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Observer-based Sensor Fault Tolerant Control with Prescribed Tracking Performance for a Class of Nonlinear Systems

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***Abstract*— In this note, a robust output feedback Fault-Tolerant Control (FTC) for a high-performance tracking problem of a Lipschitz nonlinear system under simultaneous sensor fault and disturbance is developed. The proposed scheme includes the design of an adaptive sliding mode observer which recovers the separation principle. A tangent-type barrier Lyapunov function is incorporated in the backstepping framework to maintain the system states in a prescribed performance bound. Moreover, the unknown estimation error is taken into account. Furthermore, the bounded initial condition assumption is relaxed by defining a time variable bound. The effectiveness of the proposed solution is numerically examined on a DC motor model.**

***Index Terms*— Adaptive Sliding Mode Observer, Barrier Lyapunov Function, Constrained Control, Prescribed Performance Bound, Sensor Fault, Tracking Control.**

I. Introduction

SAFETY of nonlinear systems has always been crucial and under a great deal of attention. The safety requirements are directly related to the capabilities of controllers as the non-robust and non-agile trajectory tracking performance might lead to safety violation [1, 2]. Maintaining the tracking error exactly at zero is often challenging for systems associated with unknown disturbances and faults [3, 4]. Constrained control using Barrier Lyapunov Functions (BLFs) [1, 4, 5] has been recently introduced as a promising solution. In contrast to conventional approaches, BLF-based Constrained Control (BLFCC) guarantees that the system trajectories never violate the Prescribed Performance Bound (PPB) [6].

The controller design is often based on the system states which are not necessarily all available from the measurement. Moreover, measurements can occasionally and inevitably include faults and disturbances. This inaccurate measurement leads to performance degradation or even worst, instability [7]. In this regard, Fault-Tolerant Control (FTC) schemes are suitable solutions [2, 5, 6, 8-11]. However, in practice, FTC methods, using only the available states are likely to fail [6]. So, observers are the most suitable solutions, among which Adaptive Sliding Mode Observers (ASMOs) have shown salient estimation capacity [8].

The ASMO design has been studied on a variety of systems [8, 9, 12-20]. In [12], a known bound on the time derivative of fault is assumed and the stability and convergence are proven with the assumption of zero initial estimation error, which is a restrictive assumption. In [8], the same exogenous unknown signal appears in both system dynamic and output measurement. In [8, 13], the so-called equivalent output injection is used to estimate the fault which is reliable only after the sliding surface is reached. This causes inaccuracy and high variance of the estimated fault. In [9], the common conditions required for fault estimation are relaxed for linear systems by equivalent output injection. Also, the fault and its time derivative are assumed to be bounded. In [14-16], some known bounds on disturbance and fault are assumed. In [17, 18], the ASMO estimation error is not taken into account in the FTC design which leads to a bidirectional robustness issue [8]. In [19], the ASMO is designed with a strong observability condition and repetitive differentiation of output is required. In [14] H_∞ optimization is used and all the ASMO and FTC gains are obtained in one-step optimization, which restricts design freedom and leads to an inevitable conservatism. In [21], non-faulty sensors are used to reconstruct the sensor faults, using the H_∞ optimization approach.

In the above-mentioned works, it is only guaranteed that the ASMO estimation error is bounded and ultimately approaches zero. Moreover, if the fault/disturbance effect is large, then the estimation error might be large during the transient period. Thus, PPB is not guaranteed during this period. Therefore, it is desirable to design the ASMO with a PPB. Most of the studies BLFCC assume that initial conditions lie in the PPB [1, 4-6, 11, 22]. This implies that a too-large bound must be selected for arbitrary initial conditions which are not useful in practice. Hence, it is beneficial to design the BLFCC such that it compensates for the effect of arbitrary initial conditions for a brief period of time, and thereafter the system trajectory is retained in a small PPB, i.e., vicinity of the desired trajectory.

Motivated by these considerations, in this note, an ASMO-based FTC scheme is designed for a Lipschitz nonlinear system. The ASMO is adopted to estimate the unknown states, and sensor faults and disturbances, recovering the separation principle. Then, Constrained FTC (CFTC) is designed using

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tan-type BLF. Finally, by constructing a time variable bound, the effect of arbitrary initial conditions is compensated. The main advantages of the proposed CFTC are as follows.

1) The ASMO design is decoupled from the FTC design, by recovering the separation principle, i.e., the ASMO estimation performance is separated from FTC performance.

2) The stability and PPB of the closed-loop performance are guaranteed, even in the transient period of ASMO by taking into account the unknown estimation error in CFTC design.

3) The bounded initial conditions assumption within the PPB is relaxed by constructing time variable bounds. This is to accommodate arbitrary initial conditions and to steer the system trajectory in a small PPB around the desired trajectory.

The rest of this note is organized as follows. In Section II, the system model and technical preliminaries are presented. In Section III, the ASMO structure and design algorithm are presented. In Section IV, the CFTC is designed, for arbitrary initial condition. The simulation results and conclusions are given in Sections V and VI, respectively. In this note, $\mathbb{R}$ and $\mathbb{C}$ represent real and complex number sets, respectively. $\|\bullet\|$ denotes the vector Euclidean norm and the matrix induced-norm. I_n and $0_{n\times p}$ represent the unitary matrix of size n, and zero matrix of size $n\times p$, respectively. $\mathcal{H}^-$ denotes the generalized inverse of $\mathcal{H}$ satisfying $\mathcal{H}\mathcal{H}^-\mathcal{H}=\mathcal{H}$.

II. Technical Preliminaries and System Description

Here, some technical preliminaries and nonlinear Lipschitz system with sensor fault and disturbance are presented.

A. Technical Preliminaries

Let us consider the following rectangular descriptor system.

$$\mathcal{E}\dot{x}=\mathcal{A}x+\mathcal{B}u, \qquad (1)$$

$$y=\mathcal{C}x, \qquad (2)$$

where, $x\in\mathbb{R}^n$ is state, $y\in\mathbb{R}^p$ is output, $u\in\mathbb{R}^m$ is input vectors, with matrices $\mathcal{E}\in\mathbb{R}^{q\times n}$, $\mathcal{A}\in\mathbb{R}^{q\times n}$, $\mathcal{B}\in\mathbb{R}^{q\times m}$, and $\mathcal{C}\in\mathbb{R}^{p\times n}$. As (1) is not square, it can have many solutions $x(t,x_0)$, with the initial states x_0. By using (2), we can reconstruct the impulse-free state of the system (1)-(2) [23]. The following lemmas are used in ASMO design [23].

Lemma 1: (1)-(2) are impulse observable if

$$rank\begin{bmatrix}\mathcal{E} & \mathcal{A}\\ 0_{p\times n} & \mathcal{C}\\ 0_{q\times n} & \mathcal{E}\end{bmatrix}=n+rank(\mathcal{E}). \qquad (3)$$

Lemma 2: (1)-(2) are infinitely observable if and only if (*iff*)

$$rank[\mathcal{E}^T \quad \mathcal{C}^T]^T=n. \qquad (4)$$

Lemma 3: (3) and (4) are equivalent if $rank(\mathcal{E})=q<n$.

Proof: Let's assume $\mathcal{E}=[I_q \quad 0_{q\times(n-q)}]$. Thus, matrices $\mathcal{A}$ and $\mathcal{C}$ can be partitioned according to $\mathcal{E}$ as $\mathcal{A}=[\mathcal{A}_1 \quad \mathcal{A}_2]$ and $\mathcal{C}=[\mathcal{C}_1 \quad \mathcal{C}_2]$, where $\mathcal{A}_1$ and $\mathcal{C}_1$ are the first q columns and, $\mathcal{A}_2$ and $\mathcal{C}_2$ are the last $n-q$ columns of $\mathcal{A}$ and $\mathcal{C}$, respectively. Condition (3) becomes

$$rank\begin{bmatrix}I_q & 0_{q\times(n-q)} & \mathcal{A}_1 & \mathcal{A}_2\\ 0_{p\times q} & 0_{p\times(n-q)} & \mathcal{C}_1 & \mathcal{C}_2\\ 0_{q\times q} & 0_{q\times(n-q)} & I_q & 0_{q\times(n-q)}\end{bmatrix}=2q+rank(\mathcal{C}_2)=n+q,$$

or equivalently, $q+rank(\mathcal{C}_2)=n$. Also, (4) becomes $rank\begin{bmatrix}I_q & 0_{q\times(n-q)}\\ \mathcal{C}_1 & \mathcal{C}_2\end{bmatrix}=q+rank(\mathcal{C}_2)=n$. ■

Lemma 4: The triple $(\mathcal{E},\mathcal{A},\mathcal{C})$ is detectable if (1)-(2) are impulse observable and $rank\begin{bmatrix}s\mathcal{E}-\mathcal{A}\\ \mathcal{C}\end{bmatrix}=n,\ \forall s\in\mathbb{C},\ Re(s)\geq 0$.

Lemma 5: The consistent equation $\mathcal{X}\mathcal{K}=\mathcal{W}$, with known matrices $\mathcal{K}$ and $\mathcal{W}$, has a general solution $\mathcal{X}=\mathcal{W}\mathcal{K}^- - \mathcal{Z}(I-\mathcal{K}\mathcal{K}^-)$, *iff* $rank[\mathcal{K}^T\ \mathcal{W}^T]^T=rank(\mathcal{K})$, where $\mathcal{Z}$ is an arbitrary matrix of appropriate dimensions.

The following lemmas are used in CFTC design [2].

Lemma 6: For any variable $\Upsilon(t)$ in $|\Upsilon|<1$, the inequality $tan(0.5\pi\Upsilon^2)\leq\pi\Upsilon^2 sec^2(0.5\pi\Upsilon^2)$ holds.

Lemma 7: For any variable $\chi(t)$ and positive constant κ, the inequality $0\leq|\chi|-\chi^2/\sqrt{\chi^2+\kappa^2}<c\kappa$ holds, where $c=\sqrt{0.5(5\sqrt{5}-11)}\approx 0.3$.

Finally, the following definitions and lemma are adopted in the analysis of the separation principle recovery [24].

Definition 1: A continuous function $\gamma^*:[0,a)\to[0,\infty)$ belongs to class $\mathcal{K}$ if it is strictly increasing and $\gamma^*(0)=0$. A continuous function $\beta^*:[0,a)\times[0,\infty)\to[0,\infty)$ belongs to class $\mathcal{KL}$ if for each fixed s, $\beta^*(r,s)$ belongs to class $\mathcal{K}$ with respect to r and, for each fixed r, $\beta^*(r,s)$ is decreasing with respect to s and $\beta^*(r,s)\to 0$ as $s\to\infty$.

Definition 2: $\dot{x}=f(x,u,t)$, where $f(.):\mathbb{R}^{\bar{n}}\times\mathbb{R}^{\bar{m}}\times\mathbb{R}_{\geq 0}\to\mathbb{R}^{\bar{n}}$ is Input-to-State Stable (ISS) if there exists a class of $\mathcal{KL}$ function β^* and class $\mathcal{K}$ function γ^* such that for any initial state $x(t_0)$ and any bounded input $u(t)$, the solution $x(t)$ exists for all $t\geq t_0$ and satisfies $\|x(t)\|\leq\beta^*(\|x(t_0)\|,t-t_0)+\gamma^*\left(\underset{t_0\leq\tau\leq t}{\text{Sup}}\|u(\tau)\|\right)$.

Lemma 8: Consider the following cascade system.

$$\dot{x}_1=f_1(x_1,x_2,t), \qquad (5)$$

$$\dot{x}_2=f_2(x_2,t), \qquad (6)$$

where $f_1(.):\mathbb{R}^{\bar{n}_1+\bar{n}_2}\times\mathbb{R}_{\geq 0}\to\mathbb{R}^{\bar{n}_1}$ and $f_2(.):\mathbb{R}^{\bar{n}_2}\times\mathbb{R}_{\geq 0}\to\mathbb{R}^{\bar{n}_2}$ are piecewise continuous in t and locally Lipschitz in $x=[x_1^T\ x_2^T]^T$. Then considering Definition 2, if the system (5), with x_2 as input, is ISS and the origin of (6) is Globally Uniformly Asymptotically Stable (GUAS). Then the origin of the cascade system (5) and (6) is GUAS.

B. System Description

Consider a Lipschitz nonlinear system with the sensor fault and measurement disturbance, as

$$\dot{x}(t)=Ax(t)+Bu(t)+W\phi(t,x,u), \qquad (7)$$

$$y(t)=Cx(t)+F_sf_s(t)+Dd(t),$$

with the states $x(t)\in\mathbb{R}^n$, the control input $u(t)\in\mathbb{R}^m$, the output $y(t)\in\mathbb{R}^p$, the fault $f_s(t)\in\mathbb{R}^k$, the disturbance $d(t)\in\mathbb{R}^q$, and nonlinear function $\phi(.)\in\mathbb{R}^n$, satisfying

$$\|\phi(t,x_1,u)-\phi(t,x_2,u)\|\leq\Lambda_\phi\|x_1-x_2\|, \qquad (8)$$

for $\forall(t,x_1,u),(t,x_2,u)\in\mathbb{R}_{\geq 0}\times\mathbb{R}^n\times\mathbb{R}^m$ and Lipschitz constant Λ_ϕ. Also, $A\in\mathbb{R}^{n\times n}, B\in\mathbb{R}^{n\times m}, C\in\mathbb{R}^{p\times n}, D\in\mathbb{R}^{p\times q}, F_s\in\mathbb{R}^{p\times k}$ and $W\in\mathbb{R}^{n\times n}$ are the known constant matrices. Note $\phi(t,x,u)$ it is not online computable as it is a function of the unknown $x(t)$. The system (7) is assumed to satisfy the following assumptions.

Assumption 1: The linear part of the system (7) is controllable, i.e., $rank([sI_n-A \quad B])=n\ \forall s\in\mathbb{C}$.

Assumption 2: The fault and external disturbance are norm-bounded, i.e., $\|f_s\|\leq f_{s0}$ and $\|d\|\leq d_0$, where, f_{s0} and d_0 are unknown positive constants. Also, f_s and d are differentiable.

Assumption 3: The matrix $D_s=[F_s\ D]\in\mathbb{R}^{p\times(k+q)}$ is full column rank, i.e., $rank([F_s\ D])=k+q$.

Assumption 4: The triple (A, C, D_s) is of minimum phase or $rank\begin{bmatrix} sI_n - A & 0_{n\times(k+q)} \\ C & D_s \end{bmatrix} = n + k + q, \forall s \in \mathbb{C}, Re(s) \geq 0.$

The main objective of this technical note is to design the CFTC $u(\hat{x}, x_d, k_{z_i})$ for the system (7) under Assumptions 1-4, such that i^{th} tracking error z_i is always retained in the PPB $\Phi_i = \{z_i: \|z_i\| < k_{z_i}\}$, where $\hat{x}$ is the state estimation and x_d is the desired trajectory. z_i and k_{z_i} are defined later.

Remark 1: Assumptions 1 and 2 are useful considering practical applications. f_s and d can be modelled as differentiable exogenous systems, i.e., differentiable [8, 14]. Assumption 3 is made to let the effects of f_s and d be distinguishable. Note, in contrast to [14-16], the given bounds in Assumption 2 are assumed to be unknown, and therefore, will not be used in the proposed CFTC structure.

III. ASMO Structure and Design Algorithm

Here, an ASMO structure is presented, with the design algorithm. In order to obtain accurate estimates of the states and faults, (7) is augmented into a descriptor form as

$$E\dot{\bar{x}} = \bar{A}\bar{x} + Bu + W\phi, \quad (9)$$

$$y = \bar{C}\bar{x}, \quad (10)$$

where $\bar{x} = [x^T f_s^T\ d^T]^T \in \mathbb{R}^{n+k+q}$. $E = [I_n\ 0_{n\times(k+q)}] \in \mathbb{R}^{n\times(n+k+q)}$, $\bar{A} = [A\ 0_{n\times(k+q)}] \in \mathbb{R}^{n\times(n+k+q)}$ and $\bar{C} = [C\ D_s] \in \mathbb{R}^{p\times(n+k+q)}$ are known constant matrices. It is worth noting that the descriptor form is used to avoid restrictive assumptions on the time derivative of exogenous signals [8]. Also, in (9) no extra dynamics are associated with extended state variables $\bar{x}$, i.e., in contrast to [12], where the time derivative of the fault signal is considered. As $EE^- = I_n$, (9) is consistent [23]. Assumption 4 is equivalent to conditions in Lemma 4, as $rank\begin{bmatrix} sE - \bar{A} \\ \bar{C} \end{bmatrix} = rank\begin{bmatrix} sI_n - A & 0_{n\times(k+q)} \\ C & D_s \end{bmatrix} = n + k + q, \forall s \in \mathbb{C}, Re(s) \geq 0$. Also, (3) and (4) are equivalent to Assumption 3, as $rank\begin{bmatrix} E \\ \bar{C} \end{bmatrix} = rank\begin{bmatrix} I_n & 0_{n\times(k+q)} \\ C & D_s \end{bmatrix} = n + k + q$ is equivalent to $rank(D_s) = k + q$.

Now, the ASMO structure is designed as

$$\dot{\xi} = N\xi + Hu + Jy + Fv, \quad (11)$$
$$\hat{\bar{x}} = \xi + Ly,$$
$$\hat{y} = \bar{C}\hat{\bar{x}},$$

where, $\xi \in \mathbb{R}^{n+k+q}$ is the ASMO state, $\hat{\bar{x}}$ is the estimate of $\bar{x}$, $N \in \mathbb{R}^{(n+k+q)\times(n+k+q)}$, $H \in \mathbb{R}^{(n+k+q)\times m}$, $J \in \mathbb{R}^{(n+k+q)\times p}$, $F \in \mathbb{R}^{(n+k+q)\times p}$, and $L \in \mathbb{R}^{(n+k+q)\times p}$ are design matrices. $v \in \mathbb{R}^p$ is a switching component to remove the effect of ϕ from the observer performance, designed as

$$v = \rho_v sign(e_y), \quad (12)$$

where $e_y = y - \hat{y}$, and ρ_v is a design scalar. Let $\epsilon = TE\bar{x} - \xi$, where $T \in \mathbb{R}^{(n+k+q)\times n}$ is a design matrix, and $e = \bar{x} - \hat{\bar{x}}$. Using (9)-(11), we obtain

$$\dot{\epsilon} = N\epsilon + (T\bar{A} - J\bar{C} - NTE)\bar{x} + (TB - H)u + TW\phi - Fv, \quad (13)$$

$$e = \epsilon + (I_{n+k+q} - TE - L\bar{C})\bar{x}. \quad (14)$$

Now, if

$$TE + L\bar{C} = I_{n+k+q}, \quad (15)$$
$$J\bar{C} + NTE = T\bar{A}, \quad (16)$$
$$H = TB, \quad (17)$$
$$N \text{ is Hurwitz}, \quad (18)$$

then, we obtain $\lim_{t\to\infty} e(t) = 0$ for $\phi = 0$ and $v = 0$. Equation (15) can be written as $[T\ L]\mathcal{K}_1 = \mathcal{O}_1$, where $\mathcal{K}_1 = [E^T\ \bar{C}^T]^T$ and $\mathcal{O}_1 = I_{n+k+q}$. Since D_s and $\mathcal{K}_1$ are full column rank, then $rank(\mathcal{K}_1) = rank\begin{bmatrix} \mathcal{K}_1 \\ \mathcal{O}_1 \end{bmatrix}$. So, it has the general solution as $[T\ L] = \mathcal{O}_1\mathcal{K}_1^- - Z(I_{n+p} - \mathcal{K}_1\mathcal{K}_1^-)$, using Lemma 5, where Z represents an arbitrary matrix of dimension $\mathbb{R}^{(n+k+q)\times(n+p)}$. Consequently, T and L are parameterized as

$$T = T_1 - ZT_2, \quad (19)$$
$$L = L_1 - ZL_2, \quad (20)$$

respectively, where $T_1 = \mathcal{O}_1\mathcal{K}_1^-\begin{bmatrix} I_n \\ 0_{p\times n} \end{bmatrix}$, $T_2 = (I_{n+p} - \mathcal{K}_1\mathcal{K}_1^-)\begin{bmatrix} I_n \\ 0_{p\times n} \end{bmatrix}$, $L_1 = \mathcal{O}_1\mathcal{K}_1^-\begin{bmatrix} 0_{n\times p} \\ I_p \end{bmatrix}$, and $L_2 = (I_{n+p} - \mathcal{K}_1\mathcal{K}_1^-)\begin{bmatrix} 0_{n\times p} \\ I_p \end{bmatrix}$. Now, (16) can be written

$$N(I_{n+k+q} - L\bar{C}) + J\bar{C} = T\bar{A}. \quad (21)$$

Let $K = J - NL \in \mathbb{R}^{(n+k+q)\times p}$. Then (21) becomes

$$N = T\bar{A} - K\bar{C}. \quad (22)$$

By using (19) we obtain

$$N = N_1 - YN_2. \quad (23)$$

$N_1 = T_1\bar{A}$, $N_2 = \begin{bmatrix} T_2\bar{A} \\ \bar{C} \end{bmatrix}$, and $Y = [Z\quad K] \in \mathbb{R}^{(n+k+q)\times(n+2p)}$ is a design matrix. As proven in Lemma 9, (18) is realized *iff* the pair (N_2, N_1) is detectable.

Lemma 9: There exists a parameter matrix Y, such that the matrix N is Hurwitz, *iff* Assumption 4 is satisfied.

Proof: This is equivalent to the detectability of (N_2, N_1) i.e., $rank\begin{bmatrix} sI_{n+k+q} - N_1 \\ N_2 \end{bmatrix} = n + k + q,\ \forall s \in \mathbb{C},\ Re(s) \geq 0$. On the other hand, $rank\begin{bmatrix} sE - \bar{A} \\ \bar{C} \end{bmatrix} = rank\begin{bmatrix} sE - \bar{A} \\ s\bar{C} \\ \bar{C} \end{bmatrix} = n + k + q$, which is equivalent to,

$$rank\begin{bmatrix} \mathcal{O}_1\mathcal{K}_1^- & 0_{(n+k+q)\times p} & 0_{(n+k+q)\times p} \\ I_{n+p} - \mathcal{K}_1\mathcal{K}_1^- & 0_{(n+p)\times p} & 0_{(n+p)\times p} \\ 0_{p\times(n+p)} & 0_{p\times p} & I_p \end{bmatrix}\begin{bmatrix} sE - \bar{A} \\ s\bar{C} \\ \bar{C} \end{bmatrix} =$$

$$rank\begin{bmatrix} sI_{n+k+q} - \mathcal{O}_1\mathcal{K}_1^-\begin{bmatrix} \bar{A} \\ 0_{p\times(n+k+q)} \end{bmatrix} \\ (I_{n+p} - \mathcal{K}_1\mathcal{K}_1^-)\begin{bmatrix} \bar{A} \\ 0_{p\times(n+k+q)} \end{bmatrix} \\ \bar{C} \end{bmatrix} = rank\begin{bmatrix} sI_{n+k+q} - N_1 \\ N_2 \end{bmatrix} =$$

$n + k + q,\ \forall s \in \mathbb{C},\ Re(s) \geq 0$. Accordingly, the necessary and sufficient condition for the existence of Y is Assumption 4. ■

By satisfying (15)-(17), (14) becomes

$$\dot{e} = Ne + TW\phi - Fv. \quad (24)$$

Now as $\phi \neq 0$ and $v \neq 0$, we design $F = P^{-1}\bar{C}^T$ and $TW = FQ$, where $P \in \mathbb{R}^{(n+k+q)\times(n+k+q)}$ and $Q \in \mathbb{R}^{p\times n}$ are design matrices. Consequently, the term $Q\phi$ is structurally matched with v, as it is shown in Lemma 10. Therefore, by proper design of v the effect of ϕ can be compensated on the observer estimation performance. Lemma 10 gives the conditions of the stability of the ASMO (9).

Lemma 10: Under conditions (15)-(17), the ASMO (11) is asymptotically stable and $\lim_{t\to\infty} e(t) = 0$ if there exist a symmetric Positive Definite (PD) matrix $P \in \mathbb{R}^{(n+k+q)\times(n+k+q)}$, a matrix $Q \in \mathbb{R}^{p\times n}$, and a positive constant γ such that,

$$N^T P + PN < -\gamma I_{n+k+q}, \quad (25)$$
$$2\|\bar{C}\|\|Q\|\Lambda_\phi + \alpha < \gamma, \quad (26)$$
$$PTW = \bar{C}^T Q, \quad (27)$$
with gain ρ_v designed as
$$\rho_v = \varepsille + \|Q\|\|\hat{\phi}\|, \quad (28)$$
where $\hat{\phi} = \phi(t, \hat{x}, u)$. α and ε are design positive constants. The matrix F is designed as $F = P^{-1}\bar{C}^T$.

Proof: Under conditions (15)-(17), the dynamics of $e(t)$ is given by (24). Now, choose a PD Lyapunov function V_o as
$$V_o = e^T P e, \quad (29)$$
where $P \in \mathbb{R}^{(n+k+q)\times(n+k+q)}$ is a symmetric PD design matrix. Taking the time derivative of (29) yields
$$\dot{V}_o = e^T(N^T P + PN)e + 2e^T P(TW\phi - Fv). \quad (30)$$
With matrix inequality (25), (30) can be rewritten as
$$\dot{V}_o \leq -\gamma\|e\|^2 + 2e^T P(TW\phi - Fv). \quad (31)$$
Considering $F = P^{-1}\bar{C}^T$ with (27), (31) can be rewritten as
$$\dot{V}_o \leq -\gamma\|e\|^2 + 2e^T\bar{C}^T(Q\phi - v). \quad (32)$$
It is readily seen that $Q\phi$ is matched with respect to the switching component v. Therefore, the effect of $Q\phi$ can be removed by proper design of v. Note that $e_y = \bar{C}e$. Thus, with gain ρ_v designed in (28), (32) can be rewritten as
$$\dot{V}_o \leq -\gamma\|e\|^2 - 2\|e_y\|\varepsilon + 2\|e_y\|\|Q\|(\|\phi\| - \|\hat{\phi}\|). \quad (33)$$
Using (26) and (8) with reverse triangle inequality, (33) yields,
$$\dot{V}_o \leq -\alpha\|e\|^2. \quad (34)$$
Thus, integration of (34) over $[0\ t]$ yields, $V_o(t) \leq V_o(0) - \alpha\int_0^t \|e(\varsigma)\|^2 d\varsigma$. Therefore, $\alpha\int_0^t \|e(\varsigma)\|^2 d\varsigma \leq V_o(0) < \infty$. It follows from Barbalat's Lemma that $\lim_{t\to\infty} e(t) = 0$ and the ASMO (11) is asymptotically stable. Accordingly, the sliding surface of $s = Q^T e_y$ is reachable and the ASMO (11) estimates the augmented system state $\bar{x}$ accurately. ■

Remark 2: The dynamics of $e(t)$ is governed by α in (34), i.e., the larger that α is selected, the faster $e(t)$ approaches zero. Considering (25) and (26), α replaces the eigenvalues of N away from the imaginary axis in the left half-plane. This gives design freedom such that with the appropriate design of α, the desirable behavior of the observer is achieved.

The ASMO matrices are parameterized in Theorem 1.

Theorem 1: Under Assumptions 2-4, there exists a parameter matrix Y such that the ASMO (11) is asymptotically stable and estimates the augmented system state $\bar{x}$ accurately if there exist a symmetric PD matrix $P \in \mathbb{R}^{(n+k+q)\times(n+k+q)}$, matrices $Q \in \mathbb{R}^{p\times n}$, $X_1 \in \mathbb{R}^{(n+k+q)\times(n+p)}$, $X_2 \in \mathbb{R}^{(n+k+q)\times p}$ and a positive constant γ for a given positive constant α, as a solution to the Optimization Problem 1 (OP1).
OP1: Minimize β subject to
$$\Psi_1 + \Psi_1^T \leq -\gamma I_{n+k+q}, \quad (35)$$
$$0 < \begin{bmatrix} aI_p & Q \\ Q^T & aI_n \end{bmatrix}, \quad (36)$$
$$0 < \begin{bmatrix} \beta I_{n+k+q} & \Psi_2 \\ \Psi_2^T & \beta I_n \end{bmatrix}, \quad (37)$$
where, $\Psi_1 = PN_1 - X_1T_2\bar{A} - X_2\bar{C}$, $\Psi_2 = PT_1W - X_1T_2W - \bar{C}^TQ$ and $a = 0.5(\gamma - \alpha)/\|\bar{C}\|\Lambda_\phi$. Then, matrices Z and K are then given by $Z = P^{-1}X_1$ and $K = P^{-1}X_2$.

Proof: Considering (23), inequality (25) can be written as
$$P(N_1 - ZT_2\bar{A} - K\bar{C}) + (N_1 - ZT_2\bar{A} - K\bar{C})^T P \leq -\gamma I_{n+k+q}. \quad (38)$$
Let $X_1 = PZ$ and $X_2 = PK$. Then one can easily obtain (35) from (38). The matrix inequality (26) can be written as $\|Q\| < a$, with $a = 0.5(\gamma - \alpha)/\|\bar{C}\|\Lambda_\phi$ which is a positive scalar. This is equivalent to $\lambda_{max}(Q^TQ) < a^2$ which can be written as $Q^TQ < a^2 I_n$, or equivalently $a^{-1}Q^TQ < aI_n$, since $a > 0$. Therefore, By using the Schur complement lemma, one can obtain (36). The matrix equality (27) is equivalent to $\Psi_2 = 0$. By using the method in [8], this equality can be equivalently converted into the minimizing β subject to inequality (37).■

Now, the ASMO design is summarized in Algorithm 1.

Algorithm 1: ASMO design procedure.

1) Compute the matrices T_1, T_2, N_1, and N_2.
2) For a given positive constant α, solve OP1 to obtain Y.
3) Compute $T = T_1 - ZT_2$, $L = L_1 - ZL_2$, $N = T\bar{A} - K\bar{C}$, $J = K + NL$, $H = TB$ and $F = P^{-1}\bar{C}^T$.

It is worth noting that via algorithm 1, the fault, disturbance and nonlinearity effects are removed from the ASMO performance (24). Also, the control u is decoupled by (17). In Section IV we show that in the CFTC design, the separation principle recovery is realized.

IV. CFTC DESIGN IN ADAPTIVE BACKSTEPPING FRAMEWORK

Here, the CFTC scheme is designed based on the ASMO estimations. Also, the bounded initial conditions assumption is relaxed. In the transient period of the ASMO, accurate estimation is not available. This, hence, affects inevitably the closed-loop performance. Therefore, the CFTC is designed such that it guarantees the performance is retained within PPB during this period, i.e. as long as the sliding surface of the ASMO (11) is not reached. After this period, the desired trajectory x_d is accurately tracked. It is assumed that x_d and its j^{th} time derivative, $j = 0, \ldots, n$, are bounded [2].

As the system (7) is controllable, it can be rearranged or transformed into the block-controllable strict feedback form [25]. So, the system (7) is rearranged into r subsystems, as
$$\dot{x}_i = A_i\bar{x}_i + B_i(x_{i+1} + \mathcal{W}_i\phi),\ i = 1, \ldots, r \quad (39)$$
where, $x_i \in \mathbb{R}^{n_i}$ are the new system state vectors, satisfying $x = [x_1{}^T \ldots x_r{}^T]^T$, $x_{r+1} = u$, x_1 is the system output, $\bar{x}_i = [x_1{}^T \ldots x_i{}^T]^T$, $rank(B_i) = n_i$, and $\sum_{i=1}^r n_i = n$. The matrices $\mathcal{W}_i$ are of appropriate dimensions [17]. Let $e_{x_i} = x_i - \hat{x}_i$, $e_{\bar{x}_i} = \bar{x}_i - \hat{\bar{x}}_i$, $e_\phi = \phi - \hat{\phi}$, and $z_i = \hat{x}_i - \alpha_{i-1}$ where $\hat{\phi} = \phi(t, \hat{x}, u)$ and α_{i-1} is a design virtual control, for $i = 1, \ldots, r$. Note that $z_0 = 0$, $\alpha_0 = x_d$. Define a BLF $V_{i,1}$ as
$$V_{i,1} = k_{z_i}^2 \tan(\Upsilon_i)/\pi, \quad (40)$$
for $i = 1, \ldots, r$, where $\Upsilon_i = 0.5\pi z_i^T z_i / k_{z_i}^2$, and k_{z_i} is the given bound on z_i, not to be violated. $V_{i,1}$ is PD and continuously differentiable in $\Phi_i = \{z_i : \|z_i\| < k_{z_i}\}$. Therefore, if $z_i(0) \in \Phi_i$ and $\dot{V}_{i,1}$ is bounded, then $z_i(t) \in \Phi_i$ for $\forall t \geq 0$, achieving the constrained tracking error.

Now the CFTC is designed via the following steps.

Step i $(i = 1, \ldots, r-1)$: Considering (39), the time derivative of z_i can be obtained as
$$\dot{z}_i = A_i\bar{x}_i + B_i(e_{x_{i+1}} + z_{i+1} + \alpha_i) + B_i\mathcal{W}_i\phi - \dot{e}_{x_i} - \dot{\alpha}_{i-1}, \quad (41)$$
hence, the first-time derivative of $V_{i,1}$, can be expressed as
$$\dot{V}_{i,1} = z_i^T(A_i\bar{x}_i + B_ie_{x_{i+1}} + B_iz_{i+1} + B_i\alpha_i + B_i\mathcal{W}_i\phi - \dot{e}_{x_i} - \dot{\alpha}_{i-1})\sec^2(\Upsilon_i). \quad (42)$$

Considering (42), the terms $\bar{x}_i$, $e_{x_{i+1}}$, ϕ, and $\dot{e}_{x_i}$ are contributing to $\dot{V}_{i,1}$. Therefore, by using the estimated states, the estimation error affects the closed-loop performance. Considering Lemma 10, the estimation error is bounded and asymptotically approaches zero. Accordingly, it is reasonable to assume $\left\|A_i e_{\bar{x}_i} + B_i e_{x_{i+1}} + B_i \mathcal{W}_i e_\phi - \dot{e}_{x_i}\right\| \le \rho_i$, where ρ_i is an unknown positive constant, estimated by $\hat{\rho}_i$ which is updated as

$$\dot{\hat{\rho}}_i = z_i^T z_i sec^4(\Upsilon_i)/\sqrt{z_i^T z_i sec^4(\Upsilon_i) + \gamma_{\rho_i}^2} - \mu_{\rho_i}\hat{\rho}_i, \quad (43)$$

where, γ_{ρ_i} and μ_{ρ_i} are positive design constants. Accordingly, we design the virtual control α_i as

$$\alpha_i = -B_i^{-1}\Big(A_i\hat{\bar{x}}_i + B_i\mathcal{W}_i\hat{\phi} + \gamma_i z_i - \dot{\alpha}_{i-1} + 0.5 z_i\, sec^2(\Upsilon_i) + \hat{\rho}_i z_i sec^2(\Upsilon_i)/\sqrt{z_i^T z_i sec^4(\Upsilon_i) + \gamma_{\rho_i}^2}\Big). \quad (44)$$

Consider the PD Lyapunov function V_i, as

$$V_i = V_{i,1} + 0.5\tilde{\rho}_i^2, \quad (45)$$

where $\tilde{\rho}_i$ is the estimation error of ρ_i, defined as $\tilde{\rho}_i = \hat{\rho}_i - \rho_i$. Considering (42)-(45), one can obtain that

$$\dot{V}_i \le \sum_{j=1}^{4}\mathcal{X}_{ji}, \quad (46)$$

where $\mathcal{X}_{1i} = -\gamma_i z_i^T z_i\, sec^2(\Upsilon_i)$, $\mathcal{X}_{2i} = z_i^T B_i z_{i+1}\, sec^2(\Upsilon_i) - 0.5 z_i^T z_i\, sec^4(\Upsilon_i)$, $\mathcal{X}_{3i} = \|z_i\|\rho_i\, sec^2(\Upsilon_i) - \rho_i z_i^T z_i sec^4(\Upsilon_i)/\sqrt{z_i^T z_i sec^4(\Upsilon_i) + \gamma_{\rho_i}^2}$, and $\mathcal{X}_{4i} = -\mu_{\rho_i}\hat{\rho}_i\tilde{\rho}_i$. Based on Lemmas 6 and 7, $\mathcal{X}_{1i} \le -\gamma_i k_{z_i}^2\, tan(\Upsilon_i)/\pi$, and $\mathcal{X}_{3i} < c\rho_i\gamma_{\rho_i}$. Using Young's inequality, $\mathcal{X}_{2i} \le 0.5\|B_i z_{i+1}\|^2$. Also, it is easy to obtain $\mathcal{X}_{4i} \le -0.5\mu_{\rho_i}\tilde{\rho}_i^2 + 0.5\mu_{\rho_i}\rho_i^2$. Therefore, (46) yields,

$$\dot{V}_i \le -\Xi_{1,i}V_i + \Xi_{2,i} + 0.5\|B_i z_{i+1}\|^2, \quad (47)$$

where $\Xi_{1,i} = \min\{\gamma_i, \mu_{\rho_i}\}$, $\Xi_{2,i} = c\rho_i\gamma_{\rho_i} + 0.5\mu_{\rho_i}\rho_i^2$. It is obvious that $\Xi_{1,i}$ and $\Xi_{2,i}$ are positive constants.

Step r: The time derivative of z_r can be obtained as

$$\dot{z}_r = A_r\bar{x}_r + B_r u + B_r\mathcal{W}_r\phi - \dot{e}_{x_r} - \dot{\alpha}_{r-1}. \quad (48)$$

Then, the first-time derivative of $V_{r,1}$, can be expressed as

$$\dot{V}_{r,1} = z_r^T\big(A_r\bar{x}_r + B_r u + B_r\mathcal{W}_r\phi - \dot{e}_{x_r} - \dot{\alpha}_{r-1}\big)\, sec^2(\Upsilon_r). \quad (49)$$

Considering (49), the terms $\bar{x}_r$, ϕ, and $\dot{e}_{x_r}$, are contributing to $\dot{V}_{r,1}$. As already discussed, it is reasonable to assume $\left\|A_r e_{\bar{x}_r} + B_r\mathcal{W}_r e_\phi - \dot{e}_{x_r}\right\| \le \rho_r$, where ρ_r is an unknown positive constant, estimated by $\hat{\rho}_r$ which is updated as

$$\dot{\hat{\rho}}_r = z_r^T z_r sec^4(\Upsilon_r)/\sqrt{z_r^T z_r sec^4(\Upsilon_r) + \gamma_{\rho_r}^2} - \mu_{\rho_r}\hat{\rho}_r, \quad (50)$$

where, γ_{ρ_r} and μ_{ρ_r} are positive design constants. Design u as

$$u = -B_r^{-1}\Big(A_r\hat{\bar{x}}_r + B_r\mathcal{W}_r\hat{\phi} + \gamma_r z_r - \dot{\alpha}_{r-1} + \hat{\rho}_r z_r sec^2(\Upsilon_r)/\sqrt{z_r^T z_r sec^4(\Upsilon_r) + \gamma_{\rho_r}^2}\Big). \quad (51)$$

Consider the PD Lyapunov function V_r, as

$$V_r = V_{r,1} + 0.5\tilde{\rho}_r^2, \quad (52)$$

where $\tilde{\rho}_r$ is the estimation error of ρ_r, defined as $\tilde{\rho}_r = \hat{\rho}_r - \rho_r$. Now, considering (49)-(52), one can obtain that

$$\dot{V}_r \le \sum_{j=1}^{3}\mathcal{X}_{jr}, \quad (53)$$

where $\mathcal{X}_{1r} = -\gamma_r z_r^T z_r\, sec^2(\Upsilon_r)$, $\mathcal{X}_{2r} = \|z_r\|\rho_r\, sec^2(\Upsilon_r) - \rho_r z_r^T z_r sec^4(\Upsilon_r)/\sqrt{z_r^T z_r sec^4(\Upsilon_r) + \gamma_{\rho_r}^2}$, and $\mathcal{X}_{3r} = -\mu_{\rho_r}\hat{\rho}_r\tilde{\rho}_r$. Based on Lemmas 6 and 7, one can obtain $\mathcal{X}_{1r} \le -\gamma_r k_{z_r}^2\, tan(\Upsilon_r)/\pi$, and $\mathcal{X}_{2r} < c\rho_r\gamma_{\rho_r}$. It is easy to show $\mathcal{X}_{3r} \le -0.5\mu_{\rho_r}\tilde{\rho}_r^2 + 0.5\mu_{\rho_r}\rho_r^2$. Accordingly, (53) yields,

$$\dot{V}_r \le -\Xi_{1,r}V_r + \Xi_{2,r}, \quad (54)$$

where $\Xi_{1,r} = \min\{\gamma_r, \mu_{\rho_r}\}$, $\Xi_{2,r} = c\rho_r\gamma_{\rho_r} + 0.5\mu_{\rho_r}\rho_r^2$. Obviously, $\Xi_{1,r}$ and $\Xi_{2,r}$ are positive constants.

Remark 3: One can see that by using the control law given in (51) the closed-loop system is not continuous and the classical solution to the obtained differential equation no longer exists. The main properties of the designed CFTC are given in the following theorem.

Theorem 2: Consider the system (7), rearranged as (39) and equipped with the ASMO (11) and discontinuous component (12), under Assumptions 1–4. If initial conditions satisfy $z_i(0) \in \Phi_i$ for $i = 1, \dots, r$, by using the control (51) and virtual control (44), with adaption laws (43) and (50), then, the obtained closed-loop system has a unique solution, defined for all $t \ge 0$, and the following objectives are achieved.

(i) All the states and signals are bounded,

(ii) The PPBs $z_i(t) \in \Phi_i$ are never violated for $\forall t \ge 0$,

(iii) After the sliding surface of ASMO (11) is reached, the tracking error z_1 converges to the compact set $\Phi_{z_1} = \left\{z_1: \|z_1\| \le \sqrt{2k_{z_1}^2 tan^{-1}\big(0.5\pi\|B_1\|^2\delta_1/\Xi_{1,1}k_{z_1}^2\big)/\pi}\right\}$, where $\delta_1 = \operatorname{Sup}_{\tau\in[0,t]}\|z_2(\tau)\|^2$ is unknown positive constant and by the proper choice γ_1 and μ_{ρ_1}, z_1 can be arbitrarily made small.

(iv) The recovery of the separation principle is realized.

Proof: First, we show the existence of a unique solution for the closed-loop system. Without loss of generality, assume that (39) can be obtained by a linear transformation from (7). According to (24), assume $\dot{e}_{x_i} = \aleph_i\dot{e}$. Consequently, by using (39), (41) and (51), the closed-loop system is obtained as,

$$\dot{z}_i = A_i e_{\bar{x}_i} + B_i e_{x_{i+1}} - \aleph_i N e - \aleph_i T W\phi + \aleph_i F v + B_i\mathcal{W}_i e_\phi + B_i z_{i+1} - \gamma_i z_i - 0.5 z_i\, sec^2(\Upsilon_i) - \hat{\rho}_i z_i sec^2(\Upsilon_i)/\sqrt{z_i^T z_i sec^4(\Upsilon_i) + \gamma_{\rho_i}^2}, \quad (55)$$

$$\dot{z}_r = A_r e_{x_r} + B_r\mathcal{W}_r e_\phi - \aleph_r N e - \aleph_r T W\phi + \aleph_r F v - \gamma_r z_r - \hat{\rho}_r z_r sec^2(\Upsilon_r)/\sqrt{z_r^T z_r sec^4(\Upsilon_r) + \gamma_{\rho_r}^2}. \quad (56)$$

It is obvious that the switching discontinuous component of v appears in the closed-loop dynamics. Therefore, following the procedure given in [26] Theorem 6.2, we show the existence and uniqueness of the solution of the system (55) and (56), as follows.

- The moments the sliding surface $\mathscr{s} = 0$ is not reached, at the continuity points (x, t), α_i and u, in (44) and (51), respectively, consist of a unique point and are continuously differentiable in t. Moreover, as proven in Lemma 10, ASMO (11) is asymptotically stable, and sliding motion is uniformly bounded and (24) is globally continuable on the right. Accordingly, Filippov regularization and inclusion [27], beyond the hyperplane $\mathscr{s} = 0$, the closed-loop system (55) and (56) have a unique solution.
- The moments the sliding surface $\mathscr{s} = 0$ is reached, to describe the behavior of the closed-loop system, one should use the equivalent control [26]. As proven in Lemma 10, the sliding surface $\mathscr{s} = 0$ is reached, and the observer trajectory stays on it. Also, the observer estimation error approaches zero when the sliding surface is reached. Thus, the equivalent controls are readily obtained as

$$\alpha_{i,eq} = -B_i^{-1}\Big(A_i\bar{x}_i + B_i\mathcal{W}_i\phi + \gamma_i z_i - \dot{\alpha}_{i-1} + 0.5 z_i\, sec^2(\Upsilon_i) + \hat{\rho}_i z_i sec^2(\Upsilon_i)/\sqrt{z_i^T z_i sec^4(\Upsilon_i) + \gamma_{\rho_i}^2}\Big), \quad (57)$$

$$u_{eq} = -B_r^{-1}\Big(A_r\bar{x}_r + B_r\mathcal{W}_r\phi + \gamma_r z_r - \dot{\alpha}_{r-1} + \hat{\rho}_r z_r sec^2(\Upsilon_r)/\sqrt{z_r^T z_r sec^4(\Upsilon_r) + \gamma_{\rho_r}^2}\Big), \quad (58)$$

for $i = 1, \ldots, r-1$. Obviously, the equivalent controls $\alpha_{i,eq}$ and u_{eq} are continuously differentiable in t. Therefore, taking the equivalent control into account, one can obtain

$$\dot{z}_i = B_i z_{i+1} - \gamma_i z_i - 0.5 z_i\, sec^2(\Upsilon_i) - \frac{\hat{\rho}_i z_i sec^2(\Upsilon_i)}{\sqrt{z_i^T z_i sec^4(\Upsilon_i) + \gamma_{\rho_i}^2}}, \quad (59)$$

$$\dot{z}_r = -\gamma_r z_r - \frac{\hat{\rho}_r z_r sec^2(\Upsilon_r)}{\sqrt{z_r^T z_r sec^4(\Upsilon_r) + \gamma_{\rho_r}^2}}. \quad (60)$$

Therefore, the solutions of (59) and (60) are unambiguously globally continuable on the right, when the overall system enters the discontinuity hyperplane $\mathcal{s} = 0$. Hence, the closed-loop system has a unique solution.

Now, achievement of Objectives (i)-(iv) is proven as follows.

(i) Considering (54), one can readily obtain that

$$V_r(t) \le \big(V_r(0) - \Xi_{2,r}/\Xi_{1,r}\big)e^{-\Xi_{1,r}t} + \Xi_{2,r}/\Xi_{1,r}. \quad (61)$$

Furthermore, considering $\Xi_{2,r}/\Xi_{1,r} \ge 0$ and $\lim_{t\to\infty} exp\,(-\Xi_{1,r}t) = 0$, (55) yields $V_r(t) \le \Delta_r$, where $\Delta_r = V_r(0) + \Xi_{2,r}/\Xi_{1,r}$. Accordingly, V_r is bounded. Considering (50), the boundedness of V_r implies the boundedness of $\tilde{\rho}_r$ and $V_{r,1}$, which in turn leads to the boundedness of $tan(\Upsilon_r)$ and consequently, z_r. Replacing $i = r-1$ in (47), one can obtain,

$$\dot{V}_{r-1}(t) \le -\Xi_{1,r-1}V_{r-1}(t) + \Xi_{2,r-1} + 0.5\|B_{r-1}z_r\|^2. \quad (62)$$

In a similar approach, one can readily obtain that,

$$V_{r-1}(t) \le \Xi_{2,r-1}/\Xi_{1,r-1} + \big(V_{r-1}(0) - \Xi_{2,r-1}/\Xi_{1,r-1}\big)e^{-\Xi_{1,r-1}t} + 0.5e^{-\Xi_{1,r-1}t}\int_0^t \|B_{r-1}z_r(\tau)\|^2 e^{\Xi_{1,r-1}\tau}d\tau. \quad (63)$$

Now, considering the boundedness of z_r yields $0.5e^{-\Xi_{1,r-1}t}\int_0^t \|B_{r-1}z_r(\tau)\|^2 e^{\Xi_{1,r-1}\tau}d\tau \le 0.5\|B_{r-1}\|^2\delta_{r-1}/\Xi_{1,r-1}$, where $\delta_{r-1} = \operatorname{Sup}_{\tau\in[0,t]} \|z_r(\tau)\|^2$. So, (63) can be rewritten as $V_{r-1}(t) \le \Delta_{r-1}$, where $\Delta_{r-1} = V_{r-1}(0) + \Xi_{2,r-1}/\Xi_{1,r-1} + 0.5\|B_{r-1}\|^2\delta_{r-1}/\Xi_{1,r-1}$. So, V_{r-1} is bounded. Considering (45), the boundedness of V_{r-1} implies the boundedness of $\tilde{\rho}_{r-1}$, $V_{r-1,1}$, $tan(\Upsilon_{r-1})$ and z_{r-1}. Similarly, for $i = r-2, \ldots, 1$, V_i, $\tilde{\rho}_i$, $V_{i,1}$, z_i, α_{i-1} and $\hat{\rho}_i$ are bounded, for $i = 1, \ldots, r$. Therefore, u is bounded.

(ii) Considering (40), the boundedness of $V_{i,1}$ implies $z_i(t) \in \Phi_i$ and thus PPB is never violated. Accordingly, the system output x_1 tracks x_d to the bounded set Φ_1.

(iii) After sliding surface of $\mathcal{s} = 0$ is achieved, the ASMO estimation error converges to zero. This implies that ρ_1 and $\Xi_{2,1}$ approach to zero. Therefore, it can be stated that $\|z_1\| \le \sqrt{2k_{z_1}^2 tan^{-1}\big(\pi\big(V_1(0)\,exp\big(-\Xi_{1,1}t\big) + 0.5\|B_1\|^2\delta_1/\Xi_{1,1}\big)/k_{z_1}^2\big)/\pi}$, where $\delta_1 = \operatorname{Sup}_{\tau\in[0,t]} \|z_2(\tau)\|^2$. By elapsing time, $exp\big(-\Xi_{1,1}t\big)$ converges to zero with convergence rate $\Xi_{1,1}$. Accordingly, it can be stated that ultimately z_1 converges to the compact set Φ_{z_1}. Note that $\Xi_{1,1} = \min\{\gamma_1, \mu_{\rho_1}\}$. Therefore, $\|z_1\|$ can be made arbitrarily small by the proper choice of γ_1 and μ_{ρ_1}.

(iv) Using (44), (51), (41) and (48), one can obtain

$$\dot{Z} = \Sigma_2\, Z + \Sigma_1(e), \quad (64)$$

where, $Z = [z_1^T, \ldots, z_r^T]^T$ is the tracking error vector, $\Sigma_1(e) = [\bar{\rho}_1^T(e), \ldots, \bar{\rho}_r^T(e)]^T$. $\bar{\rho}_i(e) = A_i e_{\bar{x}_i} + B_i e_{x_{i+1}} + B_i\mathcal{W}_i e_\phi - \dot{e}_{x_i}$ for $i = 1, \ldots, r-1$ and $\bar{\rho}_r(e) = A_r e_{\bar{x}_r} + B_r\mathcal{W}_r e_\phi - \dot{e}_{x_r}$ are unknown variables and a function of ASMO estimation error. Following Lemma 10, $\|\bar{\rho}_i(e)\| \le \rho_i$ where ρ_i is an unknown positive constant for $i = 1, \ldots, r$. Also,

$$\Sigma_2 = \begin{bmatrix} \varpi_1 & B_1 & 0 & \cdots & 0 \\ 0 & \varpi_2 & B_2 & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & \cdots & \cdots & \varpi_{r-1} & B_{r-1} \\ 0 & \cdots & \cdots & 0 & \varpi_r \end{bmatrix},$$

where $\varpi_i = -\gamma_i - 0.5\, sec^2(\Upsilon_i) + sec^2(\Upsilon_i)/\sqrt{z_i^T z_i sec^4(\Upsilon_i) + \gamma_{\rho_i}^2}$. Note $\|\Sigma_1\| \le \sqrt{{\rho_1}^2 + \cdots + {\rho_r}^2}$ and $\lim_{t\to\infty}\Sigma_1 = 0$, since $\lim_{t\to\infty} e(t) = 0$. Also, Σ_2 is only a function of the tracking error not the observer estimation error. Now, the estimation error dynamic (24) and tracking error dynamic (64) form a cascade system. As (64) is asymptotically stable for $e = 0$, i.e., $\Sigma_1(\mathrm{e}) = 0$, and from Lemma 10 the system (24) is asymptotically stable, then we can see that (64) is ISS. Then by using Lemma 8, we can conclude that the cascade system (24) and (64) is GUAS. This proves that the separation principle recovery is realized [24, 28]. ■

In Theorem 2, it is assumed that the initial conditions lie within the given PPB, i.e., $z_i(0) \in \Phi_i$ for $i = 1, \ldots, r$. This assumption has been made in many works [5, 7, 11, 29]. However, the initial conditions are not necessarily in the vicinity of the desired trajectory to satisfy this assumption. This requires the initial conditions to be manually set within the given PPB, which is not a practical approach. On the other hand, to satisfy this condition, in [4, 7, 22, 30] the authors have adopted too large constraints to cover the initial conditions, which is ineffective in practice. Therefore, to relax this condition systematically and automatically, the constraints are initially enlarged. Then, the constraints converge exponentially to the vicinity of the desired trajectory, in which the desired performance is achieved. In this manner, any initial condition is systematically handled and hence, the use of impractical large constraints is avoided. Also, the exponential convergence of the constraints to the given bounds offers design freedom which can be tuned based on the nature of the considered system. In this manner, the designed controller is initially tuned automatically to handle arbitrary initial conditions. Accordingly, k_{z_i} is constructed as follows:

$$k_{z_i}(t) = \bar{\gamma}_{z_i}e^{-\varrho t} + \bar{\delta}_{z_i}(t), \quad (65)$$

where $\bar{\gamma}_{z_i} = \|z_i(0)\|$ and ϱ is a positive design parameter; $\bar{\delta}_{z_i} \in \mathbb{R}_+$ is the design positive barrier, i.e. selected small distance between the desired trajectory and constraint, which can be constant or variable, hence satisfying $z_i(0) \in \Phi_i$.

Remark 4: In (65), the term $\bar{\gamma}_{z_i}\,exp(-\varrho t)$ disappears exponentially. This gives the design freedom, e.g., for slow dynamic systems, we choose $\varrho < 1$ to have enough convergence time and avoid large control effort. Also, $\bar{\delta}_{z_i}$ is a small bound in which the desired performance is achieved.

Remark 5: Considering Theorem 2, it is stated that α_{i-1} is bounded, i.e. $\|\alpha_{i-1}\| \le \bar{\alpha}_{i-1}$, $\bar{\alpha}_{i-1}$ can be obtained as the solution to an offline optimization problem [4]. Therefore, in (65), the positive barrier $\bar{\delta}_{z_i}$ is selected as $\bar{\delta}_{z_i} \geq \bar{\alpha}_{i-1}$.

In (65), k_{z_i} is time variable and thus $\dot{k}_{z_i}$ affects the closed-loop performance. This effect leads to extra terms in $\dot{V}_{i,1}$, $i = 1, \dots, r$. Therefore, (42) and (49) can be rewritten as

$$\dot{V}_{i,1} = z_i^T \dot{z}_i\, sec^2(\Upsilon_i) + 2\dot{k}_{z_i} k_{z_i}\, tan(\Upsilon_i)/\pi - z_i^T z_i \dot{k}_{z_i}\, sec^2(\Upsilon_i)/k_{z_i}. \tag{66}$$

To let the CFTC (51) be applicable, the effect of $2\dot{k}_{z_i} k_{z_i}\, tan(\Upsilon_i)/\pi - z_i^T z_i \dot{k}_{z_i}\, sec^2(\Upsilon_i)\,/k_{z_i}$ must be removed. So, α_i in (44), and control u in (51), are modified, as

$$\alpha_i = -B_i^{-1}\Big(A_i\hat{\bar{x}}_i + B_i\mathcal{W}_i\hat{\omega}_1 + \gamma_i z_i - \dot{\alpha}_{i-1} + 0.5 z_i\, sec^2(\Upsilon_i) + \hat{\rho}_i z_i sec^2(\Upsilon_i)\Big/\sqrt{z_i^T z_i sec^4(\Upsilon_i) + \gamma_{\rho_i}^2}\Big) + \vartheta_{i,1} + \vartheta_{i,2}, \tag{67}$$

$$u = -B_r^{-1}\Big(A_r\hat{\bar{x}}_r + B_r\mathcal{W}_r\hat{\omega}_1 + \gamma_r z_r - \dot{\alpha}_{r-1} + \hat{\rho}_r z_r sec^2(\Upsilon_r)\Big/\sqrt{z_r^T z_r sec^4(\Upsilon_r) + \gamma_{\rho_r}^2}\Big) + \vartheta_{r,1} + \vartheta_{r,2}, \tag{68}$$

respectively, where $\vartheta_{i,1} = -B_i^{-1} z_i k_{z_i}\dot{k}_{z_i}\, sin(2\Upsilon_i)/\pi z_i^T z_i$, and $\vartheta_{i,2} = B_i^{-1} z_i \dot{k}_{z_i}/k_{z_i}$, $i = 1, \dots, r$. The terms $\vartheta_{i,1}$ and $\vartheta_{i,2}$ remove the effect of $2\dot{k}_{z_i} k_{z_i}\, tan(\Upsilon_i)/\pi$ and $-z_i^T z_i \dot{k}_{z_i}\, sec^2(\Upsilon_i)\,/k_{z_i}$ in (66), respectively. The main properties of the modified CFTC are given in Theorem 3.

Theorem 3: Consider the system (7), rearranged as (39) and equipped with the ASMO (11), under Assumptions 1–4. With the PPB (65), by adopting the control (68) and virtual control (67), adaption laws (43) and (50), then, for any initial conditions, the closed-loop system a unique solution and the objectives (i)-(iv) in Theorem 2 are achieved.

Proof: Replacing (67) and (68) into $\dot{V}_{i,1}$, given in (66), $i = 1, \dots, r$, cancels out the extra terms $2\dot{k}_{z_i} k_{z_i}\, tan(\Upsilon_i)/\pi - z_i^T z_i \dot{k}_{z_i}\, sec^2(\Upsilon_i)\,/k_{z_i}$. The rest of the proof is similar to that of Theorem 2 and thus omitted. ■

Remark 6: Obviously $\lim_{\|z_i\|\to 0} \vartheta_{i,1} = 0$, $i = 1, \dots, r$, in (67) and (68), hence, the singularity will not happen. However, computers cannot evaluate the limit ambiguity $0/0$. To resolve this, in the implementation, the term $\vartheta_{i,1}$ in (67) and (68) are replaced with its equivalent, using the Maclaurin series, i.e. if $\|z_i\| \geq \varepsilon_i$, then $\vartheta_{i,1} = -B_i^{-1} z_i \dot{k}_{z_i} k_{z_i} \sin(2\Upsilon_i)/\pi z_i^T z_i$, otherwise, $\vartheta_{i,1} = -B_i^{-1} z_i \dot{k}_{z_i}/k_{z_i}$ and ε_i is a small positive design constant.

Remark 7: In contrast to [4, 5, 7], the proposed CFTC shows the excellent feature $\lim_{k_{z_i}\to\infty} V_{i,1} = 0.5 z_i^T z_i$, i.e. Lyapunov functions (40) capture the quadratic Lyapunov function as k_{z_i} is arbitrarily large enough. Accordingly, whenever no constraint is required on some states, the corresponding k_{z_i} is selected large enough and the proposed CFTC is still applicable, with no structural modification.

Remark 8: Note the terms $\dot{\alpha}_{i-1}$, $i = 2, \dots, r$, are readily computable [4]. This computational complexity can be easily avoided by the adoption of the dynamic surface control scheme, that is passing α_{i-1} through a low pass filter [2].

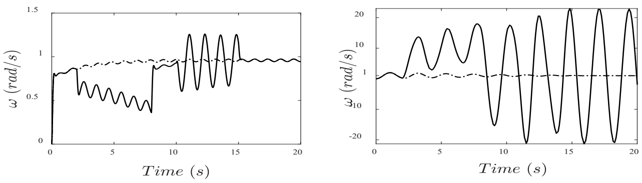
V. Simulation Results and Discussion

To investigate the performance of the proposed CFTC numerical simulations are conducted on a model of DC motor [8]. Consider a DC motor modelled by (7), where $x = [\omega \quad i_a]^T$, $u = v_a$ is the voltage, ω is the angular velocity, and i_a is the armature current. The output is $y = [\omega_s \quad i_{a,s}]^T$. Also, $A = \begin{bmatrix} -\frac{B_0}{J_i} & \frac{K_m}{J_i} \\ -\frac{K_v}{L_a} & -\frac{R_a}{L_a} \end{bmatrix}$, $B = \begin{bmatrix} 0 \\ \frac{1}{L_a} \end{bmatrix}$, $C = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, $F_s = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$, $D = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix}$, and $\phi = \begin{bmatrix} 0.1 sin(\omega + u) \\ 0.1 cos(i_a u) \end{bmatrix}$. Note that $B_0 = 0.3$ is the friction coefficient, $J_i = 0.1352\ (kgm^2)$ is the moment of inertia, $L_a = 0.05$ (H) is armature inductance, and $R_a = 1.2\ (\Omega)$ is armature resistance. $K_v = 0.6$ and $K_m = 0.6$ are motor constants [8]. $\omega_d = 1\ (rad/s)$ and $x_0 = [0 \quad 0]^T$. Also, $f_s(t) = \begin{cases} 0.3 + 0.1cos\big(2\pi(t-2)\big) + 0.05(t-2), & 2 < t < 8 \\ 0.4sin(5t), & 10 < \mathrm{t} < 15 \end{cases}$ and $d(t) = 0.2cos(6t) + 0.5sin(2\omega i_a)$. The design parameters of ASMO (11) are selected as $\alpha = 1$ and $\varepsilon = 1$. The ASMO matrices are obtained as

$$N = \begin{bmatrix} -975.4 & 895.4 & -97.3 & -82.3 \\ 82.3 & -1026.4 & 9.4 & -908.1 \\ -198.8 & -190.3 & -10.1 & -7 \\ -903.4 & -51.2 & -91.5 & -990.5 \end{bmatrix}, H = \begin{bmatrix} 0 \\ 20 \\ 200 \\ -20 \end{bmatrix},$$

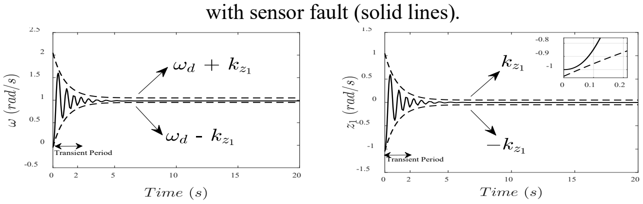
$$J = 10^{-6}\begin{bmatrix} -0.02 & 0.02 \\ 0.1 & -0.09 \\ 1.2 & -1.14 \\ -0.1 & 0.09 \end{bmatrix}, F = 10^{-4}\begin{bmatrix} 1.68 & 0 \\ 0 & 1.68 \\ 0.17 & 0 \\ 1.68 & 1.68 \end{bmatrix}, \text{and } L = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 10 & -10 \\ 0 & 1 \end{bmatrix}.$$

The CFTC parameters are $\gamma_1 = 5$, $\gamma_{\rho_r} = 0.1$, $\mu_{\rho_1} = 0.5$, $\varrho = 1$, $\bar{\delta}_{z_1} = 0.05$ and $\varepsilon_1 = 0.01$. To compare the results, a PI controller is considered, as $u(t) = P_c e_c(t) + I_c \int e_c(t)dt$, where $P_c = 4$, $I_c = 2$, and $e_c = \omega_d - \omega_s$. To further show the effectiveness of the proposed CTFC, a sliding mode controller (SMC), is used, as $u(t) = \big(-A_1\dot{\omega}_s - A_2\omega_s - A_3 i_{a,s} - A_5 - \ddot{\omega}_d - a_{sm}\dot{e}_{sm} - d_{sm}sign(s)\big)/A_4$, where, $A_1 = -B_0/J_i + 0.1\sin(\omega_s + u)$, $A_2 = -K_v K_m/L_a J_i$, $A_3 = -R_a K_m/L_a J_i$, $A_4 = -K_m/L_a J_i$, $A_5 = 0.1\cos(\omega_s + u) + 0.1K_m \cos(i_{a,s}u)/J_i$, $e_{sm} = \omega_s - \omega_d$, $s = \dot{e}_{sm} + a_{sm}e_{sm}$, with $a_{sm} = 8$ and $d_{sm} = 0.6$. The PI controller response is illustrated in Fig. 1 (a). In fault-free case, it is not able to accurately track ω_d, due to the presence of d and ϕ. On the other hand, when the sensor fault is applied, the PI performance is corrupted. The SMC tracks the desired trajectory in the fault-free situation accurately, as shown in Fig. 1 (b). In the presence of the fault, the SMC is unstable and not able to track the desired trajectory. The reason is the dependency on the computation of $\dot{e}_{sm}$ in SMC. Indeed, when the sensor fault is applied, the term of $\dot{e}_{sm}$ changes and the term $d_{sm}sign(s)$ is not able to compensate for the effect of $\dot{e}_{sm}$. The proposed CFTC response is shown in Figs. 2 (a) and (b), along with the PPB. The initial condition is set out of $\omega_d \pm \bar{\delta}_{z_1}$. Evidently, it can compensate for the effect of sensor faults and disturbance while keeping the transient and steady-state trajectory within the PPB. Also, it is clearly shown that $k_{z_1}(t)$ is automatically constructed in such a way to handle the arbitrary initial condition, i.e., out of $\omega_d \pm \bar{\delta}_{z_1}$, and to steer it toward the desired trajectory. It is worth noting that as soon as z_1 gets close to $\pm k_{z_1}(t)$, the control u, shown in Fig. 3 (a), is automatically increased to keep z_1 within PPB. This is a significant characteristic of the designed CFTC. Finally, as shown in Fig. 3 (b) the sensor fault is estimated accurately.



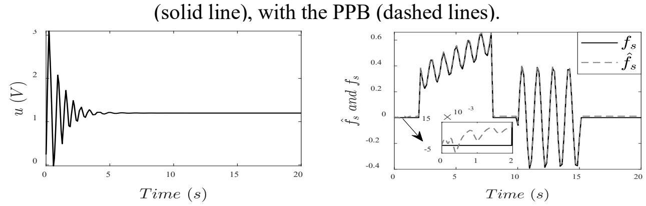
(a) (b)

Fig. 1. (a) PI and (b) SMC responses, without sensor fault (dashed line), and with sensor fault (solid lines).



(a) (b)

Fig. 2. (a) Angular velocity and (b) tracking error using proposed CFTC (solid line), with the PPB (dashed lines).



(a) (b)

Fig. 3. (a) Control input and (b) Sensor fault estimation.

VI. Conclusion

This note presented a fault-tolerant control design of a Lipschitz nonlinear system for tracking purposes with unknown states, under simultaneous sensor fault and disturbance, all of which were accurately estimated by designing an adaptive sliding mode observer. In this manner, the separation principle was recovered. By incorporating the tangent-type barrier function into the adaptive backstepping procedure, the given prescribed performance bound of the transient and steady-state tracking error was guaranteed. The effect of arbitrary initial conditions was compensated by constructing time variable bounds. Hence, the assumption of the bounded initial conditions was relaxed. Also, very large performance bounds were avoided. The designed control scheme compensates for the faults and disturbance effects, to guarantee safe and reliable operation, as it was evaluated by the numerical simulations.

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