Laurent Series and z-Transform - Geometric Series Permutations B

20241113 Wed

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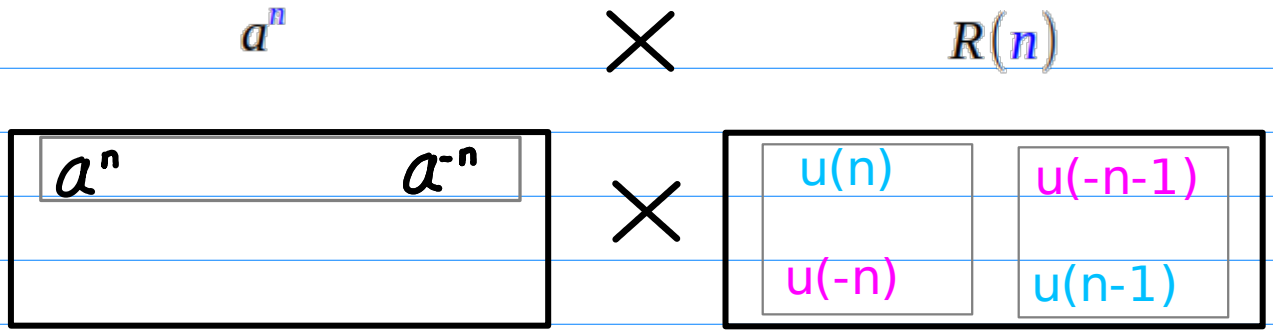
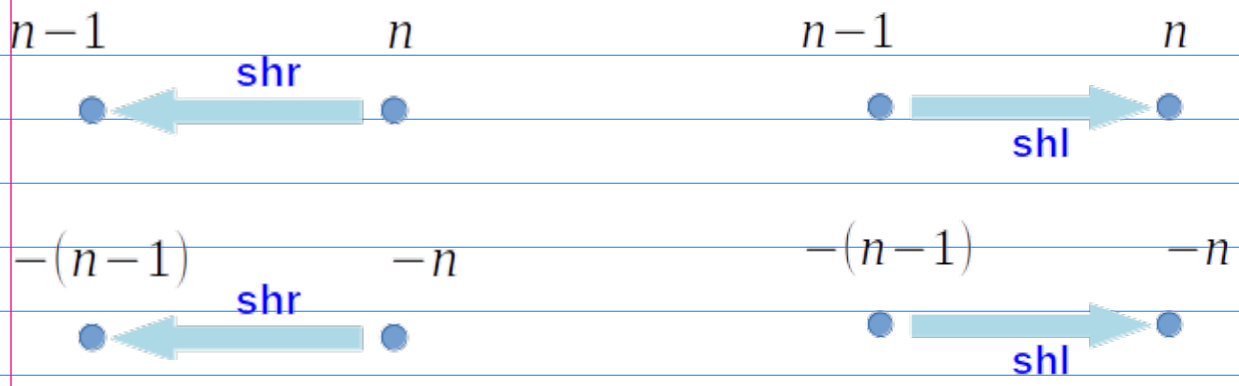


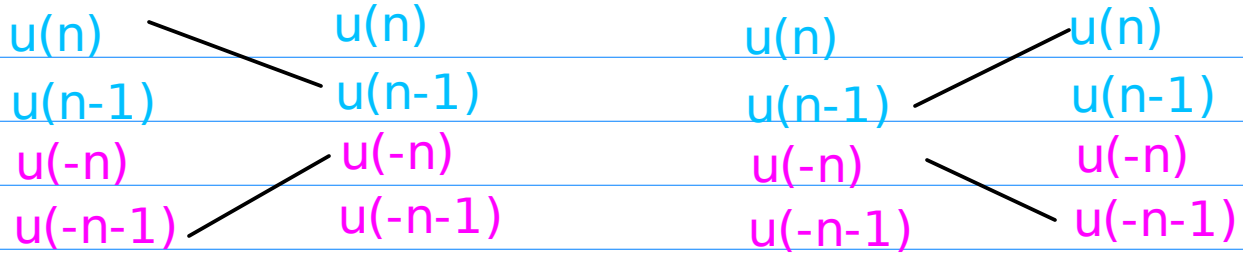
(1)	a^n u(n)	a^{-n} u(n)	(2)
(3)	a^n u(-n)	a^{-n} u(-n)	(4)
(5)	a^n u(-n-1)	a^{-n} u(-n-1)	(6)
(7)	a^n u(n-1)	a^{-n} u(n-1)	(8)

u(n)	u(-n-1)
u(-n)	u(n-1)

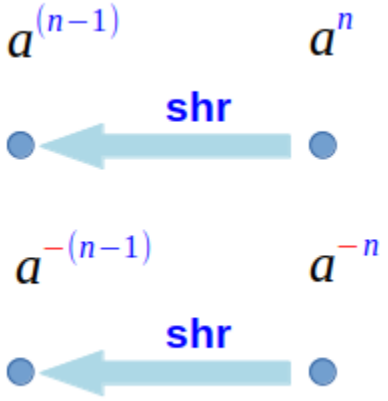
$$shl(R(n)) = R(n+1)$$
$$shr(R(n)) = R(n-1)$$

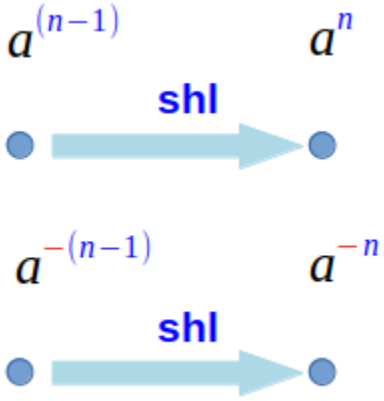
u(n)	→	u(n-1)	shr	n → n-1
u(n-1)	→	u(n)	shl	n → n+1
u(-n)	→	u(-n-1)	shl	n → n+1
u(-n-1)	→	u(-n)	shr	n → n-1





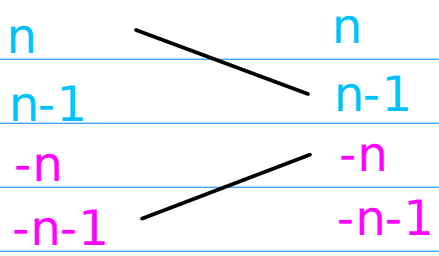
Exponent Shifting

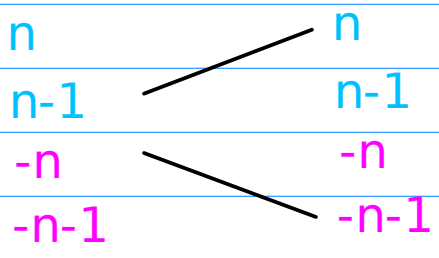




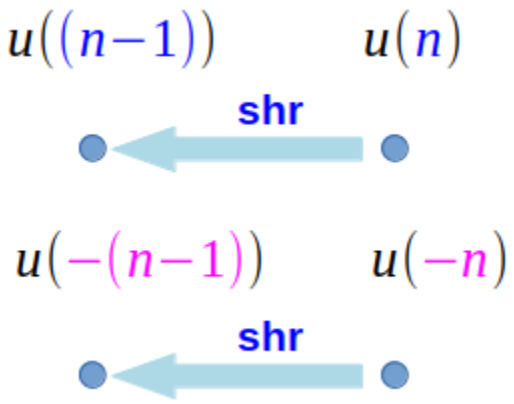
	$shl(b^n)$
b^n	b^{n+1}
a^n	$a^{(n+1)}$
a^{-n}	$a^{-(n+1)}$

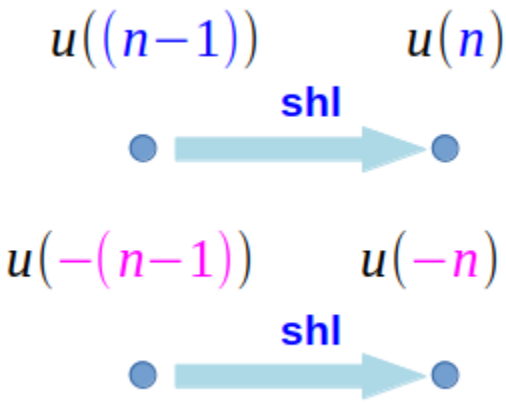
	$shr(b^n)$
b^n	b^{n-1}
$a^{(n+1)}$	a^n
$a^{-(n+1)}$	a^{-n}





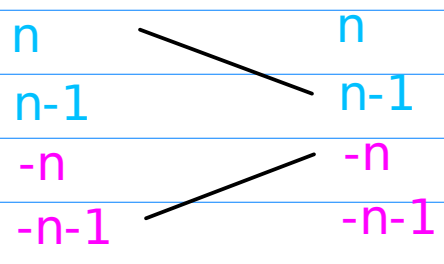
Range Shifting

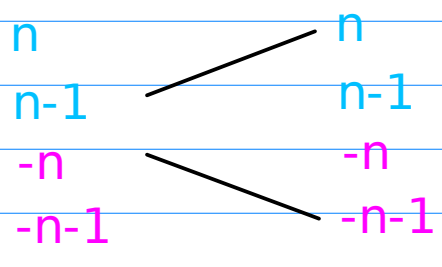




	$shl(R(n))$
$R(n)$	$R(n+1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-(n+1))$

	$shr(R(n))$
$R(n)$	$R(n-1)$
$u(n)$	$u(n-1)$
$u(-(n+1))$	$u(-n)$





$sh\,l(b^n)$

$b^n \rightarrow b^{n+1}$

$sh\,r(b^n)$

$b^n \rightarrow b^{n-1}$

b^n	$b^{sh(n)}$	
a^n	$a^{(n+1)}$	$sh\,l(b^n)$
a^n	$a^{(n-1)}$	$sh\,r(b^n)$
a^{-n}	$a^{-(n+1)}$	$sh\,l(b^n)$
a^{-n}	$a^{-(n-1)}$	$sh\,r(b^n)$

b^n	$b^{sh(n)}$	
a^n	$a^{(n-1)}$	$sh\,r(b^n)$
a^n	$a^{(n+1)}$	$sh\,l(b^n)$
a^{-n}	$a^{-(n-1)}$	$sh\,r(b^n)$
a^{-n}	$a^{-(n+1)}$	$sh\,l(b^n)$

$sh\,l(R(n))$

$R(n) \rightarrow R(n+1)$

$sh\,r(R(n))$

$R(n) \rightarrow R(n-1)$

$R(n)$	$R(sh(n))$	
$u(n)$	$u(n-1)$	$sh\,r(R(n))$
$u(-(n+1))$	$u(-n)$	$sh\,r(R(n))$

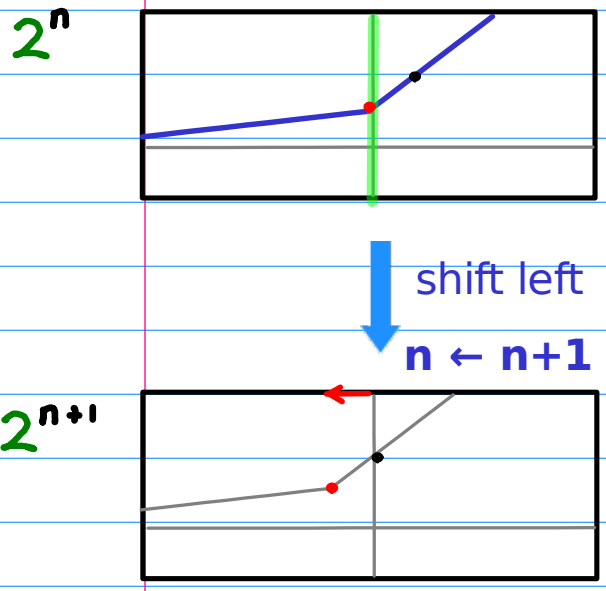
$R(n)$	$R(sh(n))$	
$u(n)$	$u(n-1)$	$sh\,r(R(n))$
$u(-(n+1))$	$u(-n)$	$sh\,r(R(n))$

Exponent Shifting

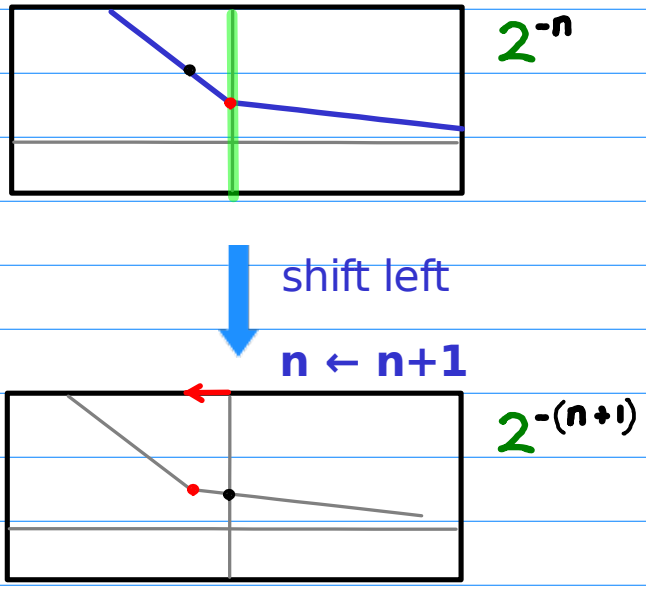
$$shl(b^n) = b^{n+1}$$

$$shr(b^n) = b^{n-1}$$

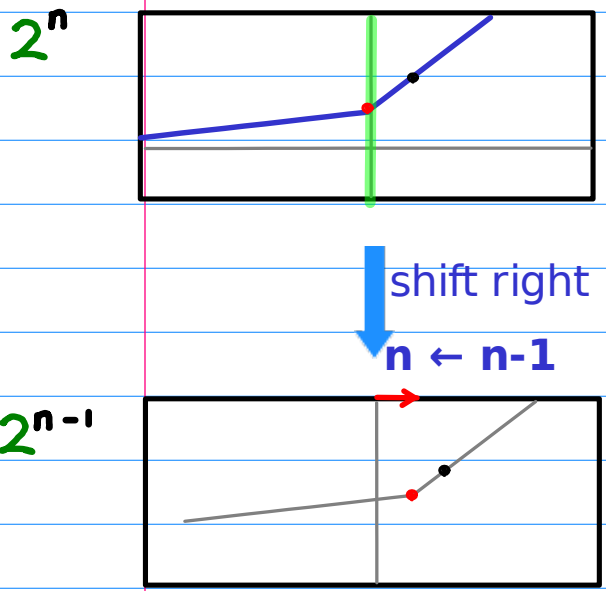
$b^n \Leftrightarrow b^{n+1}$



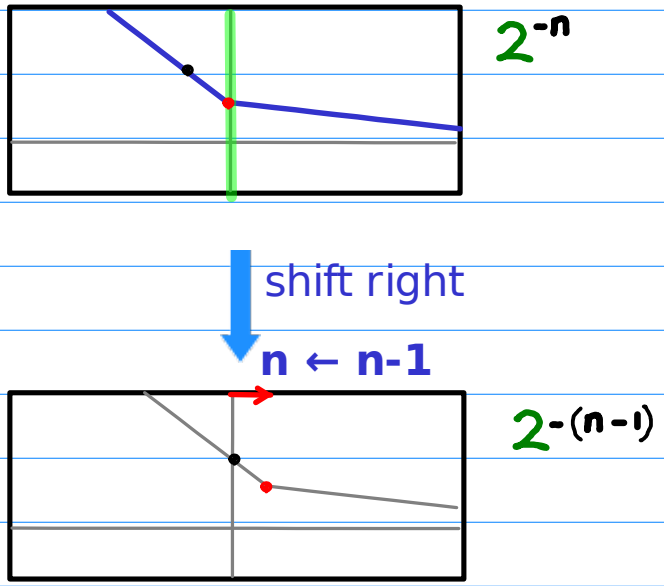
$b^n \Leftrightarrow b^{-(n+1)}$



$b^n \Leftrightarrow b^{n-1}$



$b^n \Leftrightarrow b^{-(n-1)}$

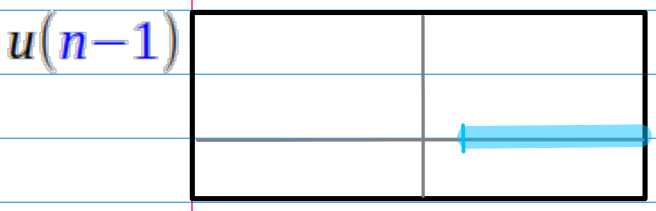


Range Shifting

$$shl(R(n)) = R(n+1)$$

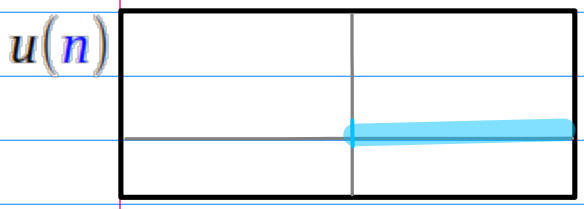
$$shr(R(n)) = R(n-1)$$

$R(n) \Longleftrightarrow R(n+1)$

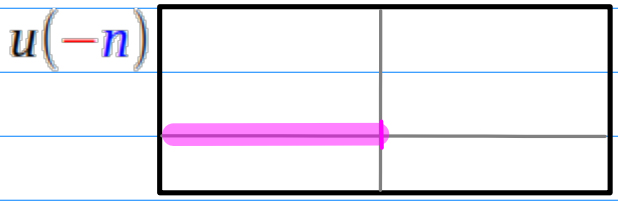


shift left

n ← n+1

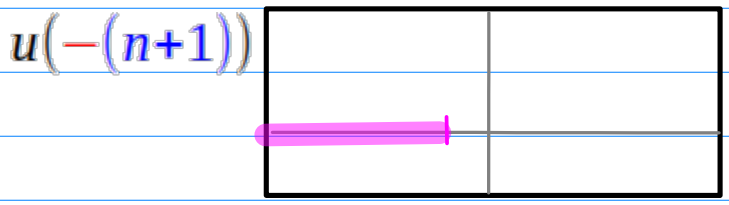


$R(n) \Longleftrightarrow R(n+1)$

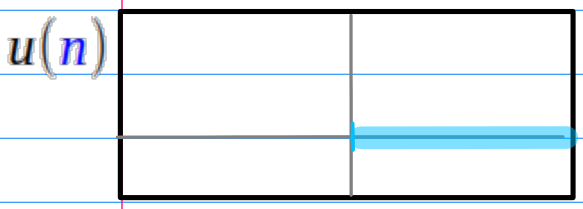


shift left

n ← n+1

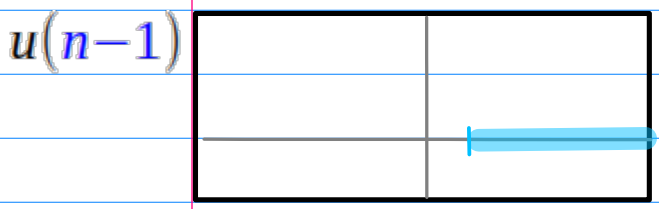


$R(n) \Longleftrightarrow R(n-1)$

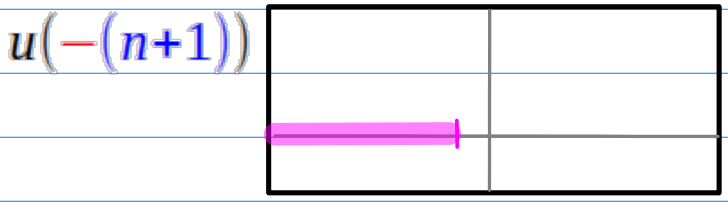


shift right

n ← n-1

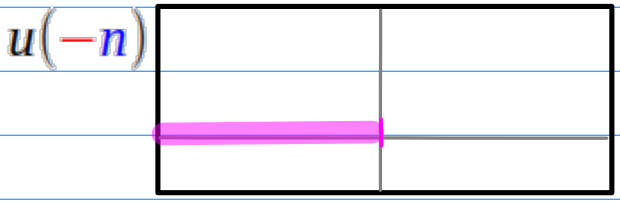


$R(n) \Longleftrightarrow R(n-1)$



shift right

n ← n-1



a^n × $R(n)$

a^{n+1}	a^{-n-1}
a^{n-1}	a^{-n+1}

×

u(n)	u(-n-1)
u(-n)	u(n-1)

(1')	a^{n+1}u(n)	a^{-n-1}u(n)	(2')
(3')	a^{n-1}u(-n)	a^{-n+1}u(-n)	(4')
(5')	a^{n+1}u(-n-1)	a^{-n-1}u(-n-1)	(6')
(7')	a^{n-1}u(n-1)	a^{-n+1}u(n-1)	(8')

(1'')	a^{n-1}u(n)	a^{-n+1}u(n)	(2'')
(3'')	a^{n+1}u(-n)	a^{-n-1}u(-n)	(4'')
(5'')	a^{n-1}u(-n-1)	a^{-n+1}u(-n-1)	(6'')
(7'')	a^{n+1}u(n-1)	a^{-n-1}u(n-1)	(8'')

a^n × $R(n)$

a^{n+1}	a^{-n-1}

×

u(n)	u(-n-1)
u(-n)	u(n-1)

(1')	$a^{n+1}u(n)$	$a^{-n-1}u(n)$	(2')
(3')	$a^{n-1}u(-n)$	$a^{-n+1}u(-n)$	(4')
(5')	$a^{n+1}u(-n-1)$	$a^{-n-1}u(-n-1)$	(6')
(7')	$a^{n-1}u(n-1)$	$a^{-n+1}u(n-1)$	(8')

a^n × $R(n)$

a^{n+1}	a^{-n-1}
a^{n-1}	a^{-n+1}

×

u(n)	u(-n-1)
u(-n)	u(n-1)

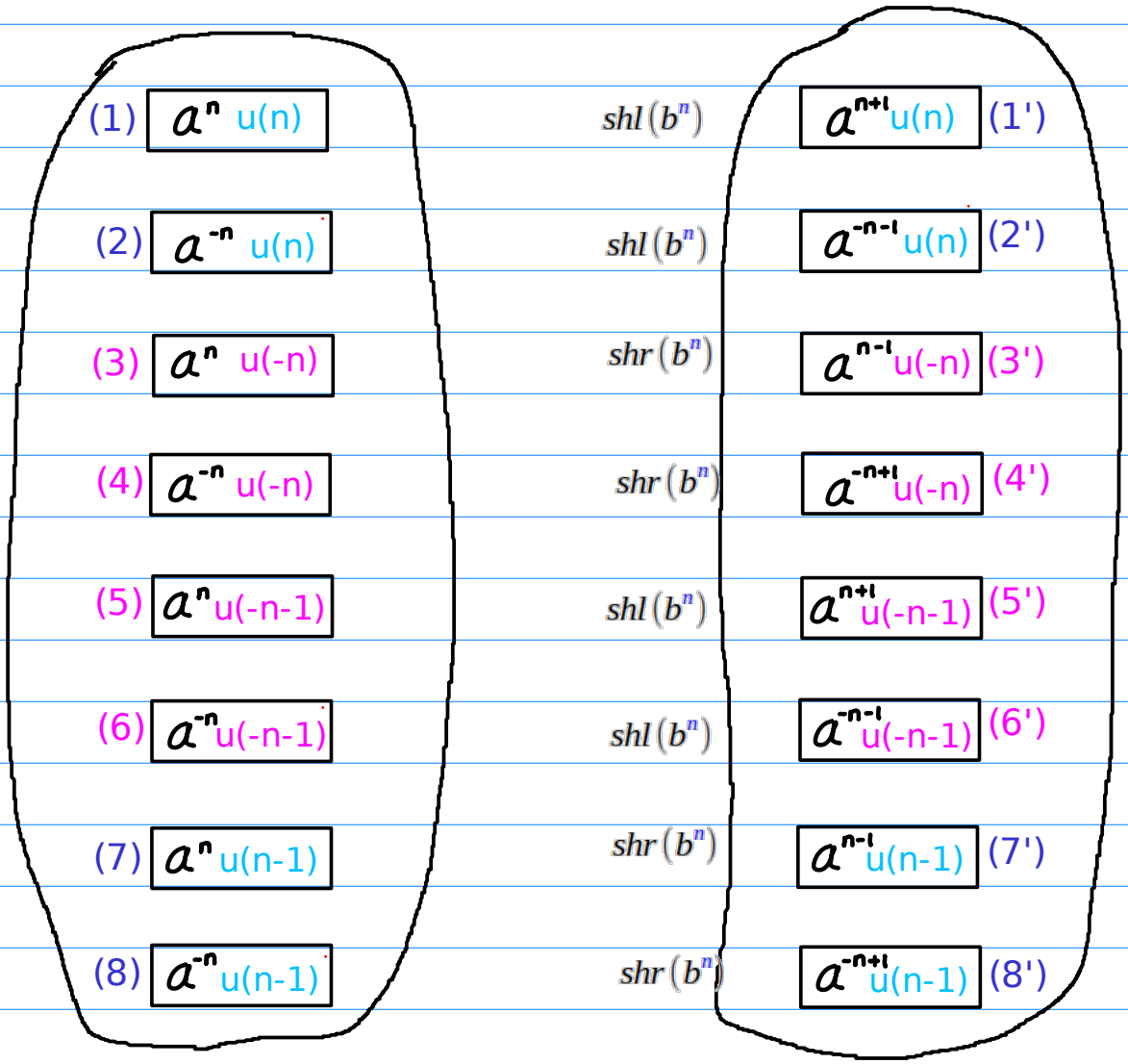
(1'')	$a^{n-1}u(n)$	$a^{-n+1}u(n)$	(2'')
(3'')	$a^{n+1}u(-n)$	$a^{-n-1}u(-n)$	(4'')
(5'')	$a^{n-1}u(-n-1)$	$a^{-n+1}u(-n-1)$	(6'')
(7'')	$a^{n+1}u(n-1)$	$a^{-n-1}u(n-1)$	(8'')

Unshifted Sequence x		Shifted Sequence 1 x'		Shifted Sequence 2 x''	
(1)	$a^{n}\,u(n)$	$shl(b^n)$	$a^{n+1}u(n)$ (1')	$shr(b^n)$	$a^{n-1}\,u(n)$ (1'')
(2)	$a^{-n}\,u(n)$	$shl(b^n)$	$a^{-n-1}u(n)$ (2')	$shr(b^n)$	$a^{-n+1}\,u(n)$ (2'')
(3)	$a^{n}\,u(-n)$	$shr(b^n)$	$a^{n-1}u(-n)$ (3')	$shl(b^n)$	$a^{n+1}\,u(-n)$ (3'')
(4)	$a^{-n}\,u(-n)$	$shr(b^n)$	$a^{-n+1}u(-n)$ (4')	$shl(b^n)$	$a^{-n-1}u(-n)$ (4'')
(5)	$a^{n}u(-n-1)$	$shl(b^n)$	$a^{n+1}u(-n-1)$ (5')	$shr(b^n)$	$a^{n-1}u(-n-1)$ (5'')
(6)	$a^{-n}u(-n-1)$	$shl(b^n)$	$a^{-n-1}u(-n-1)$ (6')	$shr(b^n)$	$a^{-n+1}u(-n-1)$ (6'')
(7)	$a^{n}u(n-1)$	$shr(b^n)$	$a^{n-1}u(n-1)$ (7')	$shl(b^n)$	$a^{n+1}u(n-1)$ (7'')
(8)	$a^{-n}u(n-1)$	$shr(b^n)$	$a^{-n+1}u(n-1)$ (8')	$shl(b^n)$	$a^{-n-1}u(n-1)$ (8'')

$$shl(b^n) \Longleftrightarrow shr(b^n)$$

$$shr(b^n) \Longleftrightarrow shl(b^n)$$

many possible permuations are possible
but consider these two

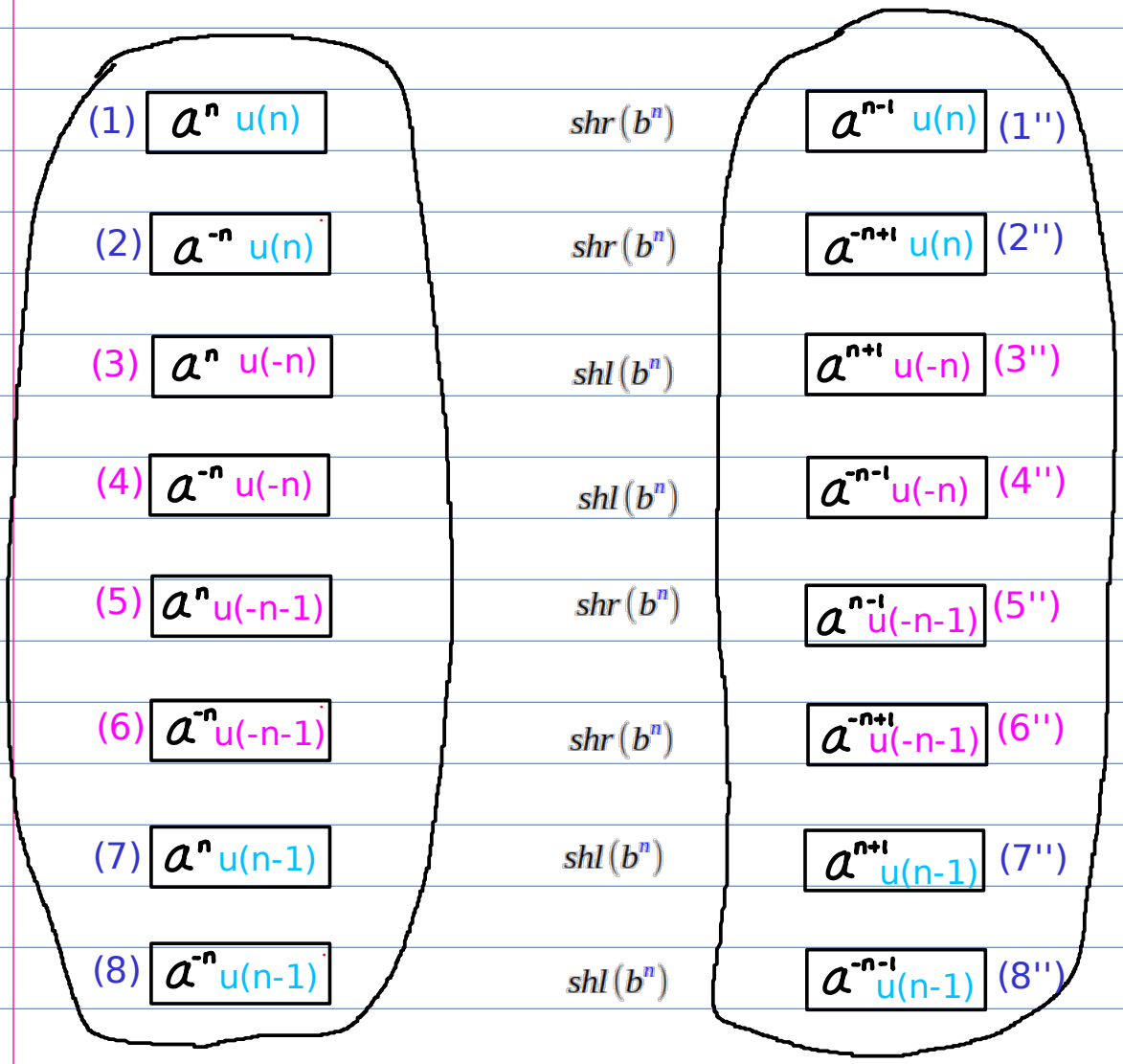


Inter-permutations over unshifted sequnce and shifted sequence

Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence

Unshifted Sequence x → **Shifted Sequence 2 x''**



Inter-permutations over unshifted sequnce and shifted sequence

Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence

(x') ⟺ **(x'')**

(1')~(8') (1'')~(8'')

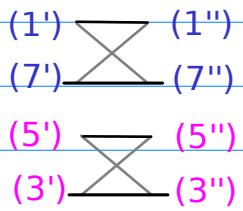
Exponent Conversion

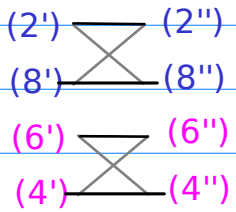
$$shl\left(b^n\right) \Longleftrightarrow shr\left(b^n\right)$$

$$shr\left(b^n\right) \Longleftrightarrow shl\left(b^n\right)$$

(1') (n+1) ——— (1'') (n−1)
(2') -(n+1) ——— (2'') -(n−1)
(3') (n−1) ——— (3'') (n+1)
(4') -(n−1) ——— (4'') -(n+1)
(5') (n+1) ——— (5'') (n−1)
(6') -(n+1) ——— (6'') -(n−1)
(7') (n−1) ——— (7'') (n+1)
(8') -(n−1) ——— (8'') -(n+1)

Identical Exponent Elements



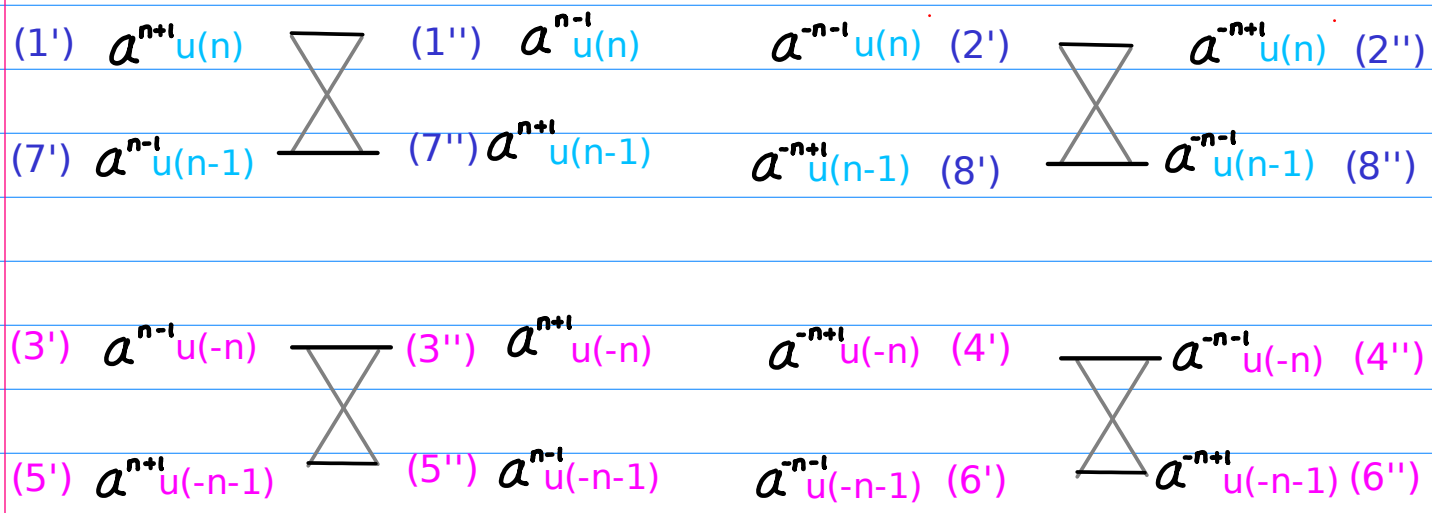


(1') (n+1) (1'') (n−1)
(2') -(n+1) (2'') -(n−1)
(3') (n−1) (3'') (n+1)
(4') -(n−1) (4'') -(n+1)
(5') (n+1) (5'') (n−1)
(6') -(n+1) (6'') -(n−1)
(7') (n−1) (7'') (n+1)
(8') -(n−1) (8'') -(n+1)

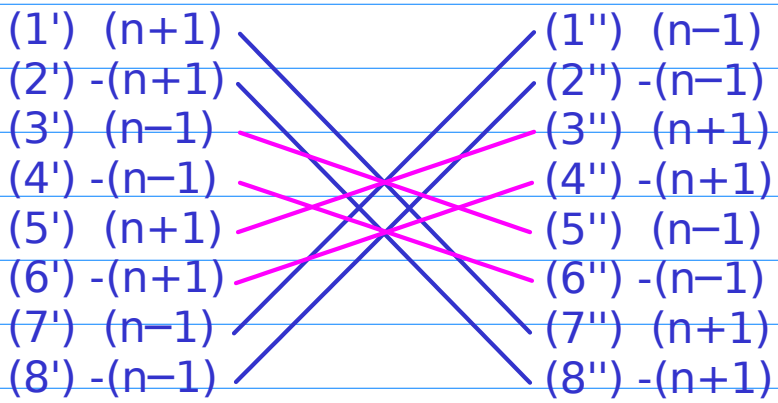
(1')	(n+1)	shl(n)	(1'')	(n−1)	shr(n)
(7')	(n−1)	shr(n)	(7'')	(n+1)	shl(n)
(2')	-(n+1)	shl(n)	(2'')	-(n−1)	shr(n)
(8')	-(n−1)	shr(n)	(8'')	-(n+1)	shl(n)
(3')	(n−1)	shr(n)	(3'')	(n+1)	shl(n)
(5')	(n+1)	shl(n)	(5'')	(n−1)	shr(n)
(4')	-(n−1)	shr(n)	(4'')	-(n+1)	shl(n)
(6')	-(n+1)	shl(n)	(6'')	-(n−1)	shr(n)

(x') ⟺ **(x'')**

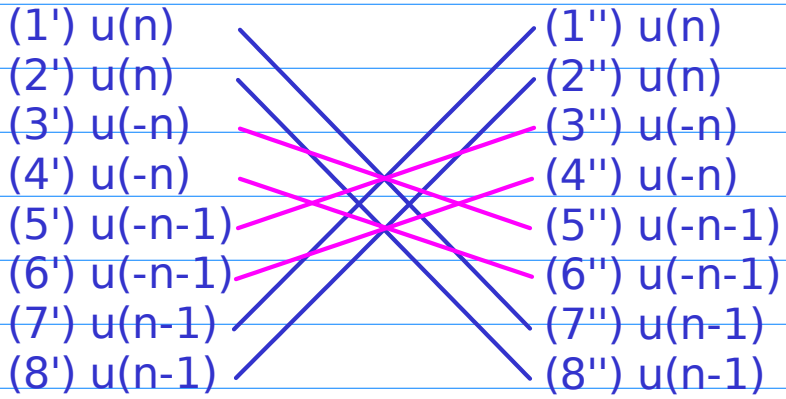
(1')~(8') (1'')~(8'')



Identical Exponent Elements



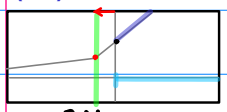
Shifted Range Elements



(x') ⟺ (x'')

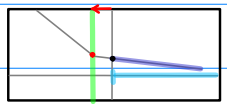
(1')~(8') (1'')~(8'')

(1') 1000



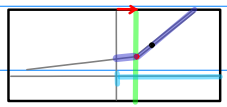
a^{n+1} u(n)

(2') 1001



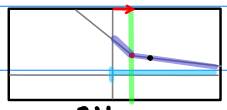
a^{-n-1} u(n)

(1'') 1000



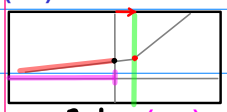
a^{n-1} u(n)

(2'') 1001



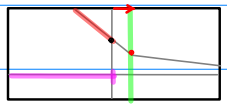
a^{-n+1} u(n)

(3') 1010



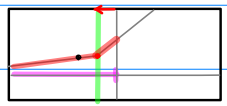
a^{n-1} u(-n)

(4') 1011



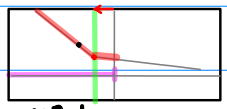
a^{-n+1} u(-n)

(3'') 1010



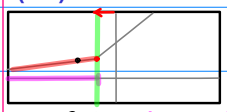
a^{n+1} u(-n)

(4'') 1011



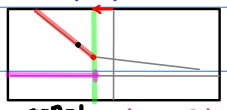
a^{-n-1} u(-n)

(5') 1100



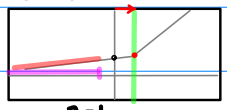
a^{n+1} u(-n-1)

(6') 1101



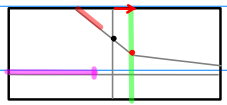
a^{-n-1} u(-n-1)

(5'') 1100



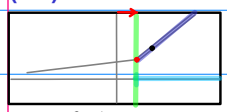
a^{n-1} u(-n-1)

(6'') 1101



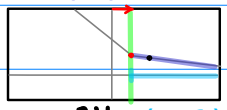
a^{-n+1} u(-n-1)

(7') 1110



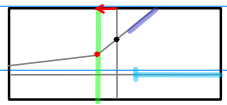
a^{n-1} u(n-1)

(8') 1111



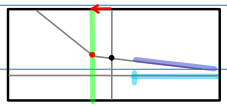
a^{-n+1} u(n-1)

(7'') 1110



a^{n+1} u(n-1)

(8'') 1111



a^{-n-1} u(n-1)

Conversion between x' and x''

$$shl(b^n) \Longleftrightarrow shr(b^n)$$

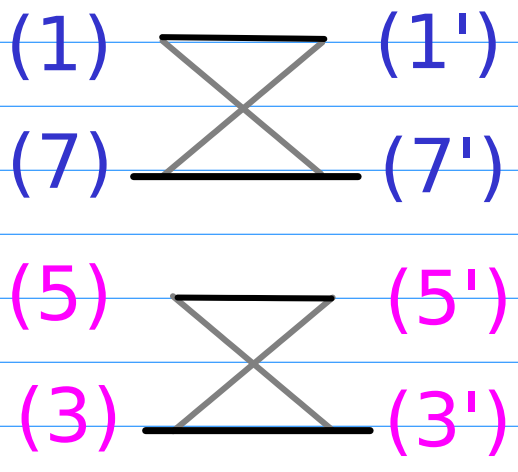
$$shr(b^n) \Longleftrightarrow shl(b^n)$$

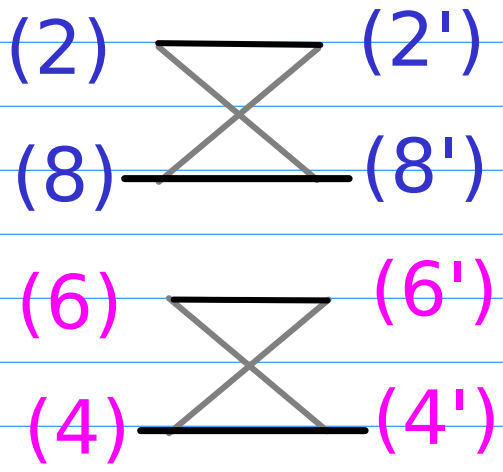
(1) - (1')
(7) - (7')
(5) - (5')
(3) - (3')

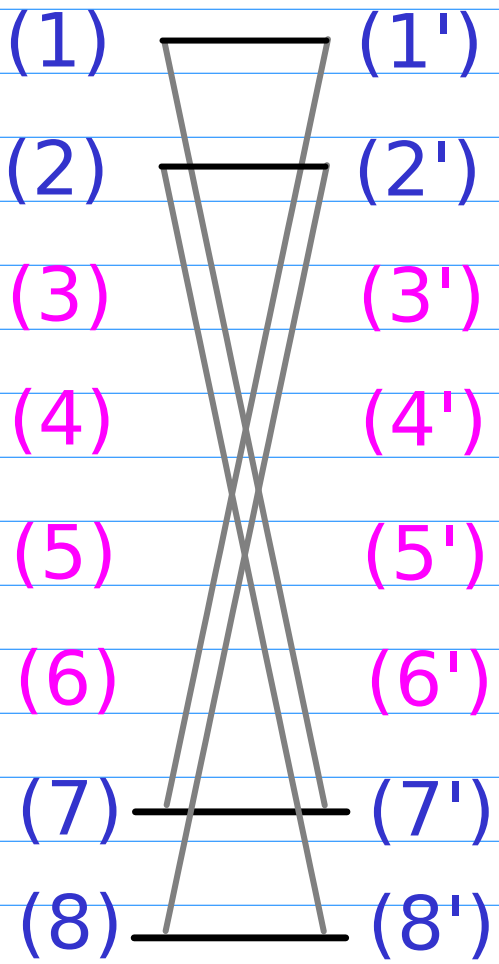
(2) - (2')
(8) - (8')
(6) - (6')
(4) - (4')

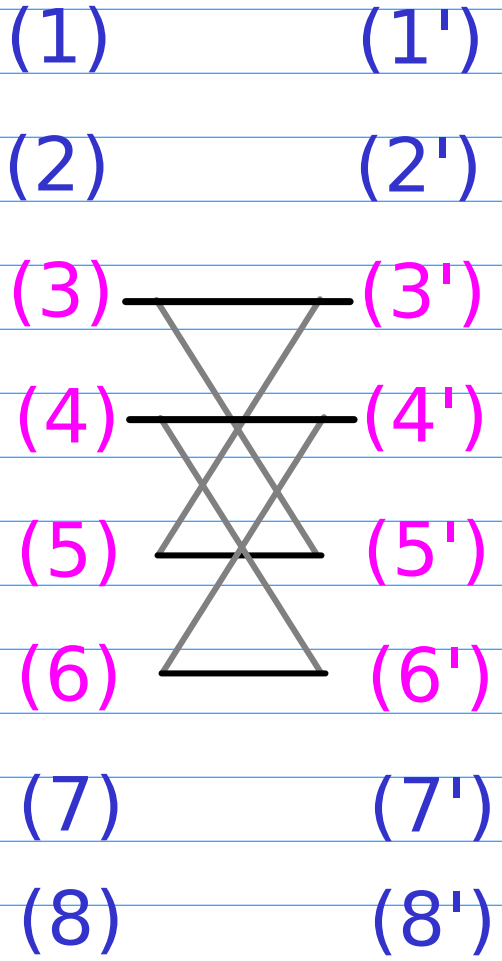
(x) → **(x')**

(1)~(8) (1')~(8')







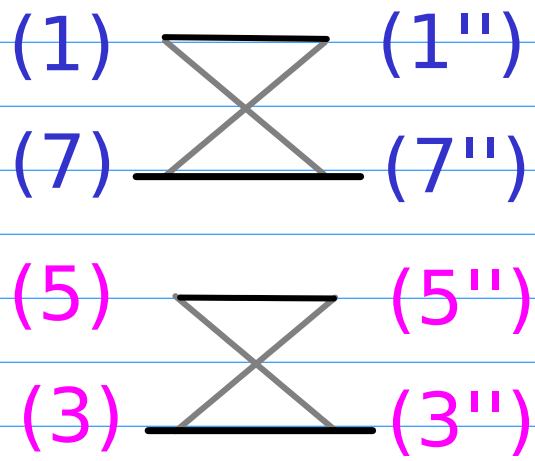


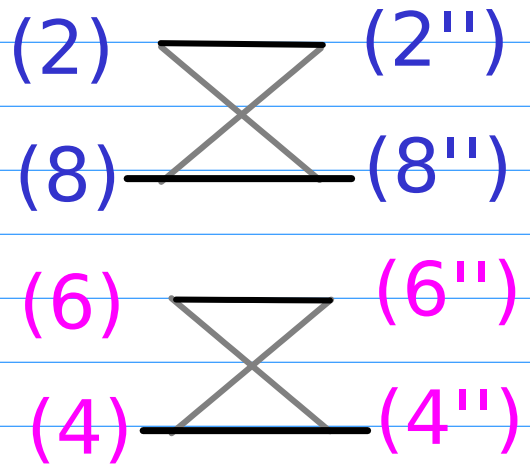
(1) - (1')
(7) - (7')
(5) - (5')
(3) - (3')

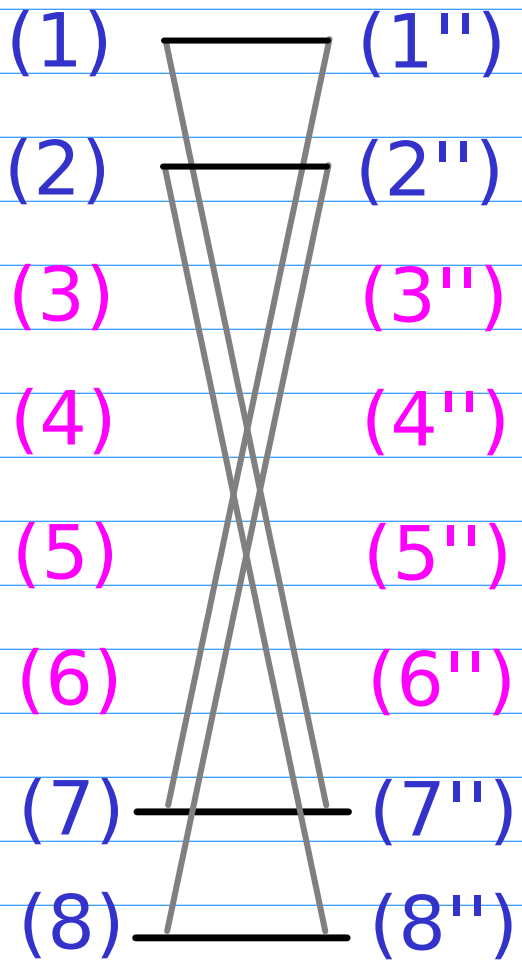
(2) - (2')
(8) - (8')
(6) - (6')
(4) - (4')

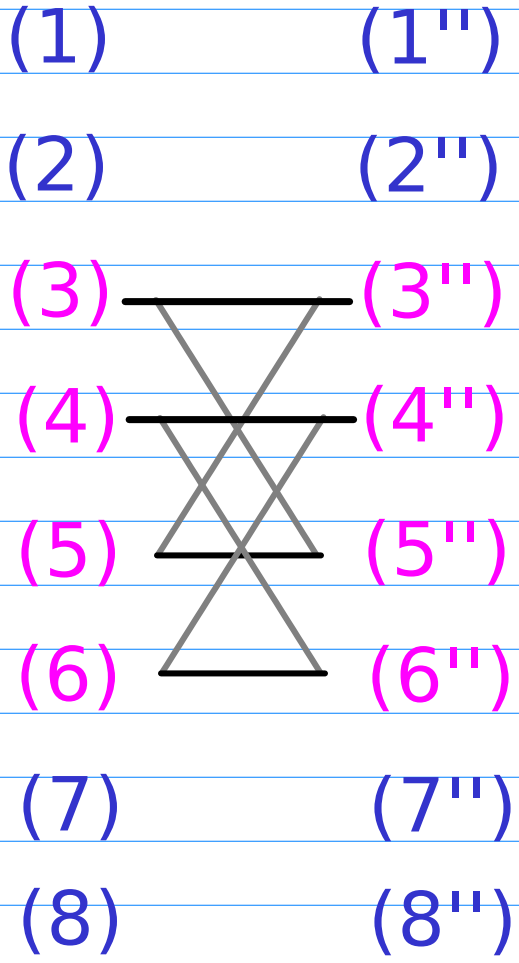
(x) → **(x'')**

(1)~(8) (1')~(8')

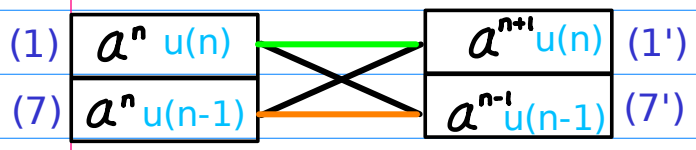


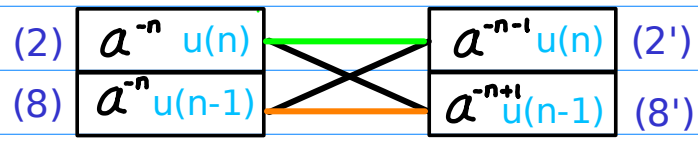


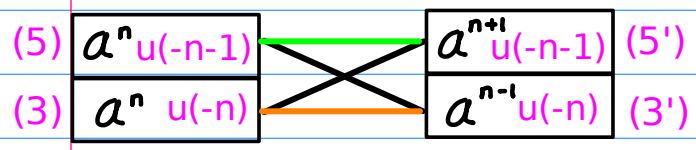




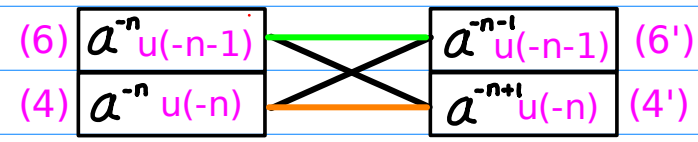
A Shifting Shifted Sequence 1





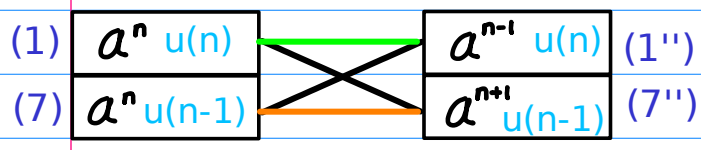


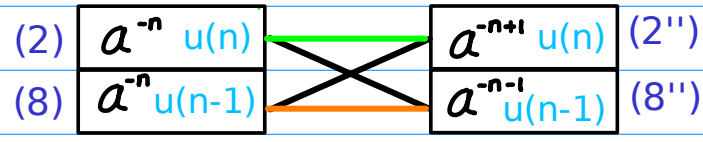
$b = a$

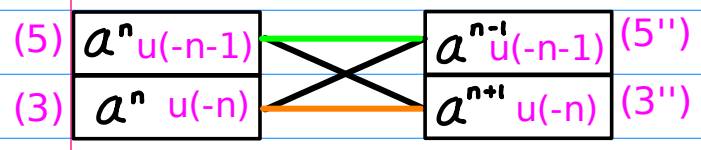


$b = a^{-1}$

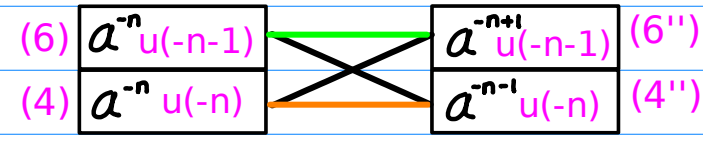
B Shifting Shifted Sequence 2





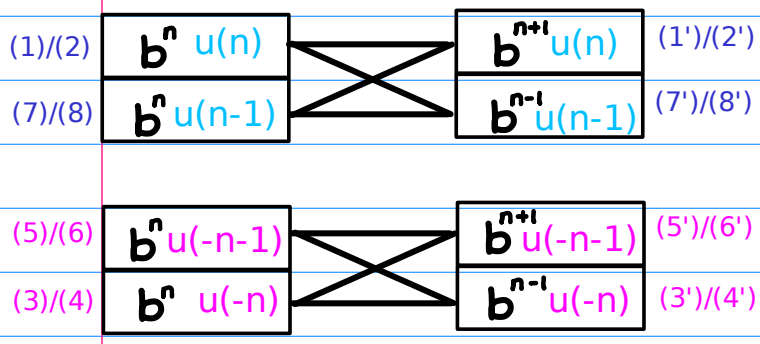


$b = a$

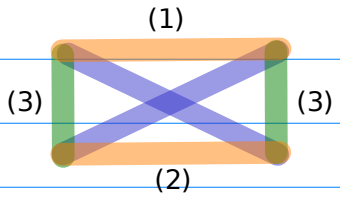


$b = a^{-1}$

A Shifting Shifted Sequence 1
(1) Exponent Shifting
(2) Range Shifting



$b = a/a^{-1}$



$$b^n \overset{(1)}{\Longleftrightarrow} b^{n+1}$$

$$b^n \overset{(2)}{\Longleftrightarrow} b^{n-1}$$

$$R(n) \overset{(3)}{\Longleftrightarrow} R(n-1)$$

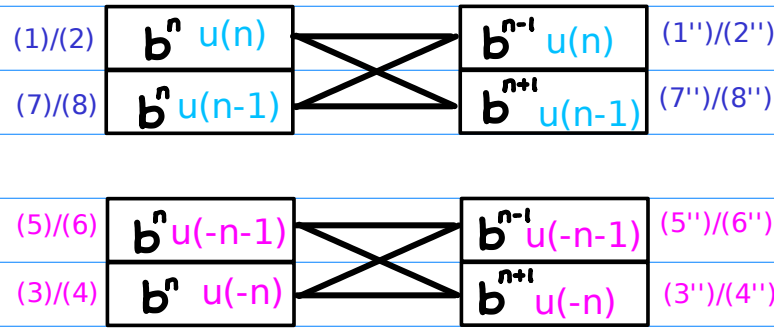
Exponent Shifting

b^n	$b^{sh(n)}$	
a^n	$a^{(n+1)}$	$shl(b^n)$
a^n	$a^{(n-1)}$	$shr(b^n)$
a^{-n}	$a^{-(n+1)}$	$shl(b^n)$
a^{-n}	$a^{-(n-1)}$	$shr(b^n)$

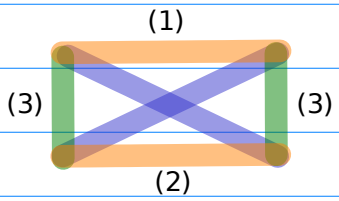
Range Shifting

$R(n)$	$R(sh(n))$	
$u(n)$	$u(n-1)$	$shr(R(n))$
$u(-(n+1))$	$u(-n)$	$shr(R(n))$

B Shifting Shifted Sequence 2
(1) Exponent Shifting
(2) Range Shifting



$b = a/a^{-1}$



$$b^n \overset{(1)}{\Longleftrightarrow} b^{n-1}$$

$$b^n \overset{(2)}{\Longleftrightarrow} b^{n+1}$$

$$R(n) \overset{(3)}{\Longleftrightarrow} R(n-1)$$

Exponent Shifting

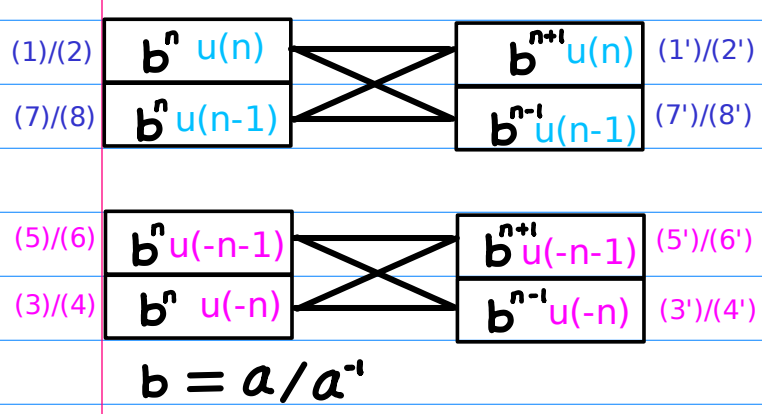
b^n	$b^{sh(n)}$	
a^n	$a^{(n-1)}$	$shr(b^n)$
a^n	$a^{(n+1)}$	$shl(b^n)$
a^{-n}	$a^{-(n-1)}$	$shr(b^n)$
a^{-n}	$a^{-(n+1)}$	$shl(b^n)$

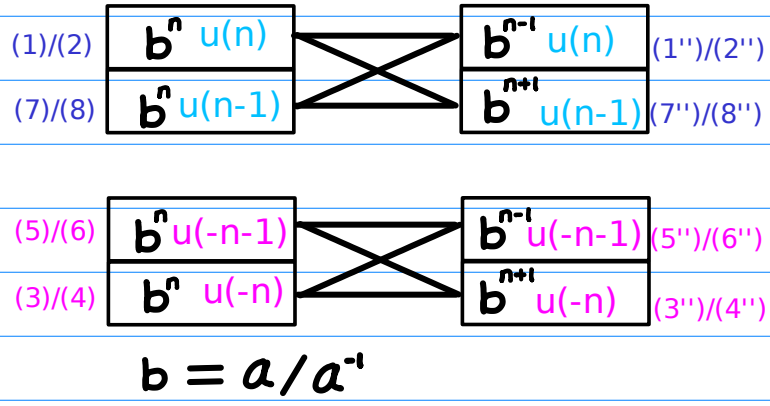
Range Shifting

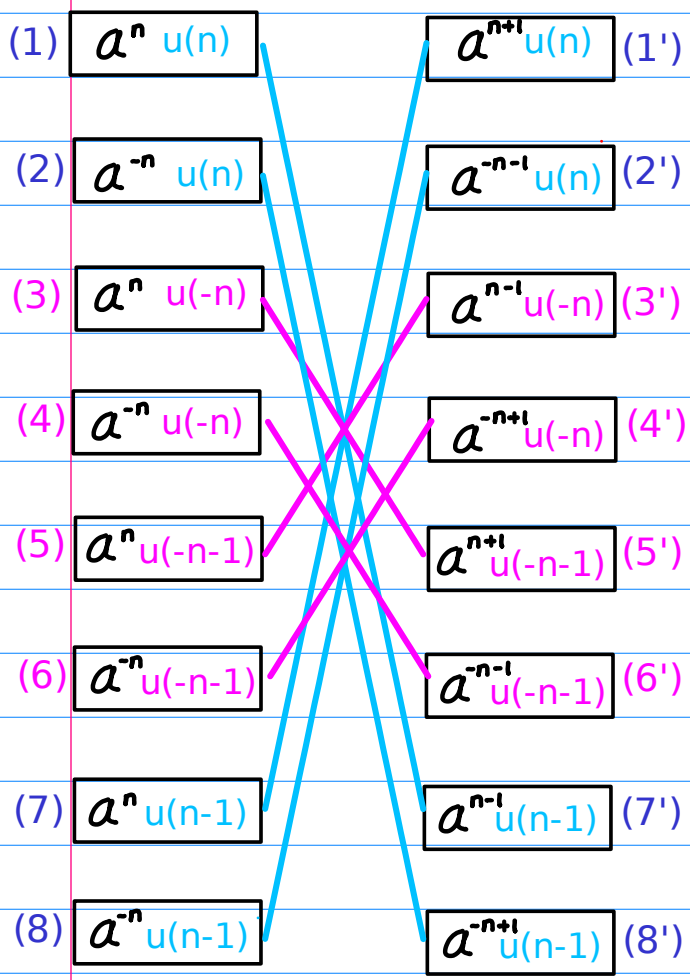
$R(n)$	$R(sh(n))$	
$u(n)$	$u(n-1)$	$shr(R(n))$
$u(-(n+1))$	$u(-n)$	$shr(R(n))$

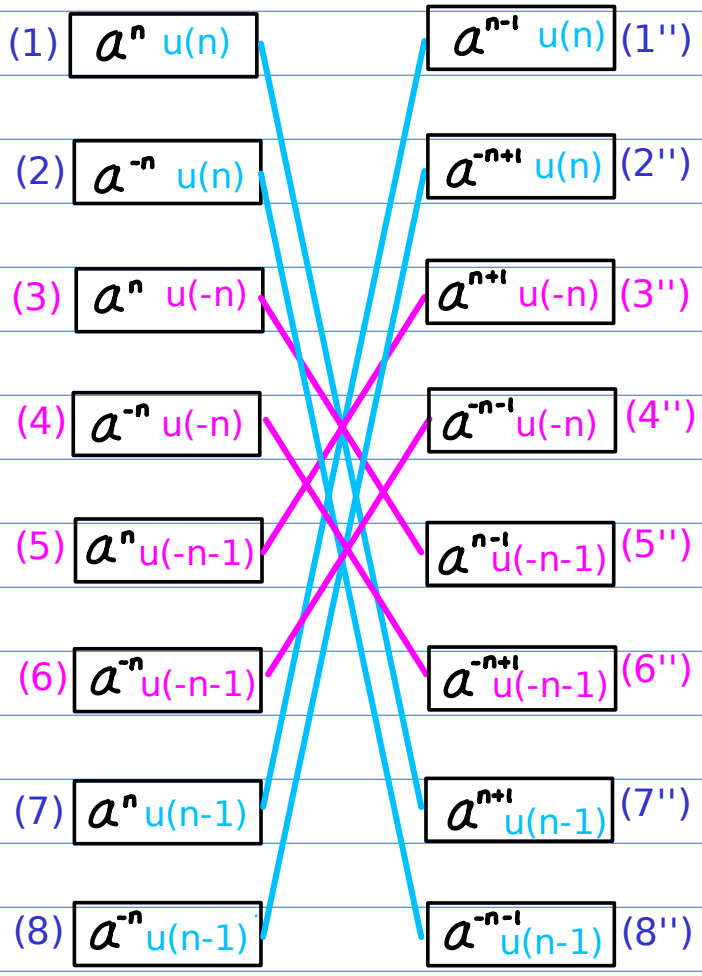
A Shifting Shifted Sequence 1

B Shifting Shifted Sequence 2

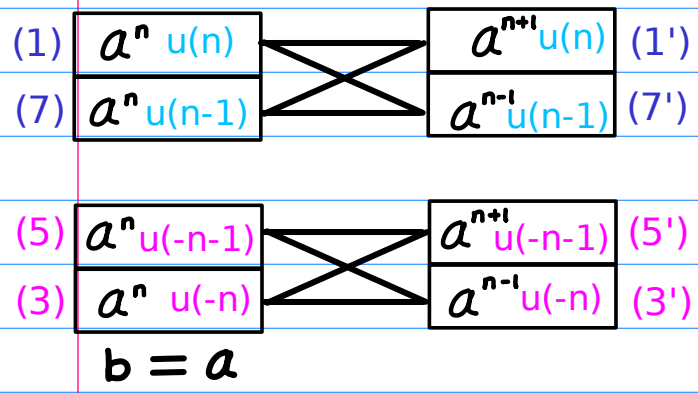


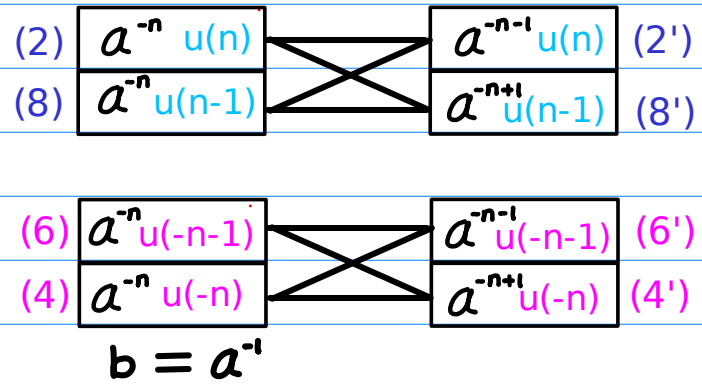


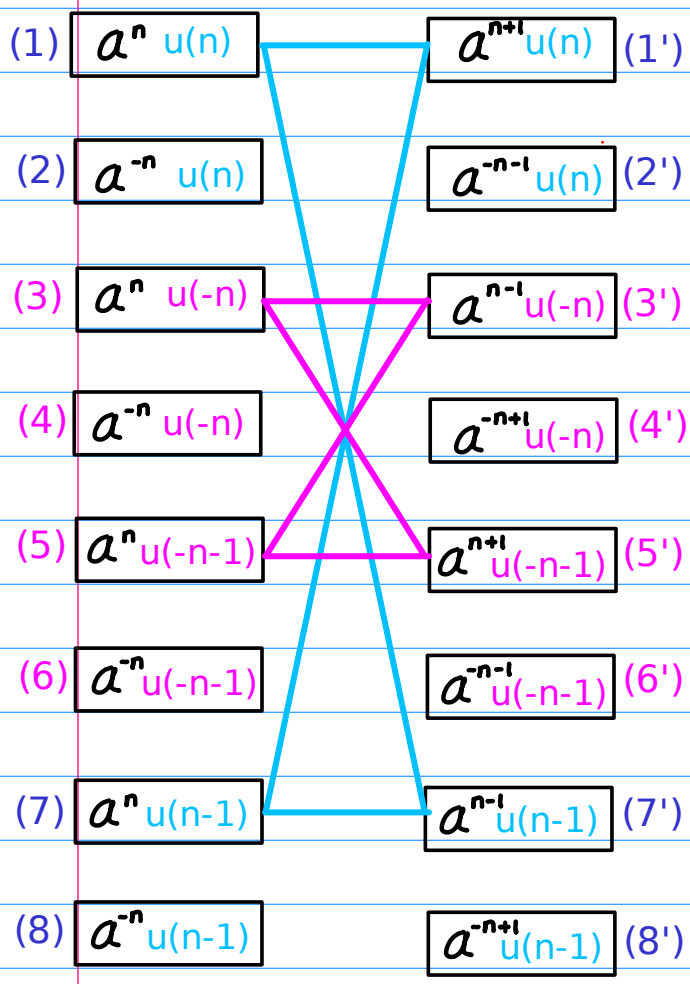


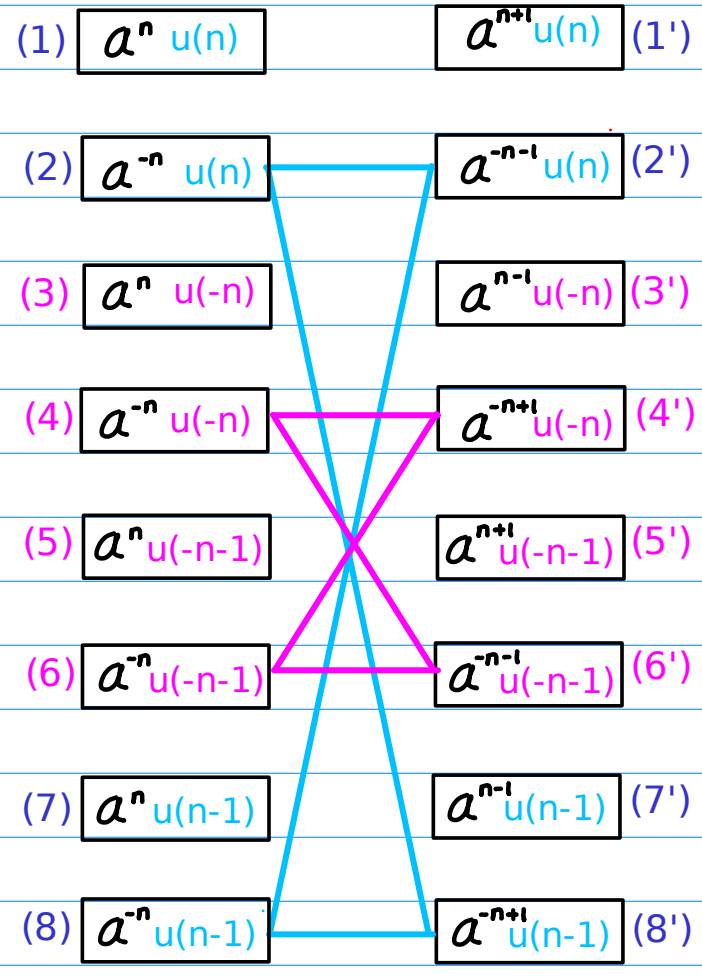


A Shifting Shifted Sequence 1

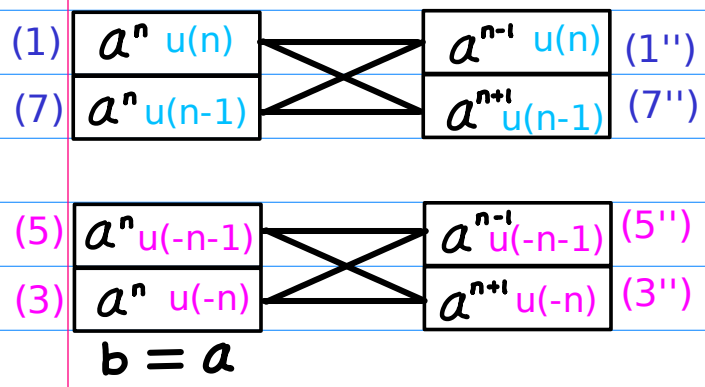


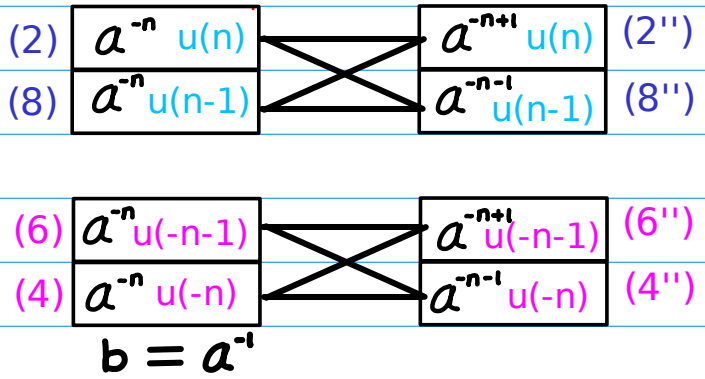


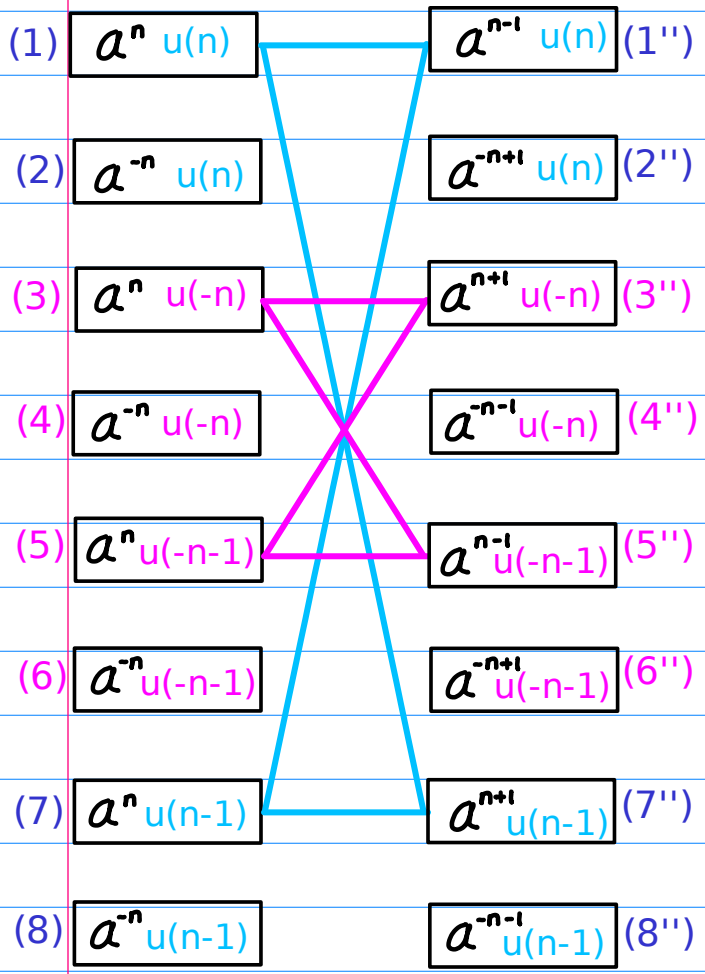


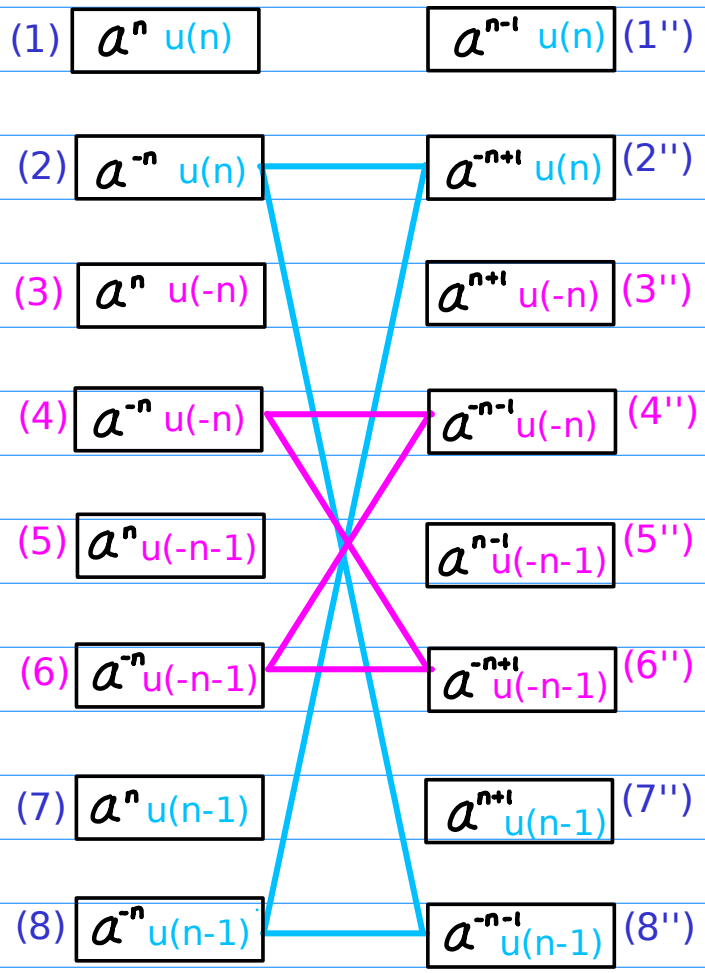


B Shifting Shifted Sequence 2



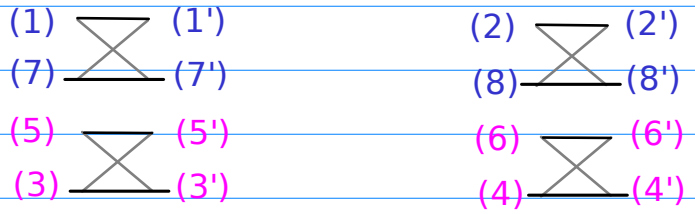


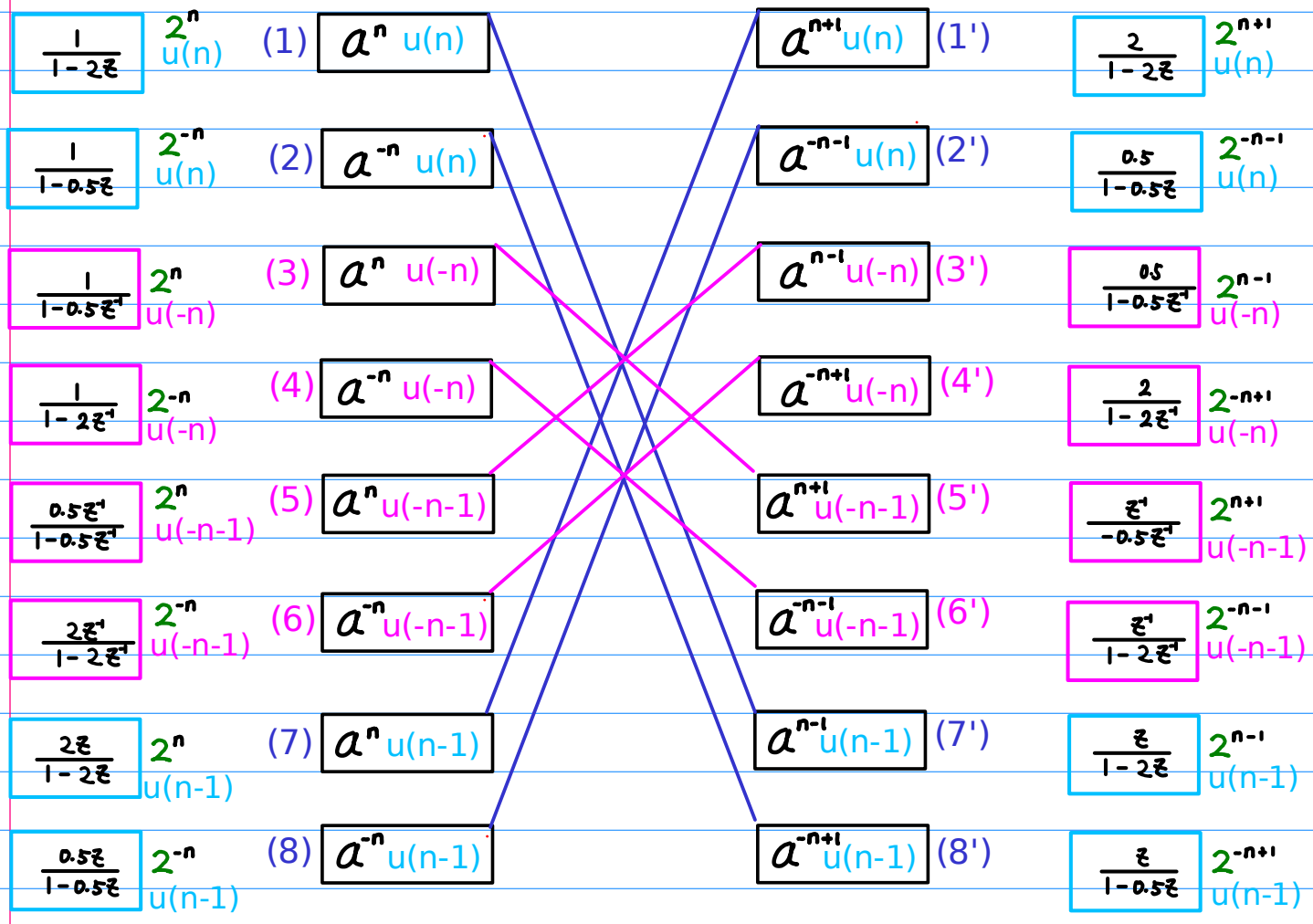




A Shifting Shifted Sequence 1

Unshifted Sequence x → **Shifted Sequence 1 x'**



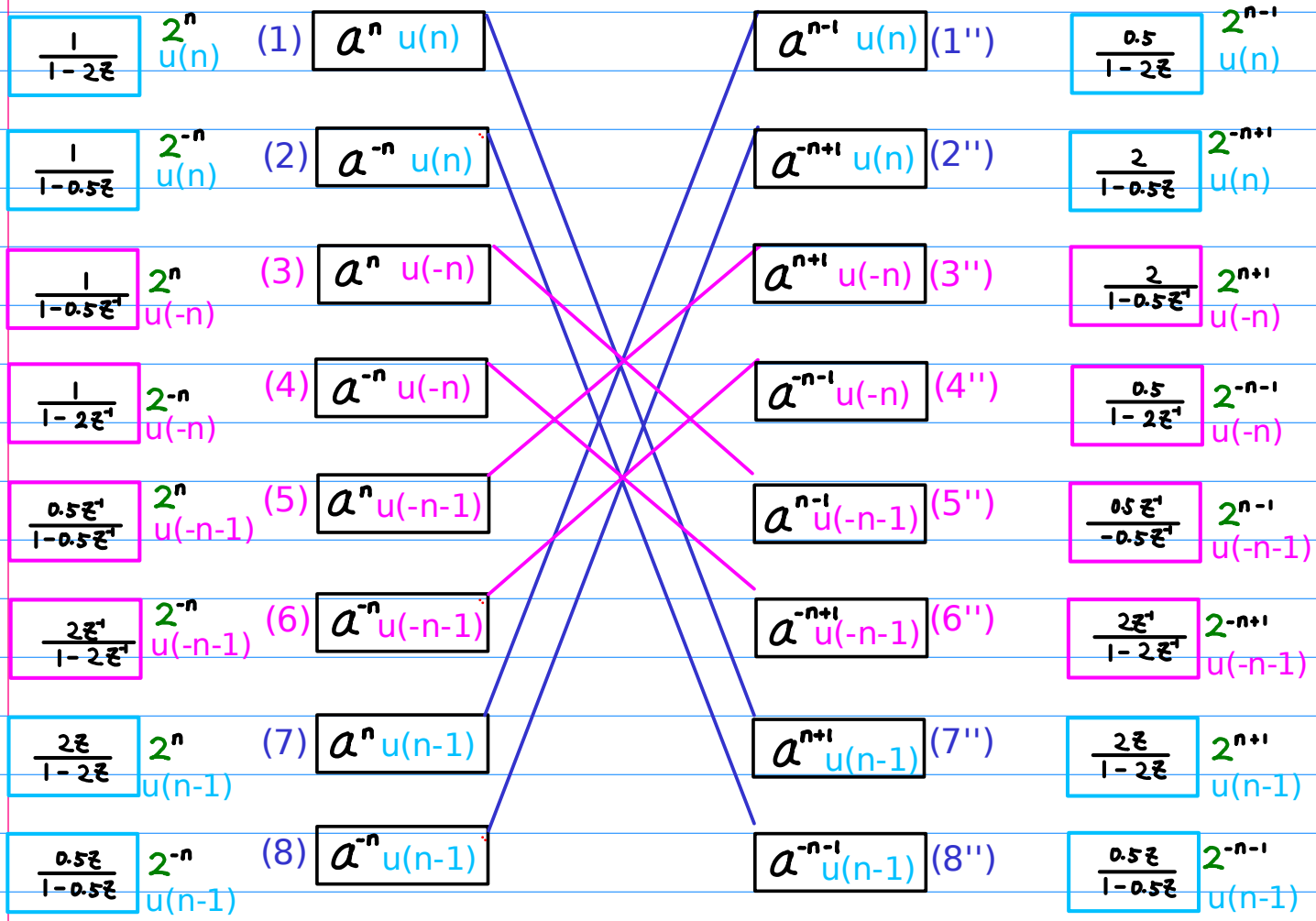


B Shifting Shifted Sequence 2

Unshifted Sequence x → **Shifted Sequence 2 x''**

(1) ⟷ (1'')
(7) ⟷ (7'')
(5) ⟷ (5'')
(3) ⟷ (3'')

(2) ⟷ (2'')
(8) ⟷ (8'')
(6) ⟷ (6'')
(4) ⟷ (4'')



Unshifted Sequence x' → **Shifted Sequence 2 x''**

(1') ⨉ (1'')
(7') ⨉ (7'')
(5') ⨉ (5'')
(3') ⨉ (3'')

(2') ⨉ (2'')
(8') ⨉ (8'')
(6') ⨉ (6'')
(4') ⨉ (4'')

$\frac{2}{1-2z}$	$2^{n+1}u(n)$	(1')	$a^{n+1}u(n)$	$a^{n-1}u(n)$ (1'')	$\frac{0.5}{1-2z}$ $2^{n-1}u(n)$
$\frac{0.5}{1-0.5z}$	$2^{-n-1}u(n)$	(2')	$a^{-n-1}u(n)$	$a^{-n+1}u(n)$ (2'')	$\frac{2}{1-0.5z}$ $2^{-n+1}u(n)$
$\frac{0.5}{1-0.5z^{-1}}$	$2^{n-1}u(-n)$	(3')	$a^{n-1}u(-n)$	$a^{n+1}u(-n)$ (3'')	$\frac{2}{1-0.5z^{-1}}$ $2^{n+1}u(-n)$
$\frac{2}{1-2z^{-1}}$	$2^{-n+1}u(-n)$	(4')	$a^{-n+1}u(-n)$	$a^{-n-1}u(-n)$ (4'')	$\frac{0.5}{1-2z^{-1}}$ $2^{-n-1}u(-n)$
$\frac{z^{-1}}{-0.5z^{-1}}$	$2^{n+1}u(-n-1)$	(5')	$a^{n+1}u(-n-1)$	$a^{n-1}u(-n-1)$ (5'')	$\frac{0.5z^{-1}}{-0.5z^{-1}}$ $2^{n-1}u(-n-1)$
$\frac{z^{-1}}{1-2z^{-1}}$	$2^{-n-1}u(-n-1)$	(6')	$a^{-n-1}u(-n-1)$	$a^{-n+1}u(-n-1)$ (6'')	$\frac{2z^{-1}}{1-2z^{-1}}$ $2^{-n+1}u(-n-1)$
$\frac{z}{1-2z}$	$2^{n-1}u(n-1)$	(7')	$a^{n-1}u(n-1)$	$a^{n+1}u(n-1)$ (7'')	$\frac{2z}{1-2z}$ $2^{n+1}u(n-1)$
$\frac{z}{1-0.5z}$	$2^{-n+1}u(n-1)$	(8')	$a^{-n+1}u(n-1)$	$a^{-n-1}u(n-1)$ (8'')	$\frac{0.5z}{1-0.5z}$ $2^{-n-1}u(n-1)$

$shl(b^n) \Longleftrightarrow shr(b^n)$

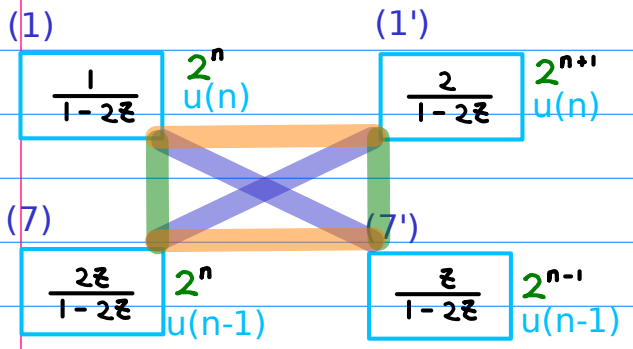
$shr(b^n) \Longleftrightarrow shl(b^n)$

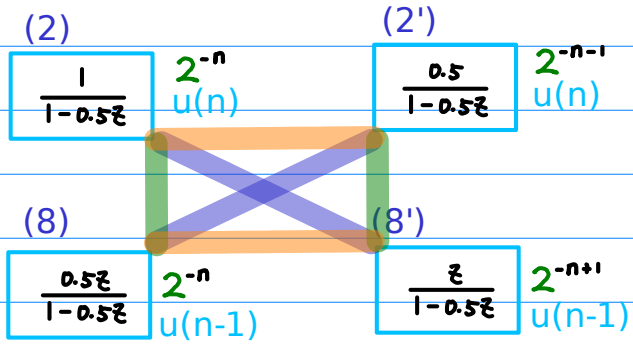
(1') (n+1)	(1'') (n-1)
(2') -(n+1)	(2'') -(n-1)
(3') (n-1)	(3'') (n+1)
(4') -(n-1)	(4'') -(n+1)
(5') (n+1)	(5'') (n-1)
(6') -(n+1)	(6'') -(n-1)
(7') (n-1)	(7'') (n+1)
(8') -(n-1)	(8'') -(n+1)

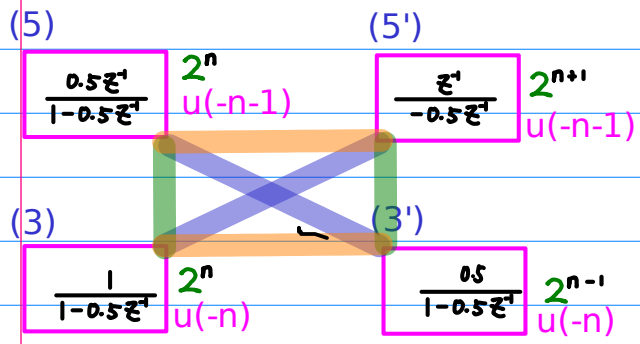
Inter-permutation (x) → (x')

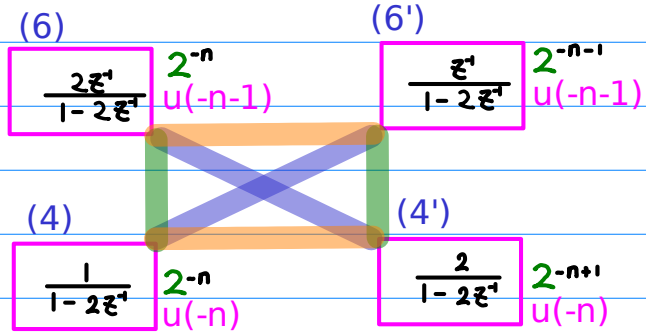
A Shifting Shifted Sequence 1

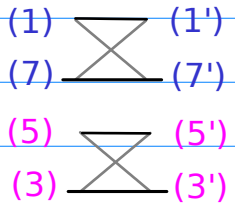
(1)~(8) (1')~(8')

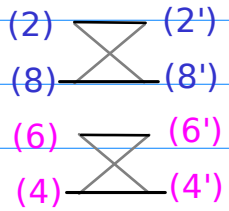


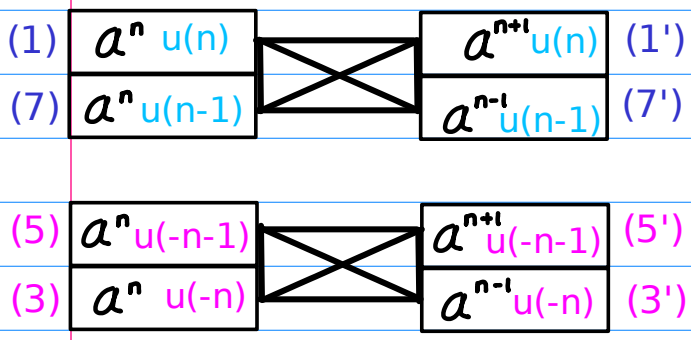


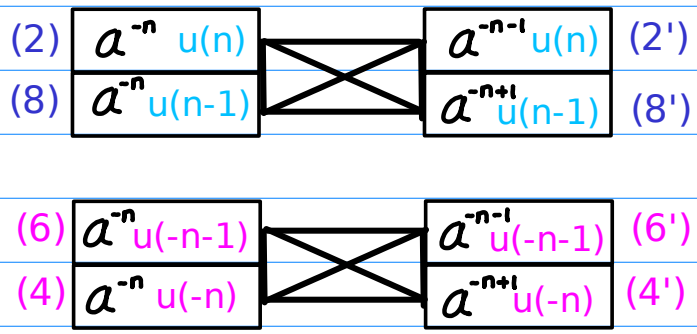








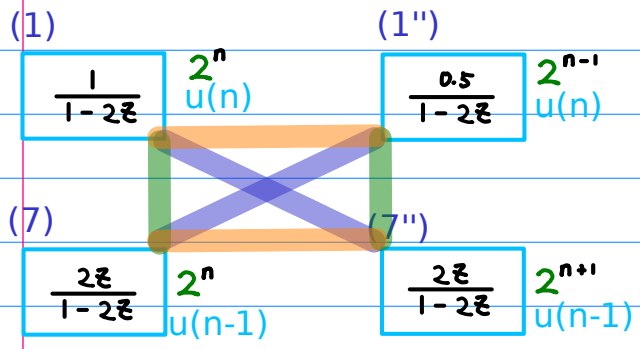


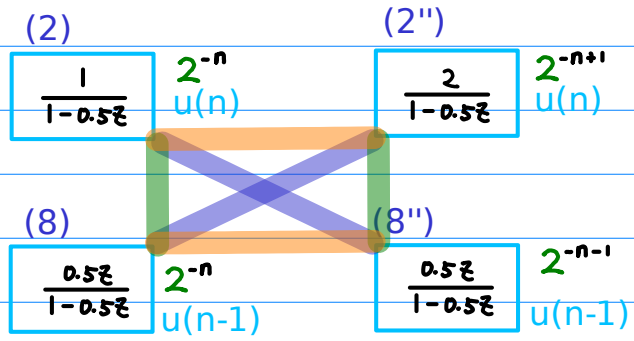


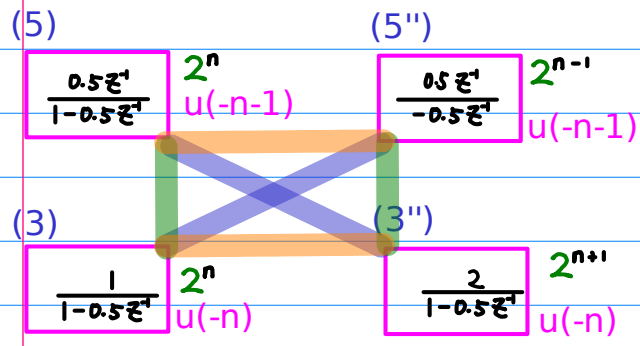
Inter-permutation (x) → (x'')

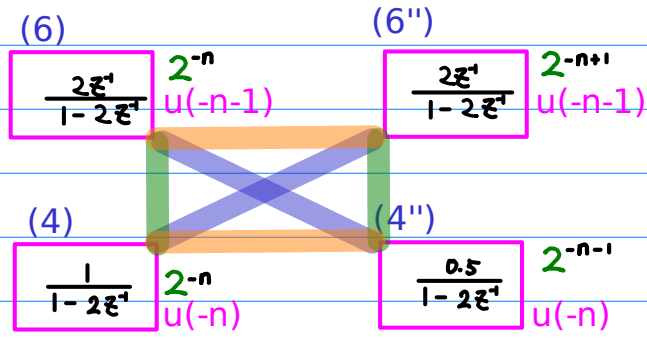
B Shifting Shifted Sequence 2

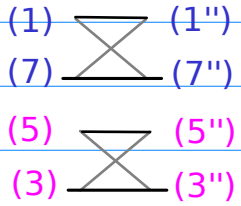
(1)~(8) → (1'')~(8'')

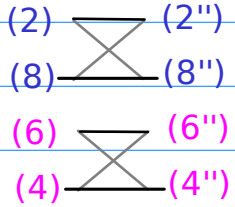


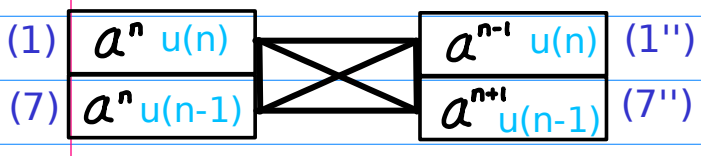


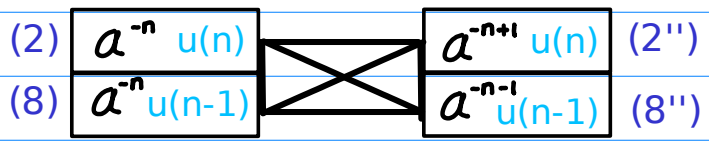


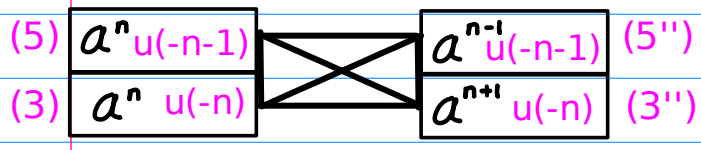


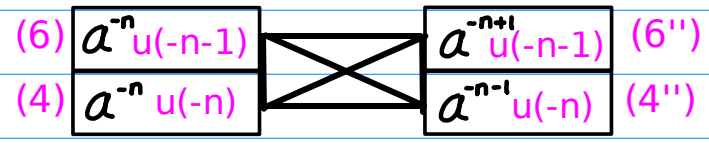




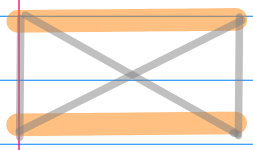




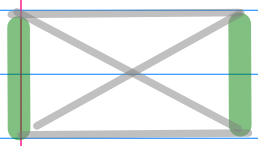




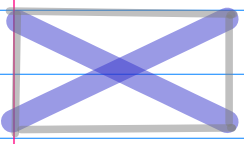
Decomposing Shift Operations



(exponent shift, identity)



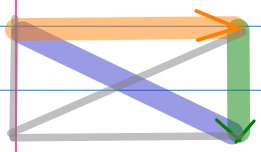
(identity, range shift)

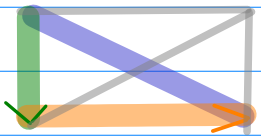


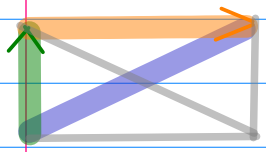
(exponent shift, range sift)

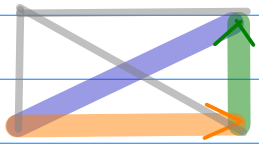
= (exponent shift, identity)

+ (identity, range shift)









A Shifting Shifted Sequence 1

(1) Exponent Shifting

(2) Range Shifting

(SR, id) + (id, SR) = (SR, SR)

(SL, id) + (id, SL) = (SL, SL)

B Shifting Shifted Sequence 2

(1) Exponent Shifting, Flipping

(2) Range Shifting

(SR, id) + (id, SL) = (SR, SL)

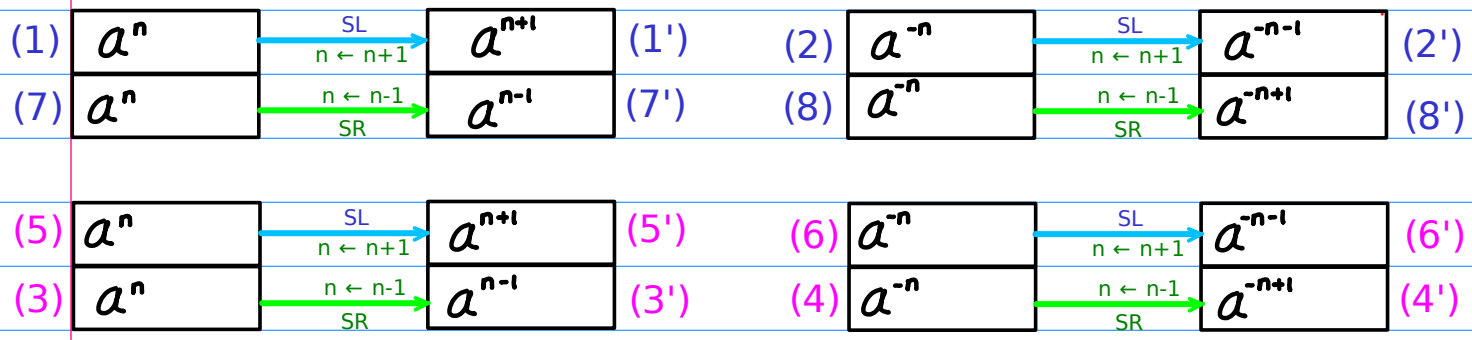
(SL, id) + (id, SR) = (SL, SR)

A Shifting Shifted Sequence 1

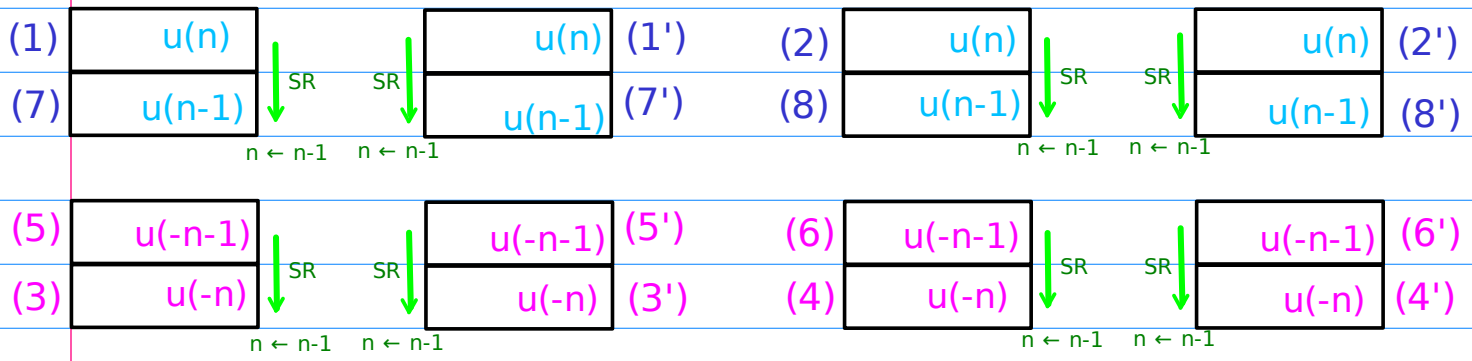
(1) Exponent Shifting (SR, id) or (SL, id)

(2) Range Shifting (id, SR) or (id, SL)

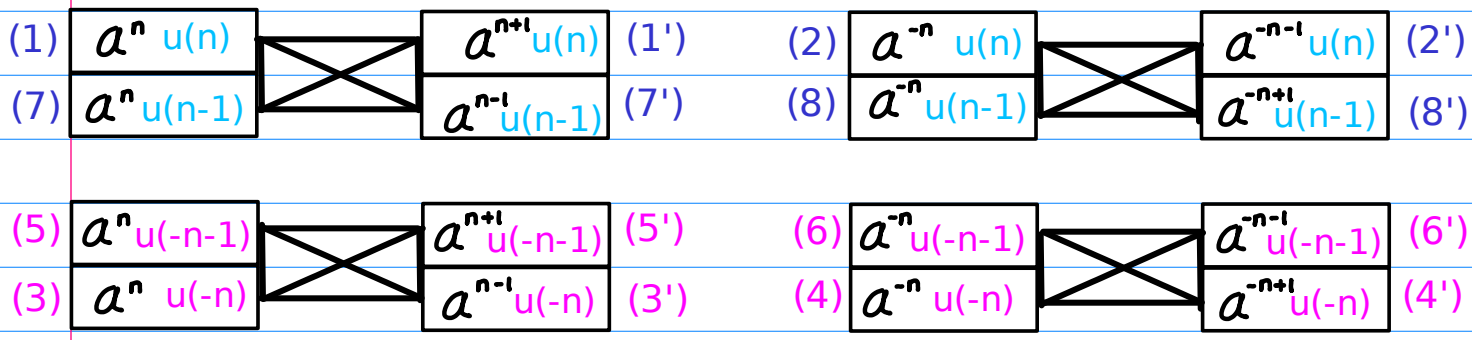
Exponent Shifts : (SR, id) or (SL, id)



Range Shifts : (id, SR) or (id, SL)



Exponent & Range Permutations



Decomposition

(EP, RP) = (EP, id) + (id, RP)

EP : Exponent Permuations

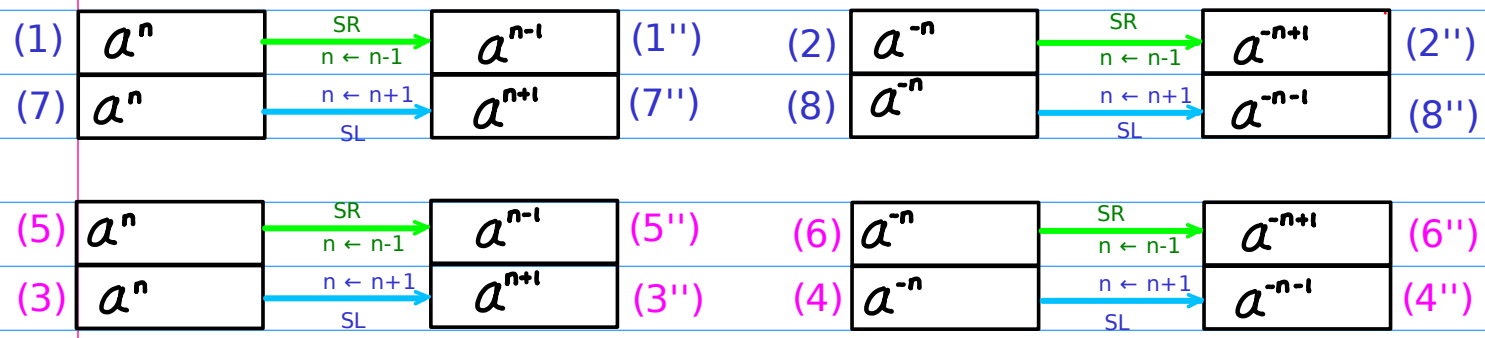
RP : Range Permutations

B Shifting Shifted Sequence 2

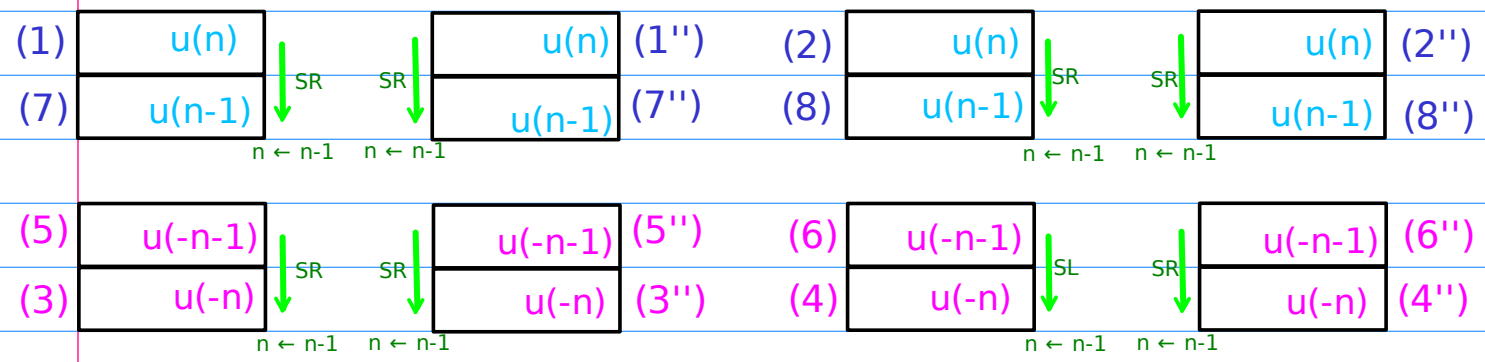
(1) Exponent Shifting, Flipping (SR, id) or (SL, id)

(2) Range Shifting (id, SR) or (id, SL)

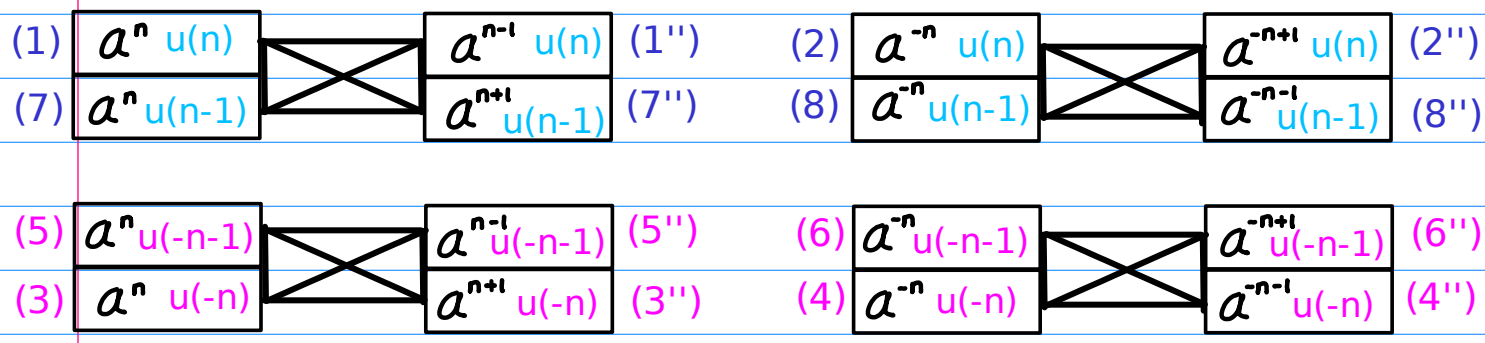
Exponent Shifts : (SR, id) or (SL, id)



Range Shifts : (id, SR) or (id, SL)



Exponent & Range Permutations



Decomposition

(EP, RP) = (EP, id) + (id, RP)

EP : Exponent Permuations

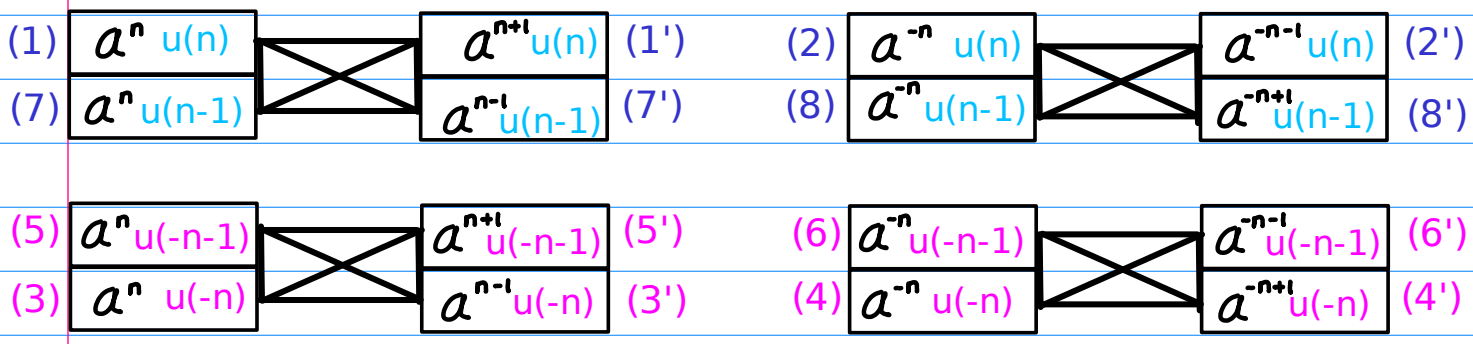
RP : Range Permutations

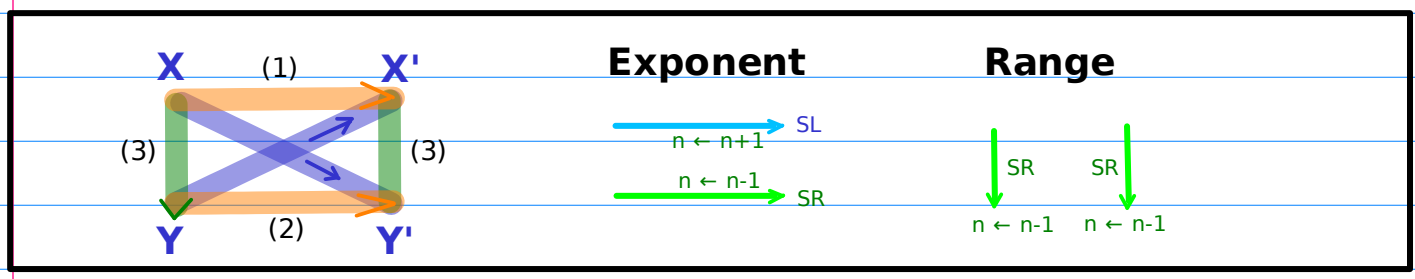
A Shifting Shifted Sequence 1

(1) Exponent Shifting
(2) Range Shifting

:

(SR, id) + (id, SR) = (SR, SR)
(SL, id) + (id, SL) = (SL, SL)

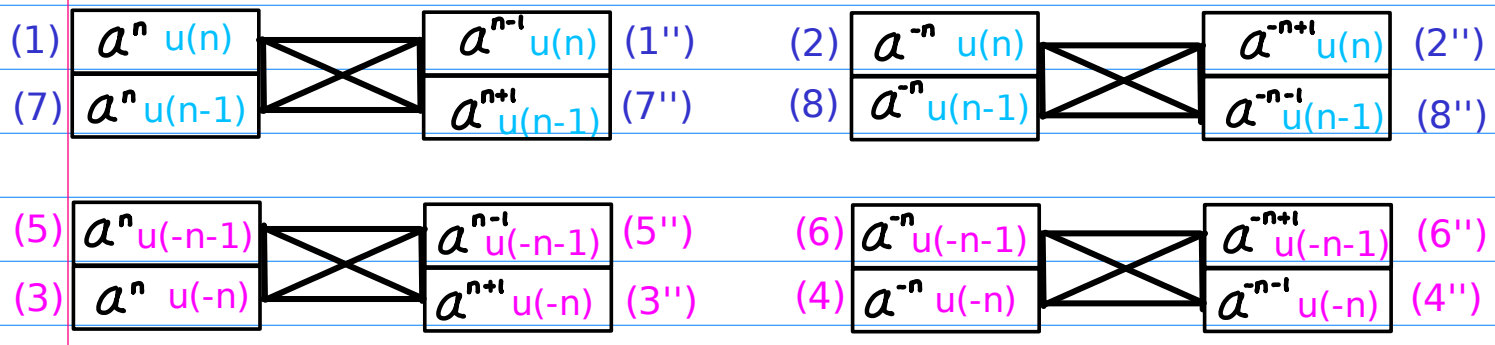


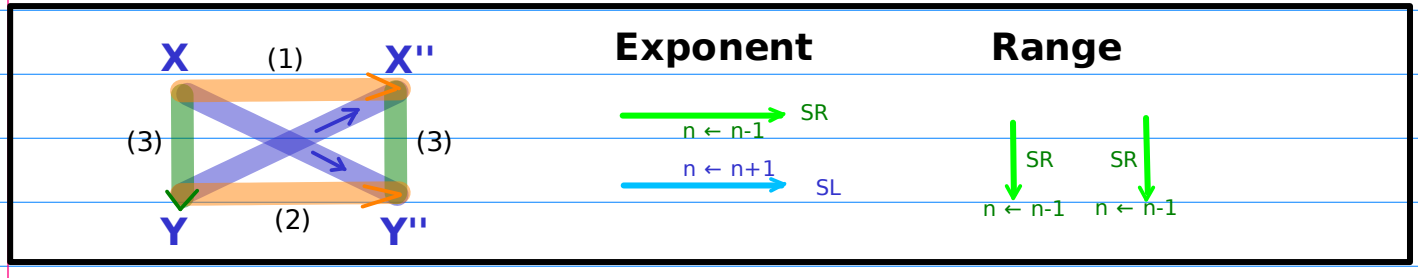


B Shifting Shifted Sequence 2

(1) Exponent Shifting
(2) Range Shifting

(SR, id) + (id, SL) = (SR, SL)
(SL, id) + (id, SR) = (SL, SR)





A Shifting **Shifted Sequence 1**

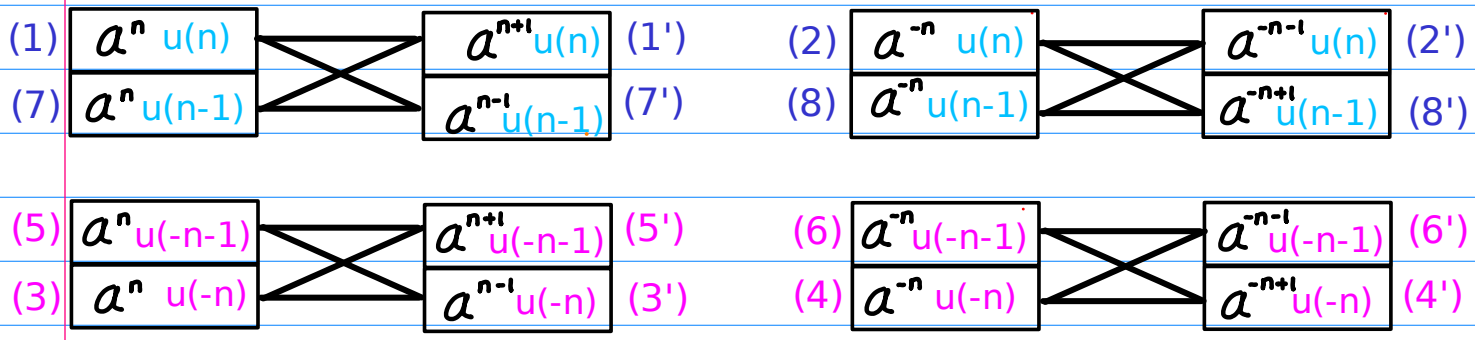
(1) Exponent Shifting

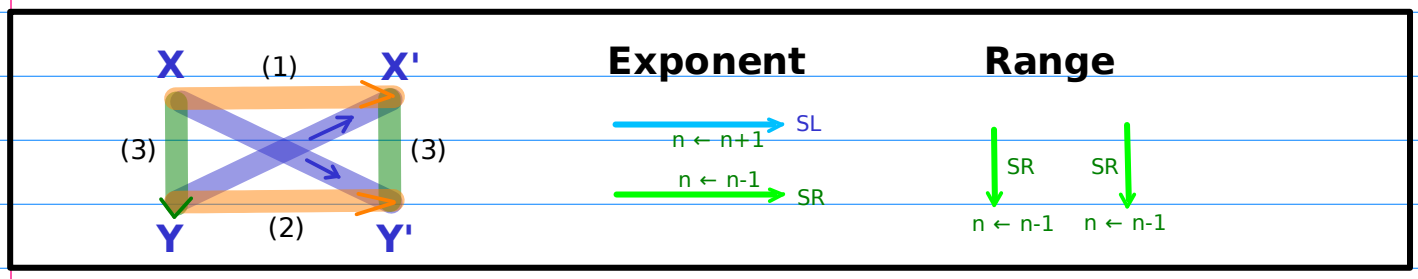
(2) Range Shifting

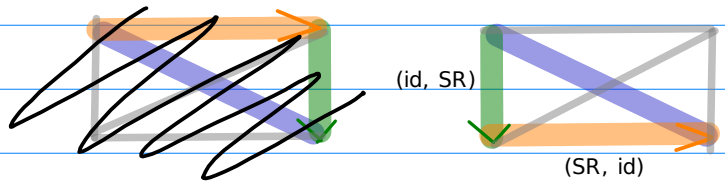
:

(SR, id) + (id, SR) = (SR, SR)

(SL, id) + (id, SL) = (SL, SL)



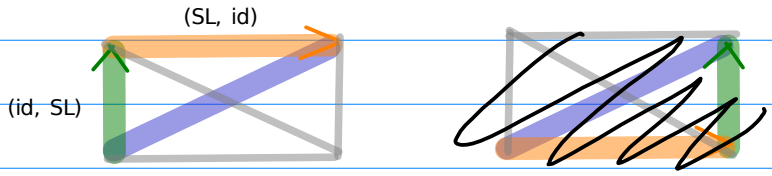




(SR, id) shift right exponent

(id, SR) shift right range

(SR, SR)



(SL, id) shift left exponent

(id, SL) shift left range

(SL, SL)

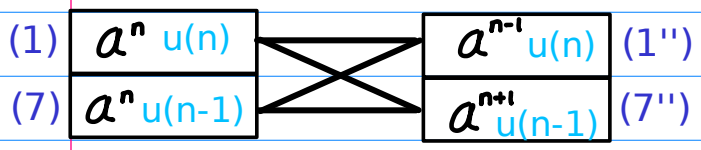
(SR, id) + (id, SR) = (SR, SR)

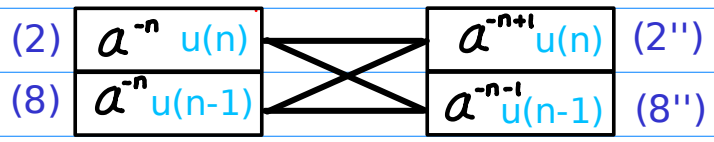
(SL, id) + (id, SL) = (SL, SL)

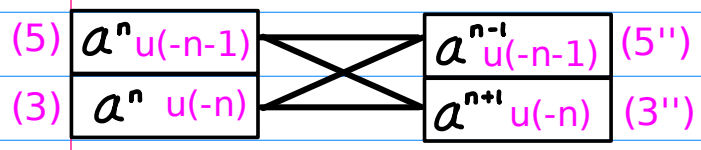
B Shifting Shifted Sequence 2

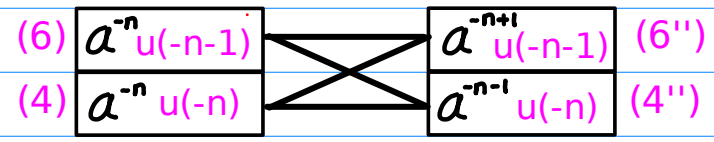
(1) Exponent Shifting
(2) Range Shifting

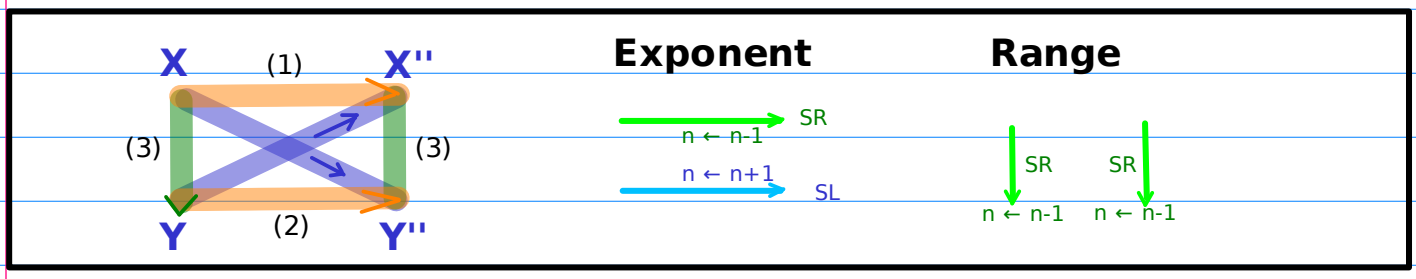
(SR, id) + (id, SL) = (SR, SL)
(SL, id) + (id, SR) = (SL, SR)

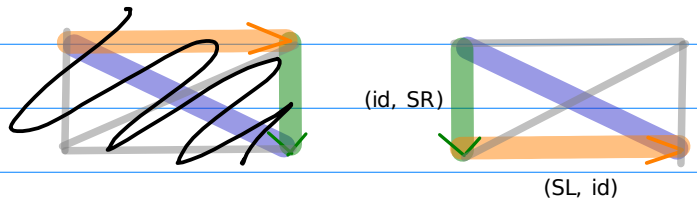








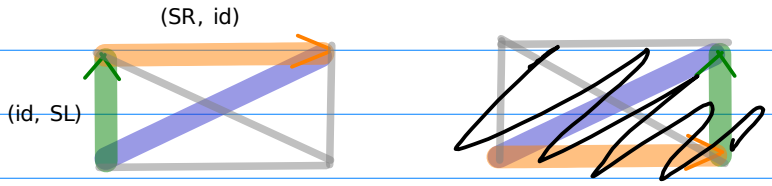




(SL, id) shift right exponent

(id, SR) shift right range

(SL, SR)



(SR, id) shift left exponent

(id, SL) shift left range

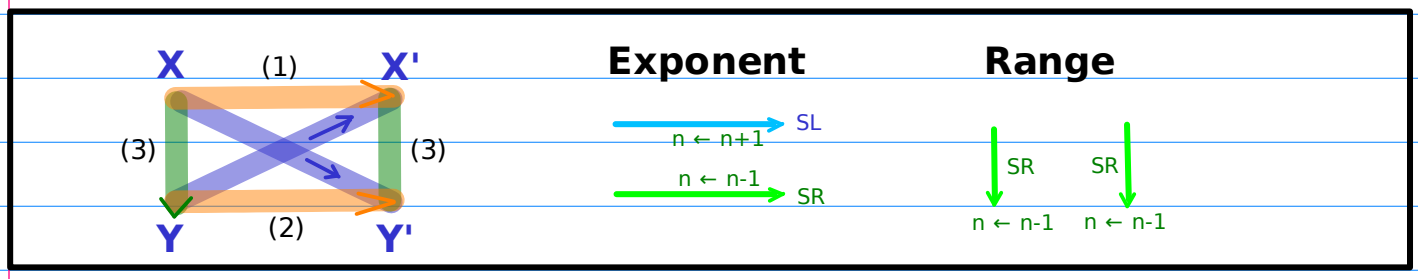
(SR, SL)

(SL, id) + (id, SR) = (SL, SR)
(SR, id) + (id, SL) = (SR, SL)

Butterfly Relations

A Shifting **Shifted Sequence 1**

Unshifted Sequence x → **Shifted Sequence 1** x'



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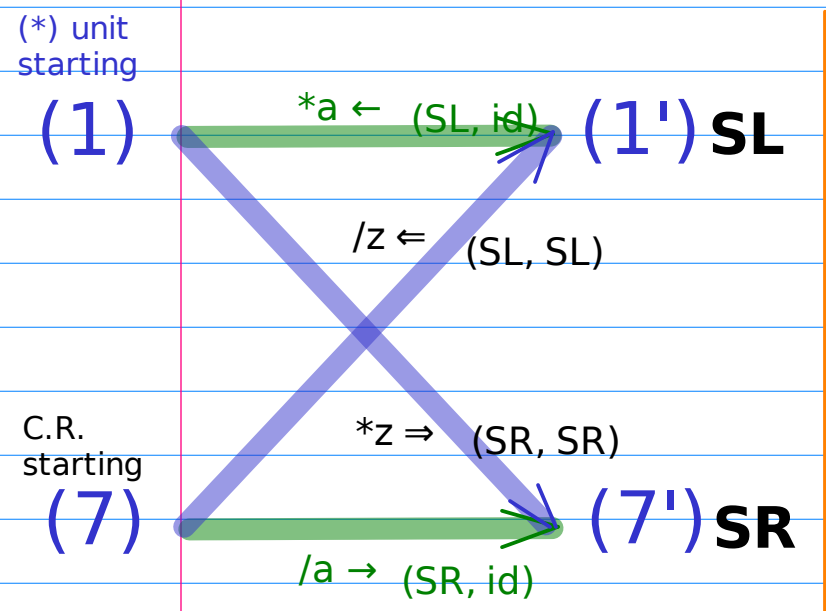
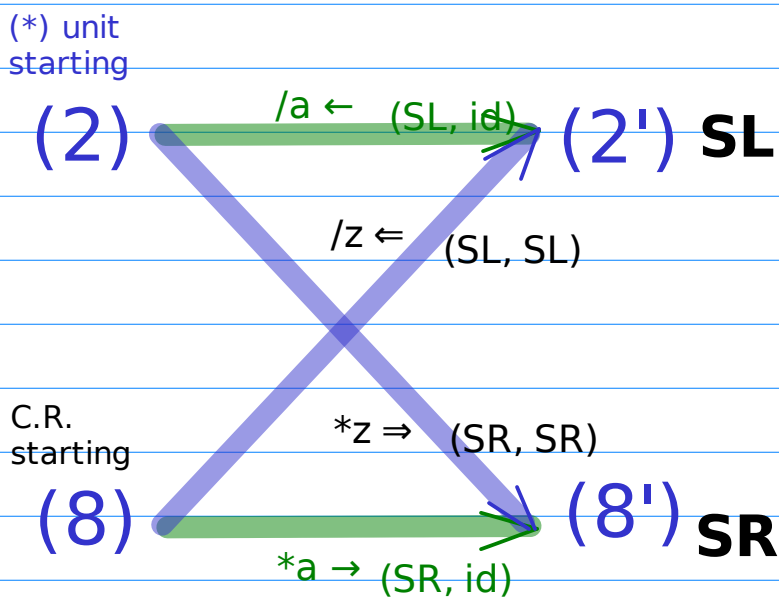
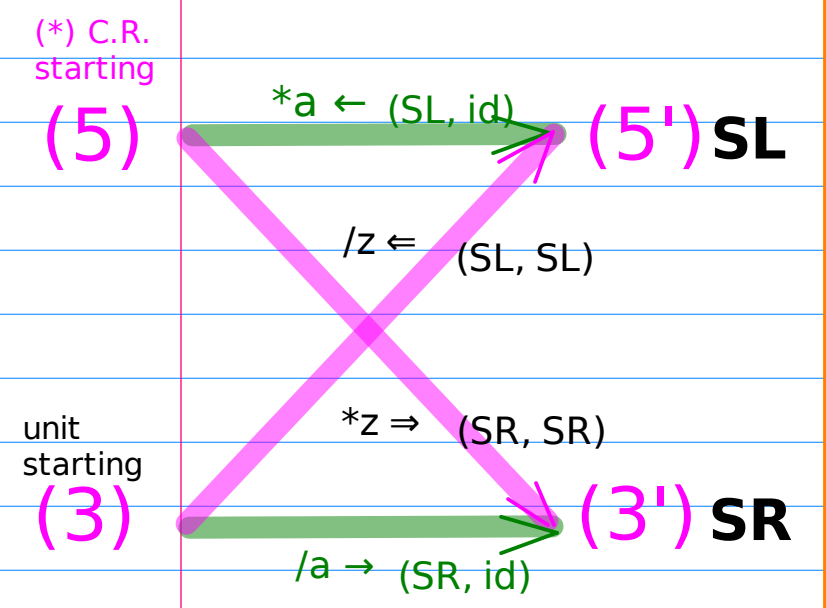
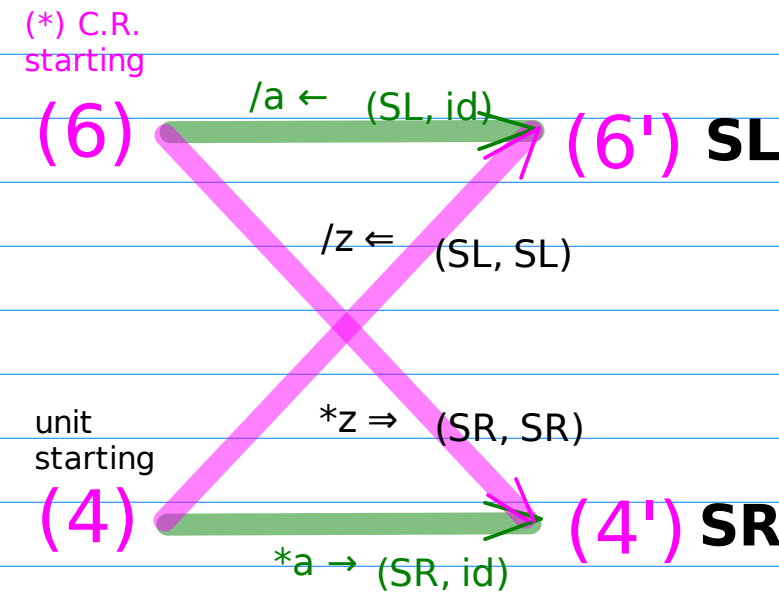




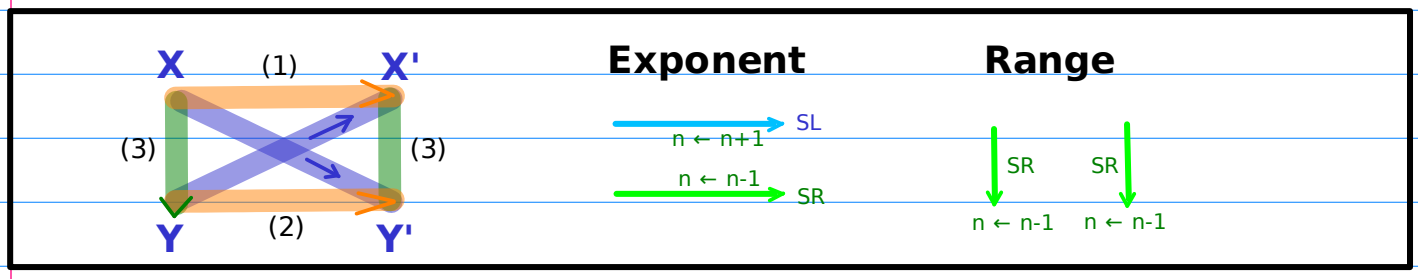


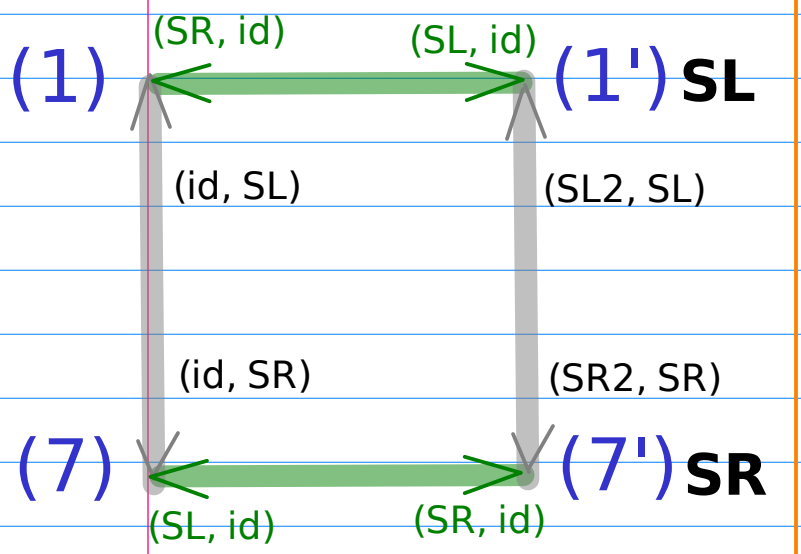


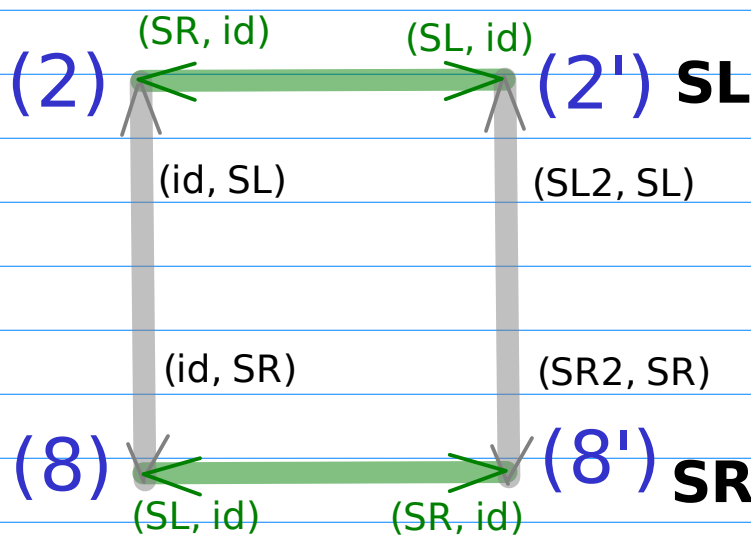
Butterfly Relations

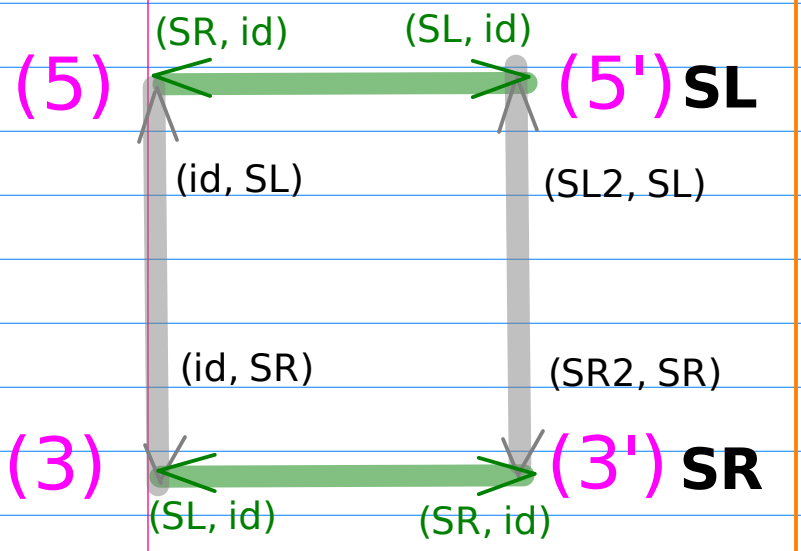
A Shifting **Shifted Sequence 1**

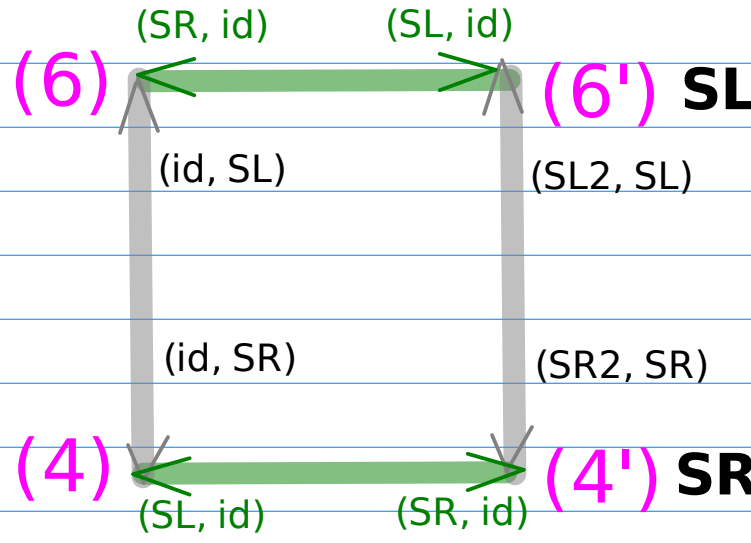
Unshifted Sequence x → **Shifted Sequence 1** x'









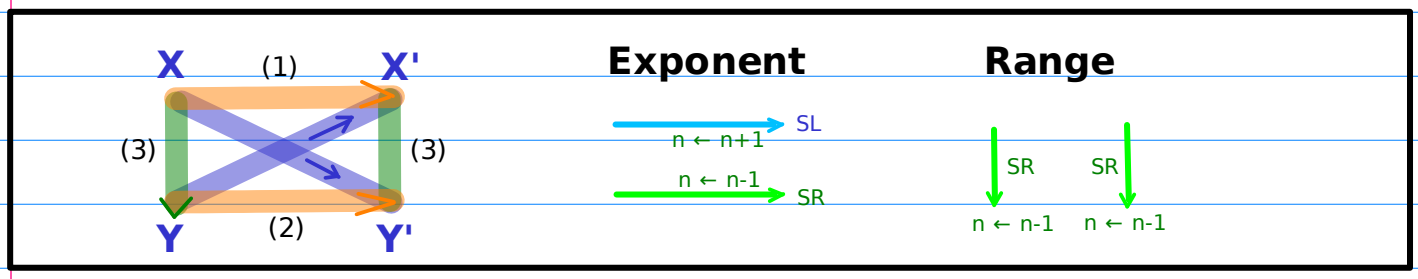


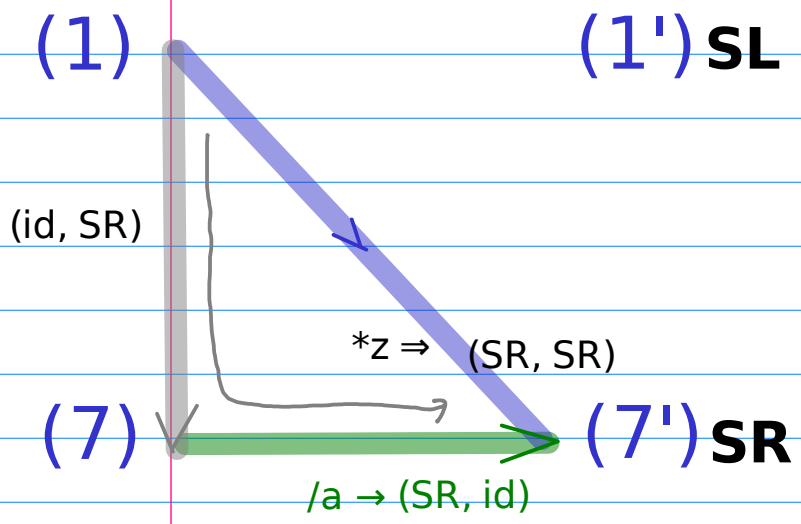
Decomposition of Exp and Rng Shifts (1)

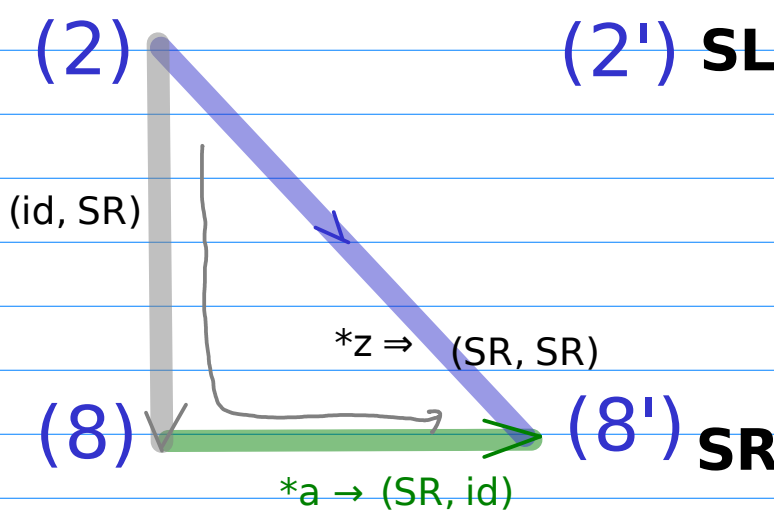
A Shifting **Shifted Sequence 1**

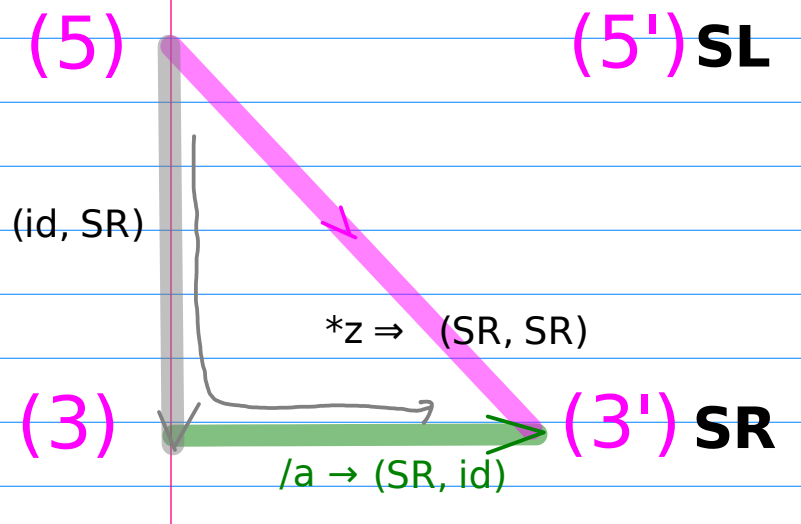
Unshifted Sequence x → **Shifted Sequence 1** x'

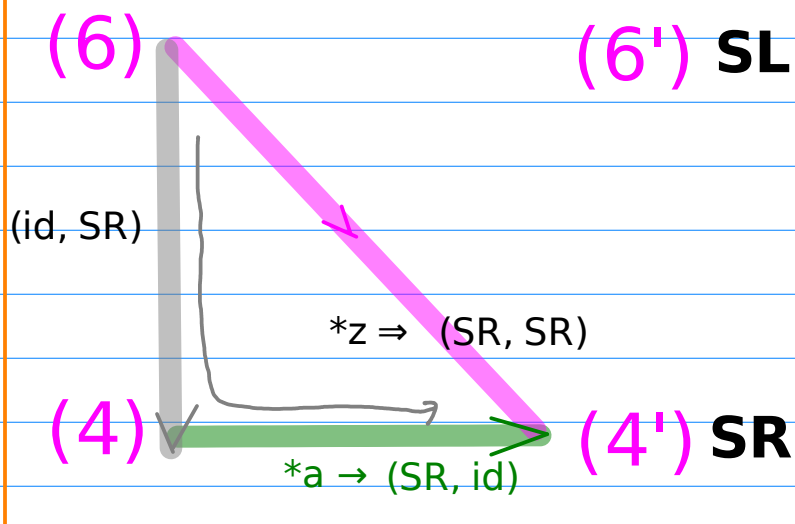
(id, SR) + (SR, id) = (SR, SR)









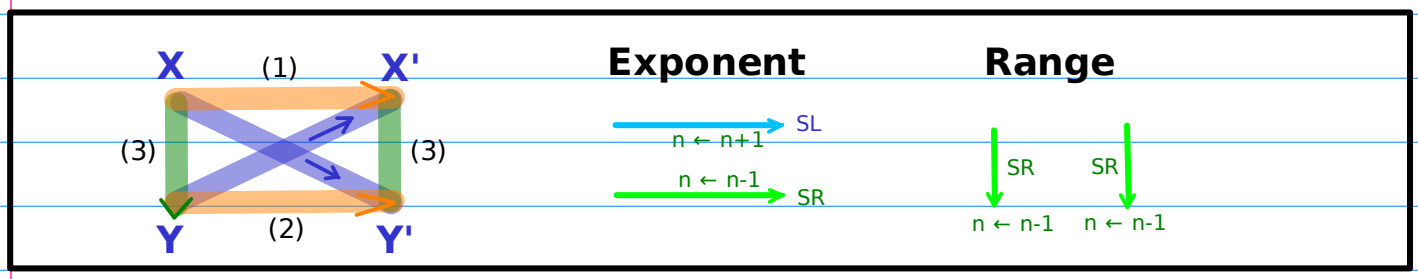


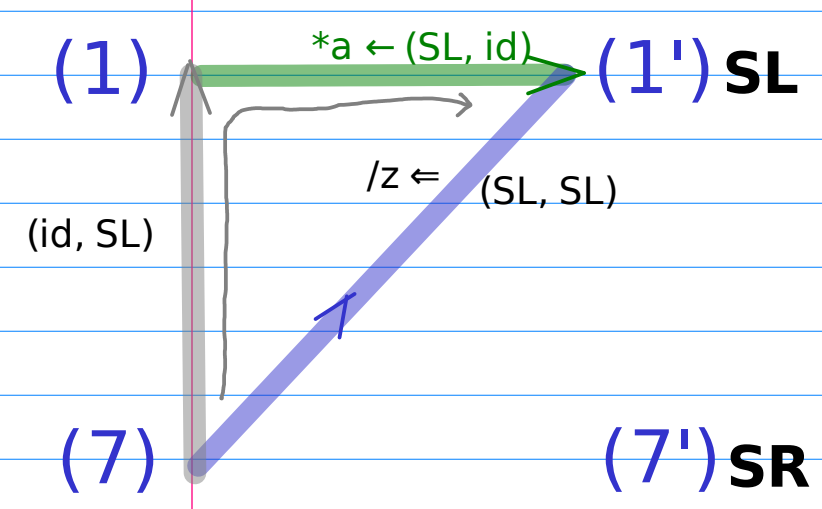
Decomposition of Exp and Rng Shifts (2)

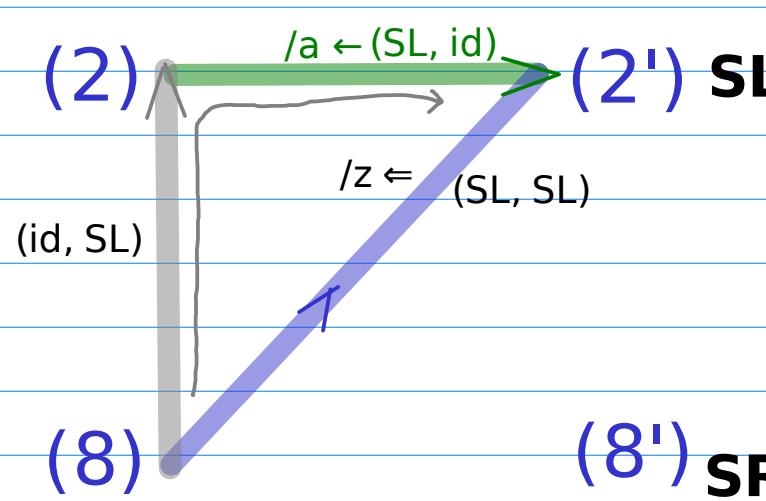
A Shifting Shifted Sequence 1

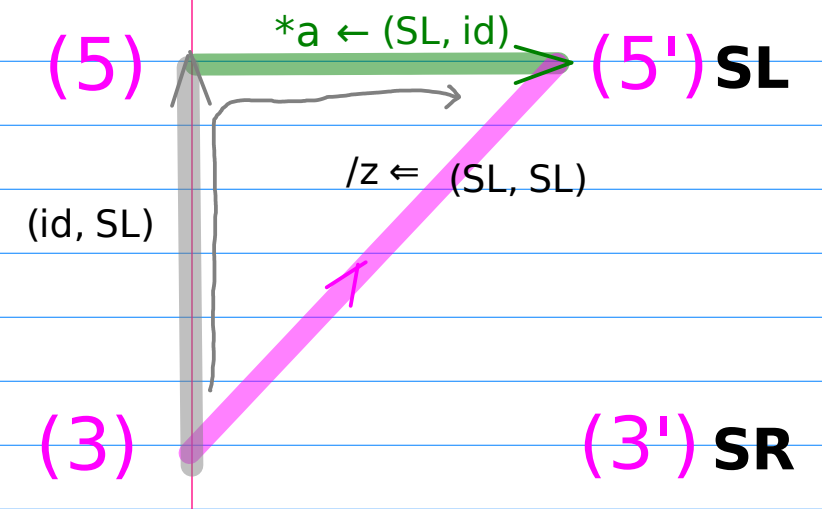
Unshifted Sequence **x** → Shifted Sequence 1 **x'**

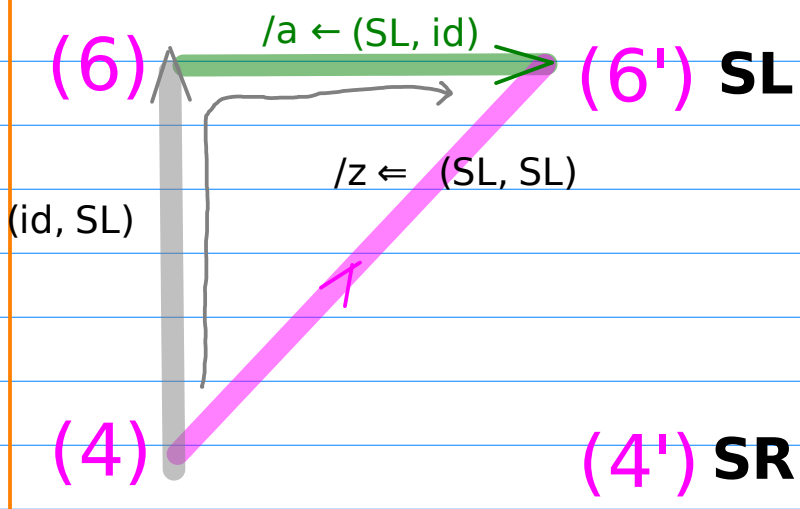
(id, SL) + (SL, id) = (SL, SL)









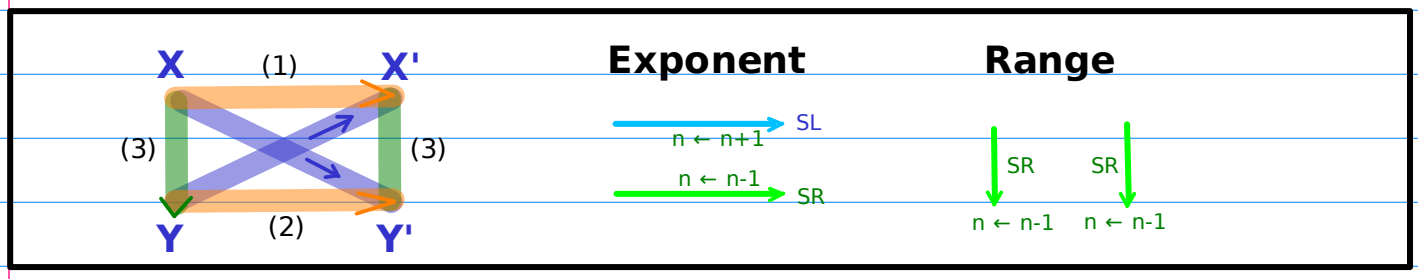


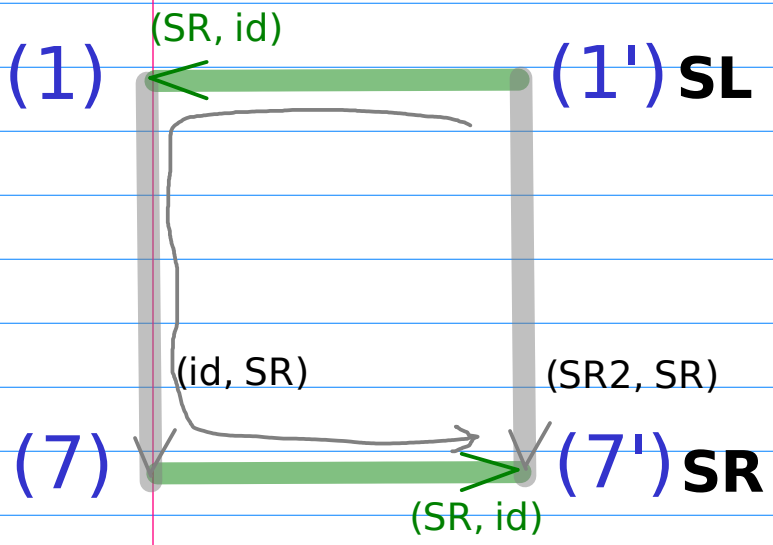
Decomposition of Exp and Rng Shifts (3)

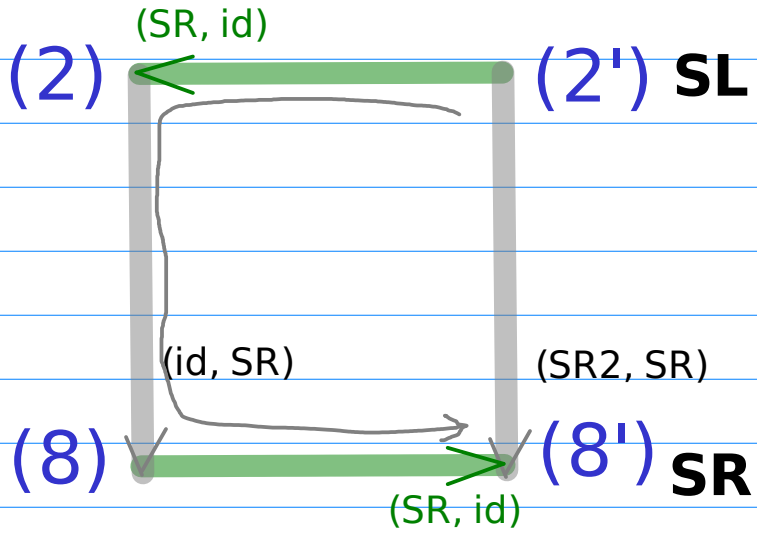
A Shifting Shifted Sequence 1

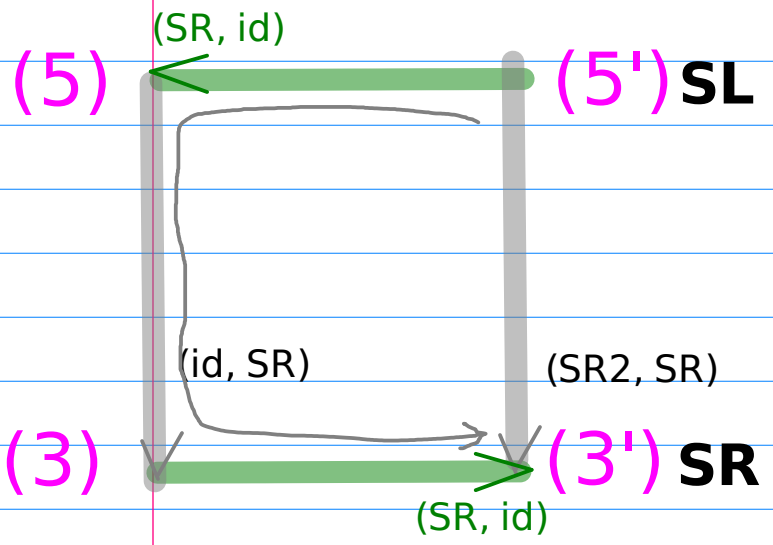
Unshifted Sequence **x** → Shifted Sequence 1 **x'**

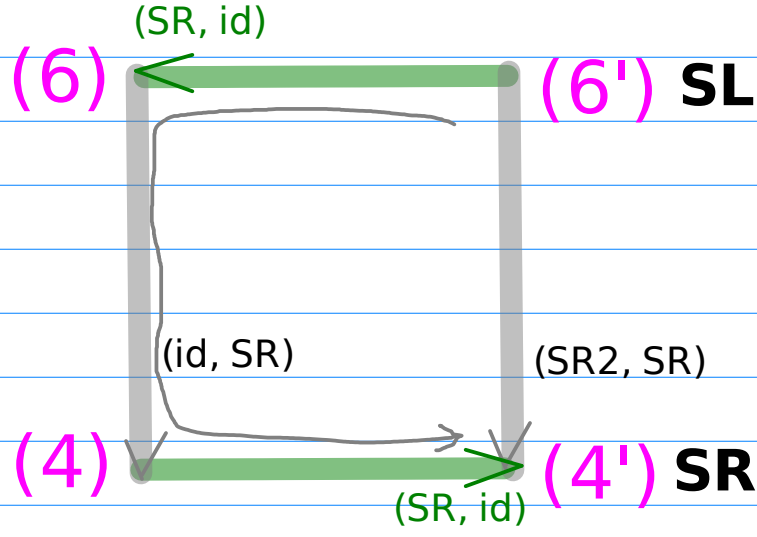
(SR, id) + (id, SR) + (SR, id) = (SR2, SR)







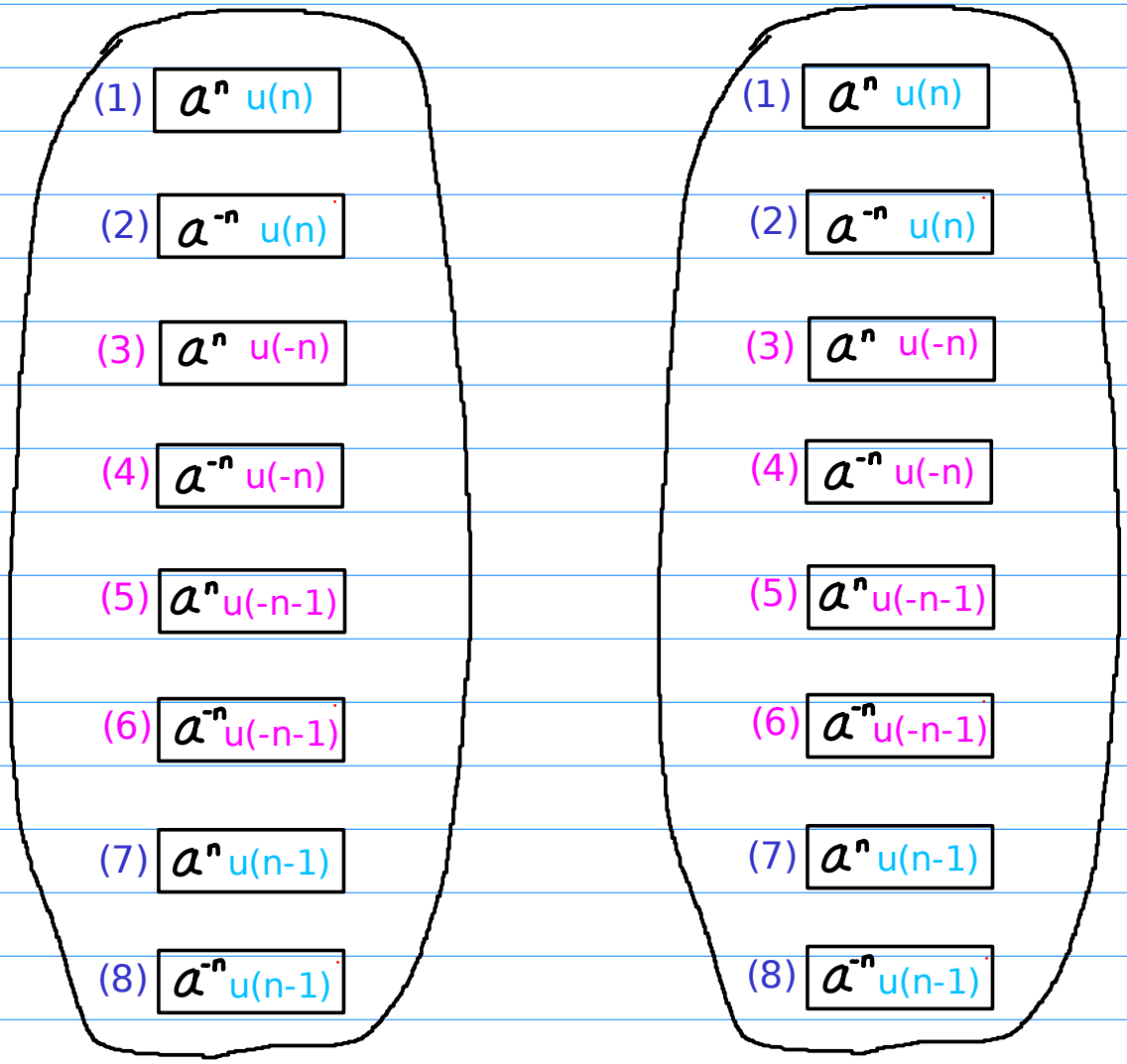




Intra-Permuations over (1) ~ (8)

Unshifted Sequence x

Unshifted Sequence x

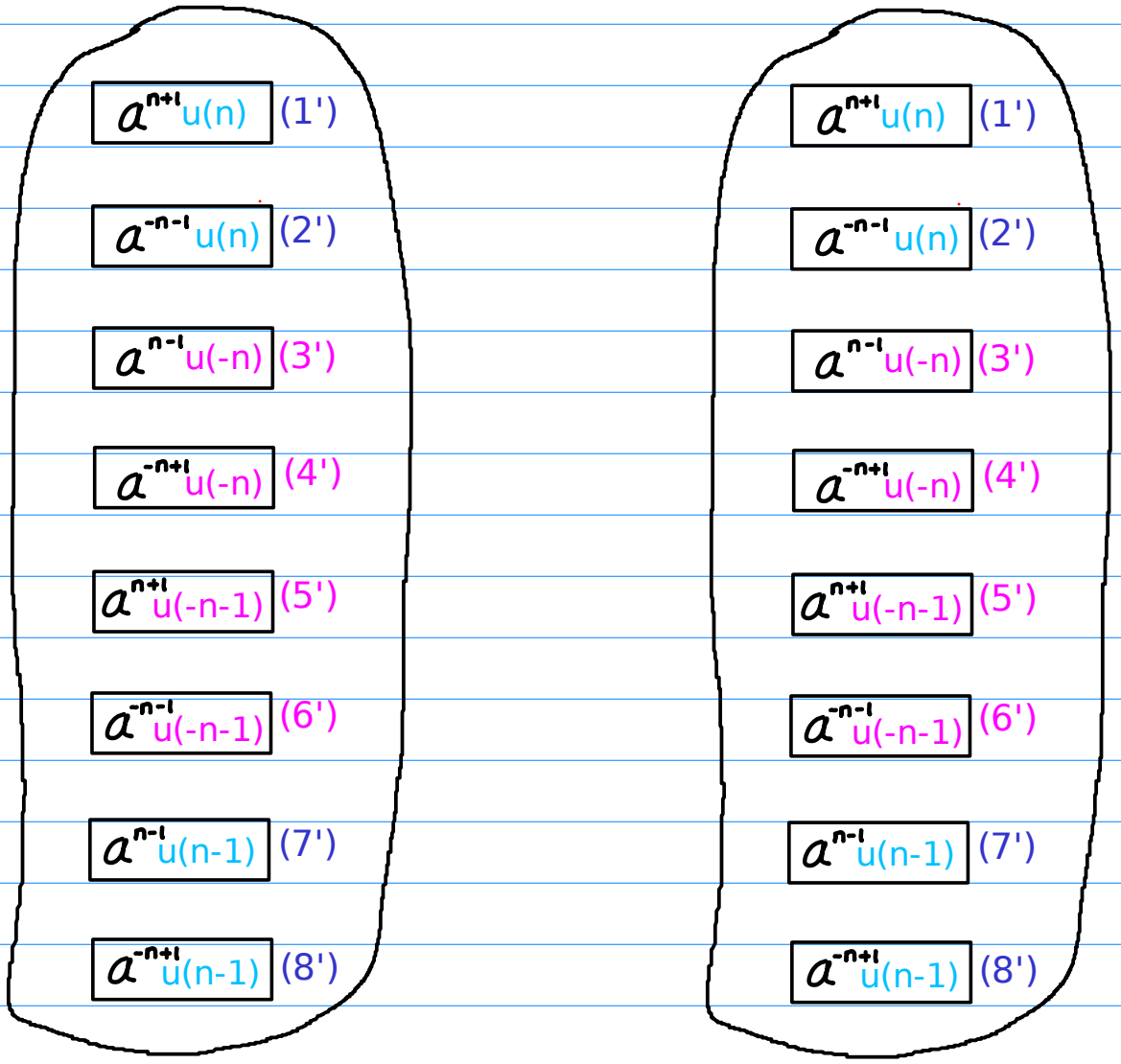


Inter-permutations over unshifted sequnce and shifted sequence

Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence

Intra-Permuations over (1') ~ (8')

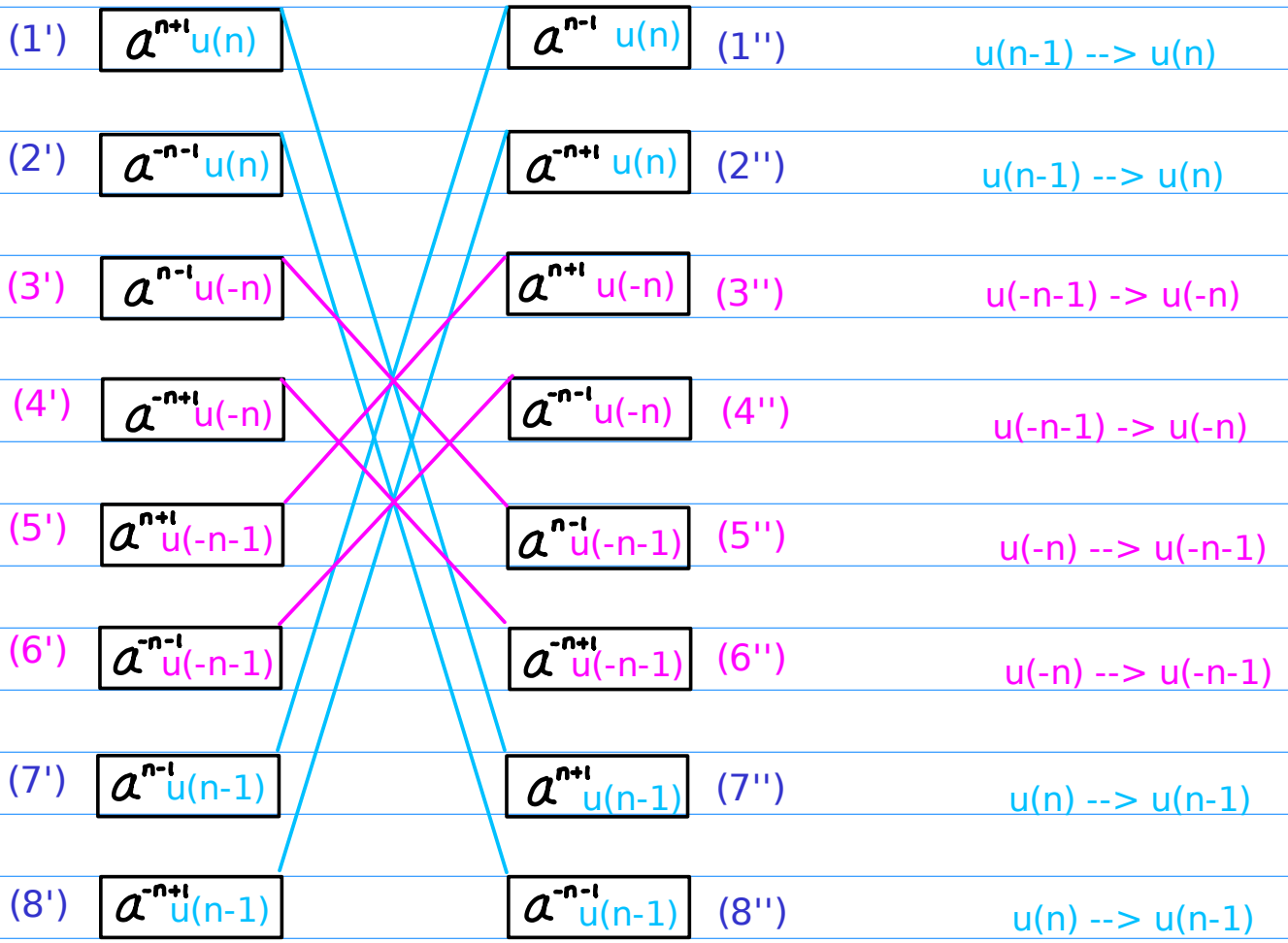


Inter-permutations over unshifted sequnce and shifted sequence

Intra-permutations over unshifted sequence

Intra-permutations over shifted sequence

Shifted
Sequence 1 x'

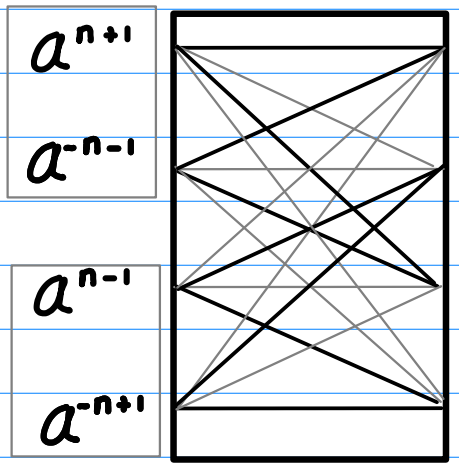


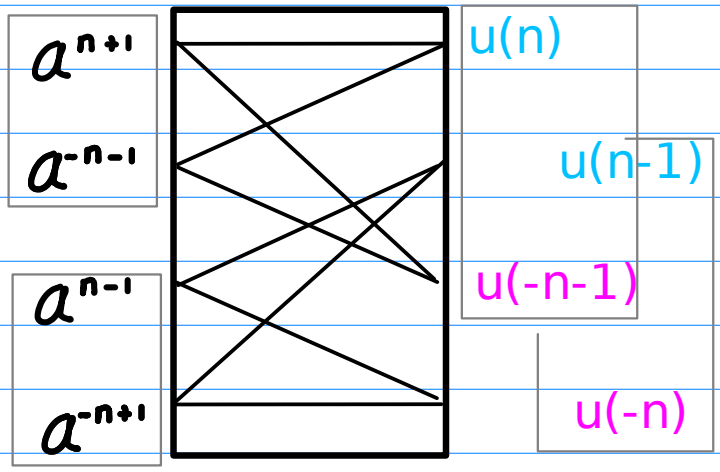
a^n × $R(n)$

a^{n+1}	a^{-n-1}
a^{n-1}	a^{-n+1}

×

u(n)	u(-n-1)
u(n-1)	u(-n)





n ← n+2 or
n ← n - 2

(1') $a^{n+1}u(n)$		$a^{-n-1}u(n)$	(2')
(3') $a^{n-1}u(-n)$		$a^{-n+1}u(-n)$	(4')
(5') $a^{n+1}u(-n-1)$		$a^{-n-1}u(-n-1)$	(6')
(7') $a^{n-1}u(n-1)$		$a^{-n+1}u(n-1)$	(8')

(1'') $a^{n-1}u(n)$		$a^{-n+1}u(n)$	(2'')
(3'') $a^{n+1}u(-n)$		$a^{-n-1}u(-n)$	(4'')
(5'') $a^{n-1}u(-n-1)$		$a^{-n+1}u(-n-1)$	(6'')
(7'') $a^{n+1}u(n-1)$		$a^{-n-1}u(n-1)$	(8'')

permutation over (1) ~ (8) (x) → (x)

A. Flipping
- Base Inverting
- Range Flipping

(1) - (4)	(5) - (8)
(2) - (3)	(6) - (7)
(3) - (2)	(7) - (6)
(4) - (1)	(8) - (5)

B. Range Shifting
- Range Flipping
- Range Complementing

(1) - (7)	(5) - (3)
(2) - (8)	(6) - (4)
(3) - (5)	(7) - (1)
(4) - (6)	(8) - (2)

C. Complementary Inverting
- Base Inverting
- Range Complementing

(1) - (6)	(5) - (2)
(6) - (1)	(6) - (1)
(2) - (5)	(7) - (4)
(5) - (2)	(8) - (3)

permutation over (1') ~ (8') (x') → (x')

D. Flipping2
- Base Inverting
- Shifted Range Flipping

(1') - (4')	(5') - (8')
(2') - (3')	(6') - (7')
(3') - (2')	(7') - (6')
(4') - (1')	(8') - (5')

E. Shifting2
- Shifted Range Flipping
- Range Complementing

(1') - (7')	(5') - (3')
(2') - (8')	(6') - (4')
(3') - (5')	(7') - (1')
(4') - (6')	(8') - (2')

F. Complementary Inverting
- Base Inverting
- Range Complementing

(1') - (6')	(5') - (2')
(6') - (1')	(6') - (1')
(2') - (5')	(7') - (4')
(5') - (2')	(8') - (3')

Shifted Range Flipping = Exponent Shifting2 + Range Flippping

Shifting2 = Shifted Range Flipping + Range Complementing
= Exponent Shifting2 + Range Flippping+ Range Complementing
= Exponent Shifting2 + Range (Flippping+Complementing)
= Exponent Shfiting2 + Range Shifting

Permutation over (1) ~ (8)

Permutations	A	B	C
Base Inverting	x		x
Range Flipping	x	x	
Range Complementing		x	x

Permutation over (1') ~ (8')

Permutations	D	E	F
Base Inverting	x		x
Shifted Range Flipping	x	x	
Range Complementing		x	x

Shifted Range Flipping = Exponent Shifting2 + Range Flippping

Shifting2 = Shifted Range Flipping + Range Complementing
= Exponent Shifting2 + Range Flippping+ Range Complementing
= Exponent Shifting2 + Range (Flippping+Complementing)
= Exponent Shfiting2 + Range Shifting

Over (1) ~ (8)

Base Inverting $a^n \Longleftrightarrow a^{-n}$

Range Flipping $R(n) \Longleftrightarrow R(-n)$

Range Complementing $R(n) \Longleftrightarrow \overline{R(n)}$

Over (1') ~ (8')

Base Inverting $a^n \Longleftrightarrow a^{-n}$

Shifted Range Flipping $a^n R(n) \Longleftrightarrow a^{sh\,2(n)} R(-n)$

Range Complementing $R(n) \Longleftrightarrow \overline{R(n)}$

A.I Flipping
Base Inverting
Range Flipping

$$a^n \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-n} R(-n)$$

B.I Range Shifting
Range Flipping
Range Complementing

$$R(n) \Longleftrightarrow R(-n)$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$R(n) \Longleftrightarrow \overline{R(-n)}$$

C.I Complementary Inverting
Base Inverting
Range Complementing

$$a^n \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n} \overline{R(n)}$$

D.I Flipping2
Base Inverting
Shifted Range Flipping

$$a^n \Longleftrightarrow a^{-n}$$
$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-sh\,2(n)} R(-n)$$

E.I Shifting2
Shifted Range Flipping
Range Complementing

$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} R(-n)$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} \overline{R(-n)}$$

F.I Complementary Inverting
Base Inverting
Range Complementing

$$a^n \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n} \overline{R(n)}$$

G.I Flipping2

Base Inverting
Shifted Range Flipping

$$a^n \Longleftrightarrow a^{-n}$$
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-sh2(n)} R(-n)$$

H.I Shifting2

Shifted Range Flipping
Range Complementing

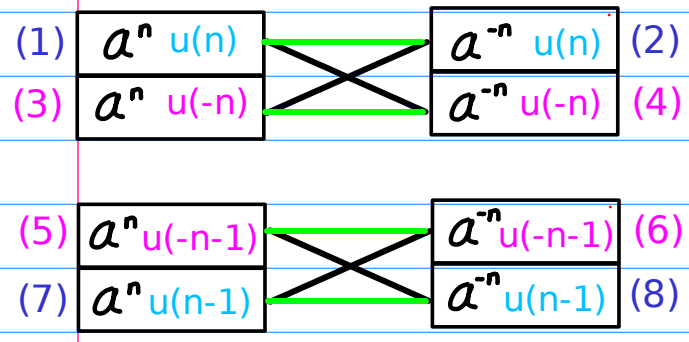
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} R(-n)$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

I.I Complementary Inverting

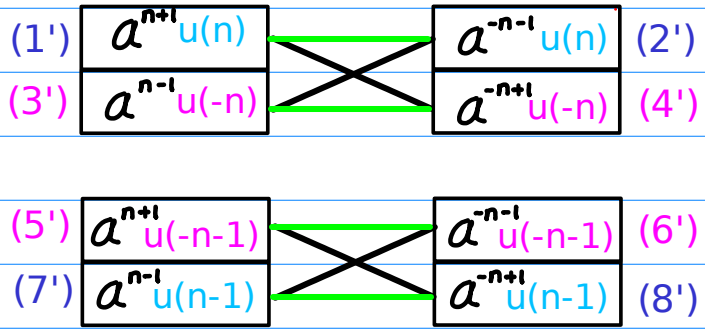
Base Inverting
Range Complementing

$$a^n \Longleftrightarrow a^{-n}$$
$$R(n) \Longleftrightarrow \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n} \overline{R(n)}$$

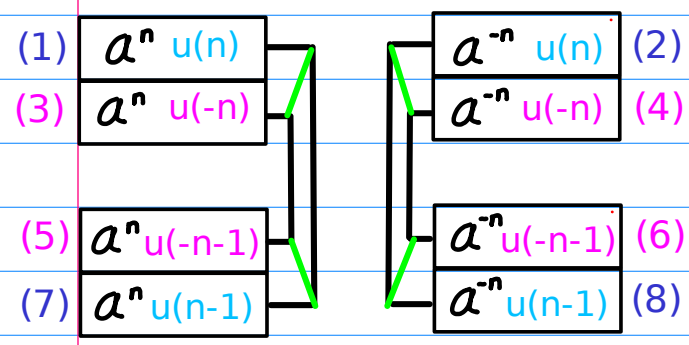
A.I Flipping
Base Inverting
Range Flipping



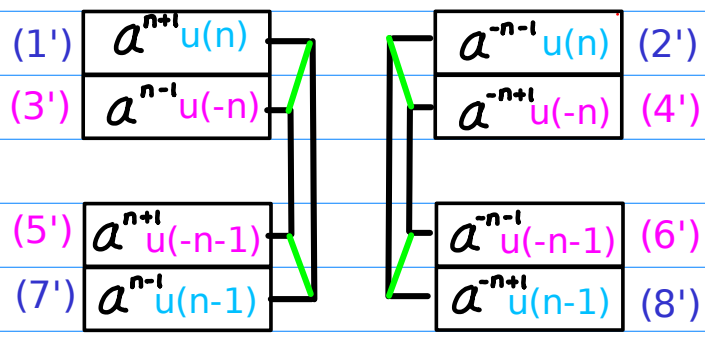
D.I Flipping2
Base Inverting
Shifted Range Flipping



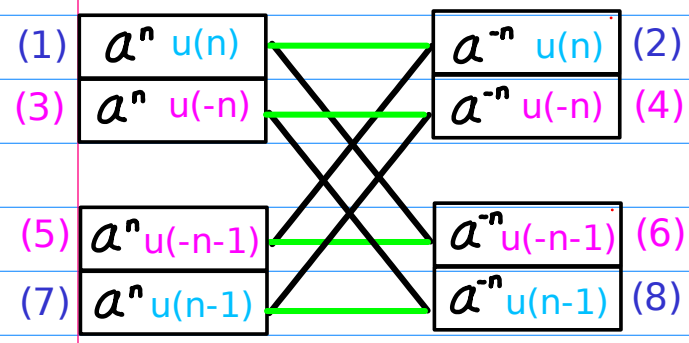
B.I Range Shifting
Range Flipping
Range Complementing



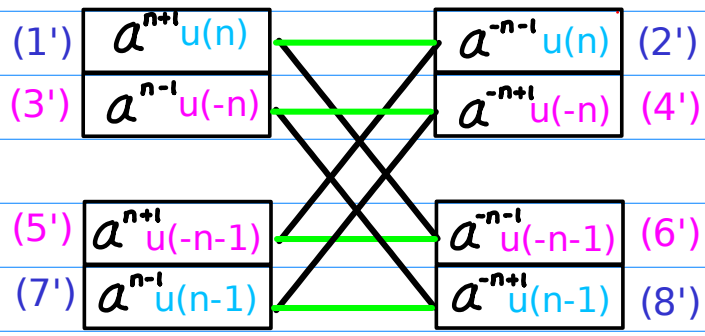
E.I Shifting2
Shifted Range Flipping
Range Complementing



C.I Complementary Inverting
Base Inverting
Range Complementing

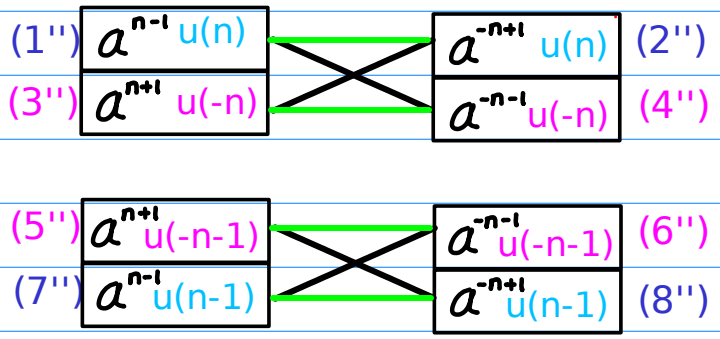


F.I Complementary Inverting
Base Inverting
Range Complementing



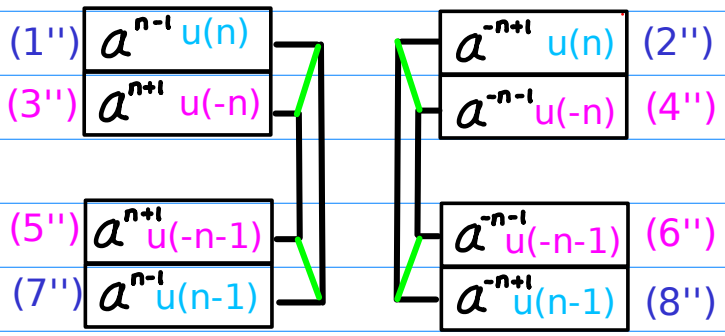
G.I Flipping2

Base Inverting
Shifted Range Flipping



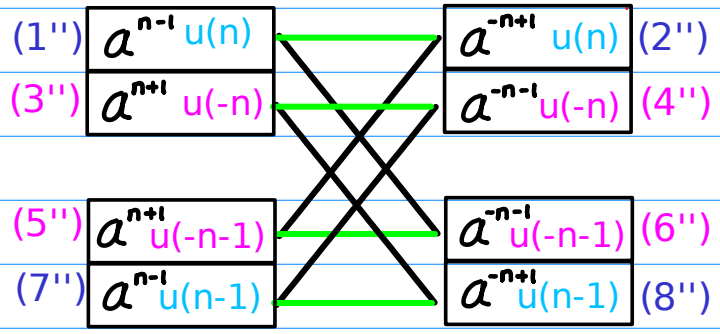
H.I Shifting2

Shifted Range Flipping
Range Complementing



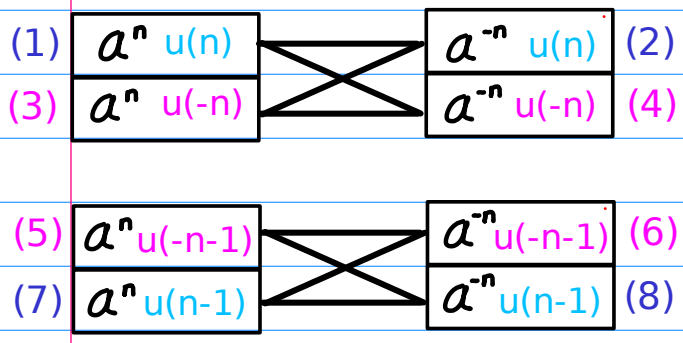
I.I Complementary Inverting

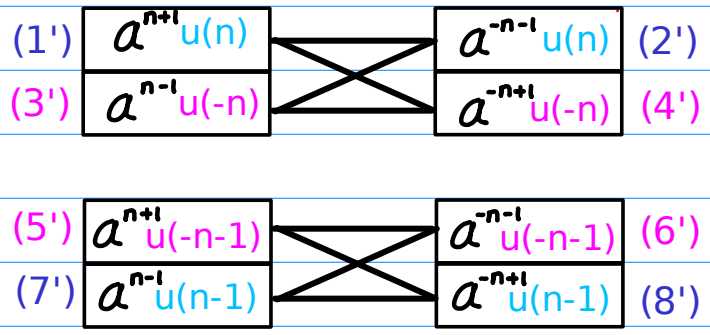
Base Inverting
Range Complementing

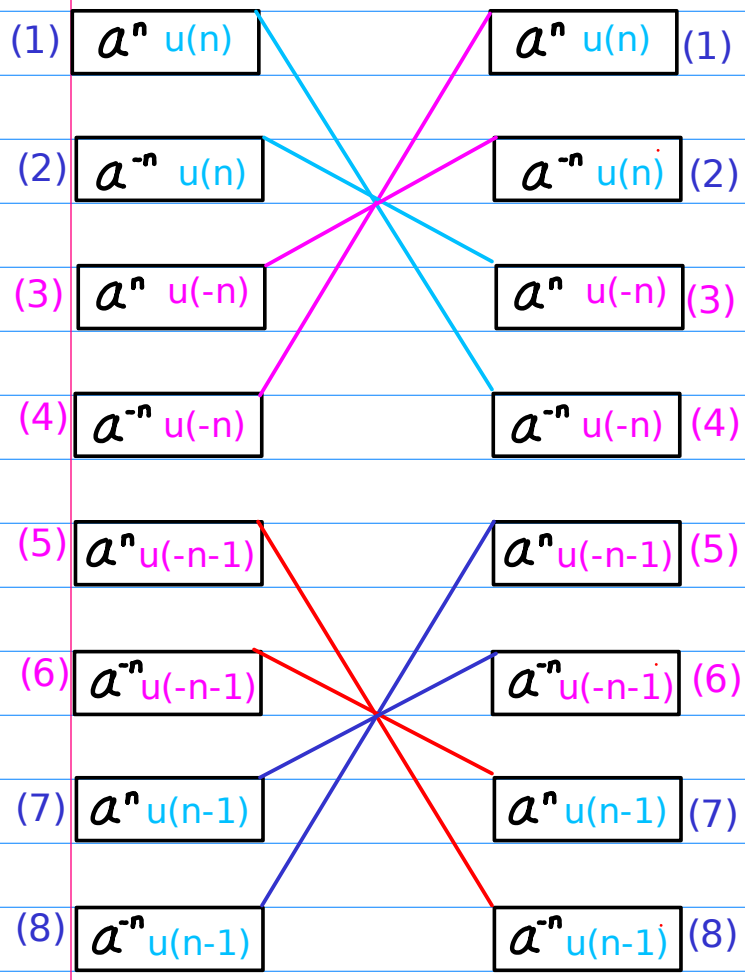


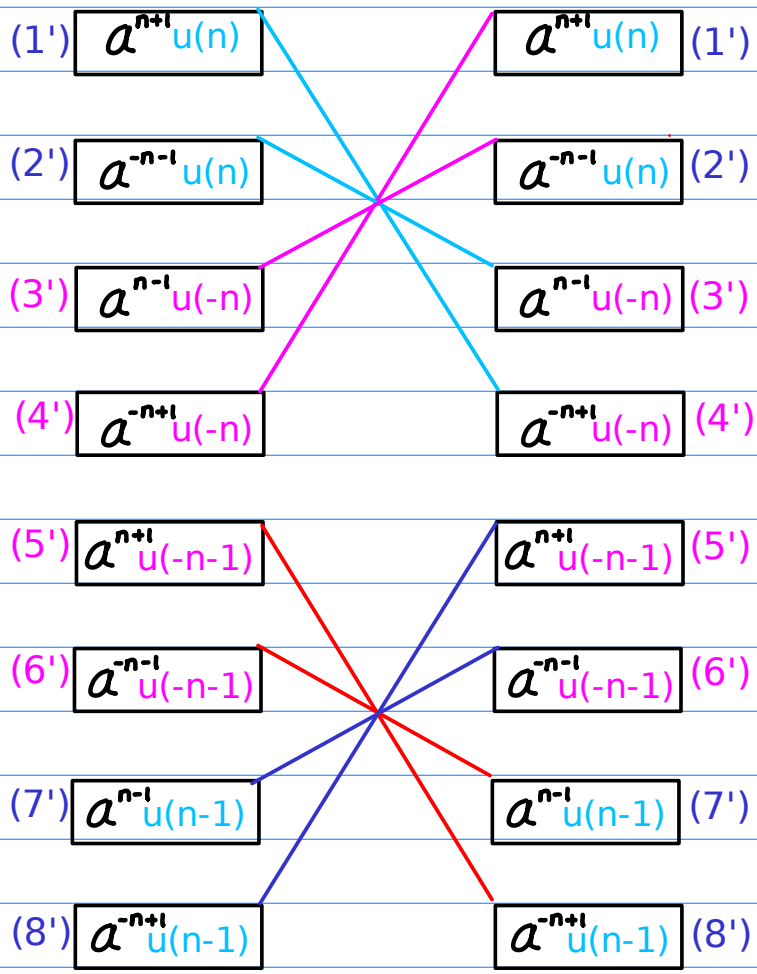
A.I Flipping
Base Inverting
Range Flipping

D.I Flipping2
Base Inverting
Shifted Range Flipping









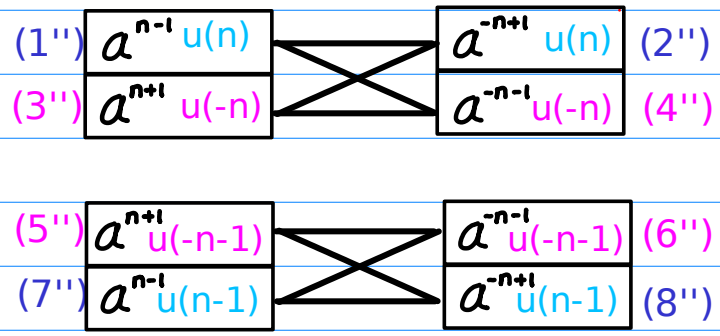
(1) - (4)
(2) - (3)
(3) - (2)
(4) - (1)

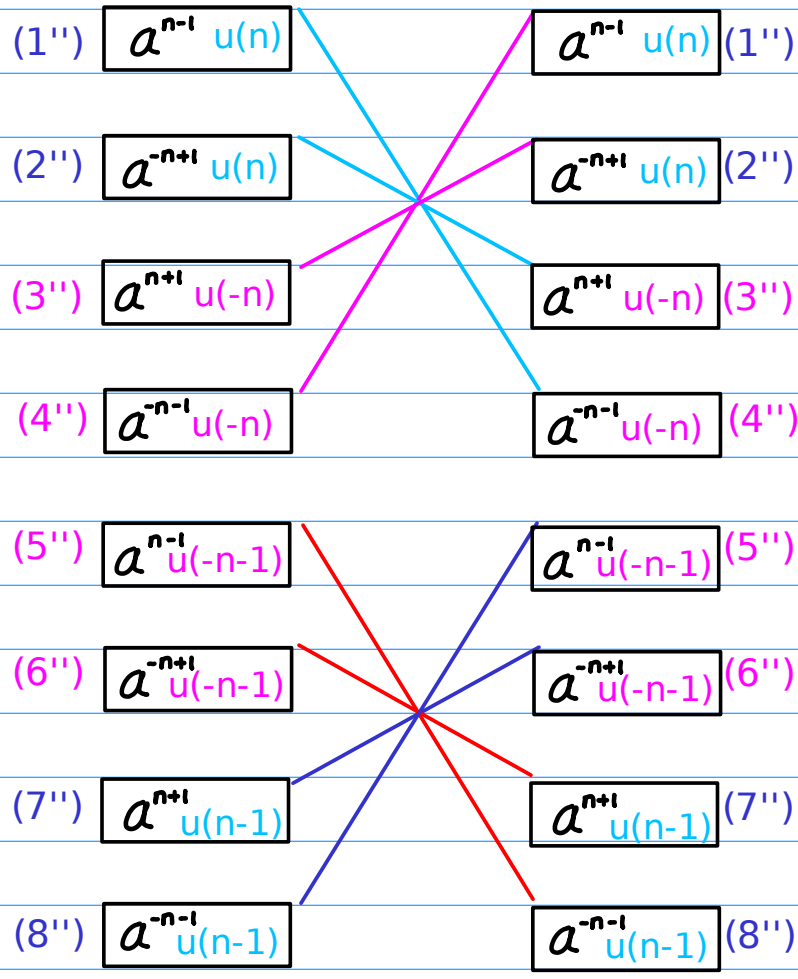
(5) - (8)
(6) - (7)
(7) - (6)
(8) - (5)

(1') - (4')
(2') - (3')
(3') - (2')
(4') - (1')

(5') - (8')
(6') - (7')
(7') - (6')
(8') - (5')

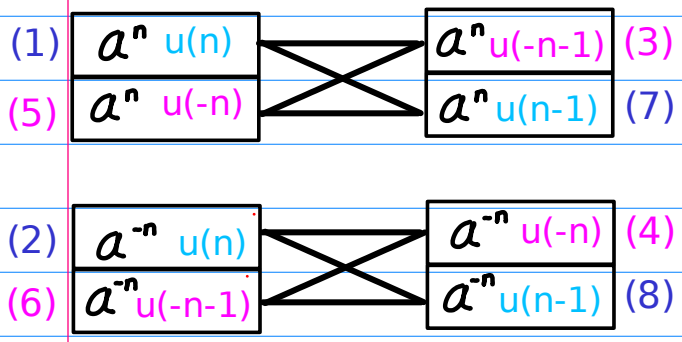
G.I Flipping2
Base Inverting
Shifted Range Flipping



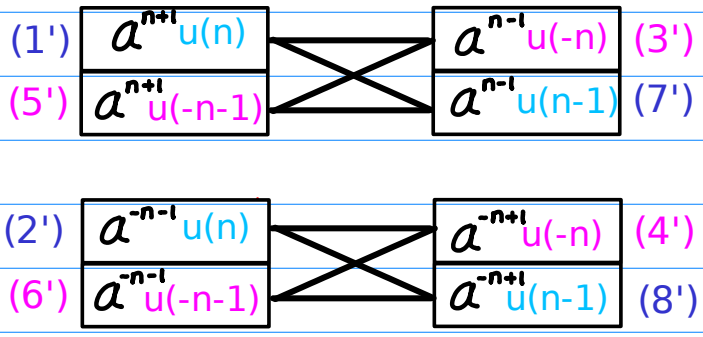


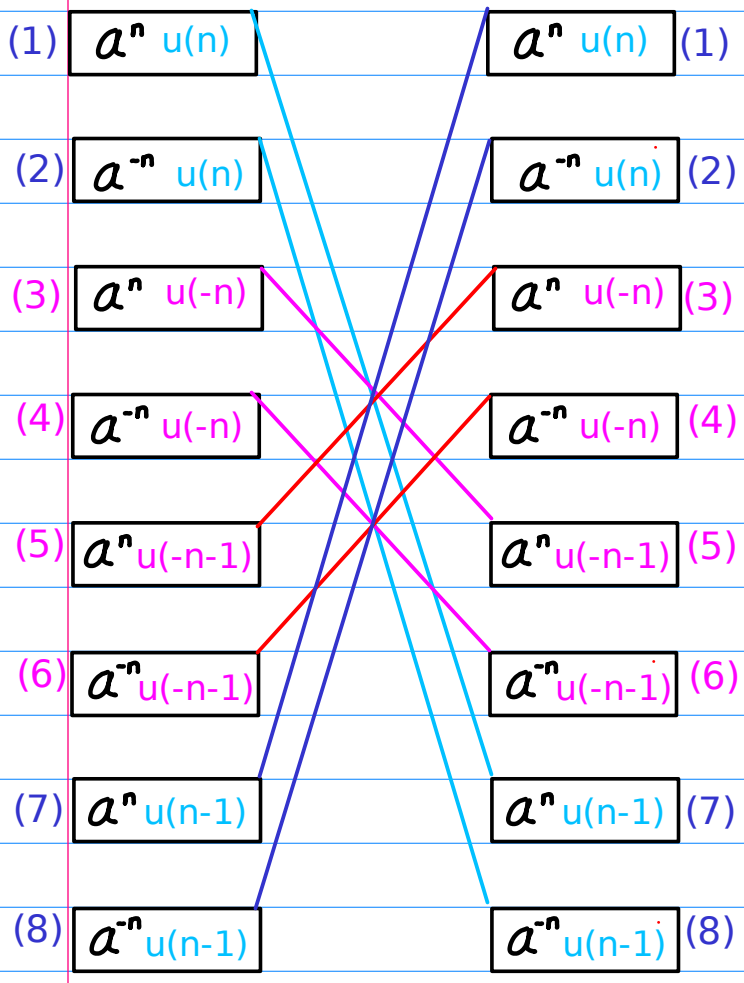
(1') - (4') (5') - (8')
(2') - (3') (6') - (7')
(3') - (2') (7') - (6')
(4') - (1') (8') - (5')

B.I Range Shifting
Range Flipping
Range Complementing



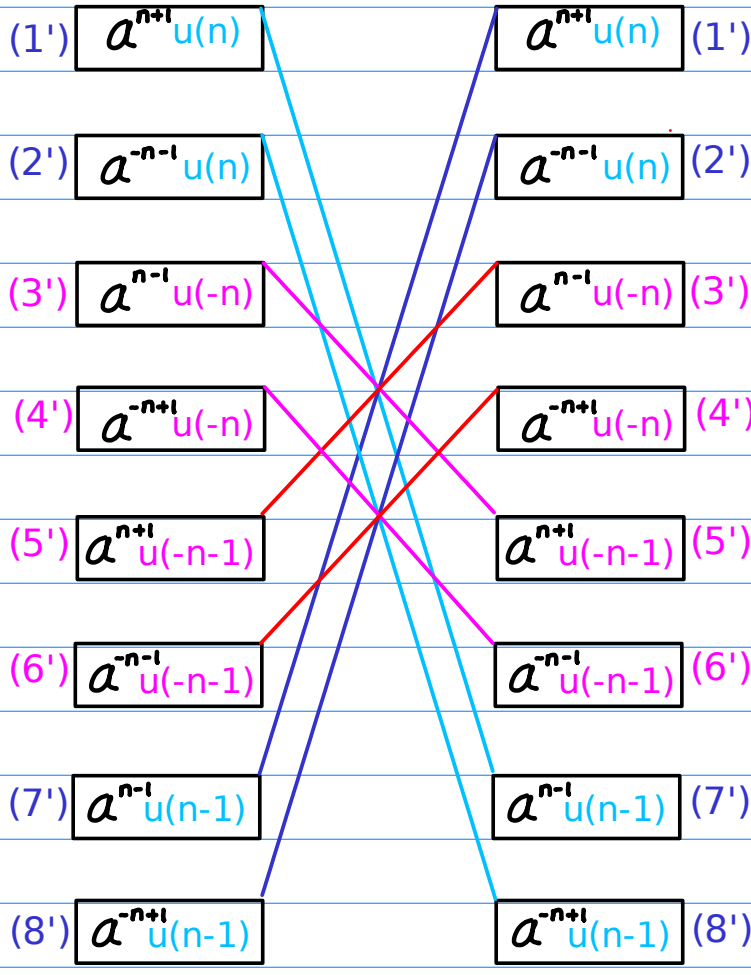
E.I Shifting2
Shifted Range Flipping
Range Complementing





(1) - (7)
(2) - (8)
(3) - (5)
(4) - (6)

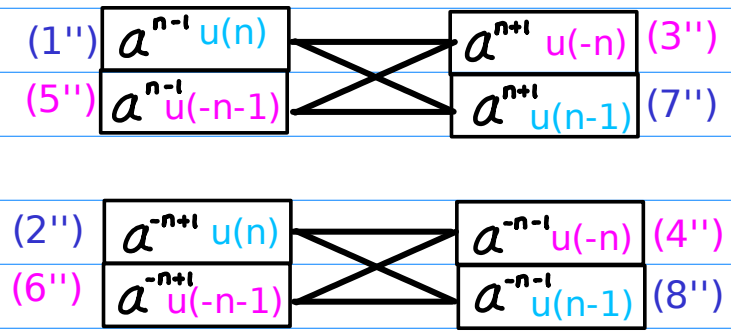
(5) - (3)
(6) - (4)
(7) - (1)
(8) - (2)

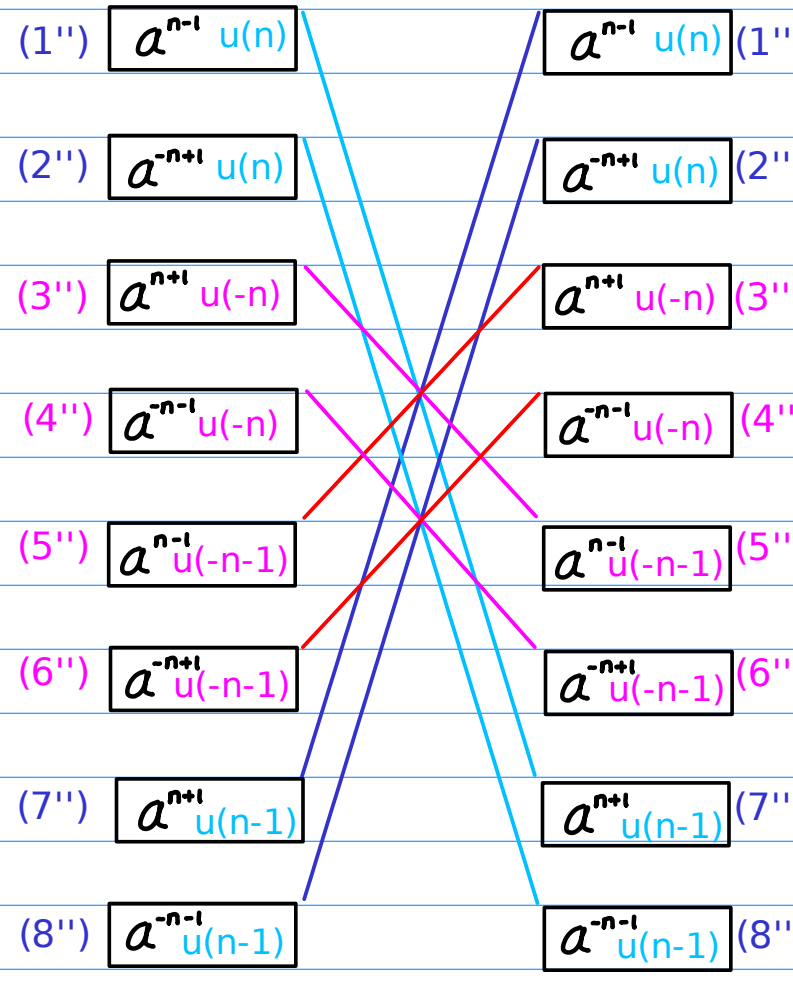


(1') - (7')
(2') - (8')
(3') - (5')
(4') - (6')

(5') - (3')
(6') - (4')
(7') - (1')
(8') - (2')

H.I Shifting2
Shifted Range Flipping
Range Complementing



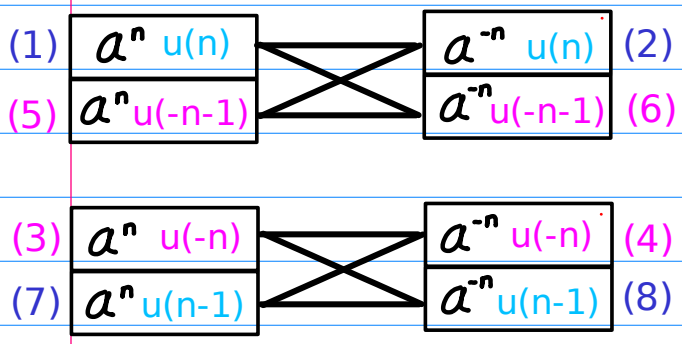


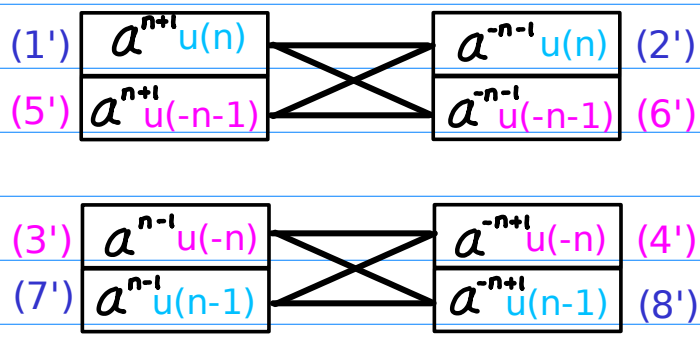
(1') - (7')
(2') - (8')
(3') - (5')
(4') - (6')

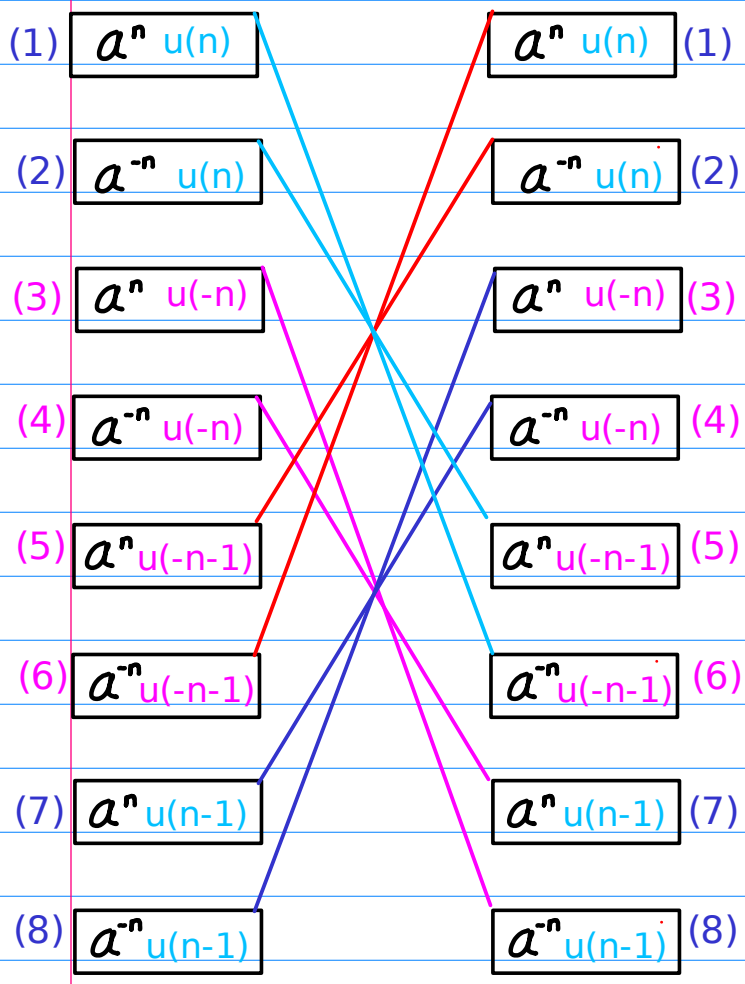
(5') - (3')
(6') - (4')
(7') - (1')
(8') - (2')

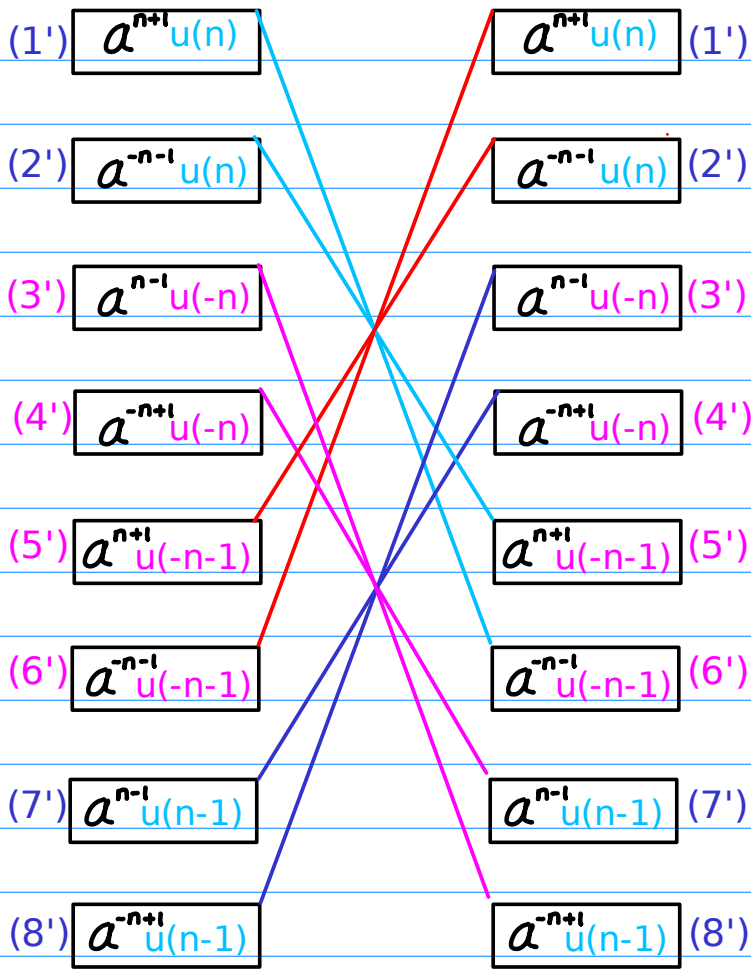
C.I Complementary Inverting
Base Inverting
Range Complementing

F.I Complementary Inverting
Base Inverting
Range Complementing









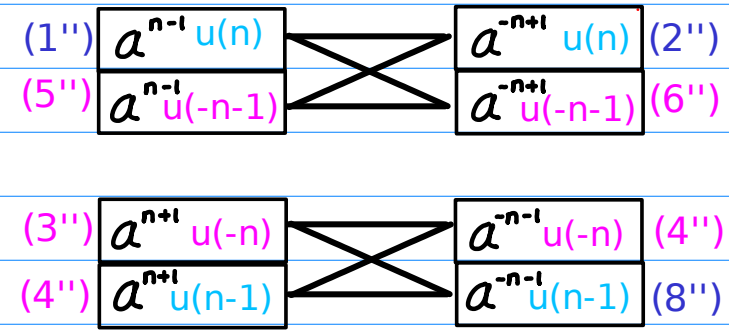
(1) - (6)
(6) - (1)
(2) - (5)
(5) - (2)

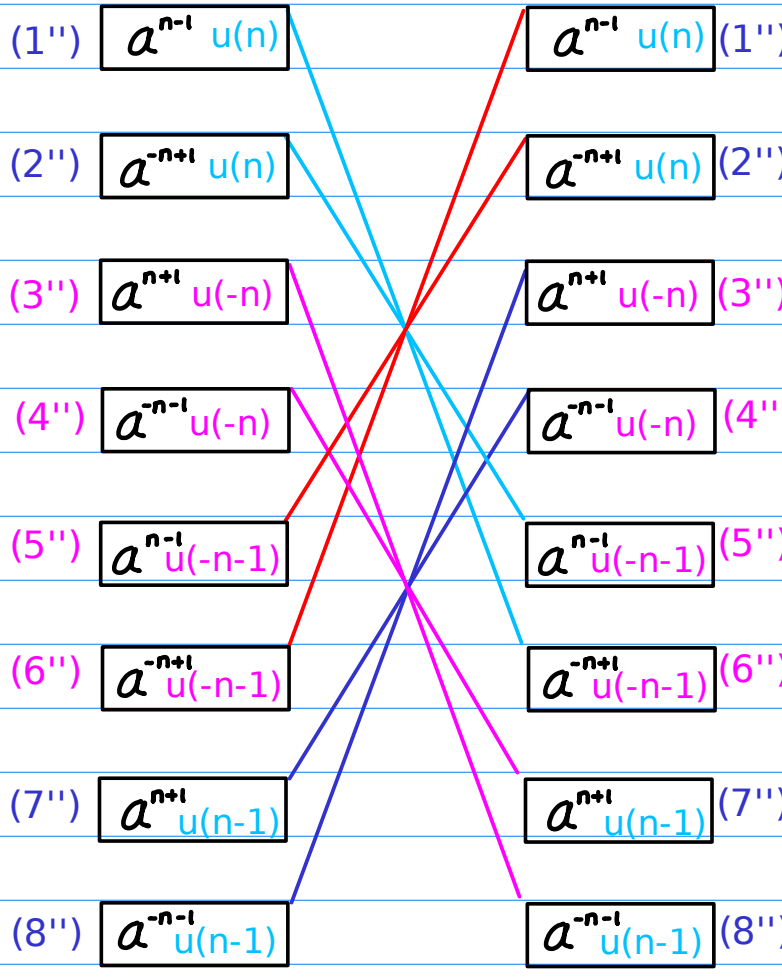
(5) - (2)
(6) - (1)
(7) - (4)
(8) - (3)

(1') - (6')
(6') - (1')
(2') - (5')
(5') - (2')

(5') - (2')
(6') - (1')
(7') - (4')
(8') - (3')

I.I Complementary Inverting
Base Inverting
Range Complementing



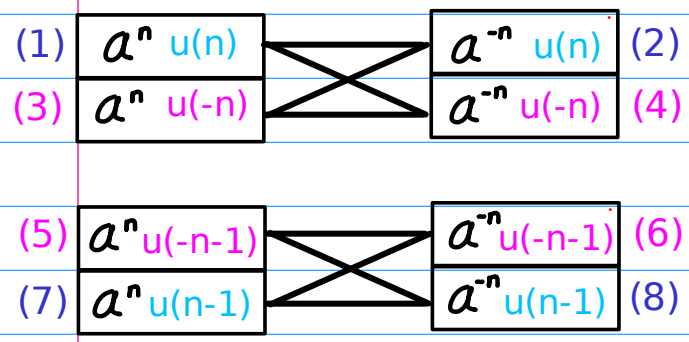


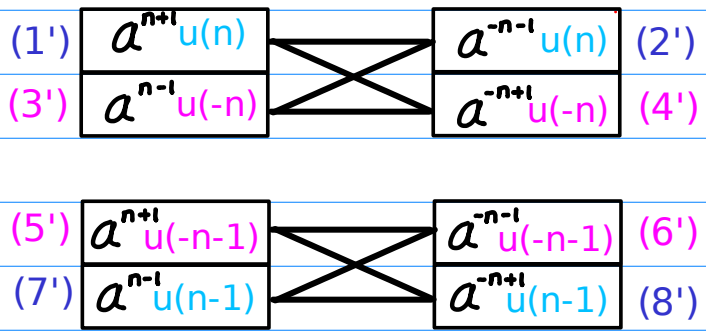
(1') - (6')
(6') - (1')
(2') - (5')
(5') - (2')

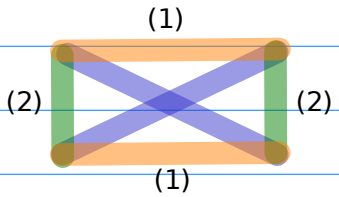
(5') - (2')
(6') - (1')
(7') - (4')
(8') - (3')

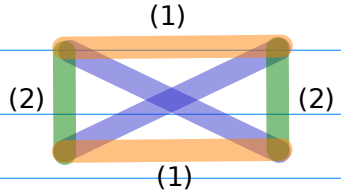
A.I Flipping
(1) Base Inverting
(2) Range Flipping

D.I Flipping2
(1) Base Inverting
(2) Shifted Range Flipping









$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-n} R(-n)$$

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$a^n R(n) \overset{(2)}{\Longleftrightarrow} a^{sh2(n)} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-sh2(n)} R(-n)$$

b^n	b^{-n}
a^n	a^{-n}
a^{-n}	a^n

b^n	$b^{-sh2(n)}$
$a^{(n+1)}$	$a^{-(n-1)}$
$a^{-(n+1)}$	$a^{(n-1)}$
$a^{(n-1)}$	$a^{-(n+1)}$
$a^{-(n-1)}$	$a^{(n+1)}$

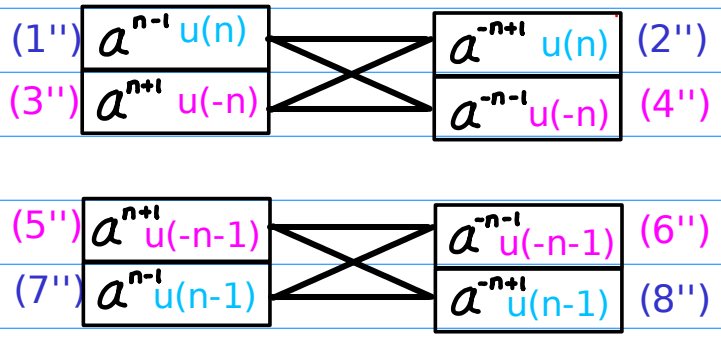
$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

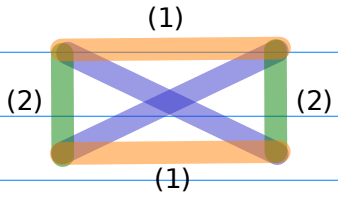
$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

G.I Flipping2

(1) Base Inverting

(2) Shifted Range Flipping





$$a^{n} \overset{(1)}{\Longleftrightarrow} a^{-n}$$

$$a^{n}R(n) \overset{(2)}{\Longleftrightarrow} a^{sh2(n)}R(-n)$$

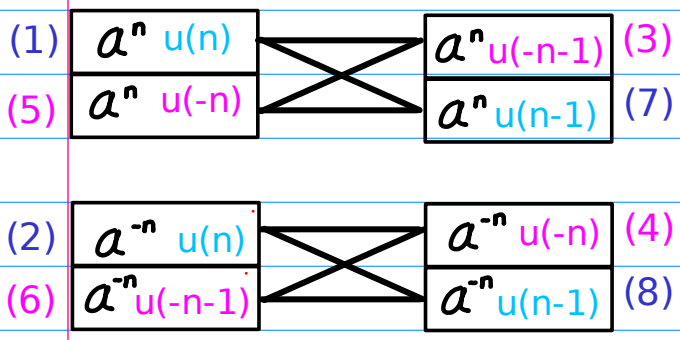
$$a^{n}R(n) \Longleftrightarrow a^{-sh2(n)}R(-n)$$

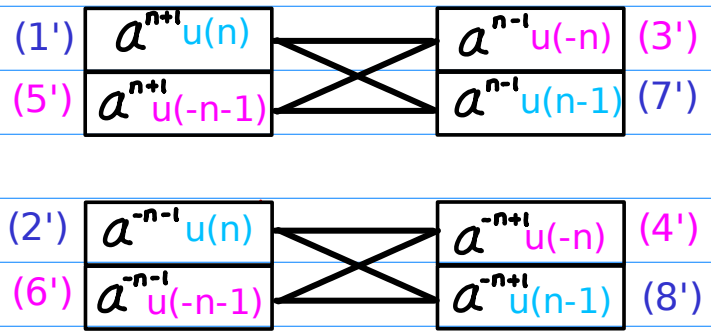
b^{n}	$b^{-sh2(n)}$
$a^{(n+1)}$	$a^{-(n-1)}$
$a^{-(n+1)}$	$a^{(n-1)}$
$a^{(n-1)}$	$a^{-(n+1)}$
$a^{-(n-1)}$	$a^{(n+1)}$

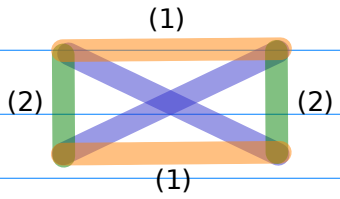
$R(n)$	$R(-n)$
$u(n)$	$u(-n)$
$u(n-1)$	$u(-n-1)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

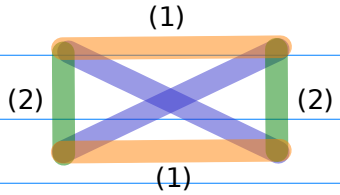
B.I Range Shifting
(1) Range Complementing
(2) Range Flipping

E.I Shifting2
(1) Shifted Range Flippin
(2) Range Complementing









$$R(n) \overset{(1)}{\Longleftrightarrow} R(-n)$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$R(n) \Longleftrightarrow \overline{R(-n)}$$

$$a^n R(n) \overset{(1)}{\Longleftrightarrow} a^{sh2(n)} R(-n)$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

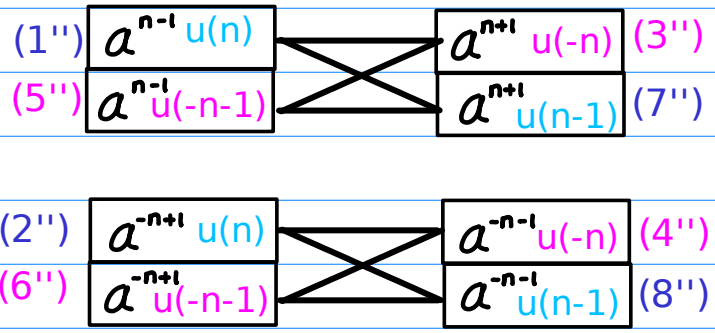
b^n	$b^{sh2(n)}$
$a^{(n+1)}$	$a^{(n-1)}$
$a^{-(n+1)}$	$a^{-(n-1)}$
$a^{(n-1)}$	$a^{(n+1)}$
$a^{-(n-1)}$	$a^{-(n+1)}$

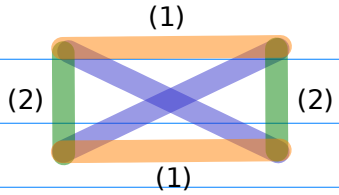
$R(n)$	$\overline{R(-n)}$
$u(n)$	$u(n-1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-n-1)$
$u(-n-1)$	$u(-n)$

$R(n)$	$\overline{R(-n)}$
$u(n)$	$u(n-1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-n-1)$
$u(-n-1)$	$u(-n)$

H.I Shifting2

(1) Shifted Range Flipping
(2) Range Complementing





$$a^n R(n) \overset{(1)}{\Longleftrightarrow} a^{sh\,2(n)} R(-n)$$

$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$

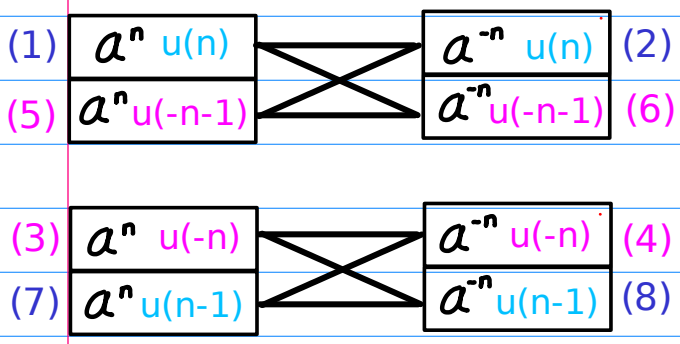
$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} \overline{R(-n)}$$

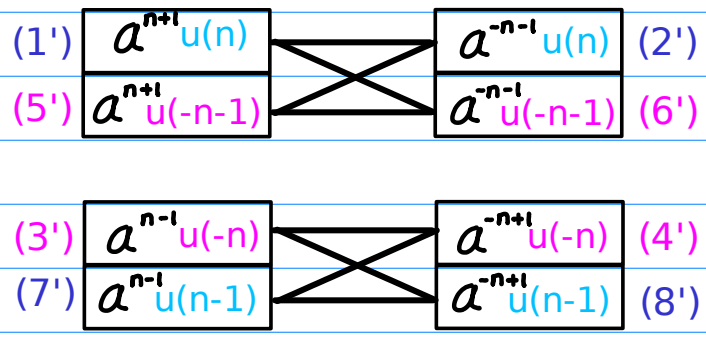
b^n	$b^{sh2(n)}$
$a^{(n+1)}$	$a^{(n-1)}$
$a^{-(n+1)}$	$a^{-(n-1)}$
$a^{(n-1)}$	$a^{(n+1)}$
$a^{-(n-1)}$	$a^{-(n+1)}$

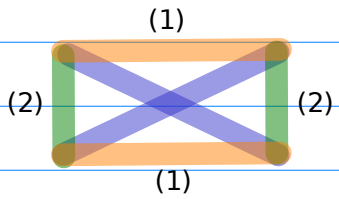
$R(n)$	$\overline{R(-n)}$
$u(n)$	$u(n-1)$
$u(n-1)$	$u(n)$
$u(-n)$	$u(-n-1)$
$u(-n-1)$	$u(-n)$

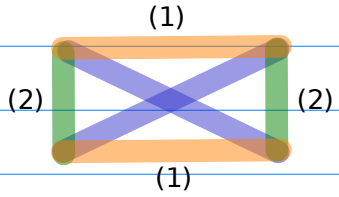
C.I Complementary Inverting
(1) Base Inverting
(2) Range Complementing

F.I Complementary Inverting
(1) Base Inverting
(2) Range Complementing









$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

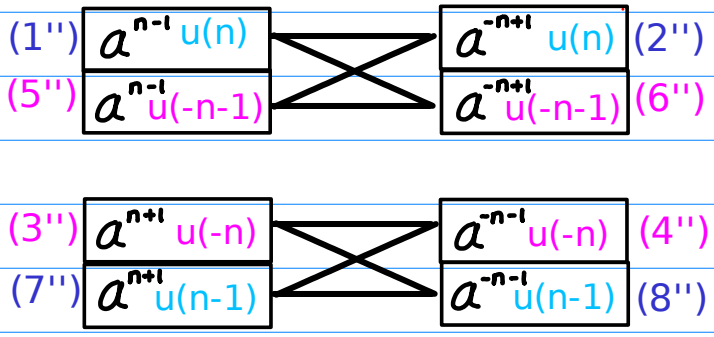
b^n	b^{-n}
a^n	a^{-n}
a^{-n}	a^n

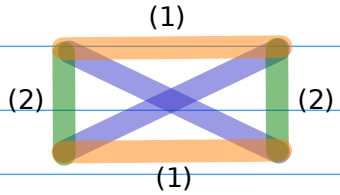
b^n	b^{-n}
$a^{(n+1)}$	$a^{-(n+1)}$
$a^{-(n+1)}$	$a^{(n+1)}$
$a^{(n-1)}$	$a^{-(n-1)}$
$a^{-(n-1)}$	$a^{(n-1)}$

$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

I.I Complementary Inverting
(1) Base Inverting
(2) Range Complementing





$$a^{n} \overset{(1)}{\Longleftrightarrow} a^{-n}$$

$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$

$$a^{n}R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

b^{n}	b^{-n}
$a^{(n+1)}$	$a^{-(n+1)}$
$a^{-(n+1)}$	$a^{(n+1)}$
$a^{(n-1)}$	$a^{-(n-1)}$
$a^{-(n-1)}$	$a^{(n-1)}$

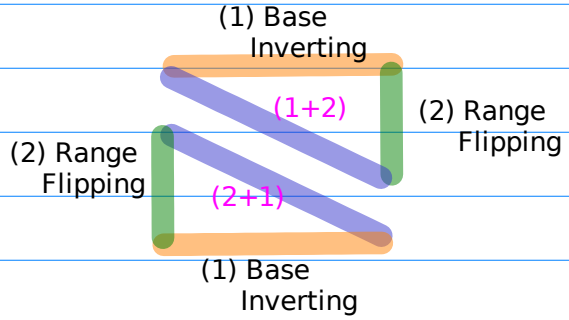
$R(n)$	$\overline{R(n)}$
$u(n)$	$u(-n-1)$
$u(n-1)$	$u(-n)$
$u(-n)$	$u(n)$
$u(-n-1)$	$u(n-1)$

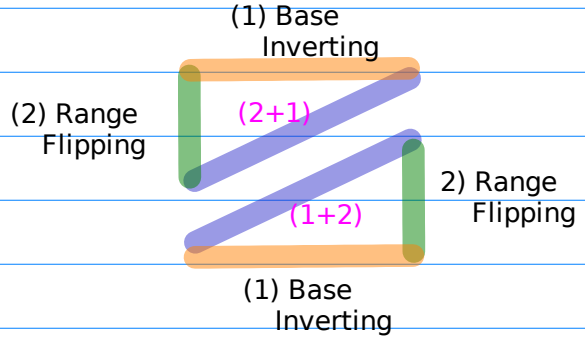
A.I Flipping
(1) Base Inverting
(2) Range Flipping

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-n} R(-n)$$

$$a^n R(n) \xrightarrow{(1)} a^{-n} R(n)$$
$$\xrightarrow{(2)} a^{-n} R(-n)$$

$$a^n R(n) \xrightarrow{(2)} a^n R(-n)$$
$$\xrightarrow{(1)} a^{-n} R(-n)$$



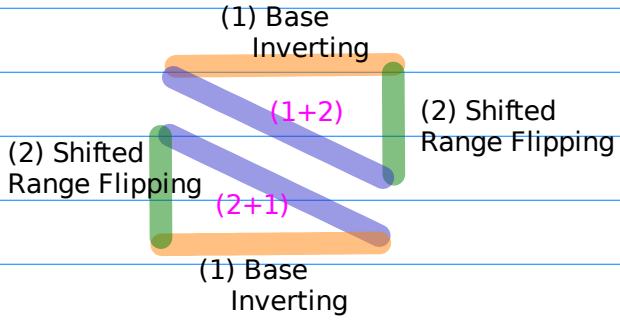


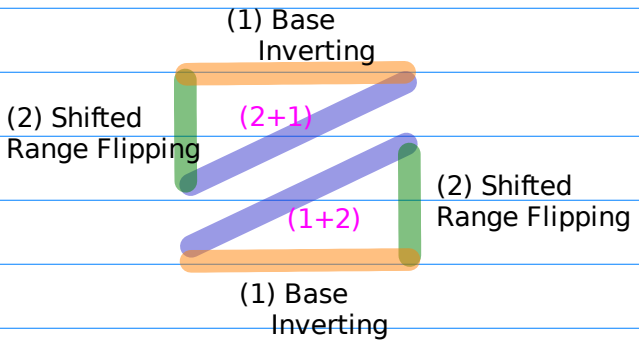
D.I Flipping2
(1) Base Inverting
(2) Shifted Range Flippin

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$a^n R(n) \overset{(2)}{\Longleftrightarrow} a^{sh2(n)} R(-n)$$
$$a^n R(n) \Longleftrightarrow a^{-sh2(n)} R(-n)$$

$$a^n R(n) \xrightarrow{(1)} a^{-n} R(n)$$
$$\xrightarrow{(2)} a^{-sh2(n)} R(-n)$$

$$a^n R(n) \xrightarrow{(2)} a^{sh2(n)} R(-n)$$
$$\xrightarrow{(1)} a^{-sh2(n)} R(-n)$$





D.I Flipping2

(1) Base Inverting

(2) Shifted Range Flipping

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$

$$a^n R(n) \overset{(2)}{\Longleftrightarrow} a^{sh2(n)} R(-n)$$

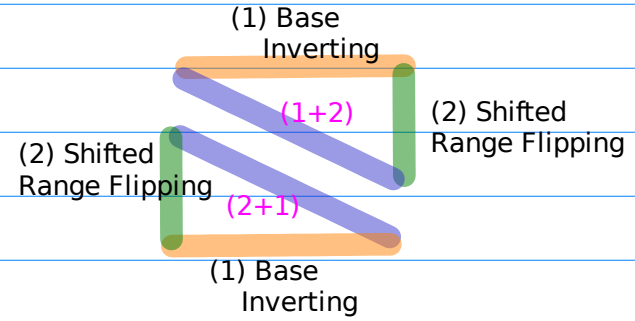
$$a^n R(n) \Longleftrightarrow a^{-sh2(n)} R(-n)$$

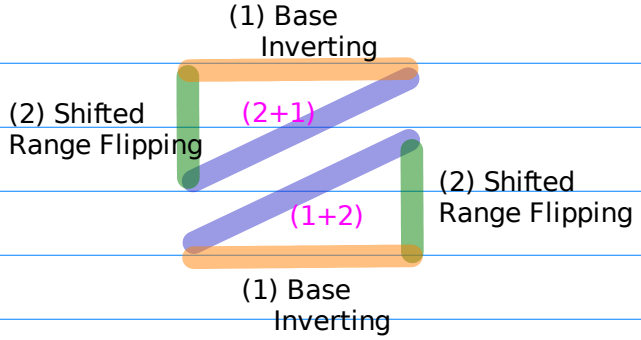
$$a^n R(n) \overset{(1)}{\Longrightarrow} a^{-n} R(n)$$

$$\overset{(2)}{\Longrightarrow} a^{-sh2(n)} R(-n)$$

$$a^n R(n) \overset{(2)}{\Longrightarrow} a^{sh2(n)} R(-n)$$

$$\overset{(1)}{\Longrightarrow} a^{-sh2(n)} R(-n)$$





B.I Range Shifting
(1) Range Complementing
(2) Range Flipping

$$R(n) \overset{(1)}{\Longleftrightarrow} R(-n)$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$R(n) \Longleftrightarrow \overline{R(-n)}$$

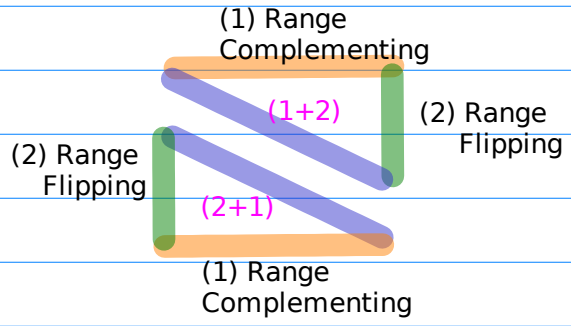
$$a^n R(n) \overset{(1)}{\Longleftrightarrow} a^{sh2(n)} R(-n)$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

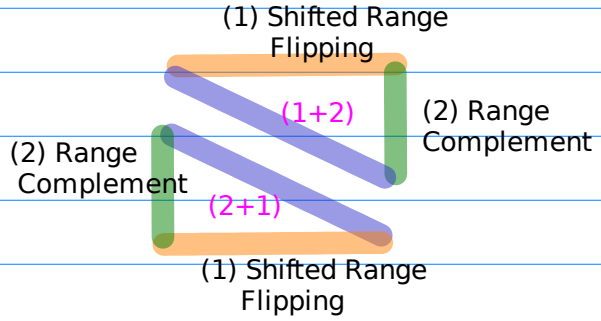
$$a^n R(n) \overset{(1)}{\Rightarrow} a^{-n} R(n)$$
$$\overset{(2)}{\Rightarrow} a^{-n} \overline{R(n)}$$

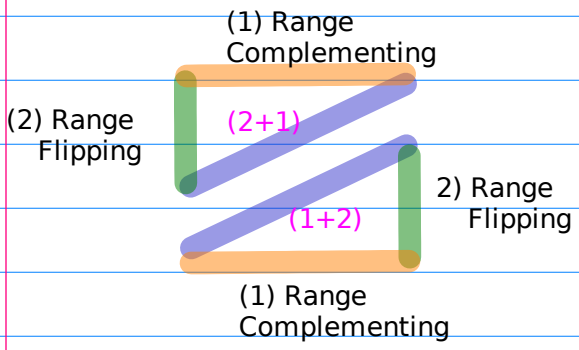
$$a^n R(n) \overset{(1)}{\Rightarrow} a^{-n} R(n)$$
$$\overset{(2)}{\Rightarrow} a^{-sh2(n)} R(-n)$$

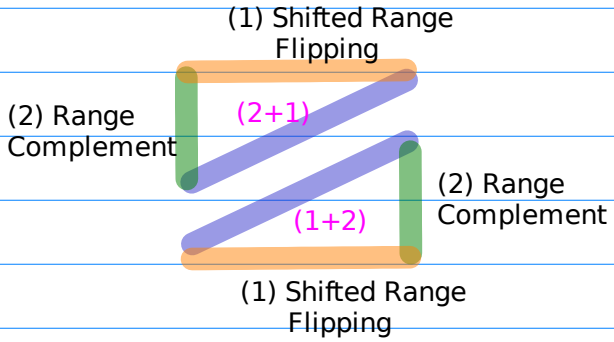
$$a^n rng(n) \overset{(2)}{\Rightarrow} a^n \overline{R(n)}$$
$$\overset{(1)}{\Rightarrow} a^{-n} \overline{R(n)}$$

$$a^n R(n) \overset{(2)}{\Rightarrow} a^{sh2(n)} R(-n)$$
$$\overset{(1)}{\Rightarrow} a^{-sh2(n)} R(-n)$$









E.I Shifting2
(1) Shifted Range Flippin
(2) Range Complementing

E.I Shifting2

(1) Shifted Range Flipping
(2) Range Complementing

$$a^n R(n) \overset{(1)}{\Longleftrightarrow} a^{sh2(n)} R(-n)$$

$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$

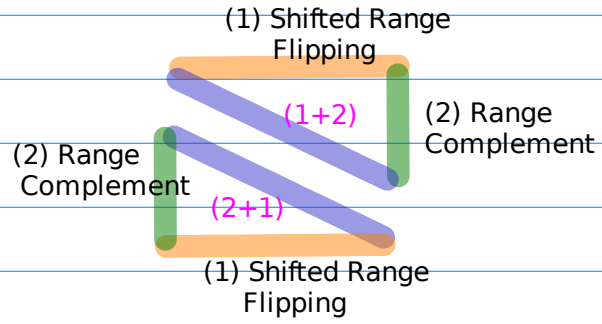
$$a^n R(n) \Longleftrightarrow a^{sh2(n)} \overline{R(-n)}$$

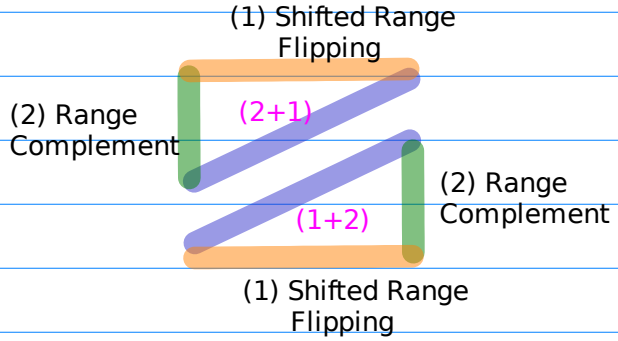
$$a^n R(n) \xrightarrow{(1)} a^{-n} R(n)$$

$$\xrightarrow{(2)} a^{-sh2(n)} R(-n)$$

$$a^n R(n) \xrightarrow{(2)} a^{sh2(n)} R(-n)$$

$$\xrightarrow{(1)} a^{-sh2(n)} R(-n)$$



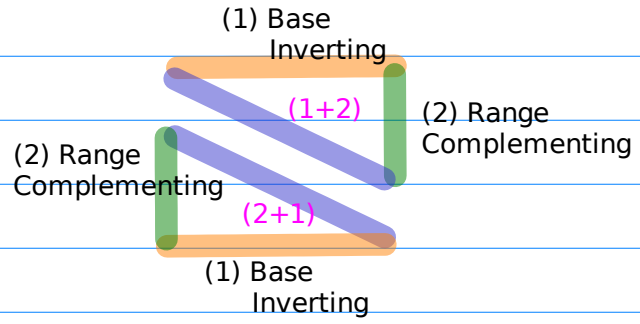


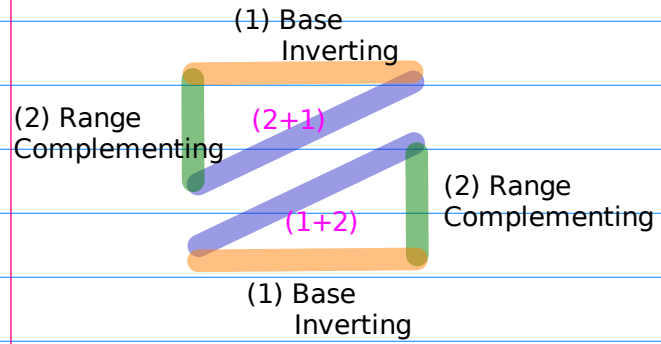
C.I Complementary Inverting
(1) Base Inverting
(2) Range Complementing

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

$$a^n R(n) \overset{(1)}{\Longrightarrow} a^{-n} R(n)$$
$$\overset{(2)}{\Longrightarrow} a^{-n}\overline{R(n)}$$

$$a^n R(n) \overset{(2)}{\Longrightarrow} a^n \overline{R(n)}$$
$$\overset{(1)}{\Longrightarrow} a^{-n}\overline{R(n)}$$



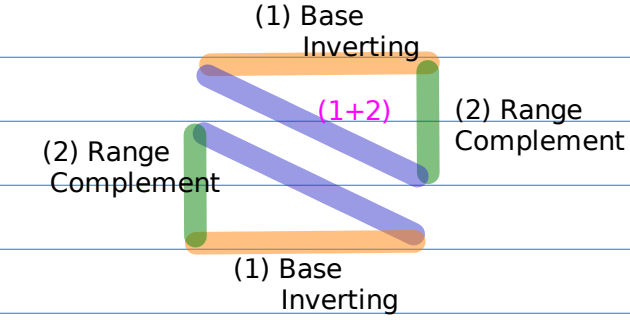


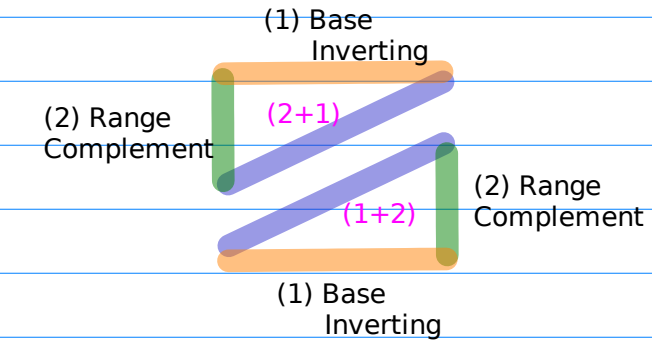
F.I Complementary Inverting
(1) Base Inverting
(2) Range Complementing

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$
$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

$$a^n R(n) \overset{(1)}{\Longrightarrow} a^{-n} R(n)$$
$$\overset{(2)}{\Longrightarrow} a^{-n}\overline{R(n)}$$

$$a^n R(n) \overset{(2)}{\Longrightarrow} a^n \overline{R(n)}$$
$$\overset{(1)}{\Longrightarrow} a^{-n}\overline{R(n)}$$





F.I Complementary Inverting
(1) Base Inverting
(2) Range Complementing

$$a^n \overset{(1)}{\Longleftrightarrow} a^{-n}$$

$$R(n) \overset{(2)}{\Longleftrightarrow} \overline{R(n)}$$

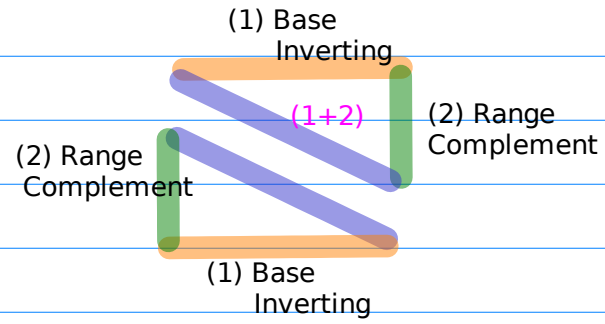
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

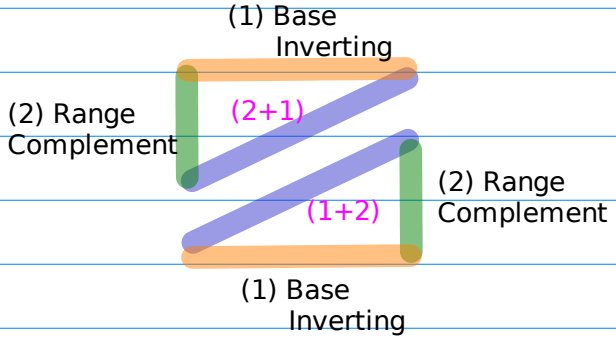
$$a^n R(n) \overset{(1)}{\Longrightarrow} a^{-n} R(n)$$

$$\overset{(2)}{\Longrightarrow} a^{-n}\overline{R(n)}$$

$$a^n R(n) \overset{(2)}{\Longrightarrow} a^n \overline{R(n)}$$

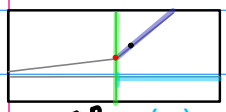
$$\overset{(1)}{\Longrightarrow} a^{-n}\overline{R(n)}$$





A.II Flipping
Base Inverting
Range Flipping

(1) 0000



a^n u(n)

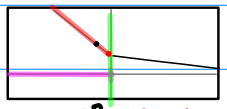
(2) 0001

a^{-n} u(n)

(3) 0010

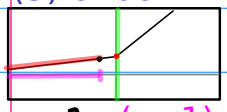
a^n u(-n)

(4) 0011



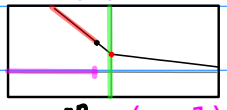
a^{-n} u(-n)

(5) 0100



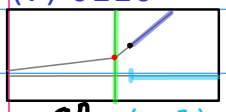
a^n u(-n-1)

(6) 0101



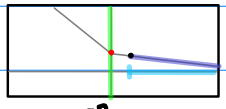
a^{-n} u(-n-1)

(7) 0110



a^n u(n-1)

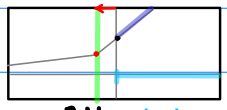
(8) 0111



a^{-n} u(n-1)

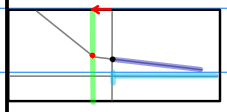
D.II Flipping2
Base Inverting
Shifted Range Flipping

(1') 1000



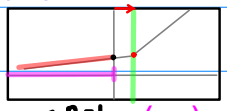
a^{n+1} u(n)

(2') 1001



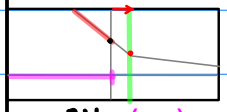
a^{-n-1} u(n)

(3') 1010



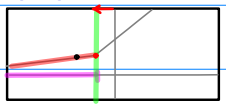
a^{n-1} u(-n)

(4') 1011



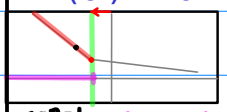
a^{-n+1} u(-n)

(5') 1100



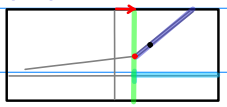
a^{n+1} u(-n-1)

(6') 1101



a^{-n-1} u(-n-1)

(7') 1110



a^{n-1} u(n-1)

(8') 1111

a^{-n+1} u(n-1)

Shifted Range Flipping
= Exponent Shifting2
+ Range Flippping

$$a^n R(n) \Longleftrightarrow a^{-n} R(-n)$$

$$a^n \Longleftrightarrow a^{-n}$$

$$R(n) \Longleftrightarrow R(-n)$$

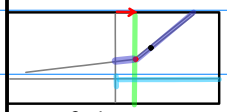
$$a^n R(n) \Longleftrightarrow a^{-sh2(n)} R(-n)$$

$$a^n \Longleftrightarrow a^{-n}$$

$$a^n R(n) \Longleftrightarrow a^{sh2(n)} R(-n)$$

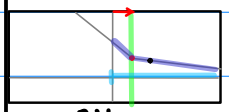
G.II Flipping2
Base Inverting
Shifted Range Flipping

(1'') 1000



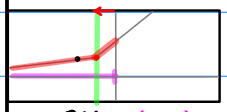
$a^{n-1}u(n)$

(2'') 1001



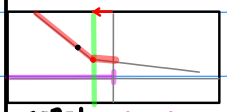
$a^{-n+1}u(n)$

(3'') 1010



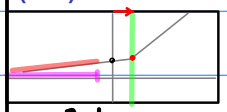
$a^{n+1}u(-n)$

(4'') 1011



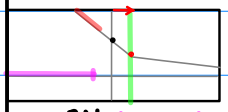
$a^{-n-1}u(-n)$

(5'') 1100



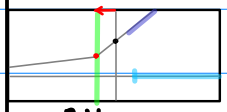
$a^{n-1}u(-n-1)$

(6'') 1101



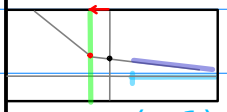
$a^{-n+1}u(-n-1)$

(7'') 1110



$a^{n+1}u(n-1)$

(8'') 1111



$a^{-n-1}u(n-1)$

Shifted Range Flipping
= Exponent Shifting2
+ Range Flippping

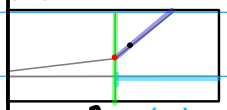
$$a^{n}R(n) \Longleftrightarrow a^{-sh2(n)}R(-n)$$

$$a^{n} \Longleftrightarrow a^{-n}$$

$$a^{n}R(n) \Longleftrightarrow a^{sh2(n)}R(-n)$$

B.II Range Shifting
Range Flipping
Range Complementing

(1) 0000

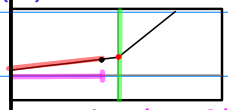


a^n u(n)

(3) 0010

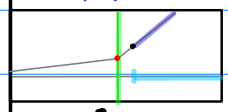
a^n u(-n)

(5) 0100



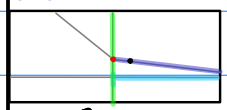
a^n u(-n-1)

(7) 0110



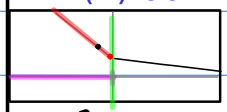
a^n u(n-1)

(2) 0001



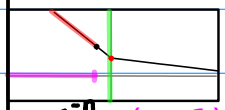
a^{-n} u(n)

(4) 0011



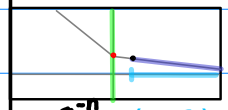
a^{-n} u(-n)

(6) 0101



a^{-n} u(-n-1)

(8) 0111



a^{-n} u(n-1)

Range Shifting
= Range Flipping
+ Range Complementing

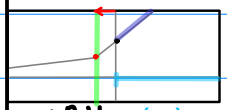
$$R(n) \Longleftrightarrow \overline{R(-n)}$$

$$R(n) \Longleftrightarrow R(-n)$$

$$R(n) \Longleftrightarrow \overline{R(n)}$$

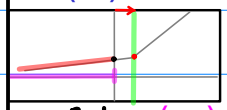
E.II Shifting2
Shifted Range Flipping
Range Complementing

(1') 1000



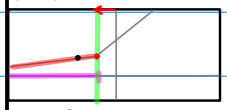
a^{n+1} u(n)

(3') 1010



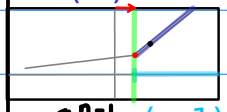
a^{n-1} u(-n)

(5') 1100



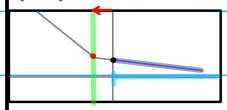
a^{n+1} u(-n-1)

(7') 1110



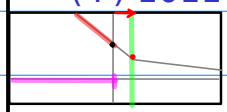
a^{n-1} u(n-1)

(2') 1001



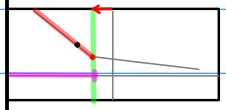
a^{-n-1} u(n)

(4') 1011



a^{-n+1} u(-n)

(6') 1101



a^{-n-1} u(-n-1)

(8') 1111

a^{-n+1} u(n-1)

Shifted Range Flipping
= Exponent Shifting2
+ Range Flippping

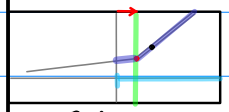
$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} \overline{R(-n)}$$

$$a^n R(n) \Longleftrightarrow a^{sh\,2(n)} R(-n)$$

$$R(n) \Longleftrightarrow \overline{R(n)}$$

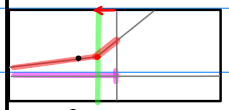
H.II Shifting2
Shifted Range Flipping
Range Complementing

(1'') 1000



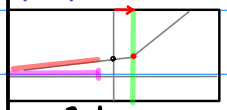
$a^{n-1}u(n)$

(3'') 1010



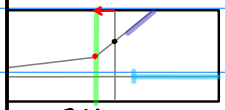
$a^{n+1}u(-n)$

(5'') 1100



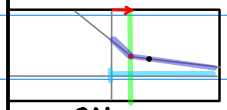
$a^{n-1}u(-n-1)$

(7'') 1110



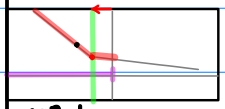
$a^{n+1}u(n-1)$

(2'') 1001



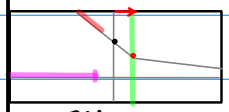
$a^{-n+1}u(n)$

(4'') 1011



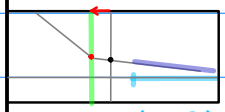
$a^{-n-1}u(-n)$

(6'') 1101



$a^{-n+1}u(-n-1)$

(8'') 1111



$a^{-n-1}u(n-1)$

Shifted Range Flipping
= Exponent Shifting2
+ Range Flippping

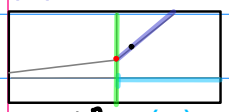
$$a^{n}R(n) \Longleftrightarrow a^{sh\,2(n)}\overline{R(-n)}$$

$$a^{n}R(n) \Longleftrightarrow a^{sh\,2(n)}R(-n)$$

$$R(n) \Longleftrightarrow \overline{R(n)}$$

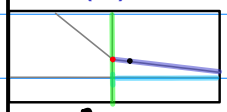
C.II Complementary Inverting
Base Inverting
Range Complementing

(1) 0000



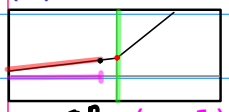
a^n u(n)

(2) 0001



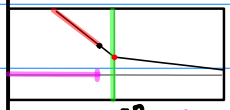
a^{-n} u(n)

(5) 0100



a^n u(-n-1)

(6) 0101

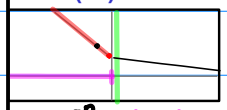


a^{-n} u(-n-1)

(3) 0010

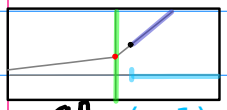
a^n u(-n)

(4) 0011



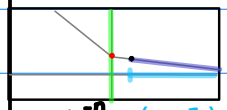
a^{-n} u(-n)

(7) 0110



a^n u(n-1)

(8) 0111



a^{-n} u(n-1)

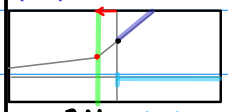
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

$$a^n \Longleftrightarrow a^{-n}$$

$$R(n) \Longleftrightarrow \overline{R(n)}$$

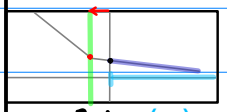
F.II Complementary Inverting
Base Inverting
Range Complementing

(1') 1000



a^{n+1} u(n)

(2') 1001

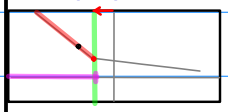


a^{-n-1} u(n)

(5') 1100

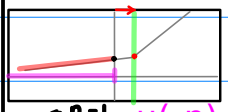
a^{n+1} u(-n-1)

(6') 1101



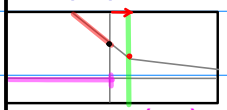
a^{-n-1} u(-n-1)

(3') 1010



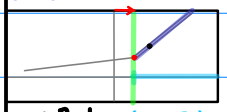
a^{n-1} u(-n)

(4') 1011



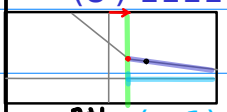
a^{-n+1} u(-n)

(7') 1110



a^{n-1} u(n-1)

(8') 1111



a^{-n+1} u(n-1)

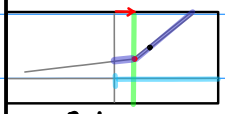
$$a^n R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

$$a^n \Longleftrightarrow a^{-n}$$

$$R(n) \Longleftrightarrow \overline{R(n)}$$

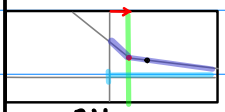
I.II Complementary Inverting
Base Inverting
Range Complementing

(1'') 1000



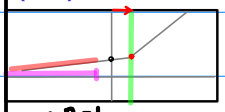
$a^{n-1}u(n)$

(2'') 1001



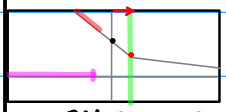
$a^{-n+1}u(n)$

(5'') 1100



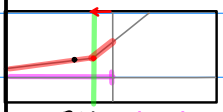
$a^{n-1}u(-n-1)$

(6'') 1101



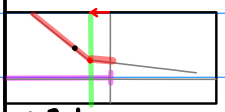
$a^{-n+1}u(-n-1)$

(3'') 1010



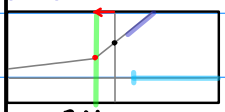
$a^{n+1}u(-n)$

(4'') 1011



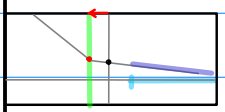
$a^{-n-1}u(-n)$

(7'') 1110



$a^{n+1}u(n-1)$

(8'') 1111



$a^{-n-1}u(n-1)$

Shifted Range Flipping
= Exponent Shifting2
+ Range Flippping

$$a^{n}R(n) \Longleftrightarrow a^{-n}\overline{R(n)}$$

$$a^{n} \Longleftrightarrow a^{-n}$$

$$R(n) \Longleftrightarrow \overline{R(n)}$$