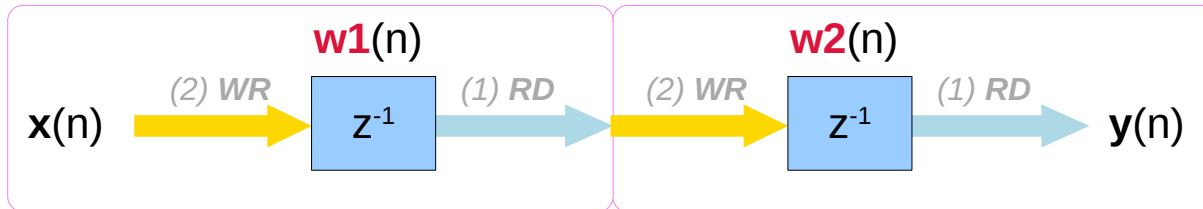
output
input



| n | x(n) | w1(n) | y(n) |
|---|------|-------|------|
| 0 | x0 | 0 | 0 |
| 1 | x1 | x0 | x0 |
| 2 | x2 | x1 | x1 |
| 3 | x3 | x2 | x2 |
| 4 | x4 | x3 | x3 |

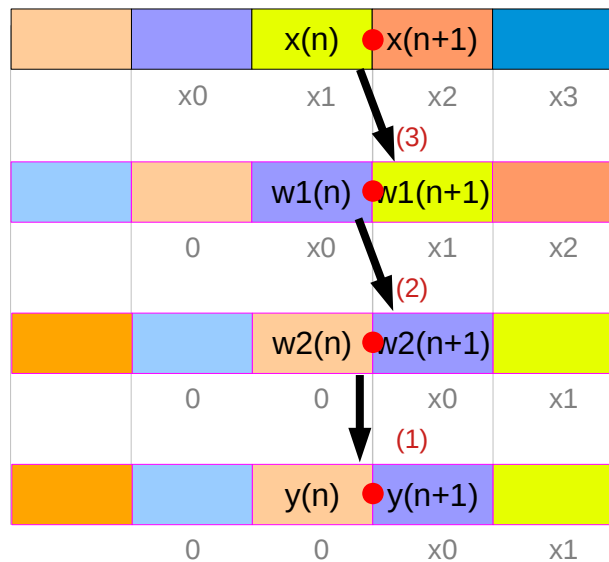


Double Delay



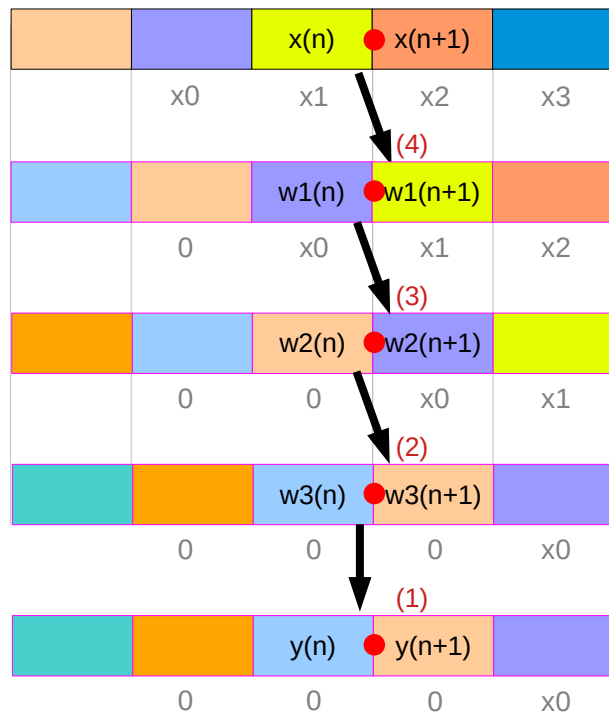
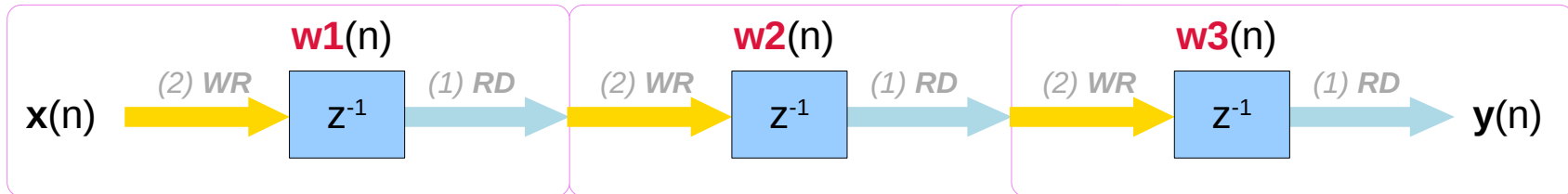
double delay

$$\begin{aligned}
 y(n) &= w2(n) && \text{output} \\
 w2(n+1) &= w1(n) \\
 w1(n+1) &= x(n) && \text{input}
 \end{aligned}$$



| n | x(n) | w1(n) | w2(n) | y(n) |
|---|------|-------|-------|------|
| 0 | x0 | 0 | 0 | 0 |
| 1 | x1 | x0 | 0 | 0 |
| 2 | x2 | x1 | x0 | x0 |
| 3 | x3 | x2 | x1 | x1 |
| 4 | x4 | x3 | x2 | x2 |

Triple Delay



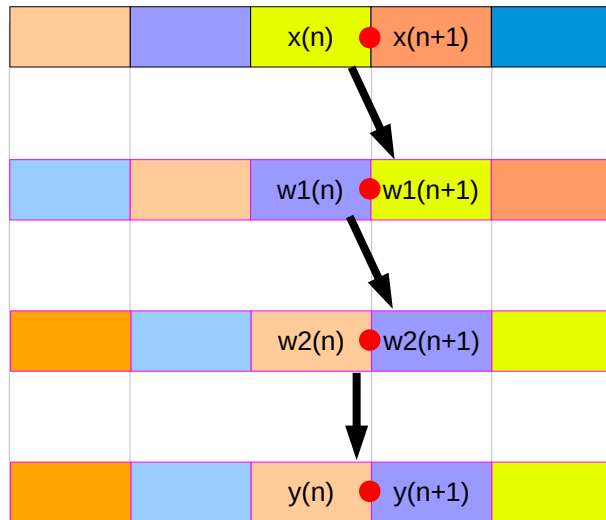
triple delay

$$\begin{aligned}
 y(n) &= w3(n) && \text{output} \\
 w3(n+1) &= w2(n) \\
 w2(n+1) &= w1(n) \\
 w1(n+1) &= x(n) && \text{input}
 \end{aligned}$$

| n | x(n) | w1(n) | w2(n) | w3(n) | y(n) |
|---|------|-------|-------|-------|------|
| 0 | x0 | 0 | 0 | 0 | 0 |
| 1 | x1 | x0 | 0 | 0 | 0 |
| 2 | x2 | x1 | x0 | 0 | 0 |
| 3 | x3 | x2 | x1 | x0 | x0 |
| 4 | x4 | x3 | x2 | x1 | x1 |

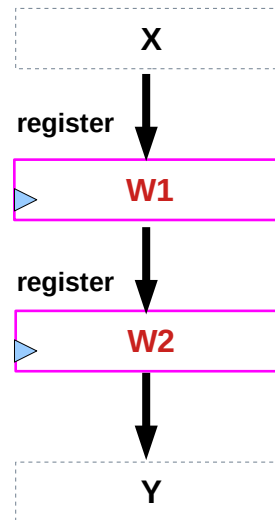
Delay C Model

Timing Chart



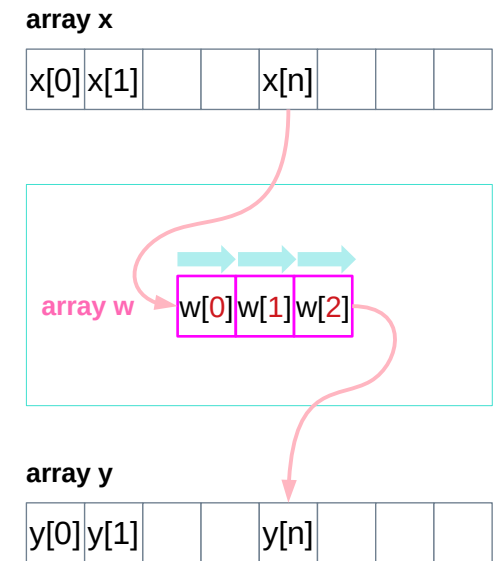
$$\begin{aligned}
 y(n) &= w2(n) \\
 w2(n+1) &= w1(n) \\
 w1(n+1) &= x(n)
 \end{aligned}$$

Register Transfer



$$\begin{aligned}
 Y &= W2 \\
 W2 &= W1 \\
 W1 &= X
 \end{aligned}$$

DSP C Model for simulation



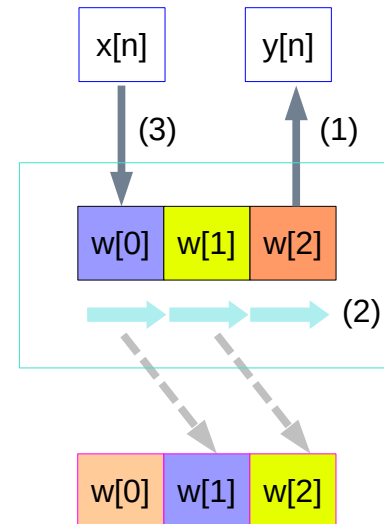
$$\begin{aligned}
 y[n] &= w[2] \\
 w[2] &= w[1] \\
 w[1] &= w[0] \\
 w[0] &= x[n]
 \end{aligned}$$

IO Equations for the Triple Delay (1)

$$\begin{aligned}y(n) &= w2(n) \\w2(n+1) &= w1(n) \\w1(n+1) &= w0(n) \\w0(n+1) &= x(n)\end{aligned}$$

$$D = 2, 1$$

$$\begin{aligned}y[n] &= w[2] && // (1) \text{ get the output} \\w[2] &= w[1] && // (2) \text{ shift} \\w[1] &= w[0] && // (2) \text{ shift} \\w[0] &= x[n] && // (3) \text{ put the input}\end{aligned}$$



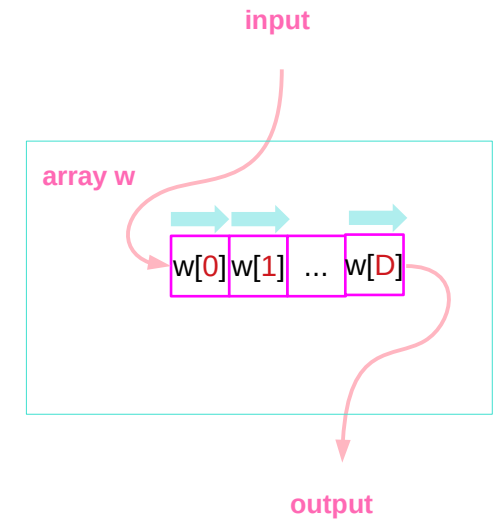
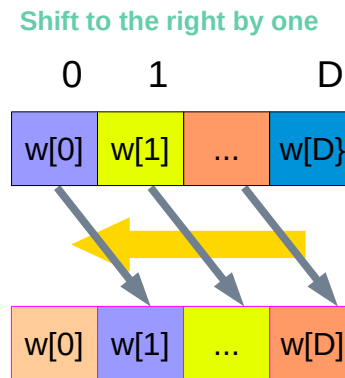
delay.c

```
/* delay.c - delay by D time samples */
/* w[0] = input, w[D] = output */

void delay(int D, double *w)
{
    int i;

    for (i=D; i>=1; i--)
        w[i] = w[i-1];

    // reverse-order updating
}
```



order of execution

$$\begin{aligned} w[D] &= w[D-1] \\ \dots & \\ w[2] &= w[1] \\ w[1] &= w[0] \end{aligned}$$

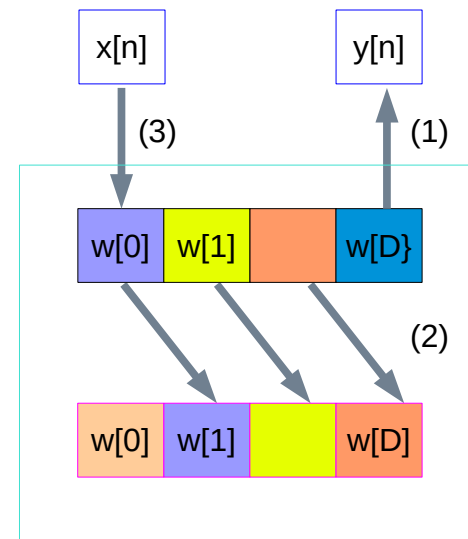
Using the delay function

```
double *w;  
w = (double *) calloc(D+1, sizeof(double)); // (D+1)-dimensional
```

```
for (n = 0; n < Ntot; n++) {  
    y[n] = w[D]; // (1) get output y[n]  
    w[0] = x[n]; // (3) read input x[n]  
    delay(D, w); // (2) update delay line  
}
```

```
/* delay.c - delay by D time samples */  
/* w[0] = input, w[D] = output */
```

```
void delay(int D, double *w)  
{  
    int i;  
  
    for (i=D; i>=1; i--)  
        w[i] = w[i-1];  
  
    // reverse-order updating  
}
```



Delay Functions

$$y(n) = w_1(n)$$

$$w_1(n+1) = x(n)$$

$$y(n) = w_2(n)$$

$$w_2(n+1) = w_1(n)$$

$$w_1(n+1) = x(n)$$

$$y(n) = w_3(n)$$

$$w_3(n+1) = w_2(n)$$

$$w_2(n+1) = w_1(n)$$

$$w_1(n+1) = x(n)$$

$$y(n) = w_D(n)$$

$$w_0(n) = x(n)$$

$$w_i(n+1) = w_{i-1}(n),$$

$$i = D, D-1, \dots, 2, 1$$

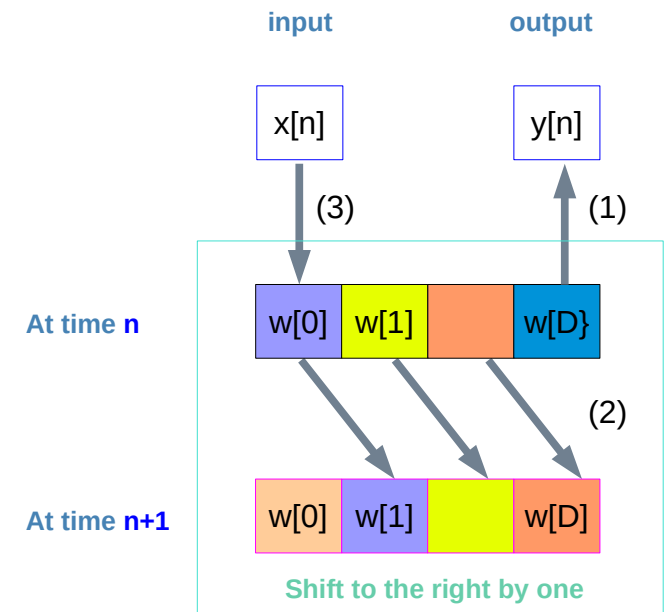
time index : n

memory location : W_i

memory index : i

$$w_i(n+1) = w_{i-1}(n)$$

the current value at w_{i-1}
will become
the next value at w_i



Holding a delayed input sequence

$$w_0(n) = x(n)$$

$$w_1(n) = x(n-1) = w_0(n-1)$$

$$w_2(n) = x(n-2) = w_1(n-1)$$

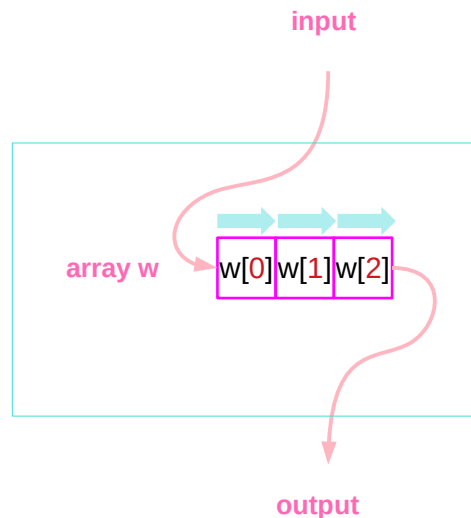
$$w_3(n) = x(n-3) = w_2(n-1)$$

$$y(n) = w_3(n)$$

$$w_3(n+1) = w_2(n)$$

$$w_2(n+1) = w_1(n)$$

$$w_1(n+1) = x(n)$$



$$y(n) = w_3(n) = w_2(n-1) = w_1(n-2) = x(n-4)$$

$$w_3(n) = w_2(n-1) = w_1(n-2) = x(n-3)$$

$$w_2(n) = w_1(n-1) = x(n-2)$$

$$w_1(n) = x(n-1)$$

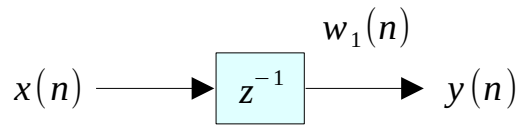
$$y(n) = w_3(n)$$

$$w_3(n) = w_2(n-1)$$

$$w_2(n) = w_1(n-1)$$

$$w_1(n) = x(n-1)$$

Single Delay (1)



$$w_1(n) = x(n-1)$$

(internal state at time n)



$$w_1(n+1) = x(n)$$

(internal state at time n+1)

$$y(n) = w_1(n)$$

$$w_1(n+1) = x(n)$$

$$y(n+1) = w_1(n+1)$$

$$w_1(n+2) = x(n+1)$$

$$w_1(0) = 0$$

$$[x_0, x_1, x_2, x_3, \dots] \rightarrow [0, x_0, x_1, x_2, \dots]$$

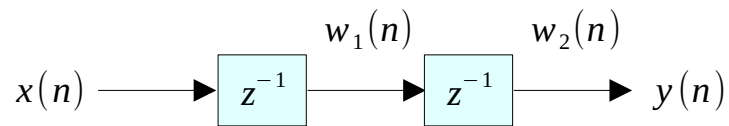
for each input sample x do:

$$y := w_1$$

$$w_1 := x$$

| n | $x(n)$ | $w_1(n)$ | $y(n)$ |
|-----|--------|----------|--------|
| 0 | x_0 | 0 | 0 |
| 1 | x_1 | x_0 | x_0 |
| 2 | x_2 | x_1 | x_1 |
| 3 | x_3 | x_2 | x_2 |
| 4 | x_4 | x_3 | x_3 |

Double Delay (1)



$$\begin{aligned} w_2(n) &= w_1(n-1) & = x((n-1)-1) & = x(n-2) \\ w_1(n) &= x(n-1) \end{aligned}$$



$$\begin{aligned} w_2(n+1) &= w_1(n) \\ w_1(n+1) &= x(n) \end{aligned}$$

$$y(n) = w_2(n)$$

$$\begin{aligned} w_2(n+1) &= w_1(n) \\ w_1(n+1) &= x(n) \end{aligned}$$

$$[x_0, x_1, x_2, x_3, \dots] \rightarrow [0, 0, x_0, x_1, \dots]$$

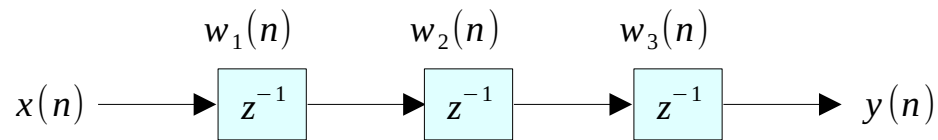
for each input sample x do:

```

y := w2
w2 := w1
w1 := x
    
```

| n | $x(n)$ | $w_1(n)$ | $w_2(n)$ | $y(n)$ |
|-----|--------|----------|----------|--------|
| 0 | x_0 | 0 | 0 | 0 |
| 1 | x_1 | x_0 | 0 | 0 |
| 2 | x_2 | x_1 | x_0 | x_0 |
| 3 | x_3 | x_2 | x_1 | x_1 |
| 4 | x_4 | x_3 | x_2 | x_2 |

Triple Delay (1)



$$\begin{aligned}
 w_3(n) &= w_2(n-1) & = w_1(n-2) & = x(n-3) \\
 w_2(n) &= w_1(n-1) \\
 w_1(n) &= x(n-1)
 \end{aligned}$$



$$\begin{aligned}
 w_3(n+1) &= w_2(n) \\
 w_2(n+1) &= w_1(n) \\
 w_1(n+1) &= x(n)
 \end{aligned}$$

$$y(n) = w_3(n)$$

$$\begin{aligned}
 w_3(n+1) &= w_2(n) \\
 w_2(n+1) &= w_1(n) \\
 w_1(n+1) &= x(n)
 \end{aligned}$$

$$[x_0, x_1, x_2, x_3, \dots] \rightarrow [0, 0, 0, x_0, \dots]$$

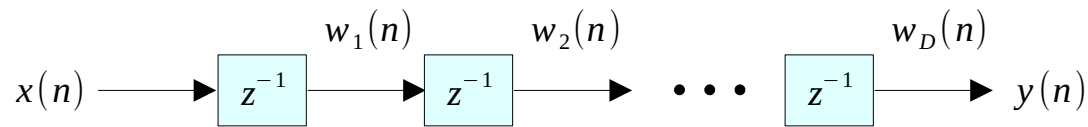
for each input sample x do:

```

y := w3
w3 := w2
w2 := w1
w1 := x
    
```

| n | $x(n)$ | $w_1(n)$ | $w_2(n)$ | $y(n)$ |
|-----|--------|----------|----------|--------|
| 0 | x_0 | 0 | 0 | 0 |
| 1 | x_1 | x_0 | 0 | 0 |
| 2 | x_2 | x_1 | x_0 | x_0 |
| 3 | x_3 | x_2 | x_1 | x_1 |
| 4 | x_4 | x_3 | x_2 | x_2 |

D Unit Delay (1)



$$w_i(n) = w_{i-1}(n-1) \quad \text{for } i = 1, 2, \dots, D$$

$$[x_0, x_1, x_2, x_3, \dots] \rightarrow [0, x_0, x_1, x_2, x_3, \dots]$$

for each input sample x do:

$y := w_D$

$w_0 := x$

$w_0 := x$

for $i = D, D-1, \dots, 1$ do:

$w_i := w_{i-1}$

for each input sample w_0 do:

for $i = D, D-1, \dots, 1$ do:

$w_i := w_{i-1}$

D Unit Delay (1)

```
/* delay.c - delay by D time samples */
void delay(int D, double *w)      w[0] = input, w[D] = output
{
    int i;

    for (i=D; i>=1; i--)          reverse-order updating
        w[i] = w[i-1];
}
```

dot

```
/* dot.c - dot product of two length-(M+1) vectors */
double dot(int M, double *h, double *w)          Usage: y = dot(M, h, w);
{
    int i;                                       h = filter vector, w = state vector
    double y;                                    M = filter order

    for (y=0, i=0; i<=M; i++)                   compute dot product
        y += h[i] * w[i];

    return y;
}
```

$$y = h_0 w_0 + h_1 w_1 + \dots + h_M w_M = [h_0, h_1, \dots, h_M] \begin{bmatrix} w_0 \\ w_1 \\ \vdots \\ w_M \end{bmatrix} = \mathbf{h}^T \mathbf{w}$$

Direct Form

Considering the widely used
Edge triggered
D-type Flip Flops

$$H(z) = \frac{N(z)}{D(z)} = \frac{b_0 + b_1 z^{-1} + b_2 z^{-2}}{1 + a_1 z^{-1} + a_2 z^{-2}}$$

$$y_n = -a_1 y_{n-1} - a_2 y_{n-2} + b_0 x_n + b_1 x_{n-1} + b_2 x_{n-2}$$

References

- [1] S. J. Ofranidis , Introduction to Signal Processing