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Central authority controlled air traffic flow management: An optimization approach

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Despite various planning efforts, airspace capacity can sometimes be exceeded, typically due to disruptive events. Air traffic flow management (ATFM) is the process of managing flights in this situation. In this paper, we present an ATFM model that accounts for different rerouting options (path rerouting and diversion) and pre-existing en-route flights. The model proposes having a central authority to control all decisions, which is then compared with current practice. We also consider inter-flight and inter-airline fairness measures in the network. We use an exact approach to solve small-to-medium-sized instances, and we propose a modified fix-and-relax heuristic to solve large-sized instances. Allowing a central authority to control all decisions increases network efficiency compared to the case where the ATFM authority and airlines control decisions independently. Our experiments show that including different rerouting options in ATFM can help reduce delays by up to 8% and cancellations by up to 23%. Moreover, ground delay cost has much more impact on network decisions than air delay cost, and network decisions are insensitive to changes in diversion cost. Furthermore, the analysis of the trade-off between total network cost and overtaking cost shows that adding costs for overtaking can significantly improve fairness at only a small increase in total system cost. A balanced total cost per flight among airlines can be achieved at a small increase in the network cost (0.2 to 3.0%) when imposing airline fairness. In conclusion, the comprehensiveness of the model makes it useful for analyzing a wide range of alternatives for efficient ATFM.

Key words: Air traffic flow management; mathematical modeling; flight cancellation; flight diversion; path rerouting; ground holding; airborne delay; flight overtaking

History:

1. Introduction

Flight schedule disruptions caused by severe climate weather, congestion, security issues, and tight airport runway availability are disruptive events that lead airline companies, at the request of the air traffic management authorities, to continuously modify their flight schedules (Deshpande and Arıkan 2012). These modifications seek to minimize negative effects on schedules, such as flight cancellations, rerouting, and delays (Barnhart and Cohn 2004, Hamdan et al. 2020).

All air traffic networks worldwide experience flight delays, diversions, cancellations, and reroutings. For instance, in 2019, the United States reported more than 95.8 million delay minutes, of which 24% were due to the airspace system (i.e. capacity issues), around 40% were due to late aircraft arrivals, and 31% were carrier delays, with the remainder caused by weather and security issues. Airspace system capacity was the trigger behind 24,775 out of the total 134,925 canceled flights. Other causes of flight cancellations were categorized as carrier-related (28%) and weather-related (18%). Furthermore, in the same year, 18,880 flights were diverted to alternative destination airports, resulting in around 4.2 million arrival delay minutes (Bureau of Transportation Statistics 2019).

Schedule modifications can be complex to handle, especially in dense traffic environments, and are not necessarily based on optimal configurations, which means that they negatively affect system performance for airspace stakeholders, resulting in financial losses, customer dissatisfaction, and unfair allocation of airspace resources (Abdi and Sharma 2008, Jafari and Zegordi 2011).

In this work, we present an air traffic flow management (ATFM) optimization model that considers different realistic features such as diversions and path reroutings. Our objective is to increase the system's efficiency by minimizing the total cost of delays, reroutings, diversions and cancellations in the network.

The ATFM resources need to be not only efficiently allocated but also fairly distributed (Bertsimas, Farias, and Trichakis 2012), since an efficient ATFM schedule might not be accepted due to fairness issues. Ways to achieve fairness should be based on the problem context (Fairbrother, Zografos, and Glazebrook 2020). A schedule that prioritizes the planned arrival sequences and allocates new arrival slots using a ration-by-schedule rule (first-scheduled first-served) is fair as agreed by airlines (Bertsimas and Gupta 2015, Fairbrother, Zografos, and Glazebrook 2020). Another way is to distribute delays among airlines based on the number of flights in each airline (Jacquillat and Vaze 2018).

Here we address fairness by limiting overtaking between flights to prioritize the scheduled sequences as far as the system capacity permits and by assigning a penalty for violating this fairness. Overtaking between two flights can be defined as the number of periods between the entering times of flights f and f' if f' enters a resource before f even though f was scheduled to enter that resource before f'. In addition to limiting overtaking, we promote fairness in the delay distribution among flights. As overtaking and delay distribution are inter-flight fairness measures, i.e. they are applied among flights without considering which airline they belong to, we also show how the model can be extended to integrate inter-airline fairness, i.e. balancing the total cost between different airlines. Thus, for a given network of resources (airports and airspace sectors), given flight schedules in a finite time horizon, the proposed mathematical model is designed to address the following questions while developing mitigation programs to respond to disruptions:

1. Which flight(s) (if any) should be held on the ground or made to reduce speed, for how long, and what are the new departure and/or arrival times?
2. Which flight(s) (if any) should be canceled, and which should be diverted to a different arrival airport?
3. Which flight(s) (if any) should be rerouted to avoid congested areas, and which alternative routes are the best?
4. What is the optimal fair delay distribution among the schedules?

This paper complements the existing literature through the following contributions:

- First, we account for all the features previously presented in the literature, i.e., continued flights (flight pairs operated by the same aircraft, where the second flight is scheduled some time after the arrival epoch of the first one), cancellation, path rerouting and speed control, all in the same model. We also add new aspects not previously studied in the literature, i.e. diversion and en-route flights that exist in the airspace at the beginning of the planning horizon. We illustrate that the formulation structure makes it possible to relax the integrality constraint on the overtaking decision variables. We then highlight the impact on the network decisions from considering different rerouting aspects.
- Second, we focus on inter-flight fairness in the developed model by considering flight overtaking and fairness in the delay distribution among different flights. The model is then extended to account for inter-airline fairness. Analysis shows that airline fairness improvements can potentially be achieved at a minimal cost.
- Third, since the ATFM authorities are responsible for the delays and the attendant overtaking while airlines are responsible for diversions, cancellations, and rerouting decisions prior to take off, we consider two groups of decisions and assign an importance weight to each group. Our method is able to analyze the trade-off between the two groups of decisions in order to achieve certain goals such as reducing delays and overtaking. The model therefore proposes that a central authority be made responsible for better assigning/suggesting decisions to better utilize the airspace network. This central authority approach is inspired by the collaborative decision-making philosophy that advocates sharing the right information with the right entity at the right time (Ball et al. 2001). It is also inline with the system-wide information management (SWIM) initiative that is currently under development. SWIM aims to make all aviation-related information available in one unified interface. It is expected to enable the establishment of a collaborative information environment where stakeholders can proactively make collaborative decisions to improve planning (International Civil Aviation Organization 2019). We compare the proposed central authority approach against current practice, and show that having a central authority to assign all decisions can reduce total system cost, particularly when considering fairness.
- Finally, we present a heuristic to solve the optimization model when it becomes intractable (e.g. instances with high maximum allowable delay). The heuristic is a modified version of the fix-and-relax algorithm that uses a modified fixing procedure. It has been validated by comparing it against the exact solution.

This paper is organized as follows. Section 2 summarizes some of the relevant work on ATFM. Section 3 discusses the problem under study. Section 4 presents the mathematical optimization model. Section 5 explains how the proposed model can be adapted to simulate current practice. Section 6 explains the heuristic approach used to solve the problem. Section 7 provides managerial insights obtained from the numerical experiments. Section 8 concludes the paper and highlights directions for future research.

2. Literature Review

In this section, we review and summarize relevant work done in ATFM, initially through the lens of the evolution of the flight scheduling problem and then by discussing the macroscopic ATFM model. We then present the microscopic ATFM model and conclude with literature findings and research gaps.

2.1. ATFM evolution

The simplest version of the flight scheduling problem is the single-airport ground holding problem that considers one airport and provides decisions on the aircraft release times (see, for instance, Ball et al. 2003, Richetta and Odoni 1993). This problem then evolved (see Figure 1) into a multi-airport ground holding problem that considers a network of airports and where delay propagates as an aircraft flies continuously (Navazio and Romanin-Jacur 1998, Vranas, Bertsimas, and Odoni 1994). Single-airport and multi-airport models lack the ability to control aircraft speed and flight path, and they also fail to consider en-route sector capacity. The ATFM problem can be seen as an extended version of the multi-airport ground holding problem in that it considers these features with an ultimate goal of balancing the flight schedules (arrivals and departures) in the airports with sector capacities (Agustín et al. 2012a,b, Bertsimas and Patterson 1998, Bertsimas and Gupta 2015). Odoni (1987) was among the first to address this problem as a mathematical optimization problem.

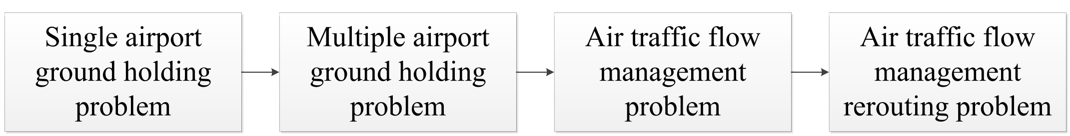


Figure 1 **Evolution of the ATFM problem in the literature**

2.2. Macroscopic ATFM models

Macroscopic ATFM models are the network models that consider the number of flights rather than a flight-by-flight case. In this variant, Andreatta, Dell'Olmo, and Lulli (2011) presented a multi-stage stochastic ATFM model that considers continued flights and the trade-off between arrival and departure capacities, and formulates interactions between different hubs. This model represents an extension of the deterministic version in Dell'Olmo and Lulli (2003). Mukherjee and Hansen (2009) developed three stochastic ATFM models, i.e. static ground holding without rerouting, static ground holding with static rerouting, and static ground holding with dynamic rerouting. These models can be seen as extensions of the model proposed in Ball et al. (2003). Chen, Chen, and Sun (2017) proposed a chance-constrained programming model to solve an ATFM problem using a polynomial approximation-based approach that is based on the Bernstein polynomial approach for large-scale problems.

2.3. Microscopic ATFM models

The rich literature investigates different aspects of a flight-by-flight ATFM and proposes different model variants. Bertsimas and Patterson (1998) formulated a mathematical optimization model that addresses the deterministic ATFM problem. They showed how the model can be reduced to a multi-airport ground delay problem, and how it can be extended to account for rerouting, the dependency between runway arrival and departure capacities, and the case of banks of flights (in a hub-and-spoke system). Lulli and Odoni (2007) introduced the fairness and equity decisions in a deterministic European ATFM model with a slight super-linear delay function that assigns airborne delays only at terminal airspace around destination airports. Bertsimas, Lulli, and Odoni (2011) developed a deterministic model to solve large-scale instances with fairness as in Lulli and Odoni (2007). Gupta and Bertsimas (2011) presented a robust and adaptive stochastic ATFM model that was solved optimally using piecewise affine policies.

Inspired by Bertsimas and Patterson (1998) and Mukherjee and Hansen (2005), Agustín et al. (2012a) proposed a deterministic multi-objective mixed-integer binary model using arc formulation for the flight-by-flight ATFM problem. They illustrated how the model can consider dependency between arrival and departure capacity, dynamic sector structure, and some airline-related constraints such as a maximum amount of delay per airline. The stochastic version is given in Agustín et al. (2012b). Bertsimas and Gupta (2015) extended the basic ATFM model proposed in Bertsimas and Patterson (1998) into a two-stage model that accounts for minimizing delays and limiting overtaking in its first stage while considering airline slot allocation in its second stage. Hamdan et al. (2018) studied the effect of rerouting on the ATFM network with fairness. García-Heredia, Alonso-Ayuso, and Molina (2019) considered dynamic sector configuration in an ATFM model.

2.4. Literature findings and research gaps

Table 1 summarizes the features of the models presented in the literature and classifies them based on information level (microscopic or macroscopic), capacity certainty (deterministic or stochastic), inclusion of fairness measures, and model decisions.

In this work, we present a flexible approach to optimally schedule flights in a given planning horizon while considering all elements in the air traffic network and a comprehensive set of decisions, i.e. rerouting, diversion, cancellation, ground delay, airborne delay and speed control. Note that this work also takes into account, for the first time, the en-route flights that already exist in

Table 1 Literature review summary

	Details level		Capacity Certainty			Model Decisions								
Paper	Microscopic	Macroscopic	Deterministic	Stochastic	Fairness in delay distribution	Speed control	Rerouting	Continued flights	Flight cancellation	Flight reversal	Flight overtaking	Slot trading	Diversion	En-route flights
Bertsimas and Patterson (1998)	✓	×	✓	×	×	✓	✓	✓	×	×	×	×	×	×
Bertsimas and Patterson (2000)	✓	×	✓	×	×	✓	✓	✓	×	×	×	×	×	×
Alonso-Ayuso, Escudero, and Ortuno (2000)	✓	×	×	✓	×	✓	×	✓	×	×	×	×	×	×
Nilim, El Ghaoui, and Duong (2003)	✓	×	×	✓	×	×	✓	×	×	×	×	×	×	×
Lulli and Odoni (2007)	✓	×	✓	×	✓	×	×	×	×	×	×	×	×	×
Bertsimas, Lulli, and Odoni (2008)	✓	×	✓	×	✓	✓	✓	✓	×	×	×	×	×	×
Mukherjee and Hansen (2009)	×	✓	×	✓	×	×	✓	×	×	×	×	×	×	×
Andreatta, Dell'Olmo, and Lulli (2011)	×	✓	×	✓	✓	×	×	✓	×	×	×	×	×	×
Bertsimas, Lulli, and Odoni (2011)	✓	×	✓	×	✓	✓	✓	✓	×	×	×	×	×	×
Agustín et al. (2012a)	✓	×	✓	×	✓	✓	✓	✓	✓	×	×	×	×	×
Agustín et al. (2012b)	✓	×	×	✓	✓	✓	✓	✓	✓	×	×	×	×	×
Gupta and Bertsimas (2011)	✓	×	×	✓	×	✓	×	✓	×	✓	×	×	×	×
Bertsimas and Gupta (2015)	✓	×	✓	×	✓	✓	×	✓	×	✓	✓	✓	×	×
Chen, Chen, and Sun (2017)	×	✓	×	✓	×	×	×	×	×	×	×	×	×	×
Hamdan et al. (2018)	✓	×	✓	×	✓	✓	✓	✓	×	✓	×	×	×	×
García-Heredia, Alonso-Ayuso, and Molina (2019)	✓	×	✓	×	✓	✓	✓	✓	×	×	×	×	×	×
This work	✓	×	✓	×	✓	✓	✓	✓	✓	✓	✓	×	✓	✓

the airspace at the beginning of the planning horizon. Moreover, and to the best of the authors' knowledge, diversion to alternative airports has never before been taken into account in previous ATFM research. Furthermore, we account for both inter-flight and inter-airline fairness, and we analyze the trade-off between ATFM authority decisions and airline decisions.

3. Problem Statement

In this section, we describe the problem studied and the different network decisions used. We then define the considered fairness measures and evaluation metrics, before going on to frame the role of decision-makers and the cost parameters.

3.1. Problem definition and network decisions

This work studies the ATFM problem that consists of a set of flights ($\mathcal{F}$) in a planning horizon ($\mathcal{T}$) of finite time periods ($t \in \mathcal{T}$) that will fly through an airspace network. The airspace network includes a set of airports ($\mathcal{K}$) and a set of sectors ($\mathcal{P}$). Each airport and sector in this network has its own capacity at a given time-period (t) in the planning horizon. Each flight has a predefined set of time-periods (time-window) for arrival at each resource ($T_j^{f,z}$). The first time-period in this set represents the earliest possible time determined by the scheduled plan and the minimum time to be spent in each resource (l_j^f). The last time-period corresponds to the latest possible time calculated based on the planning horizon or the maximum allowable delay. The ATFM authority may delay some flights to achieve better utilization of the limited capacities, and airlines may decide to reroute (prior to a flight's take-off), cancel or divert some of their flights to minimize their costs. The ATFM authority and airlines take decisions independently (although following certain information-sharing protocols and approvals), which may not necessarily result in efficient utilization of airspace. In this context, we present an ATFM model in which the decisions are taken by a central authority, in a spirit that builds on initiatives aligned to a system of collaborative air traffic management. The goal is to find a minimum-cost solution to the problem, using all the available actions, and not exceeding the capacity of airports and sectors.

In this problem, we consider that at any time in the day, in addition to the flights scheduled to depart ($\mathcal{F}^G$), there are always en-route flights ($\mathcal{F}^A$) in the airspace at the beginning of the planning horizon. These flights are accounted for by defining them as a new set of flights accompanied with a new set of constraints while considering their current sector at the beginning of the planning horizon and keeping the remaining time in this sector while forcing the model not to cancel any of them. Long-haul flights that will not be able to reach their destination by the end of the planning horizon are modeled to arrive at a virtual high-capacity airport at the end of the planning horizon.

In a flight diversion, a flight in a sector, which is usually close to its original destination airport, can start navigating to its new path leading to its alternative airport, as shown in Figure 2.

To properly model continued flights, it is necessary to include the time required to unload, clean, refuel, load, and any other preparation needed to ready the aircraft for its next flight. This time is known as the turnaround time (Bertsimas and Patterson 1998). If an aircraft is delayed, then this delay may propagate to its next flight.

3.2. Fairness measures and evaluation metrics

We consider inter-airline and inter-flight fairness measures. Inter-airline fairness is considered in the balancing of the cost per flight for each airline. Introducing a fairness coefficient (Π) makes it possibly to distribute the schedule modifications fairly. Inter-flight fairness measures include a fair allocation of delays among flights using a slight super-linear coefficient, and limiting overtaking between flights to preserve the scheduled sequences.

In some cases, the problem can have several optimal solutions, i.e. alternate optima, having the same total delays but with different allocations. We therefore raise the delay duration to a slight

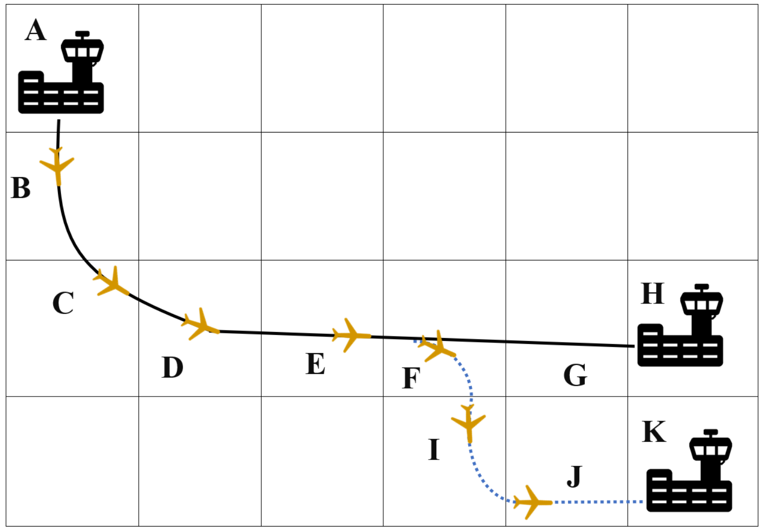


Figure 2 **Flight diversion: the original flight path is from A to H. Sector F represents the transition sector (common to the two paths). The flight diverts at F to land at an alternative airport through sectors I, J and finally K**

super-linear coefficient (1+φ) to promote fairness between flights such that instead of assigning a large amount of delay to only few flights, the delay is shared among a larger number of flights.

Definition 1. A slight super-linear coefficient (1+φ) is a weighting factor on the delay duration. It is used to distinguish between the different delay allocations in cases with the same total delay.

For example, a solution that assigns two units of delay to one flight and zero units to another flight has the same total delay $(2+0)$ as a solution that assigns one unit of delay for each of the two flights $(1+1)$. However, the second solution is assumed to provide more fairness as delays are distributed among the flights. If we raise the delay duration to a slight super-linear coefficient (1+φ), this results in $2^{(1+\varphi)}$ total delay for the first solution and $1^{(1+\varphi)}+1^{(1+\varphi)}$ for the second solution. Now, the second solution has lower weighted delay value and is consequently selected.

As part of the inter-flight fairness consideration, the model restricts the amount of overtaking between the flights by assigning a penalty to each time unit of overtaking. In other words, it assigns a penalty for violating the scheduled sequence, which preserves arrival sequences.

Definition 2. Overtaking between flight pair (f, f') at resource j is the time difference (or the number of time-periods) between the arrival of flight f' at resource j and the arrival of flight f at the same resource if and only if flight f is scheduled to arrive before flight f' at resource j but flight f' arrives before f (Bertsimas and Gupta 2015). This may occur under the following conditions:

1. The two flights share the same resource, i.e. both flight f and flight f' have resource j in their path.

2. There are at least two common time-periods between the two flights within their feasible time-sets (time-windows).

Definition 3. A flight f' is reversible if its minimum flight time is between $\overline{T}_j^{f,z} < \overline{T}_j^{f',z'} \leq \underline{T}_j^{f,z}$, where $\overline{T}_j^{f,z}$ and $\underline{T}_j^{f,z}$ are the first and last possible time-periods (earliest and latest possible times) for flight f to arrive at resource j from path z, respectively. The maximum amount of overtaking is $\underline{T}_j^{f,z} - \overline{T}_j^{f',z'} - 1$. The overtaking interval is defined by $T_j^{f,f',z,z'} = \overline{T}_j^{f',z'}, \overline{T}_j^{f',z'} + 1, ..., \underline{T}_j^{f,z} - 1$.

Figure 3 shows the landing time-window at the same airport for two flights, Flight A and Flight B. The earliest arrival times for Flights A and B are $t = 2$ and $t = 4$, respectively, and the latest arrival times for Flights A and B are $t = 6$ and $t = 9$, respectively. The common arrival time slots are $t = 4, 5$, and 6. If both flights arrive at their earliest time (Figure 3(a)) or if Flight A arrives at $t = 4$ and Flight B arrives at $t = 5$ (Figure 3(b)), then there will be no reversal as the sequence is preserved. However, if Flight B arrives at $t = 4$ and Flight A arrives at $t = 6$, then the scheduled arrival sequence is violated, where Flight B overtook Flight A by two periods (Figure 3(c)).

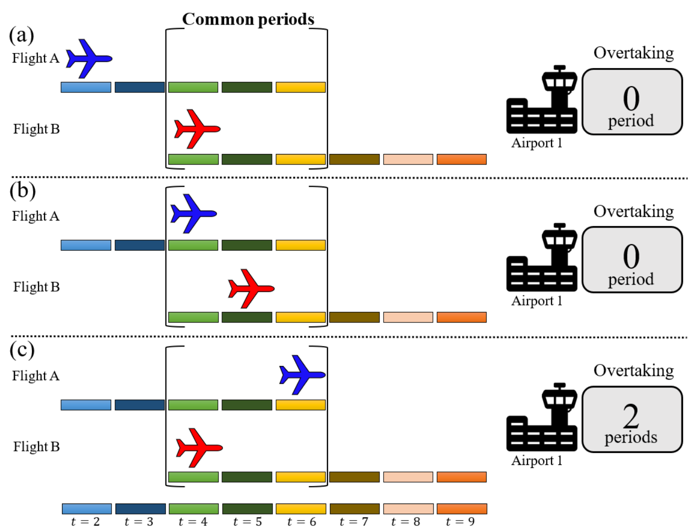


Figure 3 Possible arrival times of Flights A and B to the same airport and potential arrival scenarios

To evaluate network fairness, we use price of fairness and degree of fairness as performance indicators. The price of fairness represents the relative percentage cost increase associated with incorporating fairness in the network, i.e. the efficiency loss due to fairness (Bertsimas, Farias, and Trichakis 2011). It is measured with respect to a network case that does not impose fairness and helps to understand the trade-off between fairness and efficiency. The degree of fairness measures how close a solution is to the fairest one, and thus indicates a fairness level that ranges from 0 to 1, where 0 represents a solution that is far from the fairest one.

3.3. Role of decision-makers and the cost structure

Since decisions are controlled either by the ATFM authority (delays and consequently overtaking) or the airlines (rerouting, diversion and cancellation), we use the weighted sum approach (which is a scalarization technique in multiobjective optimization) by multiplying each group's decision costs by an importance weight. These importance weights can be seen as coefficients to control the cost parameters of each group. This approach makes it possible to study the interaction between the two sets of decisions and to find the best combination of decisions that minimizes the total network cost. It also allows to determine the trade-off between the two sets of decisions. Most of the cost parameters are obtained from a EUROCONTROL report (EUROCONTROL 2018). Delay costs (C^f_{air}, C^f_{ground}) cover fuel, safety, compensation and taxes (Bertsimas and Patterson 2000). Ground holding cost (C^f_{ground}) is generally cheaper than air holding cost (C^f_{air}) because of the fuel burn and safety issues. Cancellation cost (C^f_{cancel}) includes service recovery costs, goodwill costs, loss of revenue, passenger compensation, and crew and catering costs (EUROCONTROL 2018). Rerouting penalty cost ($C^{f,z}_{reroute}$) includes the charges for using alternative paths to the planned one (Agustín et al. 2012a). Likewise, diversion cost ($C^{f,z}_{Alt}$) accounts for the route and airport charges in addition to costs for transporting passengers to their scheduled destination. Overtaking cost ($C^f_{Rairport}$, $C^f_{Rsector}$) represents the fairness penalty, which includes missed connections and compensation, schedule integrity, and operational constraints at the airport.

4. Modeling and Optimization Formulation

In this section, we introduce the formulation of the ATFM model with different rerouting options that account for speed control, path rerouting, diversion, cancellation, and overtaking. We show how this model can be extended to account for inter-airline fairness. The following subsections define the sets, parameters, objective function and constraints of the proposed mixed-integer linear programming (MILP) model.

4.1. Sets

- $\mathcal{F}^G$: set of flights scheduled to depart from any airport in the network,
- $\mathcal{F}^A$: set of en-route flights, i.e. flights that already exist in the airspace at the beginning of the planning horizon,
- $\mathcal{F}$: set of all flights $\mathcal{F} = \mathcal{F}^G \cup \mathcal{F}^A$, indexed by f,
- $\mathcal{A}$: set of airlines, indexed by h,
- $\mathcal{G}_h$: set of flights that belongs to airline h, $\mathcal{G}_h \subseteq \mathcal{F}$,
- $\mathcal{T}$: set of discrete finite time-periods of 15-min duration, indexed by t,
- $\mathcal{K}$: set of airports in the network, indexed by k,
- $\mathcal{Z}_f$: set of routes belonging to flight f including the scheduled route, indexed by z,
- $\mathcal{P}$: set of all sectors in the network,
- $\mathcal{P}^z_f$: set of sectors that flight f is supposed to follow in route z ($\mathcal{P}^z_f \subseteq \mathcal{P}$),

- $\mathcal{P}_f^{Alt,z}$: set of sectors that flight f will use to divert to another landing airport from route z ($\mathcal{P}_f^{Alt,z} \subseteq \mathcal{P}$),
- $\mathcal{R}^j$: set of reversible flights at a resource j, $j \in \mathcal{P} \cup \mathcal{K}$. Each element in this set contains the pairs of flights that are reversible and their corresponding route (f, z, f', z'),
- $\mathcal{C}$: set of pairs of flights that are continued $(f, f') \in \mathcal{C}$, where f represents the flight that will use the same aircraft after the arrival of its preceding flight f',
- $T_j^{f,z}$: list of predefined possible flight time-periods for flight f in resource $j \in \mathcal{P}_f^z \cup \mathcal{P}_f^{Alt,z} \cup \mathcal{K}$, i.e. airport or sector, using path $z \in \mathcal{Z}_f$ according to the schedule including all possible delays,
- $T_j^{f,f',z,z'}$: set of overtaking intervals between flight f using path z and flight f' using path z', $T_j^{f,f',z,z'} = \overline{T}_j^{f',z'}, \overline{T}_j^{f',z'} + 1, ..., \underline{T}_j^{f,z} - 1$.

4.2. Parameters

The cost-related parameters are:

- C_{air}^f, C_{ground}^f: penalty cost of one-time unit for delaying a flight in the air or for delaying the departure of a flight, respectively,
- $C_{Rairport}^f$, $C_{Rsector}^f$: costs of overtaking per period at the arrival airport and sector, respectively, for flight f,
- C_{cancel}^f : penalty cost for canceling flight f,
- $C_{reroute}^{f,z}$: penalty cost for choosing path z for flight f. For the scheduled path, z^s, $C_{reroute}^{f,z^s} = 0$,
- $C_{Alt}^{f,z}$: cost of rerouting flight f to an alternative landing airport while using route z.

The flight and network parameters are:

- $origin_f$: departure airport of flight f,
- $dest_f$: scheduled destination airport of flight f,
- $dest_f^{Alt}$: alternative destination airport of flight f,
- $\mathcal{X}_{f,j}^z$, $\mathcal{Y}_{f,j}^z$: preceding and subsequent sectors, respectively, of the j^{th} sector for flight f in route z,
- $\mathcal{X}_{f,j}^{Alt,z}$, $\mathcal{Y}_{f,j}^{Alt,z}$: preceding and subsequent sectors, respectively, of the j^{th} sector in the z path to the alternative airport for flight f,
- $\mathcal{U}_f^z$: the first sector in the path of flight f,
- $\mathcal{U}_f^{Alt,z}$: the sector in set $\mathcal{P}_f^z$ after which flight f can divert to an alternative destination airport while using path z. It is the first sector in the set $\mathcal{P}_f^{Alt,z}$,
- $D_k(t)$: departure capacity of airport k at time t,
- $A_k(t)$: arrival capacity of airport k at time t,
- $S_j(t)$: capacity of sector j in the network at time t,
- $a_f^{j,z}$: scheduled arrival time of flight f to resource j using route z,

- d_f: scheduled departure time of flight f,
- z^s: index used to indicate the scheduled route of a flight,
- l_j^f: minimum time for flight f to spend in sector j, which corresponds to flying at the maximum allowable speed,
- s_f: minimum turnaround time needed for flight f to take off after the arrival of flight f' in the case of continued flights for each $(f, f') \in \mathcal{C}$,
- $\overline{T}_j^{f,z}, \underline{T}_j^{f,z}$: first and last time-period in set $T_j^{f,z}$ that represent the earliest possible time and the possible time, respectively, for flight f to enter resource j using route z. $\underline{T}_j^{f,z}$ can be calculated based on maximum allowable flight delay (a predefined value) or based on number of time-periods in the planning horizon as $\underline{T}_j^{f,z} = \overline{T}_j^{f,z} + \left(|\mathcal{T}| - a_f^{j,z}\right)$,
- φ: small value representing the fairness in delay distribution and used to have slight super-linear cost coefficients in the ground and air delays,
- π_1 and π_2: importance weights of the ATFM authority decisions and airline decisions, respectively. Note that $\pi_1 + \pi_2 = 1.0$,
- Π: airline fairness coefficient.

4.3. Decision variables

The decision variables used in the ATFM model are:

- $w_{j,t}^{f,z}$: a binary variable equal to one if flight f has arrived at resource j, $j \neq origin_f$, or taken off from $j = origin_f$ by time t using path z. Otherwise, it is equal to zero. If $w_{j,t}^{f,z} = 1$ at any period t, then it will be equal to one for all the later periods,
- $r_{j,f,f'}^{t,z,z'}$: a binary variable equal to one if a reversal occurs between flight f from route z and flight f' from route z' at resource j and at period t. Otherwise, it is equal to zero.

The additional variables used in the inter-airline fairness model (see Section 4.5) are:

- $\mathcal{B}_h$: continuous decision variable that represents the total cost per flight for airline h,
- $\overline{\mathcal{B}}$: continuous decision variable that represents the average total cost per flight,
- $\mathcal{J}_h$: continuous decision variable that represents the deviation of the total cost per flight of airline h from the average total cost per flight.

4.4. MILP model

In this section, we give the MILP model for the ATFM problem. Note that due to the formulation structure, the integrality constraint for decision variables $r_{j,f,f'}^{t,z,z'}$ can be relaxed in the case of $\pi_1 \neq 0$, $C_{Rsector}^f \neq 0$ and $C_{Rairport}^f \neq 0$ (readers can refer to Section A in the Supplementary material for further details).

$$min\ C = \pi_1 \times \sum_{h\in\mathcal{A}} \left(C_h^1 + C_h^2 + C_h^3 + C_h^4 + C_h^5\right) + \pi_2 \times \sum_{h\in\mathcal{A}} \left(C_h^6 + C_h^7 + C_h^8\right), \tag{1}$$

$$C_h^1 = \sum_{\substack{f\in\mathcal{F}^G:\\ f\in\mathcal{G}_h}} \sum_{z\in\mathcal{Z}_f} \sum_{t\in T^{f,z}_{origin_f}} \left(C^f_{ground} - C^f_{air}\right)(t-d_f)^{1+\varphi} \left(w^{f,z}_{origin_f,t} - w^{f,z}_{origin_f,t-1}\right), \tag{1a}$$

$$C_h^2 = \sum_{\substack{f\in\mathcal{F}^G:\\ f\in\mathcal{G}_h}} \sum_{z\in\mathcal{Z}_f} \sum_{k\in\{dest_f, dest_f^{Alt}\}} \sum_{t\in T_k^{f,z}} C^f_{air}\left(t - a_f^{dest_f,z^s}\right)^{1+\varphi} \left(w^{f,z}_{k,t} - w^{f,z}_{k,t-1}\right), \tag{1b}$$

$$C_h^3 = \sum_{\substack{f\in\mathcal{F}^A:\\ f\in\mathcal{G}_h}} \sum_{z\in\mathcal{Z}_f} \sum_{k\in\{dest_f, dest_f^{Alt}\}} \sum_{t\in T_k^{f,z}} C^f_{air}\left(t - a_f^{dest_f,z^s}\right)^{1+\varphi} \left(w^{f,z}_{k,t} - w^{f,z}_{k,t-1}\right), \tag{1c}$$

$$C_h^4 = \sum_{j\in\mathcal{P}} \sum_{\substack{(f,z,f',z')\in\mathcal{R}^j:\\ f'\in\mathcal{G}_h}} \sum_{t\in T_j^{f,f',z,z'}} C^f_{Rsector} \times r^{t,z,z'}_{j,f,f'}, \tag{1d}$$

$$C_h^5 = \sum_{k\in\mathcal{K}} \sum_{\substack{(f,z,f',z')\in\mathcal{R}^k:\\ f'\in\mathcal{G}_h}} \sum_{t\in T_k^{f,f',z,z'}} C^f_{Rairport} \times r^{t,z,z'}_{k,f,f'}. \tag{1e}$$

$$C_h^6 = \sum_{\substack{f\in\mathcal{F}^G:\\ f\in\mathcal{G}_h}} C^f_{cancel} \times \left(1 - \sum_{\substack{z\in\mathcal{Z}_f\\ t=\underline{T}^{f,z}_{origin_f}}} w^{f,z}_{origin_f,t}\right), \tag{1f}$$

$$C_h^7 = \sum_{\substack{f\in\mathcal{F}:\\ f\in\mathcal{G}_h}} \sum_{z\in\mathcal{Z}_f\setminus\{z^s\}} C^{f,z}_{reroute} \times \sum_{\substack{k\in\{dest_f, dest_f^{Alt}\}\\ t=\underline{T}_k^{f,z}}} w^{f,z}_{k,t}, \tag{1g}$$

$$C_h^8 = \sum_{\substack{f\in\mathcal{F}:\\ f\in\mathcal{G}_h}} \sum_{\substack{z\in\mathcal{Z}_f\\ k=dest_f^{Alt}, t=\underline{T}_k^{f,z}}} C^{f,z}_{Alt} \times w^{f,z}_{k,t}, \tag{1h}$$

Subject to:

$$\sum_{\substack{f\in\mathcal{F}^G:\\ k=origin_f}} \sum_{z\in\mathcal{Z}_f} \left(w^{f,z}_{k,t} - w^{f,z}_{k,t-1}\right) \leq D_k(t), \qquad k\in\mathcal{K},\ t\in\mathcal{T}, \tag{2}$$

$$\sum_{\substack{f\in\mathcal{F}:\\ k\in\{dest_f, dest_f^{Alt}\}}} \sum_{z\in\mathcal{Z}_f} \left(w^{f,z}_{k,t} - w^{f,z}_{k,t-1}\right) \leq A_k(t), \qquad k\in\mathcal{K},\ t\in\mathcal{T}, \tag{3}$$

$$\sum_{f\in\mathcal{F}} \sum_{\substack{z\in\mathcal{Z}_f:\\ j\in\mathcal{P}_f^z,\ j\neq\mathcal{U}_f^{Alt,z}\\ j'=\mathcal{Y}_{f,j}^z}} (w_{j,t}^{f,z} - w_{j',t}^{f,z}) + \sum_{f\in\mathcal{F}} \sum_{\substack{z\in\mathcal{Z}_f:\\ j\in\mathcal{P}_f^{Alt,z},\ j\neq\mathcal{U}_f^{Alt,z}\\ j'=\mathcal{Y}_{f,j}^{Alt,z}}} (w_{j,t}^{f,z} - w_{j',t}^{f,z}) +$$
$$\sum_{f\in\mathcal{F}} \sum_{\substack{z\in\mathcal{Z}_f:\\ j=\mathcal{U}_f^{Alt,z}\\ j'=\mathcal{Y}_{f,j}^z\\ j''=\mathcal{Y}_{f,j}^{Alt,z}}} (w_{j,t}^{f,z} - w_{j',t}^{f,z} - w_{j'',t}^{f,z}) \leq S_j(t), \qquad j\in\mathcal{P},\ t\in\mathcal{T}, \tag{4}$$

$$\sum_{\substack{k\in\{dest_f, dest_f^{Alt}\},\\ t=\underline{T}_k^{f,z}}} w_{k,t}^{f,z} = w_{k',t'}^{f,z}, \qquad f\in\mathcal{F}^G,\ z\in\mathcal{Z}_f, k'=origin_f, t'=\underline{T}_{k'}^{f,z} \tag{5}$$

$$\sum_{\substack{k\in\{dest_f, dest_f^{Alt}\},\\ t=\underline{T}_k^{f,z}}} w_{k,t}^{f,z} = w_{j,t'}^{f,z}, \qquad f\in\mathcal{F}^A,\ z\in\mathcal{Z}_f, j=\mathcal{U}_f^z, t'=\overline{T}_j^{f,z} \tag{6}$$

$$\sum_{z\in\mathcal{Z}_f} \sum_{\substack{k\in\{dest_f, dest_f^{Alt}\},\\ t=\underline{T}_k^{f,z}}} w_{k,t}^{f,z} = 1, \qquad f\in\mathcal{F}^A, \tag{7}$$

$$\sum_{z\in\mathcal{Z}_f} w_{k,t}^{f,z} \leq 1, \qquad f\in\mathcal{F}^G, k=origin_f, t=\underline{T}_k^{f,z} \tag{8}$$

$$\begin{aligned} & w_{j,t-1}^{f,z} - w_{j,t}^{f,z} \leq 0, \qquad f\in\mathcal{F},\ z\in\mathcal{Z}_f,\\ & j\in\{origin_f, dest_f, dest_f^{Alt}\}\cup\mathcal{P}_f^z\cup\mathcal{P}_f^{Alt,z},\ t\in T_j^{f,z}, \end{aligned} \tag{9}$$

$$\begin{aligned} & w_{j,t}^{f,z} - w_{j',t-l_{j'}^f}^{f,z} \ \leq\ 0,\\ & f\in\mathcal{F},\ z\in\mathcal{Z}_f,\ j\in\mathcal{P}_f^z\cup\{dest_f\},\\ & j'\neq\mathcal{U}_f^{Alt,z},\ j\neq origin_f,\ j'=\mathcal{X}_{f,j}^z,\ t\in T_j^{f,z} \end{aligned} \tag{10}$$

$$\begin{aligned} & w_{j,t}^{f,z} - w_{j',t-l_{j'}^f}^{f,z} \ \leq\ 0,\\ & f\in\mathcal{F},\ z\in\mathcal{Z}_f,\ j\in\mathcal{P}_f^{Alt,z}\cup\{dest_f^{Alt}\},\\ & j\neq origin_f,\ j'=\mathcal{X}_{f,j}^{Alt,z},\ j'\neq\mathcal{U}_f^{Alt,z},\ t\in T_j^{f,z}, \end{aligned} \tag{11}$$

$$\begin{aligned}
&w_{j,t}^{f,z} + w_{j'',t}^{f,z} - w_{j',t-l_{j'}^{f}}^{f,z} \leq 0, \\
&f \in \mathcal{F},\ z \in \mathcal{Z}_f,\ j \in \mathcal{P}_f^{Alt,z} \cup \{dest_f^{Alt}\}, \\
&j \neq origin_f,\ j' = \mathcal{U}_f^{Alt,z},\ j'' = \mathcal{Y}_{f,j}^{Alt,z},\ t \in T_j^{f,z},
\end{aligned} \tag{12}$$

$$\begin{aligned}
&w_{origin_f,t}^{f,z} - \sum_{z' \in \mathcal{Z}_{f'}} w_{dest_{f'},t-s_f}^{f',z'} \leq 0, \\
&(f,f') \in \mathcal{C},\ z \in \mathcal{Z}_f,\ t \in T_{origin_f}^{f,z},
\end{aligned} \tag{13}$$

$$\begin{aligned}
&w_{j,t}^{f',z'} - w_{j,t}^{f,z} - \left(1 - w_{j,\underline{T}_j^{f,z}}^{f,z}\right) \leq r_{j,f,f'}^{t,z,z'}, \\
&j \in \mathcal{P} \cup \mathcal{K},\ (f,z,f',z') \in \mathcal{R}^j,\ t \in T_j^{f,f',z,z'},
\end{aligned} \tag{14}$$

$$\begin{aligned}
&w_{j,t}^{f,z} - w_{j,t}^{f',z'} + r_{j,f,f'}^{t,z,z'} \leq 1, \\
&j \in \mathcal{P} \cup \mathcal{K},\ (f,z,f',z') \in \mathcal{R}^j,\ t \in T_j^{f,f',z,z'},
\end{aligned} \tag{15}$$

$$\begin{aligned}
&w_{j,t}^{f,z} + r_{j,f,f'}^{t,z,z'} \leq 1, \\
&j \in \mathcal{P} \cup \mathcal{K},\ (f,z,f',z') \in \mathcal{R}^j,\ t \in T_j^{f,f',z,z'},
\end{aligned} \tag{16}$$

$$\begin{aligned}
&- w_{j,t}^{f',z'} + r_{j,f,f'}^{t,z,z'} \leq 0, \\
&j \in \mathcal{P} \cup \mathcal{K},\ (f,z,f',z') \in \mathcal{R}^j,\ t \in T_j^{f,f',z,z'},
\end{aligned} \tag{17}$$

$$w_{j,t}^{f,z}, r_{j,f,f'}^{t,z,z'} \in \{0,1\}. \tag{18}$$

The objective function (1) minimizes the total network cost, which consists of the ground delay cost and the air delay cost for the scheduled flights for each airline h in Equations (1a) and (1b), and the air delay cost for the en-route flights (1c), the sector (1d) and destination airport (1e) overtaking penalty costs, the cancellation penalty cost for the scheduled flights (1f), the path rerouting penalty cost (1g) and the diversion penalty cost (1h). Note that Equation (1b) calculates the total delay, and so in Equation (1a) we multiply the total ground delay by parameter C_{air}^{f} so that it is subtracted from the total delay in Equation (1b) when (1a) and (1b) are combined. This will result in the total ground delay and the total air delay. Note also that the first five terms (C_h^1 through C_h^5) in the objective function are the costs resulting from ATFM authority decisions for each airline h, while the remaining terms are the costs resulting from airline decisions. Parameters π_1 and π_2 are the importance weights of the ATFM authority decisions and the airline decisions, respectively. These weights are used to obtain the weighted sum formulation. As mentioned earlier, these weights can be seen as a tool to balance the two types of decisions and find the best combination that minimizes total network cost, but also to analyze the trade-off between ATFM authority decisions and airline decisions.

Note that a dummy period ($t = 0$) needs to be added in the implementation to capture the situation where a flight may depart at $t = 1$. The value of $w_{j,t}^{f,z}$ should be set to 0 at this dummy period.

Constraints (2) and (3) ensure that the number of flights that depart from or arrive at an airport does not exceed the predefined capacity of departures or arrivals for each period in the planning horizon. While Constraint (2) is applicable only for the scheduled flights' set, Constraint (3) is for both the scheduled and the en-route flights' sets.

Constraint (4) ensures that no sector capacity is exceeded. More specifically, in every time-period t, the total number of flights in a sector that still has not entered their next sectors (in the regular path or diversion path) shall not exceed the sector capacity at that time-period. This constraint consists of three parts. The first part counts a flight if sector j is on its regular path (non-diversion path). The second part counts a flight if sector j is on its diversion path (except the common sector between the regular and diversion path). The last part counts a flight if sector j is a transition sector, i.e. the next sector is part of the flight's regular path or its diversion path.

Constraints (5) and (6) ensure that if a flight departs or if it is in its first sector (for the en-route flights), then it must arrive at its destination airport or its alternative destination airport (if any). These constraints are needed to ensure continuity in the path between starting point (departure airport/first sector) and ending point (arrival airport).

Constraints (7) and (8) are the rerouting constraints, where Constraint (7) forces en-route flights to follow one path without allowing the possibility of flight cancellation, and Constraint (8) limits the number of selected paths for any other flight to at most one, which indicates that these flights might be canceled.

Constraint (9) is the time connectivity constraint, where if $w_{j,t}^{f,z} = 1$ in period t, then it should also equal one for all the later periods. Constraints (10)-(12) represent the path connectivity that ensures a flight cannot enter the next sector in its path unless it has spent at least the minimum allowable time in its current sector. Constraint (10) is for the connectivity of the normal (non-diversion) path of a flight, and Constraint (11) is for the connectivity of the path used for diversion. Constraint (12) allows a flight to select one decision regarding diversion to another airport, i.e. either to continue its normal path to the scheduled landing airport or to divert and follow a new set of sectors to another airport.

Constraint (13) links the continued flights so that an aircraft scheduled to perform a continued flight cannot depart unless it has spent at least its specified turnaround duration after the arrival of its previous flight to the scheduled destination airport. In this case, a continued flight will be canceled if its previous flight has been canceled or diverted to another destination airport.

Equations (14)-(17) are the overtaking constraints to count if flight f' overtakes flight f at time t in a common resource j. Finally, Constraint (18) defines the decision variables as binary variables.

4.5. ATFM with airline fairness

The previous mathematical formulation can be extended to account for fairness among airlines. One way to consider the fairness measure is by balancing the total cost per flight among airlines.

$$C_9 = \Pi \times \sum_{h \in \mathcal{A}} |\mathcal{B}_h - \overline{\mathcal{B}}|, \tag{1i}$$

subject to:

$$\mathcal{B}_h = \frac{1}{|\mathcal{G}_h|} \times \left(C_h^1 + C_h^2 + C_h^3 + C_h^4 + C_h^5 + C_h^6 + C_h^7 + C_h^8\right), \qquad h \in \mathcal{A}, \tag{19}$$

$$\overline{\mathcal{B}} = \frac{1}{|\mathcal{A}|} \times \sum_{h \in \mathcal{A}} \mathcal{B}_h, \tag{20}$$

$$\mathcal{B}_h, \overline{\mathcal{B}} \ \geq \ 0, \qquad h \in \mathcal{A}. \tag{21}$$

The new objective function component (1i) minimizes the total absolute deviation of airline cost per flight from the average value. Constraint (19) calculates the airline cost per flight for each airline, Constraint (20) calculates the average cost per flight, and Constraint (21) is the non-negativity constraint. Note here that since the objective function is minimized, the non-linear objective function component can be made linear by adding one decision variable set and two constraints. Equations (22) and (23) help assign the largest value of $\left(\mathcal{B}_h - \overline{\mathcal{B}}\right)$ and $\left(\overline{\mathcal{B}} - \mathcal{B}_h\right)$ to the decision variable $\mathcal{J}_h$.

$$C_9 = \Pi \times \sum_{h \in \mathcal{A}} \mathcal{J}_h, \tag{1j}$$

subject to

$$\mathcal{J}_h \geq \mathcal{B}_h - \overline{\mathcal{B}}, \qquad h \in \mathcal{A}, \tag{22}$$

$$\mathcal{J}_h \geq \overline{\mathcal{B}} - \mathcal{B}_h, \qquad h \in \mathcal{A}. \tag{23}$$

The term C_9 can be added to the previous objective function (1). Note that minimizing term C_9 only may lead to an unnecessary increase in total cost per flight to facilitate achieving a balanced situation. Therefore, the total cost (presented in Equation (1)) needs to be considered to prevent any unnecessary increase in the cost. Note that Equation (19) can be modified if one wishes to focus on fairly distributing one cost component among airlines (e.g. delay costs or overtaking costs) rather than the total cost per flight.

5. Simulating the Current Practice

Here we illustrate how the proposed central authority ATFM model can be adapted to simulate the current practice using a two-stage ATFM. Decisions on flights are taken independently by the ATFM authority and the airlines. The process starts when airlines share their planned schedules with the ATFM authority. The ATFM authority decides on flight delays (and consequently the resulting overtaking) and shares the updated schedules with the airlines. Each airline tries to minimize its own cost using rerouting, diversion, and cancellation decisions while integrating the delays obtained from the ATFM authority. They update their schedules and share them back with the ATFM authority. The process continues until no further modifications are made. Current practice can be simulated using the model proposed in this paper with some modifications, and then compared with our proposed approach of having a central decision-making authority. This comparison will be addressed in Section 7.

The following steps illustrate how the model proposed in Section 4 can be modified to simulate the current practice:

1. The planned schedules are optimized to minimize delay and overtaking costs ($\sum_{h\in\mathcal{A}} C_h^1$ through $\sum_{h\in\mathcal{A}} C_h^5$ in Equation (1)), by setting the other costs to a number large enough to rule them out.

2. Each airline ($h \in \mathcal{A}$) tries to minimize its own costs using rerouting, diversion and cancellation decisions while setting the delay obtained from Step 1 as an upper bound. In this step, the schedules of other airlines are fixed and kept unchanged, as an airline can only modify its own schedules. The optimization model is solved for each airline individually.

3. The schedule changes, i.e. reroutings, diversions and cancellations from Step 2 from all airlines, are sent back to the ATFM authority to re-optimize the delays.

4. The process iterates until no further changes are made, and if the process does not converge to a solution, the solution with the lowest cost is selected.

6. The Solution Process

The mathematical model presented in Section 4 is solved using the branch-and-cut algorithm in CPLEX solver 12.8.0. A solution can be obtained quicker by limiting the maximum allowable delay and the number of flights that can be rerouted (as done in Section 7). However, if the maximum allowable delay, the number of flights that can be rerouted, and the other features are not limited, then computational time increases considerably, and the model becomes intractable.

In the following sections, we therefore introduce a modified version of the fix-and-relax heuristic to solve the ATFM with different rerouting options and provide decision-makers with a good trade-off between solution quality and resolution time.

6.1. Heuristic basics

Here we briefly present the proposed heuristic. Readers can refer to Section B in the Supplementary material for a detailed description.

The heuristic is a modified version of the conventional fix-and-relax heuristic, which has been widely applied with success in scheduling problems (Beraldi et al. 2008, Mohammadi et al. 2010, Uggen, Fodstad, and Nørstebø 2013). The fix-and-relax algorithm starts with a relaxed optimization problem and partitions the integer decision variables into groups. In each iteration, it fixes the binary variables of the previous iteration to their optimal values, sets the next group of variables as integers, and solves the problem until the last iteration is reached. It partitions the problem either row-wise or column-wise. Even though the modified fix-and-relax algorithm uses partitioning of the decision variables, we are only utilizing it in the first iteration so as to get a starting-point solution. In addition, we are partitioning using row-wise and column-wise simultaneously by defining a block that contains some of the rows and columns of the decision variable. In the other iterations, the fix-and-set processes are done on the variables with non-integer solutions. In addition to that, in each iteration the algorithm checks whether the resulting solution is entirely integer or not. The algorithm terminates when the resulting solution is entirely integer. These modifications save a huge amount of computational time compared with the row-wise or column-wise approaches.

6.2. Heuristic validation

The proposed heuristic performs well compared to the optimal solution of the exact approach. Performance was judged using 25 instances. The instances were chosen to cover various situations by varying the number of flights, the number of sectors, and the number of airports. The heuristic was implemented using Julia Programming Language 1.4.0 (Bezanson et al. 2017). The analysis was carried out using a computer equipped with an Intel(R) Core(TM) i7-8700 CPU @ 3.2 GHz and 16 Gb of RAM, and running Windows 10 Enterprise 64-bit operating system. A time limit of five hours was used to terminate the solver in the exact approach. Table 2 summarizes the statistics of the 25 instances, their computational time, and the optimal objective value using the exact and heuristic approaches. Figure 4 shows the gap value between the heuristic and the exact solutions and the time saving for each instance, where the gap is calculated as $100 \times \frac{Heuristic\ optimal\ value - Exact\ optimal\ value}{Exact\ optimal\ value}$

and the time saving is calculated as $100 \times \frac{Exact\ time - Heuristic\ time}{Exact\ time}$. Note that the gap between the heuristic solution and the exact solution is less than 2%. The heuristic was able to save an average 86.2% of the exact-solution time, and found good solutions in around 24 minutes for the cases where the exact approach instances reached the time limit.

Table 2 Instance details, computational times, and optimal values using the exact and heuristic approaches

Instance	$\|\mathcal{F}^G\|$	$\|\mathcal{F}^A\|$	$\|\mathcal{P}\|$	$\|\mathcal{K}\|$	APL*	Number of decision variables	Number of constraints	Exact time (minutes)	Heuristic time (minutes)	Exact objective value	Heuristic objective value
1	2000	100	80	20	6	648490	757745	56.97	13.19	2977843	2994192
2	2000	100	50	20	6	849025	1224915	49.48	10.21	1561810	1562016
3	2000	100	50	20	6	782924	1085026	37.67	21.32	1695284	1702017
4†	2500	100	50	20	6	2844778	3466085	300.00	17.49	833984	837343
5†	2500	100	50	20	6	2781608	3427712	300.00	24.68	1945268	1945267
6†	2500	100	50	20	5	2360213	2903799	300.00	5.13	2753726	2753726
7#	2500	100	50	20	5	4940592	6083877	300.00	19.49	-	1890127
8	2500	100	50	20	5	1158923	1510333	250.76	22.46	5496335	5498541
9	2500	100	50	20	6	1506510	1908330	139.25	25.21	1272724	1273430
10	2500	100	50	20	7	1481847	1863358	118.69	17.24	1812116	1812116
11	2500	100	50	20	5	1284136	1638204	107.37	4.96	5847822	5847822
12	2500	100	50	20	5	768125	1007106	34.49	19	4332753	4334124
13	2500	100	50	20	6	837702	1089997	32.28	11.57	5359030	5359042
14†	3000	100	50	20	6	1632680	2072309	300.00	26.72	1944169	1946490
15†	3000	100	50	20	6	2461675	3152897	300.00	33.15	955681	962460
16†	3000	100	50	20	6	2040735	2631817	300.00	26.80	6113883	6111600
17†	3000	100	50	20	6	1164662	1382711	300.00	30.37	827313	836799
18#	3000	100	50	20	5	1957043	2482765	300.00	32.66	-	8774690
19	3000	100	50	20	6	1725204	2192569	271.72	16.74	1032794	1051176
20	3000	100	50	20	6	1676537	2130742	207.47	21.09	5909013	5909013
21	3000	100	50	20	6	2107920	2765595	189.97	12.51	953316	953318
22	3000	100	50	20	6	2054249	2679736	143.62	5.17	2154493	2154493
23	3000	100	50	20	6	2278938	2910739	100.94	3.53	623801	623801
24	3000	100	50	20	6	2207611	2835951	98.13	4.2	1702723	1702723
25	3000	100	50	20	6	1639260	2037447	95.39	3.54	2509125	2509125

* APL: the average path length.
\# Exact solver could not find a solution within the time limit.
† Exact solver stopped due to the time limit.

7. Managerial Insights

We conducted various numerical studies in order to tease out managerial insights. The purpose of this section is to illustrate how the model with its new features can help decision-makers to achieve better airspace utilization. We used six instances with 96 time periods of 15 minutes each, which gives a total period of 24 hours and a number of flights that varies between 1000 to 3000 distributed over 25 airports with 100 en-route flights in the airspace at the beginning of the day. Each data set

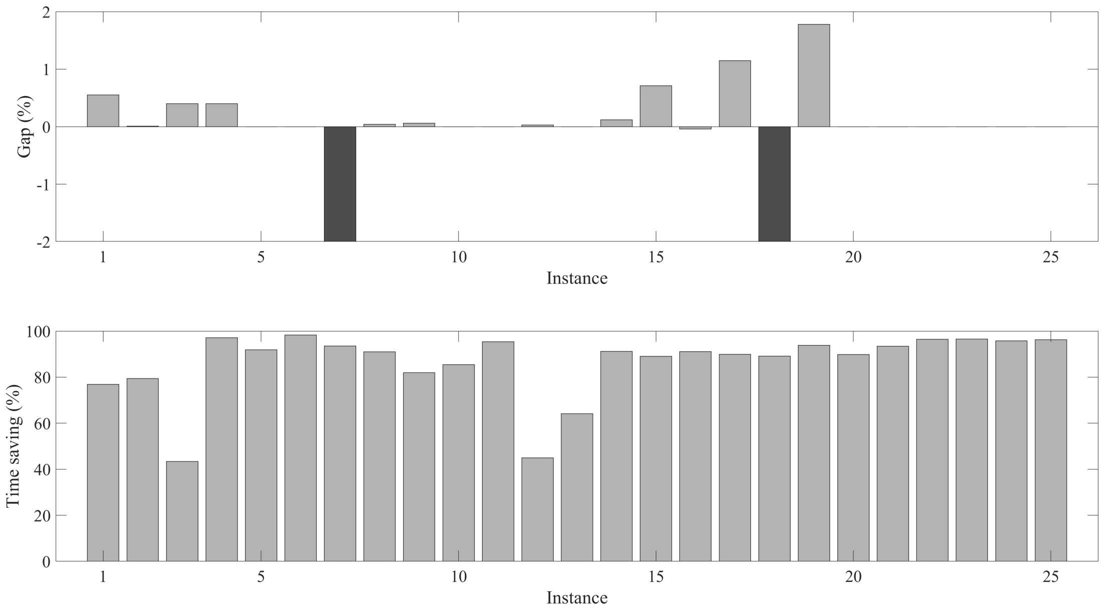


Figure 4 Heuristic performance compared to the exact solution in terms of the gap and the time saving. Dark grey bars identify the instances where no exact solution was found within the time limit and the gap is set to -100%

has 20% connecting flights. Ten major airline companies operate these flights. In each data set, 5% of the airports are considered the busiest, handling around 40% of the traffic. These instances were generated randomly, and the cost parameters were obtained from the EUROCONTROL report (EUROCONTROL 2018). The airspace is divided into 10×10 cells, each cell representing a sector. The flight trajectory from the origin to the destination airport for each flight is based on the shortest path, which yields the sequence of sectors and the corresponding distances.

Each flight is given an aircraft type, and the minimum required time in each sector is thus determined from the cruising speed (based on the aircraft type) and the distance (based on the flight trajectory). If a flight has a diversion option, then the nearest airport is identified and an additional path from a specific sector to the alternative airport is generated.

Moreover, in cases where a flight can be rerouted, two alternative paths can be generated by moving to one of the adjacent sectors (in any of the four directions) at the beginning of the flight path and then following a straight path to the destination airport. It was assumed that the maximum allowable delay for any flight is 1 hour 30 minutes, which is equivalent to 6 time-periods (of 15 minutes each). This assumption allows solving the six instances optimally using CPLEX 12.8.0 within 30 minutes. Most of the cost parameters used in this analysis were obtained from EUROCONTROL (2018), and are as follows: $C^f_{air} =$ €2190 per time-period, $C^f_{ground} =$ €1350 per time-period, $C^f_{cancel} =$ €96695 per flight, $C^{f,z}_{Alt} =$ €7400 per flight. Overtaking cost was assumed to be equal to the value of the ATFM slot swapping cost stated in the Eurocontrol report ($C^f_{R_{airport}} =$

$C^f_{R_{sector}} =$ €4600 per time-period). Rerouting cost was assumed to be $C^{f,z}_{reroute} =$ €700 per flight. The airport departure and arrival capacities and the airspace sector capacities were generated randomly in the range of 10 to 15 flights per time-period. Table 3 summarizes the remaining details of the instances.

In what follows, we compare the proposed central authority model against the current practice. We investigate the impact of the important new features presented in this model (rerouting, diversion, overtaking, and airline fairness) on the ATFM network. Furthermore, we explore the trade-off between delays and overtaking and rerouting, diversion and cancellation. The analysis of these issues is structured into six observations. Note that in Observations 1 to 5, the importance weights are set to equal ($\pi_1 = \pi_2 = 0.5$).

Table 3 Summary of Instance sets used in the numerical experiments

Instance	$\|\mathcal{F}^G\|$	$\|\mathcal{F}^A\|$	Number of flights that can be rerouted	Number of flights that can be diverted	Range of the number of sectors for each flight	Number of decision variables	Number of constraints
1	1000	100	45	32	2-19	862556	972241
2	2000	100	95	50	2-18	560235	516622
3	2000	100	95	47	2-21	605034	504265
4	2000	100	95	50	2-21	688659	601206
5	3000	100	145	70	2-19	1116316	1085664
6	3000	100	145	89	2-21	1076671	944057

Observation 1: Having a central authority to assign all decisions can reduce the total system cost, particularly when enforcing fairness.

Table 4 illustrates the results of simulating the current practice (presented in Section 5) using an iterative approach and the results of applying the proposed central authority model using equal weights ($\pi_1 = \pi_2 = 0.5$) under no airline fairness ($\Pi = 0$) and full airline fairness ($\Pi = 1000$). The iteration limit is set to 25 iterations, where one iteration includes the ATFM authority optimization and airline optimization independently and then preparing a list of all changes and sharing it with the ATFM authority for the next iteration.

The solution of instances 1 and 5 without considering airline fairness and the solution of instance 2 considering airline fairness failed to improve the performance using rerouting, diversion and cancellation when executed independently from the delay decisions (Table 4). Using rerouting, diversion and cancellation enhanced the solution of the remaining instances by an average of 34.6% when airline fairness is not considered. This percentage increases more than 10-fold when airline fairness is considered. This first observation shows the benefit of incorporating airline decisions in the process. Moreover, allowing a central authority to control all decisions results in a further 1.8%–7.6% cost reduction when not considering airline fairness, which is a relatively small

Table 4 Comparison between the current practice and the proposed approach

	Instance	1	2	3	4	5	6
Without airline fairness ($\Pi = 0$)	The current practice [iterative and independent decisions]						
	Total cost - FI[#] (€)	982225	922519	18151642	13698994	2412881	5911224
	Total cost - LI[##] (€)	982225	912666	4538699	5874394	2412881	5601533
	Iteration[*]	3	3	14	5	3	10
	Our proposed approach [central decision-making authority]						
	Total cost (€)	907166.6	896443.4	4216746	5770166	2255433	5325571
	Improvements						
	LI vs. FI (%)	0.0	1.1	75.0	57.1	0.0	5.2
	Our approach vs. LI (%)	7.6	1.8	7.1	1.8	6.5	4.9
With airline fairness ($\Pi = 1000$)	The current practice [iterative and independent decisions]						
	Total cost - FI (€)	1095423	930586	15831970	25614121	30080063	28697823
	Total cost - LI (€)	998532	930586	7151579	6896822	2426345	5668478
	Iteration[**]	3	3	8	15	3	10
	Our proposed approach [central decision-making authority]						
	Total cost (€)	933732	905090	4246439	5781051	2270050	5360701
	Improvements						
	LI vs. FI (%)	9.70	0.0	121.38	271.39	1139.73	406.27
	Our approach vs. LI (%)	6.94	2.82	68.41	19.30	6.89	5.74

[#] FI: first iteration.
[##] LI: last iteration.
[*] Instances 3 and 4 did not converge to a solution and the best cost is found at the indicated iteration.
[**] Instance 4 did not converge to a solution and the best cost is found at the indicated iteration.

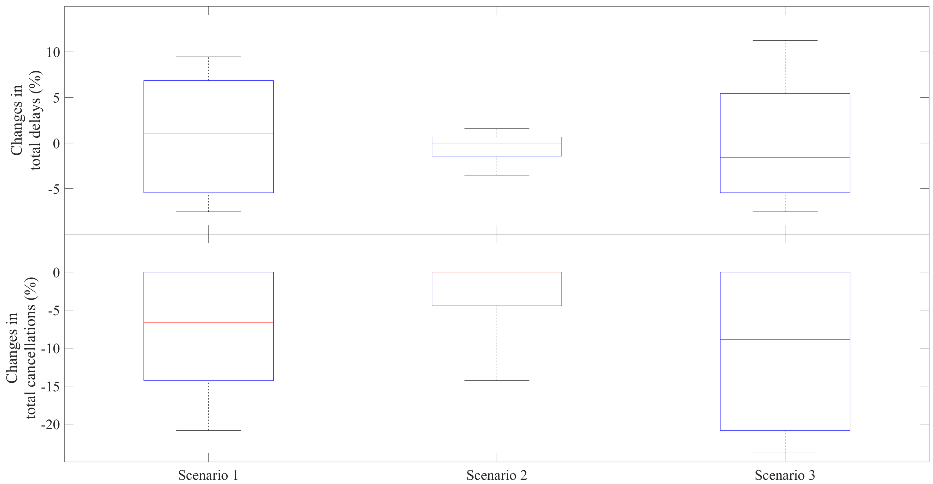
improvement. However, under airline fairness, the central authority approach can achieve a further reduction of 18.35% on average (Table 4).

This indicates that airspace may be better utilized at a lower cost if one entity controls all decisions. Having a central authority may lead to a fully transparent collaborative decision-making system, where all airlines share information about their flight preferences, such as preferred routes and other acceptable routes, and let a central authority decide on the best actions. Note that the airline shares its information with the airport authorities and the ATFM authorities but not with other airlines, in order to ensure confidentiality and fair competition.

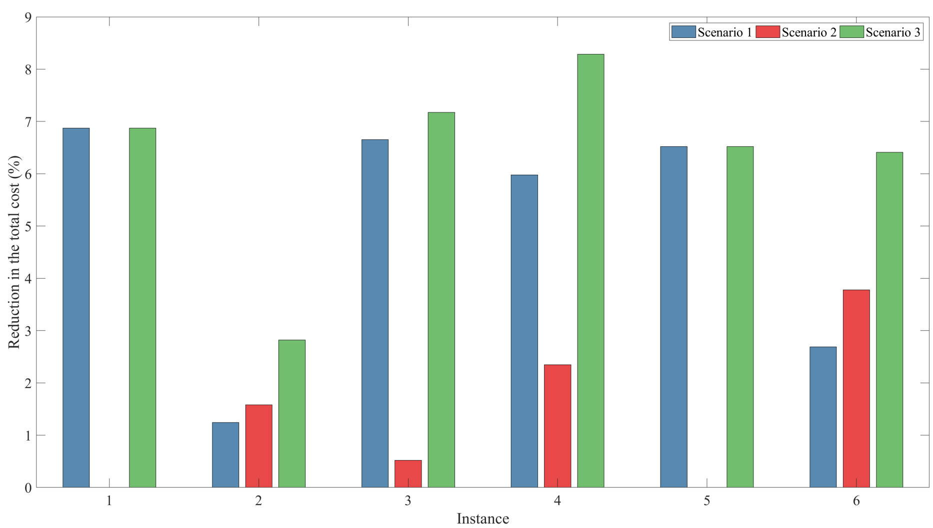
Without considering airline fairness, the feasibility of the central authority approach will be highly dependent on the readiness of communication systems from other projects such as SWIM and the required level of administrative effort. Considering airlines fairness, the savings are potentially much higher, giving more incitement to use the central authority.

Observation 2: Introducing rerouting and diversion options reduces the total cost, and the network decisions are insensitive to the increase in diversion cost.

Three scenarios were developed to examine the effect of rerouting and diversion on the total delay periods and the total number of cancellations. These scenarios are: ATFM with rerouting only, ATFM with diversion only, ATFM with rerouting and diversion. The changes in the total delay



(a) Effect of rerouting and diversion on the total delays and cancellations



(b) Effect of rerouting and diversion on the total cost

Figure 5 **Impact of rerouting and diversion decisions**

periods, total number of cancellations and total cost are observed by comparing the results of these scenarios against the 'ATFM without rerouting and diversion' scenario. Figure 5 shows the impact of considering rerouting and diversion. The boxplot (Figure 5a) summarizes the effect of considering the three scenarios—rerouting only, diversion only, and ATFM with rerouting and diversion—on the total delay periods and cancellations for the six instances. Figure 5b illustrates the reduction in total cost under the three scenarios.

Rerouting only (Scenario 1) results in up to a 7.5% reduction in delay among all instances with no canceled flights. For the instances with canceled flights, rerouting reduces cancellations by up to 20.8% with a slight increase of up to 9.5% in total delay (Figure 5a). In addition, rerouting also saves 5% in total cost (Figure 5b). On the other hand, considering diversion only (Scenario 2) can have a better effect than rerouting. For example, in instance 6, using diversion only led to less increase in total delay (0.67%) compared to the 3.6% increase in total delay using rerouting only for the same reduction in cancellations. This comes from the fact that diversion may shift flights to less-congested airports. Compared to ATFM without rerouting and diversion, considering diversion helps to reduce total cost by an average of 1.37% (Figure 5b).

Combining both decisions, i.e. rerouting and diversion, (Scenario 3), achieves better performance than the ATFM without rerouting and diversion. For example, in the instances with zero cancellations (instance 1, 2 and 5), considered rerouting and diversion together resulted in less total delay than considering only rerouting or only diversion (Table 5). The increase in total delay is always associated with a decrease in percentage of cancellations, i.e. an overall improvement. Combining rerouting and diversion decisions results in an average total cost reduction of 6.35% (Figure 5b), a maximum delay reduction of 7.5%, and a maximum cancellations reduction of 23.8% (Figure 5a).

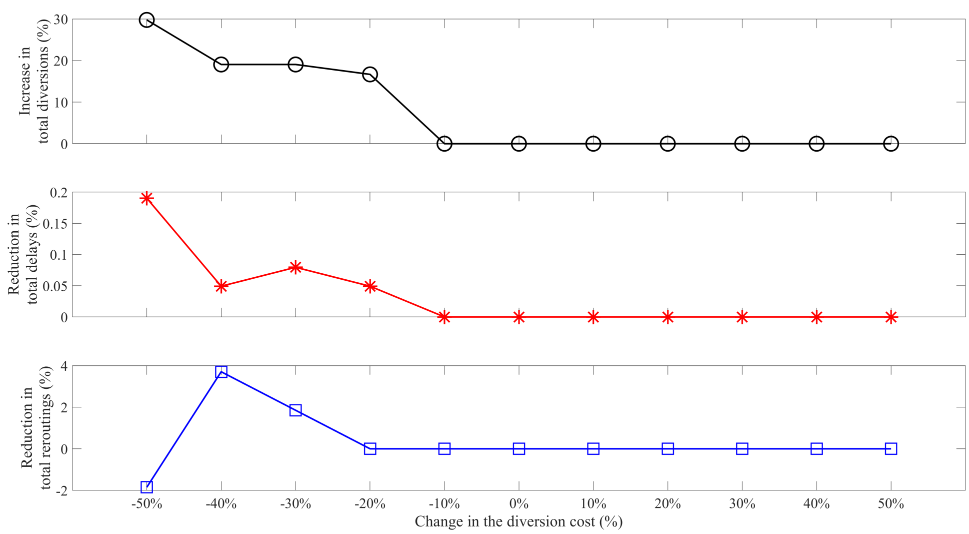


Figure 6 Impact of diversion cost on network decisions

Figure 6 illustrates how total delays, reroutings and diversions change when diversion cost changes in Scenario 3. Network decisions were found to be insensitive to the increase in diversion cost. In addition, total delay decreases when diversion cost is reduced by at least 20%, and rerouting changes when diversion cost is reduced by at least 30%. Reducing the diversion cost by 50% increases total diversions by an average of 30% and increases reroutings by 2% but decreases total delay by an average of 0.2%.

Table 5 Effect of different ATFM network features on the total delay, rerouting, diversion and cancellation decisions

	GD [1]	AD [2]	RF [3]	DF [4]	CF [5]		GD	AD	RF	DF	CF
Instance 1						Instance 2					
Baseline*	155	267	0	0	0	Baseline	605	48	0	0	0
Scenario 1	138	261	2	0	0	Scenario 1	596	48	1	0	0
Scenario 2	155	267	0	0	0	Scenario 2	580	50	0	2	0
Scenario 3	138	261	2	0	0	Scenario 3	571	50	1	2	0
Instance 3						Instance 4					
Baseline	1546	58	0	0	24	Baseline	1169	164	0	0	45
Scenario 1	1630	84	16	0	19	Scenario 1	1262	198	6	0	39
Scenario 2	1523	58	0	1	24	Scenario 2	1187	167	0	2	43
Scenario 3	1607	84	16	1	19	Scenario 3	1282	201	6	2	37
Instance 5						Instance 6					
Baseline	1558	140	0	0	0	Baseline	2699	5	0	0	21
Scenario 1	1423	147	14	0	0	Scenario 1	2791	9	6	0	18
Scenario 2	1558	140	0	0	0	Scenario 2	2718	4	0	7	18
Scenario 3	1423	147	14	0	0	Scenario 3	2743	7	5	7	16

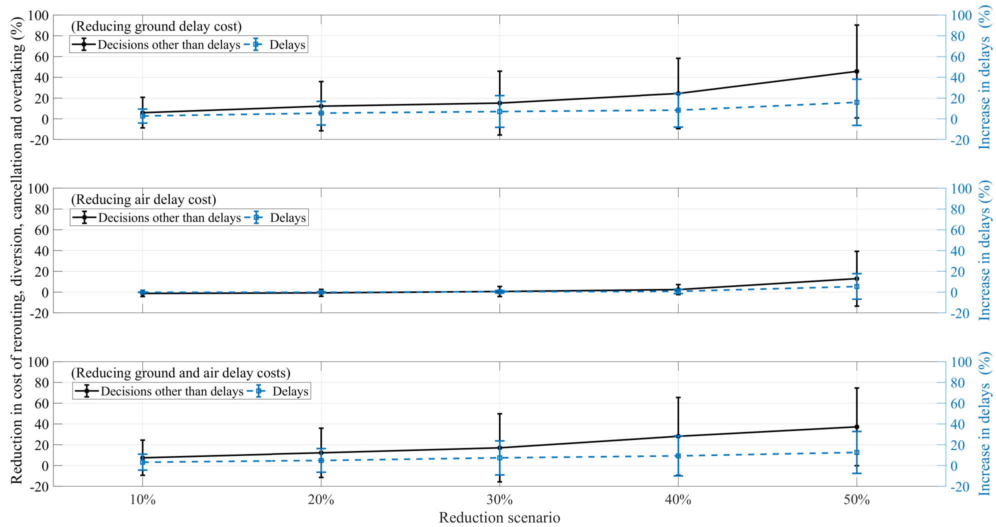
[1] GD: Total ground delay
[2] AD: Total air delay
[3] RF: Total rerouted flights
[4] DF: Total diverted flights
[5] CF: Total canceled flights
* Baseline represents ATFM without rerouting and diversion, and with overtaking

Observation 3: Reducing the ground delay cost has much more impact than reducing the air delay cost.

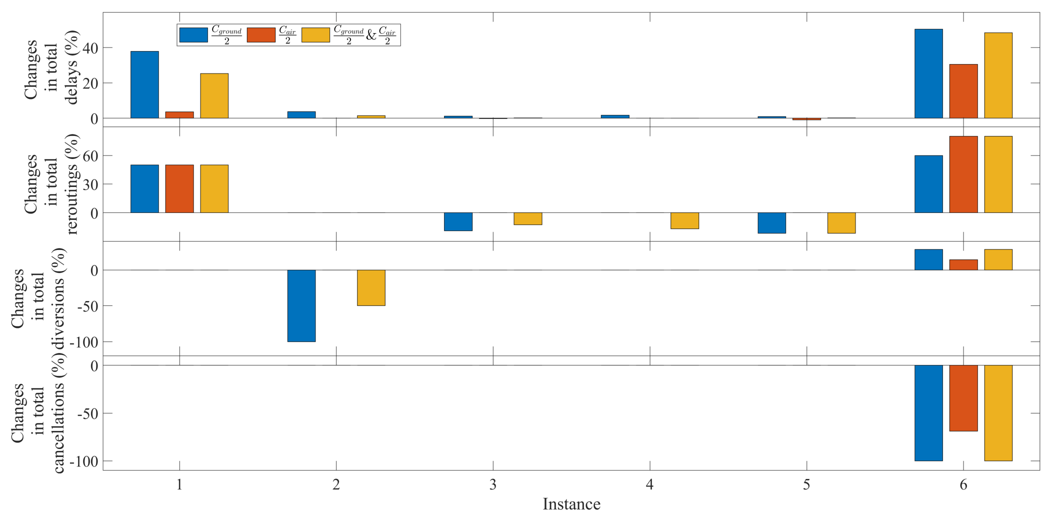
In Observation 3, we examine the effect of reducing ground delay cost, air delay cost, or both ground delay cost and air delay cost on the other network decisions (rerouting, diversion and cancellation). We tested 10%, 20%, 30%, 40% and 50% reductions in delay costs.

Figure 7 shows how delay costs interact with network decisions. Figure 7a shows the cost changes with respect to each reduction scenario, and Figure 7b illustrates the impact of reducing the delay costs by 50% on the number of total delays, reroutings, diversions and cancellations. The changes in total delays, reroutings, diversions and cancellations are calculated based on the ATFM scenario with rerouting and diversion, with $C^f_{air} =$ €2190 per time-period and $C^f_{ground} =$ €1350 per time-period.

Reducing air delay costs had less impact on network decisions than reducing ground delay costs. Reducing ground delay costs by 50% reduced the cost of rerouting, diversion, cancellation, and overtaking by 41% on average, compared to just 13% when reducing air delay costs by the same 50% (Figure 7a). In addition, reducing ground and air delay costs by 50% reduced costs from the other network decisions by an average of 37%, which is less than the effect obtained when



(a) Impact of reducing delay costs on network costs



(b) Impact of reducing delay costs by 50% on the total delays, reroutings, diversions and cancellations

Figure 7 Interactions of delay cost coefficients with reroutings, diversions and cancellations

reducing ground delays only (i.e. 41%). This slight difference is explained by the increase in delays, as reducing ground delay cost increased total delays by 16% compared to 12.6% when varying both ground and air delay costs.

Reducing the cost of ground delay by 50% reduced the need for rerouting by up to 20% and the need for diversions by up to 100% (Figure 7b) in the fully saturated network, i.e. when no further reduction in cancellations can be achieved. Note that network saturation was checked by setting the cost parameters to zero and observing the change in number of cancellations.

For the analyzed instances, we observed the following trade-offs: 1) for each 1% reduction in ground delay cost, total delays increase by 0.33% and the cost of the other decisions decreases by 1%; 2) for each 1% reduction in both ground and air delay costs, total delays increase by 0.23% and the cost of the other decisions decreases by 0.74%; 3) for each 1% reduction in air delay cost (beyond 30%), total delays increase by 0.25% and the cost of the other decisions decreases by 0.61%.

Observation 4: Overtaking cost impacts all network decisions, and a small increase in total cost can significantly increase total network fairness.

We varied the overtaking cost ($C^f_{Rairport} = C^f_{Rsector} =$ €4600) by multiplying it by the following factor $R_{OV} = (1, \frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}, \frac{1}{6}, \frac{1}{7}, \frac{1}{10}, \frac{1}{20}, \frac{1}{50}, \frac{1}{C^f_{Rairport}}, 0)$ in order to understand how network efficiency is affected by fairness between flights.

Figure 8 illustrates the price of fairness (percentage change in total cost) and the improvement in fairness in the network (percentage reduction in total overtaking). Note that the total cost is calculated based on ground and air delays, rerouting, diversion and cancellation costs. The baseline for calculating the price of fairness is the ATFM without overtaking.

Network fairness improved significantly at a very small price (e.g. the solution of $R_{OV} = \frac{1}{50}$ indicated by the red circle in the figure). In other words, an 80%-plus improvement in network fairness can be achieved at the expense of just a 5% increase in total cost for all the instances. The trade-off between improving network fairness and total network cost increase can be derived from Figure 8. For example, an average increase of 2.5% in the total cost will result in a 24.1% improvement in network fairness for instance 1, while for instance 2, an average improvement of 81.7% in network fairness is associated with an average increase of 0.15% in total cost.

Table 6 shows that varying the overtaking costs affects not only ground and air delays but also reroutings, diversions and cancellations. Rerouting, diversion and cancellation decisions are therefore used to reduce overtaking between flights and thus increase network fairness. However, assigning high overtaking costs enhances network fairness but may result in cancelling some flights to reduce overtaking, which might be undesirable (as in instance 6, Table 6). Limiting the overtaking costs to small values will help avoid any possibility of flight cancellations due to fairness.

Observation 5: Airline fairness improvements can potentially be achieved at minimal cost.

For Observation 5, we examined the effect of considering fairness among airlines by trying to balance the total cost per flight for each airline. This analysis was carried out under three fairness coefficients $\Pi = 0$, 1, 1000, which correspond to no fairness, low fairness, and high fairness among airlines. Figure 9 shows the degree of fairness in each instance and for each airline. Note that the

Table 6 Effect of overtaking costs on network decisions

R_{OV}	GD[1]	AD[2]	RF[3]	DF[4]	CF[5]	TO[6]	R_{OV}	GD	AD	RF	DF	CF	TO
Instance 1							Instance 2						
0	73	220	1	0	0	1274	0	567	50	1	2	0	356
$1/C^f_{R_{sector}}$	73	220	1	0	0	1151	$1/C^f_{R_{sector}}$	567	50	1	2	0	83
1/50	48	245	1	0	0	264	1/50	569	50	1	2	0	17
1/20	40	254	1	0	0	188	1/20	570	50	1	2	0	5
1/10	45	256	1	0	0	154	1/10	571	50	1	2	0	0
1/7	57	255	2	0	0	131	1/7	571	50	1	2	0	0
1/6	67	255	3	0	0	110	1/6	571	50	1	2	0	0
1/5	71	255	3	0	0	104	1/5	571	50	1	2	0	0
1/4	85	255	1	1	0	81	1/4	571	50	1	2	0	0
1/3	125	255	1	1	0	41	1/3	571	50	1	2	0	0
1/2	131	256	2	0	0	38	1/2	571	50	1	2	0	0
1	138	261	2	0	0	32	1	571	50	1	2	0	0
Instance 3							Instance 4						
0	1567	55	15	2	19	1883	0	1304	148	6	2	37	949
$1/C^f_{R_{sector}}$	1567	55	15	2	19	1113	$1/C^f_{R_{sector}}$	1304	148	6	2	37	200
1/50	1563	68	15	2	19	373	1/50	1302	150	6	2	37	166
1/20	1589	74	16	1	19	91	1/20	1292	160	6	2	37	93
1/10	1596	78	16	1	19	31	1/10	1283	171	6	2	37	60
1/7	1598	78	16	1	19	26	1/7	1286	171	6	2	37	53
1/6	1604	79	16	1	19	11	1/6	1288	171	6	2	37	49
1/5	1602	81	16	1	19	9	1/5	1288	173	6	2	37	44
1/4	1602	81	16	1	19	9	1/4	1281	196	6	2	37	7
1/3	1606	81	16	1	19	5	1/3	1282	198	6	2	37	3
1/2	1607	84	16	1	19	1	1/2	1282	201	6	2	37	0
1	1607	84	16	1	19	1	1	1282	201	6	2	37	0
Instance 5							Instance 6						
0	1428	111	14	0	0	62680	0	2785	7	6	7	13	8292
$1/C^f_{R_{sector}}$	1428	111	14	0	0	3085	$1/C^f_{R_{sector}}$	2785	7	6	7	13	5583
1/50	1420	142	14	0	0	101	1/50	2757	13	5	7	15	359
1/20	1420	147	14	0	0	17	1/20	2710	16	5	7	16	63
1/10	1418	150	14	0	0	3	1/10	2728	12	5	7	16	14
1/7	1424	147	14	0	0	0	1/7	2739	7	5	7	16	7
1/6	1424	147	14	0	0	0	1/6	2742	7	5	7	16	1
1/5	1424	147	14	0	0	0	1/5	2742	7	5	7	16	1
1/4	1424	147	14	0	0	0	1/4	2742	7	5	7	16	1
1/3	1424	147	14	0	0	0	1/3	2743	7	5	7	16	0
1/2	1424	147	14	0	0	0	1/2	2743	7	5	7	16	0
1	1424	147	14	0	0	0	1	2743	7	5	7	16	0

[1] GD: Total ground delay
[2] AD: Total air delay
[3] RF: Total rerouted flights
[4] DF: Total diverted flights
[5] CF: Total canceled flights
[6] TO: Total overtaking

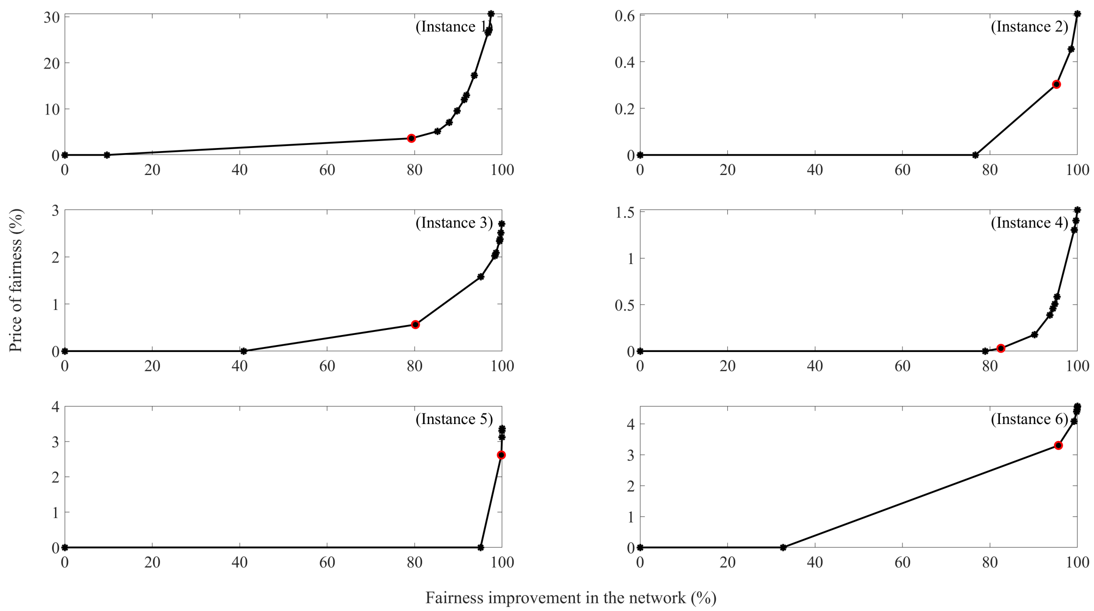


Figure 8 **Effect of changing overtaking costs on total network cost: the red circle (o) represents the solution of** $R_{OV} = 1/50$

degree of fairness (DOF_h) of airline h is calculated as $DOF_h = 1 - |\frac{\mathcal{B}_h - \overline{\mathcal{B}}}{\overline{\mathcal{B}}}|$, where $\mathcal{B}_h$ is the total cost of airline h in Equation (19) and $\overline{\mathcal{B}}$ is the average cost of all airlines in Equation (20). The degree of fairness ranges from 0 (very far from the mean) to 1 (close to the mean). The overall degree of fairness is the average degree of fairness of all airlines, and is reported in Figure 9.

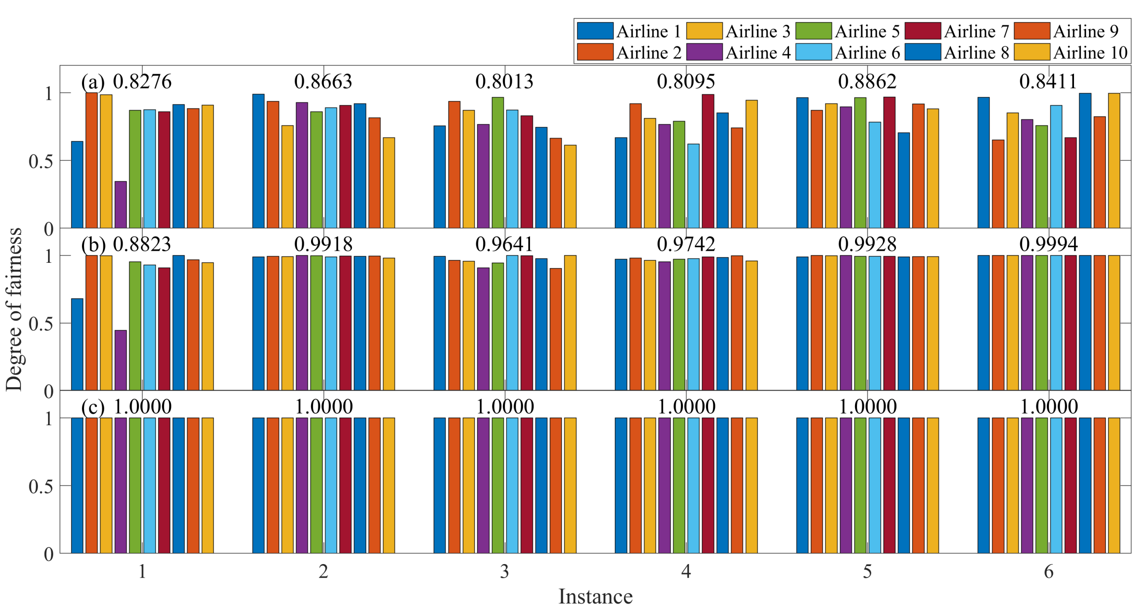


Figure 9 **Degree of fairness among airlines: (a)** $\Pi = 0$**, (b)** $\Pi = 1$**, and (c)** $\Pi = 1000$**. The value above each instance shows the average degree of fairness**

Figure 10 shows the price of fairness (calculated as $100 \times \frac{|C(\Pi=0)-C(\Pi=i)|}{C(\Pi=0)}$) for each fairness coefficient and each instance. The price of fairness is the relative percentage increase in cost associated with moving from an efficient system to a fair one (Bertsimas, Farias, and Trichakis 2012). Note that C represents the optimal value of Equation (1), and $\Pi = i$ represents the values of the fairness coefficients as stated above. The price mark-up required to achieve full fairness ($\Pi = 1000$) ranges between 0.2% and 3% of the total cost of the efficient solution (Figure 10). Moreover, considering airline fairness with a small coefficient ($\Pi = 1$) enhances fairness by an average of 13% at no added cost compared to the case of not considering airline fairness ($\Pi = 0$).

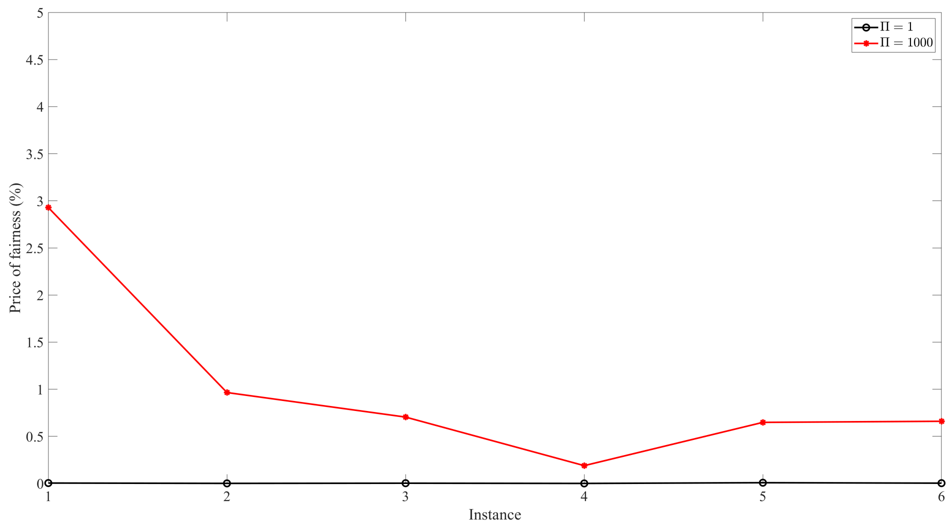


Figure 10 Boundaries of the price of fairness

Observation 6: Reducing the cost of delays and overtaking by 1% increases the costs of rerouting, diversion and cancellation by 1.74%.

As the proposed model suggests having a central authority that can control all decisions, the weights of the two groups of decisions can be used to analyze the impact of changing the decisions of one group on the other group. Here we examine the impact of reducing delays and overtaking on rerouting, diversion and cancellation. We do so by varying the weights π_1 and π_2 in the objective function (1) from 0.05 to 0.95 in increments of 0.05. Note that varying the weights can be seen as changing the costs. For instance, the case of $\pi_1 = 0.75$ and $\pi_2 = 0.25$ corresponds to reducing rerouting, diversion and cancellation costs by $\frac{\pi_2}{\pi_2} = \frac{1}{3}$.

Figure 11 shows the trade-off between the costs of delays and overtaking (x-axis) and the costs of rerouting, diversion and cancellation (y-axis). On average, reducing delays and overtaking by 1% (in terms of cost) increases the costs of rerouting, diversion and cancellation by 1.74%.

Figure 12 shows the impact of varying the importance weight (π_1) on the cost of delays and overtaking, the cost of rerouting, diversion and cancellation, and the total network cost. The lowest

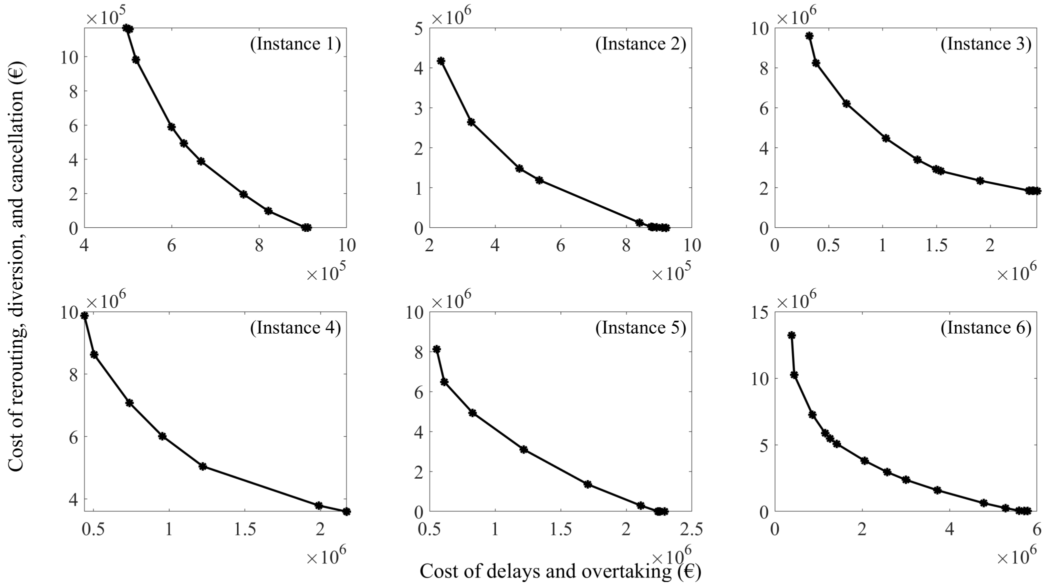


Figure 11 **Trade-off between costs of delays and overtaking and costs of rerouting, diversion and cancellation**

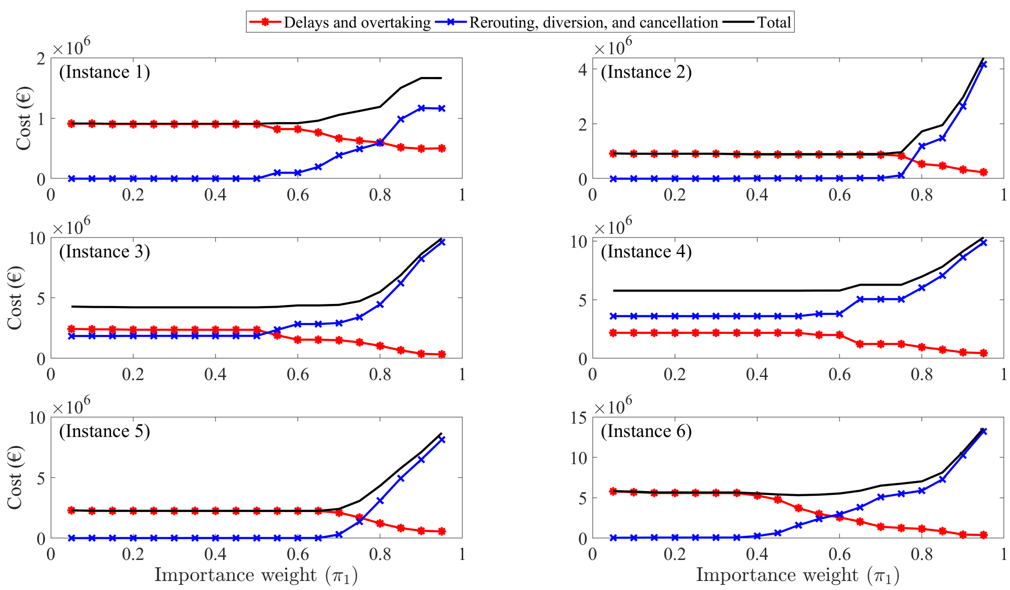


Figure 12 **Impact of changing the importance weight (π_1) on the ATFM decisions cost, airline decisions cost, and total network cost**

total network cost is achieved at $\pi_1 = 0.5$. Note that total network cost increases slightly between $\pi_1 =$ 0.05 and 0.45. This slight increase ranges from 0.5% to 1.8% for instances 1 through 5 and 9% for instance 6 compared to the total cost at $\pi_1 = 0.5$ (the lowest cost). We conclude that the increase in total network cost becomes significant when high importance is given to the delay and overtaking decisions ($\pi_1 > 0.6$), which greatly increases the need for the other decisions (rerouting, diversion and cancellation) to keep the total network cost as low as possible.

8. Conclusions

In this paper, we developed an ATFM model that allows decisions such as rerouting and diversion in addition to air delays and ground holdings. We also considered the en-route flights that already exist in airspace at a given point in time without limiting the model to only the flights scheduled to depart. Moreover, we considered fairness and equity among flights by limiting overtaking, and we considered fairness in delay distribution by using super-linear cost coefficients. A model variant that incorporated airline fairness was formulated. We provided a modified fix-and-relax heuristic to solve large-sized instances. Our numerical experiments showed that this model, with different rerouting options, can serve to significantly reduce the number of canceled flights and total delays by providing more alternatives than ground and air delays, which thus offers decision-makers a more flexible approach. The proposed model suggests having a central authority to control network decisions to increase airspace efficiency. Adopting this model can help achieve better airspace utilization, as it offers decision-makers more decisions to act on and tells them if there is a need to suggest new airspace routes in order to reduce the number of canceled flights when networks are fully saturated. Our experiments revealed that by carefully selecting the overtaking penalty, it is possible to improve inter-flight fairness without significantly increasing system cost. As for inter-airline fairness, we observed that it can be incorporated at a small increase in total network cost. The model revealed that decreasing the cost of delays and overtaking by 1% increases the cost of rerouting, diversion and cancellation by 1.74%. The combined effect of the central authority with the fairness distribution can bring better utilization of the airspace and a fairer distribution of the cost across the stakeholders.

The model is currently limited to deterministic capacities. Future research could consider weather effects on capacities. Further directions for research would be to optimize the airspace configuration in the ATFM context, and to pursue the development and analysis of a mechanism to manage and control the decisions under the central authority approach. Further investigations are needed to consider technical and administrative constraints.

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