A Simple Scheme for Delay-Tolerant Decode-and-Forward Based Cooperative Communication

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***Abstract*— In this paper, we study how to improve the performance of a decode-and-forward protocol based cooperative system over the delay constrained channels. We propose a simple transmission scheme, which makes the cooperative system tolerant of the delays caused by the poor synchronization of the relaying nodes. The proposed scheme is able to provide an improved coding gain in unsynchronized cooperative network as compared to the existing delay tolerant distributed space-time block codes.**

I. Introduction

Cooperative communications have several promising features to become a main technology in future wireless communications systems. It has been shown in [1], [2] that cooperative communications can avoid the difficulties of implementing actual antenna arrays and convert the single-input single-output (SISO) system into a virtual multiple-input multiple-output (MIMO) system. In this way, cooperation between the users allows them to exploit the diversity gain and other advantages of MIMO system at a SISO wireless network.

One of the recently discussed problems of the cooperative communication is the asynchronization of the relaying nodes. Due to the asynchronous transmissions a well designed structure of distributed space-time code is destroyed at the reception and it looses the diversity and coding gain. This point is throughly explained in [3].

In a delay constrained cooperative system, the data from different relays reach at the destination after different delays. It is shown in [4] that the received delayed distributed space-time block code loses diversity for most of the well-known codes. The first reported delay tolerant codes for asynchronous cooperative network were proposed in [3]. The work of [3] is generalized and refined in [5] to include full-diversity delay tolerant space-time trellis codes (STTC) of minimum constrained length. In [4], delay tolerant distributed space-time block codes based on threaded algebraic space-time (TAST) codes [6] are designed for unsynchronized cooperative network. The distributed TAST codes of [4] preserve the rank of the space-time codewords under arbitrary delays in the reception of different rows of the codeword matrix. A lattice based decoder is used for decoding of the delayed codeword, which is computationally more complex than the decoupled decoding. One important observation is that the TAST codes provide optimized coding gains for the *synchronized* MIMO system, where the codeword is received without any shift between the rows. In the asynchronous cooperative nework, it is not possible to obtain the optimal coding gain of the TAST codes because of the relative shifts between the rows of the received codeword.

In this paper, we propose a simple time-division multiple access (TDMA) based distributed transmission scheme for delay perturbed decode-and-forward based cooperative network which achieves full diversity under arbitrary delays. The proposed scheme also provides optimized coding gain despite of the delays. In addition, the proposed cooperative scheme performs better than same rate the existing delay tolerant distributed space-time code based cooperative scheme.

The rest of this paper is organized as follows: In Section II, the system model and assumptions are explained. Section III reviews the distributed space-time coding for delay constraint cooperative system. The proposed delay independent transmission scheme is discussed in Section IV. In Section V, the theoretical PEP performance analysis of the proposed system and code design criterion are provided. Section VI presents the simulation results and some conclusions are drawn in Section VII.

Notation: Upper (lower) bold face letters are used for matrices (row or column vector); $(\cdot)^T$, $(\cdot)^*$, and $(\cdot)^H$ are the transpose, conjugate, and Hermitian of a matrix or vector; $\otimes$ denotes Kronecker product; $K \times K$ identity matrix is shown as $\boldsymbol{I}_K$. Let $\boldsymbol{X}$ be a $v \times w$ matrix with $\boldsymbol{r}_1, \boldsymbol{r}_2, .., \boldsymbol{r}_v$ rows and $\boldsymbol{c}_1, \boldsymbol{c}_2, .., \boldsymbol{c}_w$ columns, then $\boldsymbol{X}(\boldsymbol{r}_m : \boldsymbol{r}_n, \boldsymbol{c}_p : \boldsymbol{c}_q)$ represents a matrix formed by $\boldsymbol{r}_m$ to $\boldsymbol{r}_n$, $1 \leq m < n, n \leq v$ sequential rows and $\boldsymbol{c}_p$ to $\boldsymbol{c}_q$, $1 \leq p < q, q \leq w$ sequential columns of $\boldsymbol{X}$, $\boldsymbol{X}(:, \boldsymbol{c}_p : \boldsymbol{c}_q)$ stands a matrix formed by $\boldsymbol{c}_p$ to $\boldsymbol{c}_q$, $1 \leq p < q, q \leq w$ sequential columns of $\boldsymbol{X}$, and $\boldsymbol{X}(\boldsymbol{r}_m : \boldsymbol{r}_n, :)$ denotes a matrix formed by $\boldsymbol{r}_m$ to $\boldsymbol{r}_n$, $1 \leq m < n, n \leq v$ sequential rows of $\boldsymbol{X}$, $\boldsymbol{X}(i, :)$ represents i-th row of $\boldsymbol{X}$; $\boldsymbol{0}_{a \times b}$ is an all zero matrix of size $a \times b$; $\boldsymbol{e}_c^T$ is row vector consisting 1 at c position and rest of all elements

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as 0; $\boldsymbol{D_w}$ is a diagonal matrix with the element of vector $\boldsymbol{w}$ in its diagonal.

II. System Model

We consider a cooperative communication system, which consists of one source (S), N relays $(\mathrm{R}_1, \mathrm{R}_2, \ldots, \mathrm{R}_N)$, and one destination (D) terminal as shown in Fig. 1. Each of them can either transmit or receive a signal at a time. There is no direct path between the source and the destination. The transmission of the data from S to D is furnished in two phases. In the first phase, S sequentially broadcasts the data to the relays. The relays decode the received data without any error. In the second phase, these $\mathrm{R}_i,\ i \in \{1, 2, .., N\}$ relays transmit the data to the destination. As the transmitters are distributed in different relaying terminals and there is no central local oscillator in contrast to a co-located antenna array, the cooperative network is asynchronous. As the relays are geographically distributed, there are relative timing errors between the different relays. It is assumed that these timing errors are integer multiples of the symbol duration and are perfectly known at the destination and unknown at the source and in the relays. The data transmitted by the relays is received by D with delay profile $\varDelta = (\delta_1, \delta_2, .., \delta_N)$, where δ_i denotes the relative delay of the signal received from the i-th relay as reference to the earliest received relay signal. The maximum relative delay is assumed to be $\delta_{\max}$. The channel of links are assumed to be Rayleigh distributed. Let us state the major assumptions as follows:

A1. $0 \leq \delta_1, \delta_2, .., \delta_N \leq \delta_{\max}$.

A2. The source and relays do not know the delay profile $\varDelta$ but they know $\delta_{\max}$ perfectly. However, the destination knows both the delay profile $\varDelta$ and maximum delay $\delta_{\max}$ perfectly.

A3. All channels are fast fading and can vary from one time interval to another.

A4. The destination knows the channel between the relays and itself perfectly. Similarly, each relay knows the channel between the source and itself perfectly.

A5. No errors occur in the chanels between the source and the relays.

The assumption **A3** is more general than the assumption of a block fading channel which remains constant over multiple time intervals. Therefore, a system designed under **A3** works for slow fading channels as well. This fact is used in Section VI where a flat fading channel is used for simulations. The assumption **A5** is made to convey the idea clearly. Alternatively, a group of N relays that decodes the data correctly can be chosen for cooperation in practice [3], [5], [4].

III. Distributed Space-Time Coding for Asynchronous Cooperative Network

It is clear from the assumptions **A1**-**A3** that even if all relays start transmitting all rows of a distributed space-time block code (STBC) simultaneously, different rows will reach D with different delays $\delta_i \leq \delta_{\max}, i \in \{1, 2, .., N\}$. If all relays continuously (without any pause between the transmission of

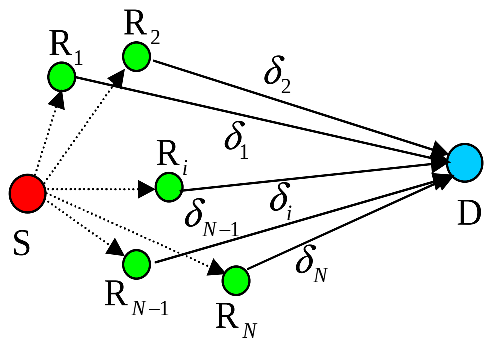

Fig. 1. Cooperative system with multiple relays.

two consecutive codewords) transmit the rows of different distributed STBC at different blocks, then the data of two consecutively transmitted STBC can be overlapped due to the timing errors. Therefore, in order to avoid this problem, the transmission of a STBC in a distributed manner from N asynchronous relays is performed by using simultaneous transmission and pause (STP) strategy as follows [4]: All relays start transmitting the assigned rows of the codeword simultaneously and as they don't know the values of the relative delays, therefore, each of them waits for $\delta_{\max}$ time intervals after the transmission of the codeword is finished. Due to the delays in the reception, an $N \times T$ transmitted STBC $\boldsymbol{S}$ is transformed into an $N \times (T + \delta_{\max})$ codeword at the receiver as follows:

$$\boldsymbol{S}^{\varDelta} = \begin{bmatrix} \mathbf{0}_{1\times\delta_1} & \boldsymbol{S}(1,:) & \mathbf{0}_{1\times(\delta_{\max}-\delta_1)} \\ \mathbf{0}_{1\times\delta_2} & \boldsymbol{S}(2,:) & \mathbf{0}_{1\times(\delta_{\max}-\delta_2)} \\ \vdots & \vdots & \vdots \\ \mathbf{0}_{1\times\delta_N} & \boldsymbol{S}(N,:) & \mathbf{0}_{1\times(\delta_{\max}-\delta_N)} \end{bmatrix}, \quad (1)$$

where a 0 represents no transmission. Let us consider the Alamouti code given as [7]

$$\boldsymbol{X} = \begin{bmatrix} x_1 & -x_2^* \\ x_2 & x_1^* \end{bmatrix}. \quad (2)$$

Due to the orthogonality of $\boldsymbol{X}$, i.e., $\boldsymbol{X}\boldsymbol{X}^{\mathcal{H}} = a\boldsymbol{I}_2$, where a is a constant which depends upon the signal constellaion, Alamouti code enables decoupled decoding of the data when used in a MIMO system with two transmit antennas as both rows of the transmitted codeword $\boldsymbol{X}$ are received aligned with each other. However, if Alamouti code $\boldsymbol{X}$ is transmitted in a distributed manner by the two asynchronous relays with $\delta_1 = 1, \delta_2 = 0, \delta_{\max} = 2$ following the STP strategy, then the received codeword will be of the following form:

$$\boldsymbol{X}^{\varDelta} = \begin{bmatrix} 0 & x_1 & -x_2^* & 0 \\ x_2 & x_1^* & 0 & 0 \end{bmatrix}. \quad (3)$$

It can be seen from (3) that $\boldsymbol{X}^{\varDelta}\left(\boldsymbol{X}^{\varDelta}\right)^{\mathcal{H}} \neq \boldsymbol{I}_2$, hence, the data *cannot* be decoded in a decoupled manner. In addition, as $\boldsymbol{X}^{\varDelta}$ is *not* unitary any more, $\boldsymbol{X}^{\varDelta}$ also looses coding gain as compared to $\boldsymbol{X}$. Further, it is shown in [4] that Alamouti code is not delay tolerant, i.e., $\boldsymbol{X}^{\varDelta}$ can be a low rank matrix under certain delay profile and it looses the diversity. The Alamouti code can be modified as follows to make it delay tolerant [4]:

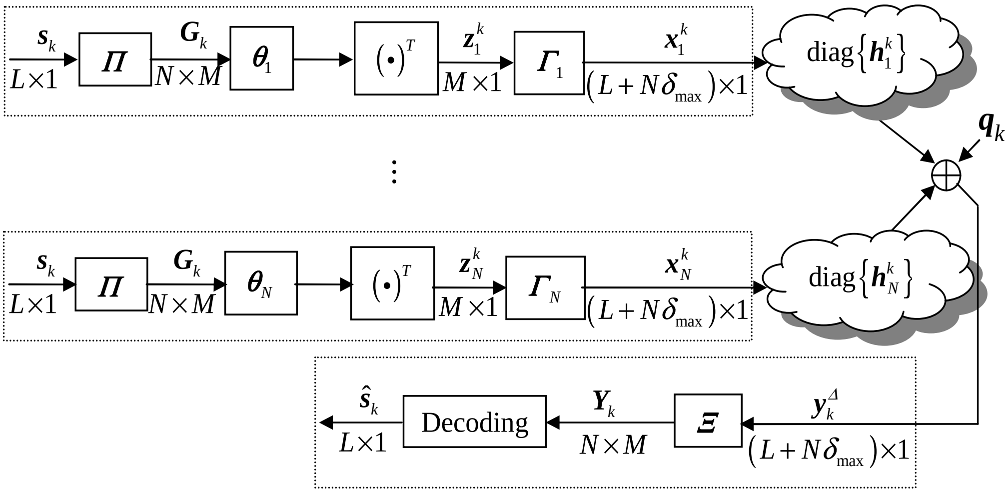

Fig. 2. Block diagram of the proposed delay independent cooperative transmission scheme.

$$\boldsymbol{X}^{\mathrm{DT}} = \begin{bmatrix} x_1 & -x_2^* & -x_2^* \\ x_2 & x_1^* & x_1^* \end{bmatrix}. \quad (4)$$

However, the delay tolerant Alamouti code of (4) is not orthogonal and the symbols x_1 and x_2 *cannot* be decoded linearly. Further, the modified delay tolerant structure is only chosen to provide maximum diversity only. Hence, optimized coding gain is not guaranteed for these codes. Let W symbols be encoded into the original STBC $\boldsymbol{S} \in \mathbb{C}^{N\times T}$, then it can be seen from (1) that by following the STP strategy it takes $T + \delta_{\max}$ time intervals for transmitting $\boldsymbol{S}$. Hence, the effective data rate in the asynchronous cooperative network is $W/(T + \delta_{\max})$, which is less than the data rate in a synchronized system W/T for which the STBC was originally designed. In [4], distributed TAST codes are designed for delay constrained asynchronous cooperative network to provide full diversity.

IV. Delay Independent Transmission Scheme

The block diagram of the proposed transmission scheme is shown in Fig. 2. Let $\boldsymbol{s}_k = \left[s_1^k, s_2^k, .., s_L^k\right]^T$, s_i^k which belongs to an arbitrary constellation $\mathcal{A}$, represents a data vector to be transmitted in k-th block/frame. It is assumed that $L = MN$, where M is a positive integer. As shown in Fig. 2 that at relay i the data vector $\boldsymbol{s}_k$ is passed through a grouping block $\boldsymbol{\Pi}$. The operation of $\boldsymbol{\Pi}$ can be compactly written as

$$\boldsymbol{G}_k = \left[\boldsymbol{\Pi}_1 \boldsymbol{s}_k, \boldsymbol{\Pi}_2 \boldsymbol{s}_k, .., \boldsymbol{\Pi}_M \boldsymbol{s}_k\right], \quad (5)$$

where $\boldsymbol{\Pi}_n$ is $N \times L$ matrix defined as $\boldsymbol{\Pi}_n = \boldsymbol{I}_L\left((n-1)N + 1 : nN; :\right)$ and $n \in \{1, 2, .., M\}$. The grouped data matrix is applied to the precoder vector $\boldsymbol{\theta}_i$, where $\boldsymbol{\theta}_i$ is $1 \times N$ row vector consisting precoding coefficients. The design of $\boldsymbol{\theta}_i$ will be discussed in Section V. The transposed precoded data vector $\boldsymbol{z}_i^k = (\boldsymbol{\theta}_i \boldsymbol{G}_k)^T$ is parsed through a $M \times (L + N\delta_{\max})$ multiplexing matrix $\boldsymbol{\Gamma}_i$ given as

$$\boldsymbol{\Gamma}_i = \left[\boldsymbol{0}_{M\times(i-1)(M+\delta_{\max})}, \boldsymbol{I}_M, \boldsymbol{0}_{M\times(N-i)M+(N-i+1)\delta_{\max}}\right]^T. \quad (6)$$

The parsed data $\breve{\boldsymbol{z}}_i^k = \boldsymbol{\Gamma}_i \boldsymbol{z}_i^k$ is transmitted through i-th relay (elements of column vector are transmitted sequentially). The data transmitted from the relays will undergo the delay profile and the destination receives delayed versions of them. The multiplexing matrix $\boldsymbol{\Gamma}_i$ introduces ordering in the transmissions from the relaying nodes. It ensures that the data transmitted from two consecutive relays is separated by $\delta_{\max}$ time intervals and each relay transmits for M non-overlapping time intervals and remain silent (transmitting a 0 signal means remaining silent here) in other times such that the received data at each time interval consists of data transmitted by one user only. We call this strategy as orthogonal transmission and pause (OTP). By using OTP, we are able to transmit L symbols in $L + N\delta_{\max}$ time intervals. Hence, the effective data rate is $L/\left(L + N\delta_{\max}\right)$ symbols per channel use (spcu), and

$$\begin{aligned} \lim_{L\to\infty} \frac{L}{L + N\delta_{\max}} &= \lim_{M\to\infty} \frac{MN}{MN + N\delta_{\max}} \\ &= \lim_{M\to\infty} \frac{M}{M + \delta_{\max}} = 1 \text{ spcu}, \end{aligned} \quad (7)$$

meaning that in OTP we can obtain the full data rate (1 spcu) if infinite delay in the decision is allowed. Nonetheless, if $N\delta_{\max} << L$, approximately full rate can be achieved with finite values of L. It is shown by the simulations that the proposed scheme achieves better coding gain than the same rate existing best delay tolerant distributed STBC [4]. Due to large delay in decoding the proposed scheme is not suitable for real-time applications, however, it is useful for applications like internet traffic. Nonetheless, the delay in decoding can be reduced by using higher order constellation and the proposed scheme can be used in practical real-time systems. For example, if $N = 2$ and $\delta_{\max} = 2$, then effective data rate for BPSK constellation with $M = 100$ will be $100/102 \approx 1$ spcu $= 1$ bit per channel use (bpcu). This means that the receiver needs to collect $MN + N\delta_{\max} = 204$ data samples before it starts decoding the data. However, the data rate of 1 bpcu can also be achieved by using QPSK constellation and $M = 2$. In this case, the delay in decoding is reduced to 8 time intervals. Apparantely, by using higher order constellation the delay in decoding $MN + N\delta_{\max}$ can be reduced considerably. However, this results into loss of the coding gain.

The received signal $\boldsymbol{y}_k^{\Delta} \in \mathbb{C}^{(L+N\delta_{\max})\times 1}$ can be written as

$$\boldsymbol{y}_k^{\Delta} = \sum_{i=1}^{N} \boldsymbol{D}_{\boldsymbol{h}_i^k} \breve{\boldsymbol{z}}_i^k + \boldsymbol{e}_k, \tag{8}$$

where

$$\tilde{\boldsymbol{h}}_i^k = \left[\boldsymbol{0}^T_{((i-1)M+\delta_i+(i-1)\delta_{\max})\times 1}, \left(\boldsymbol{h}_i^k\right)^T, \boldsymbol{0}^T_{(L-iM+(N-i+1)\delta_{\max}-\delta_i)\times 1}\right]^T,$$

is $(L+N\delta_{\max})\times 1$ column vector, which consists of the zero mean and Gaussian distributed complex channel coefficients $\boldsymbol{h}_i^k = \left[h_{i,1}^k, h_{i,2}^k, .., h_{i,M}^k\right]^T \in \mathbb{C}^{M\times 1}$ utilized during the transmission of the non-zero data from i-th relay in k-th block and $\boldsymbol{e}^k$ is $L+N\delta_{\max}\times 1$ column vector consisting additive white Gaussian noise (AWGN). At D, $\boldsymbol{y}_k^{\Delta}$ is passed through the grouping block $\boldsymbol{\Xi}$, which performs the following operation:

$$\boldsymbol{Y}_k^{\Delta} = \left[\boldsymbol{\Xi}_1\boldsymbol{y}_k^{\Delta}, \boldsymbol{\Xi}_2\boldsymbol{y}_k^{\Delta}, .., \boldsymbol{\Xi}_M\boldsymbol{y}_k^{\Delta}\right], \tag{9}$$

where $\boldsymbol{\Xi}_n$ is an $N\times(L+N\delta_{\max})$ matrix with i-th row given by $\boldsymbol{e}^T_{\delta_i+(i-1)(M+\delta_{\max})+n}$. Next, the group of symbols represented by $\boldsymbol{s}_n^k = \boldsymbol{\Pi}_n\boldsymbol{s}_k$ can be decoded from $\left(\boldsymbol{y}_n^k\right)^{\Delta} = \boldsymbol{\Xi}_n\boldsymbol{y}_k^{\Delta}$ as follows:

$$\hat{\boldsymbol{s}}_n^k = \arg\min_{\boldsymbol{s}_n^k\in\mathcal{A}} \left\|\left(\boldsymbol{y}_n^k\right)^{\Delta} - \boldsymbol{D}_{\breve{\boldsymbol{h}}_n^k}\boldsymbol{\Theta}\boldsymbol{s}_n^k\right\|^2, \tag{10}$$

where $\breve{\boldsymbol{h}}_n^k = \left[h_{1,n}^k, h_{2,n}^k, .., h_{N,n}^k\right]^T$, $h_{i,n}^k \in \boldsymbol{h}_i^k$, and $\boldsymbol{\Theta} = \left[\boldsymbol{\theta}_1^T, \boldsymbol{\theta}_2^T, .., \boldsymbol{\theta}_N^T\right]^T$.

V. Performance Analysis

Since $\boldsymbol{D}_{\breve{\boldsymbol{h}}_n^k}$ is a diagonal matrix and $\boldsymbol{\Theta}\boldsymbol{s}_n^k$ is a column vector, therefore, the ML metric (10) can be alternately written as

$$\hat{\boldsymbol{s}}_n^k = \arg\min_{\boldsymbol{s}_n^k\in\mathcal{A}} \left\|\left(\boldsymbol{y}_n^k\right)^{\Delta} - \boldsymbol{D}_{\boldsymbol{\Theta}\boldsymbol{s}_n^k}\breve{\boldsymbol{h}}_n^k\right\|^2. \tag{11}$$

The pair-wise error probability (PEP) in the case when $\boldsymbol{\Theta}\left(\boldsymbol{s}_n^k\right)^0$ is transmitted and $\boldsymbol{\Theta}\boldsymbol{s}_n^k$ is received, $\left(\boldsymbol{s}_n^k\right)^0 \neq \boldsymbol{s}_n^k$, can be given as [8, Theorem 4.2]

$$P\left(\left(\boldsymbol{s}_n^k\right)^0 \to \boldsymbol{s}_n^k \middle| \breve{\boldsymbol{h}}_n^k\right) = Q\left(\sqrt{\frac{\left\|\left(\boldsymbol{D}_{\boldsymbol{\Theta}(\boldsymbol{s}_n^k)^0} - \boldsymbol{D}_{\boldsymbol{\Theta}\boldsymbol{s}_n^k}\right)\breve{\boldsymbol{h}}_n^k\right\|}{2\sigma^2}}\right), \tag{12}$$

where σ^2 is the variance of the AWGN noise $\boldsymbol{e}_k$. A Chernoff bound over the probability of error of the proposed system can be obtained as [8]

$$P\left(\left(\boldsymbol{s}_n^k\right)^0 \to \boldsymbol{s}_n^k \middle| \breve{\boldsymbol{h}}_n^k\right) \leq \exp\left(-\frac{\left\|\left(\boldsymbol{D}_{\boldsymbol{\Theta}(\boldsymbol{s}_n^k)^0} - \boldsymbol{D}_{\boldsymbol{\Theta}\boldsymbol{s}_n^k}\right)\breve{\boldsymbol{h}}_n^k\right\|}{4\sigma^2}\right). \tag{13}$$

As $\breve{\boldsymbol{h}}_n^k \sim \mathcal{NC}\left(0, \rho^2\boldsymbol{I}_N\right)$, where ρ^2 is the transmit power, therefore, averaging (13) over $\breve{\boldsymbol{h}}_n^k$ the following upper bound of PEP can be obtained:

$$\mathbb{E}_{\breve{\boldsymbol{h}}_n^k}\left[P\left(\left(\boldsymbol{s}_n^k\right)^0 \to \boldsymbol{s}_n^k\right)\right] \leq \left|\boldsymbol{I}_N + \frac{\rho^2}{4\sigma^2}\left(\boldsymbol{D}_{\boldsymbol{\Theta}(\boldsymbol{s}_n^k)^0} - \boldsymbol{D}_{\boldsymbol{\Theta}\boldsymbol{s}_n^k}\right)\left(\boldsymbol{D}_{\boldsymbol{\Theta}(\boldsymbol{s}_n^k)^0} - \boldsymbol{D}_{\boldsymbol{\Theta}\boldsymbol{s}_n^k}\right)^H\right|^{-1}. \tag{14}$$

By using the inequalities $|\boldsymbol{A}| \leq |\boldsymbol{I}+\boldsymbol{A}|$ and equivalently $|\boldsymbol{A}|^{-1} \geq |\boldsymbol{I}+\boldsymbol{A}|^{-1}$, of a positive definite matrix $\boldsymbol{A}$ [8, Eq. (A.4.26)], we can further upper bound the PEP as follows:

$$\mathbb{E}_{\breve{\boldsymbol{h}}_n^k}\left[P\left(\left(\boldsymbol{s}_n^k\right)^0 \to \boldsymbol{s}_n^k\right)\right] \leq \left|\left(\boldsymbol{D}_{\boldsymbol{\Theta}(\boldsymbol{s}_n^k)^0} - \boldsymbol{D}_{\boldsymbol{\Theta}\boldsymbol{s}_n^k}\right)\left(\boldsymbol{D}_{\boldsymbol{\Theta}(\boldsymbol{s}_n^k)^0} - \boldsymbol{D}_{\boldsymbol{\Theta}\boldsymbol{s}_n^k}\right)^H\right|^{-1}\left(\frac{\rho^2}{4\sigma^2}\right)^{-N}. \tag{15}$$

It can be seen from (15) that the proposed scheme achieves full diversity N.

A. Code Design

From (14) the following conditions can be pointed out, which must be satisfied by $\boldsymbol{\Theta}$:

- In order to obtain full diversity, $\boldsymbol{D}_{\boldsymbol{\Theta}(\boldsymbol{s}_n^k)^0} - \boldsymbol{D}_{\boldsymbol{\Theta}\boldsymbol{s}_n^k} = \boldsymbol{D}_{\boldsymbol{\Theta}\left((\boldsymbol{s}_n^k)^0 - \boldsymbol{s}_n^k\right)}$ must be full rank matrix, i.e., $\boldsymbol{\Theta}\left(\boldsymbol{s}_n^k\right)^0$ should be different for all N elements from all possible $\boldsymbol{\Theta}\boldsymbol{s}_n^k$ provided that $\left(\boldsymbol{s}_n^k\right)^0 \neq \boldsymbol{s}_n^k$.
- $\boldsymbol{\Theta}$ must be chosen to maximize the coding gain, i.e., $\left|\boldsymbol{D}_{\boldsymbol{\Theta}\left((\boldsymbol{s}_n^k)^0 - \boldsymbol{s}_n^k\right)}\boldsymbol{D}^H_{\boldsymbol{\Theta}\left((\boldsymbol{s}_n^k)^0 - \boldsymbol{s}_n^k\right)}\right|^{1/N}$.
- In order to satisfy the average power constraint $\mathbb{E}\left[\text{Tr}\left\{\boldsymbol{D}_{\boldsymbol{\Theta}\boldsymbol{s}_n^k}\boldsymbol{D}^H_{\boldsymbol{\Theta}\boldsymbol{s}_n^k}\right\}\right] = \mathbb{E}\left[\text{Tr}\left\{\boldsymbol{s}_n^k\left(\boldsymbol{s}_n^k\right)^H\right\}\right] = N$, it must be ensured that $\text{Tr}\left\{\boldsymbol{\Theta}\boldsymbol{\Theta}^H\right\} = N$.

The optimization problem can be expressed as follows:

$$\max_{\boldsymbol{\Theta}|\text{Tr}\left\{\boldsymbol{\Theta}\boldsymbol{\Theta}^H\right\}=N} \;\min_{(\boldsymbol{s}_n^k)^0\neq\boldsymbol{s}_n^k} \Pi_{n=1}^N \left|\boldsymbol{\theta}_n\left(\left(\boldsymbol{s}_n^k\right)^0 - \boldsymbol{s}_n^k\right)\right|^{2/N}. \tag{16}$$

It can be seen from (16) that a closed form solution of $\boldsymbol{\Theta}$ is difficult to find. However, an optimized value of $\boldsymbol{\Theta}$ can be found by numerical methods. The optimization problem (16) can be seen as linear constellation precoder (LCP) design [9, Eq. (7)]. It is shown in [9] that $\boldsymbol{\Theta}$ can be chosen to be a unitary or non-unitary matrix. However, a unitary LCP preserves the distances among the the N-dimensional constellation points by introducing rotational angles. In [9], [10], LCP-A and LCP-B precoders are developed based on linear algebraic constructions. It is shown in [9, Table I], that unitary LPC-A provides better coding gain than LPC-B precoder. Further, it is shown in [9] that unitary LCP-A precoder is a Vandermonde matrix given by

$$\boldsymbol{\Theta} = \frac{1}{\sqrt{N}}\begin{bmatrix} 1 & \beta_0 & \cdots & \beta_0^{N-1} \\ 1 & \beta_1 & \cdots & \beta_1^{N-1} \\ \vdots & \vdots & \ddots & \vdots \\ 1 & \beta_N & \dots & \beta_N^{N-1} \end{bmatrix}, \tag{17}$$

where $\beta_l = e^{\jmath\left(\frac{2\pi l}{N}+\frac{\pi}{2N}\right)}$, $l = 0, 1, .., N-1$. Therefore, we can use Vandermonde matrix (17) for deciding $\boldsymbol{\Theta}$ and $\boldsymbol{\theta}_i, i \in \{1, 2, .., N\}$ is chosen as corresponding row of (17).

VI. Simulation Results

The simulations are performed with BPSK constellation, cooperative system with two asynchronous relays $N = 2$, $\delta_1 = 2$, $\delta_2 = 0$, and $\delta_{\max} = 3$. The channel between the relays and the destination is assumed Rayleigh block fading, which remains constant over six consecutive time intervals. In Fig. 3 we show the SER versus SNR performance of previously proposed delay tolerant TAST (DTTAST) codes [4] and the proposed OTP scheme. A 2×3 DTTAST code is given as [4, Eq. (17)]

$$\boldsymbol{X}^{\mathrm{DT}} = \begin{bmatrix} x_1 & \phi y_2 & \phi y_3 \\ \phi y_1 & x_2 & x_3 \end{bmatrix}, \quad (18)$$

where $[x_1, x_2, x_3]^T = \boldsymbol{\Phi}[s_1, s_2, s_3]^T$ and $[y_1, y_2, y_3]^T = \boldsymbol{\Phi}[s_4, s_5, s_6]^T$, $s_i \in \mathcal{A}$, $\phi = e^{\jmath 2\pi/15}$, and $\boldsymbol{\Phi}$ is a 3×3 optimized unitary complex rotation matrix

$$\begin{bmatrix} -0.3279852776 & -0.5910090485 & -0.7369762291 \\ -0.7369762291 & -0.3279852776 & 0.5910090485 \\ -0.5910090485 & 0.7369762291 & -0.3279852776 \end{bmatrix}.$$

The DTTAST code of (18) is used for transmission which transmits 6 BPSK symbols in one codeword block. The DTTAST code of (18) achieves data rate $R = 2$ bpcu under synchronized transmission, i.e., in MIMO system. However, in the unsynchronized cooperative network, one block of $N \times T$ DTTAST code is transmitted in $T + \delta_{\max}$ time intervals by following the STP strategy in order to avoid the overlapping of two consecutively transmitted codewords. Therefore, in the unsynchronized cooperative network under the assumed delay profile, the effective data rate of the DTTAST code in (18) reduces to $R = 1$ bpcu. In Fig. 3, the performance of DTTAST code averaged over all possible delay profiles is shown. The performance of the proposed delay independent scheme is plotted with $L = 200$ and BPSK constellation. As $N\delta_{\max} << L$, the rate of the proposed scheme $R = L/(L + N\delta_{\max}) = 0.97 \approx 1$ bpcu. It can be seen from Fig. 3 that the proposed scheme with BPSK constellation significantly outperforms the DTTAST codes at all SNRs. For example a gain of 8.5 dB is achieved at SER=10^{-3}. If we use BPSK constellation, then the receiver has to delay the decision by 206 time intervals. In order to reduce the delay in the decision process, we use QPSK constellation with $M = 3$ under the same delay profile used previously. Hence, delay in the decision is now reduced to $L + N\delta_{\max} = 12$ time intervals. The data rate of the proposed scheme is still 1 bpcu. It can be seen from Fig. 3 that utilizing higher order constellation reduces the coding gain of the proposed scheme as compared to the BPSK. However, the proposed scheme with QPSK is able to outperform the same rate DTTAST code at all SNRs. For example a gain of approximately 3 dB is achieved at SER=10^{-3} as compared to the same rate DTTAST code.

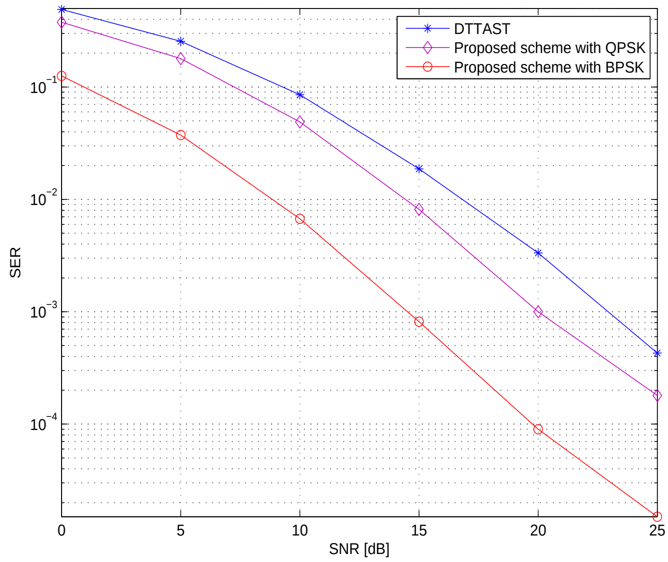

Fig. 3. Comparison of DTTAST codes with the proposed scheme.

VII. Conclusions

We have proposed a simple TDMA based transmission scheme for decode-and-forward based cooperative system. It is shown by simulations that by proper scheduling of the transmissions from the asynchronous relays, which only know the maximum delay, and optimized precoder design significant coding gain and full diversity can be achieved under arbitrary delay profile. In addition, the proposed scheme significantly outperforms same rate existing delay tolerant distributed space-time block code.

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