A Novel Amplify-and-Forward Relay Channel Model for Mobile-to-Mobile Fading Channels Under Line-of-Sight Conditions

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Mobile-to-mobile (M2M) fading channels in cooperative networks can efficiently be modeled using the multiple scattering concept. In this chapter, we propose a new second-order scattering channel model referred to as the multiple-LOS second-order scattering (MLSS) channel model 1 for M2M fading channels in amplify-and-forward relay links under line-of-sight (LOS) conditions, where the received signal comprises only the single and double scattered components. In the proposed model, LOS components exist in the direct link between the source mobile station and the destination mobile station as well as the link via the mobile relay. Analytical expressions are derived for the probability density function (PDF) of the envelope and phase of M2M fading channels. It is shown mathematically that the proposed model includes as special cases double Rayleigh, double Rice, single-LOS double-scattering (SLDS), non-line-of-sight (NLOS) second-order scattering (NLSS), and single-LOS second-order scattering (SLSS) processes. The validity of all theoretical results is confirmed by simulations. Our novel M2M channel model is important for the investigation of the overall system performance in different M2M fading environments under LOS conditions.

1. Introduction

Among several emerging wireless technologies, M2M communications in cooperative networks has gained considerable attention in recent years. The driving force behind merging M2M communications and cooperative networks is its promise to provide a better link quality (diversity gain), an improved network range, and an overall increase in the system capacity. M2M cooperative wireless networks exploit the fact that single-antenna mobile stations cooperate with each other to share their antennas in order to form a virtual multiple-input multiple-output (MIMO) system in a multi-user scenario (Dohler, 2003). Thus, in such networks, cooperative diversity (Laneman et al., 2004; Sendonaris et al., 2003a;b) is achieved by relaying the signal transmitted from a mobile station to the final destination using other mobile stations in the network. However, to cope with the problems faced within the development of such systems, a solid knowledge of the underlying multipath fading channel characteristics is essential. Therefore, the aim of this chapter is to develop a flexible M2M fading channel model for relay-based cooperative networks and to analyze its statistical

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properties. This newly developed model would help communication system designers to investigate the overall performance of cooperative communication systems.

So far, M2M amplify-and-forward relay fading channels have been modeled only for some specific communication scenarios, either assuming NLOS or partial LOS propagation conditions. It has been shown in (Patel et al., 2006) that under NLOS propagation conditions, the M2M amplify-and-forward relay fading channel can be modeled as a double Rayleigh fading channel (Erceg et al., 1997; Kovacs et al., 2002). Motivated by the studies of double Rayleigh fading channels for keyhole channels (Almers et al., 2006), the so-called double Nakagami-m fading channel model has also been proposed in (Shin & Lee, 2004). Furthermore, in amplify-and-forward relay environments, the M2M fading channel under LOS conditions can be modeled as a double Rice fading channel (Talha & Pätzold, 2007a) and/or as an SLDS fading channel (Talha & Pätzold, 2007b). In addition, M2M fading channels in cooperative networks can efficiently be modeled using the multiple scattering concept (Andersen, 2002).

In multiple scattering radio propagation environments, the received signal comprises a sum of the single, double, or generally multiple scattered components. In this chapter, we model the amplify-and-forward relay fading channel as a second-order scattering channel, i.e., the sum of only the single and the double scattered components (Salo et al., 2006). The novelty of this approach is that we have extended the NLSS (Salo et al., 2006) and the SLSS (Salo et al., 2006) channel models to an MLSS channel model (Talha & Pätzold, 2008b) by incorporating multiple LOS components in all transmission links, i.e., in the direct link between the source mobile station and the destination mobile station as well as in the link via the mobile relay. Furthermore, an important feature of the proposed MLSS channel model for M2M fading channels is that it includes several other well-known channel models as special cases, e.g., the double Rayleigh model, the double Rice model, the SLDS model, the NLSS model, and the SLSS model. Here, we derive an analytical expression of the PDF of the MLSS process, along with the PDF of the corresponding phase process. The correctness of all theoretical results would be confirmed using a high-performance channel simulator. Furthermore, all presented results provide evidence that the statistics of MLSS fading channels are entirely different from the special cases discussed above.

The chapter is structured as follows: In Section 2, the reference model for the amplifyand-forward MLSS fading channel is developed. Section 3 deals with the analysis of the statistical properties of MLSS fading processes. Section 4 confirms the validity of the analytical expressions presented in Section 3 by simulations. Finally, concluding remarks are made in Section 5.

2. The MLSS Fading Channel

Under NLOS propagation conditions, the complex time-varying channel gain of the multiple scattering radio propagation channel proposed in (Andersen, 2002) can be written as

$$
\chi(t) = \alpha_1 \mu^{(1)}(t) + \alpha_2 \mu^{(2)}(t) \mu^{(3)}(t) + \alpha_3 \mu^{(4)}(t) \mu^{(5)}(t) \mu^{(6)}(t) + \cdots
$$
 (1)

where $\mu^{(i)}\left(t\right)\,\left(i=1,2,3,\ldots\right)$ is a zero-mean complex Gaussian process that represents the scattered component of the *i*th link and α ^{*i*} ($i = 1, 2, 3, \ldots$) is a real-valued constant that

determines the contribution of *i*th scattered component. In (1), Gaussian processes $\mu^{(i)}\left(t\right)$ are mutually independent. However, when the fading channel is modeled by taking into account only the first two terms of (1), the resulting channel is referred to as the NLSS channel (Salo et al., 2006). Here, we are presenting an extension of the NLSS channel to the MLSS channel by incorporating LOS components in a novel manner for M2M amplify-and-forward relay fading channels. The considered communication scenario determined by a source mobile station, a destination mobile station and mobile relay is shown in Fig. 1.

Mobile relay

Fig. 1. The propagation scenario behind MLSS fading channels.

Starting from (1), ignoring $\mu^{(i)}(t)$ $\forall i \geq 4$, and replacing $\mu^{(i)}(t)$ by $\mu^{(i)}_{\rho}(t)$ for $i = 1,2,3,$ results in

$$
\chi_{\rho}(t) = \mu_{\rho}^{(1)}(t) + A_{MR} \mu_{\rho}^{(2)}(t) \mu_{\rho}^{(3)}(t)
$$
 (2)

where $\alpha_1 = 1$ and $\alpha_2 = A_{MR}$. The quantity A_{MR} in (2) is referred to as the relay gain. Since we have assumed fixed gain relays in our system, it follows that A_{MR} is a real constant. Furthermore, in (2), $\mu_\rho^{(1)}$ (*t*), $\mu_\rho^{(2)}$ (*t*), and $\mu_\rho^{(3)}$ (*t*) are statistically independent non-zero-mean complex Gaussian processes, which model the individual M2M fading channel in the source mobile station to the mobile relay, the mobile relay to the destination mobile station, and the source mobile station to the destination mobile station links (see Fig. 1). Each complex Gaussian process $\mu^{(i)}_\rho(t)=\mu^{(i)}_{\rho_1}(t)+j\mu^{(i)}_{\rho_2}(t)$ represents the sum of the scattered component $\mu^{(i)}(t)$ and the LOS component $m^{(i)}$ (*t*), i.e., $\mu_{\rho}^{(i)}$ (*t*) = $\mu^{(i)}$ (*t*) + $m^{(i)}$ (*t*). The scattered component $\mu^{(i)}$ (*t*) is still modeled by a zero-mean complex Gaussian process $\mu^{(i)}(t) = \mu_1^{(i)}$ $j_1^{(i)}(t) + j\mu_2^{(i)}$ $2^{(t)}(t)$ with variance $2\sigma_i^2$. The LOS component $m^{(i)}(t) = \rho_i e^{j(2\pi f_{\rho_i}t + \theta_{\rho_i})}$ assumes a fixed amplitude ρ_i , a constant Doppler frequency f_{ρ_i} , and a constant phase θ_{ρ_i} for $i = 1, 2, 3$. Furthermore, it is obvious that

the product term $\mu_{\rho}^{(2)}(t)$ $\mu_{\rho}^{(3)}(t)$ in (2) is a non-zero-mean complex double Gaussian process, i.e., $\mu_{\rho}^{(2)}(t)\,\mu_{\rho}^{(3)}(t)=\varsigma_{\rho}(t)=\varsigma_{\rho_1}(t)+j\varsigma_{\rho_2}(t)$. Furthermore, $\varsigma_{\rho}(t)$ models the overall fading in the source mobile station to the destination mobile station link via the mobile relay. It should also be noted here that the relay gain A_{MR} would just scale the mean value and variance of the complex Gaussian process $\mu_{\rho}^{(3)}(t)$, i.e., $m^{(3)}(t) = E\{A_{\text{MR}}\mu_{\rho}^{(3)}(t)\} = \rho_{A_{\text{MR}}}e^{j(2\pi f_{\rho_3}t + \theta_{\rho_3})}$ where $\rho_{A_{MR}} = A_{MR} \rho_3$, and $2\sigma_{A_{MR}}^2 = \text{Var}\{A_{MR} \mu_{\rho}^{(3)}(t)\} = 2(A_{MR} \sigma_3)^2$. Finally, the overall fading process consisting of the direct link between the source mobile station and the destination mobile station as well as the source mobile station to the destination mobile station link via the mobile relay results in the complex process $\chi_{\rho}(t) = \chi_{\rho_1}(t) + j\chi_{\rho_2}(t)$ given in (2). The absolute value of $\chi_{\rho}(t)$ defines the MLSS process, i.e., $\Xi(t) = |\chi_{\rho}(t)|$. Furthermore, the argument of $\chi_{\rho}(t)$ introduces the phase process $\Theta(t)$, i.e., $\Theta(t) = \arg{\chi_{\rho}(t)}$.

3. Statistical Analysis Of the MLSS Fading Channel

In this section, we derive the analytical expressions for the statistical properties of MLSS fading channels introduced in Section 2. The starting point for the derivation of the analytical expression of the PDF of MLSS fading channels, as well as the PDF of the corresponding phase process is the computation of the joint PDF $p_{\chi_{\rho_1}\chi_{\rho_2}\chi_{\rho_1}\chi_{\rho_2}}(u_1, u_2, u_1, u_2; t)$ of the stochastic processes $\chi_{\rho_1}(t)$, $\chi_{\rho_2}(t)$, $\chi_{\rho_1}(t)$, and $\chi_{\rho_2}(t)$ at the same time *t*. Throughout this chapter, the overdot indicates the time derivative. Equation (2) shows that the joint PDF $p_{\chi_{\rho_1}\chi_{\rho_2}\chi_{\rho_1}\chi_{\rho_2}}(u_1,u_2,u_1,u_2;t)$ can be written in terms of a 4-dimensional (4D) convolution integral as

$$
p_{\chi_{\rho_1}\chi_{\rho_2}\chi_{\rho_1}\chi_{\rho_2}}(u_1, u_2, \dot{u}_1, \dot{u}_2; t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_{\mu_{\rho_1}^{(1)}\mu_{\rho_2}^{(1)}\mu_{\rho_1}^{(1)}} \mu_{\rho_2}^{(1)} \mu_{\rho_1}^{(1)}(u_1 - y_1, u_2 - y_2, \dot{u}_1 - \dot{y}_1, \dot{u}_2 - \dot{y}_2; t) p_{\zeta_{\rho_1}\zeta_{\rho_2}\zeta_{\rho_1}\zeta_{\rho_2}}(y_1, y_2, \dot{y}_1, \dot{y}_2; t) \, dy_2 \, dy_1 \qquad (3)
$$

where $p_{\mu_{\rho_1}^{(1)}\mu_{\rho_2}^{(1)}\mu_{\rho_1}^{(1)}}(u_1, u_2, u_1, u_2; t)$ is the joint PDF of the processes $\mu_{\rho_1}^{(1)}(t)$, $\mu_{\rho_2}^{(1)}(t)$, $\mu_{\rho_1}^{(1)}(t)$, and $\mu_{\rho_2}^{(1)}(t)$ at the same time t. Similarly, $p_{\varsigma_{\rho_1}\varsigma_{\rho_2}\varsigma_{\rho_1}\varsigma_{\rho_2}}(y_1, y_2, y_1, y_2; t)$ in (3), represents the joint PDF of the processes $\varsigma_{\rho_1}(t)$, $\varsigma_{\rho_2}(t)$, $\dot{\varsigma}_{\rho_1}(t)$, and $\dot{\varsigma}_{\rho_2}(t)$ at the same time *t*. It is worth mentioning here that the processes $\mu_{\rho_i}^{(1)}(t)$, $\mu_{\rho_i}^{(1)}(t)$, $\varsigma_{\rho_i}(t)$, and $\dot{\varsigma}_{\rho_i}(t)$ $(i = 1, 2)$ are uncorrelated in pairs. Furthermore, the process pairs $\{\mu_{\rho_i}^{(1)}(t), \mu_{\rho_i}^{(1)}(t)\}$ and $\{\varsigma_{\rho_i}(t), \dot{\varsigma}_{\rho_i}(t)\}\$ $(i=1,2)$ are statistically independent, which allows us to express $p_{\varsigma_{\rho_1}\varsigma_{\rho_2}\dot{\varsigma}_{\rho_1}\dot{\varsigma}_{\rho_2}\mu_{\rho_1}^{(1)}\mu_{\rho_2}^{(1)}\mu_{\rho_1}^{(1)}\mu_{\rho_2}^{(1)}}(y_1,y_2,y_1,y_2,u_1,u_2,u_1,u_2;t)$ as given in (3). The joint PDF $p_{\mu_{\rho_1}^{(1)}\mu_{\rho_2}^{(1)}\mu_{\rho_1}^{(1)}}(u_1, u_2, u_1, u_2; t)$ can be obtained using the multivariate Gaussian distribution (see, e.g., (Simon, 2002, Eq. (3.2))). The expression for the joint PDF $p_{\varsigma_{\rho_1}\varsigma_{\rho_2}\varsigma_{\rho_1}\varsigma_{\rho_2}}(y_1,y_2,y_1,y_2;t)$ is presented in (4), where the quantity β ^{*i*} (*i* = 2, 3) is the negative curvature of the autocorrelation function of the inphase and quadrature components of $\mu^{(i)}$ (t) $(i = 2, 3)$. Under isotropic scattering conditions, *βⁱ* (*i* = 2, 3) can be expressed as (Akki, 1994; Pätzold, 2002)

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*p*ΞΞΘ ˙ Θ ˙ *x*, *x* ˙, *θ*, ˙ *θ*; *t* = *x* 2 *e* − *^x* 2 2*σ* 2 1 *e* − *m* ˙ (2) 1 (*t*) 2 + *m* ˙ (2) 2 (*t*) 2 ²*β*² *e* − *m* ˙ (3) 1 (*t*) 2 + *m* ˙ (3) 2 (*t*) 2 ²*β*³ *e* − *m* ˙ (1) 1 (*t*) 2 + *m* ˙ (1) 2 2*β*1 (2*π*) 4 *σ* 2 1 *σ* 2 2 *σ* 2 *A*MR ∞ −∞ ∞ −∞ ∞ −∞ ∞ −∞ *dz*2 *dz*1 *dy*2 *dy*1 *e* − *z*1−*m* (3) 1 (*t*) 2 + *z*2−*m* (3) 2 (*t*) 2 2*σ* 2 *^A*MR *e* − *y*1+*m* (1) 1 (*t*) 2 + *y*2+*m* (1) 2 (*t*) 2 2*σ* 2 1 *β*1 *z* 2 ¹ + *z* 2 2 + *β*³ *y* 2 ¹ + *y* 2 2 + *β*² *z* 2 ¹ + *z* 2 2 2 *e r y*1+*m* (1) 1 (*t*) ×*e* − *y* 2 1 +*y* 2 2 −2 *y*2*m* (2) 2 (*t*)+*y*1*m* (2) 1 (*t*) *z*1−2 *y*2*m* (2) 1 (*t*)−*y*1*m* (2) 2 (*t*) *z*2+ *m* (2) 1 (*t*) 2 + *m* (2) 2 (*t*) 2 (*z* 2 1 +*z* 2 2) 2*σ* 2 2 (*z* 2 1 +*z* 2 2) *e ^β*3(*^y* 2 1 +*y* 2 2) *m* ˙ (1) 1 (*t*) 2*β*1 *^β*1(*^z* 2 1 +*z* 2 2)+*β*3(*^y* 2 1 + ×*e β* 2 2 (*z* 2 1 +*z* 2 2) 2 *β*3 *m* ˙ (1) 1 (*t*) 2 + *m* ˙ (1) 2 (*t*) 2 +*β*1 *m* ˙ (3) 1 (*t*) 2 + *m* ˙ (3) 2 (*t*) 2 +*β*3 *β*1 *m* ˙ (2) 1 (*t*) 2 + *m* ˙ (2) 2 (*t*) 2 [*^β*3(*^y* 2 1 +*y* 2 2)+*β*1(*^z* 2 1 2*β*3 *β*2 *β*1 *^β*1(*^z* 2 1 +*z* 2 2)+*β*3(*^y* 2 1 +*y* 2 2)+*β*2(*^z* 2 1 +*z* 2 2) 2 ×*e z* 2 2 *m* ˙ (2) 1 (*t*)*m* ˙ (3) 1 (*t*)*y*1+*m* ˙ (2) 2 (*t*)*m* ˙ (3) 2 (*t*)*y*1+*m* ˙ (2) 2 (*t*)*m* ˙ (3) 1 (*t*)*y*2−*m* ˙ (2) 1 (*t*)*m* ˙ (3) 2 (*t*)*y*2−*z*1 *m* ˙ (1) 1 (*t*)*m* ˙ (2) 1 (*t*)+*m* ˙ (1) 2 (*t*)*m* ˙ (2) 2 (*t*) −*z*2 *m* ˙ (1) 2 (*t*)*m* ˙ (2) 1 (*t*) *^β*1(*^z* 2 1 +*z* 2 2)+*β*3(*^y* 2 1 +*y* 2 2)+*β*2(*^z* 2 1 +*z* 2 2) 2 ×*e z*2 2*z*1 *m* ˙ (2) 2 (*t*)*m* ˙ (3) 1 (*t*)*y*1−*m* ˙ (2) 1 (*t*)*m* ˙ (3) 2 (*t*)*y*1−*m* ˙ (2) 1 (*t*)*m* ˙ (3) 1 (*t*)*y*2−*m* ˙ (2) 2 (*t*)*m* ˙ (3) 2 (*t*)*y*2 −*z* 2 1 *m* ˙ (1) 2 (*t*)*m* ˙ (2) 1 (*t*)−*m* ˙ (1) 1 (*t*)*m* ˙ (2) 2 (*t*) + *m* ˙ (1) 2 (*t*)*m* ˙ (3 1 *^β*1(*^z* 2 1 +*z* 2 2)+*β*3(*^y* 2 1 +*y* 2 2)+*β*2(*^z* 2 1 +*z* 2 2) 2 ×*e z* 2 1 *m* ˙ (2) 1 (*t*)*m* ˙ (3) 2 (*t*)*y*2−*m* ˙ (2) 1 (*t*)*m* ˙ (3) 1 (*t*)*y*1−*m* ˙ (2) 2 (*t*)*m* ˙ (3) 2 (*t*)*y*1−*m* ˙ (2) 2 (*t*)*m* ˙ (3) 1 (*t*)*y*2 −*z*1 *m* ˙ (1) 1 (*t*)*m* ˙ (3) 1 (*t*)*y*1+*m* ˙ (1) 2 (*t*)*m* ˙ (3) 2 (*t*)*y*1+*m* ˙ (1) 2 (*t*)*m* ˙ (3) 1 (*t ^β*1(*^z* 2 1 +*z* 2 2)+*β*3(*^y* 2 1 +*y* 2 2)+*β*2(*^z* 2 1 +*z* 2 2) 2 ×*e* − *z* 3 1 *m* ˙ (1) 1 (*t*)*m* ˙ (2) 1 (*t*)+*m* ˙ (1) 2 (*t*)*m* ˙ (2) 2 (*t*) + *m* ˙ (3) 1 (*t*)(*^y*² *z*1−*y*1 *^z*2)+*^m* ˙ (3) 2 (*t*)(*^y*¹ *z*1+*y*2 *^z*2)⁺ *m* ˙ (1) 2 (*t*)+*m* ˙ (2) 2 (*t*)*z*1+*m* ˙ (2) 1 (*t*)*z*2 (*z* 2 1 +*z* 2 2) (*x* ˙ *θ* cos *^β*1(*^z* 2 1 +*z* 2 2)+*β*3(*^y* 2 1 +*y* 2 2)+*β*2(*^z* 2 1 +*z* 2 2) 2 ×*e m* ˙ (3) 2 (*t*)(*^y*¹ *z*2−*y*2 *^z*1)+*^m* ˙ (3) 1 (*t*)(*^y*¹ *z*1+*y*2 *^z*2)⁺ *m* ˙ (1) 1 (*t*)+*m* ˙ (2) 1 (*t*)*z*1−*m* ˙ (2) 2 (*t*)*z*2 (*z* 2 1 +*z* 2 2) (*x* ˙cos *θ*−*x* ˙ *^θ* sin *^θ*)− *m* ˙ (1) 2 (*t*)*m* ˙ (3) 2 (*t*)+*m* ˙ (1) 1 (*t*)*m ^β*1(*^z* 2 1 +*z* 2 2)+*β*3(*^y* 2 1 +*y* 2 2)+*β*2(*^z* 2 1 +*z* 2 2) 2 ×*e* − (*z* 2 1 +*z* 2){*^x* ˙ ²+(*^x* ˙ *θ*) 2 } 2 *^β*1(*^z* 2 1 +*z* 2 2)+*β*3(*^y* 2 1 +*y* 2 2)+*β*2(*^z* 2 1 +*z* 2 2) 2 *e ^β*1(*^z* 2 1 +*z* 2 2) *m* ˙ (3) 1 (*t*) 2 + *m* ˙ (3) 2 (*t*) 2 2*β*3 *^β*1(*^z* 2 1 +*z* 2 2)+*β*3(*^y* 2 1 +*y* 2 2)+*β*2(*^z* 2 1 +*z* 2 2) 2 , *x* ≥ 0, |*x* ˙| < ∞, |*θ*| ≤ *π*

˙

$$
\beta_2 = 2 (\sigma_2 \pi)^2 \left(f_{\text{max}_1}^2 + f_{\text{max}_2}^2 \right) \tag{5a}
$$

$$
\beta_3 = 2 (\sigma_{A_{MR}} \pi)^2 \left(f_{max_2}^2 + f_{max_3}^2 \right).
$$
 (5b)

The symbols f_{max_1} , f_{max_2} , and f_{max_3} appearing in (5a) and (5b) denote the maximum Doppler frequency caused by the motion of the source mobile station, the mobile relay, and the destination mobile station, respectively. Substituting the joint PDF $p_{\mu_{\rho_1}^{(1)}\mu_{\rho_2}^{(1)}}(u_1, u_2, \dot{u}_1, \dot{u}_2)$ and (4) in (3), applying the concept of transformation of random variables (Papoulis & Pillai, 2002), and doing tedious algebraic manipulations, the joint PDF $p_{\Xi\dot\Xi\Theta\dot\Theta}\left(x,\dot x,\theta,\dot\theta;t\right)$ of the processes $\Xi(t)$, $\dot{\Xi}(t)$, $\Theta(t)$, and $\dot{\Theta}(t)$ can be derived. The resulting joint PDF $p_{\Xi\dot{\Xi}\Theta\dot{\Theta}}(x,\dot{x},\theta,\dot{\theta};t)$ is presented in (5), which is of fundamental importance, because it provides the basis for the computation of the PDF, the level-crossing rate (LCR), and the average duration of fades (ADF) of MLSS processes $\Xi(t)$ as well as the PDF of the phase process $\Theta(t)$. Using (5), the analytical expressions of the LCR and the ADF of the MLSS processes Ξ (*t*) have been derived in (Talha & Pätzold, 2008a). In (5), the quantity β_1 is given by $\beta_1 = 2 (\sigma_1 \pi)^2 (f_{\max_1}^2 + f_{\max_3}^2)$ (Akki, 1994; Pätzold, 2002).

3.1 PDF of the MLSS Process

The joint PDF $p_{\Xi\Theta}(x,\theta;t)$ of the MLSS process $\Xi(t)$ and the phase process $\Theta(t)$ can be obtained by solving the integrals over the joint PDF $p_{\Xi\dot{\Xi}\Theta\dot{\Theta}}(x,\dot{x},\theta,\dot{\theta};t)$ according to

$$
p_{\Xi\Theta}(x,\theta;t) = \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} p_{\Xi\dot{\Xi}\Theta\dot{\Theta}}(x,\dot{x},\theta,\dot{\theta};t) d\dot{\theta} d\dot{x}
$$
 (6)

for $x \geq 0$ and $|\theta| \leq \pi$. Solving (6) results in the following expression

$$
p_{\Xi\Theta}(x,\theta;t) = \frac{x e^{\frac{x^2}{2\sigma_1^2}}}{(2\pi)^2 \sigma_1^2 \sigma_2^2 \sigma_{A_{MR}}^2} \int_{0}^{\infty} \int_{0-\pi}^{\infty} \frac{d\theta}{\theta} e^{-\frac{(\omega/\nu)^2 + \rho_2^2}{2\sigma_2^2}} e^{-\frac{v^2 + \rho_{A_{MR}}^2}{2\sigma_{A_{MR}}^2}} e^{-\frac{g_1(\omega,\psi;t)}{2\sigma_1^2}} e^{-\frac{xg_3(\omega,\theta,\psi;t)}{\sigma_1}} \times I_0 \left(\sqrt{g_2(\omega,\nu,\psi;t)}\right) d\psi d\omega dv, x \ge 0, |\theta| \le \pi
$$
 (7)

where

$$
g_1(\omega, \psi; t) = \omega^2 + \rho_1^2 + 2\rho_1 \omega \cos(\psi - 2\pi f_{\rho_1} t - \theta_{\rho_1})
$$
 (8a)

$$
g_2(\omega, \nu, \psi; t) = \left(\frac{\rho_2 \omega}{\sigma_2^2 \nu}\right)^2 + \left(\frac{\rho_{A_{MR}} \nu}{\sigma_{A_{MR}}^2}\right)^2 + \frac{2\rho_2 \rho_{A_{MR}} \omega}{\sigma_2^2 \sigma_{A_{MR}}^2}
$$

\n
$$
\cos\left[\psi - 2\pi (f_{\rho_2} + f_{\rho_3})t - (\theta_{\rho_2} + \theta_{\rho_3})\right]
$$

\n
$$
g_3(\omega, \theta, \psi; t) = \frac{\rho_1 \cos\left(\theta - 2\pi f_{\rho_1}t - \theta_{\rho_1}\right) + \omega \cos\left(\theta - \psi\right)}{\sigma_1}.
$$
\n(8b)

In (7), $I_0(\cdot)$ is the zeroth-order modified Bessel function of the first kind (Gradshteyn & Ryzhik, 2000).

The PDF $p_{\Xi}(x)$ of the MLSS fading process $\Xi(t)$ can be obtained by integrating (7) over *θ* in the interval $[-π, π]$. Hence,

$$
p_{\Xi}(x) = \frac{x e^{-\frac{x^2}{2\sigma_1^2}}}{2\pi \sigma_1^2 \sigma_2^2 \sigma_{A_{\text{MR}}}^2} \int\limits_{0}^{\infty} \int\limits_{-\pi}^{\infty} dv \, d\omega \, \frac{\omega}{\nu} e^{-\frac{(\omega/\nu)^2 + \rho_2^2}{2\sigma_2^2}} e^{-\frac{v^2 + \rho_{A_{\text{MR}}}^2}{2\sigma_{A_{\text{MR}}}^2}}}{\int\limits_{-\pi}^{\pi} d\psi \, e^{-\frac{84(\omega,\psi)}{2\sigma_1^2}} I_0\left(\frac{x}{\sigma_1^2} \sqrt{g_4(\omega,\psi)}\right) I_0\left(\sqrt{g_5(\omega,\nu,\psi)}\right), x \ge 0 \tag{9}
$$

where

$$
g_4(\omega, \psi) = \omega^2 + \rho_1^2 + 2\rho_1 \omega \cos(\psi)
$$
 (10a)

$$
g_5(\omega,\nu,\psi) = \left(\frac{\rho_2 \omega}{\sigma_2^2 \nu}\right)^2 + \left(\frac{\rho_{A_{MR}} \nu}{\sigma_{A_{MR}}^2}\right)^2 + \frac{2\rho_2 \rho_{A_{MR}} \omega}{\sigma_2^2 \sigma_{A_{MR}}^2} \cos{(\psi)}.
$$
 (10b)

It is worth mentioning that the joint PDF $p_{\Xi\Theta}(x,\theta;t)$ in (6) is dependent on time *t*. Nevertheless, the PDF $p_{\Xi}(x)$ in (9) is independent of time *t* showing that MLSS processes $\Xi(t)$ are first order stationary. From the PDF $p_{\Xi}(x)$ of MLSS fading processes $\Xi(t)$, the following special cases can be obtained.

Substituting $A_{MR} = 1$, $\rho_1 = \rho_2 = \rho_3 = 0$, and taking the limit $\sigma_1^2 \rightarrow 0$ in (9), reduces the PDF of MLSS processes to the PDF of double Rayleigh processes (see, e.g., (Kovacs et al., 2002; Patel et al., 2006))

 $\overline{1}$

$$
p_{\Xi}(x)\left|_{\substack{A_{\rm MR}=1\\ \rho_1, \rho_2, \rho_3=0}} = \frac{x}{\sigma_2^2 \sigma_{A_{\rm MR}}^2} K_0\left(\frac{x}{\sigma_2 \sigma_{A_{\rm MR}}}\right), \quad x \ge 0.
$$
 (11)

For the special case when $A_{MR} = 1$, $\rho_1 = 0$, and $\sigma_1^2 \rightarrow 0$, the PDF of MLSS processes given in (9) reduces to the PDF of double Rice processes (Talha & Pätzold, 2007a)

$$
p_{\Xi}(x)\begin{vmatrix} 1 \\ \frac{\partial M}{\partial t} & 1 \end{vmatrix} = \frac{x}{\sigma_2^2 \sigma_{A_{MR}}^2} \int_{0}^{\infty} \frac{1}{\nu} e^{-\frac{(x/\nu)^2 + \rho_2^2}{2\sigma_2^2}} e^{-\frac{\nu^2 + \rho_{A_{MR}}^2}{2\sigma_{A_{MR}}^2}} I_0 \left(\frac{x\rho_2}{\nu \sigma_2^2}\right) I_0 \left(\frac{\nu \rho_{A_{MR}}}{\sigma_{A_{MR}}^2}\right) d\nu, \ x \ge 0. \quad (12)
$$

Similarly, substituting $A_{MR} = 1$, $\rho_2 = \rho_3 = 0$, and $\sigma_1^2 \to 0$ in (9) allows us to write the PDF of SLDS processes in the form (Talha & Pätzold, 2007b)

$$
p_{\Xi}(x)\begin{vmatrix} \frac{x}{\sigma_2^2 \sigma_{A_{\text{MR}}^2}^2} I_0\left(\frac{x}{\sigma_2 \sigma_{A_{\text{MR}}}}\right) K_0\left(\frac{\rho_1}{\sigma_2 \sigma_{A_{\text{MR}}}}\right), x < \rho_1\\ \frac{x}{\sigma_2^2 \sigma_{A_{\text{MR}}^2}^2} K_0\left(\frac{x}{\sigma_2 \sigma_{A_{\text{MR}}}}\right) I_0\left(\frac{\rho_1}{\sigma_2 \sigma_{A_{\text{MR}}}}\right), x \ge \rho_1. \end{vmatrix}
$$
(13)

The PDF of NLSS processes (see, e.g., (Salo et al., 2006)) can be derived from the PDF of MLSS processes by substituting $\rho_1 = \rho_2 = \rho_3 = 0$ in (9), i.e.,

$$
p_{\Xi}(x)\Big|_{A_{\rm MR}=1\atop \rho_1,\rho_2,\rho_3=0} = \frac{x e^{-\frac{x^2}{2\sigma_1^2}}}{\sigma_1^2 \sigma_2^2 \sigma_{A_{\rm MR}}^2} \int_0^\infty \omega e^{-\frac{\omega^2}{2\sigma_1^2}} K_0\left(\frac{\omega}{\sigma_2 \sigma_{A_{\rm MR}}}\right) I_0\left(\frac{\omega}{\sigma_1^2}x\right) d\omega, \ x \ge 0. \tag{14}
$$

Finally, solving (9) for $\rho_2 = \rho_3 = 0$, the PDF of SLSS processes (Salo et al., 2006) is obtained as

$$
p_{\Xi}(x)\Big|_{\substack{A_{\rm MR}=1\\ \rho_2,\rho_3=0}} = \frac{x}{2\pi\sigma_1^2\sigma_2^2\sigma_{A_{\rm MR}}^2}\int\limits_{0-\pi}^{\infty}\int\limits_{0}^{\pi}d\theta\,d\omega\,\omega\,e^{-\frac{\omega^2}{2\sigma_1^2}}e^{-\frac{g_5(x,\theta)}{2\sigma_1^2}}K_0\left(\frac{\omega}{\sigma_2\sigma_{A_{\rm MR}}}\right)I_0\left(\frac{\omega}{\sigma_1^2}\sqrt{g_6(x,\theta)}\right), x\geq 0\,(15)
$$

where

$$
g_6(x,\theta) = x^2 + \rho_1^2 - 2x\rho_1 \cos \theta.
$$
 (16)

3.2 PDF of the Phase Process

Integrating (7) over x in the interval $[0, \infty)$ results in the following expression for the PDF $p_{\Theta}(\theta; t)$ of the phase process $\Theta(t)$

$$
p_{\Theta}(\theta;t) = \int_{0}^{\infty} \int_{0}^{\infty} \int_{-\pi}^{\infty} \frac{e^{-\frac{(\omega/v)^2 + \rho_2^2}{2\sigma_2^2}} e^{-\frac{v^2 + \rho_{\text{AMR}}^2}{2\sigma_1^2}}}{(2\pi)^2 \sigma_2^2 \sigma_{\text{AMR}}^2} e^{-\frac{g_1(\omega,\psi;t)}{2\sigma_1^2}} I_0\left(\sqrt{g_2(\omega,\nu,\psi;t)}\right) \left[1 + \sqrt{\frac{\pi}{2}} g_3(\omega,\theta,\psi;t)\right] \times e^{\frac{1}{2}g_3^2(\omega,\theta,\psi;t)} \left\{1 + \Phi\left(\frac{g_3(\omega,\theta,\psi;t)}{\sqrt{2}}\right)\right\} d\psi d\omega dv, |\theta| \le \pi \quad (17)
$$

where $g_1(\cdot, \cdot; t)$, $g_2(\cdot, \cdot, \cdot; t)$, and $g_3(\cdot, \cdot, \cdot; t)$ are defined in (8a), (8b), and (8c), respectively. In (17), $\Phi(\cdot)$ represents the error function (Gradshteyn & Ryzhik, 2000, Eq. (8.250.1)). From (17), it is obvious that the phase process $\Theta(t)$ is not strict sense stationary because the density $p_{\Theta}(\theta; t)$ is a function of time *t*. This time dependency is due the Doppler frequency f_{ρ_i} of the LOS component $m^{(i)}(t)$ ($i = 1, 2, 3$). However, for the special case when *f*_{*ρi*} = 0, *ρi* \neq 0 (*i* = 1, 2, 3), the phase process Θ (*t*) becomes first order stationary.

The PDF $p_{\Theta}(\theta; t)$ of the phase process $\Theta(t)$ given in (17) reduces to the PDF of the phase process corresponding to double Rayleigh (Talha & Pätzold, 2007a), double Rice (Talha & Pätzold, 2007a), SLDS (Talha & Pätzold, 2007b), NLSS, and SLSS processes by selecting *ρ*¹ , ρ_2 , ρ_3 , and σ_1^2 in a similar fashion as described in Subsection 3.1.

4. Numerical Results

In this section, we will provide sufficient evidence to support the validity of the analytical expressions presented in Section 3 with the help of simulations. Furthermore, for the sake of completeness, a detailed comparison of the PDF $p_{\Xi}(x)$ of MLSS processes $\Xi(t)$ to that of the special cases mentioned above will be presented. Similarly, a comparison of the PDF $p_{\Theta}(\theta)$ of the phase process $\Theta(t)$ with other phase PDFs will also be presented. The concept of sum-of-sinusoids (Pätzold, 2002) was employed to simulate the underlying uncorrelated Gaussian noise processes of the overall MLSS process Ξ(*t*). The number of sinusoids required to simulate the inphase and quadrature components of Gaussian processes $\mu^{(i)}(t)$ was

selected to be 20 and 21, respectively. Furthermore, the simulation model parameters were computed using the generalized method of exact Doppler spread $(GMEDS₁)$ (Pätzold & Hogstad, 2006). The maximum Doppler frequencies, i.e., f_{max_1} , f_{max_2} , and f_{max_3} were set to 91 Hz, 75 Hz, and 110 Hz, respectively. Furthermore, for simplicity, the amplitudes of the three LOS components $ρ_1$, $ρ_2$, and $ρ_3$ are assumed to be equal, i.e., $ρ_1 = ρ_2 = ρ_3 = ρ$. The quantities σ_1^2 , σ_2^2 , σ_3^2 , and the relay gain A_{MR} are set to 1, unless stated otherwise.

The results presented in Figs. 2 and 3 show a good fitting between the analytical and simulation results. In Fig. 2, the PDF $p_{\Xi}(x)$ of MLSS processes $\Xi(t)$ is compared to that of classical Rayleigh, classical Rice, double Rayleigh, double Rice, SLDS, NLSS, and SLSS p processes for $ρ = 1$, where $f_{ρ_1}$, $f_{ρ_2}$, and $f_{ρ_3}$, were set to 166 Hz, 185 Hz, and 201 Hz, respectively. It can be observed that the maximum value of the PDF $p_{\Xi}(x)$ of MLSS processes $\Xi(t)$ is lower than that of all the other processes under consideration for the same value of *ρ*. However, the PDF $p_{\Xi}(x)$ of MLSS processes $\Xi(t)$ has a higher spread as compared to the spreads of the above mentioned processes for the same value of *ρ*. The same trend can be seen when different values of ρ_i ($i = 1, 2, 3$) are selected. Furthermore, increasing the value of the relay gain A_{MR} causes a decrease in the maximum value and an increase in the spread of the PDF of MLSS processes Ξ(*t*).

Fig. 2. A comparison of the PDF $p_{\Xi}(x)$ of the MLSS process $\Xi(t)$ with that of various other stochastic processes.

Figure 3 presents a comparison of the PDF $p_{\Theta}(\theta)$ of the phase process $\Theta(t)$ with that of the phase processes corresponding to the classical Rayleigh and double Rayleigh processes, the classical Rice and double Rice processes, the SLDS process, the NLSS process, and the SLSS process for $\rho = 1$. It should be noted that the results presented in Fig. 3 are valid for the case when f_{ρ_i} $(i = 1, 2, 3)$ is set to zero. It can be observed that the PDF of the SLDS phase

process has the highest peak. The PDF $p_{\Theta}(\theta)$ of the phase process $\Theta(t)$ follows the same trend in terms of the maximum value and the spread as that of the classical Rice process for the same value of ρ . Furthermore, it is interesting to note that for the phase process $\Theta(t)$ with ρ_i (*i* = 1, 2, 3) being selected as $\rho_1 = \rho_3 = 0.5$ and $\rho_2 = 1$, the PDF $p_{\Theta}(\theta)$ is almost the same as that of the double Rice process for $\rho = 1$. Figure 3 also shows the impact of the relay gain *A*_{MR} on the PDF $p_{\Theta}(\theta)$ of the phase process $\Theta(t)$.

Fig. 3. A comparison of the PDF $p_{\Theta}(\theta)$ of the phase process $\Theta(t)$ with that of various other various processes.

5. Conclusion

In this chapter, we have proposed a new flexible M2M amplify-and-forward relay fading channel model under LOS propagation conditions. The novelty in the model is that we have considered the LOS components in both transmission links, i.e., the direct link between the source mobile station and the destination mobile station as well as the link via the mobile relay. By analogy with multiple scattering radio propagation channels, we have developed the MLSS fading channel as a second-order scattering channel, where the received signal comprises the single and double scattered components. Furthermore, the flexibility of the MLSS fading channel model comes from the fact that it can be reduced to double Rayleigh, double Rice, SLDS, NLSS, and SLSS channel models under certain assumptions.

This chapter also presents a deep analysis of the statistical properties of MLSS fading channels. The statistical properties studied, include the PDF of MLSS processes along with the PDF of the corresponding phase processes. Accurate analytical expressions have been derived for the above mentioned statistical quantities. The accuracy and validity of the analytical expressions are confirmed by simulations. The excellent fitting of the theoretical and simulation results verifies the correctness of the derived analytical expressions. The presented results show that the statistical properties of MLSS channels are quite different from those of the processes embedded in the MLSS channel model as special cases. It is also evident from the illustrated results that the relay gain has a significant impact on the statistical properties of MLSS channels.

The statistics of a fading channel dictate the choice of the transmitter and the receiver techniques, including the detection, modulation, and coding schemes, etc. Therefore, the theoretical results presented in this chapter are quite useful for the designers of the physical layer of M2M cooperative wireless networks. Furthermore, the developed M2M channel model can be employed to investigate the overall system performance of M2M communication systems under both NLOS and LOS propagation conditions.

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In the last decades the restless evolution of information and communication technologies (ICT) brought to a deep transformation of our habits. The growth of the Internet and the advances in hardware and software implementations modified our way to communicate and to share information. In this book, an overview of the major issues faced today by researchers in the field of radio communications is given through 35 high quality chapters written by specialists working in universities and research centers all over the world. Various aspects will be deeply discussed: channel modeling, beamforming, multiple antennas, cooperative networks, opportunistic scheduling, advanced admission control, handover management, systems performance assessment, routing issues in mobility conditions, localization, web security. Advanced techniques for the radio resource management will be discussed both in single and multiple radio technologies; either in infrastructure, mesh or ad hoc networks.

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