

The Carry Flag

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- 2 The Carry flag
 - TOC: Carry flag
 - Examples of signed and unsigned integer arithmetic
 - The Carry flag in unsigned and signed computations
 - Rules for the carry flag
 - Method for computing the carry flag
 - More examples of the carry flag

- The CARRY flag and OVERFLOW flag in binary arithmetic
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[https://teaching.idallen.com/dat2343/10f/notes/
040_overflow.ttx](https://teaching.idallen.com/dat2343/10f/notes/040_overflow.ttx)

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Compiling 32-bit program on 64-bit gcc

- `gcc -v`
- `gcc -m32 t.c`
- `sudo apt-get install gcc-multilib`
- `sudo apt-get install g++-multilib`
- `gcc-multilib`
- `g++-multilib`
- `gcc -m32`
- `objdump -m i386`

- Examples of signed and unsigned integer arithmetic
- Carry flag in unsigned and signed computations
- Rules for the carry flag
- Method for computing the carry flag
- More examples of the carry flag

TOC: Examples of signed and unsigned integer arithmetic

- Subtracting $0x0000618D$ from $0x0000195D$
- **Signed** and **unsigned** integer arithmetic examples
- 2's complement of the subtrahend $0x0000618D$
- Interpreting $0xFFFF9E73$ (2's complement of $0x0000618D$)
- **Unsigned** subtraction $0x0000195D - 0x0000618D$
- **Signed** subtraction $0x0000195D + (-0x0000618D)$
- 2's complement of the subtraction result $0xFFFFB7D0$
- Interpreting the subtraction result $0xFFFFB7D0$
- Summary of **signed** and **unsigned** subtractions
- Carry flag (CF) and **unsigned** integer overflows
- Overflow flag (OF) and **signed** integer overflows

Subtracting 0x0000618D from 0x0000195D (1)

- a subtraction example

$$\begin{array}{r} \text{Minuend} \quad - \text{Subtrahend} \quad = \text{Difference} \\ 0x0000195D \quad - \quad 0x0000618D \quad = \quad 0xFFFFB7D0 \end{array}$$

- 2's complement

$$\begin{array}{r} 0x0000618D \quad \leftrightarrow \quad 0xFFFF9E73 \\ 0xFFFFB7D0 \quad \leftrightarrow \quad 0x00004830 \end{array}$$

- the results of **unsigned** and **signed** subtractions have the same bit pattern 0xFFFFB7D0

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

Subtracting 0x0000618D from 0x0000195D (2)

- 2's complement

0x0000618D	↔	0xFFFF9E73 *
0xFFFFB7D0 *	↔	0x00004830

- interpreting 0xFFFF9E73

as an unsigned number	+0xFFFF9E73	+4294942323 ₁₀
as a signed number	-0x0000618D	-24973 ₁₀

- interpreting 0xFFFFB7D0 = 0x0000195D - 0x0000618D

as an unsigned number	+0xFFFFB7D0	+4294948816 ₁₀
as a signed number	-0x00004830	-18480 ₁₀

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

Signed and unsigned integer arithmetic examples

- $0x0000195D - 0x0000618D = 0xFFFFB7D0$

unsigned subtraction	$0x0000195D - 0x0000618D$ 6493 - 24973	$= 0xFFFFB7D0$ $= 16^8 - 18480 = 4294948816$
	subtraction by hand	borrow (CF=1) $16^8 = 2^{32} = 4294967296$

signed subtraction	$0x0000195D + (-0x0000618D)$ $0x0000195D + 0xFFFF9E73$ 6493 - 24973	$= -0x00004830$ $= 0xFFFFB7D0$ $= -18480$
	the <i>transformed addition</i> using the 2's complement of <u>subtrahend</u>	CF = 1 ← Cn = 0 CF = !Cn for subtraction

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

2's complement of the subtrahend 0x0000618D

- the 2's complement of 0x0000618D \rightarrow 0xFFFF9E73 ($= -24973_{10}$)

	F	F	F	F	9	E	7	3	
0xFFFF9E73	0x1111_1111_1111_1111_1001_1110_0111_0011								
0x0000618C	0x0000_0000_0000_0000_0110_0001_1000_1100								(1's complement)
0x0000618D	0x0000_0000_0000_0000_0110_0001_1000_1101								(2's complement)
	0	0	0	0	6	1	8	D	

- the 2's complement of 0xFFFF9E73 \rightarrow 0x0000618D ($= +24973_{10}$)

	0	0	0	0	6	1	8	D	
0x0000618D	0x0000_0000_0000_0000_0110_0001_1000_1101								
0xFFFF8E72	0x1111_1111_1111_1111_1001_1110_0111_0010								(1's complement)
0xFFFF8E73	0x1111_1111_1111_1111_1001_1110_0111_0011								(2's complement)
	F	F	F	F	9	E	7	3	

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

Interpreting 0xFFFF9E73

- interpreting 0xFFFF9E73
 - as an **unsigned** Number : $+0xFFFF9E73 (= +4294942323_{10})$

F	F	F	F	9	E	7	3
15	15	15	15	9	14	7	3
16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0

- as a **signed** number : $-0x0000618D (= -24973_{10})$

0	0	0	0	6	1	8	D
0	0	0	0	6	1	8	13
16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

Unsigned subtraction 0x0000195D - 0x0000618D (1)

- 0x0000195D - 0x0000618D : unsigned subtraction

subtraction by hand

```

          0  0  0  0  1  9  5  D
0x0000195D  0x0000_0000_0000_0000_0001_1001_0101_1101
- 0x0000618D  0x0000_0000_0000_0000_0110_0001_1000_1101
-----
0          0  0  0  0  6  1  8  D
-----
0xFFFFB7D0  1 0x1111_1111_1111_1111_1011_0111_1101_0000 (hand subtraction)
1          F  F  F  F  B  7  D  0
.
V borrow (CF=1) : unsigned integer overflow
```

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

Unsigned subtraction $0x0000195D - 0x0000618D$ (2)

- $0x0000195D - 0x0000618D$: unsigned subtraction subtraction by hand
 - unsigned integer overflow is indicated by the carry flag (CF)
 - a borrow is indicated by the carry flag (CF=1)
 - whenever an unsigned integer overflow happened
 - $A - B$, when $A < B$, for non-negative integers A, B

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Signed subtraction $0x0000195D + (-0x0000618D)$ (1)

- $0x0000195D + (-0x0000618D)$: signed subtraction

the *transformed addition* using the 2's complement of subtrahend

		0	0	0	0	1	9	5	D	
0x0000195D		0x0000_0000_0000_0000_0001_1001_0101_1101								(+0x0000195D)
+ 0xFFFF9E73		0x1111_1111_1111_1111_1001_1110_0111_0011								(-0x0000618D)
		F	F	F	F	9	E	7	3	

0xFFFFB7D0	0	0x1111_1111_1111_1111_1011_0111_1101_0000								(hand addition)
	0	F	F	F	F	B	7	D	0	
-0x00004830	.	0x0000_0000_0000_0000_0100_1000_0011_0000								(2's complement)
	.	0	0	0	0	4	8	3	0	
	V	no carry in the transformed addition (Cn=0) --> (CF=1)								

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

Signed subtraction $0x0000195D + (-0x0000618D)$ (2)

- $0x0000195D + (-0x0000618D)$: signed subtraction
the *transformed addition* uses the 2's complement of a subtrahend
 - signed integer overflow is indicated by the **overflow** flag (OF)
 - when a subtraction is performed by a transformed addition, the **carry** flag (CF) is set by **inverting** the carry out (C_n) of a transformed addition

ADD instruction : $CF \leftarrow c_n$ SUB instruction : $CF \leftarrow \overline{c_n}$

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2's complement of the subtraction result 0xFFFFB7D0

- $0xFFFFB7D0 = 0x0000195D - 0x0000618D$
- the 2's complement of $0xFFFFB7D0$: $0x00004830$ ($= +18480_{10}$)

	F	F	F	F	B	7	D	0
0xFFFFB7D0	0x1111_1111_1111_1111_1011_0111_1101_0000							
0x0000482F	0x0000_0000_0000_0000_0100_1000_0010_1111							(1's complement)
0x00004830	0x0000_0000_0000_0000_0100_1000_0011_0000							(2's complement)
	0	0	0	0	4	8	3	0

- the 2's complement of $0x00004830$: $0xFFFFB7D0$ ($= -18480_{10}$)

	0	0	0	0	4	8	3	0
0x00004830	0x0000_0000_0000_0000_0100_1000_0011_0000							
0xFFFFB7CF	0x1111_1111_1111_1111_1011_0111_1100_1111							(1's complement)
0xFFFFB7D0	0x1111_1111_1111_1111_1011_0111_1101_0000							(2's complement)
	F	F	F	F	B	7	D	0

<https://stackoverflow.com/questions/47333458/assembly-x86-64-setting-carry-flag-f>

Interpreting the subtraction result 0xFFFFB7D0

- interpreting $0xFFFFB7D0 = 0x0000195D - 0x0000618D$
 - as an **unsigned** Number : $+0xFFFFB7D0 (= +4294948816_{10})$

F	F	F	F	B	7	D	0
15	15	15	15	11	7	13	0
16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0

- as a **signed** number : $-0x00004830 (= -18480_{10})$

0	0	0	0	4	8	3	0
0	0	0	0	4	8	3	0
16^7	16^6	16^5	16^4	16^3	16^2	16^1	16^0

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Summary of signed and unsigned subtractions (1)

- subtracting $0x0000618D$ from $0x0000195D$
 - $0x0000195D - 0x0000618D$: unsigned integer subtraction
hand subtraction
 - $0x0000195D + (-0x0000618D)$: signed integer subtraction
the *transformed addition* using the 2's complement of the subtrahend
 - the same result : $0xFFFFB7D0$ (the same bit pattern)
 - interpreting as a unsigned integer 4294948816_{10}
 $0xFFFFB7D0$ with a borrow (CF=1) $2^{32} - 18480$
 - interpreting as a signed integer -18480_{10}
 $-0x00004830$ (meaningless CF=1)

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Summary of signed and unsigned subtractions (2)

unsigned subtraction	$0x0000195D - 0x0000618D$	$0xFFFFB7D0$ 4294948816_{10} with a borrow (CF=1)
	subtraction by hand	

signed subtraction	$0x0000195D + (-0x0000618D)$	$-0x00004830$ -18480_{10}
	the <i>transformed addition</i> using the 2's complement of <u>subtrahend</u>	

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Carry flag (CF) and **unsigned** integer overflows

- $0x0000195D - 0x0000618D$: **unsigned** subtraction
 - there is an **unsigned** integer overflow
so the **carry** flag will be set ($CF=1$) to indicate a **borrow**
 - $A - B$, when $A < B$, for non-negative integers A, B
(unsigned integers can't be negative),

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Overflow flag (OF) and signed integer overflows

- $0x0000195D + (-0x0000618D)$: signed subtraction
 - there is no signed integer overflow
the overflow flag won't be set (OF=0)
 - signed overflow occurs in the transformed addition,
 - two *positive* numbers are added and the result is a *negative*, ($P + P \rightarrow N$), or
 - two *negative* numbers are added and the result is a *positive*, ($N + N \rightarrow P$)

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TOC Carry flag in unsigned and signed computations

- 2's complement numbers : 4-bit
- Carrys in signed arithmetics
- Carrys in unsigned arithmetics
- Signed and unsigned arithmetics
- Using the carry flag as a borrow

2's complement numbers : 4-bit

	signed	unsigned
0111	(+7)	7
0110	(+6)	6
0101	(+5)	5
0100	(+4)	4
0011	(+3)	3
0010	(+2)	2
0001	(+1)	1
0000	(0)	0
1000	(-8)	8
1001	(-7)	9
1010	(-6)	10
1011	(-5)	11
1100	(-4)	12
1101	(-3)	13
1110	(-2)	14
1111	(-1)	15

Carrys in unsigned arithmetics

- a carry in an **unsigned** addition can be obtained by performing a *hand* addition
 - **CF=1** represents a carry in an **unsigned** addition
- a borrow in an **unsigned** subtraction can be obtained by performing a *hand* subtraction
 - **CF=1** represents a borrow in an **unsigned** subtraction

Carrys in signed arithmetics

- carry out in a **signed** addition / subtraction are carry out of an *n-bit* 2's complement adder
 - a **signed** subtraction $A - B$ is performed by the transformed addtion $A + \overline{B} + 1$
 - carry out is denoted by C_n

Signed and unsigned arithmetics (1)

	Addition	Subtraction
Unsigned	Hand Addition with a Carry (CF)	Hand Subtraction with a Borrow (CF)
Signed	2's complement addition with a carry out C_n $A + B$	2's complement transformed addition with a carry out C_n $A + \overline{B} + 1 \leftarrow A - B$
CF and C_n relation	Carry CF = C_n for <u>addition</u> instructions	Borrow CF = $!C_n$ for <u>subtraction</u> instructions

Signed and unsigned arithmetics (2)

CF (Carry Flag)

- a carry in an **unsigned** addition
- a borrow in an **unsigned** subtraction

C_n (the carry out of an n-bit adder)

- a carry out in a **signed** addition
 - a carry out in a **signed** subtraction
-

http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt

Signed and unsigned arithmetics (3)

- **signed** and **unsigned** additions (or subtractions)
 - share the same *n-bit* result
 - a borrow (**CF**) in an **unsigned** subtraction is the inverted value of a carry out (**C_n**) in a **signed** subtraction

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Signed and unsigned arithmetics (4)

- a borrow (CF) in an **unsigned** subtraction is the inverted value of a carry out (C_n) in a **signed** subtraction

unsigned add/sub		signed addition		signed subtraction
0011 (3)		0011 (+3)		0011 (+3)
-1110 -(14) SUB		+0010 +(2)		-1110 -(-2) SUB
-----		-----		-----
10101 (5) (-16)		00101 (+5)		00101 (+5)
CF=1		Cn=0		Cn=0
1101 (13)		1101 (-3)		1101 (-3)
-0010 -(2) SUB		+1110 +(-2)		-0010 -(+2) SUB
-----		-----		-----
01011 (11) (+11)		11011 (-5)		11011 (-5)
CF=0		Cn=1		Cn=1

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Using the carry flag as a borrow (1)

- a borrow ($CF = 1$) occurs in an **unsigned** subtraction $A - B$ when $A < B$ as unsigned numbers
- computer hardware can detect a borrow ($CF = 1$) in **unsigned** subtraction by looking at whether there is no carry out ($C_n = 0$) in the transformed addition $A + \overline{B} + 1$

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Using the carry flag as a borrow (2)

- If there is no carry out ($C_n = 0$)
in the transformed addition $A + \overline{B} + 1$ of $A - B$
then there is a borrow ($CF = 1$)
- If there is a carry out ($C_n = 1$)
in the transformed addition $A + \overline{B} + 1$ of $A - B$
then there is no borrow ($CF = 0$)
- borrows $CF = !C_n$ for subtraction instructions
- carries $CF = C_n$ for addition instructions

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Using the carry flag as a borrow (3)

- the same *addition* and *subtraction* instructions are used for both **unsigned** and **signed** integer arithmetic.
 - no special *addition* and *subtraction* instructions for **unsigned** and **signed** integer arithmetic

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Using the carry flag as a borrow (4)

- the only difference is
 - which flags you *test* afterwards
 - **CF** for **unsigned** additions and subtractions
 - **OF** for **signed** additions and subtractions
 - how you *interpret* the result
 - Carry **CF** **unsigned** additions
 - Borrow **CF** **unsigned** subtractions
 - Overflow **OF** **signed** additions and subtractions

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TOC Rules for the carry flag

- The 1st rule for setting the carry flag
- The 2nd rule for setting the carry flag
- Cases for clearing the carry flag
- Computing CF in unsigned additions and subtractions

2's complement numbers : 4-bit

	signed	unsigned
0111	(+7)	7
0110	(+6)	6
0101	(+5)	5
0100	(+4)	4
0011	(+3)	3
0010	(+2)	2
0001	(+1)	1
0000	(0)	0
1000	(-8)	8
1001	(-7)	9
1010	(-6)	10
1011	(-5)	11
1100	(-4)	12
1101	(-3)	13
1110	(-2)	14
1111	(-1)	15

The 1st rule for setting the carry flag

- 1 **CF** = 1 : carry in **unsigned** addition
 - *hand addition rule*
 - the carry flag is set if the addition of two **unsigned** numbers causes a carry out of the most significant bits added.
 - **unsigned** integer overflow in **unsigned** addition

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The 2nd rule for setting the carry flag

② **CF** = 1 : borrow in **unsigned** subtraction

- *hand subtraction rule*
- the carry flag is also set if the subtraction of two **unsigned** numbers requires a borrow into the most significant bits subtracted.
- **unsigned** integer overflow in **unsigned** subtraction

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Cases for the 1st rule (1)

① CF = 1 : carry in **unsigned** addition

- all three interpretations have the same CF=1, the same S=0000

unsigned addition		signed addition	signed subtraction
0111 (7)		0111 (+7)	0111 (+7)
+1001 +(9)		+1001 +(-7)	-0111 -(+7)
-----		-----	-----
10000 (0) +(16)		10000 (0)	10000 (0)
CF=1		Cn=1 -> CF=1	Cn=1 -> CF=1
CF means +16		CF meaningless	CF meaningless
S = 0000		S = 0000	S = 0000
* think hand		* think Cn of the corresponding addition	
addition		CF <- Cn	

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Cases for the 1st rule (2)

	unsigned	signed
0111	7	+7
1001	9	-7

signed addition	0111 + 1001	(+7) + (-7)
	0111 + (-0111)	
signed subtraction	0111 - 0111	(+7) - (+7)

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Cases for the 2nd rule (1)

2 CF = 1 : borrow in **unsigned** subtraction

- all three interpretations have the same CF=1, the same S=0000

unsigned subtraction		signed addition		signed subtraction
0111 (7)		0111 (+7)		0111 (+7)
-1001 -(9)		+0111 +(7)		-1001 -(-7)
-----		-----		-----
11110 (14) -(16)		01110 (14)		01110 (14)
CF=1		Cn=0 -> CF=1		Cn=0 -> CF=1
CF means -16		CF meaningless		CF meaningless
S = 1110		S = 1110		S = 1110
* think hand		* think Cn of the corresponding addition		
subtraction		CF <- Cn		

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Cases for the 2nd rule (2)

	unsigned	signed
0111	7	+7
1001	9	-7

signed subtraction	0111 - 1001	(+7) - (-7)
	0111 - (-0111)	
signed addition	0111 + 0111	(+7) + (+7)

http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt

Computing CF in unsigned additions and subtractions

- Computing CF in an **unsigned addition**
 - do the **signed** 2's complement addition
 - C_n is the carry out
 - $CF \leftarrow C_n$ (CF means carry)
- Computing CF in an **unsigned subtraction**
 - do the transformed **signed** 2's complement addition
 - C_n is the carry out
 - $CF \leftarrow !C_n$ (CF means borrow)

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TOC: Method for computing the carry flag

- 2's complement numbers : 4-bit
- Addend and augend in a n -bit addition
- Full adder (hardware) operation
- Internal and external carry bits
- 2's complement signed addition and subtraction
- Carry flag computation

Addend and augend in a n -bit addition

n	bits	addended	A	$\{ a_{n-1}, a_{n-2}, \dots, a_1, a_0 \}$
n	bits	augend	B	$\{ b_{n-1}, b_{n-2}, \dots, b_1, b_0 \}$
$(n+1)$	bits	carry bits	C	$\{ c_n, c_{n-1}, c_{n-2}, \dots, c_1, c_0 \}$
n	bits	sum bits	S	$\{ s_{n-1}, s_{n-2}, \dots, s_1, s_0 \}$

external carry bits : c_n carry out, c_0 carry in

$$\begin{array}{cccccc} a_{n-1} & a_{n-2} & \dots & a_1 & a_0 & \\ b_{n-1} & b_{n-2} & \dots & b_1 & b_0 & \\ \hline c_n & s_{n-1} & s_{n-2} & \dots & s_1 & s_0 \end{array}$$

Full adder (hardware) operation

full adder operation in the i^{th} bit position

$$\{c_{i+1}, s_i\} = a_i + b_i + c_i$$

$$\begin{array}{r} a_i \\ b_i \\ c_i \\ \hline c_{i+1} \quad s_i \end{array}$$

Internal and external carry bits

external carries c_n output, c_0 input
 internal carries $\{c_{n-1}, c_{n-2}, \dots, c_2, c_1\}$ output / input
 sum bits $\{s_{n-1}, s_{n-2}, \dots, s_1, s_0\}$ output

	a_{n-1}	a_{n-2}	\dots	a_1	a_0
	b_{n-1}	b_{n-2}	\dots	b_1	b_0
c_n	c_{n-1}	c_{n-2}	\dots	c_1	c_0
	s_{n-1}	s_{n-2}	\dots	s_1	s_0

	a_{n-1}	a_{n-2}	\dots	a_1	a_0
	b_{n-1}	b_{n-2}	\dots	b_1	b_0
	s_{n-1}	s_{n-2}	\dots	s_1	s_0
c_n					c_0

2's complement signed addition and subtraction

- 2's complement signed addition

$$\{c_n, S\} = A + B = A + B + 0$$

	a_{n-1}	a_{n-2}	⋯	a_1	a_0
	b_{n-1}	b_{n-2}	⋯	b_1	b_0
	c_{n-1}	c_{n-2}	⋯	c_1	0
c_n	s_{n-1}	s_{n-2}	⋯	s_1	s_0

- 2's complement signed subtraction - via a transformed addition

$$\{c_n, S\} = A - B = A + \overline{B} + 1$$

	a_{n-1}	a_{n-2}	⋯	a_1	a_0
	b_{n-1}	b_{n-2}	⋯	b_1	b_0
	c_{n-1}	c_{n-2}	⋯	c_1	1
c_n	s_{n-1}	s_{n-2}	⋯	s_1	s_0

Carry flag computation (1)

ADD (addition)	SUB (subtraction)
$CF = c_n$	$CF = \overline{c_n}$
normal carry of a 2's complement addition	inverted carry of a transformed addition
$A + B = A + B + \mathbf{0}$	$A - B = A + \overline{B} + \mathbf{1}$
$\{c_n, s_{n-1}\}$ $= a_{n-1} + b_{n-1} + c_{n-1}$	$\{c_n, s_{n-1}\}$ $= a_{n-1} + \overline{b_{n-1}} + c_{n-1}$

https://www.csie.ntu.edu.tw/~cyy/courses/assembly/12fall/lectures/handouts/lec14_1

Carry flag computation (2)

- In **unsigned** arithmetic,
 - the **carry flag** is used to detect *overflow*
 - the **carry flag** is used to extend *n-bit* result into *(n+1)-bit* result
 - for **addition**, the **carry flag** is a **carry out**
 - for **subtraction**, the **carry flag** is a **borrow in**
- In **signed** arithmetic,
 - the **carry flag** is useless
 - the **carry flag** neither detects overflow nor extends n-bit result

http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt

Carry flag computation (3)

- In **unsigned** arithmetic,

Addition	CF = 1 means carry out	when Cn = 1
Subtraction	CF = 1 means borrow in	when Cn = 0

- **CF** - Carry Flag in x86
- **Cn** - the normal carry out
 - the carry out of a 2's complement addition for **ADD**
 - the carry out of a *transformed* addition for **SUB**
- In **signed** arithmetic,
 - the **carry** flag is useless

http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt

TOC: More examples of the carry flag

- Summary I
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Summary I

unsigned add/sub			signed addition			signed subtraction			CF	OF
1101	(13)		1101	(-3)		1101	(-3)			
+1110	+(14)	ADD	+1110	+(-2)	ADD	-0010	-(-2)			
-----	-----		-----	-----		-----	-----			
11011	(11)	(+16)	11011	(-5)		11011	(-5)		1	0
0011	(3)		0011	(+3)		0011	(+3)			
-1110	-(14)	SUB	+0010	+(+2)		-1110	-(-2)	SUB		
-----	-----		-----	-----		-----	-----			
10101	(5)	(-16)	00101	(+5)		00101	(+5)		1	0
0011	(3)		0011	(+3)		0011	(+3)			
+0010	+(2)	ADD	+0010	+(+2)	ADD	-1110	-(-2)			
-----	-----		-----	-----		-----	-----			
00101	(5)	(+ 0)	00101	(+5)		00101	(+5)		0	0
1101	(13)		1101	(-3)		1101	(-3)			
-0010	-(2)	SUB	+1110	+(-2)		-0010	-(-2)	SUB		
-----	-----		-----	-----		-----	-----			
01011	(11)	(+11)	11011	(-5)		11011	(-5)		0	0

Summary II

unsigned add/sub			signed addition			signed subtraction			CF	OF
1011	(11)		1011	(-5)		1011	(-5)			
+1100	+(12)	ADD	+1100	+(-4)	ADD	-0100	-(+4)			
-----	-----		-----	-----		-----	-----			
10111	(7) (+16)		10111	(+7)		10111	(+7)		1	1
0101	(5)		0101	(+5)		0101	(+5)			
-1100	-(12)	SUB	+0100	+(+4)		-1100	-(-4)	SUB		
-----	-----		-----	-----		-----	-----			
11001	(9) (-16)		01001	(-7)		01001	(-7)		1	1
0101	(5)		0101	(+5)		0101	(+5)			
+0100	+(4)	ADD	+0100	+(+4)	ADD	-1100	-(-4)			
-----	-----		-----	-----		-----	-----			
01001	(9) (+ 0)		01001	(-7)		01001	(-7)		0	1
1011	(11)		1011	(-5)		1011	(-5)			
-0100	-(4)	SUB	+1100	+(-4)		-0100	-(+4)	SUB		
-----	-----		-----	-----		-----	-----			
00111	(7) (0)		10111	(+7)		10111	(+7)		0	1

Cases for setting the carry flag (1) CF=1, OF=0

- unsigned integer overflow (CF=1 means +16)

* unsigned addition		* signed addition		signed subtraction
1101 (13)		1101 (-3)		1101 (-3)
+1110 +(14) ADD		+1110 +(-2) ADD		-0010 -(+2)
-----		-----		-----
11011 (11) (+16)		11011 (-5)		11011 (-5)
CF=1		Cn=1 -> CF=1		Cn=1 -> CF=1
CF means 16		CF meaningless		CF meaningless
S = 0000		S = 0000		S = 0000
* think hand		* think Cn of the corresponding addition		
addition		CF <- Cn (for unsigned addition)		

* CF=1, S=1011, OF=0 for all three interpretations

http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt

Cases for setting the carry flag (2) CF=1, OF=0

- unsigned integer overflow (CF=1 means -16)

* unsigned subtraction		signed addition		* signed subtraction
0011 (3)		0011 (+3)		0011 (+3)
-1110 -(14) SUB		+0010 +(2)		-1110 -(-2) SUB
-----		-----		-----
10101 (5) (-16)		00101 (+5)		00101 (+5)
CF=1		Cn=0 -> CF=1		Cn=0 -> CF=1
CF means -16		CF meaningless		CF meaningless
S = 0101		S = 0101		S = 0101
-----		-----		-----
* think hand subtraction		* think Cn of the transformed addition		
		CF <- !Cn (for unsigned subtraction)		
-----		-----		-----

* CF=1, S=0101, OF=0 for all three interpretations

http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt

Cases for setting the carry flag (3) CF=1, OF=1

- unsigned integer overflow (CF=1 means +16)

* unsigned addition		* signed addition	signed subtraction
1011 (11)		1011 (-5)	1011 (-5)
+1100 +(12) ADD		+1100 +(-4) ADD	-0100 -(+4)
-----		-----	-----
10111 (7) (+16)		10111 (+7)	10111 (+7)
CF=1		Cn=1 -> CF=1	Cn=1 -> CF=1
CF means +16		CF meaningless	CF meaningless
S = 0111		S = 0111	S = 0111
* think hand addition		* think Cn of the corresponding addition	
		CF <- Cn (for unsigned addition)	

* CF=1, S=0111, OF=1 for all three interpretations

Cases for setting the carry flag (4) CF=1, OF=1

- unsigned integer overflow (CF=1 means -16)

* unsigned subtraction		signed addition		* signed subtraction
0101 (5)		0101 (+5)		0101 (+5)
-1100 -(12) SUB		+0100 +(4)		-1100 -(-4) SUB
-----		-----		-----
11001 (9) (-16)		01001 (-7)		01001 (-7)
CF=1		Cn=0 -> CF=1		Cn=0 -> CF=1
CF means -16		CF meaningless		CF meaningless
S = 1001		S = 1001		S = 1001
-----		-----		-----
* think hand subtraction		* think Cn of the transformed addition		CF <- !Cn (for unsigned subtraction)
-----		-----		-----

* CF=1, S=1001, OF=1 for all three interpretations

Cases for clearing the carry flag (1) CF=0, OF=0

- no unsigned integer overflow (CF=0)

* unsigned addition	* signed addition	signed subtraction
0011 (3) +0010 +(2) ADD ----- 00101 (5) (+ 0)	0011 (+3) +0010 +(2) ADD ----- 00101 (+5)	0011 (+3) -1110 -(-2) ----- 00101 (+5)
CF=0	Cn=0 -> CF=0	Cn=0 -> CF=0
CF means 0 S = 0101	CF meaningless S = 0101	CF meaningless S = 0101
* think hand addition	* think Cn of the corresponding addition CF <- Cn (for unsigned addition)	

* CF=0, S=0101, OF=0 for all three interpretations

http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt

Cases for clearing the carry flag (2) CF=0, OF=0

- no unsigned integer overflow (CF=0)

* unsigned addition	* signed addition	signed subtraction
1101 (13)	1101 (-3)	1101 (-3)
-0010 -(2) SUB	+1110 +(-2)	-0010 -(+2) SUB
-----	-----	-----
11011 (11) (-16)	11011 (-5)	11011 (-5)
CF=0	Cn=0 -> CF=0	Cn=0 -> CF=0
CF means 0	CF meaningless	CF meaningless
S = 1011	S = 1011	S = 1011
-----	-----	-----
* think hand subtraction	* think Cn of the corresponding addition	
	CF <- Cn (for unsigned addition)	

* CF=0, S=1011, OF=0 for all three interpretations

http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt

Cases for clearing the carry flag (3) CF=0, OF=1

- no unsigned integer overflow (CF=0)

* unsigned addition		* signed addition	signed subtraction
0101 (5)		0101 (+5)	0101 (+5)
+0100 +(4) ADD		+0100 +(+4) ADD	-1100 -(-4)
-----		-----	-----
01001 (9) (+ 0)		01001 (-7)	01001 (-7)
CF=0		Cn=0 -> CF=0	Cn=0 -> CF=0
CF means +0		CF meaningless	CF meaningless
S = 1001		S = 1001	S = 1001
* think hand		* think Cn of the corresponding addition	
addition		CF <- Cn (for unsigned addition)	

* CF=0, S=1001, OF=1 for all three interpretations

http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt

Cases for clearing the carry flag (4) CF=0, OF=1

- no unsigned integer overflow (CF=0)

* unsigned subtraction		signed addition		* signed subtraction
1011 (11)		1011 (-5)		1011 (-5)
-0100 -(4) SUB		+1100 +(-4)		-0100 -(+4) SUB
-----		-----		-----
00111 (7) (0)		10111 (+7)		10111 (+7)
CF=0		Cn=1 -> CF=0		Cn=1 -> CF=0
CF means 0		CF meaningless		CF meaningless
S = 0111		S = 0111		S = 0111
* think hand subtraction		* think Cn of the transformed addition		CF <- !Cn (for unsigned subtraction)

* CF=0, S=0111, OF=1 for all three interpretations

http://teaching.idallen.com/dat2343/10f/notes/040_overflow.txt