P4BID: Information Flow Control in P4

Karuna Grewal
Cornell University
USA

Loris D'Antoni
University of Wisconsin
USA

Justin Hsu
Cornell University
USA

Abstract

Modern programmable network switches can implement custom applications using efficient packet processing hardware, and the programming language P4 provides high-level constructs to program such switches. The increase in speed and programmability has inspired research in *dataplane programming*, where many complex functionalities, e.g., key-value stores and load balancers, can be implemented entirely in network switches. However, dataplane programs may suffer from novel security errors that are not traditionally found in network switches.

To address this issue, we present a new information-flow control type system for P4. We formalize our type system in a recently-proposed core version of P4, and we prove a soundness theorem: well-typed programs satisfy non-interference. We also implement our type system in a tool, P4BID, which extends the type checker in the P4C compiler, the reference compiler for the latest version of P4. We present several case studies showing that natural security, integrity, and isolation properties in networks can be captured by non-interference, and our type system can detect violations of these properties while certifying correct programs.

CCS Concepts: **• Security and privacy → Information flow control**; *Logic and verification*; *Network security*.

Keywords: Information-flow control, programmable networks

1 Introduction

The last two decades have seen an ongoing shift in how networks are programmed. The task of programming a network once consisted of manually setting configurations in specialized switch hardware that provided limited customization; low-level programming was the only way to achieve performance. Today, switches are highly programmable and provide rich functionalities for processing network packets. This increased programmability is enabling complex network functionalities, which traditionally run on slower dedicated devices, to run directly on switches and other networking hardware [18, 30]. Furthermore, new programming models and languages make it easier for network operators to define complex functionalities [6].

While the advent of programmable network switches has inspired a large number of practitioners and researchers to write complex functionalities that can run on switches, it has also brought a new level of complexity in a world where bugs can be costly. As is well known, network configuration errors have led to widespread and costly outages (e.g., [13, 33]). The problem of preventing these, and other, types of bugs has received a lot of attention in the programming languages and verification communities. For example, researchers have developed formal tools for verifying that switch configurations guarantee desirable network properties, such as node reachability, the absence of black holes, and resilience to link failures (e.g., [1, 2, 32]). While these tools are extremely useful for network operators, applications running on programmable switches may exhibit errors that are not traditionally associated with networks. In particular, there has been little work on verifying security properties for dataplane programs.

Our work. We develop a new information-flow control (IFC) type system for the network programming language P4 [6], a leading language for programming network switches. P4 is an attractive target: it is actively developed by researchers from academia and industry, and can compile to a variety of networking hardware. *Information flow control* (IFC) is a well-studied, language-based approach to verifying security properties where variables in the program are tagged with security labels, and the type system ensures that no information can flow from high-security variables (secret) to low-security ones (public). IFC is (i) *flexible*: by changing the label usage one can model security properties, like confidentiality and integrity; (ii) *general*: it can accommodate complex programming constructs; and (iii) *lightweight*: the analysis is simple, type-based, and requires minimal annotations from the programmer. Owing to these strengths, IFC has found wide adoption and has been deployed in real languages [23, 26].

Designing an IFC type system for P4 involves both technical and conceptual challenges. On the technical side, while P4 resembles a standard imperative language, it has a number of features to target the restricted computational model of networking switches. For instance, much of the computation in P4 programs happens via *tables*, which match on data in packet headers and select which actions to run. While a P4 program implements the actions, the table itself is not known until it is installed at runtime by the network controller. A second technical challenge is the size and complexity of the language. Like many languages in real-world use, P4 does not have a formal specification. To firm up the foundations of P4, Doenges et al. [10] developed a formal version of P4, called Core P4, as part of the broader PETR4 project. The formal operational model of Core P4 makes it possible to develop type systems that provably guarantee program properties. However, Core P4 is still quite large—P4 is a language intended for real-world use, with a wide variety of declarations, statements, and expressions, and Core P4 models almost all the features of P4. Our work develops an IFC system that can handle the principal features of Core P4.

On the conceptual side, IFC for dataplane programming has been little-studied and it is not know what useful properties network properties an IFC system can enforce. As part of our work, we present case studies showing that standard properties guaranteed by IFC, like *confidentiality* and *integrity*, are useful security properties for networking applications. We also show how natural network *isolation* properties can also be guaranteed with an IFC system, by adjusting the lattice of security labels.

Outline. After overviewing our approach in Section 2 and providing the necessary background on P4 and Core P4 in Section 3, we present our central contributions:

1. An *Information Flow Control (IFC) type system for Core P4* [10], a core calculus modeling the P4 language, together with a soundness theorem: well-typed programs satisfy *non-interference* (Section 4).
2. P4BID: a *type-checker* implemented on top of p4c, the reference compiler for P4. We evaluate our system through four case studies, demonstrating how properties enforced by IFC, like confidentiality and integrity, can be useful in a networking context. We implement our case studies in P4 and show that P4BID can automatically detect when these properties are violated, while correctly type-checking versions of these programs where the problems are removed (Section 5).

We conclude by surveying related work (Section 6) and outlining possible future directions (Section 7).

2 Overview

A quick introduction to P4. P4 is an actively-developed language for programming the network data plane. Computation is divided into three phases: *parser*, *pipeline*, and *deparser*. The packet processing starts at the parser, where the input packet is extracted into a typed representation given by headers using a finite state machine. The pipeline phase executes the primary logic of the switch by transforming the parsed representation of the input packet. The deparser serializes the parsed typed representation of the input packet into the output packet. Our work focuses on P4 *control blocks*, which implement the pipeline phase. To get a feel for the language, we consider a P4 program for a basic task: converting virtual addresses to physical addresses when packets enter a local network. Listing 1 begins by declaring the types of the *headers* which are carried by packets; P4 programs manipulate the state of packets by modifying the headers. In our case, there are three headers: ipv4 and ethernet carry the routing information in the original packet, while local_hdr carries information specific to the local network.

Listing 1 shows the code for the control block, which implements the core part of the logic. (The full P4 program also describes other stages of the packet-processing pipeline like parsing and deparsing, which we do not consider in our work.) The switch behavior is organized into *tables* and *actions*. Tables match data in headers (the *keys*) and apply actions. For instance, the table ipv4_lpm_forward inspects the value of the header hdr.ipv4.dstAddr and then decides whether to run action ipv4_forward or drop the packet. The concrete mapping is not specified by the P4 program; instead, the switch controller installs these mappings at runtime. Actions can inspect and modify packet headers. Actions can also be parameterized by arguments, which are supplied by the table when the action is applied. For example, the action ipv4_forward accepts a destination address and port as arguments, and then proceeds to update headers. Finally, the apply block specifies the overall behavior of the control block: here, the switch applies table virt2phys to translate virtual addresses to physical addresses, and then ipv4_lpm_forward to forward the packet.

A potential security vulnerability. Listing 1 is designed to process a packet as it enters a local network. The incoming packet refers to a *virtual address*, which must be translated to a physical address. Furthermore, the switch adjusts other packet fields, like the maximum number of hops (time-to-live, ttl), to reflect the topology of the local network. To preserve privacy, details of the local network should not leak into fields that are visible when the packet leaves the network. To accomplish this goal, the program uses a separate header of type local_hdr_t to store local information (Line 1). As the packet is routed in the local network, the switches do not touch the public ipv4 and ethernet headers; instead, they parse local_hdr and update it with the next hop route information. When the packet exits the local network, the header local_hdr is removed.

While the intended behavior is simple to describe, the program in Listing 1 has an error: Line 34 incorrectly stores the

Listing 1. Translating virtual to physical addresses.

```
1   header local_hdr_t {
2       bit<32> phys_dstAddr;
3       bit<8> phys_ttl;
4       bit<48> next_hop_MAC_addr;
5   }
6
7   header ipv4_t {
8       bit<8> ttl;
9       bit<8> protocol;
10      bit<32> srcAddr;
11      bit<32> dstAddr;
12  }
13
14  header eth_t {
15    bit<48> srcAddr;
16    bit<48> dstAddr;
17  }
18
19  struct headers {
20    ipv4_t ipv4;
21    eth_t eth;
22    local_hdr_t local_hdr;
23  }
24
25  control Obfuscate_Ingress(inout headers hdr,
26          inout standard_metadata_t std_metadata) {
27      table virtual2phys_topology {
28          key = { hdr.ipv4.dstAddr: exact; }
29          actions = { update_to_phys; }
30      }
31      action update_to_phys(bit<32> phys_dstAddr,
32                            bit<8> phys_ttl) {
33          hdr.local_hdr.phys_dstAddr = phys_dstAddr;
34          hdr.ipv4.ttl = phys_ttl;
35      }
36      table ipv4_lpm_forward {
37          key = { hdr.ipv4.dstAddr: lpm; }
38          actions = { ipv4_forward; drop; }
39      }
40      action ipv4_forward(bit<48> dstAddr, bit<9> port) {
41          hdr.eth.dstAddr = dstAddr;
42          standard_metadata.egress_spec = port;
43      }
44      action drop() { mark_to_drop(standard_metadata); }
45      apply {
46          virtual2phys_topology.apply();
47          ipv4_lpm_forward.apply();
48      }
49  }
```

Listing 2. Security-Annotated Version of Listing 1

```
1   header local_hdr_t {
2       <bit<32>, high> phys_dstAddr;
3       <bit<8>, high> phys_ttl;
4       // ...
5   }
6
7   header ipv4_t {
8       <bit<8>, low> ttl;
9       // ...
10  }
11
12  struct headers {
13    ipv4_t ipv4;
14    local_hdr_t local_hdr;
15    // ...
16  }
17
18  control Obfuscate_Ingress(inout headers hdr,
19          inout standard_metadata_t std_metadata) {
20      action update_to_phys(<bit<32>, high> phys_dstAddr,
21                            <bit<8>, high> phys_ttl) {
22          hdr.local_hdr.phys_dstAddr = phys_dstAddr;
23          // !BUG!: low <- high
24          hdr.ipv4.ttl = phys_ttl;
25          // *FIX*: high <- high
26          hdr.local_hdr.phys_ttl = phys_ttl;
27      }
28      // ...
29  }
```

local ttl in the ipv4 header, rather than the local_hdr header. Even when the local header is removed, the ipv4 header will carry private information about the local network. This kind of error unintentionally leaks local information into public headers, but it can be easy to overlook.

Security types to the rescue. We design an information-flow control type system for P4 to catch such bugs. Like standard IFC type systems, our system extends each P4 type with a *security label*: high if the data is secret, and low if the data is public. Listing 2 shows our example program annotated with security types. All data specific to the local network (e.g., phys_dstAddr, phys_ttl) are marked as *high* security. The publicly visible headers (e.g., ipv4, eth) are marked as *low* security. Our type system guarantees that information from high-security data does not influence low-security data. For instance, the information leak we saw before can be flagged in our type system: Line 24 incorrectly assigns a high-security data phys_ttl to a low-security field ipv4.ttl. The problem is corrected by assigning phys_ttl to local_hdr.ttl (Line 26), which is a high-security field.

While this kind of analysis is fairly straightforward, the design of our type system must handle unusual features from P4's programming model (e.g., actions and tables); we discuss these aspects in Section 3 and Section 4. Furthermore, while Listing 1 demonstrates a basic information leak, we will see more interesting applications of our type system to networking applications in Section 5.

3 Syntax and Semantics of Core P4

This section briefly reviews the core P4 calculus presented in the recent work on PETR4 [10], the representation of P4 programs in terms of the core calculus syntax, and the operational semantics and typing judgements for the core calculus.

3.1 Core P4 Syntax

PETR4 formalizes the semantics of various P4 primitives, like control blocks, match-action tables, and statements in a calculus called Core P4. For our information-flow control type system, we focus on the fragment of Core P4 in Figure 1. Expressions and statements are largely standard.

Core P4 programs (*prg*) are represented as a sequence of variable, object, or type declarations followed by a control block. The central construct in a P4 program is the control block, which describes how the switch processes packets in terms of table and action calls inside its apply block. A control block body (*ctrl_body*) is a sequence of declarations and statements. The *stmt* in the control block corresponds to the apply block of a P4 program.

Variable and type declarations (*var_decl*, *typ_decl*) are largely standard; the match_kind enum declares different ways tables can match on packet fields. Object declarations (*obj_decl*) declare P4 objects: tables and actions. These object declarations can have nested ordinary statements (*stmt*) that allow usual imperative primitives like mutation and control flow statements. To get a feel for these features, let's consider how they correspond to parts of the Obfuscate_Ingress control block in Listing 1. The example control block consists of three actions declarations (update_to_phys, ipv4_forward, and drop), and two table declarations (virtual2phys_topology and ipv4_lpm_forward).

Tables. A table declaration, table $x \{\overline{key}\ \overline{act}\}$, is composed of a list of expressions (usually packet header fields) that specify the lookup key, $\overline{key}$, and actions, $\overline{act}$, which the lookup table might execute. A table application uses the key to lookup the entries in the table (installed by the control plane) and invokes the action from the matched entry. For example, table virtual2phys_topology in Line 27 contains the key hdr.ipv4.dstAddr: exact (where exact specifies the match pattern, in this case, exact match on the key), and the action update_to_phys action. Applying this table, represented in Core P4 as virtual2phys_topology(), matches the table entries installed by the control plane against the corresponding keys in the current packet and returns an appropriate action to run, with all its arguments. Any optional arguments in the returned action will be supplied by the control plane. The match pattern determines the criterion for choosing a table entry based on the key. For instance, lpm specifies that a key is matched to the entry corresponding to its longest prefix; exact specifies that a key should be exactly matched to some table entry otherwise it is a match failure.

Actions. An action declaration is a special case of a function declaration, function $\tau_{ret}\ x\ (\overline{d\ y : \tau})\{stmt\}$, with no return type. For example, the action update_to_phys on Line 32 in Listing 1 has parameters phys_dstAddr and phys_ttl, of types bit⟨32⟩ and bit⟨8⟩. Parameters can have a *directionality*, d: an *in* expression can only be read from, while an *inout* expression can be both read and written to. Omitted directions in parameters default to the *in* direction; these directionless parameters are optional arguments that can be passed by the control plane. Invoking the action, which can be done directly as a statement or indirectly from a table, runs the statement *stmt* in the action body. Actions, like all Core P4 functions, do not support recursion.

Differences compared to Core P4. The language in Figure 1 is a significant fragment of Core P4, but it does not handle some of its more specialized features (e.g., generics, constant declarations, slice operation, and native functions). We consider this fragment for simplicity, but we do not foresee difficulties in extending our IFC analysis to full Core P4. We omitted some lesser-used features, like generics, because the core language is already quite large and we believe it is unlikely that omitted features lead to information-flow violations. We focus on programs with a single control block because most P4 programs encode their main functionality in a single ingress control block. Since our system already supports user-defined functions and closures, with all of their technical intricacies, we do not see any obstacle to handling multiple control blocks besides increasing the complexity of our type system.

3.2 Core P4 Semantics

To understand the semantics of Core P4 programs, we will review the evaluation judgement forms for expressions, statements, and declarations from PETR4 [10]. The main judgements are as follows:

$$\langle C, \Delta, \mu, \epsilon, exp \rangle \Downarrow \langle \mu', val \rangle$$
$$\langle C, \Delta, \mu, \epsilon, stmt \rangle \Downarrow \langle \mu', \epsilon', sig \rangle$$
$$\langle C, \Delta, \mu, \epsilon, decl \rangle \Downarrow \langle \Delta', \mu', \epsilon', sig \rangle$$

The contexts used in these judgements are defined in Figure 2. Here, Δ is the partial map from type names to types; ϵ is the partial map between variables and their memory locations; μ is the memory store mapping variable locations to their values. C models the table lookup map provided by the control plane: given a table at location l with $key = val$, and a list of actions described by a list of *PartialActionRef* (actions with optional arguments missing), C returns an action call expression with all the optional arguments of the action supplied (*ActionRef*). The judgements use *val* to denote a value; and *sig* to denote a *signal*, which indicates whether the program's control flow proceeds normally (cont), returns a value (return *val*), or errors (exit).

$$
\begin{array}{llll}
exp & ::= & b & \text{Boolean} \\
& | & n_w & \text{integers or bits of width w} \\
& | & x & \text{variable} \\
& | & exp_1[exp_2] & \text{array indexing} \\
& | & exp_1 \oplus exp_2 & \text{binary operation} \\
& | & \overline{\{f_i = exp_i\}} & \text{record} \\
& | & exp.f_i & \text{field projection} \\
& | & exp_1(\overline{exp_2}) & \text{function call} \\
\end{array}
$$

(a) Expressions

$$
\begin{array}{llll}
stmt & ::= & exp_1(\overline{exp_2}) & \text{function call} \\
& | & exp_1 := exp_2 & \text{assignment} \\
& | & \text{if } (exp_1) \; stmt_1 \text{ else } stmt_2 & \text{conditional} \\
& | & \overline{\{stmt\}} & \text{sequencing} \\
& | & \text{exit} & \text{exit} \\
& | & \text{return } exp & \text{return} \\
& | & var\_decl & \text{variable declaration} \\
\end{array}
$$

(b) Statements

$$
\begin{array}{lll}
prg & ::= & \overline{typ\_decl} \; ctrl\_body \\
ctrl\_body & ::= & \overline{decl} \; stmt \\
decl & ::= & var\_decl \mid obj\_decl \mid typ\_decl \\
var\_decl & ::= & \tau \; x := exp \mid \tau \; x \\
typ\_decl & ::= & \text{match\_kind } \{\overline{f}\} \mid \text{typedef } \tau \; X \\
obj\_decl & ::= & \text{table } x \; \{\overline{key} \; \overline{act}\} \\
& | & \text{function } \tau_{ret} \; x \; (\overline{d \; y : \tau})\{stmt\} \\
\end{array}
$$

(c) Declarations

$$
\begin{array}{lll}
d & ::= & in \mid inout \\
lval & ::= & x \\
& | & lval.f \\
& | & lval[n] \\
key & ::= & exp : x \\
act & ::= & x(\overline{exp}, \overline{x : \tau}) \\
\end{array}
$$

(d) Other constructs

Figure 1. Core P4 Expressions (fragment)

$$
\begin{array}{ll}
Var & : \text{variables} \\
TypVar & : \text{type variables} \\
Loc & : \text{locations} \\
\end{array}
\qquad
\begin{array}{ll}
Val & : \text{values} \\
Typ & : \text{types in Core P4} \\
\end{array}
$$

$$
\begin{array}{ll}
\Gamma & : Var \to Typ \\
\epsilon & : Var \to Loc \\
C & : Loc \times Val \times \overline{PartialActionRef} \to ActionRef \\
\end{array}
\qquad
\begin{array}{ll}
\Delta & : TypVar \to Typ \\
\mu & : Loc \to Val \\
\end{array}
$$

Figure 2. Typing and Evaluation Contexts

$$
\begin{array}{lll}
\rho & ::= & bool \mid int \mid bit\langle n\rangle \mid unit \\
& | & \{\overline{f : \rho}\} \mid header\{\overline{f : \rho}\} \mid \rho[n] \\
& | & match\_kind\{\overline{f}\} \\
\kappa & ::= & \rho \mid table \mid \overline{d \; \kappa} \to \kappa \\
\end{array}
$$

Figure 3. Core P4 types

Since function calls are expressions, and a function's body can update the memory store, the evaluation judgement for expressions can modify the memory store. Similarly, the statement evaluation judgement captures the updated memory store from evaluating a statement with side-effects and the environment extension on declaring a new variable. A declaration evaluation can reduce to a new memory store and environment when evaluating a variable or object declaration. Additionally, a declaration statement can update the type definition context by introducing a new type alias. Both declarations and statements evaluate to a signal *sig*, representing the result of the control flow in their sequencing blocks.

3.3 Core P4 Type System

Figure 3 recalls the types from Core P4. Core P4 divides the P4 types into two categories: base types, ρ, and general types, κ. The fields of headers and records must be base types. The simplified Core P4 typing judgements for the fragment of

Core P4 presented in Figure 1 are as follows:

$$\Gamma, \Delta \vdash exp : \kappa \; goes \; d \quad \Gamma, \Delta \vdash stmt \dashv \Gamma' \quad \Gamma, \Delta \vdash decl \dashv \Gamma', \Delta'$$

The expression typing judgement associates a directionality with expressions to indicate if the expression is read only (in) or is both readable and writable (inout). Intuitively, the contexts on the left of $\vdash$ in the statement and declaration typing rule describe the contexts before their execution, while the contexts on the right of $\dashv$ define the context after the execution of the statement and declaration.[1]

4 IFC Type System for P4

This section presents the security-type extension for the Core P4 fragment presented in Figure 1. Before presenting the security-types for our fragment of Core P4, we describe the main idea behind security type systems.

4.1 Background on Security Type Systems

A security type system lifts ordinary types to security types by annotating them with security labels [27]. These security labels are drawn from a security lattice, $(\mathbb{L}, \sqsubseteq)$, associated

---

[1]The original Core P4 typing judgements also have a constant store, to model compile-time constants. We omit this store since our fragment does not include compile-time constants.

with the type system. We illustrate the key ideas using a simple two point lattice {low, high}. Here, low identifies publicly visible values and high represents secure values, and low $\sqsubseteq$ high.

Consider a well-typed closed expression *exp* with type τ, represented by an ordinary type system as $\vdash exp : \tau$. A security-type system will additionally assign a security label, $\chi \in \mathbb{L}$ to *exp*. This can be represented by the typing judgement $\vdash exp : \langle \tau, \chi \rangle$, where the pair $\langle \tau, \chi \rangle$ is the *security type*. For instance, if *exp* evaluates to *val* and χ = high, then *val* is considered to be a secure value.

For statements (or expressions) that can mutate variables, a security type system assigns a security label $pc \in \mathbb{L}$ to the typing judgements. This label denotes the security context used to track the security level for variables that can be written at a given program point (*program counter*). Consider a conditional statement that branches on a high security guard expression:

$$\text{if } (h == 1) \{ h := set\_high(); \} \text{ else } \{ h := 1; \},$$

where the security level of h is high and the *set_high* function call in the true branch writes to only high security variables. Since the guard is at high security level, the pc for both the conditional branches becomes high. Here, both branches need to be well-typed under the high security label, which implies that no variable at security level lower than high can be mutated in either branch. For instance, we must have $\Gamma \vdash_{\text{high}} h := set\_high()$ and $\Gamma \vdash_{\text{high}} (h := 1)$. Without this restriction, there can be an implicit flow of information from the conditional guard into the statement blocks of the conditional, for instance, if the function wrote to a low variable.

The utility of a security-type system lies in the *non-interference* guarantee offered by a well-typed program. To define non-interference, suppose that all low security variables are observable while any high security variable is unobservable. Informally, non-interference can be understood as the property of a program where no unobservable input variable influences the value of any observable output.

4.2 P4 IFC Type System

This section describes our information-flow control type system for the language in Figure 1. We assume the lattice $(\mathbb{L}, \sqsubseteq)$ of security labels has $\top$ and $\bot$ elements, representing the top and bottom elements of the lattice. In our example lattice, $\bot$ = low and $\top$ = high.

Figure 4 summarizes the security types of our information-flow control system. Core P4 types are lifted to security types using a security label, χ, from the lattice $\mathbb{L}$. We also use pc to denote a security label when it is used as a security context. As in Core P4, we distinguish between base security types ρ and general security types κ. For non-base types, the security label is tracked within the type itself, for instance, the fields of headers and records are assigned security labels instead of the header or record. But to keep the shape of

$$
\begin{array}{lll}
\rho & ::= & \langle bool, \chi \rangle \mid \langle int, \chi \rangle \mid \langle bit\langle n \rangle, \chi \rangle \mid \langle unit, \bot \rangle \\
& \mid & \langle \{\overline{f : \rho}\}, \bot \rangle \mid \langle header\{\overline{f : \rho}\}, \bot \rangle \mid \langle \rho[n], \bot \rangle \\
& \mid & \langle match\_kind\{\overline{f}\}, \bot \rangle \\
\kappa & ::= & \rho \mid \langle table(pc_{tbl}), \bot \rangle \mid \langle \overline{d\ \rho} \xrightarrow{pc} \rho_{ret}, \bot \rangle \\
\tau & ::= & bool \mid int \mid bit\langle n \rangle \mid unit \\
& \mid & \{\overline{f : \rho}\} \mid header\{\overline{f : \rho}\} \\
& \mid & \rho[n] \mid match\_kind\{\overline{f}\} \\
& \mid & table(pc_{tbl}) \mid \overline{d\ \rho} \xrightarrow{pc} \rho_{ret}
\end{array}
$$

Figure 4. IFC Types

types uniform, we assign the $\bot$ security label for such types. We use the metavariable τ to denote a security type without its outer-most security label; thus, security types are of the form $\langle \tau, \chi \rangle$.

Before describing the judgement forms of the security type system, we introduce the contexts used in the typing judgements. The typing judgements use a typing context, Γ, a type definition context, Δ, and a security context, pc, which are same as Core P4's contexts Figure 2, with the difference that now *Typ* is the set of security types of the form $\langle \tau, \chi \rangle$.

For a given security label pc, variables in a typing context Γ at security level $\chi \sqsubseteq pc$ will be referred as *below-pc* variables, and variables at security level $\chi \not\sqsubseteq pc$ will be referred as *not below-pc* (or sometimes *above-pc*) variables.

Our security type system has three forms of judgements for expressions, statements, and declarations, respectively:

$$\textbf{Expressions} : \Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi \rangle \text{ goes } d$$

$$\textbf{Statements} : \Gamma, \Delta \vdash_{pc} stmt \dashv \Gamma'$$

$$\textbf{Declarations} : \Gamma, \Delta \vdash_{pc} decl \dashv \Gamma', \Delta'$$

The direction annotation *goes d* in the typing judgement for expressions is dropped when the direction is not important. The complete security typing rules can be found in Figure 5 (expressions), Figure 6 (statements), and Figure 7 (declarations). Expression typing assumes a typing oracle $\mathcal{T}$, giving the meaning of the binary operations. In statement and declaration typing, the judgement $\Delta \vdash \tau \rightsquigarrow \tau'$ converts τ to a base type by unfolding type definitions [10]. Below, we discuss the most interesting—and technically intricate—typing rules: those for functions, tables, and subtyping.

Typing rules for functions. Our system has rules for function declarations and function calls. These are also the key rules for typing actions, which are functions with no return type. The T-FnDecl rule in Figure 7 typechecks the body of the function to eliminate any leaks in the function body. The pc_{fn} security label on the function's arrow type records the lower bound on the security labels of the variables that the function mutates. For instance, in the following function:

$$\text{function insecure}()\{l := 1; h := 2; \},$$

where the security labels of l and h variable are low and high respectively, pc_{fn} will be low. The T-FnCall rule in Figure 5 enforces that a function will not be invoked in a context that is higher than the function's pc_{fn} because doing so, for instance in the example program, will implicitly flow information from a high guard expression into a low variable.

Typing rules for tables. Since a table matches on the key to select an action to invoke, the key of a table resembles the guard of a conditional. Thus, the value of a key can implicitly leak in the action's body if the invoked action writes to variables at security label lower that that of the key expression. Therefore, to declare a table of type $\langle table(pc_{tbl}), \bot \rangle$, the rule T-TblDecl in Figure 7 ensures that the security label of the most secure key, χ_k, is lower than the label of the least secure assignment, pc_a, in any action. Here, pc_{tbl} records the lower bound on the write effects associated with any keys, actions, or arguments.

The T-TblCall rule in Figure 6 prevents any implicit flow into any of the actions that a table might invoke by allowing a table to be applied only in a pc context lower than the least secure write effect associated with the table application, pc_{tbl}. This prevents implicit leaks during the evaluation of keys, arguments, or the action's body.

Subtyping rule. The T-SubType-In rule in Figure 5 allows only read-only (*in*) expressions to increase their security label. It is not safe to allow *inout* expressions to be subtyped. To see why, consider the following function:

write_to_high (*inout* h : $\langle bool, high \rangle$) {h := *true*; }

Suppose we have a low variable $l : \langle bool, low \rangle$. Since variables are *inout* expressions (T-Var in Figure 5), if *inout* expressions were allowed to increase their label, write_to_high(l) call would have been valid. In this case, the function would have written to a low variable when it should have operated with only a high variable.

4.3 Non-Interference

To define non-interference, consider two program states, $\langle C, \Delta, \mu_a, \epsilon_a \rangle$ and $\langle C, \Delta, \mu_b, \epsilon_b \rangle$, where the environments have equal domains. Suppose every below-pc variable x has equal value under both the memory stores, $\mu_a(\epsilon_a(x)) = \mu_b(\epsilon_b(x))$, but the value of any variables that are not below-pc can differ between the two stores. Non-interference is satisfied if evaluating an expression, statement, or declaration in the two program states results in two final program states that agree on below-pc variables.

The following definition formally describes a pair of below-pc equivalent memory stores and environments. The store typing context Ξ maps locations in a store to security types.

Definition 4.1. Consider two pairs of memory stores and environments $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$. Then

$$\Xi_a, \Xi_b, \Delta \models_{pc} \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$$

is satisfied when

$$\Xi_a, \Delta \models \langle \mu_a, \epsilon_a \rangle : \Gamma \quad \text{and} \quad \Xi_b, \Delta \models \langle \mu_b, \epsilon_b \rangle : \Gamma$$

and every below-pc variable x in ϵ_a and ϵ_b has equal value i.e., $\mu_a(\epsilon_a(x)) = \mu_b(\epsilon_b(x))$.

Intuitively, $\Xi, \Delta \models \langle \mu, \epsilon \rangle : \Gamma$ states that the store and environment are well-typed: recalling that the location of every variable is described by the environment ϵ and the value at valid locations is described by the memory store μ, the type assigned to a variable using the store typing Ξ must be the same as the type assigned by the typing context Γ. The formal definition for this relation is provided in Definition C.4.

The following definition of non-interference for statements requires that evaluating a statement under below-pc equivalent pairs of memory stores and environment can only reduce to pairs of final memory stores and environments that are below-pc equivalent. Technically, this is a *termination insensitive* notion of non-interference, since it does not require that both executions terminate. However, P4 programs do not allow recursion and Doenges et al. [10] prove that all well-typed Core P4 programs terminate.

Definition 4.2 (Non-interference for statements). For any security lable l, $\Gamma, \Delta \models_{pc} NI(stmt) \dashv \Gamma'$ holds for any Ξ_a, Ξ_b, $\mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b, \epsilon'_a, \epsilon'_b$ if whenever

1. $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$,
2. $\langle C; \Delta; \mu_a; \epsilon_a; stmt \rangle \Downarrow \langle \mu'_a; \epsilon'_a; sig_1 \rangle$,
3. $\langle C; \Delta; \mu_b; \epsilon_b; stmt \rangle \Downarrow \langle \mu'_b; \epsilon'_b; sig_2 \rangle$

then there exists Ξ'_a, Ξ'_b, such that

1. $\Gamma, \Delta \vdash_{pc} stmt \dashv \Gamma'$,
2. $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon'_a \rangle \langle \mu'_b, \epsilon'_b \rangle : \Gamma'$,
3. $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$,
4. for any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$,
5. for any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, we have $\mu'_a(l_a) = \mu_a(l_a)$,
6. for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, we have $\mu'_b(l_b) = \mu_b(l_b)$,
7. one of the following holds:
   - $sig_1 = sig_2 = cont$; or
   - $sig_1 = sig_2 = exit$; or
   - $sig_1 = $ return val_1 and $sig_2 = $ return val_2 such that $\Xi'_a, \Xi'_b, \Delta \models_l NI(val_1, val_2) : \langle \tau'_{ret}, \chi_{ret} \rangle$, where $\Delta \vdash \tau_{ret} \rightsquigarrow \tau'_{ret}$ and $\Gamma[\text{return}] = \langle \tau_{ret}, \chi_{ret} \rangle$,
8. we have the inclusions:
   - $\Xi_a \subseteq \Xi'_a$ and $\Xi_b \subseteq \Xi'_b$;
   - $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$ and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$; and
   - $\text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon'_a)$ and $\text{DOM}(\epsilon_b) \subseteq \text{DOM}(\epsilon'_b)$.

We present similar non-interference definitions for expressions and declarations in Definition C.5 and Definition C.10.

$$\frac{\Gamma, \Delta \vdash_{pc'} exp : \langle \tau, \chi \rangle \qquad pc \sqsubseteq pc'}{\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi \rangle} \text{ T-Subtype-PC} \qquad \frac{\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi \rangle \text{ goes in} \qquad \chi \sqsubseteq \chi'}{\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi' \rangle \text{ goes in}} \text{ T-SubType-In}$$

$$\frac{}{\Gamma, \Delta \vdash_{pc} b : \langle bool, \bot \rangle \text{ goes in}} \text{ T-Bool} \qquad \frac{}{\Gamma, \Delta \vdash_{pc} n_\infty : \langle int, \bot \rangle \text{ goes in}} \text{ T-Int} \qquad \frac{x \in \text{dom}(\Gamma) \qquad \Gamma(x) = \langle \tau, \chi \rangle}{\Gamma, \Delta \vdash_{pc} x : \langle \tau, \chi \rangle \text{ goes inout}} \text{ T-Var}$$

$$\frac{\begin{array}{cc} \Gamma, \Delta \vdash_{pc} exp_1 : \langle \rho_1, \chi_1 \rangle & \Gamma, \Delta \vdash_{pc} exp_2 : \langle \rho_2, \chi_2 \rangle \\ \mathcal{T}(\Delta; \oplus; \rho_1; \rho_2) = \rho_3 \quad \chi_1 \sqsubseteq \chi' \quad \chi_2 \sqsubseteq \chi' \end{array}}{\Gamma, \Delta \vdash_{pc} exp_1 \oplus exp_2 : \langle \rho_3, \chi' \rangle \text{ goes in}} \text{ T-BinOP} \qquad \frac{\Gamma, \Delta \vdash_{pc} \overline{\{exp : \langle \tau_i, \chi_i \rangle\}}}{\Gamma, \Delta \vdash_{pc} \{\overline{f : exp}\} : \langle \{\overline{f : \langle \tau_i, \chi_i \rangle}\}, \bot \rangle \text{ goes in}} \text{ T-Rec}$$

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle \{\overline{f_i : \langle \tau_i, \chi_i \rangle}\}, \bot \rangle \text{ goes } d}{\Gamma, \Delta \vdash_{pc} exp.f_i : \langle \tau_i, \chi_i \rangle \text{ goes } d} \text{ T-MemRec} \qquad \frac{\begin{array}{c} \Gamma, \Delta \vdash_{pc} exp_1 : \langle \langle \tau, \chi_1 \rangle[n], \bot \rangle \text{ goes } d \\ \Gamma, \Delta \vdash_{pc} exp_2 : \langle bit\langle 32 \rangle, \chi_2 \rangle \quad \chi_2 \sqsubseteq \chi_1 \end{array}}{\Gamma, \Delta \vdash_{pc} exp_1[exp_2] : \langle \tau, \chi_1 \rangle \text{ goes } d} \text{ T-Index}$$

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle header\{\overline{f_i : \langle \tau_i, \chi_i \rangle}\}, \bot \rangle \text{ goes } d}{\Gamma, \Delta \vdash_{pc} exp.f_i : \langle \tau_i, \chi_i \rangle \text{ goes } d} \text{ T-MemHdr} \qquad \frac{\begin{array}{c} \Gamma, \Delta \vdash_{pc} exp_1 : \langle \overline{d \langle \tau_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle \\ \Gamma, \Delta \vdash_{pc} \overline{exp_2 : \langle \tau_i, \chi_i \rangle \text{ goes } d} \quad pc \sqsubseteq pc_{fn} \end{array}}{\Gamma, \Delta \vdash_{pc} exp_1(\overline{exp_2}) : \langle \tau_{ret}, \chi_{ret} \rangle \text{ goes in}} \text{ T-Call}$$

Figure 5. IFC Typing Rules for Expressions

$$\frac{}{\Gamma, \Delta \vdash_{pc} \{\} \dashv \Gamma} \text{ T-Empty} \qquad \frac{}{\Gamma, \Delta \vdash_\bot exit \dashv \Gamma} \text{ T-Exit} \qquad \frac{\Gamma, \Delta \vdash_{pc} stmt_1 \dashv \Gamma_1 \qquad \Gamma_1, \Delta \vdash_{pc} \{\overline{stmt_2}\} \dashv \Gamma_2}{\Gamma, \Delta \vdash_{pc} \{stmt_1; \overline{stmt_2}\} \dashv \Gamma_2} \text{ T-Seq}$$

$$\frac{\begin{array}{c} \Gamma, \Delta \vdash_{pc} exp_1 : \langle \tau, \chi_1 \rangle \text{ goes inout} \\ \Gamma, \Delta \vdash_{pc} exp_2 : \langle \tau, \chi_2 \rangle \quad \chi_2 \sqsubseteq \chi_1 \quad pc \sqsubseteq \chi_1 \end{array}}{\Gamma, \Delta \vdash_{pc} exp_1 := exp_2 \dashv \Gamma} \text{ T-Assign} \qquad \frac{\begin{array}{c} \Gamma, \Delta \vdash_{\chi_2} stmt_1 \dashv \Gamma_1 \qquad \Gamma, \Delta \vdash_{\chi_2} stmt_2 \dashv \Gamma_2 \\ \Gamma, \Delta \vdash_{pc} exp : \langle bool, \chi_1 \rangle \quad \chi_1 \sqsubseteq \chi_2 \quad pc \sqsubseteq \chi_2 \end{array}}{\Gamma, \Delta \vdash_{pc} \text{if } (exp) \ stmt_1 \text{ else } stmt_2 \dashv \Gamma} \text{ T-Cond}$$

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi_{ret} \rangle \qquad \Gamma(\text{return}) = \langle \tau_{ret}, \chi_{ret} \rangle \qquad \Delta \vdash \tau_{ret} \rightsquigarrow \tau}{\Gamma, \Delta \vdash_\bot \text{return } exp \dashv \Gamma} \text{ T-Return} \qquad \frac{\Gamma, \Delta \vdash_{pc} var\_decl \dashv \Gamma_1, \Delta}{\Gamma, \Delta \vdash_{pc} var\_decl \dashv \Gamma_1} \text{ T-Decl}$$

$$\frac{\Gamma, \Delta \vdash_{pc} exp_1(\overline{exp_2}) : \langle \tau_{ret}, \chi_{ret} \rangle}{\Gamma, \Delta \vdash_{pc} exp_1(\overline{exp_2}) \dashv \Gamma} \text{ T-FnCallStmt} \qquad \frac{\Gamma, \Delta \vdash_{pc} exp : \langle table(pc_{tbl}), \bot \rangle \qquad pc \sqsubseteq pc_{tbl}}{\Gamma, \Delta \vdash_{pc} exp() \dashv \Gamma} \text{ T-TblCall}$$

Figure 6. IFC Typing Rules for Statements

Then, our main soundness theorem states that a well-typed program in our information-flow control type system will be non-interfering.

Theorem 4.3 (Main Soundness Theorem). *If* $\Gamma, \Delta \vdash_{pc} stmt \dashv \Gamma'$, *then* $\Gamma, \Delta \models_{pc} NI(stmt) \dashv \Gamma'$.

We present similar non-interference theorems for expressions and declarations in Theorem D.1 and Theorem D.3.

Proof Sketch. We prove non-interference theorems for statements, expressions and declarations together as a mutual induction on the typing derivation. The detailed proof of Theorem 4.3 is given in Appendix I. The most involved case is the rule for function calls (T-FnCall), where we must slightly

strengthen the non-interference definition for expressions, statements, and declarations.   □

5 Implementation and Case Studies

To evaluate our type system, we implemented a type-checker for annotated P4 programs and used it to analyze a range of example programs exhibiting different kinds of errors. We call our tool P4BID. Our information-flow control type system is implemented as an extension of the type checker in the P4C compiler [25], the reference compiler for P4$_{16}$ [24]. The target of our type checker is the simple_switch based on the BMv2 behavioral model. Our implementation adds about 700 LOC to P4C and supports the $\mathcal{L} = \{high, low\}$

$$\frac{}{\Gamma, \Delta \vdash_{pc} \langle \tau, \chi \rangle \; x \dashv \Gamma[x : \langle \tau, \chi \rangle], \Delta} \text{ T-VarDecl}$$

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle \tau', \chi \rangle \qquad \Delta \vdash \tau \rightsquigarrow \tau'}{\Gamma; \Delta \vdash_{pc} \langle \tau, \chi \rangle \; x := exp \dashv \Gamma[x : \langle \tau', \chi \rangle]; \Delta} \text{ T-VarInit}$$

$$\frac{\begin{array}{c} \Gamma, \Delta \vdash_{pc_{tbl}} \overline{exp_k : \langle \tau_k, \chi_k \rangle} \qquad \Gamma, \Delta \vdash_{pc_{tbl}} \overline{x_k : \langle match\_kind, \bot \rangle} \qquad \chi_k \sqsubseteq pc_{tbl} \text{ for all } k \\ \Gamma, \Delta \vdash_{pc_{tbl}} act_{a_j} : \overline{\langle d \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle}; \overline{\langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle} \xrightarrow{pc_{fn_j}} \langle unit, \bot \rangle, \bot \rangle \qquad pc_a \sqsubseteq pc_{fn_j} \text{ for all } j \\ \Gamma, \Delta \vdash_{pc_{tbl}} exp_{a_{ji}} : \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle \text{ goes } d \qquad \chi_k \sqsubseteq pc_{fn_j} \text{ for all } j, k \qquad pc_{tbl} \sqsubseteq pc_a \end{array}}{\Gamma, \Delta \vdash_{pc} \text{table } x \; \{ \overline{exp_k : x_k} \; \overline{act_{a_j}(\overline{exp_{a_{ji}}})} \} \dashv \Gamma[x : \langle table(pc_{tbl}), \bot \rangle], \Delta} \text{ T-TblDecl}$$

$$\frac{\begin{array}{c} \Gamma_1 = \Gamma[\overline{x_i : \langle \tau_i', \chi_i \rangle}, \text{return} : \langle \tau_{ret}', \chi_{ret} \rangle] \qquad \Gamma_1, \Delta \vdash_{pc_{fn}} stmt \dashv \Gamma_2, \\ \Delta \vdash \tau_i \rightsquigarrow \tau_i' \text{ for each } \tau_i \qquad \Delta \vdash \tau_{ret} \rightsquigarrow \tau_{ret}' \qquad \Gamma' = \Gamma[x : \langle \overline{d \langle \tau_i', \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}', \chi_{ret} \rangle, \bot \rangle] \end{array}}{\Gamma, \Delta \vdash_{pc} \text{function } \langle \tau_{ret}, \chi_{ret} \rangle \; x \; (\overline{d \; x_i : \langle \tau_i, \chi_i \rangle})\{stmt\} \dashv \Gamma', \Delta} \text{ T-FuncDecl}$$

Figure 7. IFC Typing Rules for Declaration

lattice, and a simple diamond lattice from Figure 8b, $\mathcal{L} =$ {high, alice, bob, low} for modeling isolation specifications. Standard P4 types can be annotated with a security label from the lattice; unannotated types default to low.

We evaluate our implementation by comparing the type-checking time of the secure programs presented in the case studies using the P4BID typechecker with the typechecking time of their uninstrumented insecure counterparts using the original P4C compiler. Table 1 shows that our implementation incurs an overhead of 5% (or 30ms) on average in comparison to the reference P4C compiler when evaluated on the instrumented and uninstrumented versions of the same program. We believe this overhead is reasonable for an unoptimized implementation that builds on the stock p4c compiler; developing a more optimized implementation is a direction for future work.

Table 1. Typechecking time in milliseconds.

Program	Unannotated, p4c	Annotated, P4BID
D2R	534	599
App	593	600
Lattice	495	527
Topology	554	591
Cache	538	550
Average	543	573

In the rest of the section, we present our case studies.

5.1 Dataplane Routing with Priorities

In traditional networks, the control plane is responsible for *routing*, determining how to send a packet from source to destination, while the data plane is responsible for *forwarding*, sending a packet to its next hop. Subramanian et al. [30] have shown that using programmable switches, one can handle routing in the data plane, avoiding the control plane entirely.

In their scheme, called D2R, when a switch receives a packet, it uses pre-loaded information about the network topology and local knowledge about link failures to perform a breadth-first search (BFS) and find a path to the target destination address. D2R uses P4 mechanisms (e.g., stacks) to perform the BFS computation entirely on the switch, without needing to communicate with the control plane.

We consider an extension of D2R where packets that encounter a higher number of link failures will receive higher priority. Listing 3 gives schematic code for the main headers and control block implementing this variant of data plane routing. The bfs_t headers describe the auxiliary information carried in the packets to perform the BFS, e.g., which links have been tried, while the ipv4_t headers contain information for standard packet forwarding. In the control block D2R_Ingress, the number of failures count (Line 19) can be computed from the vector of links that have been tried, hdr.bfs.tried_links, and the number of traversed links, hdr.bfs.num_hops. The table bfs_step performs one step of BFS; the details are not important for our purposes. Since P4 does not support loops, an iterative search algorithm like BFS is modeled in the apply block on Line 35 by unrolling the loop. If the BFS search has not completed, *i.e.*, the current node in the BFS search is not the destination node (Line 37), the BFS table is applied again (we elide the details of this BFS search algorithm which can be found in [30]). When the BFS search has successfully completed (Line 39), the forwarding table is applied and packet priorities are assigned based on the number of failures encountered by the packet.

Using failure information to prioritize packets may leak information. For instance, there are several potential reasons why hdr.bfs.num_hops could be secret—e.g., the packet could be transiting a private network and one might not want to reveal whether the network has reliable or unreliable links. If hdr.bfs.num_hops is annotated as high security, the program is rejected by our typechecker because the forwarding action

Listing 3. D2R: Dataplane Routing

```
1   header bfs_t {
2     <bit<32>, low> curr;
3     <bit<32>, low> tried_links;
4     <bit<32>, high> num_hops;
5     // ...
6   }
7   header ipv4_t {
8     <bit<3>, low> priority;
9     // ...
10  }
11  struct headers {
12    bfs_t bfs;
13    ipv4_t ipv4;
14    // ...
15  }
16
17  control D2R_Ingress(headers hdr) {
18    <bit<32>, high> failures
19    = num_bits_set(hdr.bfs.tried_links) – hdr.bfs.num_hops;
20
21    table bfs_step { ... }
22    table forward {
23      key = { hdr.bfs.next_node: exact; }
24      actions = { forwarding(failures); NoAction; }
25    }
26    action forwarding(in <bit<32>, high> failures) {
27      if (failures >= THRESHOLD) {
28        hdr.ipv4.priority = PRIO_1; // Leak
29      }
30      else {
31        hdr.ipv4.priority = PRIO_2; // Leak
32      }
33      // ... normal forwarding logic ...
34    }
35    apply {
36      if (hdr.bfs.curr != hdr.ipv4.dstAddr) {
37        bfs_step.apply();
38      } else {
39        forward.apply();
40      }
41      // repeat applications of bfs
42    }
43  }
```

writes data to the low-security priority after branching on the number of the failures, which is high security (Lines 28 and 31). This is an example of an indirect leak: the program branches on the secret, and then writes to public fields.

To remedy this information leak, we can modify the scheme so that the priority is computed based on non-sensitive information. For instance, we can assign priority based on the total number of links that a packet tried to cross. This count is an approximate proxy for the number of failures: as the number of failures rises, the packet tries more links.

This change can be implemented by removing hdr.bfs.num_-hops in Line 19, giving a program that is accepted by our typechecker.

A similar kind of leak can manifest in the implementation of NetChain [17], an in-network implementation of chain replication on top of a key-value store. The implementation assigns roles to the various switches in the network to determine the head, tail, or internal nodes of the chain, which among various actions determines if the node sends out a reply or not. If the roles header field is labeled as a secret field, this can give away private topological information. When instrumented with a high label on role, the typechecker flagged implicit leaks in the implementation.

5.2 Modeling Timing for In-Network Caching

Like other IFC systems, our type system can model different notions of adversary-observable data. For an example, we can consider a key-value store with an in-network cache [18]. These systems are a prominent application of data plane computing: switches can quickly retrieve hot items, keep track of which items are frequently requested, and notify the controller about which items should be stored on the switch. While the result of a query should be the same no matter where the item is stored, an observer may be able to detect variations in timing: data that is stored on the switch is returned faster, while data that is stored on the controller takes longer to access. In some cases, this *timing side-channel* may allow an adversary to learn about the state of the system.

While Core P4 does not model timing aspects of program behavior, we can still model timing information leaks by augmenting the program with new variables holding data that a timing-sensitive adversary may be able to observe. For example, Listing 4 gives a schematic P4 program implementing a simple cache. The switch first tries to fetch data locally (Line 16). If the request hits then the table runs action cache_hit, while if the request misses then the table runs action cache_miss. Both actions record the hit or miss in hdr.resp.hit. We mark this field as a low-security (publicly visible) variable, to model an adversary who can distinguish whether a request was serviced by the cache or the controller. If the query is sensitive information, hdr.req.query is declared as high security. Our typechecker rejects this program because of an information leak: the actions cache_hit and cache_miss write to the low-security field hdr.response.hit (Lines 8 and 10), but they are invoked in a table with a high-security key hdr.req.query (Line 12). This is again an indirect leak, modeling a simple timing side-channel.

5.3 Preventing Manipulation in Resource Allocation

The examples we have seen so far use IFC to guarantee confidentiality: secret information (high) should not leak into publicly visible outputs (low). As is well-known, if we interpret high-security data as "untrusted" and low-security

Listing 4. In-network cache

```
1   header request_t { <bit<8>, high> query; }
2   header response_t { <bool, low> hit; <bit<32>, low> value; }
3   struct headers { request_t req; response_t resp; eth_t eth; }
4
5   control Cache_Ingress(headers hdr) {
6      action cache_hit(<bit<32>, low> value) {
7         hdr.resp.value = value;
8         hdr.resp.hit = true;
9      }
10     action cache_miss() { hdr.resp.hit = false; }
11     table fetch_from_cache {
12        key = { hdr.req.query: exact; }
13        actions = { cache_hit; cache_miss; }
14     }
15     apply {
16        fetch_from_cache.apply();
17        // ... if miss, try to fetch from controller ...
18     }
19  }
```

Listing 5. Resource Allocation

```
1   header app_t { <bit<8>, high> appID; }
2   header ipv4_t {
3      <bit<32>, low> dstAddr;
4      <bit<32>, low> priority;
5      // ...
6   }
7   struct headers {
8      app_t app;
9      ipv4_t ipv4;
10     // ...
11  }
12
13  control App_Ingress(headers hdr) {
14     action set_priority(<bit<3>, low> priority) {
15        hdr.ipv4.priority = priority;
16     }
17     table app_resources {
18        key = { hdr.app.appID: exact; }
19        actions = { set_priority; }
20     }
21     apply {
22        set_priority.apply();
23        // ... forward the packet to hdr.ipv4.dstAddr ...
24     }
25  }
```

data as "trusted", IFC systems can also ensure *integrity*: untrusted inputs should not affect trusted outputs. To demonstrate, suppose several applications are running on separate subnetworks behind a single gateway switch, which is responsible for forwarding packets to their destination subnetwork and allocate resources to the application flows. We consider a very simple form of resource allocation, where a switch caters to the needs of latency-sensitive applications by increasing the priority of packets belonging to such applications. The P4 program in Listing 5 gives the main logic for a gateway switch that accomplishes this task. In addition to ordinary IP headers, packet headers in this setting also include an application ID hdr.app.appID indicating which application the packet belongs to. In the control block, the table app_resources matches on the application ID, and then calls set_priority with the desired priority level. This action then sets the priority level of the packet by writing to hdr.ipv4.priority (Line 15). Finally, the switch forwards the packet to the destination address hdr.ipv4.dstAddr.

While this program behaves well when clients are honest, a malicious client may manipulate the switch to increase the priority of their packets. Specifically, since hdr.app.appID is used to determine priority but not used to forward the packets, a client may report a false application ID. This issue can be detected by our IFC system if we label hdr.app.appID as untrusted (high) and hdr.ipv4.priority as trusted (low): setting priority based on application ID is an information-flow violation.

To address this problem, we can set the priority based on the destination address instead, by matching on hdr.ipv4.dstAddr instead of hdr.app.appID on Line 18. It is reasonable to model this header as trusted (low) because if a client were to manipulate this data, the packet would be delivered to the wrong
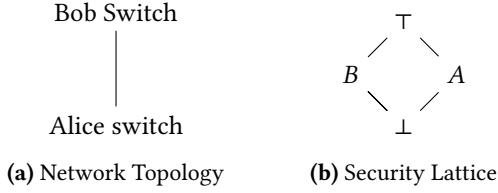
destination. In the modified program, the priority is now only computed based on trusted data in hdr.ipv4.dstAddr and the typechecker accepts this program because there is no integrity violation.

5.4 Ensuring Network Isolation

The previous example changes the interpretation of security labels in order to establish different properties with IFC. For our final case study, we show how our type system can use a richer lattice to enforce network isolation properties.

Suppose we have a private network used by two clients, Alice and Bob, who run dataplane programs on two separate nodes (the precise topology is not important, but a sketch can be see in Figure 8a). Nodes pass around a shared packet header with separate fields for Alice and for Bob, and we want to ensure that Alice does not touch Bob's fields, and vice versa. Furthermore, the network operator wants to carry telemetry data alongside the packets (*in-band network telemetry* [16]) this data may depend on Alice or Bob's data, but neither Alice nor Bob should be able to use telemetry data.

We can model this isolation property as non-interference with a four-point diamond lattice with labels $\{A, B, \top, \bot\}$ (Figure 8b). Non-interference ensures that data from level χ can flow to variables labeled χ' if and only if $\chi \sqsubseteq \chi'$. Thus, if we label Alice's fields A and label Bob's fields B, then Alice's data cannot influence Bob's fields, and vice versa. Similarly, $\top$-labeled fields can depend on all data, but cannot influence

Bob Switch

Alice switch

(a) Network Topology

⊤

B   A

⊥

(b) Security Lattice

Figure 8. Security lattice for a network topology

data below ⊤. For instance, telemetry data can be labeled ⊤: both Alice and Bob can accumulate data into ⊤-labeled fields (e.g., increment a counter), but neither Alice nor Bob are able to leak information from ⊤-labeled data into their own fields. Finally, fields labeled ⊥ contain globally visible data that cannot depend on other fields above ⊥. For example, we can pre-configure a packet's route through the private network in ⊥-labeled fields: this ensures that information from Alice or Bob does not influence routing, potentially leading to an indirect leak or isolation failure.

Labeling data from the four-point lattice can already rule out many kinds of leaks. However, it still allows some leaks involving ⊥-labeled data. For instance, Alice may write Bob's fields with ⊥-labeled data, while Bob may use ⊥-labeled data to modify ⊥-labeled data. While potentially undesirable, neither of these actions violates IFC since high data is allowed to depend on low data. To prevent these behaviors, we can additionally typecheck Alice's code with *pc* label *A*, and typecheck Bob's code with *pc* label *B*. Then, non-interference guarantees that Alice can only write to fields labeled *A* or ⊤, and Bob can only write to fields labeled *B* or ⊤.

Listing 6 shows schematic versions of programs implementing the Alice and Bob switches. Both the switches have a single action. The packet header carries one of the four security labels. In this example, we consider that hdr.alice_data and hdr.bob_data are Alice's and Bob's data, respectively; hdr.eth cannot be updated by either switch, but it can be used by both the switches; and hdr.telem can be updated by any switch but it should not be visible to Alice or Bob. Then, isolation can be established by checking two judgements:

$$\Gamma, \Delta \vdash_A \text{update\_by\_alice}() \dashv \Gamma'$$
$$\Gamma, \Delta \vdash_B \text{update\_by\_bob}() \dashv \Gamma'$$

Programs that incorrectly access packet headers will be flagged by the typechecker. For instance, in Alice_Ingress, the switch tries to write to Bob's field, Line 12 and on Line 16 it attempts to use the telemetry field hdr.telem, which can only be written to, not read. Our typechecker flags both leaks. A safe version of Alice's switch program is shown in Listing 7. In contrast, Bob_Ingress is accepted by the typechecker: it applies a table that branches on the ⊥-labeled header hdr.eth, and the action set_by_bob only modifies the ⊤-level header hdr.telem, incrementing a counter.

Listing 6. Network Isolation and Telemetry

```
1   struct headers {
2       <alice_t, A> alice_data;
3       <bob_t, B> bob_data;
4       <telem_t, top> telem;
5       <eth_t, bot> eth;
6   }
7
8   // typed at pc = A
9   control Alice_Ingress(headers hdr) {
10      action set_by_alice(<bob_t, A> value) {
11          // Error: should not have written to Bob's field
12          hdr.bob = value;
13      }
14      table update_by_alice {
15          // Error: should not have used telemetry field
16          key = { hdr.telem: exact; }
17          actions = { set_by_alice; }
18      }
19      apply { update_by_alice.apply(); }
20  }
21
22  // typed at pc = B
23  control Bob_Ingress(headers hdr) {
24      action set_by_bob() {
25          // Allowed: modify telemetry using telemetry
                information
26          hdr.telem = hdr.telem + 1;
27      }
28      table update {
29          key = { hdr.eth.dstAddr: exact; }
30          actions = { set_by_bob; NoAction; }
31      }
32      apply { update_by_bob.apply(); }
33  }
```

Listing 7. Isolation Respecting Switch Program

```
1   // typed at pc = A
2   control Alice_Ingress(headers hdr) {
3       action set_by_alice(<alice_data, A> value) {
4           hdr.alice_data = value;
5       }
6       table update_by_alice {
7           key = { hdr.alice_data: exact; }
8           action = { set_by_alice; }
9       }
10      apply { update_by_alice.apply(); }
11  }
```

While our concrete example only involves two switches and two parties, the same idea can be directly generalized to more parties by adding additional labels at the level of *A* and *B*. Then, our typechecker can ensure that programs written by different parties act on only their own packet headers. Richer dataflow policies could potentially be enforced by

using more complex lattices; this is an interesting direction for future work.

6 Related Work

Security in programmable networks. Recent works explore the security and privacy implications of programmable networks. For instance, in-network systems can be used to defend against denial-of-service attacks [37, 38], obfuscate network topology [22], mitigate covert channels [36], and enforce custom security policies [20, 30]. Tools have also been developed for helping operators test their dataplane programs against adversarial inputs (e.g., [19]). Our work complements these systems by detecting security and privacy bugs in programs running on programmable switches.

Network verification. The network verification literature is too vast to summarize here; methods have have targeted many aspects of networked systems, including routing protocols (e.g., [2–4, 35]), network configurations (e.g., [5, 28]), and network controllers (e.g., [7, 15]). Techniques have also been developed for verifying dataplane programs (e.g., [1, 14]). Some works also allow one to automatically repair faulty configurations [29] or to automatically synthesize policy-compliant ones [31, 32].

Our work focuses on dataplane programs written in the P4 language [6], building on the core version of P4 developed by Doenges et al. [10]. Perhaps the most closely related work is p4v [21], a verification system for P4 programs. Using p4v, a P4 program is verified against a logical specification by extracting a logical formula, which can be dispatched to solvers like Z3. Liu et al. [21] use p4v to verify basic correctness properties, e.g., a program does not read or write invalid headers, or a program implements the desired functionality correctly. While our system cannot verify the general properties established by p4v, our target non-interference property cannot be established in p4v since it relates a program's behavior on pairs of inputs [8]. Furthermore, our type-based analysis is lightweight and does not require automated solvers.

Two closely related type-system based works that explore properties orthogonal to non-interference properties are SafeP4 [11] and Π4 [12]. SafeP4 aims at catching invalid header access bugs, while Π4 presents a dependently-typed extension of P4 for verifying richer properties that SafeP4 could not cover. Unlike Π4, P4BID has a light-weight type-checking algorithm that does not involve constraint solving. Furthermore, our system builds on Core P4, a more realistic formal model of P4. For example, Core P4 models different calling conventions of P4 functions (e.g., pass by value and pass by reference) and control flow signals. These features introduced new opportunities for implicit leaks, which our type system rules out.

Information-flow control. Our approach belongs to a line of research on information-flow control (IFC), a type-based method of expressing and verifying a wide variety of security properties. Starting from work by Denning [9] and Volpano et al. [34], there are now many information-flow control systems ensuring different variants of non-interference against different kinds of adversaries; the survey by Sabelfeld and Myers [27] is a good introduction to this area. Existing systems target general-purpose programming languages (e.g., [23, 26]). Our work brings this idea to languages for programmable networks.

7 Conclusion and Future Directions

We have designed an information-flow control type system for P4 and demonstrated how it can verify networking properties for programs running on programmable switches.

We see several possibilities for further investigation. First, our non-interference theorems treat P4 programs as mapping a single input packet to a single output packet, but,P4 allows programming switches that can maintain internal state and recirculate packets for additional processing. These features could lead to security leaks if an adversary can observe sequences of input and output packets, and it would be interesting to establish non-interference in this richer setting. Second, it could be interesting to refine our analysis with information or assumptions about the control plane [21].

Acknowledgments

References

[1] Carolyn Jane Anderson, Nate Foster, Arjun Guha, Jean-Baptiste Jeannin, Dexter Kozen, Cole Schlesinger, and David Walker. 2014. NetKAT: semantic foundations for networks. In *ACM SIGPLAN–SIGACT Symposium on Principles of Programming Languages (POPL), San Diego, California*. 113–126. https://doi.org/10.1145/2535838.2535862

[2] Ryan Beckett, Aarti Gupta, Ratul Mahajan, and David Walker. 2017. A General Approach to Network Configuration Verification. In *Conference of the ACM Special Interest Group on Data Communication (SIGCOMM), Los Angeles, California*. 155–168. https://doi.org/10.1145/3098822.3098834

[3] Ryan Beckett, Aarti Gupta, Ratul Mahajan, and David Walker. 2018. Control plane compression. In *Conference of the ACM Special Interest Group on Data Communication (SIGCOMM), Budapest, Hungary*. 476–489. https://doi.org/10.1145/3230543.3230583

[4] Ryan Beckett, Aarti Gupta, Ratul Mahajan, and David Walker. 2020. Abstract interpretation of distributed network control planes. *Proceedings of the ACM on Programming Languages* 4, POPL (2020), 42:1–42:27. https://doi.org/10.1145/3371110

[5] Rüdiger Birkner, Dana Drachsler-Cohen, Laurent Vanbever, and Martin T. Vechev. 2020. Config2Spec: Mining Network Specifications from Network Configurations. In *USENIX Symposium on Networked Systems*

Design and Implementation (NSDI), Santa Clara, California. 969–984. https://www.usenix.org/conference/nsdi20/presentation/birkner

[6] Pat Bosshart, Dan Daly, Glen Gibb, Martin Izzard, Nick McKeown, Jennifer Rexford, Cole Schlesinger, Dan Talayco, Amin Vahdat, George Varghese, and David Walker. 2014. P4: programming protocol-independent packet processors. *Comput. Commun. Rev.* 44, 3 (2014), 87–95. https://doi.org/10.1145/2656877.2656890

[7] Eric Hayden Campbell, William T. Hallahan, Priya Srikumar, Carmelo Cascone, Jed Liu, Vignesh Ramamurthy, Hossein Hojjat, Ruzica Piskac, Robert Soulé, and Nate Foster. 2021. Avenir: Managing Data Plane Diversity with Control Plane Synthesis. In *USENIX Symposium on Networked Systems Design and Implementation (NSDI).* 133–153. https://www.usenix.org/conference/nsdi21/presentation/campbell

[8] Michael R. Clarkson and Fred B. Schneider. 2010. Hyperproperties. *J. Comput. Secur.* 18, 6 (2010), 1157–1210. https://doi.org/10.3233/JCS-2009-0393

[9] Dorothy E. Denning. 1976. A Lattice Model of Secure Information Flow. *Commun. ACM* 19, 5 (1976), 236–243. https://doi.org/10.1145/360051.360056

[10] Ryan Doenges, Mina Tahmasbi Arashloo, Santiago Bautista, Alexander Chang, Newton Ni, Samwise Parkinson, Rudy Peterson, Alaia Solko-Breslin, Amanda Xu, and Nate Foster. 2021. Petr4: formal foundations for p4 data planes. *Proceedings of the ACM on Programming Languages* 5, POPL (2021), 1–32. https://doi.org/10.1145/3434322

[11] Matthias Eichholz, Eric Hayden Campbell, Nate Foster, Guido Salvaneschi, and Mira Mezini. 2019. How to Avoid Making a Billion-Dollar Mistake: Type-Safe Data Plane Programming with SafeP4. In *European Conference on Object-Oriented Programming (ECOOP), London, England (Leibniz International Proceedings in Informatics, Vol. 134).* 12:1–12:28. https://doi.org/10.4230/LIPIcs.ECOOP.2019.12

[12] Matthias Eichholz, Eric Hayden Campbell, Matthias Krebs, Nate Foster, and Mira Mezini. 2022. Dependently-Typed Data Plane Programming. *Proceedings of the ACM on Programming Languages* 6, POPL, Article 40 (Jan. 2022), 28 pages. https://doi.org/10.1145/3498701

[13] Facebook. 2021. More details about the October 4 outage. https://engineering.fb.com/2021/10/05/networking-traffic/outage-details/

[14] Nate Foster, Dexter Kozen, Matthew Milano, Alexandra Silva, and Laure Thompson. 2015. A Coalgebraic Decision Procedure for NetKAT. In *ACM SIGPLAN–SIGACT Symposium on Principles of Programming Languages (POPL), Mumbai, India.* 343–355. https://doi.org/10.1145/2676726.2677011

[15] Nick Giannarakis, Devon Loehr, Ryan Beckett, and David Walker. 2020. NV: an intermediate language for verification of network control planes. In *ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI), London, England.* 958–973. https://doi.org/10.1145/3385412.3386019

[16] Intel. 2020. *In-band Network Telemetry Detects Network Performance Issues.* Technical Report. Intel. https://builders.intel.com/docs/networkbuilders/in-band-network-telemetry-detects-network-performance-issues.pdf

[17] Xin Jin, Xiaozhou Li, Haoyu Zhang, Nate Foster, Jeongkeun Lee, Robert Soulé, Changhoon Kim, and Ion Stoica. 2018. Netchain: Scale-Free Sub-RTT Coordination. In *USENIX Symposium on Networked Systems Design and Implementation (NSDI), Renton, Washington.* USA, 35–49.

[18] Xin Jin, Xiaozhou Li, Haoyu Zhang, Robert Soulé, Jeongkeun Lee, Nate Foster, Changhoon Kim, and Ion Stoica. 2017. NetCache: Balancing Key-Value Stores with Fast In-Network Caching. In *USENIX Symposium on Operating Systems Design and Implementation (OSDI), Shanghai, China.* 121–136. https://doi.org/10.1145/3132747.3132764

[19] Qiao Kang, Jiarong Xing, Yiming Qiu, and Ang Chen. 2021. Probabilistic profiling of stateful data planes for adversarial testing. In *International Conference on Architectural Support for Programming Langauages and Operating Systems (ASPLOS).* 286–301. https://doi.org/10.1145/3445814.3446764

[20] Qiao Kang, Lei Xue, Adam Morrison, Yuxin Tang, Ang Chen, and Xiapu Luo. 2020. Programmable In-Network Security for Context-aware BYOD Policies. In *USENIX Security Smposium (USENIX).* 595–612. https://www.usenix.org/conference/usenixsecurity20/presentation/kang

[21] Jed Liu, William T. Hallahan, Cole Schlesinger, Milad Sharif, Jeongkeun Lee, Robert Soulé, Han Wang, Calin Cascaval, Nick McKeown, and Nate Foster. 2018. p4v: practical verification for programmable data planes. In *Conference of the ACM Special Interest Group on Data Communication (SIGCOMM), Budapest, Hungary.* 490–503. https://doi.org/10.1145/3230543.3230582

[22] Roland Meier, Petar Tsankov, Vincent Lenders, Laurent Vanbever, and Martin T. Vechev. 2018. NetHide: Secure and Practical Network Topology Obfuscation. In *USENIX Security Smposium (USENIX), Baltimore, Maryland.* 693–709. https://www.usenix.org/conference/usenixsecurity18/presentation/meier

[23] Andrew C. Myers, Lantian Zheng, Steve Zdancewic, Stephen Chong, and Nathaniel Nystrom. 2006. *Jif 3.0: Java information flow.* http://www.cs.cornell.edu/jif

[24] P4Lang. 2022. P4_16 Spec. https://p4.org/p4-spec/docs/P4-16-v1.0.0-spec.html

[25] P4Lang. 2022. P4c Compiler. https://github.com/p4lang/p4c

[26] François Pottier and Vincent Simonet. 2003. Information flow inference for ML. *ACM Transactions on Programming Languages and Systems* 25, 1 (2003), 117–158. https://doi.org/10.1145/596980.596983

[27] Andrei Sabelfeld and Andrew C. Myers. 2003. Language-based information-flow security. *IEEE J. Sel. Areas Commun.* 21, 1 (2003), 5–19. https://doi.org/10.1109/JSAC.2002.806121

[28] Samuel Steffen, Timon Gehr, Petar Tsankov, Laurent Vanbever, and Martin T. Vechev. 2020. Probabilistic Verification of Network Configurations. In *Conference of the ACM Special Interest Group on Data Communication (SIGCOMM).* 750–764. https://doi.org/10.1145/3387514.3405900

[29] Kausik Subramanian, Anubhavnidhi Abhashkumar, Loris D'Antoni, and Aditya Akella. 2020. Detecting network load violations for distributed control planes. In *ACM SIGPLAN Conference on Programming Language Design and Implementation (PLDI), London, England.* 974–988. https://doi.org/10.1145/3385412.3385976

[30] Kausik Subramanian, Anubhavnidhi Abhashkumar, Loris D'Antoni, and Aditya Akella. 2021. D2R: Policy-Compliant Fast Reroute. In *ACM SIGCOMM Symposium on SDN Research (SOSR).* 148–161. https://doi.org/10.1145/3482898.3483360

[31] Kausik Subramanian, Loris D'Antoni, and Aditya Akella. 2017. Genesis: synthesizing forwarding tables in multi-tenant networks. In *ACM SIGPLAN–SIGACT Symposium on Principles of Programming Languages (POPL), Paris, France.* 572–585. https://doi.org/10.1145/3009837.3009845

[32] Kausik Subramanian, Loris D'Antoni, and Aditya Akella. 2018. Synthesis of Fault-Tolerant Distributed Router Configurations. *Proceedings of the ACM on Measurement and Analysis of Computing Systems* 2, 1 (2018), 22:1–22:26. https://doi.org/10.1145/3179425

[33] Steven J. Vaughan-Nichols. 2021. Google glitch triggers major internet outage. *ZDNet* (Nov. 2021). https://www.zdnet.com/article/google-glitch-triggers-major-internet-outage/

[34] Dennis M. Volpano, Cynthia E. Irvine, and Geoffrey Smith. 1996. A Sound Type System for Secure Flow Analysis. *J. Comput. Secur.* 4, 2/3 (1996), 167–188. https://doi.org/10.3233/JCS-1996-42-304

[35] Konstantin Weitz, Doug Woos, Emina Torlak, Michael D. Ernst, Arvind Krishnamurthy, and Zachary Tatlock. 2016. Scalable verification of border gateway protocol configurations with an SMT solver. In *ACM SIGPLAN Conference on Object Oriented Programming: Systems, Languages, and Applications (OOPSLA), Amsterdam, The Netherlands.* 765–780. https://doi.org/10.1145/2983990.2984012

[36] Jiarong Xing, Adam Morrison, and Ang Chen. 2019. NetWarden: Mitigating Network Covert Channels without Performance Loss. In

USENIX Workshop on Hot Topics in Cloud Computing (HotCloud), Renton, Washington. https://www.usenix.org/conference/hotcloud19/presentation/xing

[37] Jiarong Xing, Wenqing Wu, and Ang Chen. 2019. Architecting Programmable Data Plane Defenses into the Network with FastFlex. In

USENIX Workshop on Hot Topics in Cloud Computing (HotCloud), Princeton, New Jersey. 161–169. https://doi.org/10.1145/3365609.3365860

[38] Jiarong Xing, Wenqing Wu, and Ang Chen. 2021. Ripple: A Programmable, Decentralized Link-Flooding Defense Against Adaptive Adversaries. In *USENIX Security Smposium (USENIX).* 3865–3881. https://www.usenix.org/conference/usenixsecurity21/presentation/xing

A   Grammar

Expressions.

$$
\begin{array}{rcll}
exp & ::= & b & \text{Boolean} \\
 & | & n_w & \text{integers or bits of width w} \\
 & | & x & \text{variable} \\
 & | & exp_1[exp_2] & \text{array indexing} \\
 & | & exp_1 \oplus exp_2 & \text{binary operation} \\
 & | & \{f_i = exp_i\} & \text{record} \\
 & | & exp.f_i & \text{field projection} \\
 & | & exp_1(\overline{exp_2}) & \text{function call}
\end{array}
$$

Statements.

$$
\begin{array}{rcll}
stmt & ::= & exp_1(\overline{exp_2}) & \text{function call} \\
 & | & exp_1 := exp_2 & \text{assignment} \\
 & | & \text{if } (exp_1) \; stmt_1 \text{ else } stmt_2 & \text{conditional} \\
 & | & \{\overline{stmt}\} & \text{sequencing} \\
 & | & \text{exit} & \text{exit} \\
 & | & \text{return } exp & \text{return} \\
 & | & var\_decl & \text{variable declaration}
\end{array}
$$

Declaration.

$$
\begin{array}{rcl}
prg & ::= & \overline{typ\_decl} \; ctrl\_body \\
ctrl\_body & ::= & \overline{decl} \; stmt \\
decl & ::= & var\_decl \mid obj\_decl \mid typ\_decl \\
var\_decl & ::= & \tau \, x := exp \mid \tau \, x \\
typ\_decl & ::= & \text{match\_kind } \{\overline{f}\} \mid \text{typedef } \tau \, X \\
obj\_decl & ::= & \text{table } x \; \{\overline{key} \; \overline{act}\} \\
 & | & \text{function } \tau_{ret} \, x \; \overline{(d \, y : \tau)}\{stmt\}
\end{array}
$$

$$
\begin{array}{rcl}
d & ::= & in \mid inout \\
lval & ::= & x \\
 & | & lval.f \\
 & | & lval[n] \\
key & ::= & exp : x \\
act & ::= & x(\overline{exp}, \overline{x : \tau})
\end{array}
$$

B   Typing Rules

$\Delta \vdash \tau \leadsto \tau'$ are judgements that resolve the base types for typedefs. We use the same definition as presented in Petr4's sections A.7 and A.8 [10]. Note that the grammar that we consider doesn't support $bit\langle exp \rangle$ as we have discounted slice operations, instead we have $bit\langle n \rangle$, where n is some constant.

Expression Typing Rules.

$$\frac{\Gamma, \Delta \vdash_{pc'} exp : \langle \tau, \chi \rangle \qquad pc \sqsubseteq pc'}{\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi \rangle} \text{ T-SubType-PC}$$

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi \rangle \text{ goes in} \qquad \chi \sqsubseteq \chi'}{\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi' \rangle \text{ goes in}} \text{ T-SubType-In}$$

$$\frac{}{\Gamma, \Delta \vdash_{pc} n_\infty : \langle int, \bot \rangle \text{ goes in}} \text{ T-Int}$$

$$\frac{x \in \text{dom}(\Gamma) \qquad \Gamma(x) = \langle \tau, \chi \rangle}{\Gamma, \Delta \vdash_{pc} x : \langle \tau, \chi \rangle \text{ goes inout}} \text{ T-Var}$$

$$\frac{\begin{array}{c} \Gamma, \Delta \vdash_{pc} exp_1 : \langle \rho_1, \chi_1 \rangle \qquad \Gamma, \Delta \vdash_{pc} exp_2 : \langle \rho_2, \chi_2 \rangle \\ \mathcal{T}(\Delta; \oplus; \rho_1; \rho_2) = \rho_3 \qquad \chi_1 \sqsubseteq \chi' \qquad \chi_2 \sqsubseteq \chi' \end{array}}{\Gamma, \Delta \vdash_{pc} exp_1 \oplus exp_2 : \langle \rho_3, \chi' \rangle \text{ goes in}} \text{ T-BinOP}$$

$$\frac{\Gamma, \Delta \vdash_{pc} \overline{\{exp : \langle \tau_i, \chi_i \rangle\}}}{\Gamma, \Delta \vdash_{pc} \overline{\{f : exp\}} : \langle \{\overline{f : \langle \tau_i, \chi_i \rangle}\}, \bot \rangle \text{ goes in}} \text{ T-Rec}$$

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle \{\overline{f_i : \langle \tau_i, \chi_i \rangle}\}, \bot \rangle \text{ goes } d}{\Gamma, \Delta \vdash_{pc} exp.f_i : \langle \tau_i, \chi_i \rangle \text{ goes } d} \text{ T-MemRec}$$

$$\frac{\begin{array}{c} \Gamma, \Delta \vdash_{pc} exp_1 : \langle \langle \tau, \chi_1 \rangle[n], \bot \rangle \text{ goes } d \\ \Gamma, \Delta \vdash_{pc} exp_2 : \langle bit\langle 32 \rangle, \chi_2 \rangle \\ \chi_2 \sqsubseteq \chi_1 \end{array}}{\Gamma, \Delta \vdash_{pc} exp_1[exp_2] : \langle \tau, \chi_1 \rangle \text{ goes } d} \text{ T-Index}$$

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle header\{\overline{f_i : \langle \tau_i, \chi_i \rangle}\}, \bot \rangle \text{ goes } d}{\Gamma, \Delta \vdash_{pc} exp.f_i : \langle \tau_i, \chi_i \rangle \text{ goes } d} \text{ T-MemHdr}$$

$$\frac{\begin{array}{c} \Gamma, \Delta \vdash_{pc} exp_1 : \langle \overline{d \langle \tau_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle \\ \Gamma, \Delta \vdash_{pc} exp_2 : \langle \tau_i, \chi_i \rangle \text{ goes } d \qquad pc \sqsubseteq pc_{fn} \end{array}}{\Gamma, \Delta \vdash_{pc} exp_1(\overline{exp_2}) : \langle \tau_{ret}, \chi_{ret} \rangle \text{ goes in}} \text{ T-Call}$$

$$\frac{\begin{array}{c} \Gamma_1 = \Gamma[\overline{x_i : \langle \tau_i', \chi_i \rangle}, \text{return} : \langle \tau_{ret}', \chi_{ret} \rangle] \qquad \Gamma_1, \Delta \vdash_{pc_{fn}} stmt \dashv \Gamma_2, \\ \Delta \vdash \tau_i \rightsquigarrow \tau_i' \text{ for each } \tau_i \qquad \Delta \vdash \tau_{ret} \rightsquigarrow \tau_{ret}' \qquad \Gamma' = \Gamma[x : \langle \overline{d \langle \tau_i', \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}', \chi_{ret} \rangle, \bot \rangle] \end{array}}{\Gamma, \Delta \vdash_{pc} \text{function } \langle \tau_{ret}, \chi_{ret} \rangle \ x \ (\overline{d \ x_i : \langle \tau_i, \chi_i \rangle})\{stmt\} \dashv \Gamma', \Delta} \text{ T-FuncDecl}$$

Statement Typing Rules.

$$\frac{}{\Gamma, \Delta \vdash_{pc} \{\} \dashv \Gamma} \text{ T-Empty} \qquad\qquad \frac{}{\Gamma, \Delta \vdash_\bot \text{exit} \dashv \Gamma} \text{ T-Exit}$$

$$\frac{\begin{array}{c} \Gamma, \Delta \vdash_{pc} exp : \langle bool, \chi_1 \rangle \\ \Gamma, \Delta \vdash_{\chi_2} stmt_1 \dashv \Gamma_1 \qquad \Gamma, \Delta \vdash_{\chi_2} stmt_2 \dashv \Gamma_2 \qquad \chi_1 \sqsubseteq \chi_2 \qquad pc \sqsubseteq \chi_2 \end{array}}{\Gamma, \Delta \vdash_{pc} \text{if } (exp) \ stmt_1 \text{ else } stmt_2 \dashv \Gamma} \text{ T-conditional}$$

$$\frac{\Gamma, \Delta \vdash_{pc} stmt_1 \dashv \Gamma_1 \qquad \Gamma_1, \Delta \vdash_{pc} \{\overline{stmt_2}\} \dashv \Gamma_2}{\Gamma, \Delta \vdash_{pc} \{stmt_1; \overline{stmt_2}\} \dashv \Gamma_2} \text{ T-Seq}$$

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi_{ret} \rangle \qquad \Gamma(\text{return}) = \langle \tau_{ret}, \chi_{ret} \rangle \qquad \Delta \vdash \tau_{ret} \rightsquigarrow \tau}{\Gamma, \Delta \vdash_\bot \text{return } exp \dashv \Gamma} \text{ T-Return}$$

$$\frac{\Gamma, \Delta \vdash_{pc} exp_1 : \langle \tau, \chi_1 \rangle \text{ goes inout} \qquad \Gamma, \Delta \vdash_{pc} exp_2 : \langle \tau, \chi_2 \rangle \qquad \chi_2 \sqsubseteq \chi_1 \qquad pc \sqsubseteq \chi_1}{\Gamma, \Delta \vdash_{pc} exp_1 := exp_2 \dashv \Gamma} \text{ T-Assign}$$

$$\frac{\Gamma, \Delta \vdash_{pc} var\_decl \dashv \Gamma_1, \Delta}{\Gamma, \Delta \vdash_{pc} var\_decl \dashv \Gamma_1} \text{ T-Decl} \qquad\qquad \frac{\Gamma, \Delta \vdash_{pc} exp_1(\overline{exp_2}) : \langle \tau_{ret}, \chi_{ret} \rangle}{\Gamma, \Delta \vdash_{pc} exp_1(\overline{exp_2}) \dashv \Gamma} \text{ T-FnCallStmt}$$

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle table(pc_{tbl}), \bot \rangle \qquad pc \sqsubseteq pc_{tbl}}{\Gamma, \Delta \vdash_{pc} exp() \dashv \Gamma} \text{ T-TblCall}$$

Declaration Typing Rules.

$$\frac{}{\Gamma, \Delta \vdash_{pc} \langle \tau, \chi \rangle \; x \dashv \Gamma[x : \langle \tau, \chi \rangle], \Delta} \text{ T-VarDecl}$$

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle \tau', \chi \rangle \qquad \Delta \vdash \tau \rightsquigarrow \tau'}{\Gamma; \Delta \vdash_{pc} \langle \tau, \chi \rangle \; x := exp \dashv \Gamma[x : \langle \tau', \chi \rangle]; \Delta}$$

$$\frac{\begin{array}{c} \Gamma, \Delta \vdash_{pc_{tbl}} \overline{exp_k : \langle \tau_k, \chi_k \rangle} \qquad \Gamma, \Delta \vdash_{pc_{tbl}} \overline{x_k : \langle match\_kind, \bot \rangle} \\ \Gamma, \Delta \vdash_{pc_{tbl}} act_{a_j} : \langle \overline{d \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle} ; \overline{\langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle} \xrightarrow{pc_{fn_j}} \langle unit, \bot \rangle, \bot \rangle, \text{ for all } j \\ \Gamma, \Delta \vdash_{pc_{tbl}} \overline{exp_{a_{ji}} : \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle goes \; d} \\ \chi_k \sqsubseteq pc_{fn_j} \text{ for all } j, k \qquad pc_a \sqsubseteq pc_{fn_j}, \text{ for all } j \qquad \chi_k \sqsubseteq pc_{tbl} \text{ for all } k \qquad pc_{tbl} \sqsubseteq pc_a \end{array}}{\Gamma, \Delta \vdash_{pc} \text{table } x \; \{\overline{exp_k : x_k} \; \overline{act_{a_j}(\overline{exp_{a_{ji}}})}\} \dashv \Gamma[x : \langle table(pc_{tbl}), \bot \rangle], \Delta} \text{ T-TblDecl}$$

$$\frac{\begin{array}{c} \Gamma_1 = \Gamma[\overline{x_i : \langle \tau'_i, \chi_i \rangle}, \text{return} : \langle \tau'_{ret}, \chi_{ret} \rangle] \qquad \Gamma_1, \Delta \vdash_{pc_{fn}} stmt \dashv \Gamma_2, \\ \Delta \vdash \tau_i \rightsquigarrow \tau'_i \text{ for each } \tau_i \qquad \Delta \vdash \tau_{ret} \rightsquigarrow \tau'_{ret} \qquad \Gamma' = \Gamma[x : \langle \overline{d \langle \tau'_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau'_{ret}, \chi_{ret} \rangle, \bot \rangle] \end{array}}{\Gamma, \Delta \vdash_{pc} \text{function } \langle \tau_{ret}, \chi_{ret} \rangle \; x \; (\overline{d \; x_i : \langle \tau_i, \chi_i \rangle})\{stmt\} \dashv \Gamma', \Delta} \text{ T-FuncDecl}$$

C Definition

Let $\tau_{fn} = \langle \overline{d \; \rho} \xrightarrow{pc_{fn}} \rho_{ret}, \bot \rangle$ and $\tau_{tbl} = \langle table(pc_{tbl}), \bot \rangle$

Definition C.1 (Store typing). Store typing context Ξ is a partial map from the locations to types, $\Xi : \mathbb{L} \to \langle \tau, \chi \rangle$. A memory-store μ is well-typed in a store-typing context Ξ, which can be represented as $\Xi, \Delta \models \mu$, if for every location, $l \in \text{DOM}(\mu)$ there exists a type, $\langle \tau, \chi \rangle = \Xi(l)$ and $\Xi, \Delta \vdash \mu(l) : \langle \tau, \chi \rangle$ (value typing is defined in Appendix J).

Definition C.2 (Typing of environment). $\Xi \vdash \epsilon : \Gamma$ is defined as

$$\frac{}{\Xi \vdash [] : []} \qquad \frac{\Xi \vdash \epsilon : \Gamma \qquad \Xi(l) = \langle \tau, \chi \rangle}{\Xi \vdash (\epsilon, x \mapsto l) : (\Gamma, x : \langle \tau, \chi \rangle)} \qquad \frac{\Xi \vdash \epsilon : \Gamma}{\Xi \vdash \epsilon : \Gamma, \text{return} \mapsto l}$$

Definition C.3 (Semantic typing of store and environment). A pair of store and environment $\langle \mu, \epsilon \rangle$ is semantically well-typed $\Xi, \Delta \models \langle \mu, \epsilon \rangle : \Gamma$ if the following conditions hold:

1. $\Xi, \Delta \models \mu$
2. $\Xi \vdash \epsilon : \Gamma$
3. For any $x \in \text{DOM}(\epsilon)$, $\epsilon(x) \in \text{DOM}(\mu)$,
4. For all x in $\text{DOM}(\epsilon)$ and some $\Gamma_{fn} \subseteq \Gamma$ and any pc, if $\Gamma, \Delta \vdash_{pc} x : \tau_{fn}$, $\mu(\epsilon(x)) = clos(\epsilon_c, ...)$, and $\Xi \models \epsilon_c : \Gamma_{fn}$, then $\text{DOM}(\epsilon_c) \subseteq \text{DOM}(\epsilon)$ and $\Xi, \Delta \models \langle \mu, \epsilon_c \rangle : \Gamma_{fn}$. Here τ_{fn} is the function type. We elide the full view of the closures in this definition.
5. For all x in $\text{DOM}(\epsilon)$ and some $\Gamma_{tbl} \subseteq \Gamma$ and any pc, if $\Gamma, \Delta \vdash_{pc} x : \tau_{tbl}$, $\mu(\epsilon(x)) = table \; l \; (\epsilon_c, ...)$, and $\Xi \models \epsilon_c : \Gamma_{tbl}$, then $\text{DOM}(\epsilon_c) \subseteq \text{DOM}(\epsilon)$ and $\Xi, \Delta \models \langle \mu, \epsilon_c \rangle : \Gamma_{tbl}$. Here τ_{tbl} is the table type.

Definition C.4 (Semantic typing for a pair of memory stores and environments). $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$ holds when

1. $\Xi_a, \Delta \models \langle \mu_a, \epsilon_a \rangle : \Gamma$ and $\Xi_b, \Delta \models \langle \mu_b, \epsilon_b \rangle : \Gamma$
2. $\text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$
3. For any $x \in \text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$, $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(\mu_a(\epsilon_a(x)), \mu_b(\epsilon_b(x))) : \Gamma(x)$ (defined in Definition C.6),
4. For all x in $\text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$ and some $\Gamma_{fn} \subseteq \Gamma$ and any pc, if $\Gamma, \Delta \vdash_{pc} x : \tau_{fn}$, $\mu_a(\epsilon_a(x)) = clos(\epsilon_{c_a}, ...)$, $\mu_b(\epsilon_b(x)) = clos(\epsilon_{c_b}, ...)$, $\Xi_a \models \epsilon_{c_a} : \Gamma_{fn}$, and $\Xi_b \models \epsilon_{c_b} : \Gamma_{fn}$, then $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_{c_a} \rangle \langle \mu_b, \epsilon_{c_b} \rangle : \Gamma_{fn}$,
5. For all x in $\text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$ and some $\Gamma_{tbl} \subseteq \Gamma$ and any pc, if $\Gamma, \Delta \vdash_{pc} x : \tau_{tbl}$, $\mu_a(\epsilon_a(x)) = table \; l_a \; (\epsilon_{c_a}, ...)$, $\mu_b(\epsilon_b(x)) = table \; l_b \; (\epsilon_{c_b}, ...)$, $\Xi_a \models \epsilon_{c_a} : \Gamma_{tbl}$, and $\Xi_b \models \epsilon_{c_b} : \Gamma_{tbl}$, then $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_{c_a} \rangle \langle \mu_b, \epsilon_{c_b} \rangle : \Gamma_{tbl}$.

Definition C.5 (Non-interference for Expressions). $\Gamma, \Delta \models_{pc} \text{NI}(exp : \langle \tau, \chi \rangle)$ holds if for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b$, and any security level l,

1. *Variable at level lower than l are indistinguishable at the beginning.* $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$,
2. $\langle C; \Delta; \mu_a; \epsilon_a; exp \rangle \Downarrow \langle \mu'_a; val_a \rangle$,
3. $\langle C; \Delta; \mu_b; \epsilon_b; exp \rangle \Downarrow \langle \mu'_b; val_b \rangle$

implies there exists a Ξ'_a, Ξ'_b such that

1. *Effects on any variable at level lower than l should be indistinguishable.* $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$,

2. *PC is used to bound writes.* For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$,

3. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$,

4. For any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$,

5. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$, and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$,

6. $\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(val_a, val_b) : \langle \tau, \chi \rangle$.

Definition C.6 (Non-interference for values). $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(val_a, val_b) : \langle \tau, \chi \rangle$ holds when:

1. $\Xi_a, \Delta \vdash val_a : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash val_b : \langle \tau, \chi \rangle$ (value typing is defined in Appendix J),

2. If $\tau \notin \{\rho[n], \overline{\{f : \rho\}}, header\ \{f : \rho\}, \overline{d\ \rho} \xrightarrow{pc} \rho_{ret}, table(pc_{tbl})\}$ and $\chi \sqsubseteq l$, then $val_a = val_b$,

3. If $\tau = \rho[n], \tau = \overline{\{f : \rho\}}$ or $\tau = header\ \{f : \rho\}$, then $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(val_{ai}, val_{bi}) : \langle \tau_i, \chi_i \rangle$, for all $val_{ai} \in \overline{val_{ai}} = val_a$ and $val_{bi} \in \overline{val_{bi}} = val_b, \rho = \langle \tau_i, \chi_i \rangle$.

4. If $\langle \tau, \chi \rangle = \tau_{fn}$, then $\Xi_a, \Xi_b, \Delta \models \text{NI\_CLOS}(val_a, val_b) : \tau_{fn}$ (Definition C.7),

5. If $\langle \tau, \chi \rangle = \tau_{tbl}$, then $\Xi_a, \Xi_b, \Delta \models \text{NI\_TBL}(val_a, val_b) : \tau_{tbl}$ (Definition C.8).

Definition C.7. $\Xi_a, \Xi_b, \Delta \models \text{NI\_CLOS}(val_a, val_b) : \tau_{fn}$, where val_a and val_b are of the form $clos(\epsilon_{c_a}, \overline{dx : \langle \tau, \chi \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, \text{stmt})$ and $clos(\epsilon_{c_b}, \overline{dx : \langle \tau, \chi \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, \text{stmt})$ holds when there exists a Γ such that the following are satisfied:

1. $\Xi_a \vdash \epsilon_{c_a} : \Gamma$ and $\Xi_b \vdash \epsilon_{c_b} : \Gamma$

2. for any pc, $\Gamma, \Delta \vdash_{pc} val_a : \langle \overline{d\ \langle \tau, \chi \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle$ and $\Gamma, \Delta \vdash_{pc} val_b : \langle \overline{d\ \langle \tau, \chi \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle$.

3. $\Gamma[\overline{x : \langle \tau, \chi \rangle}, return : \langle \tau_{ret}, \chi_{ret} \rangle], \Delta \vdash_{pc_{fn}} stmt \dashv \Gamma'$

4. $val_a =_{clos} val_b$.

Here, $val_a =_{clos} val_b$ is defined as two closures with $\text{DOM}(\epsilon_{c_a}) = \text{DOM}(\epsilon_{c_b})$.

Definition C.8. $\Xi_a, \Xi_b, \Delta \models \text{NI\_TBL}(val_a, val_b) : \tau_{tbl}$, where

$$val_a = table\ l_a\ (\epsilon_a, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})})$$

and

$$val_b = table\ l_b\ (\epsilon_b, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})})$$

holds when there exists a Γ and pc_a such that the following are satisfied:

1. $\Xi_a \models \epsilon_a : \Gamma$ and $\Xi_b \models \epsilon_b : \Gamma$
2. for any pc, $\Gamma; \Delta \vdash_{pc} val_a : \langle table(pc_{tbl}), \bot \rangle$, $\Gamma; \Delta \vdash_{pc} val_b : \langle table(pc_{tbl}), \bot \rangle$.
3. $\Gamma, \Delta \vdash_{pc_{tbl}} x_k : \langle match\_kind, \bot \rangle$ for each $x_k \in \overline{x_k}$
4. $\Gamma, \Delta \vdash_{pc_{tbl}} exp_k : \langle \tau_k, \chi_k \rangle$ for each $exp_k \in \overline{exp_k}$
5. $\Gamma, \Delta \vdash_{pc_{tbl}} act_{aj} : \langle \overline{d\ \langle \tau_{aji}, \chi_{aji} \rangle}; \overline{\langle \tau_{cji}, \chi_{cji} \rangle} \xrightarrow{pc_{fn_j}} \langle unit, \bot \rangle, \bot \rangle$ for each $act_{a_j} \in \overline{act_{a_j}}$
6. $\Gamma, \Delta \vdash_{pc_{tbl}} exp_{aji} : \langle \tau_{aji}, \chi_{aji} \rangle\ goes\ d$ for each $exp_{aji} \in \overline{exp_{aji}}$
7. $val_a =_{tbl} val_b$.
8. $\chi_k \sqsubseteq pc_{fn_j}$, for all j, k
9. $pc_a \sqsubseteq pc_{fn_j}$, for all j
10. $\chi_k \sqsubseteq pc_{tbl}$ for all k
11. $pc_{tbl} \sqsubseteq pc_a$.

Here, $table\ l_a =_{tbl} table\ l_b$ is defined as two table values with $\text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$. Their control plane entries will be the same.

Definition C.9 (Non-interference for statements). For any security lable l, $\Gamma, \Delta \models_{pc} \text{NI}(stmt) \dashv \Gamma'$ holds for any Ξ_a, Ξ_b, μ_a, $\mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b, \epsilon'_a, \epsilon'_b$ if

1. $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$,
2. $\langle C; \Delta; \mu_a; \epsilon_a; stmt \rangle \Downarrow \langle \mu'_a; \epsilon'_a; sig_1 \rangle$,
3. $\langle C; \Delta; \mu_b; \epsilon_b; stmt \rangle \Downarrow \langle \mu'_b; \epsilon'_b; sig_2 \rangle$

then there exists Ξ'_a, Ξ'_b, such that

1. $\Gamma, \Delta \vdash_{pc} stmt \dashv \Gamma'$,
2. $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon'_a \rangle \langle \mu'_b, \epsilon'_b \rangle : \Gamma'$,
3. $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$,

4. *PC is used to bound writes.* For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$,

5. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$,

6. For any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$,

7. $sig_1 = sig_2 = cont$ or $sig_1 = sig_2 = exit$ or $sig_1 = \text{return } val_1;\ sig_2 = \text{return } val_2$,

8. If $sig_1 = \text{return } val_1$ and $sig_2 = \text{return } val_2$, then $\Xi'_a, \Xi'_b, \Delta \models_l NI(val_1, val_2) : \langle \tau'_{ret}, \chi_{ret} \rangle$, where $\Delta \vdash \tau_{ret} \rightsquigarrow \tau'_{ret}$ and $\Gamma[\text{return}] = \langle \tau_{ret}, \chi_{ret} \rangle$,

9. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a), \text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b), \text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon'_a)$, and $\text{DOM}(\epsilon_b) \subseteq \text{DOM}(\epsilon'_b)$.

Definition C.10 (Non-interference for declaration statements). For any security lable l, $\Gamma, \Delta \models_{pc} NI(decl) \dashv \Gamma', \Delta_1$ holds for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b, \epsilon'_a, \epsilon'_b$ if

1. $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$,
2. $\langle C; \Delta; \mu_a; \epsilon_a; decl \rangle \Downarrow \langle \Delta_1; \mu'_a; \epsilon'_a; cont \rangle$,
3. $\langle C; \Delta; \mu_b; \epsilon_b; decl \rangle \Downarrow \langle \Delta_1; \mu'_b; \epsilon'_b; cont \rangle$,

then there exists Ξ'_a, Ξ'_b such that

1. $\Gamma, \Delta \vdash_{pc} decl \dashv \Gamma', \Delta_1$,
2. $\Xi'_a, \Xi'_b, \Delta_1 \models_l \langle \mu'_a, \epsilon'_a \rangle \langle \mu'_b, \epsilon'_b \rangle : \Gamma'$ and $\Xi'_a, \Xi'_b, \Delta_1 \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$
3. *PC is used to bound writes.* For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$,
4. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$,
5. For any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$,
6. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a), \text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b), \text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon'_a)$, and $\text{DOM}(\epsilon_b) \subseteq \text{DOM}(\epsilon'_b), \Delta \subseteq \Delta_1$.

D Theorems

Theorem D.1. *If $\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi \rangle$, then $\Gamma, \Delta \models_{pc} NI(exp : \langle \tau, \chi \rangle)$.*

Theorem D.2. *If $\Gamma, \Delta \vdash_{pc} stmt \dashv \Gamma'$, then $\Gamma, \Delta \models_{pc} NI(stmt) \dashv \Gamma'$.*

Theorem D.3. *If $\Gamma, \Delta \vdash_{pc} decl \dashv \Gamma', \Delta'$, then $\Gamma, \Delta \models_{pc} NI(decl) \dashv \Gamma', \Delta'$.*

E Lemmas

Lemma E.1. *Suppose $\Xi, \Delta \models \mu$. For any l, if $l \notin \text{DOM}(\Xi)$, then $l \notin \text{DOM}(\mu)$.*

Proof. Direct proof by expanding the definition of the $\Xi, \Delta \models \mu$. $\qquad\square$

Lemma E.2. *If $\Xi, \Delta \vdash val : \langle \tau, \chi \rangle$ and $\Xi \subseteq \Xi'$, then $\Xi', \Delta \vdash val : \langle \tau, \chi \rangle$.*

Proof. By induction on the typing derivations of value typing judgement. $\qquad\square$

Lemma E.3. *Suppose $\Xi, \Delta \models \mu$ and for any $l_a \notin \text{DOM}(\Xi)$, let $\Xi' = \Xi[l_a \mapsto \langle \tau', \chi' \rangle]$, $\mu' = \mu[l_a \mapsto val]$, and $\Xi', \Delta \vdash \mu'(l_a) : \langle \tau', \chi' \rangle$. Then $\Xi', \Delta \models \mu'$.*

Proof. By Definition C.1, $l_a \notin \text{DOM}(\Xi)$ implies $l_a \notin \text{DOM}(\mu)$. For all $l \in \text{DOM}(\mu')$, there are two cases:

- $l \in \text{DOM}(\mu)$. By the definitions of μ' and Ξ' we know that for the locations in this case ($l \in \text{DOM}(\mu') \cap \text{DOM}(\mu)$), $\mu'(l) = \mu(l)$ and $\Xi'(l) = \Xi(l)$. Using $\Xi, \Delta \models \mu$, we can conclude that for the locations in this case, there exists a type, $\langle \tau, \chi \rangle = \Xi(l) = \Xi'(l)$ and $\Xi, \Delta \vdash \mu(l) : \langle \tau, \chi \rangle$. Using $\mu'(l) = \mu(l)$, we can say $\Xi, \Delta \vdash \mu'(l) : \langle \tau, \chi \rangle$. Applying Lemma E.2 with $\Xi \subseteq \Xi'$, we conclude $\Xi', \Delta \vdash \mu'(l) : \langle \tau, \chi \rangle$.
- $l = l_a$. For the last case where $l = l_a$, we can see that $\Xi'(l_a) = \langle \tau', \chi' \rangle$ and $\Xi', \Delta \vdash \mu'(l_a) : \langle \tau', \chi' \rangle$.

Therefore, we have shown that for every location, $l \in \text{DOM}(\mu')$ there exists a type, $\langle \tau, \chi \rangle = \Xi'(l)$ and $\Xi', \Delta \vdash \mu'(l) : \langle \tau, \chi \rangle$. $\qquad\square$

Lemma E.4. *If $\Xi \vdash \epsilon : \Gamma$, then for any Ξ' such that $\Xi \subseteq \Xi'$, we have $\Xi' \vdash \epsilon : \Gamma$.*

Proof. Direct proof using the definition of $\Xi' \vdash \epsilon : \Gamma$ $\qquad\square$

Lemma E.5. *Suppose $\Xi_a, \Delta \models \langle \mu_a, \epsilon_a \rangle : \Gamma$. Let $\Xi_a \subseteq \Xi'_a, \mu_a \subseteq \mu'_a$. If $\Xi'_a, \Delta \models \mu'_a$, then $\Xi'_a, \Delta \models \langle \mu'_a, \epsilon_a \rangle : \Gamma$.*

Proof. By definition, $\Xi'_a = \Xi_a \cup \{\overline{l_a \mapsto \langle \tau, \chi \rangle}\}, \mu'_a = \mu_a \cup \{\overline{l_a \mapsto val_a}\}$ and $l_a \notin \Xi_a$. This is followed by: $\mu'_a(\epsilon_a(x)) = \mu_a(\epsilon_a(x))$ and $\Xi'_a(\epsilon_a(x)) = \Xi_a(\epsilon_a(x))$, for any $x \in \epsilon_a$. We prove this lemma by induction on the $\text{DOM}(\epsilon_a)$

1. Base case. $\text{DOM}(\epsilon_a) = \emptyset$. Trivial.
2. To prove $\Xi'_a, \Delta \models \langle \mu'_a, \epsilon_a \rangle : \Gamma$, we need to show that
   a. $\Xi'_a, \Delta \models \mu'_a$ (already given),
   b. $\Xi'_a \vdash \epsilon_a : \Gamma$ (follows from Lemma E.4),
   c. for any $x \in \text{DOM}(\epsilon_a)$, we have $\epsilon_a(x) \in \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$,
   d. For all x in $\text{DOM}(\epsilon_a)$ and any $\Gamma_{clos} \subseteq \Gamma$ and any pc, if $\Gamma, \Delta \vdash_{pc} x : \tau_{clos}, \mu'_a(\epsilon_a(x)) = val'_{clos}, val_{clos} = \{clos(\epsilon_c, ...), table\ l_a(\epsilon_c, ...)\}$, and $\Xi'_a \models \epsilon_c : \Gamma_{clos}$, then $\text{DOM}(\epsilon_c) \subseteq \text{DOM}(\epsilon_a)$ and $\Xi'_a, \Delta \models \langle \mu'_a, \epsilon_c \rangle : \Gamma_{clos}$. Note that $val'_{clos} = \mu_a(\epsilon_a(x))$. This implies the ϵ_c will still satisfy $\text{DOM}(\epsilon_c) \subseteq \text{DOM}(\epsilon_a)$. By applying the induction hypothesis on $\Xi_a, \Delta \models \langle \mu_a, \epsilon_c \rangle : \Gamma$ with $\text{DOM}(\epsilon_c) \subseteq \text{DOM}(\epsilon_a)$, we can conclude that $\Xi'_a, \Delta \models \langle \mu'_a, \epsilon_c \rangle : \Gamma_{clos}$ holds.

$\square$

Lemma E.6 (Non-interference with Subtyping). *If* $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(val_a , val_b) : \langle \tau, \chi \rangle$ *and* $\chi \sqsubseteq \chi'$, *then* $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(val_a , val_b) : \langle \tau, \chi' \rangle$.

Proof. To show $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(val_a , val_b) : \langle \tau, \chi' \rangle$, we need to show the following:

1. $\Xi_a, \Delta \vdash val_a : \langle \tau, \chi' \rangle$. The above holds true using the TV-SubType rule in Appendix J, since we have the premise $\Xi_a, \Delta \vdash val_a : \langle \tau, \chi \rangle$ and $\chi \sqsubseteq \chi'$
2. $\Xi_b, \Delta \vdash val_b : \langle \tau, \chi' \rangle$. Similar to the above case.
3. if $\tau \notin \{\{\overline{f : \rho}\}, header\ \{\overline{f : \rho}\}, \overline{d\ \rho} \xrightarrow{pc} \rho_{ret}, table(pc_{tbl})\}$ and $\chi' \sqsubseteq l$, then $val_a = val_b$. Since we know $\chi \sqsubseteq \chi'$, if $\chi' \sqsubseteq l$, then $\chi \sqsubseteq l$, and $val_a = val_b$ (according to $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(val_a , val_b) : \langle \tau, \chi \rangle$).

$\square$

Lemma E.7. *If* $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(val_a , val_b) : \langle \tau, \chi \rangle$ *and* $\Xi_a \subseteq \Xi'_a$ *and* $\Xi'_b \subseteq \Xi'_b$, *then* $\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(val_a , val_b) : \langle \tau, \chi \rangle$.

Proof. Direct proof using the definition of $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(val_a , val_b) : \langle \tau, \chi \rangle$ and applying Lemma E.2, and Lemma E.4 $\square$

Definition E.8. $unused(\langle \mu, \epsilon \rangle, x, l_a)$ holds when the following are satisfied:

- if $x \in \text{DOM}(\epsilon)$, then $\epsilon(x) \neq l_a$,
- for all $y \in \text{DOM}(\epsilon)$, if $\mu(\epsilon(y)) = val_{clos}$, where $val_{clos} \in \{clos(\epsilon_c, ...), table\ l(\epsilon_c, ...)\}$, then $unused(\langle \mu, \epsilon_{clos} \rangle, x, l_a)$ holds.

Lemma E.9. *Suppose* $\Xi, \Delta \models \langle \mu, \epsilon \rangle : \Gamma$ *and* $\langle \Delta, \mu, \epsilon, stmt \rangle \Downarrow \langle \mu', \epsilon', sig \rangle$. *For some* ϵ_a *with* $\Xi_a, \Delta \models \langle \mu, \epsilon_a \rangle : \Gamma'$ *and some variable* $x \in \text{DOM}(\epsilon_a)$, *if* $unused(\langle \mu, \epsilon \rangle, x, \epsilon_a(x))$, *then* $\mu(\epsilon_a(x)) = \mu'(\epsilon_a(x))$.

Proof. By induction on the evaluation derivation of $stmt$. Involves mutual induction of Lemma E.10, Lemma E.9, and Lemma E.11. Some of the interesting bits include concluding $\Xi, \Delta' \models \langle \mu', \epsilon_a \rangle : \Gamma'$ (using the definition), $\text{UNUSED}(\langle \mu', \epsilon' \rangle, x, \epsilon_a(x))$, and the fact that declaration introduces new locations into μ'. $\square$

Lemma E.10. *Suppose* $\Xi, \Delta \models \langle \mu, \epsilon \rangle : \Gamma$ *and* $\langle \Delta, \mu, \epsilon, exp \rangle \Downarrow \langle \mu', val \rangle$. *For some* ϵ_a *with* $\Xi_a, \Delta \models \langle \mu, \epsilon_a \rangle : \Gamma'$ *and some variable* $x \in \text{DOM}(\epsilon_a)$, *if* $unused(\langle \mu, \epsilon \rangle, x, \epsilon_a(x))$, *then* $\mu(\epsilon_a(x)) = \mu'(\epsilon_a(x))$.

Proof. By induction on the evaluation derivation of exp. Involves mutual induction of Lemma E.10, Lemma E.9, and Lemma E.11. Some of the interesting bits include concluding $\Xi, \Delta \models \langle \mu', \epsilon_a \rangle : \Gamma'$ (using the definition), $\text{UNUSED}(\langle \mu', \epsilon \rangle, x, \epsilon_a(x))$. $\square$

Lemma E.11. *Suppose* $\Xi, \Delta \models \langle \mu, \epsilon \rangle : \Gamma$ *and* $\langle \Delta, \mu, \epsilon, decl \rangle \Downarrow \langle \Delta', \mu', \epsilon', cont \rangle$. *For some* ϵ_a *with* $\Xi_a, \Delta \models \langle \mu, \epsilon_a \rangle : \Gamma'$ *and some variable* $x \in \text{DOM}(\epsilon_a)$, *if* $unused(\langle \mu, \epsilon \rangle, x, \epsilon_a(x))$, *then* $\mu(\epsilon_a(x)) = \mu'(\epsilon_a(x))$.

Proof. By induction on the evaluation derivation of $decl$. Involves mutual induction of Lemma E.10, Lemma E.9, and Lemma E.11. Some of the interesting bits include concluding $\Xi, \Delta' \models \langle \mu', \epsilon_a \rangle : \Gamma'$ (using the definition), $\text{UNUSED}(\langle \mu', \epsilon' \rangle, x, \epsilon_a(x))$. $\square$

Lemma E.12. *Suppose* $\Xi_a, \Xi_b, \Delta \models_{pc} \langle \mu_a, \epsilon_{a1} \rangle \langle \mu_b, \epsilon_{b1} \rangle : \Gamma_1$, $\Xi_a, \Xi_b, \Delta \models_{pc} \langle \mu_a, \epsilon_{a2} \rangle \langle \mu_b, \epsilon_{b2} \rangle : \Gamma_2$, *where* $\epsilon_{a2} = \overline{\{x \mapsto l_a\}}$ *and* $\epsilon_{b2} = \overline{\{x \mapsto l_b\}}$ *and* $\Gamma_2 = [\overline{\{x \mapsto \langle \tau, \chi \rangle\}}]$. *Then* $\Xi_a, \Xi_b, \Delta \models_{pc} \langle \mu_a, \epsilon_{a1} \overline{[x \mapsto l_a]} \rangle \langle \mu_b, \epsilon_{b1} \overline{[x \mapsto l_a]} \rangle : \Gamma_1 \overline{[x \mapsto \langle \tau, \chi \rangle]}$.

Proof. To prove all the requirements of the Definition C.4, we use the fact that independently all $y \in \text{DOM}(\epsilon_{a1}) = \text{DOM}(\epsilon_{b1})$ and $y \in \text{DOM}(\epsilon_{a2}) = \text{DOM}(\epsilon_{b2})$ satisfy the required properties. Now, in the extended environment all $y \in \text{DOM}(\epsilon_{a1} \overline{[x \mapsto l_a]}) = \text{DOM}(\epsilon_{b1} \overline{[x \mapsto l_b]})$, will also satisfy the properties by reducing to either an element in $\text{DOM}(\epsilon_{a1})$ or $\text{DOM}(\epsilon_{a2})$ $\square$

F L-value Evaluation Rules

For a term to be a well-formed l-value, the directionality of the term should be inout. Therefore, only the following typing judgements can be used in the derivation of a well-formed l-value:

Valid L-Value Expression Typing Rules.

- T-Var
- T-Index
- T-MemHdr
- T-MemRec

Therefore, l-value is given by the following grammar:

$$base ::= x$$
$$lval ::= base \mid lval.f_i \mid lval[n]$$

lval_base. Note that only $base \in \epsilon$, where ϵ is the environment in which the l-value is evaluated. Other l-values like the ones corresponding to a header field or an array index do not map to a location in the environment. Instead, it is the header variable or the array variable that has an entry in the environment. For instance, to write to a header field $lval.f_i$ where $lval$ is the l-value of the header that needs to be updated, the value of the header variable given by $lval$ is updated and there is no variable $lval.f_i$ in ϵ. The value at location pointed by $\epsilon(lval)$ is then overwritten with the new header value. Therefore, we define a function LVAL_BASE($lval$) to return the l-value of the base variable that will be touched while writing to the $lval$. This is inductively defined using:

$$\text{LVAL\_BASE}(base) = base$$
$$\text{LVAL\_BASE}(lval.f_i) = \text{LVAL\_BASE}(lval)$$
$$\text{LVAL\_BASE}(lval[n]) = \text{LVAL\_BASE}(lval)$$

F.1 L-value Equality Relation

We inductively define an equality relation on l-value expressions as follows:

$$x =_{lval} x$$

$$\frac{lval_a =_{lval} lval_b}{lval_a.f =_{lval} lval_b.f}$$

$$\frac{lval_a =_{lval} lval_b \qquad n_a = n_b \qquad n_a : int}{lval_a[n_a] =_{lval} lval_b[n_b]}$$

Definition F.1. For any security label l, $\Gamma, \Delta \models_{pc}$ LVAL_EVAL($lval\_exp : \langle \tau, \chi \rangle$) holds for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b$, if

1. $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$,
2. $\langle C; \Delta; \mu_a; \epsilon_a; lval\_exp \rangle \Downarrow_{lval} \langle \mu'_a; lval_a \rangle$ and $\langle C; \Delta; \mu_b; \epsilon_b; lval\_exp \rangle \Downarrow_{lval} \langle \mu'_b; lval_b \rangle$

then there exists some Ξ'_a, Ξ'_b such that

1. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b$, DOM($\mu_a$) $\subseteq$ DOM(μ'_a) and DOM(μ_b) $\subseteq$ DOM(μ'_b),
2. $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$,
3. For any $l_a \in$ DOM(μ_a) such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in$ DOM(μ_b) such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$,
4. if $\chi \sqsubseteq l$, then $lval_a =_{lval} lval_b$,
5. LVAL_BASE($lval_a$) $\in$ DOM(ϵ_a), LVAL_BASE($lval_b$) $\in$ DOM(ϵ_b) and LVAL_BASE($lval_a$) = LVAL_BASE($lval_b$),
6. $\Gamma, \Delta \vdash_{pc} lval_a : \langle \tau, \chi \rangle$ and $\Gamma, \Delta \vdash_{pc} lval_b : \langle \tau, \chi \rangle$,
7. for any $l'_a \in$ DOM(μ_a) and $l'_b \in$ DOM(μ_b) such that $\Xi_a, \Delta \vdash \mu_a(l'_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l'_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_a(l'_a) = \mu_{a1}(l'_a)$ and $\mu_b(l'_b) = \mu_{b1}(l'_b)$

Lemma F.2. *For any security label l, if $\Gamma, \Delta \vdash_{pc} lval\_exp : \langle \tau, \chi \rangle$, then $\Gamma, \Delta \models_{pc}$ LVAL_EVAL($lval\_exp : \langle \tau, \chi \rangle$).*

Proof. We prove this by induction on the typing derivation of $\Gamma, \Delta \vdash_{pc} lval\_exp : \langle \tau, \chi \rangle$, where we choose the last typing rule to be the different cases.

1. **T-Var**
   Consider the case where the last typing rule in the derivation of l-value expression is T-Var

$$\frac{x \in \text{DOM}(\Gamma) \qquad \Gamma(x) = \langle \tau, \chi \rangle}{\Gamma, \Delta \vdash_{pc} x : \langle \tau, \chi \rangle \text{ goes inout}} \text{ T-Var}$$

then we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b$, if

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma \tag{1}$$

and x is evaluated to get its l-value in two initial configuration $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows:

$$\overline{\langle C, \Delta, \mu_a, \epsilon_a, x \rangle \ \Downarrow_{lval} \langle \mu_a, x \rangle} \qquad \qquad \overline{\langle C, \Delta, \mu_b, \epsilon_b, x \rangle \ \Downarrow_{lval} \langle \mu_b, x \rangle}$$

then there exists some Ξ'_a and Ξ'_b satisfying the following properties:

a. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a), \text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$, and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$, where in this case $\mu'_a = \mu_a$ and $\mu'_b = \mu_b$,

b. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$,

c. $\text{LVAL\_BASE}(lval_a) \in \text{DOM}(\epsilon_a), \text{LVAL\_BASE}(lval_b) \in \text{DOM}(\epsilon_b)$ and $\text{LVAL\_BASE}(lval_a) = \text{LVAL\_BASE}(lval_b)$.

d. $\Gamma, \Delta \vdash_{pc} lval_a : \langle \tau, \chi \rangle$ and $\Gamma, \Delta \vdash_{pc} lval_b : \langle \tau, \chi \rangle$,

e. for any $l'_a \in \text{DOM}(\mu_a)$ and $l'_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l'_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l'_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_a(l'_a) = \mu'_a(l'_a)$ and $\mu_b(l'_b) = \mu'_b(l'_b)$.

Showing that Item 1a holds for $\Xi'_a = \Xi_a, \Xi'_b = \Xi_b, \mu'_a = \mu_a$, and $\mu'_b = \mu_b$ is same as proving Equation (1), which is already given. Item 1b is trivial as the memory stores do not change. So is Item 1e. Item 1c is immediate since $lval_a =_{lval} x =_{lval} lval_b$. Additionally, $x \in \text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$ since $x \in \text{DOM}(\Gamma), \Xi_a \vdash \epsilon_a : \Gamma$, and $\Xi_b \vdash \epsilon_b : \Gamma$. Item 1d follows from $\Gamma, \Delta \vdash_{pc} x : \langle \tau, \chi \rangle$.

2. **T-MemRec**

Consider the case where the last typing rule in the derivation of l-value expression is T-MemRec

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle \{\overline{f_i : \langle \tau_i, \chi_i \rangle}\}, \bot \rangle \ goes \ d}{\Gamma, \Delta \vdash_{pc} exp.f_i : \langle \tau_i, \chi_i \rangle \ goes \ d} \ \text{T-MemRec}$$

then we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b$, if

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$$

and $exp.f_i$ is evaluated to get its l-value in two initial configuration $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows:

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, exp \rangle \ \Downarrow_{lval} \langle \mu'_a, lval_a \rangle}{\langle C, \Delta, \mu_a, \epsilon_a, exp.f_i \rangle \ \Downarrow_{lval} \langle \mu'_a, lval_a.f_i \rangle} \qquad \frac{\langle C, \Delta, \mu_b, \epsilon_b, exp \rangle \ \Downarrow_{lval} \langle \mu'_b, lval_b \rangle}{\langle C, \Delta, \mu_b, \epsilon_b, exp.f_i \rangle \ \Downarrow_{lval} \langle \mu'_b, lval_b.f_i \rangle}$$

then there exists some Ξ'_a and Ξ'_b satisfying the following properties:

a. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$ and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$. Also, $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$.

b. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$,

c. $\text{LVAL\_BASE}(lval_a.f_i) = \text{LVAL\_BASE}(lval_b.f_i), \text{LVAL\_BASE}(lval_a.f_i) = \text{LVAL\_BASE}(lval_a) \in \text{DOM}(\epsilon_a)$ and $\text{LVAL\_BASE}(lval_b.f_i) = \text{LVAL\_BASE}(lval_b) \in \text{DOM}(\epsilon_b)$,

d. $\Gamma, \Delta \vdash_{pc} lval_a.f_i : \langle \tau_i, \chi_i \rangle$ and $\Gamma, \Delta \vdash_{pc} lval_b.f_i : \langle \tau_i, \chi_i \rangle$,

e. if $\chi \sqsubseteq l$, then $lval_a.f_i =_{lval} lval_b.f_i$.

f. for any $l'_a \in \text{DOM}(\mu_a)$ and $l'_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l'_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l'_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_a(l'_a) = \mu'_a(l'_a)$ and $\mu_b(l'_b) = \mu'_b(l'_b)$

By applying induction hypothesis on the typing derivation of exp, we conclude $\Gamma, \Delta \models_{pc} \text{LVAL\_EVAL}(exp : \langle \{\overline{f_i : \langle \tau_i, \chi_i \rangle}\}, \bot \rangle)$. Since exp is evaluated to get its l-value in two initial configuration $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$, where $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$, there exists some Ξ'_{a1} and Ξ'_{b1} satisfying $\Xi_a \subseteq \Xi'_{a1}$ and $\Xi_b \subseteq \Xi'_{b1}, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a), \text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$ and the following:

$$\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \langle \mu_{a1}, \epsilon_a \rangle \langle \mu_{b1}, \epsilon_b \rangle : \Gamma, \tag{1}$$

$$\text{LVAL\_BASE}(lval_a) = \text{LVAL\_BASE}(lval_b), \text{LVAL\_BASE}(lval_a) \in \text{DOM}(\epsilon_a) \text{ and } \text{LVAL\_BASE}(lval_b) \in \text{DOM}(\epsilon_b) \tag{2}$$

$$\Gamma, \Delta \vdash_{pc} lval_a : \langle \{\overline{f_i : \langle \tau_i, \chi_i \rangle}\}, \bot \rangle \text{ and } \Gamma, \Delta \vdash_{pc} lval_b : \langle \{\overline{f_i : \langle \tau_i, \chi_i \rangle}\}, \bot \rangle \tag{3}$$

$$\text{if } \bot \sqsubseteq l, \text{ then } lval_a =_{lval} lval_b. \tag{4}$$

and For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$. With this we have shown Item 2b.

Also, for any $l'_a \in \text{DOM}(\mu_a)$ and $l'_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l'_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l'_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_a(l'_a) = \mu_{a1}(l'_a)$ and $\mu_b(l'_b) = \mu_{b1}(l'_b)$. This proves Item 2f. Equation (1) proves Item 2a. Proof of Item 2c follows from the definition of LVAL_BASE and Equation (2). Using Equation (3) and T-MemRec we conclude Item 2d. Equation (4) with the definition Appendix F.1 proves Item 2e. Note that we have proved that $lval_a.f_i =_{lval} lval_b.f_i$.

3. **T-MemHdr**

   Consider the case where the last typing rule in the derivation of l-value expression is T-MemHdr

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle header\{\overline{f_i : \langle \tau_i, \chi_i \rangle}\}, \bot \rangle \; goes \; d}{\Gamma, \Delta \vdash_{pc} exp.f_i : \langle \tau_i, \chi_i \rangle \; goes \; d} \; \text{T-MemHdr}$$

   then showing all the required properties for an evaluation rule as follows is similar to the T-MemRec case.

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, exp \rangle \Downarrow_{lval} \langle \mu_{a1}, lval_a \rangle}{\langle C, \Delta, \mu_a, \epsilon_a, exp.f \rangle \Downarrow_{lval} \langle \mu_{a1}, lval_a.f \rangle} \qquad \frac{\langle C, \Delta, \mu_b, \epsilon_b, exp \rangle \Downarrow_{lval} \langle \mu_{b1}, lval_b \rangle}{\langle C, \Delta, \mu_b, \epsilon_b, exp.f \rangle \Downarrow_{lval} \langle \mu_{b1}, lval_b.f \rangle}$$

4. **T-Index**

   Consider the case where the last typing rule in the derivation of l-value expression is T-Index

$$\frac{\begin{array}{c}\Gamma, \Delta \vdash_{pc} exp_1 : \langle \langle \tau, \chi_1 \rangle [n], \bot \rangle \; goes \; d \\ \Gamma, \Delta \vdash_{pc} exp_2 : \langle bit \langle 32 \rangle, \chi_2 \rangle \\ \chi_2 \sqsubseteq \chi_1 \end{array}}{\Gamma, \Delta \vdash_{pc} exp_1[exp_2] : \langle \tau, \chi_1 \rangle \; goes \; d} \; \text{T-Index}$$

   then we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b$, if

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$$

   and $exp_1[exp_2]$ is evaluated to get its l-value in two initial configuration $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows:

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, exp_1 \rangle \Downarrow_{lval} \langle \mu_{a1}, lval_a \rangle \qquad \langle C, \Delta, \mu_{a1}, \epsilon_a, exp_2 \rangle \Downarrow \langle \mu_{a2}, n_a \rangle}{\langle C, \Delta, \mu_a, \epsilon_a, exp_1[exp_2] \rangle \Downarrow_{lval} \langle \mu_{a2}, lval_a[n_a] \rangle}$$

$$\frac{\langle C, \Delta, \mu_b, \epsilon_b, exp_1 \rangle \Downarrow_{lval} \langle \mu_{b1}, lval_b \rangle \qquad \langle C, \Delta, \mu_{b1}, \epsilon_b, exp_2 \rangle \Downarrow \langle \mu_{b2}, n_b \rangle}{\langle C, \Delta, \mu_b, \epsilon_b, exp_1[exp_2] \rangle \Downarrow_{lval} \langle \mu_{b2}, lval_b[n_b] \rangle}$$

   then there exists some Ξ'_a and Ξ'_b satisfying the following properties:

   a. $\Xi_a \subseteq \Xi'_a$, $\Xi_b \subseteq \Xi'_b$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$ and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$, where $\mu'_a = \mu_{a2}$ and $\mu'_b = \mu_{b2}$. Also, $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$.
   b. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$,
   c. LVAL_BASE$(lval_a[n_a]) = $ LVAL_BASE$(lval_b[n_b])$, LVAL_BASE$(lval_a[n_a]) \in \text{DOM}(\epsilon_a)$ and LVAL_BASE$(lval_b[n_b]) \in \text{DOM}(\epsilon_b)$,
   d. $\Gamma, \Delta \vdash_{pc} lval_a[n_a] : \langle \tau, \chi_1 \rangle$ and $\Gamma, \Delta \vdash_{pc} lval_b[n_b] : \langle \tau, \chi_1 \rangle$,
   e. if $\chi_1 \sqsubseteq l$, then $lval_a[n_a] =_{lval} lval_b[n_b]$.
   f. for any $l'_a \in \text{DOM}(\mu_a)$ and $l'_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l'_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l'_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_a(l'_a) = \mu_{a2}(l'_a)$ and $\mu_b(l'_b) = \mu_{b2}(l'_b)$

   By applying induction hypothesis on the typing derivation of exp_1, we conclude $\Gamma, \Delta \models_{pc}$ LVAL_EVAL$(exp_1 : \langle \langle \tau, \chi_1 \rangle [n], \bot \rangle)$. Since exp_1 is evaluated to get its l-value in two initial configuration $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$, where $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$, there exists some Ξ'_{a1} and Ξ'_{b1} satisfying $\Xi_a \subseteq \Xi'_{a1}$ and $\Xi_b \subseteq \Xi'_{b1}$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_{a1})$, $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu_{b1})$ and the following:

$$\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \langle \mu_{a1}, \epsilon_a \rangle \langle \mu_{b1}, \epsilon_b \rangle : \Gamma, \tag{1}$$

$$\text{LVAL\_BASE}(lval_a) = \text{LVAL\_BASE}(lval_b), \text{LVAL\_BASE}(lval_a) \in \text{DOM}(\epsilon_a) \text{ and LVAL\_BASE}(lval_b) \in \text{DOM}(\epsilon_b) \tag{2}$$

$$\Gamma, \Delta \vdash_{pc} lval_a : \langle \langle \tau, \chi_1 \rangle [n], \bot \rangle \text{ and } \Gamma, \Delta \vdash_{pc} lval_b : \langle \langle \tau, \chi_1 \rangle [n], \bot \rangle \tag{3}$$

$$\text{if } \bot \sqsubseteq l, \text{ then } lval_a =_{lval} lval_b. \tag{4}$$

   and For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{a1}(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{b1}(l_b) = \mu_b(l_b)$. Also, for any $l'_a \in \text{DOM}(\mu_a)$ and $l'_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l'_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l'_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_a(l'_a) = \mu_{a1}(l'_a)$ and $\mu_b(l'_b) = \mu_{b1}(l'_b)$.

By applying induction hypothesis of Theorem D.1 on the exp_2, we get $\Gamma, \Delta \models_{pc} \text{NI}(exp_2 : \langle bit\langle 32\rangle, \chi_2\rangle)$. Since exp_2 is evaluated in an initial configuration satisfying Equation (1), we can conclude that there exists some Ξ'_{a2} and Ξ'_{b2} satisfying $\Xi'_{a1} \subseteq \Xi'_{a2}$, $\Xi'_{b1} \subseteq \Xi'_{b2}$, and $\text{DOM}(\mu_{a1}) \subseteq \text{DOM}(\mu_{a2})$, $\text{DOM}(\mu_{b1}) \subseteq \text{DOM}(\mu_{b2})$ and the following:

$$\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_a\rangle \langle \mu_{b2}, \epsilon_b\rangle : \Gamma$$

$$\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \text{NI}(n_a , n_b) : \langle bit\langle 32\rangle, \chi_2\rangle$$

And finally, For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{a2}(l_a) = \mu_{a1}(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{b2}(l_b) = \mu_{b1}(l_b)$. This equation proves Item 4b, since $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_{a1}) \subseteq \text{DOM}(\mu_{a2})$ and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu_{b1}) \subseteq \text{DOM}(\mu_{b2})$. Similarly, we prove Item 4f

Since the type of n_a is $\tau = bit\langle 32\rangle$, we can say that $n_a = n_b$ if the $\chi_1 \sqsubseteq l$. Therefore, Equation (4) proves Item 4e. Using the definition of LVAL_BASE along with Equation (2) we can conclude Item 4c. Equation (3) with T-Index proves Item 4d.

□

G L-value Writing

Lemma G.1. *Let* $\Gamma, \Delta \vdash_{pc} lval\_exp : \langle \tau, \chi\rangle$, $\langle C; \Delta; \mu; \epsilon; lval\_exp\rangle \Downarrow_{lval} \langle \mu'; lval\rangle$. *Suppose* $\langle C, \Delta, \mu_1, \epsilon_1, lval\rangle \Downarrow \langle \mu_2, val\rangle$, $\text{DOM}(\mu') \subseteq \text{DOM}(\mu_1)$ *and* $\text{DOM}(\epsilon) \subseteq \text{DOM}(\epsilon_1)$. *Then* $\mu_2 = \mu_1$.

Proof. By induction on the typing derivation of $lval\_exp$. Intuitively, $lval$ has no unevaluated expression, so evaluating a normalized value will not have side-effects.   □

Lemma G.2. *Let* $\Gamma, \Delta \vdash_{pc} lval\_exp : \langle \tau, \chi\rangle$, $\langle C; \Delta; \mu_a; \epsilon_a; lval\_exp\rangle \Downarrow_{lval} \langle \mu'_a; lval_a\rangle$ *and* $\langle C; \Delta; \mu_b; \epsilon_b; lval\_exp\rangle \Downarrow_{lval} \langle \mu'_b; lval_b\rangle$.
Suppose $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a\rangle \langle \mu_b, \epsilon_b\rangle : \Gamma$, $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \langle \mu_{a1}, \epsilon_a\rangle \langle \mu_{b1}, \epsilon_b\rangle : \Gamma$, *and* $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(val_a , val_b) : \langle \tau, \chi\rangle$, *where* $\Xi_a \subseteq \Xi_{a1}$, $\Xi_b \subseteq \Xi_{b1}$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_{a1})$, $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu_{b1})$.
If $\langle C, \Delta, \mu_{a1}, \epsilon_a, lval_a := val_a\rangle \Downarrow_{write} \mu'_{a1}$, $\langle C, \Delta, \mu_{b1}, \epsilon_b, lval_b := val_b\rangle \Downarrow_{write} \mu'_{b1}$, *then*

1. $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(\mu'_{a1}(\epsilon_a(\text{LVAL\_BASE}(lval_a))) , \mu'_{b1}(\epsilon_b(\text{LVAL\_BASE}(lval_b))) : \Gamma(\text{LVAL\_BASE}(lval_b))$,
2. *For any* $l_a \in \text{DOM}(\mu_a)$ *such that* $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, *where* $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, *then* $\mu'_{a1}(l_a) = \mu_{a1}(l_a)$. *Similarly for any* $l_b \in \text{DOM}(\mu_b)$ *such that* $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, *where* $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, *then* $\mu'_{b1}(l_b) = \mu_{b1}(l_b)$.
3. *for any* $l_a \in \text{DOM}(\mu_{a1})$ *and* $l_b \in \text{DOM}(\mu_{b1})$ *such that* $l_b \neq \epsilon_b(\text{LVAL\_BASE}(lval_b))$ *and* $l_a \neq \epsilon_a(\text{LVAL\_BASE}(lval_a))$, *we have* $\mu'_{a1}(l_a) = \mu_{a1}(l_a)$ *and* $\mu'_{b1}(l_b) = \mu_{b1}(l_b)$,

Proof. By induction hypothesis on the typing derivation of $lval\_exp$.

1. **T-Var**
   If the l-value expression's typing derivation ends with a variable typing rule, then write to the l-value follows the following evaluation, where $\mu'_{a1} = \mu_{a1}[l_a := val_a]$, and $\mu'_{b1} = \mu_{b1}[l_b := val_b]$.

$$\frac{\epsilon_a(x) = l_a}{\langle C, \Delta, \mu_{a1}, \epsilon_a, x := val_a\rangle \Downarrow_{write} \mu_{a1}[l_a := val_a]}$$

$$\frac{\epsilon_b(x) = l_b}{\langle C, \Delta, \mu_{b1}, \epsilon_b, x := val_b\rangle \Downarrow_{write} \mu_{b1}[l_b := val_b]}$$

   According to the evaluation rule, $\mu'_{a1}(\epsilon_a(\text{LVAL\_BASE}(x)) = \mu'_{a1}(\epsilon_a(x)) = val_a$ and $\mu'_{b1}(\epsilon_b(\text{LVAL\_BASE}(x)) = \mu'_b(\epsilon_b(x)) = val_b$. Since we already know that $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(val_a , val_b) : \langle \tau, \chi\rangle$, we have proved the requirement. Since the memory store doesn't change for other location's besides that of x, showing the other two requirements are direct.

2. **T-Mem**
   If the l-value expression's $\Gamma, \Delta \vdash_{pc} exp.f_i : \langle \tau_i, \chi_i\rangle$ typing derivation ends with a T-MemRec rule, then write to the l-value follows the following evaluation, where $\mu'_{a1} = \mu_{a3}$ and $\mu'_{b1} = \mu_{b3}$.

$$\frac{\langle C, \Delta, \mu_{a1}, \epsilon_a, lval_a\rangle \Downarrow \langle \mu_{a2}, \{\overline{f_j = val_{f_a}}\}\rangle \qquad \langle C, \Delta, \mu_{a2}, \epsilon_a, lval_a := \{f_i = val_a, \overline{f_{j\neq i} = val_{f_a}}\}\rangle \Downarrow_{write} \mu_{a3}}{\langle C, \Delta, \mu_{a1}, \epsilon_a, lval_a.f_i := val_a\rangle \Downarrow_{write} \mu_{a3}}$$

$$\frac{\langle C, \Delta, \mu_{b1}, \epsilon_b, lval_b\rangle \Downarrow \langle \mu_{b2}, \{\overline{f_j = val_{f_b}}\}\rangle \qquad \langle C, \Delta, \mu_{b2}, \epsilon_b, lval_b := \{f_i = val_b, \overline{f_{j\neq i} = val_{f_b}}\}\rangle \Downarrow_{write} \mu_{b3}}{\langle C, \Delta, \mu_{b1}, \epsilon_b, lval_b.f_i := val_b\rangle \Downarrow_{write} \mu_{b3}}$$

   We know that $\text{LVAL\_BASE}(lval_a.f_i) = \text{LVAL\_BASE}(lval_b.f_i)$ using Lemma F.2. We can have two cases:

- $\chi_i \sqsubseteq l$. According to Lemma F.2, this implies that $lval_a.f_i =_{lval} lval_b.f_i$. Therefore, by inversion of the equality defined in Appendix F.1, $lval_a =_{lval} lval_b$. By using Lemma G.1 to get the value of the respective l-value, we get $\mu_{a2} = \mu_{a1}$ and $\mu_{b2} = \mu_{b1}$. This implies $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma_1$. $lval_a$ and $lval_b$ are returned by sub-expression exp of $lval\_exp = exp.f_i$, therefore, by Lemma F.2, we have $\Gamma, \Delta \vdash_{pc} lval_a : \langle \{f : \langle \tau_i, \chi_i \rangle\}, \bot \rangle$ and $\Gamma, \Delta \vdash_{pc} lval_b : \langle \{f : \langle \tau_i, \chi_i \rangle\}, \bot \rangle$. Therefore, we can apply induction hypothesis of Theorem D.1 to evaluate the value of a well-typed expression under two different configurations. By applying induction hypothesis of Theorem D.1 on evaluating $lval_a =_{lval} lval_b$, we get $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(\{\overline{f_j = val_{f_a}}\}, \{\overline{f_j = val_{f_b}}\}) : \langle \{f : \langle \tau_i, \chi_i \rangle\}, \bot \rangle$. Since val_a and val_b are given to be non-interfering, we have

$$\Xi_a, \Xi_b, \Delta \models_l \text{NI}(\{f_i = val_a, \overline{f_{j \neq i} = val_{f_a}}\}, \{f_i = val_b, \overline{f_{j \neq i} = val_{f_b}}\}) : \langle \{f : \langle \tau_i, \chi_i \rangle\}, \bot \rangle$$

We can apply the induction hypothesis of this lemma to write to two l-value expressions generated from a well-typed exp, and conclude that

$$\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(\mu_{a3}(\epsilon_a(\text{LVAL\_BASE}(lval_a)), \mu_{b3}(\epsilon_b(\text{LVAL\_BASE}(lval_b))) : \Gamma(\text{LVAL\_BASE}(lval_b))$$

Since $\text{LVAL\_BASE}(lval_a.f_i) = \text{LVAL\_BASE}(lval_a)$ and $\text{LVAL\_BASE}(lval_b.f_i) = \text{LVAL\_BASE}(lval_b)$, we have proved the necessary. Also, the two other requirements follow from the results of this induction hypothesis.

- $\chi_i \not\sqsubseteq l$. Since $lval_a$ and $lval_b$ are evaluated from exp, by using Lemma F.2, we can conclude that $lval_a =_{lval} lval_b$ because $\bot \sqsubseteq l$. Also, the type of both $lval_a$ and $lval_b$ is $\langle \{f : \langle \tau_i, \chi_i \rangle\}, \bot \rangle$. Now, by applying induction hypothesis of Theorem D.1 to evaluate the value of a well-typed expression $lval_a =_{lval} lval_b$ under two different configurations, we conclude $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(\{\overline{f_j = val_{f_a}}\}, \{\overline{f_j = val_{f_b}}\}) : \langle \{f : \langle \tau_i, \chi_i \rangle\}, \bot \rangle$. Similar to the previous case, we can apply the induction hypothesis of this lemma on the lval-write to $lval_a$ and $lval_b$ because they are both evaluated from sub-expression exp of $exp.f_i$ (this can be checked from the lval-evaluation derivation). This can conclude that $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(\mu_{a3}(\epsilon_a(\text{LVAL\_BASE}(lval_a)), \mu_{b3}(\epsilon_b(\text{LVAL\_BASE}(lval_b))) : \Gamma(\text{LVAL\_BASE}(lval_b))$. Also, the two other requirements follow from the results of this induction hypothesis.

3. **T-Hdr**
   Follows similarly.

$$\frac{\langle C, \Delta, \mu, \epsilon, lval \rangle \Downarrow \langle \mu_1, \text{header}\{valid = true, \overline{f = val_f}\} \rangle \quad \langle C, \Delta, \mu_1, \epsilon, lval := \text{header}\{valid = true, f_i = val, \overline{f_{\neq i} = val_f}\} \rangle \Downarrow_{write} \mu_2}{\langle C, \Delta, \mu, \epsilon, lval.f_i := val \rangle \Downarrow_{write} \mu_2}$$

4. **T-Index**
   If the l-value expression's typing derivation ends with a T-Index rule

$$\frac{\begin{array}{c} \Gamma, \Delta \vdash_{pc} exp_1 : \langle \langle \tau, \chi_1 \rangle[n], \bot \rangle \text{ goes } d \\ \Gamma, \Delta \vdash_{pc} exp_2 : \langle bit\langle 32 \rangle, \chi_2 \rangle \\ \chi_2 \sqsubseteq \chi_1 \end{array}}{\Gamma, \Delta \vdash_{pc} exp_1[exp_2] : \langle \tau, \chi_1 \rangle \text{ goes } d} \text{ T-Index}$$

then write to the l-value follows the following evaluation, where $\mu'_{a1} = \mu_{a3}$ and $\mu'_{b1} = \mu_{b3}$.

$$\frac{\langle C, \Delta, \mu_{a1}, \epsilon_a, lval_a \rangle \Downarrow \langle \mu_{a2}, stack\ \tau\ \{\overline{val_a}\} \rangle \quad \langle C, \Delta, \mu_{a2}, \epsilon_a, lval_a := stack\ \tau\ \{..., val_{a_{n_a-1}}, val_a, val_{a_{n_a+1}}, ...\} \rangle \Downarrow_{write} \mu_{a3}}{\langle C, \Delta, \mu_{a1}, \epsilon_a, lval_a[n_a] := val_a \rangle \Downarrow_{write} \mu_{a3}}$$

$$\frac{\langle C, \Delta, \mu_{b1}, \epsilon_b, lval_b \rangle \Downarrow \langle \mu_{b2}, stack\ \tau\ \{\overline{val_b}\} \rangle \quad \langle C, \Delta, \mu_{b2}, \epsilon_b, lval_b := stack\ \tau\ \{..., val_{b_{n_b-1}}, val_b, val_{b_{n_b+1}}, ...\} \rangle \Downarrow_{write} \mu_{b3}}{\langle C, \Delta, \mu_{b1}, \epsilon_b, lval_b[n_b] := val_b \rangle \Downarrow_{write} \mu_{b3}}$$

We know that $\text{LVAL\_BASE}(lval_a[n_a]) = \text{LVAL\_BASE}(lval_b[n_b])$ using Lemma F.2 on exp_1. We can have two cases:

- $\chi_1 \sqsubseteq l$. Lemma F.2 implies that $lval_a[n_a] =_{lval} lval_b[n_b]$. Therefore, $lval_a =_{lval} lval_b$ and $n_a = n_b$. Using similar argument to the "record" case, we can show that $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(\mu_{a3}(\epsilon_a(\text{LVAL\_BASE}(lval_a)), \mu_{b3}(\epsilon_b(\text{LVAL\_BASE}(lval_b))) : \Gamma(\text{LVAL\_BASE}(lval_b))$. And by using the definition of LVAL_BASE, we conclude

$$\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(\mu_{a3}(\epsilon_a(\text{LVAL\_BASE}(lval_a[n_a])), \mu_{b3}(\epsilon_b(\text{LVAL\_BASE}(lval_b[n_b]))) : \Gamma(\text{LVAL\_BASE}(lval_b))$$

- $\chi_1 \not\sqsubseteq l$. We can have the following cases:

– Case $\chi_2 \sqsubseteq l$. In this case $n_a = n_b$. (using induction hypothesis of Theorem D.1 on exp_2 evaluation in $\Downarrow_{lval}$ of $exp_1[exp_2]$). Observe that $\Gamma, \Delta \vdash_{pc} exp_1 : \langle\langle \tau, \chi_1\rangle[n], \bot\rangle$, and because $\bot \sqsubseteq l$, we have $lval_a =_{lval} lval_b$ (by using Lemma F.2, $lval_a$ is the lvalue generated from exp_1). By applying Theorem D.1 on $lval_a$'s evaluation, we get $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(stack\ \tau\ \{\overline{val_a}\}\ ,\ stack\ \tau\ \{\overline{val_b}\}) : \langle\langle \tau, \chi_1\rangle[n], \bot\rangle$. Similar to the previous cases, by applying induction hypothesis of this lemma on the lval-write to $lval_a$ and $lval_b$ that are generated from the same $lval\_exp$, we can conclude that

$$\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(\mu_{a3}(\epsilon_a(\text{LVAL\_BASE}(lval_a)))\ ,\ \mu_{b3}(\epsilon_b(\text{LVAL\_BASE}(lval_b)))) : \Gamma(\text{LVAL\_BASE}(lval_b))$$

– Case $\chi_2 \not\sqsubseteq l$. In this case n_a and n_b can be $n_a \neq n_b$ (using induction hypothesis of Theorem D.1 on exp_2 evaluation in $\Downarrow_{lval}$ of $exp_1[exp_2]$). As $\bot \sqsubseteq pc$, $lval_a =_{lval} lval_b$. By applying Theorem D.1 on $lval_a$'s evaluation, we get $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(stack\ \tau\ \{\overline{val_a}\}\ ,\ stack\ \tau\ \{\overline{val_b}\}) : \langle\langle \tau, \chi_1\rangle[n], \bot\rangle$. However, with $\chi_2 \not\sqsubseteq l$ and $\chi_1 \not\sqsubseteq l$ and according to Definition C.6, we have $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(stack\ \tau\ \{..., val_{a_{n_a-1}}, val_a, val_{a_{n_a+1}}, ...\}\ ,\ stack\ \tau\ \{..., val_{b_{n_b-1}}, val_b, val_{b_{n_b+1}}, ...\}) : \langle \tau, \chi_1\rangle[n]$. Similar to the previous case, by applying induction hypothesis of this lemma on the lval-write to $lval_a$ and $lval_b$ that are generated from the same $lval\_exp$, we can conclude that

$$\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(\mu_{a3}(\epsilon_a(\text{LVAL\_BASE}(lval_a)))\ ,\ \mu_{b3}(\epsilon_b(\text{LVAL\_BASE}(lval_b)))) : \Gamma(\text{LVAL\_BASE}(lval_b))$$

$\square$

Lemma G.3. *Let* $\Gamma, \Delta \vdash_{pc} lval\_exp : \langle \tau, \chi\rangle, \langle C; \Delta; \mu_a; \epsilon_a; lval\_exp\rangle \Downarrow_{lval} \langle \mu_a'; lval_a\rangle$ *and* $\langle C; \Delta; \mu_b; \epsilon_b; lval\_exp\rangle \Downarrow_{lval} \langle \mu_b'; lval_b\rangle$.
Suppose $\Xi_a, \Xi_b, \Delta \models_l \langle\mu_a, \epsilon_a\rangle \langle\mu_b, \epsilon_b\rangle : \Gamma$, $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \langle\mu_{a1}, \epsilon_a\rangle \langle\mu_{b1}, \epsilon_b\rangle : \Gamma$, *and* $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(val_a\ ,\ val_b) : \langle \tau, \chi\rangle$, *where* $\Xi_a \subseteq \Xi_{a1}, \Xi_b \subseteq \Xi_{b1}, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_{a1}), \text{DOM}(\mu_b) \subseteq \text{DOM}(\mu_{b1})$. *If* $\langle C, \Delta, \mu_{a1}, \epsilon_a, lval_a := val_a\rangle \Downarrow_{write} \mu_{a1}'$ *and* $\langle C, \Delta, \mu_{b1}, \epsilon_b, lval_b := val_b\rangle \Downarrow_{write} \mu_{b1}'$, *then* $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \langle\mu_{a1}', \epsilon_a\rangle \langle\mu_{b1}', \epsilon_b\rangle : \Gamma$.

Proof. Follows from Lemma G.2. $\square$

H   Function Evaluation Strategy

Lemma H.1. *Consider the following well-typed expressions* $\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi'\rangle$, *and* $d\ x : \langle \tau, \chi\rangle := exp$, *where* $\chi' \sqsubseteq \chi$ *is evaluated in two different initial configurations* $\langle\mu_a, \epsilon_a\rangle$ *and* $\langle\mu_b, \epsilon_b\rangle$ *satisfying* $\Xi_a, \Xi_b, \Delta \models_l \langle\mu_a, \epsilon_a\rangle \langle\mu_b, \epsilon_b\rangle : \Gamma$ *as follows:*

$$\langle C; \Delta; \mu_a; \epsilon_a; d\ x : \langle \tau, \chi\rangle := exp\rangle \Downarrow_{copy} \langle\mu_a'; x \mapsto l_a; lval_a \mapsto l_a\rangle$$

and

$$\langle C; \Delta; \mu_b; \epsilon_b; d\ x : \langle \tau, \chi\rangle := exp\rangle \Downarrow_{copy} \langle\mu_b'; x \mapsto l_b; lval_b \mapsto l_b\rangle$$

then

1. $\Xi_a', \Xi_b', \Delta \models_l \langle\mu_a', x \mapsto l_a\rangle \langle\mu_b', x \mapsto l_b\rangle : \Gamma'$, *for some* $\Xi_a', \Xi_b', \mu_a', \mu_b'$ *such that* $\Xi_a \subseteq \Xi_a'$ *and* $\Xi_b \subseteq \Xi_b', \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_a')$, $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu_b')$ *and* $\Gamma' = \{x \mapsto \langle \tau, \chi\rangle\}$.
2. $\Xi_a', \Xi_b', \Delta \models_l \langle\mu_a', \epsilon_a\rangle \langle\mu_b', \epsilon_b\rangle : \Gamma$,
3. $\text{LVAL\_BASE}(lval_a) \in \text{DOM}(\epsilon_a), \text{LVAL\_BASE}(lval_b) \in \text{DOM}(\epsilon_b)$, *and* $\text{LVAL\_BASE}(lval_a) = \text{LVAL\_BASE}(lval_b)$,
4. $l_a \in \text{DOM}(\mu_a')$ *and* $l_b \in \text{DOM}(\mu_b')$,
5. l_a *and* l_b *are fresh locations,* $l_a \notin \Xi_a$ *and* $l_b \notin \Xi_b$,
6. *For any* $l_a \in \text{DOM}(\mu_a)$ *such that* $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, *where* $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, *then* $\mu_a'(l_a) = \mu_a(l_a)$,
7. *For any* $l_b \in \text{DOM}(\mu_b)$ *such that* $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, *where* $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, *then* $\mu_b'(l_b) = \mu_b(l_b)$,
8. PC *is used to bound writes. For any* $l_a \in \text{DOM}(\mu_a)$ *and* $l_b \in \text{DOM}(\mu_b)$ *such that* $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi\rangle$ *and* $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi\rangle$ *and* $pc \not\sqsubseteq \chi$, *we have* $\mu_a'(l_a) = \mu_a(l_a)$ *and* $\mu_b'(l_b) = \mu_b(l_b)$.

Note. *By Definition C.4,* $\text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$.

Proof. Case analysis on the possible directionalities d for the arguments.

1. **Copy In** If the statement $in\ x : \langle \tau, \chi\rangle := exp$ is evaluated in two different initial configurations $\langle\mu_a, \epsilon_a\rangle$ and $\langle\mu_b, \epsilon_b\rangle$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle\mu_a, \epsilon_a\rangle \langle\mu_b, \epsilon_b\rangle : \Gamma$$

as follows:

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, exp\rangle \Downarrow \langle\mu_{a1}, val_a\rangle \qquad l_a\ fresh}{\langle C, \Delta, \mu_a, \epsilon_a, in\ x : \langle \tau, \chi\rangle = exp\rangle \Downarrow_{copy} \langle\mu_{a1}[l_a \mapsto val_a], x \mapsto l_a, []\rangle}$$

$$\frac{\langle C, \Delta, \mu_b, \epsilon_b, exp \rangle \Downarrow \langle \mu_{b1}, val_b \rangle \qquad l_b \ fresh}{\langle C, \Delta, \mu_b, \epsilon_b, in \ x : \langle \tau, \chi \rangle = exp \rangle \Downarrow_{copy} \langle \mu_{b1}[l_b \mapsto val_b], x \mapsto l_b, [] \rangle}$$

then we need to show each of the following, where $\mu_a' = \mu_{a1}[l_a \mapsto val_a]$ and $\mu_b' = \mu_{b1}[l_b \mapsto val_b]$, $\Gamma' = \{x \mapsto \langle \tau, \chi \rangle\}$.

a. $\Xi_a', \Xi_b', \Delta \models_l \langle \mu_a', x \mapsto l_a \rangle \langle \mu_b', x \mapsto l_b \rangle : \Gamma'$, for some $\Xi_a', \Xi_b', \mu_a', \mu_b'$ such that $\Xi_a \subseteq \Xi_a'$ and $\Xi_b \subseteq \Xi_b'$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_a'), \text{DOM}(\mu_b) \subseteq \text{DOM}(\mu_b')$.

b. $\Xi_a', \Xi_b', \Delta \models_l \langle \mu_a', \epsilon_a \rangle \langle \mu_b', \epsilon_b \rangle : \Gamma$

c. Since the set of l-values, *i.e.,* the third element of the final tuple is empty, vacuously we have $\text{LVAL\_BASE}(lval_a) \in \text{DOM}(\epsilon_a)$, $\text{LVAL\_BASE}(lval_b) \in \text{DOM}(\epsilon_b)$, and $\text{LVAL\_BASE}(lval_a) = \text{LVAL\_BASE}(lval_b)$.

d. $l_a \in \text{DOM}(\mu_a'), l_b \in \text{DOM}(\mu_b'), l_a$ and l_b are fresh locations, $l_a \notin \Xi_a$ and $l_b \notin \Xi_b$,

e. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_a'(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_b'(l_b) = \mu_b(l_b)$,

f. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_a'(l_a) = \mu_a(l_a)$ and $\mu_b'(l_b) = \mu_b(l_b)$.

By applying the induction hypothesis of Theorem D.1 on $\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi' \rangle$, we conclude that $\Gamma, \Delta \models_{pc} \text{NI}(exp : \langle \tau, \chi' \rangle)$. This can be expanded to show that there exist some $\Xi_{a1}, \Xi_{b1}, \mu_{a1}, \mu_{b1}$ satisfying $\Xi_a \subseteq \Xi_{a1}, \Xi_b \subseteq \Xi_{b1}, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_{a1})$, $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu_{b1})$ and the following:

$$\Xi_{a1}, \Xi_{b1}, \Delta \models_l \langle \mu_{a1}, \epsilon_a \rangle \langle \mu_{b1}, \epsilon_b \rangle : \Gamma \tag{1}$$

$$\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(val_a \ , \ val_b) : \langle \tau, \chi' \rangle \tag{2}$$

and for any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{a1}(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{b1}(l_b) = \mu_b(l_b)$, Also, for any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_{a1}(l_a) = \mu_a(l_a)$ and $\mu_{b1}(l_b) = \mu_b(l_b)$,

Using Lemma E.6, we can reduce the Equation (2) as follows since $\chi' \sqsubseteq \chi$:

$$\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(val_a \ , \ val_b) : \langle \tau, \chi \rangle \tag{3}$$

To prove Item 1a, we take $\Xi_a' = \Xi_{a1}[l_a \mapsto \langle \tau, \chi \rangle], \Xi_b' = \Xi_{b1}[l_b \mapsto \langle \tau, \chi \rangle]$, and $\mu_a' = \mu_{a1}[l_a \mapsto val_a]$ and $\mu_b' = \mu_{b1}[l_b \mapsto val_b]$. Now to prove $\Xi_a', \Xi_b', \Delta \models_l \langle \mu_a', x \mapsto l_a \rangle \langle \mu_b', x \mapsto l_a \rangle : \Gamma'$, we need to show:

a. $\Xi_a', \Delta \models \langle \mu_a', \{x \mapsto l_a\} \rangle : \{x \mapsto \langle \tau, \chi \rangle\}$ and $\Xi_b', \Delta \models \langle \mu_b', \{x \mapsto l_b\} \rangle : \{x \mapsto \langle \tau, \chi \rangle\}$. $\Xi_a', \Delta \models \langle \mu_a', \{x \mapsto l_a\} \rangle : \{x \mapsto \langle \tau, \chi \rangle\}$ holds as $\Xi_a', \Delta \models \mu_a'$ (using Lemma E.3) and $\Xi_a' \vdash \{x \mapsto l_a\} : \{x \mapsto \langle \tau, \chi \rangle\}$ (by definition). Since x is not of function type (as we do not support higher-order function), we do not need to prove the third/ fourth property of Definition C.3. Similarly, $\Xi_b', \Delta \models \langle \mu_b', \{x \mapsto l_b\} \rangle : \{x \mapsto \langle \tau, \chi \rangle\}$ also holds.

b. $\text{DOM}(\{x \mapsto l_a\}) = \text{DOM}(\{x \mapsto l_b\})$. Trivial.

c. $\Xi_a', \Xi_b', \Delta \models_l \text{NI}(\mu_a'(l_a) \ , \ \mu_b'(l_b)) : \langle \tau, \chi \rangle)$
   Applying Lemma E.7 on Equation (3) with $\Xi_{a1} \subseteq \Xi_a'$ and $\Xi_{b1} \subseteq \Xi_b'$, we conclude $\Xi_a', \Xi_b', \Delta \models_l \text{NI}(val_a \ , \ val_b) : \langle \tau, \chi \rangle$. As $\mu_a'(l_a) = val_a$ and $\mu_b'(l_b) = val_b$, we have shown the necessary.

With this we have shown Item 1a. Observe that we do not need to show properties related to closure variables because x is not a closure variable in our setting.

Can't this be proved by saying that old locations have same value? To prove Item 1b, we apply Lemma E.5 on Equation (1) with $\Xi_{a1} \subseteq \Xi_a', \Xi_{b1} \subseteq \Xi_b', \mu_{a1} \subseteq \mu_a'$, and $\mu_{b1} \subseteq \mu_b'$, to conclude $\Xi_a', \Xi_b', \Delta \models_l \langle \mu_a', \epsilon_a \rangle \langle \mu_b', \epsilon_b \rangle : \Gamma$.

Item 1d can be seen in the final configuration of the evaluation rule. Item 1e is satisfied using the result of applying induction hypothesis of non-interference for expression.

2. **Copy out**
   If the statement, $out \ x : \langle \tau, \chi \rangle := exp$, is evaluated in two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ satisfying $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$ as follows:

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, exp \rangle \Downarrow_{lval} \langle \mu_{a1}, lval_a \rangle \qquad l_a \ fresh}{\langle C, \Delta, \mu_a, \epsilon_a, out \ x : \langle \tau, \chi \rangle = exp \rangle \Downarrow_{copy} \langle \mu_{a1}[l_a \mapsto init_\Delta \tau], x \mapsto l_a, [lval_a := l_a] \rangle}$$

$$\frac{\langle C, \Delta, \mu_b, \epsilon_b, exp \rangle \Downarrow_{lval} \langle \mu_{b1}, lval_b \rangle \qquad l_b \ fresh}{\langle C, \Delta, \mu_b, \epsilon_b, out \ x : \langle \tau, \chi \rangle = exp \rangle \Downarrow_{copy} \langle \mu_{b1}[l_b \mapsto init_\Delta \tau], x \mapsto l_b, [lval_b := l_b] \rangle}$$

Then we need to show each of the following, where $\mu_a' = \mu_{a1}[l_a \mapsto init_\Delta \tau]$ and $\mu_b' = \mu_{b1}[l_b \mapsto init_\Delta \tau]$:

a. $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, x \mapsto l_a \rangle \langle \mu'_b, x \mapsto l_b \rangle : \Gamma'$, for some $\Xi'_a, \Xi'_b, \mu'_a, \mu'_b$ such that $\Xi_a \subseteq \Xi'_a$ and $\Xi_b \subseteq \Xi'_b$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$, $\text{DOM}(\mu'_b) \subseteq \text{DOM}(\mu'_b)$ and $\Gamma' = \{x \mapsto \langle \tau, \chi \rangle\}$.

b. $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$

c. $\text{LVAL\_BASE}(lval_a) \in \text{DOM}(\epsilon_a)$, $\text{LVAL\_BASE}(lval_b) \in \text{DOM}(\epsilon_b)$, and $\text{LVAL\_BASE}(lval_a) = \text{LVAL\_BASE}(lval_b)$.

d. $l_a \in \text{DOM}(\mu'_a)$ and $l_b \in \text{DOM}(\mu'_b)$. l_a and l_b are fresh locations, $l_a \notin \Xi_a$ and $l_b \notin \Xi_b$,

e. for any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$,

f. for any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$.

The proof for this case is similar to Copy-in, with the difference that here we use Lemma F.2 to conclude $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \langle \mu_{a1}, \epsilon_a \rangle \langle \mu_{b1}, \epsilon_b \rangle : \Gamma$, for some $\Xi_{a1}, \Xi_{b1}, \mu_{a1}$ and μ_{b1} satisfying $\Xi_a \subseteq \Xi_{a1}, \Xi_b \subseteq \Xi_{b1}, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_{a1})$ and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu_{b1})$, Item 2c, and Item 2d. From here proving all the cases is similar to the Copy-in.

3. **Copy inout**

If the statement, *inout $x : \langle \tau, \chi \rangle := exp$*, is evaluated in two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ satisfying $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$ as follows:

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, exp \rangle \Downarrow_{lval} \langle \mu_{a1}, lval_a \rangle \qquad \langle C, \Delta, \mu_{a1}, \epsilon_a, lval_a \rangle \Downarrow \langle \mu_{a2}, val_a \rangle \qquad l_a \; fresh}{\langle C, \Delta, \mu_a, \epsilon_a, inout \; x : \langle \tau, \chi \rangle = exp \rangle \Downarrow_{copy} \langle \mu_{a2}[l_a \mapsto val_a], x \mapsto l_a, [lval_a := l_a] \rangle}$$

$$\frac{\langle C, \Delta, \mu_b, \epsilon_b, exp \rangle \Downarrow_{lval} \langle \mu_{b1}, lval_b \rangle \qquad \langle C, \Delta, \mu_{b1}, \epsilon_b, lval_b \rangle \Downarrow \langle \mu_{b2}, val_b \rangle \qquad l_b \; fresh}{\langle C, \Delta, \mu_b, \epsilon_b, inout \; x : \langle \tau, \chi \rangle = exp \rangle \Downarrow_{copy} \langle \mu_{b2}[l_b \mapsto val_b], x \mapsto l_b, [lval_b := l_b] \rangle}$$

Then we need to show each of the following, where $\mu'_a = \mu_{a2}[l_a \mapsto val_a], \mu'_b = \mu_{b2}[l_b \mapsto val_b]$:

a. $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, x \mapsto l_a \rangle \langle \mu'_b, x \mapsto l_b \rangle : \Gamma'$, for some $\Xi'_a, \Xi'_b, \mu'_a, \mu'_b$ such that $\Xi_a \subseteq \Xi'_a$ and $\Xi_b \subseteq \Xi'_b$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$, $\text{DOM}(\mu'_b) \subseteq \text{DOM}(\mu'_b)$ and $\Gamma' = \{x \mapsto \langle \tau, \chi \rangle\}$.

b. $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$,

c. $\text{LVAL\_BASE}(lval_a) \in \text{DOM}(\epsilon_a)$, $\text{LVAL\_BASE}(lval_b) \in \text{DOM}(\epsilon_b)$, and $\text{LVAL\_BASE}(lval_a) = \text{LVAL\_BASE}(lval_b)$.

d. $l_a \in \text{DOM}(\mu'_a)$ and $l_b \in \text{DOM}(\mu'_b)$. l_a and l_b are fresh locations, $l_a \notin \Xi_a$ and $l_b \notin \Xi_b$,

e. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$.

f. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$.

By applying the induction hypothesis of Definition F.1, we conclude that

a. $\Xi_a \subseteq \Xi_{a1}, \Xi_b \subseteq \Xi_{b1}, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_{a1})$ and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu_{b1})$,

b. $\Xi_{a1}, \Xi_{b1}, \Delta \models_l \langle \mu_{a1}, \epsilon_a \rangle \langle \mu_{b1}, \epsilon_b \rangle : \Gamma$

c. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{a1}(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{b1}(l_b) = \mu_b(l_b)$,

d. if $\chi' \sqsubseteq pc$, then $lval_a =_{lval} lval_b$,

e. $\text{LVAL\_BASE}(lval_a) \in \text{DOM}(\epsilon_a)$, $\text{LVAL\_BASE}(lval_b) \in \text{DOM}(\epsilon_b)$ and $\text{LVAL\_BASE}(lval_a) = \text{LVAL\_BASE}(lval_b)$.

f. $\Gamma, \Delta \vdash_{pc} lval_a : \langle \tau, \chi \rangle$ and $\Gamma, \Delta \vdash_{pc} lval_b : \langle \tau, \chi \rangle$

g. For any $l_a \in \text{DOM}(\mu_{a1})$ and $l_b \in \text{DOM}(\mu_{b1})$ such that $\Xi_{a1}, \Delta \vdash \mu_{a1}(l_a) : \langle \tau, \chi \rangle$ and $\Xi_{b1}, \Delta \vdash \mu_{b1}(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_{a1}(l_a) = \mu_{a2}(l_a)$ and $\mu_{b1}(l_b) = \mu_{b2}(l_b)$.

By applying Theorem D.1 on the expressions, $lval_a$ and $lval_b$ (which satisfy $lval_a lval_b$), where $\Gamma, \Delta \vdash_{pc} lval_a : \langle \tau, \chi \rangle$ and $\Gamma, \Delta \vdash_{pc} lval_b : \langle \tau, \chi \rangle$ we get $\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \text{NI}(val_a, val_b) : \langle \tau, \chi' \rangle$. Here, $\Xi_{a1} \subseteq \Xi'_{a1}$ and $\Xi_{b1} \subseteq \Xi'_{b1}$. By applying Lemma G.1, we conclude that $\mu_{a2} = \mu_{a1}$ and $\mu_{b2} = \mu_{b1}$. Now similar to the proof for copy-in, we can prove that Item 3a and Item 3b. The other parts directly follow from the above induction results.

□

Lifting the copy-in-out rules to a list of statements, we arrive at the following lemma:

Lemma H.2. *Consider well-typed expressions $\overline{\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi' \rangle}$ and the statement, $\overline{d \; x : \langle \tau, \chi \rangle := exp}$, , where $\chi' \sqsubseteq \chi$ that is evaluated in two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ satisfying $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$ as follows:*

$\langle C; \Delta; \mu_{a1}; \epsilon_a; \overline{d \; x : \langle \tau, \chi \rangle := exp} \rangle \Downarrow_{copy} \langle \mu_{a2}; \overline{x \mapsto l_a}; \overline{lval_a \mapsto l_a} \rangle$ *and*

$\langle C; \Delta; \mu_{b1}; \epsilon_b; \overline{d \; x : \langle \tau, \chi \rangle := exp} \rangle \Downarrow_{copy} \langle \mu_{b2}; \overline{x \mapsto l_b}; \overline{lval_b \mapsto l_b} \rangle$

Then:

1. $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu_{a2}, \overline{x \mapsto l_a} \rangle \langle \mu_{b2}, \overline{x \mapsto l_b} \rangle : \Gamma'$, for some $\Xi'_a, \Xi'_b, \mu'_a, \mu'_b$ such that $\Xi_a \subseteq \Xi'_a$ and $\Xi_b \subseteq \Xi'_b$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$, $\text{DOM}(\mu'_b) \subseteq \text{DOM}(\mu'_b)$ and $\Gamma' = \{ \overline{x \mapsto \langle \tau, \chi \rangle} \}$,

2. $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma$,

3. $\text{LVAL\_BASE}(lval_a) \in \text{DOM}(\epsilon_a)$, $\text{LVAL\_BASE}(lval_b) \in \text{DOM}(\epsilon_b)$, and $\text{LVAL\_BASE}(lval_a) = \text{LVAL\_BASE}(lval_b)$ for each $lval_a$ and $lval_b$,

4. $\overline{l_a} \in \text{DOM}(\mu_{a2})$ and $\overline{l_b} \in \text{DOM}(\mu_{b2})$

5. $\overline{l_a}$ and $\overline{l_b}$ are fresh locations, $\overline{l_a} \notin \Xi_a$ and $\overline{l_b} \notin \Xi_b$,

6. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$,

7. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$.

Note. By Definition C.4, $\text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$.

I Proof of Non-Interference

Proof of Theorem D.1. The proof is given by induction on the typing derivation of the expression and the cases are given by the last typing rule in the expression's typing derivation.

1. **T-Int** If the typing derivation ends with the following last rule

$$\frac{}{\Gamma, \Delta \vdash_{pc} n_w : \langle int, \bot \rangle \text{ goes in}} \text{ T-Int}$$

then we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma \tag{1}$$

if the expression n_w is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows:

$$\frac{}{\langle C, \Delta; \mu_a; \epsilon_a; n_w \rangle \Downarrow \langle \mu_a, n_w \rangle} \text{ Eval 1}$$

$$\frac{}{\langle C, \Delta; \mu_b; \epsilon_b; n_w \rangle \Downarrow \langle \mu_b, n_w \rangle} \text{ Eval 2}$$

then there exists some Ξ'_a and Ξ'_b, such that the following hold:

a. $\Gamma, \Delta \vdash_{pc} n_w : \langle int, \bot \rangle$. Already given in the hypothesis of this theorem,

b. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$, and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$ and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$. Here, $\mu'_a = \mu_a$ and $\mu'_b = \mu_b$,

c. $\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(val_a, val_b) : \langle int, \bot \rangle$,

d. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$. This is trivial, since memory store doesn't change.

e. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$. This is trivial, since memory store doesn't change.

First we will prove Item 1b. Let $\Xi'_a = \Xi_a$ and $\Xi'_b = \Xi_b$, now showing Item 1b is same as showing

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma \tag{2}$$

From the evaluation rule, we know $\mu'_a = \mu_a$ and $\mu'_b = \mu_b$. Therefore, showing Equation (2) is same as showing

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$$

This is what we had started out with in Equation (1). Therefore we have shown Item 1b.

Next to show Item 1c, we first expand the definition for NI for values and prove each of its requirement. Since $\tau = \langle int, \bot \rangle$, using the syntactic typing, $\Gamma, \Delta \vdash_{pc} n_w : \langle int, \bot \rangle$ goes in we can show that $\Xi'_a, \Delta \vdash_{pc} n_w : \langle int, \bot \rangle$ and $\Xi'_b, \Delta \vdash_{pc} n_w : \langle int, \bot \rangle$. Also, since both integers have equal value $val_a = n_w = val_b$, we have shown NI for values .

2. **T-Bool** Similar to **E-Int**.

3. **T-Var** If the typing derivation ends with the following last rule

$$\frac{x \in \text{DOM}(\Gamma) \qquad \Gamma(x) = \langle \tau, \chi \rangle}{\Gamma, \Delta \vdash_{pc} x : \langle \tau, \chi \rangle \text{ goes inout}} \text{ T-Var}$$

then we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma, \tag{1}$$

if the expression x is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows:

$$\frac{\epsilon_a(x) = l_a \qquad \mu_a(l_a) = val_a}{\langle C, \Delta; \mu_a; \epsilon_a; x \rangle \Downarrow \langle \mu_a, val_a \rangle} \text{ EVAL 1}$$

$$\frac{\epsilon_b(x) = l_b \qquad \mu_b(l_b) = val_b}{\langle C, \Delta; \mu_b; \epsilon_b; x \rangle \Downarrow \langle \mu_b, val_b \rangle} \text{ EVAL 2}$$

then there exists some Ξ'_a and Ξ'_b, such that the following hold:

a. $\Gamma, \Delta \vdash_{pc} x : \langle \tau, \chi \rangle$. Already given in the hypothesis of this theorem,

b. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$, and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$ and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$. Here, $\mu'_a = \mu_a$ and $\mu'_b = \mu_b$,

c. $\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(val_a , val_b) : \langle \tau, \chi \rangle$,

d. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$.

e. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$.

First we will prove Item 3b. Let $\Xi'_a = \Xi_a$ and $\Xi'_b = \Xi_b$, now showing Item 3b is same as showing

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma \tag{2}$$

From the evaluation rule, we know $\mu'_a = \mu_a$ and $\mu'_b = \mu_b$. Therefore, showing Equation (2) is same as showing

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma \tag{3}$$

This is what we had started out with in Equation (1). Therefore we have shown Item 1b.

Next, we use the following property from the definition of Equation (3)

$$\text{for any } x, \ \Xi_a, \Xi_b, \Delta \models_l \text{NI}(\mu_a(\epsilon_a(x)) , \ \mu_b(\epsilon_b(x))) : \Gamma(x) \tag{4}$$

to conclude that $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(val_a , val_b) : \langle \tau, \chi \rangle$.

4. **T-SubType-In** In case the last typing rule is the following and we need to prove that $\Gamma, \Delta \models_{pc} \text{NI}(exp : \langle \tau, \chi' \rangle)$.

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi \rangle \ goes \ in \qquad \chi \sqsubseteq \chi'}{\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi' \rangle \ goes \ in} \text{ T-SUBTYPE-IN}$$

By applying the induction hypothesis of this theorem, we get $\Gamma, \Delta \models_{pc} \text{NI}(exp : \langle \tau, \chi \rangle)$. Now we need to show that if $\chi \sqsubseteq \chi'$, then $\Gamma, \Delta \models_{pc} \text{NI}(exp : \langle \tau, \chi' \rangle)$. To show NI of expression, we need to first show that the final memory stores are below-pc equivalent. This is already available from the expansion of $\Gamma, \Delta \models_{pc} \text{NI}(exp : \langle \tau, \chi \rangle)$. In addition, we need to show that the value that this expression evaluates to is still respecting non-interference of values with the security label χ' as defined in Definition C.6. To do this we use Lemma E.6.

5. **T-BinOp** If the typing derivation ends with the following last rule

$$\frac{\begin{array}{cc} \Gamma, \Delta \vdash_{pc} exp_1 : \langle \rho_1, \chi_1 \rangle & \Gamma, \Delta \vdash_{pc} exp_2 : \langle \rho_2, \chi_2 \rangle \\ \mathcal{T}(\Delta; \oplus; \rho_1; \rho_2) = \rho_3 & \chi_1 \sqsubseteq \chi' \qquad \chi_2 \sqsubseteq \chi' \end{array}}{\Gamma, \Delta \vdash_{pc} exp_1 \oplus exp_2 : \langle \rho_3, \chi' \rangle \ goes \ in} \text{ T-BINOP}$$

then we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma, \tag{1}$$

if the expression $exp_1 \oplus exp_2$ is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows:

$$\frac{\langle C, \Delta; \mu_a; \epsilon_a; exp_1 \rangle \Downarrow \langle \mu_{a1}, val_{a1} \rangle \qquad \langle C, \Delta; \mu_{a1}; \epsilon_a; exp_2 \rangle \Downarrow \langle \mu_{a2}, val_{a2} \rangle}{\langle C, \Delta; \mu_a; \epsilon_a; exp_1 \oplus exp_2 \rangle \Downarrow \langle \mu_{a2}, \mathbb{E}(\oplus, val_{a1}, val_{a2}) \rangle} \text{ EVAL 1}$$

$$\frac{\langle C, \Delta; \mu_b; \epsilon_b; exp_1 \rangle \Downarrow \langle \mu_{b1}, val_{b1} \rangle \qquad \langle C, \Delta; \mu_{b1}; \epsilon_b; exp_2 \rangle \Downarrow \langle \mu_{b2}, val_{b2} \rangle}{\langle C, \Delta; \mu_b; \epsilon_b; exp_1 \oplus exp_2 \rangle \Downarrow \langle \mu_{b2}, \mathbb{E}(\oplus, val_{b1}, val_{b2}) \rangle} \text{ EVAL 2}$$

then there exists some Ξ'_a and Ξ'_b, such that the following hold:

a. $\Gamma, \Delta \vdash_{pc} exp_1 \oplus exp_2 : \langle \rho_3, \chi' \rangle$. Already given in the hypothesis of this theorem,

b. $\Xi_a \subseteq \Xi'_a$, $\Xi_b \subseteq \Xi'_b$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$, and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$ and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$. Here, $\mu'_a = \mu_a$ and $\mu'_b = \mu_b$,

c. $\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(\mathbb{E}(\oplus, val_{a1}, val_{a2}), \mathbb{E}(\oplus, val_{b1}, val_{b2})) : \langle \rho_3, \chi' \rangle$,

d. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$.

e. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$.

We repeatedly apply induction hypothesis on the typing derivation of exp_1 and exp_2 to get:

$$\Gamma, \Delta \models_{pc} \text{NI}(exp_1 : \langle \rho_1, \chi_1 \rangle) \tag{2}$$

and

$$\Gamma, \Delta \models_{pc} \text{NI}(exp_2 : \langle \rho_2, \chi_2 \rangle) \tag{3}$$

Expanding Equation (2) we get:

$$\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \langle \mu_{a1}, \epsilon_a \rangle \langle \mu_{b1}, \epsilon_b \rangle : \Gamma, \tag{4}$$

where $\Xi_a \subseteq \Xi'_{a1}$ and $\Xi_b \subseteq \Xi'_{b1}$

$$\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \text{NI}(val_{a1}, val_{b1}) : \langle \rho_1, \chi_1 \rangle \tag{5}$$

Similarly expanding Equation (3) we get:

$$\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma, \tag{6}$$

where $\Xi'_{a1} \subseteq \Xi'_{a2}$ and $\Xi'_{b1} \subseteq \Xi'_{b2}$

$$\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \text{NI}(val_{a2}, val_{b2}) : \langle \rho_2, \chi_2 \rangle \tag{7}$$

Using Equation (4) and Equation (6), we conclude $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma$, where $\Xi'_a = \Xi'_{a2}$ and $\Xi'_b = \Xi'_{b2}$. This proves Item 5b.

We assume the following about $\mathbb{E}$:

$$if\ x_1 = x_2\ and\ y_1 = y_2,\ then\ \mathbb{E}(\oplus, x_1, y_1) = \mathbb{E}(\oplus, x_2, y_2) \tag{8}$$

Thus, if the parameters to the evaluation function $\mathbb{E}$ are non-interfering, $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(val_{a1}, val_{b1}) : \langle \rho_1, \chi_1 \rangle$ and $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(val_{a2}, val_{b2}) : \langle \rho_2, \chi_2 \rangle$, then the resultant value will also be non-interfering

$$\Xi_a, \Xi_b, \Delta \models_l \text{NI}(\mathbb{E}(\oplus, val_{a1}, val_{a2}), \mathbb{E}(\oplus, val_{b1}, val_{b2})) : \langle \rho_3, \chi' \rangle,$$

where $\chi_1 \sqsubseteq \chi'$ and $\chi_2 \sqsubseteq \chi'$ and $\rho_3 = \mathcal{T}(\oplus, \rho_1, \rho_2)$.

We consider only binary operations returning integers, bit vectors and booleans.

Using Equation (5), $\Xi'_{a1} \subseteq \Xi'_{a2}$ and the Lemma E.7 we have:

$$\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \text{NI}(val_{a1}, val_{b1}) : \langle \rho_1, \chi_1 \rangle \tag{9}$$

Using the above equation with Equation (7) and the above assumption about the $\mathbb{E}$ function, we conclude:

$$\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \text{NI}(\mathbb{E}(\oplus, val_{a1}, val_{a2}), \mathbb{E}(\oplus, val_{b1}, val_{b2})) : \langle \rho_3, \chi' \rangle$$

Now, we prove Item 5d. We know from Equation (2) that for any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{a1}(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{b1}(l_b) = \mu_b(l_b)$. Equation (3) also implies that for any $l_a \in \text{DOM}(\mu_{a1})$ such that $\Xi_{a2}, \Delta \vdash \mu_{a1}(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{a2}(l_a) = \mu_{a1}(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_{b1})$ such that $\Xi_{b2}, \Delta \vdash \mu_{b1}(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{b2}(l_b) = \mu_{b1}(l_b)$. We also know that $\Xi_a \subseteq \Xi_{a1}$, this implies that $l_a \in \text{DOM}(\mu_a)$ will also be present in $\text{DOM}(\mu_{a1})$. Therefore, we can show Item 5d. Item 5e can be similarly shown.

6. **T-Rec** If the typing derivation ends with the following last rule

$$\frac{\Gamma, \Delta \vdash_{pc} \overline{\{exp : \langle \tau_i, \chi_i \rangle\}}}{\Gamma, \Delta \vdash_{pc} \overline{\{f : exp\}} : \langle \{\overline{f : \langle \tau_i, \chi_i \rangle}\}, \bot \rangle\ goes\ in} \text{ T-Rec}$$

then we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma, \tag{1}$$

if the expression $\{\overline{f : exp}\}$ is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows:

$$\frac{\langle C, \Delta; \mu_a; \epsilon_a; \overline{exp} \rangle \Downarrow \langle \mu'_a, \overline{val_a} \rangle}{\langle C, \Delta; \mu_a; \epsilon_a; \{\overline{f = exp}\} \rangle \Downarrow \langle \mu'_a, \{\overline{f = val_a}\} \rangle} \text{ EVAL 1}$$

$$\frac{\langle C, \Delta; \mu_b; \epsilon_b; \overline{exp} \rangle \Downarrow \langle \mu'_b, \overline{val_b} \rangle}{\langle C, \Delta; \mu_b; \epsilon_b; \{\overline{f = exp}\} \rangle \Downarrow \langle \mu'_b, \{\overline{f = val_b}\} \rangle} \text{ EVAL 2}$$

then there exists some Ξ'_a and Ξ'_b, such that the following hold:

a. $\Gamma, \Delta \vdash_{pc} \{\overline{f = exp}\} : \langle \{\overline{f : \langle \tau_i, \chi_i \rangle}\}, \perp \rangle$. Already given in the hypothesis of this theorem,

b. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$, and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$ and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$,

c. $\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(\{\overline{f = val_a}\}, \{\overline{f = val_b}\}) : \langle \{\overline{f : \langle \tau_i, \chi_i \rangle}\}, \perp \rangle$,

d. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$.

e. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$,

We repeatedly apply induction hypothesis on each $\Gamma, \Delta \vdash_{pc} exp : \langle \tau_i, \chi_i \rangle$ in the sequence $\Gamma, \Delta \vdash_{pc} \overline{exp : \langle \tau, \chi \rangle}$. The last memory store we arrive at is given by μ'_a and μ'_b in the two evaluations. Therefore, after repeated application of induction hypothesis we get,

$$\Gamma, \Delta \models_{pc} \text{NI}(\overline{exp : \langle \tau_i, \chi_i \rangle}) \tag{2}$$

Since we evaluate $\overline{exp}$ in initial configurations satisfying Equation (1), this can be expanded to conclude that there exists some Ξ'_a and Ξ'_b satisfying $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$, $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$ and all of the following:

$$\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma, \tag{3}$$

$$\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(\overline{val_a}, \overline{val_b}) : \overline{\langle \tau_i, \chi_i \rangle} \tag{4}$$

This is to be interpreted as a sequence of non-interfering values. Equation (3) proves the goal in Item 6b.
Equation (4) can be interpreted as satisfying $\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(val_a, val_b) : \langle \tau_i, \chi_i \rangle$ for each val_a, val_b.

We use the TV-rec rule with Equation (4) to conclude that $\Xi'_a, \Delta \vdash \{\overline{f = val_a}\} : \langle \{\overline{f : \langle \tau_i, \chi_i \rangle}\}, \perp \rangle$ and $\Xi'_b, \Delta \vdash \{\overline{f = val_b}\} : \langle \{\overline{f : \langle \tau_i, \chi_i \rangle}\}, \perp \rangle$. Therefore, we have shown Item 6c. Item 6d and Item 6e follows from Equation (2).

7. **T-MemRec** If the typing derivation ends with the following last rule

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle \{\overline{f_i : \langle \tau_i, \chi_i \rangle}\}, \perp \rangle \text{ goes } d}{\Gamma, \Delta \vdash_{pc} exp.f_i : \langle \tau_i, \chi_i \rangle \text{ goes } d} \text{ T-MEMREC}$$

then we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma, \tag{1}$$

if the expression $exp.f_i$ is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows:

$$\frac{\langle C, \Delta; \mu_a; \epsilon_a; exp \rangle \Downarrow \langle \mu'_a, \{\overline{f_i : \langle \tau, \chi \rangle = val_{ai}}\} \rangle}{\langle C, \Delta; \mu_a; \epsilon_a; exp.f_i \rangle \Downarrow \langle \mu'_a, val_{ai} \rangle} \text{ EVAL 1}$$

$$\frac{\langle C, \Delta; \mu_b; \epsilon_b; exp \rangle \Downarrow \langle \mu'_b, \{\overline{f_i : \langle \tau, \chi \rangle = val_{bi}}\} \rangle}{\langle C, \Delta; \mu_b; \epsilon_b; exp.f_i \rangle \Downarrow \langle \mu'_b, val_{bi} \rangle} \text{ EVAL 2}$$

then there exists some Ξ'_a and Ξ'_b, such that the following hold:

a. $\Gamma, \Delta \vdash_{pc} exp.f_i : \langle \tau_i, \chi_i \rangle$. Already given in the hypothesis of this theorem,

b. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$, and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$ and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$.

c. $\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(val_{ai}, val_{bi}) : \langle \tau_i, \chi_i \rangle$,

d. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$.

e. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$,

By applying induction hypothesis on the typing derivation of exp, which is evaluated in an initial configuration satisfying Equation (1), we get:

$$\Gamma, \Delta \models_{pc} \text{NI}(exp \; : \; \langle\{\overline{f_i : \langle \tau_i, \chi_i \rangle}\}, \bot\rangle)$$

This implies that there exists a Ξ'_a, Ξ'_b, such that $\Xi_a \subseteq \Xi'_a$, $\Xi_b \subseteq \Xi'_b$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$, $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$ and the following:

$$\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma \, , \tag{2}$$

This proves Item 7b.

$$\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(\{\overline{f_i : \langle \tau, \chi \rangle = val_{ai}}\} \, , \, \{\overline{f_i : \langle \tau, \chi \rangle = val_{bi}}\}) : \langle\{\overline{f_i : \langle \tau_i, \chi_i \rangle}\}, \bot\rangle \tag{3}$$

Using the Definition C.6, we can observe that for each val_{ai} and val_{bi} the following holds:

$$\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(val_{ai} \, , \, val_{bi}) : \langle \tau_i, \chi_i \rangle \tag{4}$$

This proves Item 7c. Item 7d and Item 7e is also a conclusion of applying the induction hypothesis.

8. **T-Index** If the typing derivation ends with the following last rule

$$\frac{\begin{array}{c} \Gamma, \Delta \vdash_{pc} exp_1 : \langle \langle \tau, \chi_1 \rangle [n], \bot \rangle \text{ goes } d \\ \Gamma, \Delta \vdash_{pc} exp_2 : \langle bit\langle 32 \rangle, \chi_2 \rangle \\ \chi_2 \sqsubseteq \chi_1 \end{array}}{\Gamma, \Delta \vdash_{pc} exp_1[exp_2] : \langle \tau, \chi_1 \rangle \text{ goes } d} \text{ T-Index}$$

then we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma, \tag{1}$$

if the expression $exp_1[exp_2]$ is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows. Observe that if exp_2 evaluates to a value within the array bounds the following rule will be used; otherwise Eval 1 error.

$$\frac{\langle C, \Delta; \mu_a; \epsilon_a; exp_1 \rangle \Downarrow \langle \mu_{a1}, stack \, \tau\{\overline{val_a}\}\rangle \quad \langle C, \Delta; \mu_{a1}; \epsilon_a; exp_2 \rangle \Downarrow \langle \mu_{a2}, n_a \rangle \quad 0 \le n_a < len(\overline{val_a})}{\langle C, \Delta; \mu_a; \epsilon_a; exp_1[exp_2] \rangle \Downarrow \langle \mu_{a2}, val_{a \, n_a} \rangle} \text{ Eval 1}$$

$$\frac{\langle C, \Delta; \mu_a; \epsilon_a; exp_1 \rangle \Downarrow \langle \mu_{a1}, stack \, \tau\{\overline{val_a}\}\rangle \quad \langle C, \Delta; \mu_{a1}; \epsilon_a; exp_2 \rangle \Downarrow \langle \mu_{a2}, n_a \rangle \quad n_a \ge len(\overline{val_a})}{\langle C, \Delta; \mu_a; \epsilon_a; exp_1[exp_2] \rangle \Downarrow \langle \mu_{a2}, havoc(\tau) \rangle} \text{ Eval 1 error}$$

If exp_2 evaluates to a value within the array bounds the following rule will be used; otherwise $Eval_{2error}$

$$\frac{\langle C, \Delta; \mu_b; \epsilon_b; exp_1 \rangle \Downarrow \langle \mu_{b1}, stack \, \tau\{\overline{val_b}\}\rangle \quad \langle C, \Delta; \mu_{b1}; \epsilon_b; exp_2 \rangle \Downarrow \langle \mu_{b2}, n_b \rangle \quad 0 \le n_b < len(\overline{val_b})}{\langle C, \Delta; \mu_b; \epsilon_b; exp_1[exp_2] \rangle \Downarrow \langle \mu_{b2}, val_{b \, n_b} \rangle} \text{ Eval 2}$$

$$\frac{\langle C, \Delta; \mu_b; \epsilon_b; exp_1 \rangle \Downarrow \langle \mu_{b1}, stack \, \tau\{\overline{val_b}\}\rangle \quad \langle C, \Delta; \mu_{b1}; \epsilon_b; exp_2 \rangle \Downarrow \langle \mu_{b2}, n_b \rangle \quad n_b \ge len(\overline{val_b})}{\langle C, \Delta; \mu_b; \epsilon_b; exp_1[exp_2] \rangle \Downarrow \langle \mu_{b2}, havoc(\tau) \rangle} \text{ Eval 2 error}$$

then there exists some Ξ'_a and Ξ'_b, such that the following hold:
a. $\Gamma, \Delta \vdash_{pc} exp_1[exp_2] : \langle \tau, \chi_1 \rangle$. Already given in the hypothesis of this theorem,
b. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$, and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$ and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$. Here, $\mu'_a = \mu_{a2}$ and $\mu'_b = \mu_{b2}$,
c. $\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(val'_a \, , \, val'_b) : \langle \tau, \chi_1 \rangle$, where $val'_a \in \{val_{a \, n_a}, havoc(\tau)\}$ and $val'_b \in \{val_{b \, n_b}, havoc(\tau)\}$,
d. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$.
e. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$,

By applying induction hypothesis on the typing derivation of exp_1 that is evaluated in configuration satisfying Equation (1), we conclude that there exist some Ξ'_{a1} and Ξ'_{b1} satisfying $\Xi_a \subseteq \Xi'_{a1}, \Xi_b \subseteq \Xi'_{b1}$ and all of the following:

$$\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \langle \mu_{a1}, \epsilon_a \rangle \langle \mu_{b1}, \epsilon_b \rangle : \Gamma \, , \tag{2}$$

$$\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \text{NI}(stack \, \tau\{\overline{val_a}\} \, , \, stack \, \tau\{\overline{val_b}\}) : \langle \langle \tau, \chi_1 \rangle [n], \bot \rangle \tag{3}$$

Using the Definition C.6, we can observe that for each val_a and val_b the following holds:

$$\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \text{NI}(val_a \, , \, val_b) : \langle \tau, \chi_1 \rangle \tag{4}$$

By applying induction hypothesis on the typing derivation of exp_2 that is evaluated in configuration satisfying Equation (2), we conclude that there exist some Ξ'_{a2} and Ξ'_{b2} satisfying $\Xi'_{a1} \subseteq \Xi'_{a2}$, $\Xi'_{b1} \subseteq \Xi'_{b2}$ and all of the following:

$$\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma , \tag{5}$$

$$\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \text{NI}(n_a , n_b) : \langle bit\langle 32\rangle, \chi_2 \rangle \tag{6}$$

Equation (5) proves the requirement of Item 8b. To prove Item 8c we consider the following cases for the final values val'_a and val'_b:

- Index within bound. In this case both the evaluations use the same evaluation rules.
  If $\chi_1 \sqsubseteq l$, then $\chi_2 \sqsubseteq l$, which implies that $n_a = n_b = n_{32}$. We can observe that in this case we will have $val'_a = val_{a\ n_{32}}$ and $val'_b = val_{b\ n_{32}}$. Using Equation (4), we conclude that $\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \text{NI}(val'_a , val'_b) : \langle \tau, \chi_1 \rangle$. By applying Lemma E.7, we will get $\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \text{NI}(val'_a , val'_b) : \langle \tau, \chi_1 \rangle$. We have shown Item 8c.
  If $\chi_1 \not\sqsubseteq l$, according to the Definition C.6, $\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \text{NI}(val'_a , val'_b) : \langle \tau, \chi_1 \rangle$ will hold even if $val'_a \neq val'_b$. Therefore, even if $n_a \neq n_b$, $val'_a = val_{a\ n_a}$ and $val'_b = val_{b\ n_b}$, we will have $\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \text{NI}(val'_a , val'_b) : \langle \tau, \chi_1 \rangle$.
- One index is out-of-bound. In this case one of the evaluation will yield the $havoc(\tau)$ and n_a and n_b should have differed. This implies $\chi_2 \not\sqsubseteq l$, which implies $\chi_1 \not\sqsubseteq l$. As described in the previous case, $\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \text{NI}(val_{a\ n_a} , havoc(\tau)) : \langle \tau, \chi_1 \rangle$ is true according to the Definition C.6.
- Both indices are out-of-bound. In this case the values will be of the form $val'_a = havoc(\tau) = val'_b$. According to the Definition C.6, $\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \text{NI}(havoc(\tau) , havoc(\tau)) : \langle \tau, \chi_1 \rangle$ is satisfied.

9. **T-HdrMem** If the typing derivation ends with the following last rule

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle header\{\overline{f_i : \langle \tau_i, \chi_i \rangle}\}, \bot \rangle \text{ goes } d}{\Gamma, \Delta \vdash_{pc} exp.f_i : \langle \tau_i, \chi_i \rangle \text{ goes } d} \text{ T-MemHdr}$$

then we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma, \tag{1}$$

if the expression $exp.f_i$ is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows:

$$\frac{\langle C, \Delta; \mu_a; \epsilon_a; exp \rangle \Downarrow \langle \mu'_a, header\{valid, \overline{f : \tau = val_a}\} \rangle}{\langle C, \Delta; \mu_a; \epsilon_a; exp.f_i \rangle \Downarrow \langle \mu'_a, val_{ai} \rangle} \text{ Eval 1}$$

$$\frac{\langle C, \Delta; \mu_b; \epsilon_b; exp \rangle \Downarrow \langle \mu'_b, header\{valid, \overline{f : \tau = val_b}\} \rangle}{\langle C, \Delta; \mu_b; \epsilon_b; exp.f_i \rangle \Downarrow \langle \mu'_b, val_{bi} \rangle} \text{ Eval 2}$$

then there exists some Ξ'_a and Ξ'_b, such that the following hold:
a. $\Gamma, \Delta \vdash_{pc} exp.f_i : \langle \tau_i, \chi_i \rangle$. Already given in the hypothesis of this theorem.
b. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \text{dom}(\mu_a) \subseteq \text{dom}(\mu'_a)$, and $\text{dom}(\mu_b) \subseteq \text{dom}(\mu'_b)$ and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$,
c. $\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(val_{ai} , val_{bi}) : \langle \tau_i, \chi_i \rangle$,
d. For any $l_a \in \text{dom}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{dom}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$.
We consider only valid headers in this information-flow control system. Similar to case 7, we apply induction hypothesis on typing derivation of exp followed by inverting the value typing for headers.

10. **T-FuncCall** If the typing derivation ends with the following last rule

$$\frac{\Gamma, \Delta \vdash_{pc} exp_1 : \langle \overline{d \langle \tau_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle \quad \Gamma, \Delta \vdash_{pc} \overline{exp_2 : \langle \tau_i, \chi_i \rangle \text{ goes } d} \quad pc \sqsubseteq pc_{fn}}{\Gamma, \Delta \vdash_{pc} exp_1(\overline{exp_2}) : \langle \tau_{ret}, \chi_{ret} \rangle \text{ goes in}} \text{ T-Call}$$

then we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma \tag{1}$$

if the function call expression $exp_1(\overline{exp_2})$ is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows:

$$\frac{\begin{array}{c} \langle C, \Delta, \mu_a, \epsilon_a, exp_1 \rangle \Downarrow \langle \mu_{a1}, clos(\epsilon_{c_a}, \overline{d\ x : \langle \tau, \chi \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, stmt) \rangle \\ \langle \Delta, \mu_{a1}, \epsilon_a, \overline{dx : \langle \tau, \chi \rangle := exp_2} \rangle \Downarrow_{copy} \langle \mu_{a2}, \overline{x \mapsto l_a}, \overline{lval_a := l_a} \rangle \\ \langle C, \Delta, \mu_{a2}, \epsilon_{c_a}[\overline{x \mapsto l_a}], stmt \rangle \Downarrow \langle \mu_{a3}, \epsilon_{a2}, \text{return } val_a \rangle \qquad \langle C, \Delta, \mu_{a3}, \epsilon_a, \overline{lval := \mu_{a3}(l)} \rangle \Downarrow_{write} \mu_4 \end{array}}{\langle C, \Delta, \mu_a, \epsilon_a, exp_1(\overline{exp_2}) \rangle \Downarrow \langle \mu_{a4}, val_a \rangle}$$

$$\frac{\begin{array}{c} \langle C, \Delta, \mu_b, \epsilon_b, exp_1 \rangle \Downarrow \langle \mu_{b1}, clos(\epsilon_{c_b}, \overline{d\ x : \langle \tau, \chi \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, stmt) \rangle \\ \langle \Delta, \mu_{b1}, \epsilon_b, \overline{dx : \langle \tau, \chi \rangle := exp_2} \rangle \Downarrow_{copy} \langle \mu_{b2}, \overline{x \mapsto l_b}, \overline{lval_b := l_b} \rangle \\ \langle C, \Delta, \mu_{b2}, \epsilon_{c_b}[\overline{x \mapsto l_b}], stmt \rangle \Downarrow \langle \mu_{b3}, \epsilon_{b2}, \text{return } val_b \rangle \qquad \langle C, \Delta, \mu_{b3}, \epsilon_b, \overline{lval_b := \mu_{b3}(l_b)} \rangle \Downarrow_{write} \mu_{b4} \end{array}}{\langle C, \Delta, \mu_b, \epsilon_b, exp_1(\overline{exp_2}) \rangle \Downarrow \langle \mu_{b4}, val_b \rangle}$$

then there exists some Ξ_a' and Ξ_b', such that the following hold:

a. $\Gamma, \Delta \vdash_{pc} exp_1(\overline{exp_2}) : \langle \tau_{ret}, \chi_{ret} \rangle$. Already given in the hypothesis of this theorem.

b. $\Xi_a \subseteq \Xi_a'$, $\Xi_b \subseteq \Xi_b'$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_a')$, $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu_b')$, and $\Xi_a', \Xi_b', \Delta \models_l \langle \mu_a', \epsilon_a \rangle \langle \mu_b', \epsilon_b \rangle : \Gamma$. Here, $\mu_a' = \mu_{a4}$ and $\mu_b' = \mu_{b4}$,

c. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_a'(l_a) = \mu_a(l_a)$ and $\mu_b'(l_b) = \mu_b(l_b)$,

d. $\Xi_a', \Xi_b', \Delta \models_l \text{NI}(val_a, val_b) : \langle \tau_{ret}, \chi_{ret} \rangle$,

e. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_a'(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_b'(l_b) = \mu_b(l_b)$.

By applying induction hypothesis of Theorem D.1 on exp_1, which is evaluated in an initial configuration satisfying Equation (1), we get: $\Gamma, \Delta \models_l \text{NI}(exp_1 : \langle \overline{d\ \langle \tau_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle)$. This implies that there exists some Ξ_{a1}, Ξ_{b1}, μ_{a1}, μ_{b1} satisfying $\Xi_a \subseteq \Xi_{a1}$ and $\Xi_b \subseteq \Xi_{b1}$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_{a1})$, and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu_{b1})$ and the following:

$$\Xi_{a1}, \Xi_{b1}, \Delta \models_l \langle \mu_{a1}, \epsilon_a \rangle \langle \mu_{b1}, \epsilon_b \rangle : \Gamma \tag{2}$$

$$\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(val_{a1}, val_{b1}) : \tau_{fn} \tag{3}$$

Here $val_{a1} = clos(\epsilon_{c_a}, \overline{dx : \langle \tau, \chi \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, stmt)$ and $val_{b1} = clos(\epsilon_{c_b}, \overline{dx : \langle \tau, \chi \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, stmt)$.

Since $\langle \tau, \chi \rangle = \tau_{fn}$, by using Equation (3) we conclude that $\Xi_{a1}, \Xi_{b1}, \Delta \models \text{NI\_CLOS}(val_{a1}, val_{b1}) : \langle \overline{d\langle \tau_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle$. Expanding the non-interference definition for closure (Definition C.7), we conclude that there exists some Γ_{fn}, such that the following properties are satisfied:

$$\Xi_{a1}, \Delta \models \epsilon_{c_a} : \Gamma_{fn}$$

$$\Xi_{b1}, \Delta \models \epsilon_{c_b} : \Gamma_{fn}$$

$$\Gamma_{fn}; \Delta \vdash_{pc} clos(\epsilon_{c_a}, \overline{dx : \langle \tau, \chi \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, stmt) : \langle \overline{d\langle \tau, \chi \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle$$

$$\Gamma_{fn}; \Delta \vdash_{pc} clos(\epsilon_{c_b}, \overline{dx : \langle \tau, \chi \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, stmt) : \langle \overline{d\langle \tau, \chi \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle$$

$$\Gamma_{fn}[\overline{x : \langle \tau, \chi \rangle}, \text{return} : \langle \tau_{ret}, \chi_{ret} \rangle], \Delta \vdash_{pc_{fn}} stmt \dashv \Gamma_{fn2} \tag{4}$$

Application of the induction hypothesis on exp_1 also grantees that the closure values do not change in the transition from μ_a to μ_{a1} and μ_b to μ_{b1}. Therefore, we can apply the property of closure values in the state given by Equation (2) to the closure values returned after the evaluation of exp_1. Equation (2) concludes that for any $x \in \text{DOM}(\epsilon_a)$, satisfying $\Gamma \vdash x : \tau_{fn}$, $\mu_{a1}(\epsilon_a(x)) = clos(\epsilon_c, ...)$, and $\Xi_{a1} \vdash \epsilon_c : \Gamma_c$, we will have $\text{DOM}(\epsilon_c) \subseteq \text{DOM}(\epsilon_a)$ and $\Xi_{a1}, \Delta \models \langle \mu_{a1}, \epsilon_c \rangle : \Gamma_c$. Here $\tau_f = \langle \overline{d\langle \tau, \chi \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle$, for any $\tau, \chi, \tau_{ret}, \chi_{ret}$. This implies that $\text{DOM}(\epsilon_{c_a}) \subseteq \text{DOM}(\epsilon_a)$ and $\Xi_{a1}, \Delta \models \langle \mu_{a1}, \epsilon_{c_a} \rangle : \Gamma_{fn}$. Similarly $\text{DOM}(\epsilon_{c_b}) \subseteq \text{DOM}(\epsilon_b)$ and $\Xi_{b1}, \Delta \models \langle \mu_{b1}, \epsilon_{c_b} \rangle : \Gamma_{fn}$.

Using Lemma H.2 for the evaluation of $\overline{dx : \langle \tau, \chi \rangle := exp_2}$ in the initial configuration satisfying Equation (2), we conclude the following:

a. $\Xi_{a2}, \Xi_{b2}, \Delta \models_l \langle \mu_{a2}, \overline{x \mapsto l_a} \rangle \langle \mu_{b2}, \overline{x \mapsto l_b} \rangle : \Gamma'$, for some $\Xi_{a2}, \Xi_{b2}, \mu_{a2}, \mu_{b2}$ such that $\Xi_{a1} \subseteq \Xi_{a2}$ and $\Xi_{b1} \subseteq \Xi_{b2}$, $\text{DOM}(\mu_{a1}) \subseteq \text{DOM}(\mu_{a2})$, $\text{DOM}(\mu_{b1}) \subseteq \text{DOM}(\mu_{b2})$ and $\Gamma' = \{\overline{x \mapsto \langle \tau, \chi \rangle}\}$.

b. $\Xi_{a2}, \Xi_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma$

c. For any $l_a' \in \text{DOM}(\mu_{a1})$ and $l_b' \in \text{DOM}(\mu_{b1})$ such that $\Xi_{a1}, \Delta \vdash \mu_{a1}(l_a') : \langle \tau, \chi \rangle$ and $\Xi_{b1}, \Delta \vdash \mu_{b1}(l_b') : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_{a1}(l_a') = \mu_{a2}(l_a')$ and $\mu_{b2}(l_b') = \mu_{b1}(l_b')$,

    d. $\text{LVAL\_BASE}(lval_a) \in \text{DOM}(\epsilon_a)$, $\text{LVAL\_BASE}(lval_b) \in \text{DOM}(\epsilon_b)$, and $\text{LVAL\_BASE}(lval_a) = \text{LVAL\_BASE}(lval_b)$ for each $lval_a$
        and $lval_b$.

    e. $\overline{l_a} \in \text{DOM}(\mu_{a2})$ and $\overline{l_b} \in \text{DOM}(\mu_{b2})$

    f. l_a and l_b are fresh locations, $l_a \notin \Xi_{a1}$ and $l_b \notin \Xi_{b1}$

    g. For any $l_a \in \text{DOM}(\mu_{a1})$ such that $\Xi_{a1}, \Delta \vdash \mu_{a1}(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{a1}(l_a) = \mu_{a2}(l_a)$. Similarly for
        any $l_b \in \text{DOM}(\mu_{b1})$ such that $\Xi_b, \Delta \vdash \mu_{b1}(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{b1}(l_b) = \mu_{b2}(l_b)$.

    Given Item 10g, we can observe that some closure variable x that evaluated to the closures returned on evaluating exp_1
    will have the same value in μ_{a2}. Therefore, by expanding Item 10b we conclude

$$\Xi_{a2}, \Xi_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_{c_a} \rangle \langle \mu_{b2}, \epsilon_{c_b} \rangle : \Gamma_{fn} \tag{5}$$

Combining Item 10a and Equation (5) using Lemma E.12 we get:

$$\Xi_{a2}, \Xi_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_{c_a}[\overline{x \mapsto l_a}] \rangle \langle \mu_{b2}, \epsilon_{c_b}[\overline{x \mapsto l_b}] \rangle : \Gamma_{fn}[\overline{x \mapsto \langle \tau, \chi \rangle}] \tag{6}$$

Note that Item 10f enforces that l_a and l_b are present in μ_{a2} and μ_{b2}.

By using the induction hypothesis of Theorem D.2 on $stmt$ that is evaluated in the initial configuration satisfying
Equation (6), we conclude $\Gamma_{fn}[\overline{x : \langle \tau, \chi \rangle}, \text{return} : \langle \tau_{ret}, \chi_{ret} \rangle], \Delta \models_{pc} \text{NI}(stmt) = \Gamma_{fn1}$ or there exist some $\Xi_{a3}, \Xi_{b3}, \mu_{a3}$,
μ_{b3}, ϵ_{a2}, and ϵ_{b2} such that $\Xi_{a2} \subseteq \Xi_{a3}$ and $\Xi_{b2} \subseteq \Xi_{b3}$, $\text{DOM}(\mu_{a2}) \subseteq \text{DOM}(\mu_{a3})$, $\text{DOM}(\mu_{b2}) \subseteq \text{DOM}(\mu_{b3})$, $\text{DOM}(\epsilon_{c_a}[\overline{x \mapsto l_a}]) \subseteq$
$\text{DOM}(\epsilon_{a2})$, and $\text{DOM}(\epsilon_{c_b}[\overline{x \mapsto l_b}]) \subseteq \text{DOM}(\epsilon_{b2})$ satisfying:

$$\Xi_{a3}, \Xi_{b3}, \Delta \models_l \langle \mu_{a3}, \epsilon_{a2} \rangle \langle \mu_{b3}, \epsilon_{b2} \rangle : \Gamma_{fn1} \tag{7}$$

$$\Xi_{a3}, \Xi_{b3}, \Delta \models_l \langle \mu_{a3}, \epsilon_{c_a}[\overline{x \mapsto l_a}] \rangle \langle \mu_{b3}, \epsilon_{c_b}[\overline{x \mapsto l_b}] \rangle : \Gamma_{fn}[\overline{x : \langle \tau, \chi \rangle}, \text{return} : \langle \tau_{ret}, \chi_{ret} \rangle] \tag{8}$$

and none of the locations with security label $pc \sqsubseteq \chi$ will be updated between μ_{a2} and μ_{a3}, μ_{b2} and μ_{b3}.

We know that $\Xi_{a2}, \Xi_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma$. Any $y \in \text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$ can satisfy one of the following:

    a. $\epsilon_a(y) = \epsilon_{c_a}[\overline{x \mapsto l_a}](y)$ and $\epsilon_b(y) = \epsilon_{c_b}[\overline{x \mapsto l_b}](y)$, then $\mu_{a3}(\epsilon_a(y)) = \mu_{a3}(\epsilon_{c_a}[\overline{x \mapsto l_a}](y))$ and $\mu_{b3}(\epsilon_b(y)) = \mu_{b3}(\epsilon_{c_b}[\overline{x \mapsto l_b}](y))$. This variable has non-interfering value (Equation (8)).

    b. $\text{UNUSED}(\mu_{a2}, \epsilon_{c_a}, y, \epsilon_a(y))$ (Definition E.8) and $\text{UNUSED}(\mu_{b2}, \epsilon_{c_b}, y, \epsilon_b(y))$, then $\mu_{a3}(y) = \mu_{a2}(y)$ and $\mu_{b3}(y) = \mu_{b2}(y)$. $\Xi_{a2}, \Xi_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma$ guarantees this value to be non-interfering.

    c. $\neg\text{UNUSED}(\mu_{a2}, \epsilon_{c_a}, y, \epsilon_a(y))$ and $\neg\text{UNUSED}(\mu_{b2}, \epsilon_{c_b}, y, \epsilon_b(y))$, then there exists some closure value with ϵ'_{c_a} and $\text{DOM}(\epsilon'_{c_a}) \subseteq$
        $\text{DOM}(\epsilon_{c_a})$, and ϵ'_{c_b} and $\text{DOM}(\epsilon'_{c_b}) \subseteq \text{DOM}(\epsilon_{c_b})$ where $\epsilon_a(y) = \epsilon'_{c_a}(y)$ and $\epsilon_b(y) = \epsilon'_{c_b}(y)$. From Equation (8), we know
        that $\Xi_{a3}, \Xi_{b3}, \Delta \models_l \langle \mu_{a3}, \epsilon'_{c_a} \rangle \langle \mu_{b3}, \epsilon'_{c_b} \rangle : \Gamma_{clos}$.

    To conclude that

$$\Xi_{a3}, \Xi_{b3}, \Delta \models_l \langle \mu_{a3}, \epsilon_a \rangle \langle \mu_{b3}, \epsilon_b \rangle : \Gamma \tag{9}$$

we also need to ensure that for all x in $\text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$ and some $\Gamma_{clos} \subseteq \Gamma$ and any pc, if $\Gamma, \Delta \vdash_{pc} x : \tau_{clos}$ with
closure environments ϵ'_{c_a} and ϵ'_{c_b} in the two states, then $\Xi_{a3}, \Xi_{b3}, \Delta \models_l \langle \mu_{a3}, \epsilon'_{c_a} \rangle \langle \mu_{b3}, \epsilon'_{c_b} \rangle : \Gamma_{clos}$. For closure variables
satisfying Item 10c, this will follow from closure properties in Equation (8). For variables satisfying Item 10b, this
will follow from the fact that the variables in their closure environments can again be UNUSED (implies unchanged
between μ_{a2} and μ_{a3}, μ_{b2} and μ_{b3}) or USED (in this case we already know from Equation (8) that such variables satisfy
non-interference of values).

Using Lemma G.3 on μ_{a3}, μ_{b3} to assign non-interfering values (Equation (9) implies that the store has non-interfering
values) to l-values, we conclude

$$\Xi_{a4}, \Xi_{b4}, \Delta \models_l \langle \mu_{a5}, \epsilon_a \rangle \langle \mu_{b5}, \epsilon_b \rangle : \Gamma \tag{10}$$

Since $\Xi_a \subseteq \Xi_{a1} \subseteq \Xi_{a2} \subseteq \Xi_{a3} \subseteq \Xi_{a4}$ and $\Xi_b \subseteq \Xi_{b1} \subseteq \Xi_{b2} \subseteq \Xi_{b3} \subseteq \Xi_{b4}$, showing the above equation is same as showing
Item 10b. Proof of Item 10c and Item 10e follows from the results of the application of the theorem for NI for expression,
statements above and the fact that domain of memory stores have increasing domains.

11. **T-MatchKind** Trivial

$$\frac{match\_kind\{\overline{f}\} \in \Delta(match\_kind) \qquad f_i \in \overline{f}}{\Gamma, \Delta \vdash_{pc} match\_kind.f_i : \langle match\_kind\{\overline{f}\}, \bot \rangle \text{ goes in}} \text{ T-MemHdr}$$

Evaluation rule

$$\frac{match\_kind\{\overline{f}\} \in \Delta(match\_kind)}{\langle C, \Delta; \mu_a; \epsilon_a; match\_kind.f_i \rangle \Downarrow \langle \mu_a, f_i \rangle} \text{ Eval 1}$$

$$\frac{match\_kind\{\overline{f}\} \in \Delta(match\_kind)}{\langle C, \Delta; \mu_b; \epsilon_b; exp.f_i \rangle \Downarrow \langle \mu_b, f_i \rangle} \text{ Eval 2}$$

Proof on Theorem D.2. The non-interference theorem for statements is given in Theorem D.2.

1. **T-Empty** The last typing rule in the derivation of an empty statement will be:

$$\overline{\Gamma, \Delta \vdash_{pc} \{\} \dashv \Gamma}$$

Given the above typing judgement holds for, $\{\}$, statement, we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b, \epsilon'_a, \epsilon'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma \tag{1}$$

if the statement, $\{\}$ is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows:

$$\overline{\langle C, \Delta, \mu_a, \epsilon_a, \{\} \rangle \Downarrow \langle \mu_a, \epsilon_a, cont \rangle} \qquad \overline{\langle C, \Delta, \mu_b, \epsilon_b, \{\} \rangle \Downarrow \langle \mu_b, \epsilon_b, cont \rangle}$$

Then there exists some Ξ'_a and Ξ'_b, such that the following hold:
   a. $\Gamma, \Delta \vdash_{pc} \{\} \dashv \Gamma'$. This is already the theorem's hypothesis.
   b. We have $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$, $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$, $\text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon'_a)$, and $\text{DOM}(\epsilon_b) \subseteq \text{DOM}(\epsilon'_b)$, and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon'_a \rangle \langle \mu'_b, \epsilon'_b \rangle : \Gamma'$. In this case $\mu'_a = \mu_a, \mu'_b = \mu_b, \epsilon'_a = \epsilon_a, \epsilon'_b = \epsilon_b$.
   With $\Xi'_a = \Xi_a, \Xi'_b = \Xi_b$, the above equation reduces to showing Equation (1).
   c. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$. This is evident as the memory store remains unchanged.
   d. sig in any two evaluations are of the same form. In this case $sig_1 = cont = sig_2$.
   e. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$. The stores remain unchanged.
2. **T-Exit**

$$\overline{\langle C, \Delta, \sigma, \epsilon, exit \rangle \Downarrow \langle \sigma, \epsilon, exit \rangle}$$

Similar to the empty statement case. This time the $sig_1 = sig_2 = exit$
3. **T-Cond** The last rule in the typing derivation of a conditional statement will be:

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle bool, \chi_1 \rangle \qquad \qquad \qquad}{\Gamma, \Delta \vdash_{\chi_2} stmt_1 \dashv \Gamma_1 \qquad \Gamma, \Delta \vdash_{\chi_2} stmt_2 \dashv \Gamma_2 \qquad \chi_1 \sqsubseteq \chi_2 \qquad pc \sqsubseteq \chi_2}{\Gamma, \Delta \vdash_{pc} \text{if } (exp) \ stmt_1 \text{ else } stmt_2 \dashv \Gamma} \text{ T-Cond}$$

Given the above typing judgement holds for, if $(exp) \ stmt_1$ else $stmt_2$, statement, we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b, \epsilon'_a, \epsilon'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma \tag{1}$$

if the statement, if $(exp) \ stmt_1$ else $stmt_2$ is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows (in a given evaluation, a conditional statement can have the exp evaluate to true or false):

Boolean guard evaluates to false.

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, exp \rangle \Downarrow \langle \mu_{a1}, false \rangle \qquad \langle C, \Delta, \mu_{a1}, \epsilon_a, stmt_2 \rangle \Downarrow \langle \mu_{a2}, \epsilon_{a1}, sig_{a1} \rangle}{\langle C, \Delta, \mu_a, \epsilon_a, \text{if } (exp) \ stmt_1 \text{ else } stmt_2 \rangle \Downarrow \langle \mu_{a2}, \epsilon_a, sig_{a1} \rangle}$$

$$\frac{\langle C, \Delta, \mu_b, \epsilon_b, exp \rangle \Downarrow \langle \mu_{b1}, false \rangle \qquad \langle C, \Delta, \mu_{b1}, \epsilon_b, stmt_2 \rangle \Downarrow \langle \mu_{b2}, \epsilon_{b1}, sig_{b1} \rangle}{\langle C, \Delta, \mu_b, \epsilon_b, \text{if } (exp) \ stmt_1 \text{ else } stmt_2 \rangle \Downarrow \langle \mu_{b2}, \epsilon_b, sig_{b1} \rangle}$$

Boolean guard evaluates to true.

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, exp \rangle \Downarrow \langle \mu_{a1}, true \rangle \qquad \langle C, \Delta, \mu_{a1}, \epsilon_a, stmt_1 \rangle \Downarrow \langle \mu_{a2}, \epsilon_{a1}, sig_{a2} \rangle}{\langle C, \Delta, \mu_a, \epsilon_a, \text{if } (exp) \ stmt_1 \text{ else } stmt_2 \rangle \Downarrow \langle \mu_{a2}, \epsilon_a, sig_{a2} \rangle}$$

$$\frac{\langle C, \Delta, \mu_b, \epsilon_b, exp \rangle \Downarrow \langle \mu_{b1}, true \rangle \qquad \langle C, \Delta, \mu_{b1}, \epsilon_b, stmt_1 \rangle \Downarrow \langle \mu_{b2}, \epsilon_{b1}, sig_{b2} \rangle}{\langle C, \Delta, \mu_b, \epsilon_b, \text{if } (exp) \ stmt_1 \text{ else } stmt_2 \rangle \Downarrow \langle \mu_{b2}, \epsilon_b, sig_{b2} \rangle}$$

Then there exists some Ξ'_a and Ξ'_b, such that the following hold:

a. $\Gamma, \Delta \vdash_{pc} \text{if } (exp) \ stmt_1 \text{ else } stmt_2 \dashv \Gamma'$. This is already the theorem's hypothesis.

b. We have $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a), \text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b), \text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon'_a)$, and $\text{DOM}(\epsilon_b) \subseteq \text{DOM}(\epsilon'_b)$, and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon'_a \rangle \langle \mu'_b, \epsilon'_b \rangle : \Gamma'$. In this case $\mu'_a = \mu_{a2}, \mu'_b = \mu_{b2}, \epsilon'_a = \epsilon_a, \epsilon'_b = \epsilon_b$.

c. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$.

d. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$,

e. Final sig in any two evaluations are of the same form. We will show this by proving that despite both the branches yielding independent sig_{a1}, sig_{a2} (similarly for b), the typing rule will ensure that the final sig will be of the same form.

In the following part, we prove the last four requirements. By applying induction hypothesis of Theorem D.1 on the well-typed exp that is evaluated in an initial state satisfying Equation (1), we conclude that there exists some Ξ'_{a1} and Ξ'_{b1} such that $\Xi_a \subseteq \Xi'_{a1}$ and $\Xi_b \subseteq \Xi'_{b1}, \text{DOM}(\mu_{a1}) \supseteq \text{DOM}(\mu_a), \text{DOM}(\mu_{b1}) \supseteq \text{DOM}(\mu_b)$ and the following hold:

$$\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \langle \mu_{a1}, \epsilon_a \rangle \langle \mu_{b1}, \epsilon_b \rangle : \Gamma , \tag{2}$$

for any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{a1}(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{b1}(l_b) = \mu_b(l_b)$,

for any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_{a1}(l_a) = \mu_a(l_a)$ and $\mu_{b1}(l_b) = \mu_b(l_b)$,

$$\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \text{NI}(val_{a1} , val_{b1}) : \langle bool, \chi_1 \rangle \tag{3}$$

To interpret this judgement, we consider two cases for χ_1:

- $\chi_1 \sqsubseteq l$. This implies $val_{a1} = val_{b1}$. Therefore, both the evaluations will either take *true* branch or both take *false* branch. We prove the required results for the *true* case; proof for the other case follows similarly. By applying the current theorem's induction hypothesis on the well-typed $stmt_1$ that is evaluated in an initial configuration satisfying Equation (2), we conclude that given $\langle C; \Delta; \mu_{a1}; \epsilon_a; stmt_1 \rangle \Downarrow \langle \mu_{a2}; \epsilon_{a1}; sig_a \rangle$ and $\langle C; \Delta; \mu_{b1}; \epsilon_b; stmt_1 \rangle \Downarrow \langle \mu_{b2}; \epsilon_{b1}; sig_b \rangle$ there exists some Ξ'_{a2} and Ξ'_{b2}, such that $\Xi'_{a1} \subseteq \Xi'_{a2}, \Xi'_{b1} \subseteq \Xi'_{b2}, \text{DOM}(\mu_{a2}) \supseteq \text{DOM}(\mu_{a1}), \text{DOM}(\mu_{b2}) \supseteq \text{DOM}(\mu_{b1}), \text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon_{a1}), \text{DOM}(\epsilon_b) \subseteq \text{DOM}(\epsilon_{b1})$, the signals satisfy the property of being of the same form (this proves the requirement in Item 3e) and

$$\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_{a1} \rangle \langle \mu_{b2}, \epsilon_{b1} \rangle : \Gamma', \tag{4}$$

$$\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma, \tag{5}$$

We already know from above that $\Xi_a \subseteq \Xi'_{a1} \subseteq \Xi'_{a2}, \Xi_b \subseteq \Xi'_{b1} \subseteq \Xi'_{b2}, \text{DOM}(\mu_{a2}) \supseteq \text{DOM}(\mu_{a1}) \supseteq \text{DOM}(\mu_a), \text{DOM}(\mu_{b2}) \supseteq \text{DOM}(\mu_{b1}) \supseteq \text{DOM}(\mu_b)$. Therefore, the Equation (5) proves the results needed to show Item 3b. Applying the induction hypothesis also concludes that for any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{a2}(l_a) = \mu_{a1}(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{b2}(l_b) = \mu_{b1}(l_b) = \mu_b(l_b)$. This proves the result needed to show Item 3c.

Applying the induction hypothesis also gives us that for any $l_a \in \text{DOM}(\mu_{a1})$ and $l_b \in \text{DOM}(\mu_{b1})$ such that $\Xi'_{a1}, \Delta \vdash \mu_{a1}(l_a) : \langle \tau, \chi \rangle$ and $\Xi'_{b1}, \Delta \vdash \mu_{b1}(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_{a1}(l_a) = \mu_{a2}(l_a)$ and $\mu_{b1}(l_b) = \mu_{b2}(l_b)$. As $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_{a1}) \subseteq \text{DOM}(\mu_{a2})$, this proves the result needed to show Item 3d.

- $\chi_1 \not\sqsubseteq l$. In this case the conditional guards might differ causing different branches to be taken. However, $\chi_1 \not\sqsubseteq l$ implies $\chi' \not\sqsubseteq l$. Since we know that $stmt_1$ and $stmt_2$ are well-typed at χ', which means store locations at $\chi' \not\sqsubseteq \chi$ remain unchanged across μ_{a1} and μ_{a2}, and μ_{b1} and μ_{b2}. This implies locations at $\chi \sqsubseteq l$ remain unchanged. Therefore, we can conclude from Equation (2) that

$$\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma \tag{6}$$

$stmt_1$ and $stmt_2$ are well-typed at $pc = \chi'$. Since $\chi' \not\sqsubseteq l$ and $\bot \sqsubseteq l$, we know that $\chi' \not\sqsubseteq \bot$. This implies that return and exit statements cannot be in these statement block because these two statements are well typed at the $pc = \bot$ only.

Therefore, only *sig* that can be returned in these statement blocks are *cont*. With this we prove that the final *sig* are of the same kind.

4. **T-Seq-1** The last rule in the typing derivation of a block of statements will be:

$$\frac{\Gamma, \Delta \vdash_{pc} stmt_1 \dashv \Gamma_1 \qquad \Gamma_1, \Delta \vdash_{pc} \{\overline{stmt_2}\} \dashv \Gamma_2}{\Gamma, \Delta \vdash_{pc} \{stmt_1; \overline{stmt_2}\} \dashv \Gamma_2} \text{ T-Seq}$$

Given the above typing judgement holds for the statement, $\{stmt_1; \overline{stmt_2}\}$, we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b, \epsilon'_a, \epsilon'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma \tag{1}$$

If the statement, $\{stmt_1; \overline{stmt_2}\}$ is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$, then there exists some Ξ'_a and Ξ'_b, such that the following hold:

a. $\Gamma, \Delta \vdash_{pc} \{stmt_1, \overline{stmt_2}\} \dashv \Gamma'$,. This is already the theorem's hypothesis.

b. We have $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a)$, $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b)$, $\text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon'_a)$, and $\text{DOM}(\epsilon_b) \subseteq \text{DOM}(\epsilon'_b)$, and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon'_a \rangle \langle \mu'_b, \epsilon'_b \rangle : \Gamma'$. In this case $\mu'_a = \mu_{a2}, \mu'_b = \mu_{b2}, \epsilon'_a = \epsilon_{a2}, \epsilon'_b = \epsilon_{b2}$. We also need to show that $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$.

c. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$.

d. For any $l'_a \in \text{DOM}(\mu_a)$ and $l'_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l'_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l'_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_a(l'_a) = \mu'_a(l'_a)$ and $\mu_b(l'_b) = \mu'_b(l'_b)$,

e. *sig* in any two evaluations are of the same form.

There are three cases for this evaluation: involving return statement, exit statement, or ordinary statements. We explain the ordinary statements case in detail, and the other two follow similarly.

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, stmt_1 \rangle \Downarrow \langle \mu_{a1}, \epsilon_{a1}, cont \rangle \qquad \langle C, \Delta, \mu_{a1}, \epsilon_{a1}, \{\overline{stmt_2}\} \rangle \Downarrow \langle \mu_{a2}, \epsilon_{a2}, sig_a \rangle}{\langle C, \Delta, \mu_a, \epsilon_a, \{stmt_1, \overline{stmt_2}\} \rangle \Downarrow \langle \mu_{a2}, \epsilon_{a2}, sig_a \rangle}$$

$$\frac{\langle C, \Delta, \mu_b, \epsilon_b, stmt_1 \rangle \Downarrow \langle \mu_{b1}, \epsilon_{b1}, cont \rangle \qquad \langle C, \Delta, \mu_{b1}, \epsilon_{b1}, \{\overline{stmt_2}\} \rangle \Downarrow \langle \mu_{b2}, \epsilon_{b2}, sig_b \rangle}{\langle C, \Delta, \mu_b, \epsilon_b, \{stmt_1, \overline{stmt_2}\} \rangle \Downarrow \langle \mu_{b2}, \epsilon_{b2}, sig_b \rangle}$$

In the following part, we prove the last three requirements. Since $stmt_1$ is evaluated in an initial configuration satisfying Equation (1), by applying induction hypothesis on the typing derivation of $stmt_1$, we conclude that given $\langle C; \Delta; \mu_a; \epsilon_a; stmt_1 \rangle \Downarrow \langle \mu_{a1}; \epsilon_{a1}; cont \rangle$ and $\langle C; \Delta; \mu_b; \epsilon_b; stmt_1 \rangle \Downarrow \langle \mu_{b1}; \epsilon_{b1}; cont \rangle$ there exists some Ξ'_{a1} and Ξ'_{b1}, such that $\Xi'_a \subseteq \Xi'_{a1}, \Xi'_b \subseteq \Xi'_{b1}$, $\text{DOM}(\mu_{a1}) \supseteq \text{DOM}(\mu_a)$, $\text{DOM}(\mu_{b1}) \supseteq \text{DOM}(\mu_b)$, $\text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon_{a1})$, $\text{DOM}(\epsilon_b) \subseteq \text{DOM}(\epsilon_{b1})$, the signals satisfy the property of being of the same form (in both case it is *cont*) and

$$\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \langle \mu_{a1}, \epsilon_{a1} \rangle \langle \mu_{b1}, \epsilon_{b1} \rangle : \Gamma_1, \tag{2}$$

$$\Xi'_{a1}, \Xi'_{b1}, \Delta \models_l \langle \mu_{a1}, \epsilon_a \rangle \langle \mu_{b1}, \epsilon_b \rangle : \Gamma, \tag{3}$$

and for any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{a1}(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{b1}(l_b) = \mu_b(l_b)$.

$\overline{stmt_2}$ is a sequence of statements, so we apply induction hypothesis repeatedly on each statement and conclude that the final states after evaluation of the sequence of statements $\langle C; \Delta; \mu_{a1}; \epsilon_{a1}; \overline{stmt_2} \rangle \Downarrow \langle \mu_{a2}; \epsilon_{a2}; sig_a \rangle$ and $\langle C; \Delta; \mu_{b1}; \epsilon_{b1}; \overline{stmt_2} \rangle \Downarrow \langle \mu_{b2}; \epsilon_{b2}; sig_b \rangle$ there exists some Ξ'_{a2} and Ξ'_{b2}, such that $\Xi'_{a1} \subseteq \Xi'_{a2}, \Xi'_{b1} \subseteq \Xi'_{b2}$, $\text{DOM}(\mu_{a2}) \supseteq \text{DOM}(\mu_{a1})$, $\text{DOM}(\mu_{b2}) \supseteq \text{DOM}(\mu_{b1})$, $\text{DOM}(\epsilon_{a1}) \subseteq \text{DOM}(\epsilon_{a2})$, $\text{DOM}(\epsilon_{b1}) \subseteq \text{DOM}(\epsilon_{b2})$, the signals satisfy the property of being of the same form (this proves the requirement in Item 4e) and

$$\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_{a2} \rangle \langle \mu_{b2}, \epsilon_{b2} \rangle : \Gamma_2 \tag{4}$$

$$\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_{a1} \rangle \langle \mu_{b2}, \epsilon_{b1} \rangle : \Gamma_1 \tag{5}$$

and for any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{a2}(l_a) = \mu_{a1}(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{b2}(l_b) = \mu_{b1}(l_b) = \mu_b(l_b)$. This proves the result needed to show Item 3c. Since we know that $\text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon_{a1})$, any $x \in \text{DOM}(\epsilon_a)$ will also be in $\text{DOM}(\epsilon_{a1})$. Similarly for ϵ_b. There can be two cases due to shadowing a variable name:

a. $\epsilon_a(x) = \epsilon_{a1}(x)$, $\epsilon_b(x) = \epsilon_{b1}(x)$. In this case, $\mu_{a2}(\epsilon_a(x)) = \mu_{a2}(\epsilon_{a1}(x))$ and $\mu_{b2}(\epsilon_b(x)) = \mu_{b2}(\epsilon_{b1}(x))$. We know that these variables satisfy non-interference in μ_{a2} and μ_{b2} from Equation (5).

   b. $\epsilon_a(x) \neq \epsilon_{a1}(x)$ and $\epsilon_b(x) \neq \epsilon_{b1}(x)$.

     i. If $\textsc{unused}(\langle \mu_{a1}, \epsilon_{a1} \rangle, x, \epsilon_a(x))$ and $\textsc{unused}(\langle \mu_{b1}, \epsilon_{b1} \rangle, x, \epsilon_b(x))$, then $\mu_{a2}(\epsilon_a(x)) = \mu_{a1}(\epsilon_a(x))$ and $\mu_{b2}(\epsilon_b(x)) = \mu_{b1}(\epsilon_b(x))$, which we know are non-interfering from Equation (3).

     ii. If $\neg\textsc{unused}(\langle \mu_{a1}, \epsilon_{a1} \rangle, x, \epsilon_a(x))$ and $\neg\textsc{unused}(\langle \mu_{b1}, \epsilon_{b1} \rangle, x, \epsilon_b(x))$, then there exists some closure value with environment ϵ'_{c_a} and $\textsc{dom}(\epsilon'_{c_a}) \subseteq \textsc{dom}(\epsilon_{a1})$, and ϵ'_{c_b} and $\textsc{dom}(\epsilon'_{c_b}) \subseteq \textsc{dom}(\epsilon_{b1})$ where $x \in \textsc{dom}(\epsilon'_{c_a})$ and $\epsilon_a(x) = \epsilon'_{c_a}(x)$. Also, $\epsilon_b(y) = \epsilon'_{c_b}(y)$. From Equation (5), we know that $\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon'_{c_a} \rangle \langle \mu_{b2}, \epsilon'_{c_b} \rangle : \Gamma_{clos}$. This implies that this variable x will have non-interfering values in μ_{a2} and μ_{b2}.

We also need to ensure that for all x in $\textsc{dom}(\epsilon_a) = \textsc{dom}(\epsilon_b)$ and some $\Gamma_{clos} \subseteq \Gamma$ and any pc, if $\Gamma, \Delta \vdash_{pc} x : \tau_{clos}$ with closure environments ϵ'_{c_a} and ϵ'_{c_b} in the two states, then $\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon'_{c_a} \rangle \langle \mu_{b2}, \epsilon'_{c_b} \rangle : \Gamma_{clos}$. For closure variables satisfying Item 4(b)ii, this will follow from closure properties in Equation (5). For variables ratifying Item 4(b)i, this will follow from the fact that the variables in their closure environments can again be $\textsc{unused}$ (implies unchanged between μ_{a1} and μ_{a2}, μ_{b1} and μ_{b2}) or $\textsc{used}$ (in this case we already know from Equation (5) that such variables satisfy non-interference of values). By combining the observation that all variables in ϵ_a and ϵ_b are non-interfering, we can conclude

$$\Xi'_{a2}, \Xi'_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma$$

This proves Item 4b.

For reference, the evaluation rules for the other two cases are as follows:

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, stmt_1 \rangle \Downarrow \langle \mu_{a1}, \epsilon_{a1}, return\ val_a \rangle}{\langle C, \Delta, \mu_a, \epsilon_a, \{stmt_1, \overline{stmt_2}\} \rangle \Downarrow \langle \mu_{a1}, \epsilon_{a1}, return\ val_a \rangle} \qquad \frac{\langle C, \Delta, \mu_b, \epsilon_b, stmt_1 \rangle \Downarrow \langle \mu_{b1}, \epsilon_{b1}, return\ val_b \rangle}{\langle C, \Delta, \mu_b, \epsilon_b, \{stmt_1, \overline{stmt_2}\} \rangle \Downarrow \langle \mu_{b1}, \epsilon_{b1}, return\ val_b \rangle}$$

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, stmt_1 \rangle \Downarrow \langle \mu_{a1}, \epsilon_{a1}, return\ val_a \rangle}{\langle C, \Delta, \mu_a, \epsilon_a, \{stmt_1, \overline{stmt_2}\} \rangle \Downarrow \langle \mu_{a1}, \epsilon_{a1}, return\ val_a \rangle} \qquad \frac{\langle C, \Delta, \mu_b, \epsilon_b, stmt_1 \rangle \Downarrow \langle \mu_{b1}, \epsilon_{b1}, exit \rangle}{\langle C, \Delta, \mu_b, \epsilon_b, \{stmt_1, \overline{stmt_2}\} \rangle \Downarrow \langle \mu_{b1}, \epsilon_{b1}, exit \rangle}$$

5. **T-Return** The last rule in the typing derivation of a return will be:

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi_{ret} \rangle \qquad \Gamma(return) = \langle \tau_{ret}, \chi_{ret} \rangle \qquad \Delta \vdash \tau_{ret} \rightsquigarrow \tau}{\Gamma, \Delta \vdash_{pc} return\ exp \dashv \Gamma} \text{ T-Return}$$

Given the above typing judgement holds for, *return exp*, we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b, \epsilon'_a, \epsilon'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma \tag{1}$$

if the statement, *return exp* is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows:

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, exp \rangle \Downarrow \langle \mu_{a1}, val_a \rangle}{\langle C, \Delta, \mu_a, \epsilon_a, return\ exp \rangle \Downarrow \langle \mu_{a1}, \epsilon_a, return\ val_a \rangle} \qquad \frac{\langle C, \Delta, \mu_b, \epsilon_b, exp \rangle \Downarrow \langle \mu_{b1}, val_b \rangle}{\langle C, \Delta, \mu_b, \epsilon_b, return\ exp \rangle \Downarrow \langle \mu_{b1}, \epsilon_b, return\ val_b \rangle}$$

Then there exists some Ξ'_a and Ξ'_b, such that the following hold:

a. $\Gamma, \Delta \vdash_{pc} return\ exp \dashv \Gamma'$, where $\Gamma' = \Gamma$. This is already the theorem's hypothesis.

b. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \textsc{dom}(\mu_a) \subseteq \textsc{dom}(\mu'_a), \textsc{dom}(\mu_b) \subseteq \textsc{dom}(\mu'_b), \textsc{dom}(\epsilon_a) \subseteq \textsc{dom}(\epsilon'_a), \textsc{dom}(\epsilon_b) \subseteq \textsc{dom}(\epsilon'_b)$, and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon'_a \rangle \langle \mu'_b, \epsilon'_b \rangle : \Gamma'$. In this case $\mu'_a = \mu_{a1}, \mu'_b = \mu_{b1}, \epsilon'_a = \epsilon_a, \epsilon'_b = \epsilon_b$.

c. For any $l_a \in \textsc{dom}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \textsc{dom}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$.

d. *sig* in any two evaluations are of the same form.

e. For any $l'_a \in \textsc{dom}(\mu_a)$ and $l'_b \in \textsc{dom}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l'_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l'_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_a(l'_a) = \mu'_a(l'_a)$ and $\mu_b(l'_b) = \mu'_b(l'_b)$,

Since *exp* is evaluated in an initial configuration satisfying Equation (1), by applying induction hypothesis of Theorem D.1 on the typing derivation of *exp*, we conclude that there exists some Ξ'_a, Ξ'_b, μ_{a1}, and μ_{b1} such that $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \textsc{dom}(\mu_a) \subseteq \textsc{dom}(\mu_{a1}), \textsc{dom}(\mu_b) \subseteq \textsc{dom}(\mu_{b1})$ and the following holds:

$$\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu_{a1}, \epsilon_a \rangle \langle \mu_{b1}, \epsilon_b \rangle : \Gamma, \tag{2}$$

For any $l_a \in \textsc{dom}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{a1}(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \textsc{dom}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{b1}(l_b) = \mu_b(l_b)$. This proves Item 5b and Item 5c. Also, for any $l'_a \in \textsc{dom}(\mu_a)$ and $l'_b \in \textsc{dom}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l'_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l'_b) : \langle \tau, \chi \rangle$

and $pc \not\sqsubseteq \chi$, we have $\mu_a(l'_a) = \mu_{a1}(l'_a)$ and $\mu_b(l'_b) = \mu_{b1}(l'_b)$. This proves Item 5e. The above applying of the induction hypothesis also shows

$$\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(val_a, val_b) : \langle \tau_{ret}, \chi_{ret} \rangle. \tag{3}$$

Since the signal in this case is of the form $ret\ val$, we need to show that

$$\Xi'_a, \Xi'_b, \Delta \models_{pc} NI(val_a, val_b) : \langle \tau_{ret}, \chi_{ret} \rangle$$

This is already given by Equation (3).

6. **T-Assign** The last rule in the typing derivation of an assignment statement will be:

$$\frac{\Gamma, \Delta \vdash_{pc} exp_1 : \langle \tau, \chi_1 \rangle\ goes\ inout \qquad \Gamma, \Delta \vdash_{pc} exp_2 : \langle \tau, \chi_2 \rangle \qquad \chi_2 \sqsubseteq \chi_1 \qquad pc \sqsubseteq \chi_1}{\Gamma, \Delta \vdash_{pc} exp_1 := exp_2 \dashv \Gamma} \text{ T-Assign}$$

Given the above typing judgement holds for, $exp_1 := exp_2$, we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b, \epsilon'_a, \epsilon'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma \tag{1}$$

if the statement, $exp_1 := exp_2$ is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows:

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, exp_1 \rangle \Downarrow_{lval} \langle \mu_{a1}, lval_a \rangle \qquad \langle C, \Delta, \mu_{a1}, \epsilon_a, exp_2 \rangle \Downarrow \langle \mu_{a2}, val_a \rangle \qquad \langle C, \Delta, \mu_{a2}, \epsilon_a, lval_a := val_a \rangle \Downarrow_{write} \mu_{a3}}{\langle C, \Delta, \mu_a, \epsilon_a, \{exp_1 := exp_2\} \rangle \Downarrow \langle \mu_{a3}, \epsilon_a, cont \rangle}$$

$$\frac{\langle C, \Delta, \mu_b, \epsilon_b, exp_1 \rangle \Downarrow_{lval} \langle \mu_{b1}, lval_b \rangle \qquad \langle C, \Delta, \mu_{b1}, \epsilon_b, exp_2 \rangle \Downarrow \langle \mu_{b2}, val_b \rangle \qquad \langle C, \Delta, \mu_{b2}, \epsilon_b, lval_b := val_b \rangle \Downarrow_{write} \mu_{b3}}{\langle C, \Delta, \mu_b, \epsilon_b, \{exp_1 := exp_2\} \rangle \Downarrow \langle \mu_{a3}, \epsilon_a, cont \rangle}$$

Then there exists some Ξ'_a and Ξ'_b, such that the following hold:

a. $\Gamma, \Delta \vdash_{pc} exp_1 := exp_2 \dashv \Gamma'$,. This is already the theorem's hypothesis.

b. We have $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a), \text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b), \text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon'_a)$, and $\text{DOM}(\epsilon_b) \subseteq \text{DOM}(\epsilon'_b)$, and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon'_a \rangle \langle \mu'_b, \epsilon'_b \rangle : \Gamma'$. In this case $\mu'_a = \mu_{a2}, \mu'_b = \mu_{b2}, \epsilon'_a = \epsilon_a, \epsilon'_b = \epsilon_b$.

c. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$.

d. For any $l'_a \in \text{DOM}(\mu_a)$ and $l'_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l'_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l'_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_a(l'_a) = \mu'_a(l'_a)$ and $\mu_b(l'_b) = \mu'_b(l'_b)$,

e. sig in any two evaluations are of the same form. In this case $sig_1 = cont = sig_2$.

By applying Lemma F.2 on exp_1, which is evaluated in an initial configuration satisfying Equation (1), we conclude: There exists some $\Xi_{a1}, \Xi_{b1}, \mu_{a1}$ and μ_{b1} satisfying $\Xi_a \subseteq \Xi_{a1}, \Xi_b \subseteq \Xi_{b1}, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_{a1})$ and $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu_{b1})$ and the following:

$$\Xi_{a1}, \Xi_{b1}, \Delta \models_l \langle \mu_{a1}, \epsilon_a \rangle \langle \mu_{b1}, \epsilon_b \rangle : \Gamma \tag{2}$$

For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{a1}(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_{b1}(l_b) = \mu_b(l_b)$.

Also, for any $l'_a \in \text{DOM}(\mu_a)$ and $l'_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l'_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l'_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_a(l'_a) = \mu_{a1}(l'_a)$ and $\mu_b(l'_b) = \mu_{b1}(l'_b)$,

Also, if $\chi_1 \sqsubseteq l$, then $lval_a =_{lval} lval_b$. Also, $\text{LVAL\_BASE}(lval_a) \in \text{DOM}(\epsilon_a)$ and $\text{LVAL\_BASE}(lval_b) \in \text{DOM}(\epsilon_b)$.

By applying induction Theorem D.1 on exp_2, which is evaluated in an initial configuration satisfying Equation (2), we can conclude: There exists some $\Xi_{a2}, \Xi_{b2}, \mu_{a2}$, and μ_{b2} such that $\Xi_{a1} \subseteq \Xi_{a2}, \Xi_{b1} \subseteq \Xi_{b2}, \text{DOM}(\mu_{a1}) \subseteq \text{DOM}(\mu_{a2}), \text{DOM}(\mu_{b1}) \subseteq \text{DOM}(\mu_{b2})$ and the following hold:

$$\Xi_{a2}, \Xi_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma, \tag{3}$$

$$\Xi_{a2}, \Xi_{b2}, \Delta \models_l \text{NI}(val_a, val_b) : \langle \tau, \chi_2 \rangle \tag{4}$$

Using Lemma G.3 on l-value write in expressions $lval_a := val_a$ and $lval_b := val_b$, we get that

$$\Xi_{a2}, \Xi_{b2}, \Delta \models_l \langle \mu_{a3}, \epsilon_a \rangle \langle \mu_{b3}, \epsilon_b \rangle : \Gamma, \tag{5}$$

Since $\Xi_a \subseteq \Xi_{a1} \subseteq \Xi_{a2}$ and $\Xi_b \subseteq \Xi_{b1} \subseteq \Xi_{b2}$, showing the above equation is same as showing Item 6b. Observe that the $lval_a$ and $lval_b$ have security level $pc \sqsubseteq \chi_1$, and Lemma G.3 states that only the location given by $\epsilon_a(\text{LVAL\_BASE}(lval_a))$ is updated in the μ_{a3} and similarly μ_{b3}. Therefore, we have proved Item 6d. Proof of Item 6c follows similarly from the results of applying the above induction hypothesis.

7. **T-VarDecl** A well-formed declaration statement will satisfy the following typing rule:

$$\frac{\Gamma; \Delta \vdash_{pc} \text{var\_decl} \dashv \Gamma'; \Delta_1}{\Gamma; \Delta \vdash_{pc} \text{var\_decl} \dashv \Gamma'}$$

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, \text{var\_decl} \rangle \Downarrow \langle \Delta_1, \mu'_a, \epsilon'_a, cont \rangle}{\langle C, \Delta, \mu_a, \epsilon_a, \text{var\_decl} \rangle \Downarrow \langle \mu'_a, \epsilon'_a, cont \rangle}$$

$$\frac{\langle C, \Delta, \mu_b, \epsilon_b, \text{var\_decl} \rangle \Downarrow \langle \Delta_1, \mu'_b, \epsilon'_b, cont \rangle}{\langle C, \Delta, \mu_b, \epsilon_b, \text{var\_decl} \rangle \Downarrow \langle \mu'_b, \epsilon'_b, cont \rangle}$$

The proof of this case follows from applying the induction hypothesis for NI for declarations. In case of var_decl $\Delta_1 = \Delta$.

8. **T-TblCall**

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle table(pc_{tbl}), \bot \rangle \qquad pc \sqsubseteq pc_{tbl}}{\Gamma, \Delta \vdash_{pc} exp() \dashv \Gamma} \text{ T-TblCall}$$

Given the above typing judgement holds for, $exp()$ statement, we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b, \epsilon'_a, \epsilon'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma \tag{1}$$

If the statement, $exp()$ is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows,

$$\frac{\begin{array}{c} \langle C, \Delta, \mu_a, \epsilon_a, exp \rangle \Downarrow \langle \mu_{a1}, table\ l_a(\epsilon_{c_a}, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_c : \langle \tau_c, \chi_c \rangle})}) \rangle \\ \langle C, \Delta, \mu_{a1}, \epsilon_c, \overline{exp_k} \rangle \Downarrow \langle \mu_{a2}, \overline{val_{ka}} \rangle \qquad \langle C, l_a, \overline{val_{ka} : x_k}, act_{a_j}(y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle) \rangle \Downarrow_{match} \langle act_{a_j}(\overline{exp_{c_{ji}}}) \rangle \\ \langle C, \Delta, \mu_{a2}, \epsilon_{c_a}, act_{aj}(\overline{exp_{a_{ji}}}, \overline{exp_{c_{ji}}}) \rangle \Downarrow \langle \mu_{a3}, \epsilon'_{c_a}, cont \rangle \end{array}}{\langle C, \Delta, \mu_a, \epsilon_a, exp() \rangle \Downarrow \langle \mu_{a3}, \epsilon_a, cont \rangle}$$

$$\frac{\begin{array}{c} \langle C, \Delta, \mu_b, \epsilon_b, exp \rangle \Downarrow \langle \mu_{b1}, table\ l_b(\epsilon_{c_b}, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_j} : \langle \tau_{c_j}, \chi_{c_j} \rangle})}) \rangle \\ \langle C, \Delta, \mu_{b1}, \epsilon_{c_b}, \overline{exp_k} \rangle \Downarrow \langle \mu_{b2}, \overline{val_{kb}} \rangle \qquad \langle C, l_b, \overline{val_{kb} : x_k}, act_{a_j}(y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle) \rangle \Downarrow_{match} \langle act_{a_j}(\overline{exp_{c_{ji}}}) \rangle \\ \langle C, \Delta, \mu_{b2}, \epsilon_{c_b}, act_{aj'}(\overline{exp_{a_{ji'}}}, \overline{exp_{c_{ji'}}}) \rangle \Downarrow \langle \mu_{b3}, \epsilon'_{c_b}, cont \rangle \end{array}}{\langle C, \Delta, \mu_b, \epsilon_b, exp() \rangle \Downarrow \langle \mu_{b3}, \epsilon_b, cont \rangle}$$

Then there exists some Ξ'_a and Ξ'_b, such that the following hold:

a. $\Gamma, \Delta \vdash_{pc} exp() \dashv \Gamma'$, where $\Gamma' = \Gamma$. This is already the theorem's hypothesis.

b. We have $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \text{dom}(\mu_a) \subseteq \text{dom}(\mu'_a), \text{dom}(\mu_b) \subseteq \text{dom}(\mu'_b), \text{dom}(\epsilon_a) \subseteq \text{dom}(\epsilon'_a)$, and $\text{dom}(\epsilon_b) \subseteq \text{dom}(\epsilon'_b)$ and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon'_a \rangle \langle \mu'_b, \epsilon'_b \rangle : \Gamma'$. In this case $\epsilon'_a = \epsilon_a, \epsilon'_b = \epsilon_b, \mu'_a = \mu_{a3}$ and $\mu'_b = \mu_{b3}$.

c. For any $l_a \in \text{dom}(\mu_a)$ and $l_b \in \text{dom}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$,

d. For any $l_a \in \text{dom}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{dom}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$.

e. sig in any two evaluations are of the same form. In this case $sig_1 = cont = sig_2$.

To show that the final state satisfies Item 8b we start by showing that final state after evaluating all the sub-step in the table evaluation satisfies Item 8b.

Evaluating table expression. By applying induction hypothesis of Theorem D.1 on the well-typed exp, we get

$$\Gamma, \Delta \models_{pc} \text{NI}(exp : \langle table(pc_{tbl}), \bot \rangle).$$

Since exp is evaluated in an initial configuration satisfying Equation (1), we can expand the NI for expression definition to conclude that there exists some Ξ_{a1}, Ξ_{b1}, satisfying $\Xi_a \subseteq \Xi_{a1}, \Xi_b \subseteq \Xi_{b1}, \text{dom}(\mu_a) \subseteq \text{dom}(\mu_{a1})$ and $\text{dom}(\mu_b) \subseteq \text{dom}(\mu_{b1})$ and the following:

$$\Xi_{a1}, \Xi_{b1}, \Delta \models_l \langle \mu_{a1}, \epsilon_{a1} \rangle \langle \mu_{b1}, \epsilon_{b1} \rangle : \Gamma, \tag{2}$$

$$\Xi_{a1}, \Xi_{b1}, \Delta \models_l \text{NI}(val_a, val_b) : \langle table(pc_{tbl}), \bot \rangle \tag{3}$$

where

$$val_a = table\ l_a\ (\epsilon_a, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})})$$

and
$$val_b = table\ l_b\ (\epsilon_b, \overline{exp_k : x_k}, \overline{act_{b_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})}).$$

Equation (3) expands to give $\Xi_{a1}, \Xi_{b1}, \Delta \models_{pc} \text{NI\_TBL}(val_a, val_b) : \langle table(pc_{tbl}), \bot \rangle$, which implies that there exists a Γ_{tbl} and pc_a such that

a. $\Xi_{a1} \models \epsilon_{c_a} : \Gamma_{tbl}$ and $\Xi_{b1} \models \epsilon_{c_b} : \Gamma_{tbl}$

b. Well-typed. $\Gamma_{tbl}; \Delta \vdash_{pc} table\ l_a\ (\epsilon_a, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})}) : \langle table(pc_a, pc_{tbl}), \bot \rangle$. Similarly, we have

$\Gamma_{tbl}; \Delta \vdash_{pc} table\ l_b\ (\epsilon_b, \overline{exp_k : x_k}, \overline{act_{b_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})}) : \langle table(pc_a, pc_{tbl}), \bot \rangle$

c. $\Gamma_{tbl}, \Delta \vdash_{pc_{tbl}} x_k : \langle match\_kind, \bot \rangle$ for each $x_k \in \overline{x_k}$

d. $\Gamma_{tbl}, \Delta \vdash_{pc_{tbl}} exp_k : \langle \tau_k, \chi_k \rangle$ for each $exp_k \in \overline{exp_k}$

e. $\Gamma_{tbl}, \Delta \vdash_{pc_{tbl}} act_{aj} : \langle \overline{d\langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle} ; \overline{\langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle} \xrightarrow{pc_{fn_j}} \langle unit, \bot \rangle, \bot \rangle$ for each $act_{a_j} \in \overline{act_{a_j}}$

f. $\Gamma_{tbl}, \Delta \vdash_{pc_{tbl}} exp_{a_{ji}} : \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle$ goes d for each $exp_{a_{ji}} \in \overline{exp_{a_{ji}}}$

g. $val_a =_{tbl} val_b$.

h. $\chi_k \sqsubseteq pc_{fn_j}$, for all j, k

i. $pc_a \sqsubseteq pc_{fn_j}$, for all j

j. $\chi_k \sqsubseteq pc_{tbl}$ for all k.

k. $pc_{tbl} \sqsubseteq pc_a$

From Equation (1), we already know that for all x in $\text{DOM}(\epsilon)$ and some $\Gamma_{tbl} \subseteq \Gamma$, if $\Gamma, \Delta \vdash_{pc} x : \tau_{tbl}, \mu(\epsilon(x)) = table\ l\ (\epsilon_c, ...)$, and $\Xi \models \epsilon_c : \Gamma_{tbl}$, then $\text{DOM}(\epsilon_c) \subseteq \text{DOM}(\epsilon)$ and $\Xi, \Delta \models \langle \mu, \epsilon_c \rangle : \Gamma_{tbl}$.

This implies that $\text{DOM}(\epsilon_{c_a}) \subseteq \text{DOM}(\epsilon_a)$ and $\text{DOM}(\epsilon_{c_b}) \subseteq \text{DOM}(\epsilon_b)$. Also, $\Xi_{a1}, \Delta \models \langle \mu_{a1}, \epsilon_{c_a} \rangle : \Gamma_{tbl}$ and $\Xi_{b1}, \Delta \models \langle \mu_{b1}, \epsilon_{c_b} \rangle : \Gamma_{tbl}$. Since closure values do not change across μ_a, μ_{a1}, and μ_b, μ_{b1}, the variable that would have evaluated to the table closure value under μ_a will have the same value under μ_{a1}. By using the property of closures implied by Equation (2), we conclude $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_{a1}, \epsilon_{c_a} \rangle \langle \mu_{b1}, \epsilon_{c_b} \rangle : \Gamma_{tbl}$.

Evaluating key expression. By repeatedly applying the induction hypothesis of Theorem D.1 on $\Gamma_{tbl}, \Delta \vdash_{pc_{tbl}} exp_k : \langle \tau_k, \chi_k \rangle$ for each $exp_k \in \overline{exp_k}$, implies that there exists some $\Xi_{a2}, \Xi_{b2}, \mu_{a2}$ and μ_{b2} satisfying $\Xi_{a1} \subseteq \Xi_{a2}, \Xi_{b1} \subseteq \Xi_{b2}$, $\mu_{a1} \subseteq \mu_{a2}$ and $\mu_{b1} \subseteq \mu_{b2}$ and the following:

$$\Xi_{a2}, \Xi_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_{c_a} \rangle \langle \mu_{b2}, \epsilon_{c_b} \rangle : \Gamma_{tbl}, \tag{4}$$

$$\Xi_{a2}, \Xi_{b2}, \Delta \models_l \text{NI}(\overline{val_{ka}}, \overline{val_{kb}}) : \overline{\langle \tau_k, \chi_k \rangle} \tag{5}$$

This can be read as "if $\chi_k \sqsubseteq l$ then $val_{ka} = val_{kb}$".

Also, none of the variables at security label $pc \not\sqsubseteq \chi$ are updated between μ_{a1}, μ_{a2}, and μ_{b1}, μ_{b2}. Similar to the argument used in function call case to prove Equation (9), we can also conclude

$$\Xi_{a2}, \Xi_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma \tag{6}$$

Table match. The $\Downarrow_{match}$ depends on some assumption about the control plane, C that it will ensure that only well-typed arguments, $\Gamma_{tbl}, \Delta \vdash_{pc_{tbl}} exp_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle$ for each $exp_{c_{ji}} \in \overline{exp_{c_{ji}}}$ are passed to partially-applied actions (this is same as Petr4's assumption around the control plane). In addition, considering that the table entries are fixed, matching on a equal $\overline{val_{ka} = val_{kb}}$ will return the same action and arguments, *i.e.*, the matched action will be the same $act_{aj} = act_{aj'}$ and $exp_{c_{ji}} = exp_{c'_{ji}}$. exp_k at security-level $\chi_k \not\sqsubseteq l$ might not evaluate to equal values. Therefore, we have two cases for the *match* evaluation, either same actions, $act_{aj} = act_{aj'}$, with same parameter expressions, $\overline{exp_{c_{ji}} = exp_{c'_{ji}}}$ are returned or $act_{aj} \neq act_{aj'}$ and their parameter expression can also differ.

Invoking the matched action. In case $act_{aj} = act_{aj'}$, $\overline{exp_{c_{ji}} = exp_{c'_{ji}}}$ and $\overline{exp_{a_{ji}} = exp_{a'_{ji}}}$, then the last premise of the evaluation rule is equivalent to evaluating a function expression with same parameter expression. By using induction hypothesis of Theorem D.2 for a well-typed function call statement, we arrive at a final state involving $\Xi_{a3}, \Xi_{b3}, \mu_{a3}, \mu_{b3}$, $\epsilon'_{c_a}, \epsilon'_{c_b}$ satisfying $\Xi_{a2} \subseteq \Xi_{a3}, \Xi_{b2} \subseteq \Xi_{b3}, \text{DOM}(\mu_{a2}) \subseteq \text{DOM}(\mu_{a3})$ and $\text{DOM}(\mu_{b2}) \subseteq \text{DOM}(\mu_{b3}), \text{DOM}(\epsilon_{c_a}) \subseteq \text{DOM}(\epsilon'_{c_a})$, and $\text{DOM}(\epsilon_{c_b}) \subseteq \text{DOM}(\epsilon'_{c_b})$ and the following:

$$\Xi_{a3}, \Xi_{b3}, \Delta \models_l \langle \mu_{a3}, \epsilon'_{c_a} \rangle \langle \mu_{b3}, \epsilon'_{c_b} \rangle : \Gamma_{tbl}, \tag{7}$$

In case of a function call statement, $\epsilon_{c_a} = \epsilon'_{c_a}$, and $\epsilon_{c_b} = \epsilon'_{c_b}$.

Similar to the argument used in function call case to prove Equation (9), since we have Equation (6) we can also conclude

$$\Xi_{a3}, \Xi_{b3}, \Delta \models_l \langle \mu_{a3}, \epsilon_a \rangle \langle \mu_{b3}, \epsilon_b \rangle : \Gamma \tag{8}$$

This proves Item 8b.

In case $act_{aj} \neq act_{aj'}$, $\overline{exp_{c_{ji}} \neq exp_{c'_{ji}}}$ and $\overline{exp_{a_{ji}} \neq exp_{a'_{ji}}}$, then there exists some $val_{ka} \neq val_{kb}$. This implies $\chi_k \not\sqsubseteq l$. Since $\chi_k \sqsubseteq pc_{tbl}$, we can conclude $pc_{tbl} \not\sqsubseteq l$. Although, the function call statements are different in the two cases, we know that both the function call statements are well-typed at pc_{tbl}. This implies that when the function call statement in μ_{a2} and ϵ_{c_a} is evaluated, then variables at $pc \not\sqsubseteq \chi$ will have unchanged value in μ_{a3}. Similarly, the other function call statement despite being different guarantees that the values of variables at $pc \not\sqsubseteq \chi$ in μ_{b2} and will have unchanged value in μ_{b3}. We already know that

$$\Xi_{a2}, \Xi_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma$$

By using the fact that none of the variables at $\chi \sqsubseteq l$ are updated between μ_{a2} and μ_{a3}, and similarly μ_{b2} and μ_{b3}, we can conclude that

$$\Xi_{a3}, \Xi_{b3}, \Delta \models_l \langle \mu_{a3}, \epsilon_a \rangle \langle \mu_{b3}, \epsilon_b \rangle : \Gamma$$

A consistent state requires that any variables at $\chi \sqsubseteq l$ are indistinguishable; this holds in

$$\Xi_{a2}, \Xi_{b2}, \Delta \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma$$

and with no changes to the variables at $\chi \sqsubseteq l$, it will continue to hold in the final memory store.

Proof of Theorem D.3. By induction on typing derivation of declaration statements.

1. **T-VarDecl**

$$\frac{\Delta \vdash \tau \rightsquigarrow \tau'}{\Gamma; \Delta \vdash_{pc} \langle \tau, \chi \rangle\ x \dashv \Gamma[x : \langle \tau', \chi \rangle]; \Delta}$$

Given the above typing judgement holds for, $\langle \tau, \chi \rangle\ x$, we need to show that $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b, \epsilon'_a, \epsilon'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma, \tag{1}$$

if the declaration, $\langle \tau, pc \rangle\ x$ is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows,

$$\frac{l_a\ \text{fresh} \qquad \langle \Delta, \mu_a, \epsilon_a, \tau \rangle \Downarrow_\tau \tau'}{\langle C, \Delta, \mu_a, \epsilon_a, \langle \tau, \chi \rangle\ x \rangle \Downarrow \langle \Delta, \mu_a[l_a := init_\Delta \tau'], \epsilon_a[x \mapsto l_a], cont \rangle}$$

$$\frac{l_b\ \text{fresh} \qquad \langle \Delta, \mu_b, \epsilon_b, \tau \rangle \Downarrow_\tau \tau'}{\langle C, \Delta, \mu_b, \epsilon_b, \langle \tau, \chi \rangle\ x \rangle \Downarrow \langle \Delta, \mu_b[l_b := init_\Delta \tau'], \epsilon_b[x \mapsto l_b], cont \rangle}$$

then there exists Ξ'_a, Ξ'_b such that

a. $\Gamma, \Delta \vdash_{pc} \langle \tau, \chi \rangle\ x \dashv \Gamma', \Delta$. This is already the hypothesis of the theorem.

b. $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon'_a \rangle \langle \mu'_b, \epsilon'_b \rangle : \Gamma'$, where $\mu'_a = \mu_a[l_a := init_\Delta \tau']$, $\mu'_b = \mu_b[l_b := init_\Delta \tau']$, $\epsilon'_a = \epsilon_a[x \mapsto l_a]$, $\epsilon'_b = \epsilon_b[x \mapsto l_b]$. Also, $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$,

c. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$,

d. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a), \text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b), \text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon'_a)$, and $\text{DOM}(\epsilon_b) \subseteq \text{DOM}(\epsilon'_b)$.

e. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$.

With $\Xi'_a = \Xi_a \cup \{l_a \mapsto \langle \tau', \chi \rangle\}, \Xi'_b = \Xi_b \cup \{l_b \mapsto \langle \tau', \chi \rangle\}$ the equation in Item 1d is evident. To show Item 1b, we need to show the following:

$$\Xi'_a, \Delta \models \langle \mu'_a, \epsilon'_a \rangle : \Gamma' \tag{2}$$

$$\Xi'_b, \Delta \models \langle \mu'_b, \epsilon'_b \rangle : \Gamma' \tag{3}$$

$$\text{DOM}(\epsilon'_a) = \text{DOM}(\epsilon'_b) \tag{4}$$

For any $x \in \text{DOM}(\epsilon'_a) = \text{DOM}(\epsilon'_b)$ we have $\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(\mu'_a(\epsilon'_a(x)),\ \mu'_b(\epsilon'_b(x))) : \Gamma'(x)$ (5)

For all x in $\text{DOM}(\epsilon'_a) = \text{DOM}(\epsilon'_b)$ and some $\Gamma_{fn} \subseteq \Gamma$ and any pc, if $\Gamma', \Delta \vdash_{pc} x : \tau_{fn}, \mu'_a(\epsilon'_a(x)) = clos(\epsilon_{c_a}, ...), \mu'_b(\epsilon'_b(x)) = clos(\epsilon_{c_b}, ...), \Xi'_a \models \epsilon_{c_a} : \Gamma_{fn}$, and $\Xi'_b \models \epsilon_{c_b} : \Gamma_{fn}$, then $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_{c_a} \rangle \langle \mu'_b, \epsilon_{c_b} \rangle : \Gamma_{fn}$.

For all x in $\text{DOM}(\epsilon'_a) = \text{DOM}(\epsilon'_b)$ and some $\Gamma_{tbl} \subseteq \Gamma'$ and any pc, if $\Gamma', \Delta \vdash_{pc} x : \tau_{tbl}, \mu'_a(\epsilon'_a(x)) = table\ l_a\ (\epsilon_{c_a}, ...), \mu'_b(\epsilon'_b(x)) = table\ l_b\ (\epsilon_{c_b}, ...), \Xi'_a \models \epsilon_{c_a} : \Gamma_{tbl}$, and $\Xi'_b \models \epsilon_{c_b} : \Gamma_{tbl}$, then $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_{c_a} \rangle \langle \mu'_b, \epsilon_{c_b} \rangle : \Gamma_{tbl}$.

Equation (4) is evident from the definitions of ϵ'_a, and ϵ'_b and the given fact that $\text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$. First, we begin by showing Equation (2). This requires us to in turn prove the following:

a. $\Xi'_a, \Delta \models \mu'_a$. This is shown in Lemma E.3.

b. $\Xi_a' \vdash \epsilon_a' : \Gamma'$. We are given $\Xi_a \vdash \epsilon_a : \Gamma$. Using Lemma E.4, we can say that $\Xi_a' \vdash \epsilon_a : \Gamma$. Since $\Gamma' = \Gamma[x \mapsto \langle \tau', \chi \rangle]$, $\epsilon_a' = \epsilon_a[x \mapsto l_a]$, $\Xi_a' = \Xi_a \cup \{l_a \mapsto \langle \tau', \chi \rangle\}$, by using the typing judgements for $\Xi \vdash \epsilon : \Gamma$, we can show that

$$\frac{\Xi_a' \vdash \epsilon_a : \Gamma \qquad \Xi_a'(l_a) = \langle \tau', \chi \rangle}{\Xi_a' \vdash \epsilon_a[x \mapsto l_a] : \Gamma[x \mapsto \langle \tau', \chi \rangle]}$$

This gives us the proof for $\Xi_a' \vdash \epsilon_a' : \Gamma'$.

c. For all x in $\text{DOM}(\epsilon_a')$ and some $\Gamma_{fn} \subseteq \Gamma'$ and any pc, if $\Gamma', \Delta \vdash_{pc} x : \tau_{fn}$, $\mu_a'(\epsilon_a'(x)) = clos(\epsilon_c, ...)$, and $\Xi_a' \models \epsilon_c : \Gamma_{fn}$, then $\text{DOM}(\epsilon_c) \subseteq \text{DOM}(\epsilon_a')$ and $\Xi_a', \Delta \models \langle \mu_a', \epsilon_c \rangle : \Gamma_{fn}$. Here τ_{fn} is the function type. We elide the full view of the closures in this definition. Observe that the function closure variables in $\text{DOM}(\epsilon_a')$ are variables that were also in $\text{DOM}(\epsilon_a)$ and are not shadowed by the new declaration x. We already know for such closure variables that $\text{DOM}(\epsilon_c) \subseteq \text{DOM}(\epsilon_a)$. Therefore, we can conclude that $\text{DOM}(\epsilon_c) \subseteq \text{DOM}(\epsilon_a')$. Also, we know that $\Xi_a, \Delta \models \langle \mu_a, \epsilon_c \rangle : \Gamma_{fn}$. Using the proof in Lemma E.5 we can conclude that $\Xi_a', \Delta \models \langle \mu_a', \epsilon_c \rangle : \Gamma_{fn}$.

d. For all x in $\text{DOM}(\epsilon_a')$ and some $\Gamma_{tbl} \subseteq \Gamma'$ and any pc, if $\Gamma', \Delta \vdash_{pc} x : \tau_{tbl}$, $\mu_a'(\epsilon_a'(x)) = table\ l\ (\epsilon_c, ...)$, and $\Xi_a' \models \epsilon_c : \Gamma_{tbl}$, then $\text{DOM}(\epsilon_c) \subseteq \text{DOM}(\epsilon_a')$ and $\Xi_a', \Delta \models \langle \mu_a', \epsilon_c \rangle : \Gamma_{tbl}$. Here τ_{tbl} is the table type. Proof for this is similar to the function closures case.

Proof of Equation (3) follows similarly.

To show Equation (5), we again use the fact that any $y \in \text{DOM}(\epsilon_a')$ will be either in $\text{DOM}(\epsilon_a)$ or be the new variable. The new variable already satisfies $\Xi_a', \Xi_b', \Delta \models_l \text{NI}(\mu_a'(\epsilon_a'(x)), \mu_b'(\epsilon_b'(x))) : \Gamma'(x)$, since the value is $init_\Delta \tau'$ which is not a function closure. For the other case where $y \in \text{DOM}(\epsilon_a)$, we already know that for any $y \in \text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$ we have $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(\mu_a(\epsilon_a(x)), \mu_b(\epsilon_b(x))) : \Gamma(x)$. This concludes $\Xi_a', \Xi_b', \Delta \models_l \text{NI}(\mu_a'(\epsilon_a'(x)), \mu_b'(\epsilon_b; (x))) : \Gamma'(x)$ because for variables in ϵ_a not equal to this new variable the memory store remains unchanged.

The last requirement is to prove that for all x in $\text{DOM}(\epsilon_a') = \text{DOM}(\epsilon_b')$ and some $\Gamma_{fn} \subseteq \Gamma$ and any pc, if $\Gamma', \Delta \vdash_{pc} x : \tau_{fn}$, $\mu_a'(\epsilon_a'(x)) = clos(\epsilon_{c_a}, ...)$, $\mu_b'(\epsilon_b'(x)) = clos(\epsilon_{c_b}, ...)$, $\Xi_a' \models \epsilon_{c_a} : \Gamma_{fn}$, and $\Xi_b' \models \epsilon_{c_b} : \Gamma_{fn}$, then $\Xi_a', \Xi_b', \Delta \models_l \langle \mu_a', \epsilon_{c_a} \rangle \langle \mu_b', \epsilon_{c_b} \rangle : \Gamma_{fn}$. This holds true because we already know that these closures satisfied $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_{c_a} \rangle \langle \mu_b, \epsilon_{c_b} \rangle : \Gamma_{fn}$ and because the memory stores haven't changed for any of the locations in $\text{DOM}(\mu_a)$ or $\text{DOM}(\mu_b)$, we can conclude that $\Xi_a', \Xi_b', \Delta \models_l \langle \mu_a', \epsilon_{c_a} \rangle \langle \mu_b', \epsilon_{c_b} \rangle : \Gamma_{fn}$ is also true. Similarly, we can show this for table closures as well.

This proves that $\Xi_a', \Xi_b', \Delta \models_l \langle \mu_a', \epsilon_a' \rangle \langle \mu_b', \epsilon_b' \rangle : \Gamma'$, where $\mu_a' = \mu_a[l_a := init_\Delta \tau']$, $\mu_b' = \mu_b[l_b := init_\Delta \tau']$, $\epsilon_a' = \epsilon_a[x \mapsto l_a]$, $\epsilon_b' = \epsilon_b[x \mapsto l_b]$. We also have $\Xi_a', \Xi_b', \Delta \models_l \langle \mu_a', \epsilon_a \rangle \langle \mu_b', \epsilon_b \rangle : \Gamma$, which follows from the definition (using induction in a similar manner as shown in Lemma E.5). Item 1c and Item 1e is satisfied because only the value of the new variable x is updated across μ_a and μ_a' and similarly μ_b and μ_b'.

2. **T-VarInit**

$$\frac{\Gamma, \Delta \vdash_{pc} exp : \langle \tau, \chi \rangle \qquad \Delta \vdash \tau' \rightsquigarrow \tau}{\Gamma; \Delta \vdash_{pc} \langle \tau', \chi \rangle\ x := exp \dashv \Gamma[x : \langle \tau, \chi \rangle]; \Delta}$$

Given the above typing judgement holds for $\langle \tau, \chi \rangle\ x := exp$ declaration, we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu_a', \mu_b', \epsilon_a', \epsilon_b'$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma \tag{1}$$

if the declaration, $\langle \tau, \chi \rangle\ x := exp$ is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows,

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, exp \rangle \Downarrow \langle \mu_{a1}, val_a \rangle \qquad l_a\ \text{fresh}}{\langle C, \Delta, \mu_a, \epsilon_a, \tau\ x := exp \rangle \Downarrow \langle \Delta, \mu_{a1}[l_a := val_a], \epsilon_a[x \mapsto l_a], cont \rangle}$$

$$\frac{\langle C, \Delta, \mu_b, \epsilon_b, exp \rangle \Downarrow \langle \mu_{b1}, val_b \rangle \qquad l_b\ \text{fresh}}{\langle C, \Delta, \mu_b, \epsilon_b, \langle \tau, \chi \rangle\ x := exp \rangle \Downarrow \langle \Delta, \mu_{b1}[l_b := val_b], \epsilon_b[x \mapsto l_b], cont \rangle}$$

then there exists some Ξ_a', Ξ_b' such that

a. $\Gamma, \Delta \vdash_{pc} \langle \tau, \chi \rangle\ x := exp \dashv \Gamma', \Delta$. This is already the hypothesis of the theorem.

b. $\Xi_a', \Xi_b', \Delta \models_l \langle \mu_a', \epsilon_a' \rangle \langle \mu_b', \epsilon_b' \rangle : \Gamma'$, where $\mu_a' = \mu_{a1}[l_a := val_a]$, $\mu_b' = \mu_{b1}[l_b := val_b]$, $\epsilon_a' = \epsilon_a[x \mapsto l_a]$, and $\epsilon_b' = \epsilon_b[x \mapsto l_b]$. Also, $\Xi_a', \Xi_b', \Delta \models_l \langle \mu_a', \epsilon_a \rangle \langle \mu_b', \epsilon_b \rangle : \Gamma$,

c. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_a'(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_b'(l_b) = \mu_b(l_b)$,

d. $\Xi_a \subseteq \Xi_a'$, $\Xi_b \subseteq \Xi_b'$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_a')$, $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu_b')$, $\text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon_a')$, and $\text{DOM}(\epsilon_b) \subseteq \text{DOM}(\epsilon_b')$.

e. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_a'(l_a) = \mu_a(l_a)$ and $\mu_b'(l_b) = \mu_b(l_b)$.

We can show the last three requirements similar to the previous case. In this case, we additionally know using the induction hypothesis of Theorem D.1 that exp evaluates to values that satisfy NI for values.

3. **T-FuncDecl**

$$\frac{\begin{array}{c} \Gamma_1 = \Gamma[\overline{x_i : \langle \tau'_i, \chi_i \rangle}, \text{return} : \langle \tau'_{ret}, \chi_{ret} \rangle] \qquad \Gamma_1, \Delta \vdash_{pc_{fn}} stmt \dashv \Gamma_2, \\ \Delta \vdash \tau_i \rightsquigarrow \tau'_i \text{ for each } \tau_i \qquad \Delta \vdash \tau_{ret} \rightsquigarrow \tau'_{ret} \qquad \Gamma' = \Gamma[x : \langle \overline{d \langle \tau'_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau'_{ret}, \chi_{ret} \rangle, \bot \rangle] \end{array}}{\Gamma, \Delta \vdash_{pc} \text{function } \langle \tau_{ret}, \chi_{ret} \rangle \ x \ (\overline{d\ x_i : \langle \tau_i, \chi_i \rangle})\{stmt\} \dashv \Gamma', \Delta} \text{ T-FuncDecl}$$

Given the above typing judgement holds for function declaration, we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b, \epsilon'_a, \epsilon'_b, \Delta$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma \tag{1}$$

if the function declaration is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows,

$$\frac{\begin{array}{c} l_a \ fresh \qquad val_a = clos(\epsilon_a, \overline{d\ x_i : \langle \tau'_i, \chi_i \rangle}, \langle \tau'_{ret}, \chi_{ret} \rangle, stmt) \\ \langle \Delta, \mu_a, \epsilon_a, \overline{\tau_i} \rangle \Downarrow_\tau \overline{\tau'_i} \qquad \langle \Delta, \mu_a, \epsilon_a, \tau_{ret} \rangle \Downarrow_\tau \overline{\tau'_{ret}} \end{array}}{\langle C, \Delta, \mu_a, \epsilon_a, \text{function } \langle \tau_{ret}, \chi_{ret} \rangle \ x \ (\overline{d\ x_i : \langle \tau_i, \chi_i \rangle})\{\overline{decl}\ stmt\} \rangle \Downarrow \langle \Delta, \mu_a[l_a \mapsto val_a], \epsilon_a[x \mapsto l_a], cont \rangle}$$

$$\frac{\begin{array}{c} l_b \ fresh \qquad val_b = clos(\epsilon_b, \overline{d\ x_i : \langle \tau'_i, \chi_i \rangle}, \langle \tau'_{ret}, \chi_{ret} \rangle, stmt) \\ \langle \Delta, \mu_a, \epsilon_a, \overline{\tau_i} \rangle \Downarrow_\tau \overline{\tau'_i} \qquad \langle \Delta, \mu_a, \epsilon_a, \tau_{ret} \rangle \Downarrow_\tau \overline{\tau'_{ret}} \end{array}}{\langle C, \Delta, \mu_b, \epsilon_b, \text{function } \langle \tau_{ret}, \chi_{ret} \rangle \ x \ (\overline{d\ x_i : \langle \tau_i, \chi_i \rangle})\{\overline{decl}\ stmt\} \rangle \Downarrow \langle \Delta, \mu_b[l_b \mapsto val_b], \epsilon_b[x \mapsto l_b], cont \rangle}$$

then there exists some Ξ'_a, Ξ'_b such that:

a. $\Gamma, \Delta \vdash_{pc} function\ declaration \dashv \Gamma', \Delta$. This is already the hypothesis of the theorem.

b. $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon'_a \rangle \langle \mu'_b, \epsilon'_b \rangle : \Gamma'$ and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$, where $\mu'_a = \mu_a[l_a \mapsto val_a]$, $\mu'_b = \mu_b[l_b \mapsto val_b]$, $\epsilon'_a = \epsilon_a[x \mapsto l_a]$ and $\epsilon'_b = \epsilon_b[x \mapsto l_b]$.

c. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$.

d. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a), \text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b), \text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon'_a)$, and $\text{DOM}(\epsilon_b) \subseteq \text{DOM}(\epsilon'_b), \Delta \subseteq \Delta_1$.

e. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$,

With $\Xi'_a = \Xi_a \cup \{l_a \mapsto \tau_{fn}\}, \Xi'_b = \Xi_b \cup \{l_b \mapsto \tau_{fn}\}, \mu'_a = \mu_a \cup \{l_a \mapsto val_a\}, \mu'_b = \mu_b \cup \{l_b \mapsto val_b\}, \text{DOM}(\epsilon'_a) = \text{DOM}(\epsilon_a) \cup \{x\}$ $\text{DOM}(\epsilon'_b) = \text{DOM}(\epsilon_b) \cup \{x\}$ the equation in Item 1d is evident.

To prove $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$, we need to show the following:

$$\Xi'_a, \Delta \models \langle \mu'_a, \epsilon_a \rangle : \Gamma \tag{2}$$

$$\Xi'_b, \Delta \models \langle \mu'_b, \epsilon_b \rangle : \Gamma \tag{3}$$

$$\text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b) \tag{4}$$

For any $x \in \text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$ we have $\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(\mu'_a(\epsilon_a(x)) , \ \mu'_b(\epsilon_b(x))) : \Gamma(x) \tag{5}$

and for all x in $\text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$ and some $\Gamma_{fn} \subseteq \Gamma$ and any pc, if $\Gamma, \Delta \vdash_{pc} x : \tau_{fn}, \mu'_a(\epsilon_a(x)) = clos(\epsilon_{c_a}, ...)$, $\mu'_b(\epsilon_b(x)) = clos(\epsilon_{c_b}, ...), \Xi'_a \models \epsilon_{c_a} : \Gamma_{fn}$, and $\Xi'_b \models \epsilon_{c_b} : \Gamma_{fn}$, then $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_{c_a} \rangle \langle \mu'_b, \epsilon_{c_b} \rangle : \Gamma_{fn}$.

First, we begin by showing Equation (2). This requires us to in turn prove the following:

a. $\Xi'_a, \Delta \models \mu'_a$. This is shown in Lemma E.3.

b. $\Xi'_a \vdash \epsilon_a : \Gamma$. This follows from $\Xi_a \vdash \epsilon_a : \Gamma$ and weakening of store typing context.

c. Next, we need to show that any closure value has $\text{DOM}(\epsilon_{clos}) \subseteq \text{DOM}(\epsilon)$ (already known from Equation (1)) and $\Xi'_a, \Delta \models \langle \mu'_a, \epsilon_{clos} \rangle : \Gamma_{clos}$, where ϵ_{clos} is the environment bound to the closure. We already know that $\Xi_a, \Delta \models \langle \mu_a, \epsilon_{clos} \rangle : \Gamma_{clos}$. Using Lemma E.5, we have $\Xi'_a, \Delta \models \langle \mu'_a, \epsilon_{clos} \rangle : \Gamma_{clos}$.

Similarly, we can prove Equation (3). Since value of no location besides the fresh l_a and l_b changes between μ_a and μ'_a and μ_b and μ'_b, we can show Equation (5) and the one following it using the results from Equation (1). All the variables referenced by closures that were declared until ϵ_a or ϵ_b have unchanged values.

To prove $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon'_a \rangle \langle \mu'_b, \epsilon'_b \rangle : \Gamma$, we need to show the following:

$$\Xi'_a, \Delta \models \langle \mu'_a, \epsilon'_a \rangle : \Gamma' \tag{6}$$

$$\Xi'_b, \Delta \models \langle \mu'_b, \epsilon'_b \rangle : \Gamma' \tag{7}$$

$$\text{DOM}(\epsilon'_a) = \text{DOM}(\epsilon'_b) \tag{8}$$

For any $x \in \text{DOM}(\epsilon'_a) = \text{DOM}(\epsilon'_b)$ we have $\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(\mu'_a(\epsilon'_a(x)), \mu'_b(\epsilon'_b(x))) : \Gamma'(x)$ \hfill (9)

and for all x in $\text{DOM}(\epsilon'_a) = \text{DOM}(\epsilon'_b)$ and some $\Gamma_{fn} \subseteq \Gamma$ and any pc, if $\Gamma', \Delta \vdash_{pc} x : \tau_{fn}$, $\mu'_a(\epsilon'_a(x)) = clos(\epsilon_{c_a}, ...)$, $\mu'_b(\epsilon'_b(x)) = clos(\epsilon_{c_b}, ...)$, $\Xi'_a \models \epsilon_{c_a} : \Gamma_{fn}$, and $\Xi'_b \models \epsilon_{c_b} : \Gamma_{fn}$, then $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_{c_a} \rangle \langle \mu'_b, \epsilon_{c_b} \rangle : \Gamma_{fn}$.

First, we begin by showing Equation (6). This requires us to in turn prove the following:

a. $\Xi'_a, \Delta \models \mu'_a$. This is shown in Lemma E.3.

b. $\Xi'_a \vdash \epsilon'_a : \Gamma'$. Since we are given $\Xi_a \vdash \epsilon_a : \Gamma$, by using Lemma E.4, we can say that $\Xi'_a \vdash \epsilon_a : \Gamma$. Since $\Gamma' = \Gamma[x \mapsto \overline{\langle d \langle \tau_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle]$, $\epsilon'_a = \epsilon_a[x \mapsto l_a]$, $\Xi'_a = \Xi_a \cup \{l_a \mapsto \langle \tau, pc \rangle\}$, by using the rules for $\Xi \vdash \epsilon : \Gamma$, we can show that $\Xi'_a \vdash \epsilon'_a : \Gamma'$.

$$\frac{\Xi'_a \vdash \epsilon_a : \Gamma \qquad \Xi'_a(l_a) = \overline{\langle d \langle \tau_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle}{\Xi'_a \vdash \epsilon_a[x \mapsto l_a] : \Gamma[x \mapsto \overline{\langle d \langle \tau_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle]}$$

c. Next, we need to show that any closure value has $\text{DOM}(\epsilon_{clos}) \subseteq \text{DOM}(\epsilon')$ and $\Xi'_a, \Delta \models \langle \mu'_a, \epsilon_{clos} \rangle : \Gamma_{clos}$, where ϵ_{clos} is the environment bound to the closure. Since $\epsilon'_a = \epsilon_a[x \mapsto l_a]$, where $\Gamma', \Delta \vdash_{pc} x : \tau_{fn}$, we need to show the above property for $y \in \text{DOM}(\epsilon_a)$ that is not equal to x and the new closure variable x. Since

$$\text{for any } y \in \text{DOM}(\Gamma) \text{ such that } y \neq x, \text{ we have } \Gamma'(y) = \Gamma(y)$$

$$\text{for any } y \in \text{DOM}(\epsilon_a) \text{ such that } y \neq x, \text{ we have } \epsilon'_a(y) = \epsilon_a(y)$$

and from Equation (1), we already know that for all $\Gamma, \Delta \vdash_{pc} y : \tau_{clos} \in \text{DOM}(\epsilon_a)$ such that $y \neq x$ and $\mu'_a(\epsilon'_a(y)) = val_{clos}$, we have $\text{DOM}(\epsilon_c) \subseteq \text{DOM}(\epsilon_a)$, $\Xi_a, \Delta \models \langle \mu_a, \epsilon_c \rangle : \Gamma_{clos}$ and $\Xi_b, \Delta \models \langle \mu_b, \epsilon_c \rangle : \Gamma_{clos}$. Using Lemma E.5, we can conclude that $\Xi'_a, \Delta \models \langle \mu'_a, \epsilon_c \rangle : \Gamma_{clos}$ and $\Xi'_b, \Delta \models \langle \mu'_b, \epsilon_c \rangle : \Gamma_{clos}$.

Since $\mu'_a(\epsilon'_a(x)) = clos(\epsilon_a, \overline{d\ x_i : \langle \tau'_i, \chi_i \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, stmt)$, $\text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon'_a)$, and $\Xi'_a, \Delta \models \langle \mu'_a, \epsilon_a \rangle : \Gamma$ we can conclude that for all closure values, we have $\text{DOM}(\epsilon_c) \subseteq \text{DOM}(\epsilon'_a)$, $\Xi'_a, \Delta \models \langle \mu'_a, \epsilon_c \rangle : \Gamma_{clos}$ and $\Xi'_b, \Delta \models \langle \mu'_b, \epsilon_c \rangle : \Gamma_{clos}$.

This proves Equation (6). Proof of Equation (7) follows similarly.

To show Equation (9), we again use the fact that any $y \in \text{DOM}(\epsilon'_a)$ will be either in $\text{DOM}(\epsilon_a)$ or be the function name, x. For the case where $y \neq x \in \text{DOM}(\epsilon_a)$, we already know that $\Xi_a, \Xi_b, \Delta \models_l \text{NI}(\mu_a(\epsilon_a(x)), \mu_b(\epsilon_b(x))) : \Gamma(x)$. This implies that for any $y \neq x \in \text{DOM}(\epsilon_a) = \text{DOM}(\epsilon_b)$, we have $\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(\mu'_a(\epsilon'_a(x)), \mu'_b(\epsilon'_b(x))) : \Gamma'(x)$, since such y satisfies $\mu'_a(\epsilon'_a(x)) = \mu_a(\epsilon_a(x))$, $\mu'_b(\epsilon'_b(x)) = \mu_b(\epsilon_b(x))$, $\Gamma(x) = \Gamma'(x)$ and $\Xi'_a(x) = \Xi_a(x)$, and $\Xi'_b(x) = \Xi_b(x)$.

For the case when $y = x$, we need to show the following, where $\tau_{fn} = \overline{\langle d \langle \tau_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle$.

$$\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(clos(\epsilon_a, \overline{d\ x_i : \langle \tau'_i, \chi_i \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, stmt), clos(\epsilon_b, \overline{d\ x_i : \langle \tau'_i, \chi_i \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, stmt)) : \tau_{fn} \hfill (10)$$

To show this, we need to first prove that $\Xi'_a, \Delta \vdash clos(\epsilon_a, \overline{d\ x_i : \langle \tau'_i, \chi_i \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, \overline{decl}\ stmt) : \overline{\langle d \langle \tau_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle$. For this we need to look at the value typing rule for function closures.

TV-CLOS
$$\frac{\Xi \vdash \epsilon : \Gamma \qquad \Gamma[\overline{x : \langle \tau, \chi \rangle}, \text{return} = \langle \tau_{ret}, \chi_{ret} \rangle], \Delta \vdash_{pc_{fn}} stmt \dashv \Gamma'}{\Xi, \Delta \vdash clos(\epsilon, \overline{d\ x : \langle \tau, \chi \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, \overline{decl}\ stmt) : \langle \overline{\langle d\ \tau, \chi \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle}$$

These premises are:

a. We need to show $\Xi'_a \vdash \epsilon_a : \Gamma$. We already know from Equation (1) that $\Xi_a \vdash \epsilon_a : \Gamma$. Using Lemma E.4, we can conclude $\Xi'_a \vdash \epsilon_a : \Gamma$.

b. We need to show $\Gamma[\overline{x_i : \langle \tau_i, \chi_i \rangle}, \text{return} : \langle \tau_{ret}, \chi_{ret} \rangle], \Delta \vdash_{pc_{fn}} stmt \dashv \Gamma_1$,

This is satisfied as a part of the premise in the typing rule for function declaration. This concludes

$$\Xi'_a, \Delta \vdash clos(\epsilon_a, \overline{d\ x_i : \langle \tau'_i, \chi_i \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, stmt) : \overline{\langle d \langle \tau_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle$$

$$\Xi'_b, \Delta \vdash clos(\epsilon_b, \overline{d\ x_i : \langle \tau'_i, \chi_i \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, stmt) : \overline{\langle d \langle \tau_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle$$

Next, we show that for Γ, the following properties hold:

a. $\Xi'_a \vdash \epsilon_a : \Gamma$, $\Xi'_b \vdash \epsilon_b : \Gamma$. Already shown above.

b. $\Gamma, \Delta \vdash_{pc} clos(\epsilon_a, \overline{d\ x_i : \langle \tau'_i, \chi_i \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, stmt) : \overline{\langle d \langle \tau_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle$. We already know this by the typing derivation. Similarly, $\Gamma, \Delta \vdash_{pc} clos(\epsilon_b, \overline{d\ x_i : \langle \tau'_i, \chi_i \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, stmt) : \overline{\langle d \langle \tau_i, \chi_i \rangle} \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle$.

c. $\Gamma[\overline{x : \langle \tau, \chi \rangle}, \text{return} : \langle \tau_{ret}, \chi_{ret} \rangle], \Delta \vdash_{pc_{fn}} stmt \dashv \Gamma'$

d. Also, we have $val_a =_{clos} val_b$.

We also need to show that for all $y \in \text{DOM}(\epsilon'_a) = \text{DOM}(\epsilon'_b)$ and some $\Gamma_{clos} \subseteq \Gamma'$ and any pc, if $\Gamma', \Delta \vdash_{pc} y : \tau_{clos}$, $\mu'_a(\epsilon'_a(x)) = val_{clos}$ with environment ϵ_{c_a}, $\mu'_b(\epsilon'_b(y)) = val_{clos}$ with environment ϵ_{c_b}, $\Xi'_a \models \epsilon_{c_a} : \Gamma_{clos}$, and $\Xi'_b \models \epsilon_{c_b} : \Gamma_{clos}$, then $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_{c_a} \rangle \langle \mu'_b, \epsilon_{c_b} \rangle : \Gamma_{clos}$. For $y \neq x$, we know that a closure value would satisfy $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_{c_a} \rangle \langle \mu_b, \epsilon_{c_b} \rangle : \Gamma_{clos}$. Since value of no variable referenced by any of the closure defined until ϵ_a is updated between μ'_a and μ'_b, we can say that $\Xi_a, \Xi_b, \Delta \models_l \langle \mu'_a, \epsilon_{c_a} \rangle \langle \mu'_b, \epsilon_{c_b} \rangle : \Gamma_{clos}$. By weakening the store typing context, we can also say $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_{c_a} \rangle \langle \mu'_b, \epsilon_{c_b} \rangle : \Gamma_{clos}$. For the new closure variable x, we already have $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$. This finally proves Item 3b. Item 3e is satisfied because only the value of the location pointed by the function name, x is updated in the memory store. Item 3c is trivial since no location besides the fresh l_a and l_b are updated.

4. **T-TblDecl**

$$\frac{\begin{array}{c} \Gamma, \Delta \vdash_{pc_{tbl}} \overline{exp_k : \langle \tau_k, \chi_k \rangle} \qquad \Gamma, \Delta \vdash_{pc_{tbl}} \overline{x_k : \langle match\_kind, \bot \rangle} \\ \Gamma, \Delta \vdash_{pc_{tbl}} act_{a_j} : \langle \overline{d\langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle} ; \overline{\langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle} \xrightarrow{pc_{fn_j}} \langle unit, \bot \rangle, \bot \rangle, \text{ for all } j \\ \Gamma, \Delta \vdash_{pc_{tbl}} \overline{exp_{a_{ji}} : \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle goes\ d} \\ \chi_k \sqsubseteq pc_{fn_j} \text{ for all } j, k \qquad pc_a \sqsubseteq pc_{fn_j}, \text{ for all } j \qquad \chi_k \sqsubseteq pc_{tbl} \text{ for all } k \qquad pc_{tbl} \sqsubseteq pc_a \end{array}}{\Gamma, \Delta \vdash_{pc} table\ x\ \{\overline{exp_k : x_k}\ \overline{act_{a_j}(\overline{exp_{a_{ji}}})}\} \dashv \Gamma[x : \langle table(pc_{tbl}), \bot \rangle], \Delta} \text{ T-TblDecl}$$

Given the above typing judgement holds for table declaration, we need to show that for any $\Xi_a, \Xi_b, \mu_a, \mu_b, \epsilon_a, \epsilon_b, \mu'_a, \mu'_b, \epsilon'_a, \epsilon'_b$ satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma \tag{1}$$

if the table declaration is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$ as follows,

$$\frac{l_a\ fresh \qquad val_a = table\ l_a\ (\epsilon_a, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})})}{\langle C, \Delta, \mu_a, \epsilon_a, table\ x\ \{\overline{exp_k : x_k}\ \overline{act_{a_j}(\overline{exp_{a_{ji}}})}\} \rangle \Downarrow \langle \Delta, \mu_a[l_a \mapsto val_a], \epsilon_a[x \mapsto l_a], cont \rangle}$$

$$\frac{l_b\ fresh \qquad val_b = table\ l_b\ (\epsilon_b, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})})}{\langle C, \Delta, \mu_b, \epsilon_b, table\ x\ \{\overline{exp_k : x_k}\ \overline{act_{a_j}(\overline{exp_{a_{ji}}})}\} \rangle \Downarrow \langle \Delta, \mu_b[l_b \mapsto val_b], \epsilon_b[x \mapsto l_b], cont \rangle}$$

then there exists some Ξ'_a, Ξ'_b such that

a. $\Gamma, \Delta \vdash_{pc} table\ declaration \dashv \Gamma', \Delta$. This is already the hypothesis of the theorem.

b. $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon'_a \rangle \langle \mu'_b, \epsilon'_b \rangle : \Gamma'$ and $\Xi'_a, \Xi'_b, \Delta \models_l \langle \mu'_a, \epsilon_a \rangle \langle \mu'_b, \epsilon_b \rangle : \Gamma$, where $\mu'_a = \mu_a[l_a \mapsto val_a]$, $\mu'_b = \mu_b[l_b \mapsto val_b]$, $\epsilon'_a = \epsilon_a[x \mapsto l_a]$ and $\epsilon'_b = \epsilon_b[x \mapsto l_b]$, $\Gamma' = \Gamma[x \mapsto \langle table(pc_{tbl}), \bot \rangle]$.

c. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_a(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu'_b(l_b) = \mu_b(l_b)$,

d. $\Xi_a \subseteq \Xi'_a, \Xi_b \subseteq \Xi'_b, \text{DOM}(\mu_a) \subseteq \text{DOM}(\mu'_a), \text{DOM}(\mu_b) \subseteq \text{DOM}(\mu'_b), \text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon'_a)$, and $\text{DOM}(\epsilon_b) \subseteq \text{DOM}(\epsilon'_b)$.

e. For any $l_a \in \text{DOM}(\mu_a)$ and $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b) : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu'_a(l_a) = \mu_a(l_a)$ and $\mu'_b(l_b) = \mu_b(l_b)$.

With $\mu'_a = \mu_a[l_a \mapsto val_a]$, $\epsilon'_a = \epsilon_a[x \mapsto l_a]$, $\text{DOM}(\epsilon'_a) = \text{DOM}(\epsilon_a) \cup \{x\}$, $\mu'_b = \mu_b[l_b \mapsto val_b]$, $\epsilon'_b = \epsilon_b[x \mapsto l_b]$, $\text{DOM}(\epsilon'_b) = \text{DOM}(\epsilon_b) \cup \{x\}$, $\Xi'_a = \Xi_a[l_a \mapsto \langle texttable(pc_{tbl}), \bot \rangle]$, and $\Xi'_b = \Xi_b[l_b \mapsto \langle table(pc_{tbl}), \bot \rangle]$ Item 4d is evident.

Proof of Item 4b follows similar to the function declaration case. The interesting bit is to show that the freshly added table name x satisfies the following property. For the case when $y = x$, we need to show that

$$\Xi'_a, \Xi'_b, \Delta \models_l \text{NI}(table\ l_a\ (\epsilon_a, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})}), table\ l_b\ (\epsilon_a, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})})) : \tau_{tbl}$$

where $\tau_{tbl} = \langle table(pc_{tbl}), \bot \rangle$. For this, we need to first show that

$$\Xi'_a, \Delta \vdash table\ l_a\ (\epsilon_a, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})}) : \langle table(pc_{tbl}), \bot \rangle$$

$$\Xi'_b, \Delta \vdash table\ l_b\ (\epsilon_b, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})}) : \langle table(pc_{tbl}), \bot \rangle$$

To show this, we need to prove that the premises of the following value typing rule are satisfied,

TV-Tbl

$$\frac{\Xi \vdash \epsilon : \Gamma \qquad \Gamma, \Delta \vdash_{pc_{tbl}} \overline{exp_k : \langle \tau_k, \chi_k \rangle} \qquad \Gamma, \Delta \vdash_{pc_{tbl}} \overline{x_k : \langle match\_kind, \bot \rangle}}{\Gamma, \Delta \vdash_{pc_{tbl}} act_{a_j} : \langle \overline{d \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle} ; \overline{\langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle} \xrightarrow{pc_{fn_j}} \langle unit, \bot \rangle, \bot \rangle, \text{ for all } j}$$

$$\frac{\Gamma, \Delta \vdash_{pc_{tbl}} exp_{a_{ji}} : \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle goes \, d}{\chi_k \sqsubseteq pc_{fn_j} \text{ for all } j, k \qquad pc_a \sqsubseteq pc_{fn_j}, \text{for all } j \qquad \chi_k \sqsubseteq pc_{tbl} \text{ for all } k \qquad pc_{tbl} \sqsubseteq pc_a}{\Xi, \Delta \vdash table \, l \, \{\epsilon, \overline{exp_k : x_k} \, \overline{act_{a_j}(\overline{exp_{a_{ji}}})}\} : \langle table(pc_{tbl}), \bot \rangle}$$

a. We need to show $\Xi'_a, \Delta \vdash \epsilon_a : \Gamma$. We already know from Equation (1) that $\Xi_a \vdash \epsilon_a : \Gamma$. Using Lemma E.4, we can conclude $\Xi'_a \vdash \epsilon_a : \Gamma$.

b. We need to show $\Gamma, \Delta \vdash_{pc_{tbl}} \overline{exp_k : \langle \tau_k, \chi_k \rangle}$

c. We need to show $\Gamma, \Delta \vdash_{pc_{tbl}} \overline{x_k : \langle match\_kind, \bot \rangle}$

d. We need to show $\Gamma, \Delta \vdash_{pc_{tbl}} act_{a_j} : \langle \overline{d \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle} ; \overline{\langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle} \xrightarrow{pc_{fn_j}} \langle unit, \bot \rangle, \bot \rangle$

e. We need to show $\Gamma, \Delta \vdash_{pc_{tbl}} exp_{a_{ji}} : \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle \, goes \, d$

The last four properties are satisfied as a part of the premise for the typing rule for table declaration. This concludes:

$$\Xi'_a, \Delta \vdash table \, l_a \, (\epsilon_a, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})}) : \langle table(pc_{tbl}), \bot \rangle$$

$$\Xi'_b, \Delta \vdash table \, l_b \, (\epsilon_a, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})}) : \langle table(pc_{tbl}), \bot \rangle$$

Next we show that for Γ the following properties hold (by expanding the definition of NI for table closures)

a. $\Xi'_a \vdash \epsilon_a : \Gamma, \Xi'_b \vdash \epsilon_b : \Gamma$. Already shown.

b. $\Gamma, \Delta \vdash_{pc} table \, l_a \, (\epsilon_a, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})}) : \langle table(pc_{tbl}), \bot \rangle$. We already know this by the typing

derivation. Similarly, $\Gamma, \Delta \vdash_{pc} table \, l_b \, (\epsilon_a, \overline{exp_k : x_k}, \overline{act_{a_j}(\overline{exp_{a_{ji}}}, \overline{y_{c_{ji}} : \langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle})}) : \langle table(pc_{tbl}), \bot \rangle$.

c. $\Gamma, \Delta \vdash_{pc_{tbl}} x_k : \langle match\_kind, \bot \rangle$ for each $x_k \in \overline{x_k}$.

d. $\Gamma, \Delta \vdash_{pc_{tbl}} exp_k : \langle \tau_k, \chi_k \rangle$. for each $exp_k \in \overline{exp_k}$.

e. $\Gamma, \Delta \vdash_{pc_{tbl}} act_{aj} : \langle \overline{d \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle} ; \overline{\langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle} \xrightarrow{pc_{fn_j}} \langle unit, \bot \rangle, \bot \rangle$ for each $act_{a_j} \in \overline{act_{a_j}}$.

f. $\Gamma, \Delta \vdash_{pc_{tbl}} exp_{a_{ji}} : \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle \, goes \, d$ for each $exp_{a_{ji}} \in \overline{exp_{a_{ji}}}$.

g. $val_a =_{tbl} val_b$.

h. $\chi_k \sqsubseteq pc_{fn_j}$, for all j, k

i. $pc_a \sqsubseteq pc_{fn_j}$, for all j

j. $\chi_k \sqsubseteq pc_{tbl}$ for all k

k. $pc_{tbl} \sqsubseteq pc_a$

These properties are can be shown using the premise in the typing derivation.

5. **T-Typedef**

$$\frac{}{\Gamma, \Delta \vdash_{pc} typedef \, \tau \, X \; \dashv \Gamma, \Delta[X = \tau]} \text{ T-Typedef}$$

$$\frac{}{\langle C, \Delta; \mu_a; \epsilon_a; typedef \, \tau \, X \rangle \Downarrow \langle \Delta[X = \tau], \mu_a, \epsilon_a, cont \rangle}$$

The proof of this case is trivial. The only interesting part is to show that $\Xi'_a, \Xi'_b, \Delta[X = \tau] \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$. We already know that $\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma$. By definition of this judgement for a pair of consistent state, we can observe that we can prove this for the extended Δ as it is a case of weakening the context.

6. **T-MatchKind**

$$\frac{}{\Gamma, \Delta \vdash_{pc} match\_kind\{\overline{f}\} \; \dashv \Gamma, \Delta[match\_kind = match\_kind\{\overline{f}\}]} \text{ T-MemHdr}$$

Evaluation rule is

$$\frac{}{\langle C, \Delta; \mu_a; \epsilon_a; match\_kind\{\overline{f}\} \rangle \Downarrow \langle \Delta[match\_kind = match\_kind\{\overline{f}\}], \mu_a, \epsilon_a, cont \rangle} \text{ Eval 1}$$

The proof is similar to the typedef case.

7. T-Seq-2

$$\frac{\Gamma, \Delta \vdash_{pc} decl \dashv \Gamma_1, \Delta_1 \qquad \Gamma_1, \Delta_1 \vdash_{pc} stmt \dashv \Gamma_2}{\Gamma, \Delta \vdash_{pc} decl\ stmt \dashv \Gamma_2, \Delta_1} \text{ T-Seq}$$

$$\frac{\langle C, \Delta, \mu_a, \epsilon_a, decl \rangle \Downarrow \langle \Delta_1, \mu_{a1}, \epsilon_{a1}, cont \rangle \qquad \langle C, \Delta_1, \mu_{a1}, \epsilon_{a1}, stmt \rangle \Downarrow \langle \mu_{a2}, \epsilon_{a2}, sig_a \rangle}{\langle C, \Delta, \mu_a, \epsilon_a, decl\ stmt \rangle \Downarrow \langle \Delta_1, \mu_{a2}, \epsilon_{a2}, sig_a \rangle}$$

$$\frac{\langle C, \Delta, \mu_b, \epsilon_b, decl \rangle \Downarrow \langle \Delta_1, \mu_{b1}, \epsilon_{b1}, cont \rangle \qquad \langle C, \Delta_1, \mu_{b1}, \epsilon_{b1}, stmt \rangle \Downarrow \langle \mu_{b2}, \epsilon_{b2}, sig_b \rangle}{\langle C, \Delta, \mu_b, \epsilon_b, decl\ stmt \rangle \Downarrow \langle \Delta_1, \mu_{b2}, \epsilon_{b2}, sig_b \rangle}$$

Given the above typing judgement holds for the statement,$decl\ stmt$, we need to show that for any Ξ_a, Ξ_b, μ_a, μ_b, ϵ_a, ϵ_b, μ_a', μ_b', ϵ_a', ϵ_b' satisfying

$$\Xi_a, \Xi_b, \Delta \models_l \langle \mu_a, \epsilon_a \rangle \langle \mu_b, \epsilon_b \rangle : \Gamma \tag{1}$$

If the statement, $decl\ stmt$ is evaluated under two different initial configurations $\langle \mu_a, \epsilon_a \rangle$ and $\langle \mu_b, \epsilon_b \rangle$, then there exists some Ξ_a' and Ξ_b', such that the following hold:

a. $\Gamma, \Delta \vdash_{pc} decl\ stmt \dashv \Gamma_2, \Delta_1$. This is already the theorem's hypothesis.

b. We have $\Xi_a \subseteq \Xi_a'$, $\Xi_b \subseteq \Xi_b'$, $\text{DOM}(\mu_a) \subseteq \text{DOM}(\mu_a')$, $\text{DOM}(\mu_b) \subseteq \text{DOM}(\mu_b')$, $\text{DOM}(\epsilon_a) \subseteq \text{DOM}(\epsilon_a')$, and $\text{DOM}(\epsilon_b) \subseteq \text{DOM}(\epsilon_b')$, $\Delta \subseteq \Delta_1$, and $\Xi_a', \Xi_b', \Delta_1 \models_l \langle \mu_a', \epsilon_a' \rangle \langle \mu_b', \epsilon_b' \rangle : \Gamma'$. In this case $\mu_a' = \mu_{a2}$, $\mu_b' = \mu_{b2}$, $\epsilon_a' = \epsilon_{a2}$, $\epsilon_b' = \epsilon_{b2}$. We also need to show that $\Xi_a', \Xi_b', \Delta \models_l \langle \mu_a', \epsilon_a \rangle \langle \mu_b', \epsilon_b \rangle : \Gamma$.

c. For any $l_a \in \text{DOM}(\mu_a)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_a'(l_a) = \mu_a(l_a)$. Similarly for any $l_b \in \text{DOM}(\mu_b)$ such that $\Xi_b, \Delta \vdash \mu_b(l_b) : \tau_{clos}$, where $\tau_{clos} \in \{\tau_{fn}, \tau_{tbl}\}$, then $\mu_b'(l_b) = \mu_b(l_b)$.

d. For any $l_a' \in \text{DOM}(\mu_a)$ and $l_b' \in \text{DOM}(\mu_b)$ such that $\Xi_a, \Delta \vdash \mu_a(l_a') : \langle \tau, \chi \rangle$ and $\Xi_b, \Delta \vdash \mu_b(l_b') : \langle \tau, \chi \rangle$ and $pc \not\sqsubseteq \chi$, we have $\mu_a(l_a') = \mu_a'(l_a')$ and $\mu_b(l_b') = \mu_b'(l_b')$,

e. sig in any two evaluations are of the same form.

The proof is direct by applying induction hypothesis on the $decl$ and $stmt$. We will highlight the most interesting part. By applying induction hypothesis of Theorem D.3 on $decl$, we conclude that NI decl. This implies $\Xi_{a1}, \Xi_{b1}, \Delta_1 \models_l \langle \mu_{a1}, \epsilon_{a1} \rangle \langle \mu_{b1}, \epsilon_{b1} \rangle : \Gamma$ and $\Xi_{a1}, \Xi_{b1}, \Delta_1 \models_l \langle \mu_{a1}, \epsilon_a \rangle \langle \mu_{b1}, \epsilon_b \rangle : \Gamma$ By applying induction hypothesis of Theorem D.2 on $stmt$, we conclude that $\Xi_{a2}, \Xi_{b2}, \Delta_1 \models_l \langle \mu_{a2}, \epsilon_{a2} \rangle \langle \mu_{b2}, \epsilon_{b2} \rangle : \Gamma$ and $\Xi_{a2}, \Xi_{b2}, \Delta_1 \models_l \langle \mu_{a2}, \epsilon_{a1} \rangle \langle \mu_{b2}, \epsilon_{b1} \rangle : \Gamma$. To prove $\Xi_{a2}, \Xi_{b2}, \Delta_1 \models_l \langle \mu_{a2}, \epsilon_a \rangle \langle \mu_{b2}, \epsilon_b \rangle : \Gamma$, we use the same approach from T-Seq-1 case (Item 4).

J   Value Typing Rule

TV-Rec
$$\frac{\Xi, \Delta \vdash \overline{val : \langle \tau, \chi \rangle}}{\Xi, \Delta \vdash \{\overline{f = val}\} : \langle \{\overline{f : \langle \tau, \chi \rangle}\}, \bot \rangle}$$

TV-Hdr
$$\frac{\Xi, \Delta \vdash \overline{val : \langle \tau, \chi \rangle}}{\Xi, \Delta \vdash header\{valid, \overline{f : \langle \tau, \chi \rangle = val}\} : \langle header\{\overline{f : \langle \tau, \chi \rangle}\}, \bot \rangle}$$

TV-Stack
$$\frac{len(\overline{val}) = n \qquad \Xi, \Delta \vdash \overline{val} : \langle \tau, \chi \rangle}{\Xi, \Delta \vdash stack \, \langle \tau, \chi \rangle \, \{\overline{val}\} : \langle \langle \tau, \chi \rangle [n], \bot \rangle}$$

TV-Clos
$$\frac{\Xi \vdash \epsilon : \Gamma \qquad \Gamma[\overline{x : \langle \tau, \chi \rangle}, return = \langle \tau_{ret}, \chi_{ret} \rangle], \Delta \vdash_{pc_{fn}} stmt \dashv \Gamma'}{\Xi, \Delta \vdash clos(\epsilon, \overline{d \, x : \langle \tau, \chi \rangle}, \langle \tau_{ret}, \chi_{ret} \rangle, \overline{decl} \, stmt) : \langle \langle \overline{d \, \tau, \chi} \rangle \xrightarrow{pc_{fn}} \langle \tau_{ret}, \chi_{ret} \rangle, \bot \rangle}$$

TV-Tbl
$$\frac{\begin{array}{c} \Xi \vdash \epsilon : \Gamma \qquad \Gamma, \Delta \vdash_{pc_{tbl}} \overline{exp_k : \langle \tau_k, \chi_k \rangle} \qquad \Gamma, \Delta \vdash_{pc_{tbl}} \overline{x_k : \langle match\_kind, \bot \rangle} \\ \Gamma, \Delta \vdash_{pc_{tbl}} act_{a_j} : \langle \overline{d \, \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle} ; \overline{\langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle} \xrightarrow{pc_{fn_j}} \langle unit, \bot \rangle, \bot \rangle, \text{ for all } j \\ \Gamma, \Delta \vdash_{pc_{tbl}} exp_{a_{ji}} : \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle goes \, d \\ \chi_k \sqsubseteq pc_{fn_j} \text{ for all } j, k \qquad pc_a \sqsubseteq pc_{fn_j}, \text{ for all } j \qquad \chi_k \sqsubseteq pc_{tbl} \text{ for all } k \qquad pc_{tbl} \sqsubseteq pc_a \end{array}}{\Xi, \Delta \vdash table \, l \, \{\epsilon, \overline{exp_k : x_k} \, \overline{act_{a_j}(\overline{exp_{a_{ji}}})}\} : \langle table(pc_{tbl}), \bot \rangle}$$

TV-PartialApp
$$\frac{\Gamma, \Delta \vdash_{pc} x_{act} : \langle \overline{d \, \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle} ; \overline{\langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle} \xrightarrow{pc_{fn}} \langle unit, \bot \rangle, \bot \rangle \qquad \Gamma, \Delta \vdash_{pc} \overline{exp : \langle \tau, \chi \rangle goes \, d}}{\Xi, \Delta \vdash x_{act}(\overline{exp}, \overline{x_c : \langle \tau, \chi \rangle}) : \langle \overline{d \, \langle \tau_{a_{ji}}, \chi_{a_{ji}} \rangle} ; \overline{\langle \tau_{c_{ji}}, \chi_{c_{ji}} \rangle} \xrightarrow{pc_{fn}} \langle unit, \bot \rangle, \bot \rangle}$$

Match
$$\frac{\Delta(match\_kind) = match\_kind\{\overline{f}\}}{\Xi, \Delta \vdash match\_kind.f : match\_kind}$$

TV-SubType
$$\frac{\Xi, \Delta \vdash val : \langle \tau, \chi \rangle \qquad \chi \sqsubseteq \chi'}{\Xi, \Delta \vdash val : \langle \tau, \chi' \rangle}$$