Robust Formation Control of Robot Manipulators with Inter-agent Constraints over Undirected Signed Networks

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***Abstract*— We address the problem of distributed control of a network of cooperative and competitive robot manipulators in end-effector coordinates. We propose a distributed bipartite formation controller that guarantees collision avoidance of the end-effectors. In the considered setting two groups are formed and reach inter-group bipartite consensus or disagreement. On the other hand, the end-effectors achieve intra-group formation. To ensure that the end-effectors do not collide, we design gradient-based control laws using barrier-Lyapunov functions. In addition, the proposed controller ensures that the closed-loop system is robust to external disturbances. The latter are assumed to be generated by an exosystem, so they are effectively rejected by an internal-model-based compensator. More precisely, we establish asymptotic stability of the bipartite formation manifold. Finally, we illustrate our theoretical results via numerical simulations.**

***Index Terms*— Formation consensus, signed networks, robotic manipulators, barrier-Lyapunov functions.**

I. Introduction

Formation control consists, roughly speaking, in making a group of physical systems adopt a formation and remain stable at an equilibrium, or move along a path, describing a common trajectory. This problem has been extensively studied, often relying on the bulk of literature on consensus control. However, the greater part of the literature considers only cooperative agents (in which case these form a network that can be modeled by a graph containing only links with positive weights) or considers linear models. Yet, robot manipulators are inherently nonlinear and there are many scenarios in which some agents may be competitive, so their interactions carry negative weights. Beyond applications involving robot manipulators, other scenarii that pertain to coopetitive networks include herding control [1], [2]; social-networks theory [3], and aerospace applications [4].

The cooperative *vs* competitive nature of the links may be analyzed using the formalism of *signed networks* [3], in which the edges have both positive and negative weights. For some of such networks, called structurally balanced[1], the achievable goal is *bipartite consensus* [3], in which all the agents converge to the same state in modulus but opposite in signs. See, *e.g.*, [3], [5], [6], and [7].

In all of the previous references, however, generic first-, second-, or higher-order linear models are used. These are less suitable for robot manipulators, which are most commonly modeled by the Euler-Lagrange equations. In that regard, the literature on control of multi-agent Euler-Lagrange systems is also rich, but most often only cooperative networks are considered. For instance, in [8], [9] the tracking-consensus problem for mobile robots with nonholonomic constraints is addressed, in [10] the formation control of flying spacecrafts, in [11] the synchronization of multi-Lagrangian systems, and in [12]–[15] the synchronization of multiple robot manipulators. Now in all of these references, the synchronization problem is studied in joint coordinates. Formation of manipulators in end-effector coordinates is considered in [16]–[18]. Nonetheless, in all of the previously cited references only networks of cooperative agents are considered. For signed networks, the bipartite consensus of networked robot manipulators is addressed, *e.g.*, in [19]–[23], while the leader-follower bipartite consensus is studied in [24]–[27] (in the latter parametric uncertainty is also considered). In end-effectors coordinates the bipartite formation-control problem is considered in [28].

Now, besides the two aspects previously described, which relate to the network and systems' model (*i.e.*, the sign of the interconnections and the agents' dynamics), there are others that must be taken into account in the control of multi-agent robot systems. Two of these are the existence of constraints and the effect of external disturbances. Considering that a disturbance may be modeled by a multi-periodic signal [18] an effective method to compensate for its effect is the internal-model-based approach. See, *e.g.*, [15], [18], [29]–[31] for works on consensus among cooperative robots, and [22]–[24] and [28] for works on coopetitive networks of robot manipulators. Yet, none of the references cited above considers the presence of constraints.

Collision avoidance and maintenance of information exchange objectives are typically expressed as inter-agent constraints and are commonly addressed using artificial potential

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[1]A signed network is structurally balanced if all the nodes may be split into two disjoint subsets, where agents cooperative with each other are in the same subset and agents competitive with each other are in different ones [3].

functions—see *e.g.*, [32], [33] for single integrators, and [34] for unicycles. Now, several articles address constrained consensus problems for first-order systems [35], second-order systems [36], and for underactuated UAVs [37], but only a few works focus on constrained control problems for networks containing competitive interactions. For instance, for first-order systems, connectivity-constrained multi-swarm herding of unicycle robots is studied in [1]; for second-order systems, non-cooperative herding with connectivity maintenance is achieved in [2], and bipartite flocking with collision avoidance and connectivity maintenance is achieved in [38] using artificial potential functions. A barrier-Lyapunov-function-based controller is proposed in [39], which is a preliminary version of this paper, devoted to the constrained leaderless bipartite formation problem over undirected signed networks of double integrators. As a matter of fact, all of the references mentioned above, in which competitive interactions are considered, concern only first- and second-order integrators.

In this paper, we consider the distributed bipartite formation-control problem of robot manipulators' end-effectors under relative distance constraints and in the presence of disturbances. We consider a networked system of cooperative-competitive robot manipulators modeled by the Euler-Lagrange equations and interconnected over a structurally balanced undirected signed graph [3]. The desired formation goal is imposed on the manipulators' end-effectors. Such scenarios are motivated, for example, by applications in industrial robotics where robots share the same workspace but are assigned symmetric tasks by the team, such as working on opposite surfaces of an object. Ideally, the robot manipulators should occupy the minimum space while evolving with guaranteed safety and increased reactivity.

Relative to [16]–[18], our results apply to networks having both cooperative and competitive interactions. Contrary to [19]–[28], in which the bipartite consensus problem of robot manipulators over signed networks is studied, we address the problem under inter-agent constraints. We consider inter-agent distance constraints on the end-effectors to ensure to keep a minimum safety distance between any pair of interconnected end-effectors to avoid collisions and maximum distance maintenance to make certain the task requirements for cooperative end-effectors are guaranteed. However, the problem of avoiding link collisions is not addressed. Relative to [1], [2], in which the control strategies rely on optimization techniques and to [38], in which artificial potential functions are used, we base our controller on the gradient of a barrier-Lyapunov function. In contrast to [1], [2], our controller applies to signed networks and in contrary to [38], a minimum safety distance between agents is ensured. Relative to [39], we consider Euler-Lagrange systems, not simple integrators, and we establish robustness with respect to external perturbations. To that end, we follow the frameworks of [15], [18], [29], to use an internal model to reject the disturbances, but contrary to these references, our work considers signed networks. Relative to [22]–[24] and [28], in which the presence of disturbances is considered, we also address minimum and maximum distance constraints on the end-effectors. Our control design and analysis rely on the edge-based formulation for signed networks [40], which allows to recast the problem into one of stabilization of the origin in error coordinates. We establish asymptotic stability of the bipartite formation manifold using Lyapunov's direct method.

Thus, relative to the existing literature, we contribute with a robust bipartite formation control law that ensures that the manipulator's end-effectors achieve the desired formation while avoiding inter-agent collisions and staying within a maximum distance imposed by the task requirements. To the best of our knowledge, similar results are not available in the literature for robot manipulators containing competitive interactions.

II. Model and Problem Formulation

We describe in detail every aspect of the problem of bipartite formation of end-effectors with constraints and under perturbations, and present the models that we use.

A. Agents' dynamics

Consider a network of N n-degrees-of-freedom robot manipulators modeled by the Euler-Lagrange equations.

$$M_i(q_i)\ddot{q}_i + C_i(q_i,\dot{q}_i)\dot{q}_i + \frac{\partial}{\partial q_i}U_i(q_i) = \tau_i + d_i, \quad i \le N, \tag{1}$$

where $q_i, \dot{q}_i, \ddot{q}_i \in \mathbb{R}^n$ are the generalized joint position, velocity, and acceleration respectively, $M_i(q_i) \in \mathbb{R}^{n\times n}$ is the inertia matrix, $U : \mathbb{R}^n \to \mathbb{R}$ is the potential energy function, $\tau_i \in \mathbb{R}^p$ is the control input and $d_i \in \mathbb{R}^n$ is an external disturbance generated by an exosystem. As it is customary, we assume the following.

Assumption 1: The following properties hold.

1. There exist $\underline{c}_i$ and $\bar{c}_i > 0$ such that, $\underline{c}_i I \le M_i(q_i) \le \bar{c}_i I$ for all $q_i \in \mathbb{R}^n$.
2. The matrix $\dot{M}_i(q_i) - 2C_i(q_i,\dot{q}_i)$ is skew-symmetric.
3. The Coriolis matrix $C_i(q_i,\dot{q}_i)$ is uniformly bounded in q_i. Moreover $|C_i(q_i,\dot{q}_i)\dot{q}_i| \le k_{c_i}|\dot{q}_i|^2$ for $k_{c_i} > 0$.

As in [15] and [18], we consider that the external disturbances are modeled by

$$d_i = d_{M,i} + J_i(q_i)^\top d_{E,i}, \tag{2}$$

where $d_{M,i} \in \mathbb{R}^n$, $d_{E,i} \in \mathbb{R}^p$ and $J_i(q_i) \in \mathbb{R}^{n\times p}$ is the Jacobian matrix. The disturbance d_i is generated by an exosystem of the form

$$\dot{w}_{M,i} = S_{M,i}w_{M,i}, \quad d_{M,i} = C_{M,i}w_{M,i}, \tag{3a}$$
$$\dot{w}_{E,i} = S_{E,i}w_{E,i}, \quad d_{E,i} = C_{E,i}w_{E,i}, \quad i \le N \tag{3b}$$

where $w_{M,i}, w_{E,i} \in \mathbb{R}^{l_i}$, $S_{M,i}, S_{E,i} \in \mathbb{R}^{l_i\times l_i}$ and $C_{M,i}, C_{E,i} \in \mathbb{R}^{n\times l_i}$. As in [29], we assume the following.

Assumption 2: The exosystems $S_{M,i}$ and $S_{E,i}$ are assumed to be neutrally stable, that is, all the eigenvalues of $S_{M,i}$ and $S_{E,i}$ are different and lie on the imaginary axis, and they are nonsingular. Moreover, they are assumed to be known.

Such an assumption is realistic for various human-robot-environment interactions because the disturbance is expressed as a sum of sinusoidals—cf. [29], which is a truncated finite Fourier approximation of general external bounded disturbances.

B. Problem statement

We define now the problem of bipartite formation of manipulators' end-effectors. Let $x_i \in \mathbb{R}^p$ be the position of the ith manipulator's end-effector in the task space. The end-effector's position x_i can be mapped to its generalized joint coordinates via the nonlinear forward kinematics mapping [41]

$$x_i = x_{i_0} + h_i(q_i), \tag{4}$$

where x_{i_0} is the position of the manipulator's base and $h_i : \mathbb{R}^n \to \mathbb{R}^p$ is the mapping from joint-space to the task space. Differentiating (4) with respect to time, we obtain the relation between the task-space velocity and joint velocity [41]

$$\dot{x}_i = J_i(q_i)\dot{q}_i, \quad J_i(q_i) := \frac{\partial h_i(q_i)}{\partial q_i}\dot{q}_i, \tag{5}$$

with $J_i(q_i) \in \mathbb{R}^{n\times p}$ the Jacobian matrix of the forward kinematics.

The bipartite formation control problem consists in the end-effectors' positions of the cooperative agents reaching a desired geometric shape around a consensus value, while the end-effectors' positions of non-cooperative agents converge to another spatial configuration. The characteristics of the formation shape are defined through the relative biases b_i and b_j with respect to the consensus points. Hence, we define the bipartite formation control objective as

$$\lim_{t\to\infty} \bar{x}_i(t) - \text{sgn}(a_{ij})\bar{x}_j(t) \to 0, \quad i, j \leq N, \tag{6}$$

where

$$\bar{x}_i := x_i - b_i, \tag{7}$$

and $a_{ij} \in \mathbb{R}$ is the adjacency weight between the two agents.

In an all-cooperative-agents setting, consensus means that all $\bar{x}_i$ converge to the same value, but in this case, since some robot manipulators are cooperative and others are competitive, all of the end-effectors reach two symmetrical consensus values. For the purpose of control design and analysis, this boils down to making some synchronization errors to converge to zero. These errors correspond to the edges on the graph and are defined as

$$\bar{e}_k := \bar{x}_i - \text{sgn}(a_{ij})\bar{x}_j, \quad k \leq M, \tag{8}$$

where $\bar{x}_i$ is defined in (7) and k denotes the index of the interconnection between the ith and jth end-effectors. Since a_{ij} is either positive or negative, the resulting network is modeled by a signed graph [3], and we assume that the following holds.

Assumption 3: The systems described in (1), which are interconnected via inputs τ_i, form a structurally balanced (see below), undirected, and connected signed graph.

Remark 1: Recall that a signed graph is *structurally balanced* if it may be split into two disjoint sets of vertices $\mathcal{V}_1$ and $\mathcal{V}_2$, where $\mathcal{V}_1 \cup \mathcal{V}_2 = \mathcal{V}$, $\mathcal{V}_1 \cap \mathcal{V}_2 = \emptyset$ such that for every $i, j \in \mathcal{V}_p, p \in \{1, 2\}$, if $a_{ij} \geq 0$, while for every $i \in \mathcal{V}_p, j \in \mathcal{V}_q$, with $p, q \in \{1, 2\}, p \neq q$, if $a_{ij} \leq 0$. Otherwise, it is *structurally unbalanced* [3]. •

In addition, it is imposed that the controller ensure that the end-effectors do not collide and remain within a maximum distance imposed by the task requirements. This comes to ensuring that for any pair of communicating nodes ν_i and $\nu_j \in \mathcal{V}$, the corresponding positions remain in certain constraint sets, which are defined as follows. Let $\mathcal{E}_m$ denote the set of indices k corresponding to edges containing pairs of cooperative agents, *i.e.,* $i, j \in \mathcal{V}_l$ with $l \in \{1, 2\}$, $\delta_k := x_i - x_j$. In addition, for each $k \leq M$, let $R_k > 0$ and $\Delta_k > 0$. Then, we define

$$\mathcal{I}_r := \{\delta_k \in \mathbb{R}^n : |\delta_k| < R_k, \quad k \in \mathcal{E}_m\} \tag{9a}$$

$$\mathcal{I}_c := \{\delta_k \in \mathbb{R}^n : |\delta_k| > \Delta_k \quad k \leq M\}, \tag{9b}$$

where $\mathcal{I}_r$ is the set of proximity constraints and $\mathcal{I}_c$ is the set of collision-avoidance constraints. Under these conditions, it is required to design a distributed bipartite formation control law of the form

$$\dot{\chi}_i = f_{1_i}(\bar{e}_k, q_i, \dot{q}_i, \chi_i)$$
$$\tau_i = f_{2_i}(\bar{e}_k, q_i, \dot{q}_i, \chi_i),$$

where χ_i is the disturbance compensator to be designed later, to achieve bipartite formation of end-effectors, such that

$$\lim_{t\to\infty} \bar{e}_k(t) = 0, \quad \lim_{t\to\infty} \dot{q}_i(t) = 0, \quad k \leq M, i \leq N, \tag{10}$$

and the manipulators' end-effector's trajectories satisfy the proximity and collision-avoidance constraints. That is, it must hold that $\delta(t) \in \mathcal{I}$ for all $t \geq 0$, with $\delta := [\delta_1\ \delta_2\ \cdots\ \delta_M]^\top$, $\mathcal{I} := \mathcal{I}_r \cap \mathcal{I}_c$ for cooperative agents and $\mathcal{I} := \mathcal{I}_c$ for competitive agents.

III. Control design

We deal with the considered problem posed above as one of stabilization of the origin in edge coordinates [40], [42], which correspond exactly to the synchronization errors in (8). In order to respect the inter-agent constraints the control input is designed as the gradient of a so-called barrier-Lyapunov function (BLF)—cf. [34], [42], [43]. Then, in order to cope with disturbance, we use an internal model approach, similar to [15], [18], [29]. Next, we discuss in more detail each aspect of the control design, and we recall the definition of BLF for convenience.

Definition 1: Consider the system $\dot{x} = f(x)$ and let $\mathcal{I}$ be an open set containing the origin. A BLF is a positive definite $\mathcal{C}^1$ function $W : \mathcal{I} \to \mathbb{R}_{\geq 0}$, $x \mapsto W(x)$, such that $\nabla W(x)f(x) \leq 0$, where $\nabla W(x) := \partial V/\partial x$, and having the property that $W(x) \to \infty$, and $\nabla W(x) \to \infty$ as $x \to \partial\mathcal{I}$, where $\partial\mathcal{I}$ denotes the boundary of $\mathcal{I}$.

A. Gradient-based control design

Let $k \leq M$ be arbitrarily fixed and consider the following BLF in terms of the synchronization errors, $W_k : \mathbb{R} \to \mathbb{R}_{\geq 0}$, defined as

$$W_k(s) = \frac{1}{2}[|s|^2 + B_k(s)], \tag{11}$$

where $B_k(s)$ is the sum of two functions[2] satisfying Definition 1, each of them encoding the constraints in (9), respectively. More precisely, $B_k(s) = \frac{1}{2}(1+\sigma_k)B_{r_k}(s) + B_{c_k}(s)$, where $B_{c_k}(s) \to \infty$ as $|s| \to \Delta_k$ and $B_{r_k}(s) \to \infty$ as $|s| \to R_k$ for all k. In the latter, $\sigma_k = 1$ if $k \in \mathcal{E}_m$, i.e., if the interaction is cooperative, and $\sigma_k = -1$ otherwise. Furthermore, $B_k(s)$ is non-negative, it satisfies $B_k(0) = 0$, and it tends to infinity as $|s| \to \Delta_k$ for all edges and as $|s| \to R_k$ for $k \in \mathcal{E}_m$. However, the constraints are imposed on the physical distance between any two end-effectors, that is $\delta_k = x_i - x_j$, which in function of the synchronization errors $\bar{e}_k$ defined in (8) are given by

$$\delta_k = \bar{e}_k + \bar{b}_k, \quad i, j \in \mathcal{V}_p, \tag{12}$$

for a pair of cooperative agents, and as

$$\delta_k = \bar{e}_k + \bar{b}_k - 2x_j, \quad i \in \mathcal{V}_p, j \in \mathcal{V}_q, \tag{13}$$

for a pair of competitive agents, where $p, q \in \{1, 2\}$, $p \neq q$ and $\bar{b}_k = b_i - \text{sgn}(a_{ij})b_j$. That is, in terms of the synchronization errors, the constraint sets in (9) take the form

$$\mathcal{I}_r = \{\bar{e}_k \in \mathbb{R}^n : |\bar{e}_{x_k} + \alpha_k| < R_k, \ k \in \mathcal{E}_m\}, \tag{14a}$$
$$\mathcal{I}_c = \{\bar{e}_k \in \mathbb{R}^n : \Delta_k < |\bar{e}_{x_k} + \alpha_k|, \ k \leq M\}, \tag{14b}$$

where

$$\alpha_k := \delta_k - \bar{e}_k, \quad \forall k \leq M. \tag{15}$$

For the purpose of designing a gradient-based controller that ensures that the constraints are satisfied, the BLF must be such that $W_k(\bar{e}_k) = 0$ if $\bar{e}_k = 0$, but it must also hold that $W_k(\bar{e}_k) \to \infty$ as $|\bar{e}_{x_k} + \alpha_k| \to \Delta_k$ or $|\bar{e}_{x_k} + \alpha_k| \to R_k$. Then, for the BLF to be effective both for cooperative and competitive agents, we shift its minimum by introducing a so-called *gradient recentered barrier function* [44]. We define

$$\widehat{W}_k(\alpha_k, \bar{e}_k) := W_k(\bar{e}_k + \alpha_k) - W_k(\alpha_k) - \frac{\partial W_k}{\partial s}(\alpha_k)\bar{e}_k, \tag{16}$$

which has the desired properties by the definition of W_k in (11), since it satisfies $\widehat{W}_k(\alpha_k, 0) \equiv 0$, $\nabla_{\bar{e}_k}\widehat{W}_k(\alpha_k, 0) \equiv 0$, where $\nabla_{\bar{e}_k}\widehat{W}_k = \partial\widehat{W}_k/\partial\bar{e}_k$, and $\widehat{W}_k(\alpha_k, \bar{e}_k) \to \infty$ as $|\delta_k| \to \Delta_k$ for all $k \leq M$, and as $|\delta_k| \to R_k$ for all $k \in \mathcal{E}_m$. Also, we note that $\{\bar{e}_k = 0\}$ is a minimum of $\widehat{W}(\alpha_k, \cdot)$ and, as a matter of fact, it is also a unique minimum even though $\widehat{W}(\alpha_k, \cdot)$ has a second critical point, which we denote e_k^*. Then, defining $\mathcal{W}_k := \{0, e_k^*\}$ for any $k \leq M$, we have

$$\frac{\kappa_1}{2}|\bar{e}_k|^2_{\mathcal{W}_k} \leq \widehat{W}_k(\alpha_k, \bar{e}_k) \tag{17}$$

for all $\alpha_k \in \mathbb{R}^n$ and $\bar{e}_k$ such that $\bar{e} \in \mathcal{I}$, where $|\bar{e}_k|_{\mathcal{W}_k} := \min\{|\bar{e}_k|, |\bar{e}_k - e_k^*|\}$.

[2]A particular choice for $B_k(s)$ is given in Section V.

Based on $\widehat{W}_k$ we define the gradient-based bipartite-formation control law as

$$\tau_i^* = -k_{1_i}J_i(q_i)^\top \left[\sum_{k=1}^{M}[E_s]_{ik}\nabla_{\bar{e}_k}\widehat{W}_k + \sum_{k=1}^{M}[\mathbb{E}]_{ik}\nabla_{\alpha_k}\widehat{W}_k\right] - k_{2_i}\dot{q}_i + \frac{\partial}{\partial q_i}U_i(q_i), \tag{18}$$

where $k_{1_i} > 0$, $k_{2_i} > 0$ for all $i \leq N$,

$$\mathbb{E} = E - E_s, \tag{19}$$

E is the incidence matrix of the cooperative version of the considered network[3], and E_s the incidence matrix of the considered signed network. We recall that E_s describes the interaction topology of the network and is defined as follows for a structurally balanced signed network:

$$[E_s]_{ik} := \begin{cases} +1, & \text{if } v_i \text{ is the initial node of the edge } \varepsilon_k; \\ -1, & \text{if } v_i, v_j \text{ are cooperative such that } v_i, v_j \in \mathcal{V}_l, l \in \{1,2\} \text{ and } v_i \text{ is the terminal node of the edge } \varepsilon_k; \\ +1, & \text{if } v_i, v_j \text{ are competitive such that } v_i \in \mathcal{V}_p, v_j \in \mathcal{V}_q, p, q \in \{1,2\}, p \neq q \text{ and } v_i \text{ is the terminal node of the edge } \varepsilon_k; \\ 0, & \text{otherwise,} \end{cases}$$

where $\varepsilon_k = \{v_i, v_j\}$, $k \leq M$, $i, j \leq N$ are arbitrarily oriented edges and $\mathcal{V}_1$ and $\mathcal{V}_2$ are the two disjoint sets of vertices.

The first two terms in the control law in (18) are needed to ensure the bipartite formation of end-effectors while respecting the inter-agent constraints imposed on the task space. The second term is needed specifically because of the use of the gradient recentered barrier function and the presence of competitive interactions between agents. The third term is needed to control the joint velocity. It consists of a damping term to stabilize the joint velocity at zero. The last term is to compensate for the gravitational force. We show in the next Section that the control law τ^* in (18) achieves bipartite formation and deals with inter-agent distance constraints for the system (1), but only in the absence of disturbances, that is, if $d_i = 0$. In the next Subsection, an internal model-based approach is presented to cope with the disturbance.

B. Robust control redesign

In order to deal with the disturbances modeled by (2) and (3), we design an estimator of d_i. For that, we use an internal model-based approach [15], [18], [29]. Let

$$\dot{\chi}_{1_i} = A_{M,i}\chi_{1_i} - B_{M,i}u_i, \tag{20a}$$
$$\dot{\chi}_{2_i} = A_{E,i}\chi_{2_i} - B_{E,i}J_i(q_i)u_i, \tag{20b}$$

where $\chi_{1_i} \in \mathbb{R}^{l_i}$, $\chi_{2_i} \in \mathbb{R}^{l_i}$, $A_{M,i} \in \mathbb{R}^{l_i \times l_i}$, $A_{E,i} \in \mathbb{R}^{l_i \times l_i}$, $B_{M,i} \in \mathbb{R}^{l_i \times n}$, $B_{E,i} \in \mathbb{R}^{l_i \times n}$, $u_i \in \mathbb{R}^n$ is the input to the internal model dynamics, which is defined later, $A_{M,i} + A_{M,i}^\top = 0$, $A_{E,i} + A_{E,i}^\top = 0$ and the pairs $(B_{M,i}^\top, A_{M,i})$ and $(B_{E,i}^\top, A_{E,i})$ are observable. We also assume, as in [15]

[3]A structurally balanced graph may be transformed into a traditional cooperative one using the gauge transformation—see [3], [40].

and [18], that the eigenvalues of the matrix $S_{M,i}$ in (3) and $A_{M,i}$ and the eigenvalues of $S_{E,i}$ in (3) and $A_{E,i}$ are identical. Under these conditions, there exist transformation matrices $T_{M,i} \in \mathbb{R}^{l_i \times p_i}$ and $T_{E,i} \in \mathbb{R}^{l_i \times p_i}$, such that—cf. [30, Section 4.2]

$$T_{M,i}S_{M,i} = A_{M,i}T_{M,i}, \quad B_{M,i}^\top T_{M,i} + C_{M,i} = 0 \tag{21a}$$
$$T_{E,i}S_{E,i} = A_{E,i}T_{E,i}, \quad B_{E,i}^\top T_{E,i} + C_{E,i} = 0. \tag{21b}$$

Then, we can rewrite (20) in the compact form as

$$\dot{\chi}_i = A_i \chi_i - B_i(q_i)u_i, \tag{22}$$

where $\chi_i = \begin{bmatrix} \chi_{1_i} & \chi_{2_i} \end{bmatrix}^\top$, $A_i = \begin{bmatrix} A_{M,i} & 0 \\ 0 & A_{E,i} \end{bmatrix}$ and $B_i(q_i) = \begin{bmatrix} B_{M,i} & J_i(q_i)^\top B_{E,i} \end{bmatrix}$.

Next, the control law is redesigned using χ_i, and the input u_i will be defined later, using the internal model. Then, we define the following estimation error coordinates: $\tilde{\chi}_i$, for the estimate of the disturbance, and $\tilde{d}_i$, for the disturbance.

$$\tilde{\chi}_i = \chi_i - T_i w_i \tag{23a}$$
$$\tilde{d}_i = B_i^\top(q_i)\chi_i + d_i, \tag{23b}$$

where $T_i = \begin{bmatrix} T_{M,i} & T_{E,i} \end{bmatrix}$ and $w_i = \begin{bmatrix} w_{M,i} & w_{E,i} \end{bmatrix}^\top$. Taking the derivative of (23a) and using (3) for (23b), we obtain

$$\dot{\tilde{\chi}}_i = \dot{\chi}_i - T_i \dot{w}_i$$
$$\dot{\tilde{d}}_i = B_i(q_i)^\top \chi_i + C_i w_i.$$

Replacing (22) in the first equation and using (21) in the second, we obtain

$$\begin{aligned} \dot{\tilde{\chi}}_i &= A_i\chi_i - B_i(q_i)u_i - T_i S_i w_i \\ &= A_i \tilde{\chi}_i - B_i(q_i)u_i \end{aligned} \tag{24a}$$
$$\begin{aligned} \tilde{d}_i &= B_i(q_i)^\top \chi_i - B_i(q_i)^\top T_i w_i \\ &= B_i(q_i)^\top(\chi_i - T_i w_i) = B_i(q_i)^\top \tilde{\chi}_i. \end{aligned} \tag{24b}$$

The equations in (24) are important because they define a passive map from u_i to $\tilde{d}_i$. To see that, consider the storage function $H_i(\tilde{\chi}_i) = \frac{1}{2}|\tilde{\chi}_i|^2$. Its derivative gives

$$\begin{aligned} \dot{H}_i(\tilde{\chi}_i) = \tilde{\chi}_i^\top \dot{\tilde{\chi}}_i &= \frac{1}{2}\tilde{\chi}_i^\top (A_i + A_i^\top)\tilde{\chi}_i - \tilde{\chi}_i^\top B_i(q_i)u_i \\ &= -\tilde{\chi}_i^\top B_i(q_i)u_i, \end{aligned}$$

since $A_i + A_i^\top = 0$. Thus, the system in (24) is lossless (passive) from the input u_i to the output $\tilde{d}_i = B_i^\top \tilde{\chi}_i$. We use this observation in the control analysis.

Thus, to robustify the controller, the control law in (18) is redesigned into

$$\tau_i = \tau_i^* + B_i^\top(q_i)\chi_i, \tag{25}$$

where the last term counteracts the effect of external disturbances.

IV. Stability analysis

A. Asymptotic stability in the absence of disturbance

We analyze the stability of the bipartite formation manifold for the closed-loop system (1) interconnected by the control law (18). To that end, using the definition of the incidence matrix, we represent the synchronization errors in (8) and α_k defined in (15), in the vector form

$$\bar{e} = [E_s^\top \otimes I_n]\bar{x}, \tag{26a}$$
$$\alpha = [E^\top \otimes I_n]x - [E_s^\top \otimes I_n]\bar{x}. \tag{26b}$$

Then, after (16), we define

$$\bar{W}(\alpha, \bar{e}) = \sum_{k=1}^{M} \widehat{W}_k(\alpha_k, \bar{e}_k). \tag{27}$$

Finally, we see that the closed-loop system (1)-(18) in the compact form is

$$\begin{aligned} \ddot{q} = &-M(q)^{-1}\Big[C(q,\dot{q})\dot{q} + K_1 J(q)^\top [E_s \otimes I_n]\nabla_{\bar{e}}\bar{W}(\alpha,\bar{e}) \\ &+ K_1 J(q)^\top [\mathbb{E} \otimes I_n]\nabla_\alpha \bar{W}(\alpha,\bar{e}) + [K_2 \otimes I_n]\dot{q}\Big], \end{aligned} \tag{28}$$

where $q = [q_i]$, $M(q) = \text{blkdiag}[M_i(q_i)]$, $C(q,\dot{q}) = \text{blkdiag}[C_i(q_i,\dot{q}_i)]$, $K_1 = \text{diag}(k_{1_i})$, $K_2 = \text{diag}(k_{2_i})$ and $J(q)^\top = \text{blkdiag}[J_i(q_i)^\top]$, $\forall i \leq N$.

Proposition 1: Consider N robot manipulators modeled by (1), with $d_i = 0$ and satisfying the Assumptions 1 and 3, in closed loop with the distributed control law (18), with k_{1i}, $k_{2i} > 0$, for all $i \leq N$ and $\widehat{W}_k$ as defined in (16). Then, for any given formation shape reachable by the end-effectors, the set $\{(\bar{e},\dot{q}) = (0,0)\}$ is asymptotically stable for almost all initial conditions such that $(\bar{e}(0),\dot{q}(0)) \in \mathcal{I} \times \mathbb{R}^{nN}$ and $|\alpha_k(0)| > \Delta_k$ for any $k \leq M$. □

Proof: After Assumption 3, the considered graph is undirected and connected, so it contains a spanning tree. Then, as for the more ordinary scenario of consensus, the result may be assessed by analyzing the dynamics of the agents that belong to the spanning-tree—see [40], [42], [43]. To obtain the closed-loop equations in spanning-tree coordinates, following the latter, we first recall that

$$E_s = [E_{t_s} \quad E_{c_s}], \tag{29}$$

where $E_{t_s} \in \mathbb{R}^{N \times N-1}$ is the incidence matrix representing the edges of the spanning tree, corresponding to the spanning-tree graph $\mathcal{G}_t$, and $E_{c_s} \in \mathbb{R}^{N \times M-(N-1)}$ is the incidence matrix representing the remaining edges, corresponding to $\mathcal{G}_c := \mathcal{G} \backslash \mathcal{G}_t$. Consequently, after (26a) and (29), the errors can be expressed as $\bar{e} = [(E_{t_s}^\top \bar{x})^\top \ (E_{c_s}^\top \bar{x})^\top]^\top$, which gives $\bar{e} := [\bar{e}_t^\top \quad \bar{e}_c^\top]^\top$. Furthermore, for a structurally balanced signed graph, there exists a matrix R_s such that

$$E_s = E_{t_s} R_s, \tag{30}$$

where $R_s := [I_{N-1} \quad T_s]$ and $T_s := (E_{t_s}^\top E_{t_s})^{-1}E_{t_s}^\top E_{c_s}$—see Proposition 1 in [39]. Notably, the following relationship between the synchronization errors $\bar{e}$ and the spanning-tree errors $\bar{e}_t$ holds:

$$\bar{e} = [(E_{t_s}R_s)^\top \otimes I_n]\bar{x} = [R_s^\top \otimes I_n]\bar{e}_t, \tag{31}$$

so the bipartite formation objective (10) is achieved if $\bar{e}_t \to 0$ and $\dot{q} \to 0$. On the other hand, a similar relation holds for α defined in (26b):

$$\alpha = [\mathbb{E}^\top \otimes I_n]x + [E_s^\top \otimes I_n]b, \tag{32}$$

where $\mathbb{E}$ is defined in (19). The matrix $\mathbb{E}$ corresponds only to competitive edges. Thus, akin to (29), we can write $\mathbb{E} = [\mathbb{E}_t \quad \mathbb{E}_c]$ and $\alpha = [\alpha_t^\top \ \alpha_c^\top]^\top$. Thus,

$$\mathbb{E} = \mathbb{E}_t R_s \tag{33}$$

and

$$\alpha = R_s^\top \left[[\mathbb{E}_t^\top \otimes I_n]x + [E_{t_s}^\top \otimes I_n]b\right] = [R_s^\top \otimes I_n]\alpha_t. \tag{34}$$

Next, to express the control law in spanning-tree coordinates, we introduce

$$\tilde{W}(\alpha_t, \bar{e}_t) := \bar{W}(R_s^\top \alpha_t, R_s^\top \bar{e}_t). \tag{35}$$

That is, in view of (31) and (34), $\tilde{W}(\alpha_t, \bar{e}_t)$ denotes the same quantity as the right-hand-side of (27), but in spanning-tree coordinates, so $\tilde{W}$ maps $\mathcal{I}_t \times \mathbb{R}^{nN} \to \mathbb{R}^{nN}_{\geq 0}$, where $\mathcal{I}_t := \mathcal{I}_{r_t} \cap \mathcal{I}_{c_t}$ for cooperative agents and $\mathcal{I}_t := \mathcal{I}_{c_t}$ for competitive agents, and

$$\mathcal{I}_{r_t} := \{\bar{e}_{t_k} \in \mathbb{R}^n : |[r_{s_k}^\top \otimes I_n][\bar{e}_{t_k} + \alpha_{t_k}]| < R_k \ , k \in \mathcal{E}_{m_t}\}, \tag{36}$$

$$\mathcal{I}_{c_t} := \{\bar{e}_{t_k} \in \mathbb{R}^n : \Delta_k < |[r_{s_k}^\top \otimes I_n][\bar{e}_{t_k} + \alpha_{t_k}]|, \ k \leq N-1\}, \tag{37}$$

$\mathcal{E}_{m_t}$ denotes the set of indices k corresponding to the $N-1$ edges of the spanning-tree graph containing pairs of cooperative agents, and r_{s_k} is the kth column of R_s. The set $\mathcal{I}_t$ defines the constraints in spanning-tree coordinates. Using $\tilde{W}$, we define the gradient-based control terms

$$\nabla_{\bar{e}_t} \tilde{W} \equiv \frac{\partial \bar{W}(\alpha, \bar{e})}{\partial \bar{e}}^\top \frac{\partial \bar{e}}{\partial \bar{e}_t} = \nabla_{\bar{e}} \bar{W}^\top [R_s^\top \otimes I_n],$$
$$\nabla_{\alpha_t} \tilde{W} \equiv \frac{\partial \bar{W}(\alpha, \bar{e})}{\partial \alpha}^\top \frac{\partial \alpha}{\partial \alpha_t} = \nabla_{\alpha} \bar{W}^\top [R_s^\top \otimes I_n]. \tag{38}$$

Thus, in spanning-tree edge coordinates, Eq. (28) becomes

$$\ddot{q} = -M(q)^{-1} \Big[C(q,\dot{q})\dot{q} + K_1 J(q)^\top [E_{t_s} \otimes I_n] \nabla_{\bar{e}_t} \tilde{W}(\alpha_t, \bar{e}_t) + K_1 J(q)^\top [\mathbb{E}_t \otimes I_n] \nabla_{\alpha_t} \tilde{W}(\alpha_t, \bar{e}_t) + [K_2 \otimes I_n]\dot{q}\Big]. \tag{39}$$

The rest of the proof consists in establishing asymptotic stability of the origin $\{(\bar{e}_t, \dot{q}) = (0,0)\}$ and forward invariance of the set $\mathcal{I}_t \times \mathbb{R}^{nN}$, for the trajectories of (39). First, consider the Lyapunov function candidate

$$V(\alpha_t, \bar{e}_t, \dot{q}) = \tilde{W}(\alpha_t, \bar{e}_t) + \frac{1}{2}\dot{q}^\top M(q)\dot{q}, \tag{40}$$

where $M(q) = M(q)^\top$. Also, we remark that V is positive definite in $\bar{e}_t$ and $\dot{q}$, and bounded from above uniformly in α_t. More precisely, there exist $\mu_1 > 0$ such that $\mu_1 \left[|\bar{e}_t|^2 + |\dot{q}|^2\right] \leq V(\alpha_t, \bar{e}_t, \dot{q})$, and $V(\alpha_t, \bar{e}_t, \dot{q}) \to 0$ as $|\bar{e}_t| \to 0$ and $|\dot{q}| \to 0$.

Now, its derivative satisfies

$$\begin{aligned}\dot{V} =& \nabla_{\bar{e}_t} \tilde{W}^\top [E_{t_s} \otimes I_n]^\top J\dot{q} + \nabla_{\alpha_t} \tilde{W}^\top [\mathbb{E}_t \otimes I_n]^\top J\dot{q} \\ &+ \frac{1}{2}\dot{q}^\top \dot{M}\dot{q} - \dot{q}^\top C(q,\dot{q})\dot{q} - \dot{q}^\top [K \otimes I_n]\dot{q} \\ &- \dot{q}^\top J(q)^\top [E_{t_s} \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} - \dot{q}^\top J(q)^\top [\mathbb{E}_t \otimes I_n] \nabla_{\alpha_t} \tilde{W} \\ =& -\frac{1}{2}\dot{q}^\top \left[\dot{M} - 2C(q,\dot{q})\right]\dot{q} - \dot{q}^\top [K \otimes I_n]\dot{q}.\end{aligned}$$

But since $\dot{M} - 2C(q,\dot{q})$ is skew-symmetric, we obtain

$$\dot{V}(\alpha_t, \bar{e}_t, \dot{q}) = -\dot{q}^\top [K \otimes I_n]\dot{q} \leq 0, \tag{41}$$

for all $(\bar{e}_t, \dot{q}) \in \mathcal{I}_t \times \mathbb{R}^{nN}$ so the origin is stable and the solutions are uniformly bounded. Next, we use LaSalle's invariance theorem. To that end, we first note that on the set $\{\dot{q} \in \mathbb{R}^{nN} : \dot{V} = 0\}$, we have $\dot{q} = 0$, and consequently, $\ddot{q} = 0$. In view of (5), it follows that $\dot{x} = 0$ because $\dot{x} = J(q)\dot{q}$. In turn, since all the functions on the right-hand-side of (39) are continuous, we have

$$J(q)^\top [E_{t_s} \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} + J(q)^\top [\mathbb{E}_t \otimes I_n] \nabla_{\alpha_t} \tilde{W} = 0. \tag{42}$$

On the one hand, after (16), we have

$$\nabla_{\alpha_t} \tilde{W} = \nabla_{\bar{e}_t} \tilde{W} - \frac{\partial}{\partial \alpha_t} \left\{ \frac{\partial W}{\partial \alpha_t}(\alpha_t) \right\} \bar{e}_t, \tag{43}$$

and, because $\alpha = [E^\top \otimes I_n]x - [E_s^\top \otimes I_n]\bar{x}$, then $\dot{\alpha} = [\mathbb{E} \otimes I_n]^\top \dot{x}$ and $\dot{\alpha}_t = [\mathbb{E}_t \otimes I_n]^\top \dot{x}$. Thus, $\dot{\alpha}_t = 0$, which is equivalent to $\alpha_t \equiv$ const on $\{\dot{V} = 0\}$. In turn, the last term of the right-hand-side of (43) equals to zero. Then, from (42) and using (19), $J(q)^\top [(E_{t_s} + E_t - E_{t_s}) \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} = J(q)^\top [E_t \otimes I_n] \nabla_{\bar{e}_t} \tilde{W} = 0$. Now, since E_t is full rank (because it corresponds to the incidence matrix of a spanning tree) it follows that $\nabla_{\bar{e}_t} \tilde{W} = 0$, which holds if and only if $\bar{e}_t \in \mathcal{W}^t$, where $\mathcal{W}^t = \{0, e_t^*\}$ and e_t^* is the saddle point of $\tilde{W}$. Therefore, the solutions converge to the set $\mathcal{W}^t \times \{0\}$. However, since e_t^* is a saddle point of $\tilde{W}$, the set of initial conditions generating solutions that converge to $(e_t^*, 0)$ has zero Lebesgue measure. Thus, almost all initial conditions generate trajectories that converge to the origin. Asymptotic stability follows.

Next, we show that asymptotic stability holds for almost all initial conditions in $\mathcal{I}_t \times \mathbb{R}^{nN}$. To that end, we first remark that, from (36), $\bar{e}_t \in \mathcal{I}_t$ implies $\bar{e} \in \mathcal{I}$, so we must show that $\mathcal{I}_t \times \mathbb{R}^{nN}$ is forward invariant. To that end, for any r and $\epsilon \geq 0$, let us define

$$\mathcal{D}_{r,\epsilon} := \{\dot{q} \in B_r, \ \bar{e}_t \in \mathcal{I}_\epsilon\},$$

where $\mathcal{I}_\epsilon = \mathcal{I}_{c_\epsilon} \cap \mathcal{I}_{r_\epsilon}$ for cooperative agents and $\mathcal{I}_\epsilon = \mathcal{I}_{c_\epsilon}$ for competitive agents, while $\mathcal{I}_{\epsilon_r} := \{\bar{e}_{t_k} \in \mathbb{R}^n : |\bar{e}_{t_k} + \alpha_{t_k}| < R_k - \epsilon, \ k \in \mathcal{E}_{m_t}\}$, $\mathcal{I}_{\epsilon_c} := \{\bar{e}_{t_k} \in \mathbb{R}^n : \Delta_k + \epsilon < |\bar{e}_{t_k} + \alpha_{t_k}|, \ k \leq N-1\}$. From the definition of $\tilde{W}(\alpha_t, \bar{e}_t)$, $V(\alpha_t, \bar{e}_t, \dot{q})$ is positive definite on $\mathcal{I}_\epsilon$ for all $\bar{e}_t \in \mathcal{I}_\epsilon$, $\dot{q} \in \mathbb{R}^{nN}$ and for all ϵ. From (41), we have $|\bar{e}_t(t)|^2 + |\dot{q}(t)|^2 \leq \frac{1}{\mu_1} V(\alpha_t(0), \bar{e}_t(0), \dot{q}(0))$ then

$$(\bar{e}_t(0), \dot{q}(0)) \in \mathcal{D}_{r,\epsilon} \Rightarrow (\bar{e}_t(t), \dot{q}(t)) \in \mathcal{D}', \tag{44}$$

where

$$\mathcal{D}' := \left\{ \ (\bar{e}_t, \dot{q}) \in \mathcal{I}_t \times \mathbb{R}^{nN} : V(\alpha_t, \bar{e}_t, \dot{q}) \leq \gamma_{r,\epsilon} \right\},$$

and

$$\gamma_{r,\epsilon} := \sup_{\substack{(\bar{e}_t,\dot{q})\in\mathcal{D}_{r,\epsilon}\\ \alpha_t\in\mathbb{R}^{nN}}} \sqrt{\frac{V(\alpha_t,\bar{e}_t,\dot{q})}{\mu_1}}.$$

Note that $\gamma_{r,\epsilon}$ is well defined because V is uniformly bounded in α_t and $\mathcal{D}_{r,\epsilon}$ is bounded. After LaSalle's invariance principle, we conclude that all the trajectories contained in $\mathcal{D}'$ converge to the set $\mathcal{W}^t \times \{0\}$. After (44), that is all the trajectories starting in $\mathcal{D}_{r,\epsilon}$. This holds for any $r > 0$ arbitrarily large and $\epsilon > 0$ arbitrarily small. Thus, again because $\mathcal{W}^t = \{0, e_t^*\}$, and e_t^* is a saddle point, we conclude that the origin is asymptotically stable for almost all trajectories starting in $\mathcal{I}_t \times \mathbb{R}^{nN}$. □

B. Asymptotic stability in the presence of disturbance

Now we analyze the system (1) in the presence of disturbances and driven by the control law (25), where χ_i is defined by (22), with $u_i = \dot{q}_i$. We have the following.

Proposition 2: Consider N robot manipulators modeled by (1) and satisfying the Assumptions 1 and 3 in closed-loop with the distributed controller defined by (25), (18), and (22), with $u_i = \dot{q}_i$ and k_{1i}, $k_{2i} > 0$, for all $i \leq N$. Then, for any given formation shape reachable by the end-effectors, the set $\{(\bar{e}, \dot{q}) = (0, 0)\}$ is asymptotically stable for almost all initial conditions such that $(\bar{e}(0), \dot{q}(0)) \in \mathcal{I} \times \mathbb{R}^{nN}$ and $|\alpha_k(0)| > \Delta_k$ for any $k \leq M$. □

Proof: As for Proposition 1 the statement follows if we establish asymptotic stability of the origin in spanning-tree coordinates and forward invariance of $\mathcal{I}_t \times \mathbb{R}^{nN}$.

First, proceeding as in Section IV-A, we obtain that the closed-loop equations now read

$$\begin{aligned}\ddot{q} = &- M(q)^{-1}\Big[C(q,\dot{q})\dot{q} + K_1 J(q)^\top [E_{t_s}\otimes I_n]\nabla_{\bar{e}_t}\tilde{W}(\alpha_t,\bar{e}_t)\\ &+ K_1 J(q)^\top [\mathbb{E}_t \otimes I_n]\nabla_{\alpha_t}\tilde{W}(\alpha_t,\bar{e}_t) + [K_2\otimes I_n]\dot{q}\\ &-[B(q)\otimes I_n]^\top \chi - d\Big],\end{aligned} \tag{45}$$

where $d := \mathrm{col}[d_i]$, $i \leq N$.

Next, we consider the Lyapunov function candidate

$$V(\alpha_t,\bar{e}_t,\dot{q},\tilde{\chi}) = \tilde{W}(\alpha_t,\bar{e}_t) + \frac{1}{2}\left[\dot{q}^\top M(q)\dot{q} + |\tilde{\chi}|^2\right]. \tag{46}$$

The derivative of (46) gives

$$\begin{aligned}\dot{V} =&\nabla_{\bar{e}_t}\tilde{W}^\top [E_{t_s}\otimes I_n]^\top J(q)\dot{q} + \nabla_{\alpha_t}\tilde{W}^\top [\mathbb{E}_t\otimes I_n]^\top J(q)\dot{q}\\ &+ \frac{1}{2}\dot{q}^\top \dot{M}\dot{q} - \dot{q}^\top C(q,\dot{q})\dot{q} - \dot{q}^\top [K\otimes I_n]\dot{q}\\ &- \dot{q}^\top J(q)^\top \Big[[E_{t_s}\otimes I_n]\nabla_{\bar{e}_t}\tilde{W} + [\mathbb{E}_t\otimes I_n]\nabla_{\alpha_t}\tilde{W}\Big]\\ &- \tilde{\chi}^\top [B(q)\otimes I_n]u + \dot{q}^\top [B(q)\otimes I_n]^\top \chi + \dot{q}^\top d,\end{aligned} \tag{47}$$

where we used (23) to obtain

$$\begin{aligned}\dot{V} =& - \tilde{\chi}^\top [B(q)\otimes I_n]u + \frac{1}{2}\dot{q}^\top \Big[\dot{M} - 2C(q,\dot{q})\Big]\dot{q}\\ &+ \dot{q}^\top [B(q)\otimes I_n]^\top \chi - \dot{q}^\top [K\otimes I_n]\dot{q}\\ &+ \dot{q}^\top \Big[\tilde{d} - [B(q)\otimes I_n]^\top \chi\Big]\\ =& - \dot{q}^\top [K\otimes I_n]\dot{q} \leq 0,\end{aligned} \tag{48}$$

for which we used the skew symmetry of $\dot{M} - 2C(q,\dot{q})$ and $u = \dot{q}$. Note that on the set $\{\dot{q} \in \mathbb{R}^{nN} : \dot{V} = 0\}$, we have $\dot{q} = 0$ and $\ddot{q} = 0$. In turn, after (45), we have

$$\begin{aligned}K_1 J(q)^\top [E_{t_s}\otimes I_n]\nabla_{\bar{e}_t}\tilde{W} + K_1 J(q)^\top [\mathbb{E}_t\otimes I_n]\nabla_{\alpha_t}\tilde{W}&\\ -[B(q)\otimes I_n]^\top \chi - d = 0.&\end{aligned} \tag{49}$$

As in the Proof of Proposition 1, we have $\dot{x} = 0$ and $\dot{\alpha}_t = 0$ on $\{\dot{V} = 0\}$. Consequently, α_t is constant and, after (43), $\nabla_{\alpha_t}\tilde{W} = \nabla_{\bar{e}_t}\tilde{W}$. Then, from (49) and (19), we obtain $K_1 J(q)^\top [E_t\otimes I_n]\nabla_{\bar{e}_t}\tilde{W} - [B(q)\otimes I_n]^\top \chi - d = 0$. Replacing the estimation error coordinates in (23b) in the latter equation, we obtain

$$\begin{aligned}K_1 J(q)^\top [E_t\otimes I_n]\nabla_{\bar{e}_t}\tilde{W} - [B(q)\otimes I_n]^\top \chi&\\ - \Big[\tilde{d} - [B(q)\otimes I_n]^\top \chi\Big] = 0.&\end{aligned}$$

so $K_1 J(q)^\top [E_t\otimes I_n]\nabla_{\bar{e}_t}\tilde{W} - \tilde{d} = 0$. Then, replacing (24b) in the previous equation, we obtain

$$K_1 J(q)^\top [E_t\otimes I_n]\nabla_{\bar{e}_t}\tilde{W} = [B(q)\otimes I_n]^\top \tilde{\chi}. \tag{50}$$

Differentiating on both sides of the latter, we obtain

$$\begin{aligned}&K_1 J(q)^\top [E_t\otimes I_n]\frac{\partial^2 \tilde{W}}{\partial \bar{e}_t^2}\dot{\bar{e}}_t + K_1 \dot{J}(q)^\top [E_t\otimes I_n]\nabla_{\bar{e}_t}\tilde{W}\\ &= [B(q)\otimes I_n]^\top \dot{\tilde{\chi}} + [\dot{B}(q)\otimes I_n]^\top \tilde{\chi}.\end{aligned}$$

As $\dot{\bar{e}}_t = E_{t_s}^\top J\dot{q} = 0$, $\dot{J}(q) = \frac{\partial J(q)}{\partial q}\dot{q} = 0$ and $\dot{B}(q) = \frac{\partial B(q)}{\partial q}\dot{q} = 0$, we have $[B(q)\otimes I_n]^\top \dot{\tilde{\chi}} = 0$. Then, replacing (22) in the latter, we obtain $[B(q)\otimes I_n]^\top \left[[A\otimes I_n]\tilde{\chi} - [B(q)\otimes I_n]u\right] = \left[[B(q)^\top A]\otimes I_n\right]\tilde{\chi} = 0$, since $u = \dot{q} = 0$. Differentiating the latter again, we have

$$\begin{cases}\left[[B(q)^\top A]\otimes I_n\right]\dot{\tilde{\chi}} = \left[[B(q)^\top A^2]\otimes I_n\right]\tilde{\chi} = 0\\ \left[[B(q)^\top A^2]\otimes I_n\right]\dot{\tilde{\chi}} = \left[[B(q)^\top A^3]\otimes I_n\right]\tilde{\chi} = 0\\ \qquad\vdots\\ \left[[B(q)^\top A^{l_i-1}]\otimes I_n\right]\dot{\tilde{\chi}} = \left[[B(q)^\top A^{l_i}]\otimes I_n\right]\tilde{\chi} = 0.\end{cases} \tag{51}$$

Next, let

$$p(\lambda) = \lambda^{l_i} + c_{l_{i-1}}\lambda^{l_i-1} + \cdots + c_1\lambda + c_0 \tag{52}$$

denote the characteristic polynomial of A. On the one hand, after the Cayley-Hamilton Theorem $p(A) = 0$. Therefore, $\frac{1}{c_0}B(q)^\top [p(A)\otimes I_n]\tilde{\chi} = 0$, that is,

$$\begin{aligned}-\frac{1}{c_0}\left[B(q)^\top \left[A^{l_i} + c_{l_i-1}A^{l_i-1} + \cdots + c_1 A\right]\otimes I_n\right]\tilde{\chi}&\\ = [B(q)\otimes I_n]^\top \tilde{\chi}.&\end{aligned} \tag{53}$$

On the other hand, the equations in (51) continue to hold if the left-hand sides are multiplied by the coefficients c_p with $p \leq l_i$. Therefore, by observing that $c_{l_i} = 1$,

$$\left[B(q)^\top \left[A^{l_i} + c_{l_{i-1}}A^{l_i-1} + \cdots + c_2A^2 + c_1A\right]\otimes I_n\right]\tilde{\chi} = 0.$$

From the latter and (53), we conclude that $[B(q)\otimes I_n]^\top \tilde{\chi} = 0$. In turn, from (50) we have $J(q)^\top [E_t\otimes I_n]\nabla_{\bar{e}_t}\tilde{W} = 0$. The rest of the proof follows as for Proposition 1. □

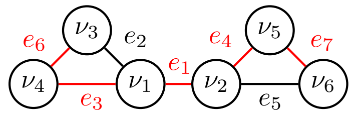

Fig. 1. An undirected signed network of 6 robot manipulators. The black lines (e_2 and e_5) represent cooperative edges, and the red lines the competitive edges.

V. Simulation results

We provide a numerical example to show the performance of our control laws, first the one in (18) in the absence of disturbance and then the one in (25) in the presence of disturbance. For that, we consider a system of $N = 6$ two-link robot manipulators interconnected over a structurally balanced undirected signed network, modeled by a graph as the one depicted in Figure 1. For the corresponding graph, we define the orientation of the seven edges as $e_1 = \nu_1 + \nu_2$, $e_2 = \nu_1 - \nu_3$, $e_3 = \nu_1 + \nu_4$, $e_4 = \nu_2 + \nu_5$, $e_5 = \nu_2 - \nu_6$, $e_6 = \nu_3 + \nu_4$, and $e_7 = \nu_5 + \nu_6$. The set of nodes may be split into two disjoint subgroups as $\mathcal{V}_1 = \{\nu_1, \nu_3, \nu_5\}$ and $\mathcal{V}_2 = \{\nu_2, \nu_4, \nu_6\}$, so the network is structurally balanced. From (29), the edges $e_i, i \leq 5$ correspond to the edges of the spanning tree, and the remaining edges, e_6 and e_7, correspond to the cycles. The corresponding incidence matrix is given by

$$E_s = \begin{bmatrix} 1 & 1 & 1 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & -1 & 0 & 1 \end{bmatrix}.$$

Each manipulator is modeled by the Euler-Lagrange equations in (1), with inertia and Coriolis matrices given by

$$M_i(q_i) = \begin{bmatrix} \alpha_i + 2\beta_i \cos(q_{2_i}) & \delta_i + \beta_i \cos(q_{2_i}) \\ \delta_i + \beta_i \cos(q_{2_i}) & \delta_i \end{bmatrix},$$

$$C_i(q_i, \dot{q}_i) = \delta_i \begin{bmatrix} -\sin(q_{2_i})\dot{q}_{2_i} & -\sin(q_{2_i})(\dot{q}_{1_i} + \dot{q}_{2_i}) \\ -\sin(q_{2_i})\dot{q}_{1_i} & 0 \end{bmatrix},$$

where $\alpha_i = l_{2_i}^2 m_{2_i} + l_{1_i}^2(m_{1_i} + m_{2_i})$, $\beta_i = l_{1_i} l_{2_i} m_{2_i}$ and $\delta_i = l_{2_i}^2 m_{2_i}$ with l_{1_i}, l_{2_i} and m_{1_i}, m_{2_i} are the length and the mass of links 1 and 2. The physical parameters are $m_1 = 1.2$kg, $m_2 = 1$kg, and $l_1 = l_2 = 1$m for all $i \leq N$. The kinematic model for each manipulator is given by

$$x_i = \begin{bmatrix} l_1 \cos(q_{i1}) + l_2 \cos(q_{1_i} + q_{2_i}) \\ l_1 \sin(q_{i1}) + l_2 \sin(q_{1_i} + q_{2_i}) \end{bmatrix} + x_{i0},$$

and the Jacobian matrix

$$J_i(q_i) = \begin{bmatrix} -l_1 \sin(q_{i1}) - l_2 \sin(q_{1_i} + q_{2_i}) & -l_2 \sin(q_{1_i} + q_{2_i}) \\ l_1 \cos(q_{i1}) + l_2 \cos(q_{1_i} + q_{2_i}) & l_2 \cos(q_{1_i} + q_{2_i}) \end{bmatrix}.$$

First, consider the system (1), where $d_i = 0$ for all $i \leq N$, with the bipartite formation control law (18), where $k_{1_i} = 20$, $k_{2_i} = 15$ for all $i \leq N$ and the barrier-Lyapunov function in (16), with $B_{r_k}(s) = \ln\left(\frac{R_k^2}{R_k^2 - |s|^2}\right), B_{c_k}(s) = \ln\left(\frac{|s|^2}{|s|^2 - \Delta_k^2}\right)$. The bases of six robot manipulators are located at $x_{1_0} = [0,\ 0.5]^\top$, $x_{2_0} = [2.5,\ 0]^\top$, $x_{3_0} = [-1,\ 0]^\top$, $x_{4_0} = [0,\ -2]^\top$, $x_{5_0} = [-3,\ 0.5]^\top$, $x_{6_0} = [2,\ -2]^\top$. The initial conditions for each agent are $q_1(0) = [\pi,\ \pi/3]^\top$, $q_2(0) = [2\pi/3,\ \pi/3]^\top$, $q_3(0) = [-\pi,\ \pi/3]^\top$, $q_4(0) = [-\pi/2,\ 0]^\top$, $q_5(0) = [\pi,\ \pi/3]^\top$, $q_6(0) = [0,\ \pi/3]^\top$, $\dot{q}_1(0) = \dot{q}_2(0) = \dot{q}_3(0) = \dot{q}_4(0) = \dot{q}_5(0) = \dot{q}_6(0) = [0,\ 0]^\top$, with $q = [q_1,\ q_2]^\top$ and $\dot{q} = [\dot{q}_1,\ \dot{q}_2]^\top$ and the relative displacements of the end-effectors are $b_1 = [0,\ 0.4]^\top$, $b_2 = [-0.4,\ 0]^\top$, $b_3 = [0.4,\ 0]^\top$, $b_4 = [0,\ -0.4]^\top$, $b_5 = [-0.4,\ 0]^\top$, $b_6 = [0.4,\ 0]^\top$, with $b = [b_x,\ b_y]^\top$. The constraint sets are $\Delta_k = 0.2$m for all edges and the maximum distance constraints for the two cooperative edges e_2 and e_5 are given as $R_2 = 2.5$m and $R_5 = 3.5$m.

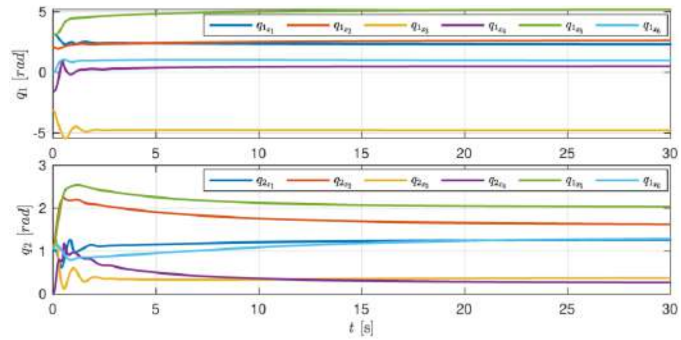

Fig. 2. Bipartite formation of system (1) with control input (18) on joint trajectories.

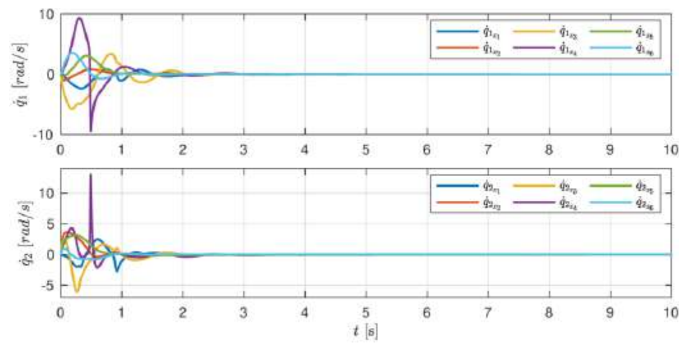

Fig. 3. Bipartite formation of system (1) with control input (18) on joint velocities.

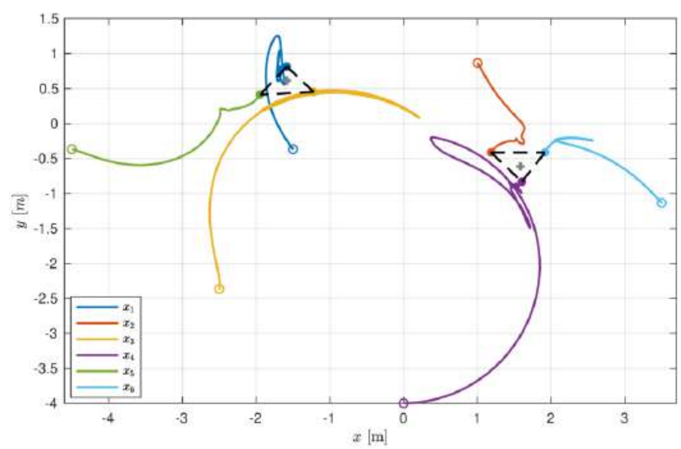

Fig. 4. Evolution of the manipulators' end-effector from the initial positions (o) to the final positions (*). Each group of end-effectors forms a triangle around the symmetric consensus points +.

The joint positions and velocities are depicted in Figures 2 and 3, respectively, and all velocities converge to zero. The paths of each end-effector up to bipartite formation are

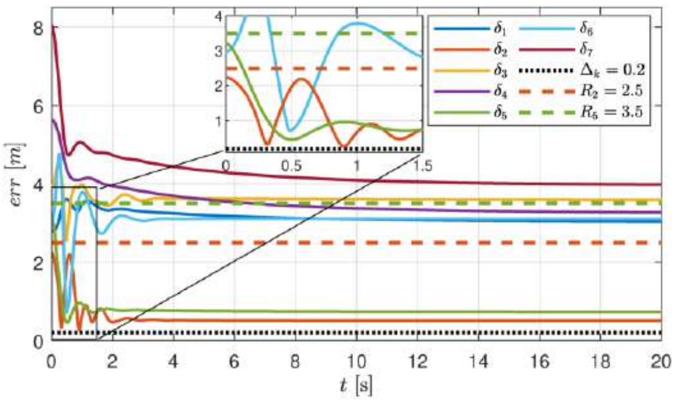

Fig. 5. Trajectories of the norm of inter-agent distances with control input (18). The black dashed line is the minimum distance constraint for a pair of end-effectors corresponding to each edge, and the red and green dashed lines are the maximum distance constraints for the edges e_2 and e_5, respectively.

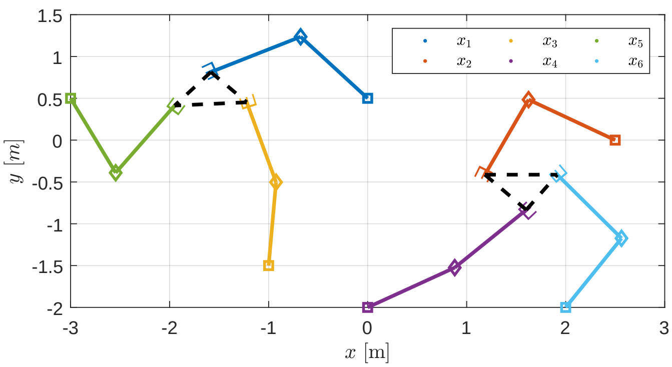

Fig. 6. Final positions of the manipulators and their end-effector.

depicted in Figure 4, and their final configuration is depicted in Figure 6[4]. Moreover, it is clear from Figure 5 that the minimum safety distance between each pair of end-effectors is respected. In addition, for the two cooperative edges, e_2 and e_5, the maximum distance between the end-effectors is also respected. Thus, collision avoidance and maximum distance maintenance among the manipulators' end-effectors are both guaranteed.

In a second run of simulations, we consider the system (1), where $d_i \neq 0$ and with the robust bipartite formation control law in (25). We take the same initial conditions as before. Let $k_{1_i} = 200$ and $k_{2_i} = 300$ for all $i \leq N$. The matrices in (3) of the exosystem generating the disturbance are given as

$$S_{M_i} = S_{E_i} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \; C_{M_i} = C_{E_i} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

The matrices of the internal model in (22) are given as

$$A_{M_i} = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \; A_{E_i} = \begin{bmatrix} 0 & \pi/2 \\ -\pi/2 & 0 \end{bmatrix},$$
$$B_{M_i} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \; B_{E_i} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}.$$

[4]A video of the simulation is available at: http://tinyurl.com/simulationRM.

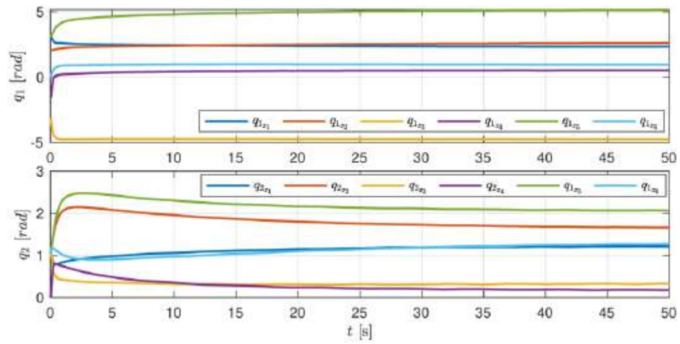

Fig. 7. Bipartite formation of system (1) with control input (25) on joint trajectories.

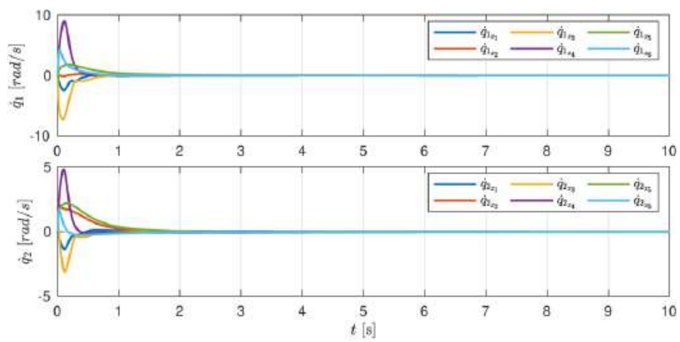

Fig. 8. Bipartite formation of system (1) with control input (25) on joint velocities.

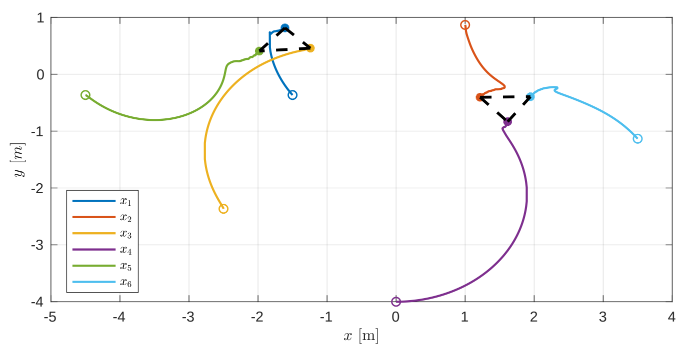

Fig. 9. Evolution of the manipulators' end-effector from the initial positions (o) to the final positions (*). Each group of end-effectors forms a triangle around the symmetric consensus points.

The joint positions and velocities are depicted in Figures 7 and 8, respectively, and all velocities converge to zero. The paths of each end-effector up to bipartite formation are depicted in Figure 9. Their final configuration is the same as in Figure 6. Moreover, it is clear from Figure 10 that collision avoidance and maximum distance maintenance are guaranteed among the manipulators' end-effectors.

VI. Conclusions

We addressed the problem of constrained bipartite formation of cooperative-competitive robot manipulators' end-effectors, modeled by Euler-Lagrange equations. We considered a structurally balanced and undirected signed graph. First, we presented a bipartite formation control law based on the

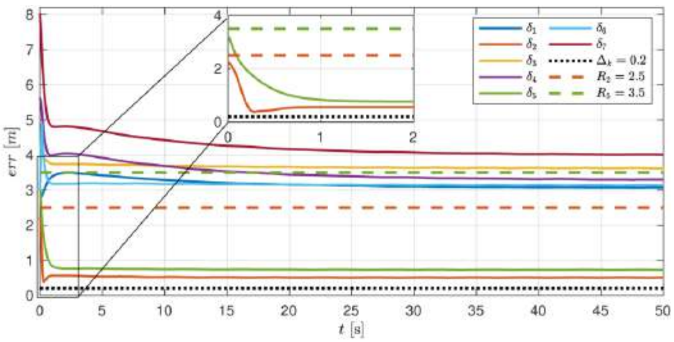

Fig. 10. Trajectories of the norm of inter-agent distances with control input (25). The black dashed line is the minimum distance constraint, and the red dashed line is the maximum distance constraint for end-effectors.

gradient of a barrier-Lyapunov function that guarantees that end-effectors do not collide and stay within the maximum distance imposed by the task requirements. Then, in order to deal with perturbed robot manipulators, we robustified our controller with an internal model-based approach to reject disturbances. We established the asymptotic stability of the bipartite formation manifold both in the absence and the presence of disturbance. Further research aims to extend these results to consider general directed signed networks, as well as guaranteeing collision avoidance among links.

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