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On the Design of Cognitive-Radio-Inspired Asymmetric Network Coding Transmissions in MIMO Systems

ON THE DESIGN OF COGNITIVE RADIO INSPIRED ASYMMETRIC NETWORK CODING TRANSMISSIONS IN MIMO SYSTEMS

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Abstract

In this paper, a cognitive radio inspired asymmetric network coding (CR-AsNC) scheme is proposed for multiple-input and multiple-output (MIMO) cellular transmissions, where information exchange among users and base station broadcasting can be accomplished simultaneously. The key idea is to apply the concept of cognitive radio in network coding transmissions, where the base station tries to send new information while helping users' transmissions as a relay. In particular, we design an asymmetric network coding method for information exchange between the base station and the users, while many existing works consider the design of network coding in symmetric scenarios. To approach the optimal performance, an iterative precoding design for CR-AsNC is first developed. Then a channel diagonalization-based precoding design with low complexity is proposed, to which power allocation can be optimized with a closed form solution. The simulation results show that the proposed CR-AsNC scheme with precoding optimization can significantly improve the system transmission performance.

Index Terms

multiple-input and multiple-output (MIMO), network coding, precoding design, multi-way transmission

I. Introduction

Due to the limitation of radio resources, spectrum scarcity becomes a major issue in cellular mobile communication systems [1], which has drawn a lot of attention in the research field. To further improve the spectral efficiency, many feasible methods have been proposed for cellular communications, such as the multiple-input and multiple-output (MIMO) technique [2]. By applying appropriate precoding design, MIMO transmission systems allow spatial multiplexing of spectrum resource, and the spectral efficiency can increase linearly as the the number of antennas increases [3], [4].

Network coding has been introduced as another promising method to improve the spectral efficiency and throughput of wireless communications [6]–[8]. Unlike conventional MIMO techniques, where the interference is removed completely, network coding encourages wireless systems to make full use of manageable interference by mixing the messages from different sources at the intermediate nodes. Hence combining network coding with MIMO has naturally been considered to achieve significant gains of transmission performance for cellular communications [9]–[13]. Most existing work focuses on MIMO two-way relay systems, to which network coding has been shown to be a promising technique for the improvement of system throughput. For example [12] considers a network coding scheme in two-way MIMO amplify-and-forward (AF) relay systems, and demonstrates that the use of network coding yields better transmission performance than the conventional schemes. In [9], the joint design of relay and user precoding matrices based on the MSE criterion is studied for AF based network coding schemes. [10] considers the space-time coding design for two-way relay networks, which can improve the diversity gains with less demanding CSI assumptions. In [11], the precoding design to maximize the sum rate is studied, where the closed-form solution is given when all the nodes are equipped with two antennas. [13] extends the joint design of MIMO and network coding into two-way MIMO decode-and-forward relay scenarios, and proposes a denoise-and-forward (DNF) strategy. A simplified version of such a DNF network coding scheme is proposed in [18], where a linear receiver and a simpler remapping function are applied at the DNF relay. Apart from two-way relaying networks, the application of network coding to other communication scenarios is much less understood, which is the motivation of this paper.

In this paper, we consider a cellular communication network, where multiple users want to exchange information with each other, and the base station also wants to broadcast information to the users. A conventional method is to apply time sharing approaches and accomplish the two tasks in different time slots. Inspired by the idea of cognitive radio in distributed communication systems, where the licensed spectrum for primary users can also be allocated for transmissions of unlicensed secondary users, a new network coding scheme is proposed in this paper to accomplish the two tasks simultaneously. The main contributions can be summarized as follows:

Firstly, a cognitive radio inspired asymmetric network coding (CR-AsNC) scheme is designed for cellular MIMO systems. The key idea of the proposed CR-AsNC scheme is similar to that of cognitive radio (CR). In particular, the base station can be viewed as the secondary user in CR, and broadcasts a message by sharing the spectrum resources with the all the users, which act as the primary users in CR. The principle for designing both proposed scheme and CR is to improve spectrum multiplexing with the least performance loss at the licensed users. But different from conventional CR, the base station is acting as a secondary user, and also as a relay to help the multi-way transmissions among the primary users. Network coding is used to ensure that the message broadcasted by the base station will not cause interference to information exchanging among the users. Unlike the related work in [14] and [15], which consider two-way relay channels, our proposed CR-AsNC supports multi-way relaying and broadcasting with multiple antennas settings, which are two common cellular communication scenarios.

Secondly, a new form of network coding is proposed to accommodate asymmetrical scenarios, where different transmitters can use different modulation constellations. Due to the different capabilities and channel conditions of the base station and the users, it is not practical to require that all nodes use the same constellations, and thus conventional physical-layer network coding for symmetric message exchange, such as presented in [6] and [17], is not applicable. Hence we design a new form of network coding for asymmetric symbols. Compared with the maximum likelihood detection based scheme in [13], our proposed method provides a low complexity solution based on linear MIMO receivers.

Thirdly, iterative precoding design for both phases of the CR-AsNC scheme is studied. By alternately

updating the precoding and the detection matrices, our proposed algorithms can approach the optimal solution of the established optimization problems, which are neither convex or linear. *Finally*, to seek a better tradeoff between performance and complexity, a precoding design with low computational complexity is presented based on channel diagonalization. With a fixed precoding structure, the precoding design can be transformed into power allocation problems, for which closed-form solutions can be obtained.

The rest of this paper is organized as follows. Section II describes the system model, and introduces the CR-AsNC scheme. In Section III, the iterative precoding optimization for proposed CR-AsNC is studied. Section IV designs the precoding for CR-AsNC based on channel diagonalization with low complexity. The simulation results are shown in Section V, and followed by the conclusions in Section VI.

Notation: Vectors and matrices are denoted as boldface small and capital letters respectively, e.g., $\mathbf{A}$ and $\mathbf{b}$. The matrix $\mathbf{I}_L$ is defined as the L-dimension identity matrix, and $\mathbf{A} = \mathrm{diag}\{a_1 \cdots a_n\}$ is a diagonal matrix with diagonal elements $a_1 \cdots a_n$. The matrix $\mathbf{A}^H$ and $\mathbf{A}^T$ are the Hermitian and the transpose matrices of $\mathbf{A}$, and $\tilde{\mathbf{A}} = \mathbf{A}[R_1 : R_2, C_1 : C_2]$ is a $(R_2 - R_1) \times (C_2 - C_1)$ submatrix of $\mathbf{A}$, which is composed by the elements located on the cross from the row R_1 to R_2, and the column C_1 to C_2 in $\mathbf{A}$. The trace for $\mathbf{A}$ is denoted as $\mathrm{tr}(\mathbf{A})$, $\mathbf{e}_i$ is an unitary vector whose i-th element is 1, and $\mathbb{E}\{x\}$ is the expectation of the variable x.

II. SYSTEM MODEL AND PROTOCOL DESCRIPTION

Consider a MIMO transmission system with multiple users, where each user node UE_k is required to transmit a message to all the other users with the help of one base station, and a common message is broadcasted to all the users by the base station as well ($k = 1, \cdots, K$). As illustrated in Fig. 1, all the nodes are equipped with multiple antennas, and the numbers of antennas at the users and the base station are denoted as L and N, respectively. To ensure that the interference can be removed completely, the numbers of antennas must satisfy that both $L, N \geqslant K$. For notation simplify, the noise power at all the nodes is set as σ^2 in this paper. For combatting with co-channel interference, global channel state information (CSI) is required at the base station as well as at the users to perform precoding and detecting. The transmission can be divided into two phases, which are named as the multiple access (MAC) phase

and the broadcast (BC) phase respectively, which are introduced in the following part.
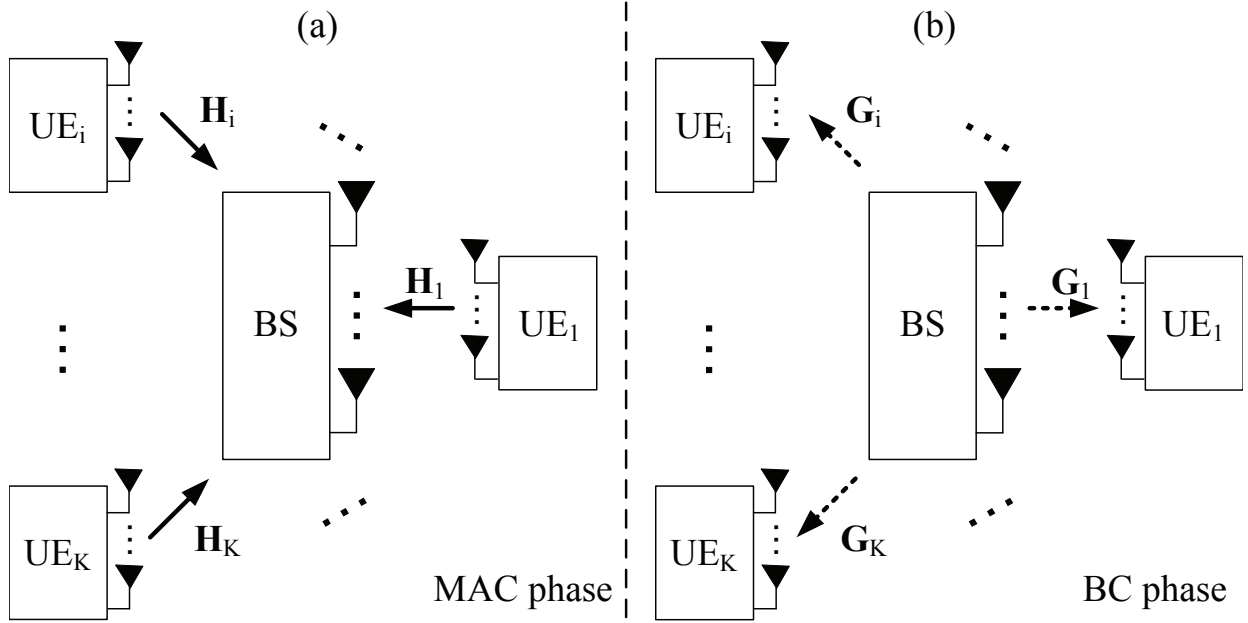
Fig. 1. System model; part (a) shows the multiple access (MAC) phase, and part (b) shows the broadcast (BC) phase

A. Protocol Description

1) Multiple Access Phase: All the user nodes transmit the precoded version of their messages to the base station simultaneously, and its observation can be written as

$$\mathbf{y}_B = \sum_{k=1}^{K} \mathbf{H}_k \mathbf{v}_k x_k + \mathbf{n}_B = \mathbf{F}\mathbf{x} + \mathbf{n}_B, \tag{1}$$

where x_k is the transmit message from UE_k, $\mathbf{H}_k$ is the $N \times L$ channel matrix from UE_k to the base station, $\mathbf{v}_k$ is the $L \times 1$ precoding vector at UE_k, $\mathbf{n}_B$ is the $N \times 1$ additive Gaussian noise at the base station, $\mathbf{x} = [x_1 \cdots x_K]^T$ and $\mathbf{F} = [\mathbf{H}_1 \mathbf{v}_1 \cdots \mathbf{H}_K \mathbf{v}_K]$. In particular, M-ary quadrature amplitude modulation (M-QAM) is applied for all the nodes, and $x_k = \alpha_k s_k$ is the M_k-QAM symbol with unit power from UE_k, where α_k is the power normalizing factor at UE_k, and s_k is a conventional M_k-QAM symbol with integer coordinates. By using minimum mean square error (MMSE) detection, the base station can obtain the messages from all the users. The MMSE detection matrix at the base station can be given as [24]

$$\mathbf{D}_B = \mathbf{F}^H (\mathbf{F}\mathbf{F}^H + \sigma^2 \mathbf{I}_N)^{-1}, \tag{2}$$

and the base station can detect the messages by using the following criterion,

$$\hat{\mathbf{x}} = \arg \min_{x_k \in \Omega_k} \|\mathbf{D}_B \mathbf{y}_R - \mathbf{x}\|^2, \ k = 1, \cdots, K, \tag{3}$$

where Ω_k is the constellation set for x_k.

2) Broadcast Phase (Asymmetric Network Coding & Decoding): The base station broadcasts a precoded network coding message vector to all users. Specifically, the combined message can be obtained through network coding of the base station broadcast message with each detected user's message respectively, which is the superposition of two QAM symbols. For such a multi-user scenario, it is important to note that each node is allowed to chose different constellations, and thus the conventional network coding scheme for asymmetric messages is not applicable, such as the schemes in [6] and [13]. Based on the network coding scheme for symmetric symbols in [13], we develop a practical symbol-level network coding method for the messages with different constellations in this paper. Assuming that an M_B-QAM constellation with integer coordinates is utilized for the broadcast message s_B, the following symbol-level network coding form for asymmetric messages can be proposed,

$$s_k^{\oplus} = 2\left[\left(\frac{1}{2}(\hat{s}_k + s_B) - 1\right) \mod \sqrt{\mathcal{M}_k}\right] - \left(\sqrt{\mathcal{M}_k} - 1\right), \tag{4}$$

where $\mathcal{M}_k = \max\{M_k, M_B\}$ and $\hat{s}_k = \hat{x}_k/\alpha_k$. Note that the modulo operation for a complex number is defined to calculate the modulo for the real and imaginary parts separately in this paper, i.e, $d = c \mod M$, $c = c_R + c_I j$ and $d = d_R + d_I j$ are two complex numbers, then $d_R = c_R \mod M$ and $d_I = c_I \mod M$. To obtain the unified expression given by (4), the constellation of binary phase shift keying (BPSK) need to be adjusted to become $\Omega_{\text{BPSK}} = \{1 + j, -1 - j\}$. Compared to the existing modulo-based physical-layer network coding as introduced in [13], our proposed scheme reduces the computational complexity significantly since linear receiver can be applied, while maximum likelihood detection is required to obtained the network coding message in [13]. Moreover, we extend the applications of physical-layer network coding to more general scenarios where the participants can be asymmetric.

Next the base station broadcasts the network coding message vector $\mathbf{x}^{\oplus} = [x_1^{\oplus} \cdots x_K^{\oplus}]^T$ to all the user nodes after precoding, $x_k^{\oplus} = \beta_k s_k^{\oplus}$ and β_k is the power normalizing factor for $s_k^{\oplus}$. Without loss of

generality, we take UE_k as an example, whose observation can be expressed as

$$\mathbf{y}_k = \mathbf{G}_k \mathbf{P} \mathbf{x}^{\oplus} + \mathbf{n}_k, \tag{5}$$

where $\mathbf{P}$ is the $N \times K$ precoding matrix at the base station, $\mathbf{G}_k$ is the $L \times N$ channel matrix from the base station to UE_k, and $\mathbf{n}_k$ is the $L \times 1$ additive Gaussian noise vector at UE_k. Similar to the MAC phase given in (2), MMSE detection is employed at each user, and the detection matrix at UE_k can be derived as

$$\mathbf{D}_k = (\mathbf{G}_k \mathbf{P})^H (\mathbf{G}_k \mathbf{P} \mathbf{P}^H \mathbf{G}_k^H + \sigma^2 \mathbf{I}_L)^{-1}. \tag{6}$$

After obtaining the detected network coding message vector $\hat{\mathbf{x}}_k^{\oplus}$, UE_k first decodes the broadcast message s_B from the base station. Assuming that M_k and M_B are accessible at the user nodes, s_B can be decoded by subtracting self-interference from $\hat{s}_{k-k}^{\oplus}$,

$$\hat{s}_B = 2\left[\left(\frac{1}{2}(\hat{s}_{k-k}^{\oplus} - s_k) \right) \mod \sqrt{\mathcal{M}_k} \right] - \left(\sqrt{\mathcal{M}_k} - 1 \right). \tag{7}$$

where $\hat{s}_{k-k}^{\oplus} = \hat{x}_{k-k}^{\oplus}/\beta_k$ and $\hat{x}_{k-k}^{\oplus}$ is the k-th element of $\hat{\mathbf{x}}_k^{\oplus}$. Next UE_k can decode the messages from other users as follows by using the detected broadcasting mesasage $\hat{s}_B$,

$$\hat{s}_i = 2\left[\left(\frac{1}{2}(\hat{s}_{k-i}^{\oplus} - \hat{s}_B) \right) \mod \sqrt{\mathcal{M}_i} \right] - \left(\sqrt{\mathcal{M}_i} - 1 \right), \ i \neq k, \tag{8}$$

where $\hat{s}_{k-i}^{\oplus} = \hat{x}_{k-i}^{\oplus}/\beta_i$ and $\hat{x}_{k-i}^{\oplus}$ is the i-th element of $\hat{\mathbf{x}}_k^{\oplus}$. Then the transmission is accomplished.

B. Further Discussion on CR-AsNC

Inspired by the idea of cognitive radio, our proposed CR-AsNC scheme employs network coding at the base station to improve the spectral efficiency. As shown in Fig. 2, by carrying the broadcast message s_B with each user message through network coding, it does not need to allocate a separate radio resource for base station's broadcast. Moreover, each user can detect all the messages without suffering co-channel interference, which is realized by subtracting side information from the network coded messages.

The key step of CR-AsNC is to implement the network coding for asymmetric messages at the base station, which can guarantee performance gains in terms of higher spectral efficiency and lower error probability. Unlike the conventional physical-layer network coding for symmetric symbols, our proposed
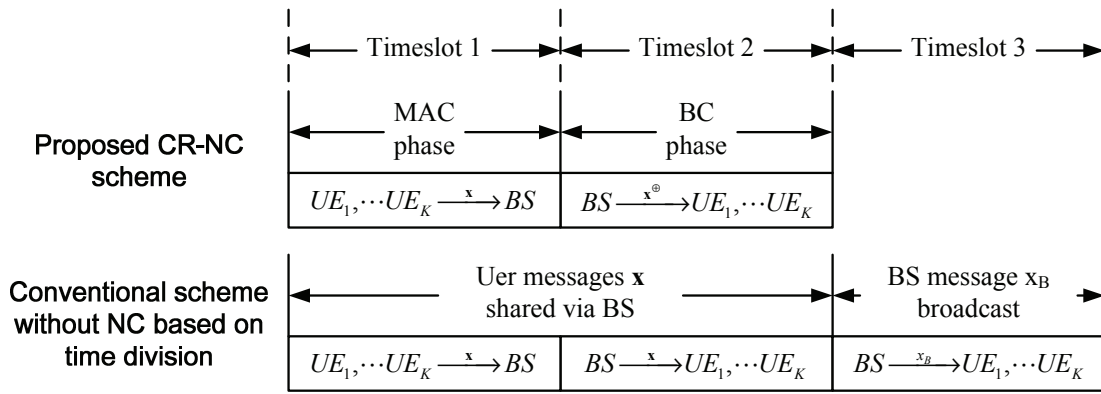
Fig. 2. Illustration of frame structure for proposed CR-AsNC scheme.

scheme allows both participants to chose appropriate rates due to their own channel conditions and service requirements, respectively, and thus the performance loss caused by symmetric symbol matching can be avoided. As illustrated in Fig. 3, where the asymmetric network coding for BPSK and QPSK is shown as an example, the superposed constellations of such asymmetric symbols are a subset of those of superposing two QPSK symbols. Recalling (4), the parameter $\mathcal{M}_k$ for the modulo operation can be set as 4, and the superposition can be remapped as a QSPK symbol. Then the network coded message can be remapped as a high-order QAM symbol. Since the proposed scheme matches the asymmetric messages by choosing different orders of modulations and uses linear detectors, its implementations are easy and with low complexity.

Fig. 3. Illustration of asymmetric network coding at the base station.

III. PRECODING OPTIMIZATION FOR CR-ASNC BASED ON MSE WITH ITERATION

In this section, we focus on the precoding design for the proposed CR-AsNC scheme, whose objective is to minimize the MSEs at the destinations. To maximize the multiplexing gains, we focus on a special case that $L = N = K$ in this section. Due to the complicated relationship between precoding matrices and MSEs, the precoding optimization problems for the two phases are non-convex, which makes it challenging to find the optimal solutions. We propose iterative algorithms instead, where the precoders and the the detectors are alternately updated until convergence.
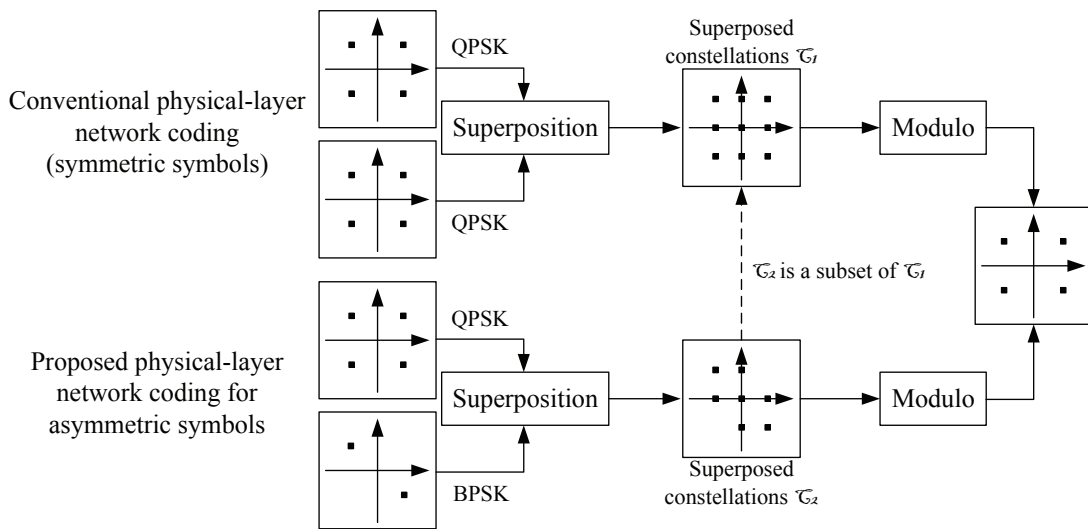
A. Iterative Precoding Design for MAC Phase

Based on the application of MMSE detector, the MSE at the base station can be derived as

$$
\begin{aligned}
\epsilon_B &= \mathbb{E}\big\{\|\mathbf{D}_B\mathbf{y}_R - \mathbf{x}\|^2\big\} = \mathrm{tr}\big(\mathbf{D}_B\mathbf{F}\mathbf{F}^H\mathbf{D}_B^H - \mathbf{D}_B\mathbf{F} - \mathbf{F}^H\mathbf{D}_B^H + \sigma^2\mathbf{I}_K\big) \\
&\simeq \sum_{k=1}^{K}\bigg(\sum_{i=1}^{K}\mathbf{v}_k^H\mathbf{H}_k^H\mathbf{d}_i\mathbf{d}_i^H\mathbf{H}_k\mathbf{v}_k - \mathbf{d}_k^H\mathbf{H}_k\mathbf{v}_k - \mathbf{v}_k^H\mathbf{H}_k^H\mathbf{d}_k\bigg),
\end{aligned}
\tag{9}
$$

where $\mathbf{d}_i$ is the i-th column of the matrix $\mathbf{D}_B^H$, and the last equation in (9) follows from a high SNR approximation, which leads σ^2 approaching 0. In fact, the noise part of ϵ_B is not related to $\mathbf{v}_k$, and thus it does not impact on the precoding design. Then the precoding optimization for the MAC phase can be established as

$$
\begin{aligned}
\min_{\mathbf{v}_k} \quad & \epsilon_B = \sum_{k=1}^{K}\bigg(\sum_{i=1}^{K}\mathbf{v}_k^H\mathbf{H}_k^H\mathbf{d}_i\mathbf{d}_i^H\mathbf{H}_k\mathbf{v}_k - \mathbf{d}_k^H\mathbf{H}_k\mathbf{v}_k - \mathbf{v}_k^H\mathbf{H}_k^H\mathbf{d}_k\bigg) \\
\text{s.t.} \quad & \sum_{k=1}^{K}\mathrm{tr}\big(\mathbf{v}_k\mathbf{v}_k^H\big) \leqslant p_{UE},
\end{aligned}
\tag{10}
$$

where p_{UE} is the total transmit power at the users. The use of the total power constraint can yield a performance gain compared to the one with the individual power constraints. Such a conclusion has been verified by Fig.4 in [18]. Moreover, since users' power is allocated by their base station, the use of the total power constraint is more reasonable, particularly in cellular systems. Note that similar strategies have been applied in similar multi-user scenarios, as shown in [19]–[21]. Furthermore, the total power constraint for the uplink transmissions in each cell attracts more attention in the practical cellular networks, since it has a direct impact on the the inter-cell interference. To solve the precoding design problem for the

MAC phase efficiently, an iterative method is proposed by alternately updating $\mathbf{D}_B$ and $\mathbf{v}_k$, and a close to optimal solution can be obtained.

First, we focus on the design of the detection matrix at the base station. Since the MMSE detection matrix can be derived from an unconstrained convex optimization problem, $\mathbf{D}_B$ given by (2) is the optimal solution for (10) with specified precoding vectors $\mathbf{v}_k$. Next we turn to study the optimization of precoding vectors at the users. Recalling (9), the Hessian matrices for ϵ_B, which are the second derivatives of the precoding vectors, can be derived as

$$\nabla_{\mathbf{v}_k,\mathbf{v}_k^H}\epsilon_B = \sum_{i=1}^{K}\mathbf{H}_k^H\mathbf{d}_i\mathbf{d}_i^H\mathbf{H}_k, \ \nabla_{\mathbf{v}_k,\mathbf{v}_i^H}\epsilon_B = \nabla_{\mathbf{v}_i,\mathbf{v}_k^H}\epsilon_B = \mathbf{0}, \ i \neq k. \tag{11}$$

Then the Hessian matrices provided by (11) can formulated in matrix form as follows,

$$\boldsymbol{\Phi}_{\epsilon_B} = \begin{bmatrix} \sum_{i=1}^{K}\mathbf{H}_1^H\mathbf{d}_i\mathbf{d}_i^H\mathbf{H}_1 & \cdots & \mathbf{0} \\ \vdots & \ddots & \vdots \\ \mathbf{0} & \cdots & \sum_{i=1}^{K}\mathbf{H}_K^H\mathbf{d}_i\mathbf{d}_i^H\mathbf{H}_K \end{bmatrix}. \tag{12}$$

It can be seen that $\boldsymbol{\Phi}_{\epsilon_B}$ is a block diagonal matrix, and $\nabla_{\mathbf{v}_k,\mathbf{v}_k^H}\epsilon_B$ is the sum of K Gram matrices, which are positive semidefinite. Therefore, we can state that the optimization problem presented by (10) is a convex optimization problem. Such a problem can be solved efficiently by applying Karush-Kuhn-Tucker (KKT) conditions, and its Lagrangian function can be given as

$$\mathfrak{L}_{\epsilon_B} = \epsilon_B + \lambda\left(\sum_{k=1}^{K}\text{tr}\left(\mathbf{v}_k\mathbf{v}_k^H\right) - p_{UE}\right), \tag{13}$$

where λ is the Lagrangian multiplier, and $\lambda \geqslant 0$. Hence the KKT conditions can be expressed as

$$\frac{\partial}{\partial\mathbf{v}_k^H}\mathfrak{L}_{\epsilon_B} = \left(\sum_{i=1}^{K}\mathbf{H}_k^H\mathbf{d}_i\mathbf{d}_i^H\mathbf{H}_k\right)\mathbf{v}_k - \mathbf{H}_k^H\mathbf{d}_k + \lambda\mathbf{v}_k = \mathbf{0}, k = 1,\cdots,K, \tag{14}$$

$$\lambda\left(\sum_{k=1}^{K}\text{tr}\left(\mathbf{v}_k\mathbf{v}_k^H\right) - p_{UE}\right) = 0, \tag{15}$$

$$\sum_{k=1}^{K}\text{tr}\left(\mathbf{v}_k\mathbf{v}_k^H\right) \leqslant p_{UE}. \tag{16}$$

Then the relation between the solution of (10) and λ can be shown as follows,

$$\mathbf{v}_k^{\text{opt}} = \left(\sum_{i=1}^{K}\mathbf{H}_k^H\mathbf{d}_i\mathbf{d}_i^H\mathbf{H}_k + \lambda\mathbf{I}_L\right)^{-1}\mathbf{H}_k^H\mathbf{d}_k, \ k = 1,\cdots,K. \tag{17}$$

Specifically, $\mathbf{v}_k^{\mathrm{opt}}$ can be further simplified when $\lambda = 0$,

$$\mathbf{v}_k^{\mathrm{opt}} = \left(\sum_{i=1}^{K} \mathbf{H}_k^H \mathbf{d}_i \mathbf{d}_i^H \mathbf{H}_k \right)^{-1} \mathbf{H}_k^H \mathbf{d}_k, \ k = 1, \cdots, K. \tag{18}$$

Note that $\mathbf{v}_k^{\mathrm{opt}}$ must satisfy the KKT conditions given in (15) and (16). If (18) does not meet the condition (16), a proper value of λ needs to be found to meet the KKT conditions. To demonstrate that such a Lagrangian multiplier is tractable, the following lemma is presented.

Lemma *1:* The source transmit power is a decreasing function of λ, and the upper bound of qualified λ can be given as

$$\lambda^{\mathrm{up}} = \sqrt{\frac{\sum_{k=1}^{K} \mathrm{tr}\left(\mathbf{H}_k^H \mathbf{d}_k \mathbf{d}_k^H \mathbf{H}_k \right)}{p_{UE}}}. \tag{19}$$

Proof: Pease see Appendix A. ∎

Although the exact value of the Lagrangian multiplier can be obtained numerically, the computational complexity can be reduced significantly by using Lemma 1. In particular, we can apply the bisection search to find a proper λ based on the upper bound λ^{up} given in Lemma 1, and the computational complexity can be estimated as $\mathcal{O}(\log |\lambda^{\mathrm{up}}|)$ [23]. On the other hand, a matrix function with λ needs to be solved when the conventional numerical method is used, and its computational complexity is at least $\mathcal{O}(L^3)$ [26].

Based on the forgoing analysis in this part, the iterative precoding generation scheme for MAC phase can be introduced as Table I. Particularly, denote $\triangle \epsilon_B^{\mathrm{th}}$ as a given threshold for detecting MSE at the base station and $\mathrm{ite}_{\mathrm{max}}$ as the maximum iteration time in Table I. Our proposed algorithm stops until the MSE gap for the latest two iterations satisfies $\triangle \epsilon_B \leqslant \triangle \epsilon_B^{\mathrm{th}}$, or the iteration number reaches the limit $\mathrm{ite}_{\mathrm{index}} = \mathrm{ite}_{\mathrm{max}}$. In each iteration, the detection matrix $\mathbf{D}_B$ is first updated with fixed precoding vectors at the users. Then the precoding vectors $\mathbf{v}_1, \cdots, \mathbf{v}_K$ can be updated with a fixed $\mathbf{D}_B$, where a proper value of λ needs to be searched. Because of Lemma 1, it can be obtained by using bisection search until the total transmit power of the users satisfies $\tilde{p}_{UE} \leqslant p_{UE}$ and $\triangle p_{UE} = |\tilde{p}_{UE} - p_{UE}| \leqslant \triangle p_{UE}^{\mathrm{th}}$, where $\triangle p_{UE}^{\mathrm{th}}$ is a given threshold.

As introduced previously, both subproblems in the proposed iterative precoding design are convex, and thus the solutions for these two subproblems in each iteration are optimal for the specified precoding matrix. Therefore, the MSE ϵ_B decreases as the iterations increase, and it can be lower bounded by 0. It

TABLE I

ITERATIVE PRECODING ALGORITHM FOR MAC PHASE

Initialize $\mathbf{v}_1, \cdots, \mathbf{v}_K$, $\triangle\epsilon_B^{\text{th}}$, $\triangle\epsilon_B = \triangle\epsilon_B^{\text{th}}$, $\epsilon_{B-1} = \epsilon_{B-2} = 0$, the maximum iteration time $\text{ite}_{\max}$ and $\text{ite}_{\text{index}} = 1$;

while $\text{ite}_{\text{index}} < \text{ite}_{\max}$ and $\triangle\epsilon_B > \triangle\epsilon_B^{\text{th}}$

    Update the detection matrix $\mathbf{D}_B$ at the base station according to (2);

    Initialize $\lambda = 0$, $\triangle p_{UE}^{\text{th}}$, source transmit power $\tilde{p}_{UE} = p_{UE} + \triangle p_{UE}^{\text{th}}$ and $\triangle p_{UE} = \triangle p_{UE}^{\text{th}}$;

    while $\tilde{p}_{UE} > p_{UE}$ and $\triangle p_{UE} \geqslant \triangle p_{UE}^{\text{th}}$

      for $k = 1$ to K

        Update the precoding vectors $\mathbf{v}_1, \cdots, \mathbf{v}_K$ using (17);

      end for

      Update the transmit power $\tilde{p}_{UE} = \sum_{k=1}^{K} \text{tr}\big(\mathbf{v}_k \mathbf{v}_k^H\big)$ and $\triangle p_{UE} = \big| \sum_{k=1}^{K} \text{tr}\big(\mathbf{v}_k \mathbf{v}_k^H\big) - p_{UE} \big|$;

      Update the Lagrange multiplier $\lambda = \frac{1}{2}(\lambda + \lambda^{\text{up}})$;

    end while

    Update ϵ_{B-1} by using (9), and $\triangle\epsilon_B = \epsilon_{B-2} - \epsilon_{B-1}$, $\epsilon_{B-2} = \epsilon_{B-1}$;

    $\text{ite}_{\text{index}} = \text{ite}_{\text{index}} + 1$;

end while

Return $\mathbf{v}_k^{\text{opt}} = \mathbf{v}_k$, $k = 1, \cdots, K$.

---

means that the proposed algorithm is convergent, and there exists a stationary point of (10) after iteration. Then the following corollary can be presented.

Corollary *2:* Algorithm II, which is our proposed iterative algorithm for the precoding design of CR-AsNC scheme in MAC phase, is convergent, and there exists a stationary point of (10).

B. Iterative Precoding Design for BC Phase

To minimize the total MSEs at the users, the precoding design for BC phase can be can be formulated as the following optimization problem,

$$\min_{\mathbf{P}} \quad \epsilon_{UE} = \sum_{k=1}^{K} \text{tr}\big(\mathbf{D}_k \mathbf{G}_k \mathbf{P}\mathbf{P}^H \mathbf{G}_k^H \mathbf{D}_k^H - \mathbf{D}_k \mathbf{G}_k \mathbf{P} - \mathbf{P}^H \mathbf{G}_k^H \mathbf{D}_k^H + \sigma^2 \mathbf{I}_L\big)$$
$$s.t. \quad \text{tr}\big(\mathbf{P}\mathbf{P}^H\big) \leqslant p_B, \tag{20}$$

where p_B is the transmit power at the base station. Similar to the MAC phase, such a complicated problem can be solved iteratively. We focus on the precoding design for BC phase with a fixed detection matrix

$\mathbf{D}_k$, and the Hessian matrix of ϵ_{UE} in (20) can be derived as follows,

$$\mathbf{\Phi}_{\epsilon_{UE}} = \begin{bmatrix} \nabla_{\mathbf{P}^H, \mathbf{P}} \epsilon_{UE} & \nabla_{\mathbf{P}, \mathbf{P}} \epsilon_{UE} \\ \nabla_{\mathbf{P}^H, \mathbf{P}^H} \epsilon_{UE} & \nabla_{\mathbf{P}, \mathbf{P}^H} \epsilon_{UE} \end{bmatrix} = \begin{bmatrix} \sum_{k=1}^{K} \mathbf{G}_k^H \mathbf{D}_k^H \mathbf{D}_k \mathbf{G}_k & \mathbf{0} \\ \mathbf{0} & \sum_{k=1}^{K} (\mathbf{G}_k^H \mathbf{D}_k^H \mathbf{D}_k \mathbf{G}_k)^T \end{bmatrix}. \quad (21)$$

It can be easily proved that $\mathbf{\Phi}_{\epsilon_{UE}}$ is positive semidefinite. Thus it can be solved by KKT conditions as well, which can be expressed as

$$\frac{\partial}{\partial \mathbf{P}^H} \mathfrak{L}_{\epsilon_{UE}} = \left(\sum_{k=1}^{K} \mathbf{G}_k^H \mathbf{D}_k^H \mathbf{D}_k \mathbf{G}_k \right) \mathbf{P} - \sum_{k=1}^{K} \mathbf{D}_k \mathbf{G}_k + \mu \mathbf{P} = \mathbf{0}, \quad (22)$$

$$\mu \left(\mathrm{tr}(\mathbf{P}\mathbf{P}^H) - p_B \right) = 0, \quad (23)$$

$$\mathrm{tr}(\mathbf{P}\mathbf{P}^H) \leqslant p_B \quad (24)$$

where $\mathfrak{L}_{\epsilon_{UE}} = \epsilon_{UE} + \mu \left(\mathrm{tr}(\mathbf{P}\mathbf{P}^H) - p_B \right)$, and $\mu \geqslant 0$ is the Lagrangian multiplier. Then the solution of (20) can be expressed as a function of μ,

$$\mathbf{P}^{\mathrm{opt}} = \left(\sum_{k=1}^{K} \mathbf{G}_k^H \mathbf{D}_k^H \mathbf{D}_k \mathbf{G}_k + \mu \mathbf{I}_N \right)^{-1} \left(\sum_{k=1}^{K} \mathbf{D}_k \mathbf{G}_k \right). \quad (25)$$

To prove the existence of $\mathbf{P}^{\mathrm{opt}}$ which can meet the KKT conditions, the following lemma is provided.

Lemma 3: $\mathrm{tr}\left[\mathbf{P}^{\mathrm{opt}} (\mathbf{P}^{\mathrm{opt}})^H \right]$ is decreasing as μ increases, and the value of μ which can follow the KKT conditions are bounded as

$$\mu^{\mathrm{up}} = \sqrt{\frac{\mathrm{tr}\left[(\sum_{k=1}^{K} \mathbf{D}_k \mathbf{G}_k)(\sum_{k=1}^{K} \mathbf{D}_k \mathbf{G}_k)^H \right]}{p_B}}. \quad (26)$$

Proof: Please See Appendix B. ∎

Then the precoding generation algorithm for BC phase can be summarized as Table II. The proposed iterative algorithm stops until $\mathrm{ite}_{\mathrm{index}} = \mathrm{ite}_{\mathrm{max}}$, or the MSE gap for the latest two iterations satisfies $\triangle \epsilon_B \leqslant \triangle \epsilon_B^{\mathrm{th}}$, where $\triangle \epsilon_B^{\mathrm{th}}$ is a given threshold for the gap of MSE. In each iteration, the detection matrix $\mathbf{D}_k$ at UE_k is updated with a fixed $\mathbf{P}$, and then the precoding matrix $\mathbf{P}$ can be updated, where a proper value of μ can be obtained by using the bisection search until the transmit power at the base station $\tilde{p}_B$ is no longer larger than its constraint p_B, and their gap is no longer larger than $\triangle p_B^{\mathrm{th}}$ as well. The convergence of introduced algorithm can be proved by following a similar analysis to Corollary 2.

TABLE II

ITERATIVE PRECODING GENERATION ALGORITHM FOR BC PHASE

Initialize P, $\triangle\epsilon_{UE}^{\text{th}}$, $\triangle\epsilon_{UE} = \triangle\epsilon_{UE}^{\text{th}}$, $\epsilon_{UE-1} = \epsilon_{UE-2} = 0$, the maximum iteration time $\text{ite}_{\max}$ and $\text{ite}_{\text{index}} = 1$;

while $\text{ite}_{\text{index}} < \text{ite}_{\max}$ and $\triangle\epsilon_{UE} > \triangle\epsilon_{UE}^{\text{th}}$

  for $k = 1$ to K

    Update the detection matrix $\mathbf{D}_k$ at UE_k according to (6);

  end for

  Initialize $\mu = 0$, $\triangle p_B^{\text{th}}$, base station transmit power $\tilde{p}_B = p_B + \triangle p_B^{\text{th}}$ and $\triangle p_B = \triangle p_B^{\text{th}}$;

  while $\tilde{p}_B > p_B$ and $\triangle p_B \geqslant \triangle p_B^{\text{th}}$

    Update the precoding matrix $\mathbf{P}$ based on (25), $\tilde{p}_B = \text{tr}\left(\mathbf{P}\mathbf{P}^H\right)$ and $\triangle p_B = \left|\text{tr}\left(\mathbf{P}\mathbf{P}^H\right) - p_B\right|$;

    Update the Lagrange multiplier $\mu = \frac{1}{2}(\mu + \mu^{\text{up}})$;

  end while

end while

Update ϵ_{UE-1} by using (20), and $\triangle\epsilon_{UE} = \epsilon_{UE-2} - \epsilon_{UE-1}$, $\epsilon_{UE-2} = \epsilon_{UE-1}$;

$\text{ite}_{\text{index}} = \text{ite}_{\text{index}} + 1$;

Return $\mathbf{P}^{\text{opt}} = \mathbf{P}$.

IV. PRECODING DESIGN BASED ON CHANNEL DIAGONALIZATION FOR CR-AsNC WITH LOW COMPLEXITY

In the previous section, the precoding design for proposed CR-AsNC scheme has been studied, and iterative algorithms for both phases were introduced, which can approach the globally optimal solution. Although the convergence of proposed algorithms is guaranteed, the iterations impose a high computational burden on the transmission system. To provide a better tradeoff between performance and complexity, sub-optimal precoding design schemes will be of interest in this section.

As in other typical multi-user scenarios, the main issue of precoding design for CR-AsNC is how to combat co-channel interference which could degrade the performance severely. Motivated by the study for multi-user transmissions in [16] and [13], we design the precoding matrices to diagonalize the channel matrices for the MAC and BC phases. Based on the proposed precoding structure, the precoding optimization problem can be simplified as power allocation, to which closed-form solutions can be obtained. Therefore the channel-diagonlized precoding design scheme can reduce the computational

complexity by avoiding iterative methods, and the transmission performance can also be guaranteed, as verified in the next section.

A. Precoding Design for Channel Diagonalized CR-AsNC

Due to the description of MAC phase transmission, it is a multiuser uplink scenario, where co-channel interference exists. For such a scenario, source precoding can be designed by diagonalizing the channels. To generate the precoding vector for UE_k, the singular value decomposition (SVD) is first computed for the following matrix,

$$\tilde{\mathbf{H}}_k^{(k)} = \tilde{\mathbf{U}}_k \tilde{\boldsymbol{\Sigma}}_k \big[\tilde{\mathbf{V}}_k^{(1)} \quad \tilde{\mathbf{v}}_k\big]^H, \tag{27}$$

where $\tilde{\mathbf{H}}_i^{(j)}$ is a $(N-1) \times L$ submatrix of $\mathbf{H}_i$ generated by removing its j-th row, $\tilde{\mathbf{V}}_k^{(1)}$ holds the first $(N-1)$ right singular vectors with respect to the non-zero singular values, and the rest are in the null space of $\tilde{\mathbf{H}}_k^{(k)}$. To ensure the existence of the null space for $\tilde{\mathbf{H}}_k^{(k)}$, which aims to cope with the interference for UE_k, the base station antennas should follow the constraint that $N < L+1$. Following the assumption that $L = N = K$ in Section III, $\tilde{\mathbf{v}}_k$ in (27) is the only singular vector in the null space of $\mathbf{H}_k^{(k)}$. Using $\tilde{\mathbf{v}}_k$ as the precoding vector for UE_k, the co-channel interference can be canceled at the base station, and its observation is rewritten as

$$\mathbf{y}_B = \sum_{k=1}^{K} \mathbf{H}_k \tilde{\mathbf{v}}_k w_k^{UE} x_k + \mathbf{n}_B = \tilde{\mathbf{F}} \mathbf{W}_{UE} \mathbf{x} + \mathbf{n}_B, \tag{28}$$

where $\mathbf{W}_{UE} = \mathrm{diag}\big\{w_1^{UE}, \cdots, w_K^{UE}\big\}$ is a diagonal matrix for power allocation at the sources, and $\tilde{\mathbf{F}} = \mathrm{diag}\big\{\mathbf{H}_1 \tilde{\mathbf{v}}_1, \cdots, \mathbf{H}_K \tilde{\mathbf{v}}_K\big\}$. Then the detection matrix at the base station can be given as

$$\mathbf{D}_B = \big(\tilde{\mathbf{F}} \mathbf{W}_{UE}\big)^H \big[\big(\tilde{\mathbf{F}} \mathbf{W}_{UE}\big)\big(\tilde{\mathbf{F}} \mathbf{W}_{UE}\big)^H + \sigma^2 \mathbf{I}_N\big]^{-1}. \tag{29}$$

Combining (29) with (3), the base station can detect all the messages from the users.

During the BC phase, the base station first generates the network coding messages based on (4), and then broadcasts the messages to all the users, and thus it is equivalent to virtual point to point MIMO channels equivalently. Specifically, all the channels can be reconstructed as follows,

$$\mathbf{G} = [\mathbf{G}_1^T \quad \cdots \quad \mathbf{G}_K^T]^T. \tag{30}$$

To diagonalize channel matrices in the BC phase, $\mathbf{G}$ can be decomposed as follows by applying the SVD,

$$\mathbf{G} = \mathbf{V}_G \mathbf{\Sigma} \mathbf{U}_G^H, \tag{31}$$

where $\mathbf{\Sigma} = [\mathbf{\Lambda}^T, \mathbf{0}_{(K^2-K) \times K}^T]^T$, $\mathbf{\Lambda}$ is a $K \times K$ nonzero diagonal matrix, $\mathbf{V}_G$ and $\mathbf{U}_G$ are two unitary matrices, respectively. Note that the null space of $\mathbf{\Sigma}$ has no contribution to the decomposition result of each channel matrix. Without loss of generality, $\mathbf{G}_k$ is taken as an example, whose decomposition can be derived as,

$$\mathbf{G}_k = \tilde{\mathbf{V}}^k \mathbf{\Lambda} \mathbf{U}_G^H, \ k = 1, \cdots, K, \tag{32}$$

where $\tilde{\mathbf{V}}^k = \mathbf{V}_G[(k-1)K + 1 : kK, 1 : K]$ is a $K \times K$ submatrix of $\mathbf{V}_G$, which is composed by the elements located on the cross of first K columns, and from the $((k-1)K+1)$-th row to the kK-th row in $\mathbf{V}_G$. Since all decompositions of each channel matrix share a common right matrix $\mathbf{U}_G^H$, the precoding matrix at the base station can be generated as follows to diagonalize the channels,

$$\mathbf{P} = \mathbf{U}_G \mathbf{W}_B, \tag{33}$$

where $\mathbf{W}_B$ is a diagonal matrix for base station power allocation. Substituting (32) and (33) into (5), the received signal at UE_k can be written as

$$\mathbf{y}_k = \tilde{\mathbf{V}}^k \mathbf{\Lambda} \mathbf{W}_B \mathbf{x}^\oplus + \mathbf{n}_k. \tag{34}$$

The detection matrix $\mathbf{D}_k$ at UE_k can be expressed as

$$\mathbf{D}_k = (\tilde{\mathbf{V}}^k \mathbf{\Lambda} \mathbf{W}_B)^H [(\tilde{\mathbf{V}}^k \mathbf{\Lambda} \mathbf{W}_B)(\tilde{\mathbf{V}}^k \mathbf{\Lambda} \mathbf{W}_B)^H + \sigma^2 \mathbf{I}_K]^{-1}. \tag{35}$$

Then each user can detect the network coding messages by using MMSE detection, and they obtain the desired messages based on the decoding rules given in (7) and (8).

B. Power Allocation for Channel Diagonalized CR-AsNC

As introduced in the previous part, the precoding structure for the channel diagonalized CR-AsNC scheme has been fixed, and the precoding design problems can be simplified as two independent power

allocation problems. By substituting (29) into (9), the power optimization for MAC phase can be formulated as the following problem,

$$
\begin{aligned}
\min_{w_k^{UE}} \quad & \epsilon_B = \sum_{k=1}^{K} \frac{\sigma^2}{\sigma^2 + (\tilde{\mathbf{v}}_k^H \mathbf{H}_k^H \mathbf{H}_k \tilde{\mathbf{v}}_k) p_{UE-k}} \\
s.t. \quad & \sum_{k=1}^{K} p_{UE-k} \leqslant p_{UE},
\end{aligned}
\tag{36}
$$

where $p_{UE-k} = w_{UE-k}^2$ is the power allocated to UE_k, and w_{UE-k} is the k-th diagonal element of $\mathbf{W}_{UE}$ which denotes the power allocation factor for UE_k. The problem can be solved efficiently due to its convexity, and the closed-form expression of power allocation factors at the sources can be presented in the following theorem.

Theorem *4:* The power allocation for the MAC phase of the channel diagonalized CR-AsNC, which is given by (36), is a convex optimization problem, and the power allocation factor w_{UE-k} can be obtained as

$$
w_{UE-k}^{\text{opt}} = \left(\sqrt{\frac{1}{(\tilde{\mathbf{v}}_k^H \mathbf{H}_k^H \mathbf{H}_k \tilde{\mathbf{v}}_k)\alpha}} p_{UE} - \sum_{i \neq k} \frac{\sigma^2}{\tilde{\mathbf{v}}_i^H \mathbf{H}_i^H \mathbf{H}_i \tilde{\mathbf{v}}_i} \right)^{1/2},
\tag{37}
$$

where $\alpha = \sum_{k=1}^{K} [1/(\tilde{\mathbf{v}}_k^H \mathbf{H}_k^H \mathbf{H}_k \tilde{\mathbf{v}}_k)]$.

Proof: Please See Appendix C. ∎

Next we focus on the power allocation for BC phase. By substituting (6), (33) and (32) into the objective function of (20), the total MSEs at the users can be derived as

$$
\epsilon_{UE} = \sum_{k=1}^{K} \text{tr} \left[\left(\mathbf{I}_K + \frac{1}{\sigma^2} \mathbf{W}_B^H \mathbf{\Lambda}^H (\tilde{\mathbf{V}}^k)^H \tilde{\mathbf{V}}^k \mathbf{\Lambda} \mathbf{W}_B \right)^{-1} \right].
\tag{38}
$$

Compared with the original objective function, (38) reduces the computation complexity by narrowing its feasible set down to a set that only consists of K-dimensional positive diagonal matrices. However, it is still hard to get the closed-form solution from (38), so we can simplify the problem by providing a manageable bound,

$$
\epsilon_{UE} \leqslant \sum_{i=1}^{K} \sum_{k=1}^{K} \frac{\tilde{v}_k^i \sigma^2}{\tilde{v}_k^i \sigma^2 + \lambda_k^2 w_k^2},
\tag{39}
$$

where $\tilde{v}_k^i$ is the k-th diagonal element of $(\tilde{\mathbf{V}}^i)^H \tilde{\mathbf{V}}^i$, λ_k and w_k are defined similarly for $\mathbf{\Lambda}$ and $\mathbf{W}_R$ respectively, and the last inequity in (40) follows Lemma 5 in [9]. Then the power allocation problem for

BC phase can be reformulated as follows

$$\min_{\mathbf{p}_k} \quad \epsilon_{UE}^{\text{up}} = \sum_{i=1}^{K} \sum_{k=1}^{K} \frac{\tilde{v}_k^i \sigma^2}{\tilde{v}_k^i \sigma^2 + \lambda_k^2 p_k}$$

$$s.t. \quad \sum_{k=1}^{K} p_k \leqslant p_B, \tag{40}$$

where $p_k = w_k^2$ is the power allocated to the k-th antenna of base station, and the closed-form solution for (40) is given in the following theorem.

Theorem *5:* The power optimization problem given by (40) is convex, and its closed-form solution can be presented as follows for the high SNR region, in which σ^2 approaches 0,

$$w_k^{\text{opt}} = \left(\sum_{i=1}^{K} \frac{\tilde{v}_k^i p_B^2}{\beta \lambda_k^2} \right)^{1/4}, \tag{41}$$

where $\beta = \sum_{k=1}^{K} \sqrt{\frac{\tilde{v}_k^i}{\lambda_k^2}}$.

Proof: Please See Appendix D. ∎

Recalling the proposed closed-form power allocation strategy, antennas which have better channel conditions are allocated with less power. It is because that our objective is to minimize the total MSEs, so the link with worst channel conditions becomes the bottleneck. Its performance can be improved by increasing the transmit power. Note that this approach is very different to the conventional water-filling algorithm, where more power is allocated to the transmitters with better channel condition, which is to maximize the channel capacity.

V. SIMULATION RESULTS

To evaluate the transmission performance of the proposed CR-AsNC scheme with precoding optimization, simulation results are provided in this section. Specifically the channels are modeled as quasi-static Rayleigh fading channels. The power of noise is $\sigma^2 = 1$, and the transmit power of the users and the base station are set as $p_{UE} = Kp$ and $p_B = p$ respectively. The average SNR can be defined as $\rho = p/\sigma^2$. The number of users and the numbers of antennas at each user and the base station are fixed as $K = L = N = 3$.
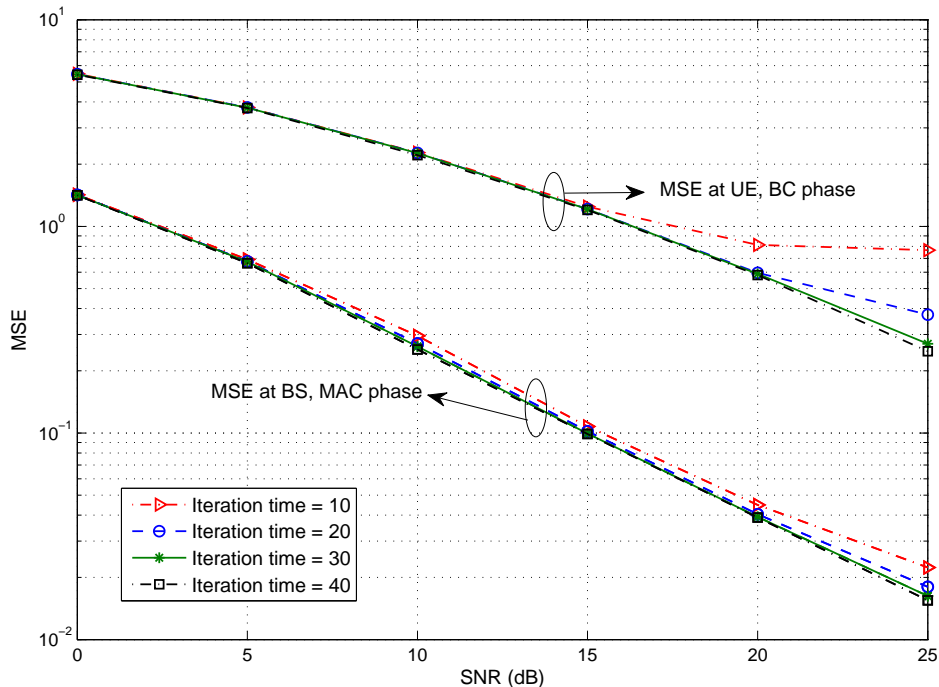
Fig. 4. The convergence performance of proposed iterative precoding optimization algorithms.

In Fig. 4, the MSEs of proposed CR-AsNC with iterative precoding design are presented for both phases, which demonstrates the convergence of the iterative algorithms in Section III. As shown in the figure, the gaps between 30 iterations and 40 iterations for both phases are narrower than those between 10 iterations and 20 iterations respectively, and the proposed algorithms always converge after 20 iterations in the low SNR region, while 40 iterations are required for the high SNR region. These simulation results show that both algorithms converge as the number of iterations increase.

The MSE performance is shown as a function of SNR in Fig. 5, and the MSEs for MAC and BC phases are plotted in Fig. 5(a) and Fig. 5(b) respectively. To demonstrate the performance of precoding optimization, random precoding design and the channel diagonalization-based precoding design with equal power are selected as two baseline schemes. The simulation results demonstrate that our proposed optimized precoding design can improve the robustness, especially for the MAC phase. Moreover, as introduced in Section IV, the transmission for BC phase can be constructed as one virtual point-to-point system, and thus the channel diagonalization-based precoding design based on SVD is quite close to the

optimal performance [24], which coincides with the simulation results.

In Fig. 6, the average BER performance for both the base station and the users are provided. QPSK modulation is utilized at the base station and the users for CR-AsNC, while 8-PSK is applied for the conventional time division scheme, and thus the spectral efficiencies for different schemes are the same for fairness. The simulation results show that when the spectral efficiencies for the different schemes are the same, the two proposed CR-AsNC schemes outperform the conventional time division scheme after precoding optimization. The iterative CR-AsNC scheme achieves the lowest BER performance, followed by the CR-AsNC scheme with channel diagonalization-based precoding design, which provides a better balance between computational complexity and transmission performance.

Fig. 7 illustrates the spectral efficiency performance of the proposed CR-AsNC schemes, and the conventional scheme based on time division multiple access without network coding, as shown in Fig. 2, is chosen as a baseline scheme. The spectral efficiency for the proposed MIMO system is defined as

$$\eta = \frac{(1 - \mathrm{BER}_B)M_B + (1 - \mathrm{BER}_{UE}) \sum_{k=1}^{K} M_k}{\gamma}, \tag{42}$$

where BER_B and BER_{UE} are the average bit-error-rates (BER) at the base station and each user respectively, and γ is the number of time slot occupied for transmissions. As introduced in Section II, γ should be set as 2 and 3 for CR-AsNC schemes and the conventional time division scheme respectively. As shown in the figure, the spectral efficiencies are compared with different constellations. Two proposed CR-AsNC schemes always achieve higher spectral efficiencies than the conventional time division scheme. The CR-AsNC with iterative precoding design achieves the best spectral efficiency performance, and the performance of channel diagonalization-based precoding design, which has lower complexity, approaches to that of iterative precoding design in the high SNR region. Specifically the proposed CR-AsNC schemes can improve the spectral efficiency by 1.3 Bit/s/Hz at 20 dB when QPSK is applied at both the base station and the users. When the higher-order constellation is used, i.e., 16-QAM at the base station or the users, the simulation results show that the performance gains can be further enlarged.

We also compare the proposed CR-AsNC scheme with some relevant existing schemes in Fig. 8. Since the two-way relay scenario is a special case of information exchange between the users when
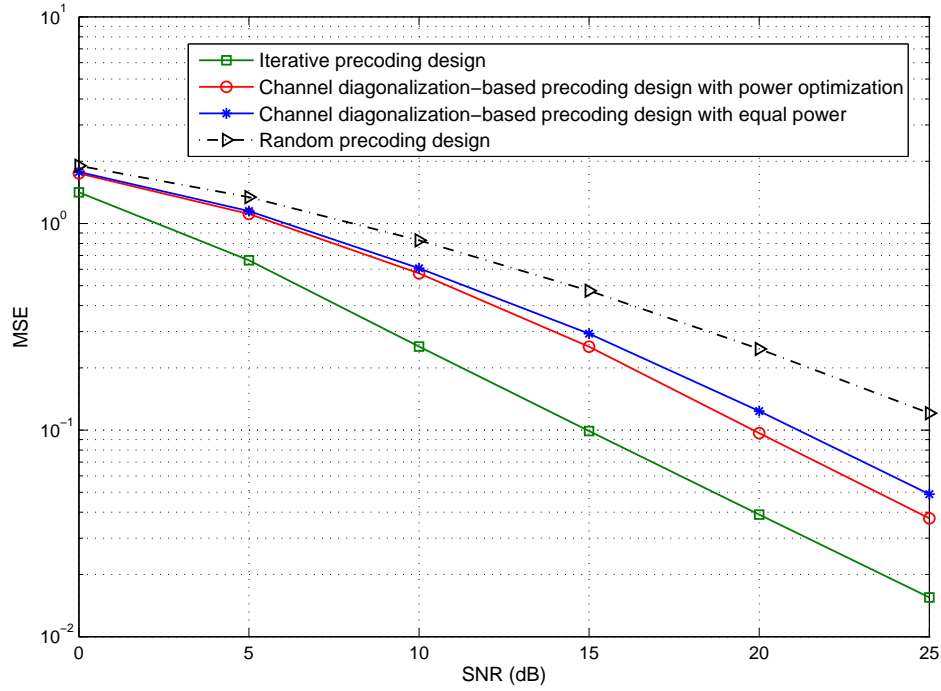
(a) The MSE vs. SNR for MAC phase
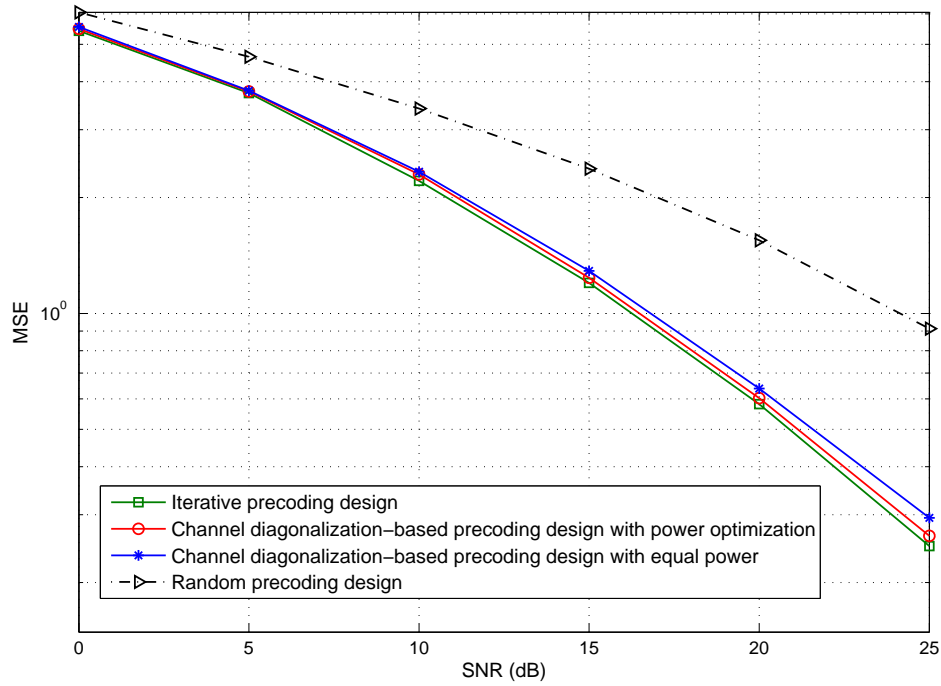
(b) The MSE vs. SNR for BC phase

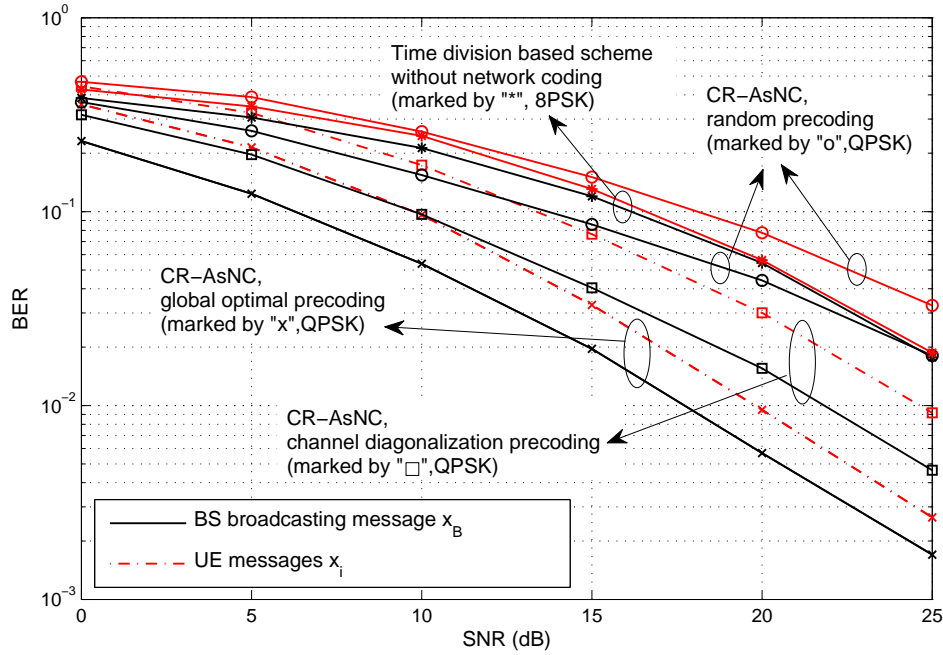Fig. 5.   MSE performance comparison for CR-AsNC with different precoding design.

Fig. 6. The BER performance of proposed CR-AsNC. (QPSK at UE and BS for CR-AsNC, 8PSK for conventional scheme based on time division without network coding for fairness).
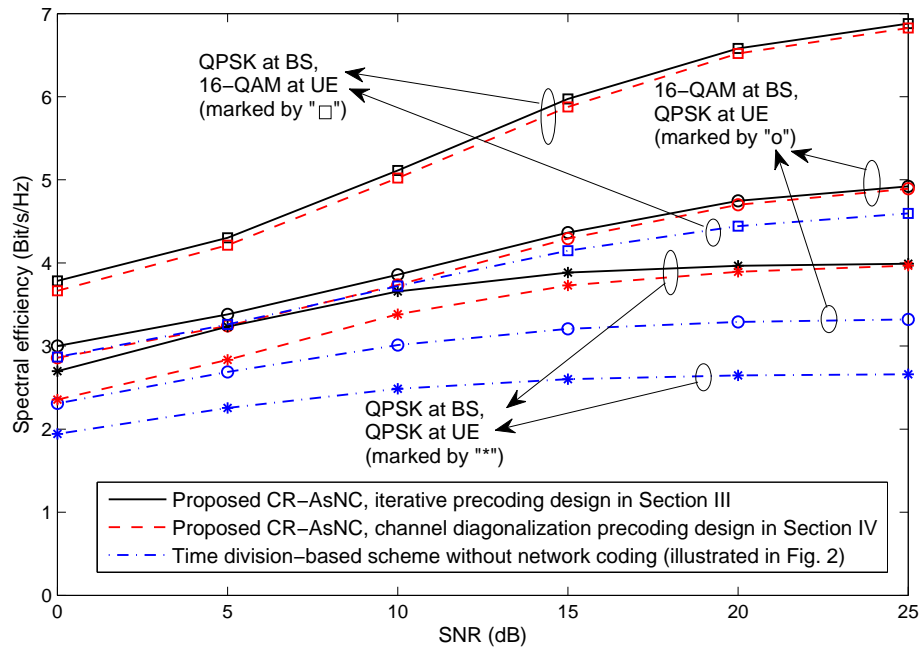

Fig. 7. The spectral efficiency performance of proposed CR-AsNC, $K = L = N = 3$.
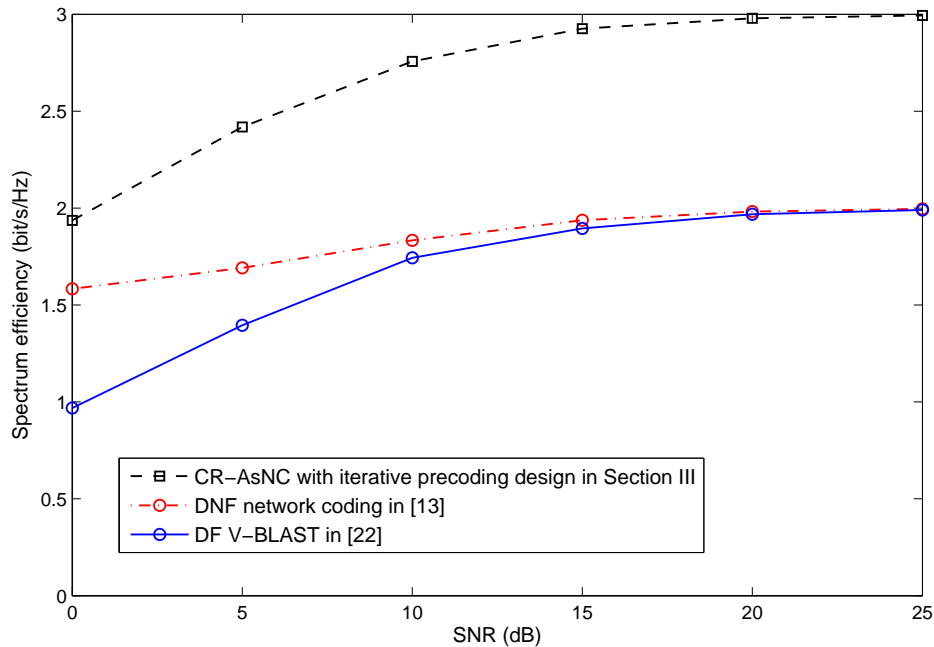
Fig. 8. The spectrum efficiency comparison with relevant schemes in two-way relay scenario, $K = L = N = 2$.

$K = 2$, the denoise-and-forward network coding (DNF-NC) scheme in [13] and the DF-based V-BLAST bidirectional relaying scheme in [22] are chosen as benchmarks. QPSK is applied at the all transmitters, and the performance of different schemes is compared by using the spectrum efficiency as the criterion. Since our proposed CR-AsNC scheme transmits an extra broadcasting message due to cogitative-inspired network coding, it can achieve higher spectrum efficiency than both benchmarks. As shown in the figure, based on the use of the same modulation at all transmitters, the proposed scheme can ensure that the spectrum efficiency is improved by 50% in the high SNR region.

VI. CONCLUSION

In this paper, we proposed a CR-AsNC scheme in cellular MIMO systems. The broadcast message from the base station is network coded with each user message, and thus the spectral efficiency can be improved by avoiding allocating extra radio resources. The precoding design was first formulated as an optimization problem, where an iterative algorithm based on alternating optimization was developed. To achieve a better tradeoff between system complexity and performance, a simplified precoding scheme based

on channel diagonalization was also proposed. The simulation results show that the spectral efficiency can be improved by 1.3 Bit/s/Hz at 20 dB when QPSK is applied, and the performance gain can be further enlarged when the higher order constellations are used. Moreover, the BER performance demonstrates that our proposed precoding optimization can ensure the transmission robustness of CR-AsNC, and can achieve better reliability than the conventional time division scheme with the same spectral efficiency.

APPENDIX A

PROOF OF LEMMA 1

Firstly, we will prove that the source transmit power is decreasing with λ. By substituting (17) into the constraint of (10), the total power at the source can be expressed as

$$\sum_{k=1}^{K} \text{tr}\left(\mathbf{v}_k \mathbf{v}_k^H\right) = \sum_{k=1}^{K} \mathbf{v}_k^H \mathbf{v}_k = \sum_{k=1}^{K} \mathbf{d}_k^H \mathbf{H}_k \left(\sum_{i=1}^{K} \mathbf{H}_k^H \mathbf{d}_i \mathbf{d}_i^H \mathbf{H}_k + \lambda \mathbf{I}_L\right)^{-2} \mathbf{H}_k^H \mathbf{d}_k. \tag{43}$$

It can be easily proved that (43) is a positive semidefinite quadratic form, whose standard form can be given as

$$\sum_{k=1}^{K} \text{tr}\left(\mathbf{v}_k \mathbf{v}_k^H\right) = \sum_{k=1}^{K} \sum_{l=1}^{L} \frac{c_{k,l}^2}{(\lambda + \mu_{k,l})^2}, \tag{44}$$

where $c_{k,l}$ is a constant unrelated to λ, and $\mu_{k,l}$ is an eigenvalue of $\sum_{i=1}^{K} \mathbf{H}_k^H \mathbf{d}_i \mathbf{d}_i^H \mathbf{H}_k$, $c_{k,l}, \mu_{k,l}, \lambda \geqslant 0$, $k = 1, \cdots, K$ and $l = 1, \cdots, L$. As shown in (44), the source transmit power is a decreasing function of λ.

Then we try to derive the upper bound of λ, and the following equations can be obtained based on (17),

$$\sum_{k=1}^{K} \frac{1}{\lambda^2} \text{tr}\left(\mathbf{H}_k^H \mathbf{d}_k \mathbf{d}_k^H \mathbf{H}_k\right)$$

$$= \sum_{k=1}^{K} \frac{1}{\lambda^2} \text{tr}\left[\left(\sum_{i=1}^{K} \mathbf{H}_k^H \mathbf{d}_i \mathbf{d}_i^H \mathbf{H}_k + \lambda \mathbf{I}_L\right) \mathbf{v}_k^{\text{opt}} \left(\mathbf{v}_k^{\text{opt}}\right)^H \left(\sum_{i=1}^{K} \mathbf{H}_k^H \mathbf{d}_i \mathbf{d}_i^H \mathbf{H}_k + \lambda \mathbf{I}_L\right)^H\right]$$

$$= \sum_{k=1}^{K} \text{tr}\left[\mathbf{v}_k^{\text{opt}} \left(\mathbf{v}_k^{\text{opt}}\right)^H\right] + 2\text{tr}\left[\left(\sum_{i=1}^{K} \mathbf{H}_k^H \mathbf{d}_i \mathbf{d}_i^H \mathbf{H}_k\right) \mathbf{v}_k^{\text{opt}} \left(\mathbf{v}_k^{\text{opt}}\right)^H\right]$$

$$+ \text{tr}\left[\left(\sum_{i=1}^{K} \mathbf{H}_k^H \mathbf{d}_i \mathbf{d}_i^H \mathbf{H}_k\right)^2 \mathbf{v}_k^{\text{opt}} \left(\mathbf{v}_k^{\text{opt}}\right)^H\right], \tag{45}$$

where the last equality in (45) follows from the fact that $\sum_{i=1}^{K} \mathbf{H}_k^H \mathbf{d}_i \mathbf{d}_i^H \mathbf{H}_k$ is a Hermitian matrix, and $\mathrm{tr}(\mathbf{AB}) = \mathrm{tr}(\mathbf{BA})$. Since $\mathbf{v}_k^{\mathrm{opt}}(\mathbf{v}_k^{\mathrm{opt}})^H$ and $\sum_{i=1}^{K} \mathbf{H}_k^H \mathbf{d}_i \mathbf{d}_i^H \mathbf{H}_k$ are both positive semidefinite matrices, all the three terms in (45) are positive or at least equal to zero [9]. Moreover, due to the KKT condition given by (15), the equation that $\sum_{k=1}^{K} \mathrm{tr}[\mathbf{v}_k^{\mathrm{opt}}(\mathbf{v}_k^{\mathrm{opt}})^H] = p_{UE}$ must be satisfied when $\lambda > 0$. Then the following inequality can be obtained from (45)

$$\sum_{k=1}^{K} \frac{1}{\lambda^2} \mathrm{tr}(\mathbf{H}_k^H \mathbf{d}_k \mathbf{d}_k^H \mathbf{H}_k) \geqslant p_{UE}. \tag{46}$$

And the proof has been finished.

APPENDIX B

PROOF OF LEMMA 3

Following the proof of Lemma 1, it can be easily proved that $\mathrm{tr}[\mathbf{P}^{\mathrm{opt}}(\mathbf{P}^{\mathrm{opt}})^H]$ is a positive semidefinite quadratic form, which decreases as μ increases. Then we can derive the following upper bound of $\mathrm{tr}[(\sum_{k=1}^{K} \mathbf{D}_k \mathbf{G}_k)(\sum_{k=1}^{K} \mathbf{D}_k \mathbf{G}_k)^H]/\mu^2$,

$$\begin{aligned}
\frac{1}{\mu^2} \mathrm{tr}\left[\left(\sum_{k=1}^{K} \mathbf{D}_k \mathbf{G}_k\right)\left(\sum_{k=1}^{K} \mathbf{D}_k \mathbf{G}_k\right)^H\right] &= \mathrm{tr}[\mathbf{P}^{\mathrm{opt}}(\mathbf{P}^{\mathrm{opt}})^H] + \frac{2}{\mu^2} \mathrm{tr}\left[\left(\sum_{k=1}^{K} \mathbf{D}_k \mathbf{G}_k\right)\mathbf{P}^{\mathrm{opt}} + (\mathbf{P}^{\mathrm{opt}})^H\right] + \\
&\quad \frac{1}{\mu^2} \mathrm{tr}\left[\left(\sum_{k=1}^{K} \mathbf{D}_k \mathbf{G}_k\right)\mathbf{P}^{\mathrm{opt}}(\mathbf{P}^{\mathrm{opt}})^H \left(\sum_{k=1}^{K} \mathbf{D}_k \mathbf{G}_k\right)^H\right] \geqslant p_B, \tag{47}
\end{aligned}$$

where the inequality in (47) is based on the fact that $\mathrm{tr}[\mathbf{P}^{\mathrm{opt}}(\mathbf{P}^{\mathrm{opt}})^H] = p_B$, and the three terms of the first equation are the traces of three positive semidefinite matrices. The lemma has been proved.

APPENDIX C

PROOF OF THEOREM 4

To verify the convexity of (36), we first derive the second derivative for ϵ_B to formulate its Hessian matrix,

$$\nabla_{p_{UE-k}, p_{UE-k}}^2 \epsilon_B = \frac{2(\tilde{\mathbf{v}}_k^H \mathbf{H}_k^H \mathbf{H}_k \tilde{\mathbf{v}}_k)^2 \sigma^2}{[\sigma^2 + (\tilde{\mathbf{v}}_k^H \mathbf{H}_k^H \mathbf{H}_k \tilde{\mathbf{v}}_k) p_{UE-k}]^3}, \quad \nabla_{p_{UE-k}, p_{UE-i}}^2 \epsilon_B = 0, \; i \neq k. \tag{48}$$

The nonnegativity of $\nabla_{p_{UE-k}, p_{UE-k}}^2 \epsilon_B$ can be ensured, and then the Hessian matrix can be presented as $\boldsymbol{\Psi}_{\epsilon_B} = \mathrm{diag}\{\nabla_{p_{UE-1}, p_{UE-1}}^2 \epsilon_B, \cdots, \nabla_{p_{UE-K}, p_{UE-K}}^2 \epsilon_B\}$. Since $\boldsymbol{\Psi}_{\epsilon_B}$ is a positive semidefinite matrix, it can

be proved that (36) is a convex optimization problem. To solve the problem with KKT conditions, we first formulate its Lagrangian function as follows,

$$\mathcal{L}_{\epsilon_B} = \sum_{k=1}^{K} \frac{\sigma^2}{\sigma^2 + (\tilde{\mathbf{v}}_k^H \mathbf{H}_k^H \mathbf{H}_k \tilde{\mathbf{v}}_k)p_{UE-k}} + \tilde{\lambda}\left(\sum_{k=1}^{K} p_{UE-k} - p_{UE} \right), \tag{49}$$

where $\tilde{\lambda}$ is the Lagrangian multiplier. Then the KKT conditions for (36) can be presented as

$$\nabla_{p_{UE-k}}\mathcal{L}_{\epsilon_B} = \frac{(\tilde{\mathbf{v}}_k^H \mathbf{H}_k^H \mathbf{H}_k \tilde{\mathbf{v}}_k)\sigma^2}{[\sigma^2 + (\tilde{\mathbf{v}}_k^H \mathbf{H}_k^H \mathbf{H}_k \tilde{\mathbf{v}}_k)p_{UE-k}]^2} - \tilde{\lambda} = 0, \tag{50}$$

$$\sum_{k=1}^{K} p_{UE-k} - p_{UE} = 0. \tag{51}$$

Based on (50) and (51), the closed-form solution for (36) can be derived as

$$p_{UE-k}^{\text{opt}} = \sqrt{\frac{1}{(\tilde{\mathbf{v}}_k^H \mathbf{H}_k^H \mathbf{H}_k \tilde{\mathbf{v}}_k)\alpha}p_{UE} - \sum_{i\neq k} \frac{\sigma^2}{\tilde{\mathbf{v}}_i^H \mathbf{H}_i^H \mathbf{H}_i \tilde{\mathbf{v}}_i}} \tag{52}$$

Then w_{UE-k}^{opt} can be easily obtained due to the relationship that $p_{UE-k}^{\text{opt}} = (w_{UE-k}^{\text{opt}})^2$, and the proof is complete.

APPENDIX D

PROOF OF THEOREM 5

The Hessian matrix for the objective function of (40) can be derived as

$$\boldsymbol{\Psi}_{\epsilon_{UE}^{\text{up}}} = \text{diag}\left\{\nabla_{p_1,p_1}^2 \epsilon_{UE}^{\text{up}}, \cdots, \nabla_{p_K,p_K}^2 \epsilon_{UE}^{\text{up}}\right\} = \text{diag}\left\{ \sum_{i=1}^{K} \frac{2\lambda_1^4 \tilde{v}_1^i \sigma^2}{(\tilde{v}_1^i \sigma^2 + \lambda_1^2 p_1)^3}, \cdots, \sum_{i=1}^{K} \frac{2\lambda_K^4 \tilde{v}_K^i \sigma^2}{(\tilde{v}_K^i \sigma^2 + \lambda_K^2 p_K)^3}\right\}. \tag{53}$$

(53) follows from the fact that $\nabla_{p_k,p_i}^2 \epsilon_{UE}^{\text{up}} = \nabla_{p_i,p_k}^2 \epsilon_{UE}^{\text{up}} = 0$ $(i \neq k)$. Since $\boldsymbol{\Psi}_{\epsilon_{UE}^{\text{up}}}$ is a positive semidefinite matrix, (40) is a convex optimization problem, and its KKT conditions can be presented as

$$\nabla_{p_k}\mathcal{L}_{\epsilon_{UE}^{\text{up}}} = \frac{\tilde{v}_k^i \lambda_k^2 \sigma^2}{(\tilde{v}_k^i \sigma^2 + \lambda_k^2 p_k)^2} - \nu_{\epsilon_{UE}^{\text{up}}} = 0, \tag{54}$$

$$\sum_{k=1}^{K} p_k - p_B = 0, \tag{55}$$

where $\mathcal{L}_{\epsilon_{UE}^{\text{up}}} = \epsilon_{UE}^{\text{up}} + \nu_{\epsilon_{UE}^{\text{up}}}(\sum_{k=1}^{K} p_k - p_B)$ is the Lagrangian function of (40), and $\nu_{\epsilon_{UE}^{\text{up}}}$ is its Lagrangian multiplier. Then the relation between p_k and $\nu_{\epsilon_{UE}^{\text{up}}}$ can be obtained based on (54),

$$p_k = \sqrt{\frac{\tilde{v}_k^i \sigma^2}{\nu_{\epsilon_{UE}^{\text{up}}} \lambda_k^2} - \frac{\tilde{v}_k^i \sigma^2}{\lambda_k^2}} \simeq \sqrt{\frac{\tilde{v}_k^i \sigma^2}{\nu_{\epsilon_{UE}^{\text{up}}} \lambda_k^2}}, \tag{56}$$

where the approximation follows from the second term in (56) is a higher order infinitesimal in the high SNR region. Combining (55) and (56), the closed-form power allocation factor can be derived as

$$w_k^{\text{opt}} = \sqrt{p_k^{\text{opt}}} = \left(\sum_{i=1}^{K} \frac{\tilde{v}_k^i p_B^2}{\beta \lambda_k^2} \right)^{1/4}. \tag{57}$$

And the theorem has been proved.

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