

Fraenkel 1922,
Skolem 1922
Stated: *Replacement*

Zermelo 1908 (Zermelo set theory)
Sets
Extensionality (sets), Elementary sets, Union, Power set, Infinity, Separation, Choice



← Elementary sets
← Separation, Choice

Von Neumann 1925, 1928
Functions, Arguments
Extensionality (functions), Ordered pair, Function existence axioms, Union, Power set, Infinity, Limitation of size
Stated but not adopted: *Regularity*

→ Ordered pair
→ Function existence axioms
→ Limitation of size (implies Replacement)



← Limitation of size
← Replacement
→ Von Neumann choice

Von Neumann 1929
Functions, Arguments
Extensionality (functions), Ordered pair, Function existence axioms, Union, Power set, Infinity, Replacement, Von Neumann choice
Proved relatively consistent: *Regularity*



← Ordered pair
← Function existence axioms
→ Unordered pair
→ Class existence axioms
→ Separation, Regularity

Bernays 1931 [letter to Gödel], 1937, 1941 [axioms published]
Classes, Sets (two sorts)
Extensionality (classes), Unordered pair, Class existence axioms, Union, Power set, Infinity, Separation, Replacement, Von Neumann choice, Regularity



← Separation
← Von Neumann choice
→ Global choice

Gödel 1940 (NBG)
Classes, Sets (one sort)
Extensionality (classes), Unordered pair, Class existence axioms, Union, Power set, Infinity, Replacement, Global choice, Regularity

Legend:

Approach
Primitives
Axioms