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Dynamic response of clamped axially moving beams: Integral transform solution

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A R T I C L E I N F O

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A B S T R A C T

The generalized integral transform technique (GITT) is employed to obtain a hybrid analytical–numerical solution for dynamic response of clamped axially moving beams. The use of the GITT approach in the analysis of the transverse vibration equation leads to a coupled system of second order differential equations in the dimensionless temporal variable. The resulting transformed ODE system is then solved numerically with automatic global accuracy control by using the subroutine DIVPAG from IMSL Library. Excellent convergence behavior is shown by comparing the vibration displacement of different points along the beam length. Numerical results are presented for different values of axial translation velocity and flexural stiffness. A set of reference results for the transverse vibration displacement of axially moving beam is provided for future co-validation purposes.

1. Introduction

Dynamic behavior of axially moving beams has been extensively studied because of its technological relevance in various applications such as data storage tapes [1], textile machines [2], automotive belts [3], band saws [4] and fluid conveying pipes [5,6].

Many experimental, analytical and numerical approaches have been employed to investigate the vibration characteristics and dynamic stability of axially moving beams. By simplifying the tape drive to a fixed–fixed Euler–Bernoulli beam model with axial velocity, Hayes and Bhushan [1] determined the natural frequencies and mode shapes of the gyroscopic system, performed parametric studies on axial velocity, tension, free span length and tape thickness, and compared the numerical results to experimental data measured by both static and dynamic methods. Lee and Mote [5] presented the energy expression of translating tensioned beams and fluid conveying pipes, and discussed the dynamic stability of the translating continua under both symmetric and asymmetric boundary configurations. Ni et al. [6] demonstrated the application of the technique of differential transformation method to the free vibration problem of pipes conveying fluid with several typical boundary conditions, where the natural frequencies and critical flow velocities were obtained. Based on the method of multiple scales, Öz and Pakdemirli [7] and Öz [8] studied the stability boundaries of an axially moving Euler–Bernoulli beam with time-dependent velocity under simply supported and clamped boundary condition, respectively. Chen and Zhao [9] obtained a conserved quantity in the free nonlinear transverse vibration of axially moving nonlinear beams with simple or fixed supports, which was applied to verify the Lyapunov stability of the straight equilibrium configuration of a beam moving with low axial speed. Chen et al. [10] applied the Galerkin method to descretize the governing equation of the axially accelerating viscoelastic tensioned beam under the fixed–fixed boundary conditions and employed the method of averaging to analyze

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the dynamic stability of the 2-term truncated system. As a continuation of their research, Chen and Yang [11] derived the governing equation of axially accelerating viscoelastic beams from Newton's second law, the constitution relation and the strain–displacement relation, used the method of multiple scales to investigate the stability conditions for combination and principal resonance, and obtained the stability boundaries for the beams with simple supports and fixed supports.

There are acomprehensive studies on the dynamic response of axially moving structures. Chen and Zhao [12] presented a modified finite difference method to simulate transverse vibration of an axially moving string, where the techniques of alternating computation and discretization at frictional knots were adopted to improve the efficiency and stability of the algorithm. To enhance the effectiveness of the approach, Chen et al. [13] developed another modified finite difference algorithm to simulate the transverse vibrations of an axially moving string, where only the spatial variable was discretized by finite difference. Using Galerkin's method, Behdinan and Tabarrok [14] computed the transient response of axially inextensible sliding beams and further extended the analysis to the non-linear case in the fixed domain. Čepon and Boltežar [15] utilized an approximate Galerkin finite-element method to study the dynamic response of a viscously damped axially moving pre-tensioned beam to arbitrary support excitations, then the solution was compared with the results obtained by the finite-difference and Galerkin methods. Based on extended Hamilton's principle, Wang et al. [16] performed a dynamic analysis for an axially translating cantilever beam simulating the spacecraft antenna with changing length at a time-variant velocity, where the assumed modes method and the separation of variables were applied for discretizing the equation of motion. Extending previous work, Wang et al. [17] solved the dynamics problem of axially moving beams using the radial basis collocation method for spatial discretization and Newmark method for temporal discretization. Besides, Lee and Oh [18] developed a spectral element model for the axially moving viscoelastic beams subjected to axial tension and examined the effects of viscoelasticity and moving speed on the dynamics of an example axially moving beam.

Recently, a hybrid numerical-analytical approach, known as GITT, has been successfully developed in heat and fluid flow applications [19–21]. The most interesting feature of this technique is the automatic and straightforward global error control procedure, which makes it particularly suitable for benchmarking purposes, and the only mild increase in overall computational effort with increasing number of independent variables. Ma et al. [22] applied the GITT to solve a transverse vibration problem of an axial moving string, where the convergence behavior of the integral transform solution was examined. However, to the authors' best knowledge, there are no previous study endeavored to perform the vibration analysis of the axially moving beams based on GITT approach. To address the lack of research in this aspect, the GITT approach is adopted to analyzes dynamic response of a clamped axially moving beam in the present paper. The rest of the paper is organized as follows. In the next section, the mathematical formulation of the problem of transverse vibration of a clamped axially moving beam is presented. In Section 3, the hybrid numerical-analytical solution is obtained by carrying out integral transform. Numerical results with automatic global accuracy control are presented in Section 4, where the convergence of the present approach is assessed. Parametric study are then performed to investigate the effects of moving speed and flexural stiffness on the dynamic response of an axially moving beam. Finally, Section 5 concludes the paper.

2. Mathematical formulation

The governing equation for linear free vibration of a tensioned Euler–Bernoulli beam travelling at constant speed v with two fixed supports can be written as [5]:

$$\rho u_{tt} + 2\rho v u_{xt} - (P_0 - \rho v^2)u_{xx} + EIu_{xxxx} = 0, \quad 0 < x < L, \tag{1a}$$

subjected to the following boundary conditions

$$u(0,t) = 0, \quad \frac{\partial u(0,t)}{\partial x} = 0, \quad u(L,t) = 0, \quad \frac{\partial u(L,t)}{\partial x} = 0, \tag{1b-e}$$

where $u(x,t)$ is the transverse displacement, ρ the mass density, P_0 the axial tension, EI the flexural rigidity of the beam and L the beam length. The following dimensionless variables are introduced

$$x^* = \frac{x}{L}, \quad u^* = \frac{u}{L}, \quad t^* = t\sqrt{\frac{P_0}{\rho L^2}}, \quad v^* = v\sqrt{\frac{\rho}{P_0}}, \quad \xi = \frac{EI}{P_0 L^2}. \tag{2a-e}$$

Substituting Eq. (2) into Eq. (1) gives the dimensionless equation (dropping the superposed asterisks for simplicity)

$$u_{tt} + 2vu_{xt} - (1 - v^2)u_{xx} + \xi u_{xxxx} = 0, \quad 0 < x < 1, \tag{3a}$$

together with the boundary conditions

$$u(0,t) = 0, \quad \frac{\partial u(0,t)}{\partial x} = 0, \quad u(1,t) = 0, \quad \frac{\partial u(1,t)}{\partial x} = 0, \tag{3b-e}$$

The initial conditions are defined as follows:

$$u(x,0) = 0, \quad \dot{u}(x,0) = v_0 \sin(\pi x), \tag{4a, b}$$

which are the same initial conditions adopted by Wang et al. [16] to analyze the transverse vibration of an axially moving simply supported beam.

3. Integral transform solution

According to the principle of the generalized integral transform technique, the auxiliary eigenvalue problem needs to be chosen for the dimensionless governing Eq. (3a) with the homogenous boundary conditions (3b-e). The eigenvalue problem, previously studied in [19], is adopted for the transverse displacement representation as follows:

$$\frac{d^4X_i(x)}{dx^4} = \mu_i^4 X_i(x), \quad 0 < x < 1, \tag{5a}$$

with the following boundary conditions

$$X_i(0) = 0, \quad \frac{dX_i(0)}{dx} = 0, \tag{5b,c}$$

$$X_i(1) = 0, \quad \frac{dX_i(1)}{dx} = 0, \tag{5d,e}$$

where $X_i(x)$ and μ_i are, respectively, the eigenfunctions and eigenvalues of problem (5). The eigenfunctions satisfy the following orthogonality property

$$\int_0^1 X_i(x)X_j(x)dx = \delta_{ij}N_i, \tag{6}$$

with $\delta_{ij} = 0$ for $i \neq j$, and $\delta_{ij} = 1$ for $i = j$. The norm, or normalization integral, is written as

$$N_i = \int_0^1 X_i^2(x)dx. \tag{7}$$

Problem (5) is readily solved analytically to yield

$$X_i(x) = \begin{cases} \frac{\cos[\mu_i(x-1/2)]}{\cos(\mu_i/2)} - \frac{\cosh[\mu_i(x-1/2)]}{\cosh(\mu_i/2)}, & \text{for } i \text{ odd}, \\ \frac{\sin[\mu_i(x-1/2)]}{\sin(\mu_i/2)} - \frac{\sinh[\mu_i(x-1/2)]}{\sinh(\mu_i/2)}, & \text{for } i \text{ even}, \end{cases} \tag{8a,b}$$

where the eigenvalues are obtained from the transcendental equations:

$$\tanh(\mu_i/2) = \begin{cases} -\tan(\mu_i/2), & \text{for } i \text{ odd}, \\ \tan(\mu_i/2), & \text{for } i \text{ even}, \end{cases} \tag{9a,b}$$

and the normalization integral is evaluated as

$$N_i = 1, \quad i = 1, 2, 3, \ldots \tag{10}$$

Therefore, the normalized eigenfunction coincides, in this case, with the original eigenfunction itself, i.e.

$$\widetilde{X}_i(x) = \frac{X_i(x)}{N_i^{1/2}}. \tag{11}$$

Note that the eigenfunction employed here is different from the stationary eigenfunctions of a clamped–clamped beam presented by Bishop and Johnson [23].

The solution methodology proceeds towards the proposition of the integral transform pair for the potentials, the integral transformation itself and the inversion formula. For the transverse displacement:

$$\bar{u}_i(t) = \int_0^1 \widetilde{X}_i(x)u(x,t)dx, \quad \text{transform}, \tag{12a}$$

$$u(x,t) = \sum_{i=1}^{\infty} \widetilde{X}_i(x)\bar{u}_i(t), \quad \text{inverse}. \tag{12b}$$

The integral transformation process is now employed through operation of (3a) with $\int_0^1 \widetilde{X}_i(x)dx$ to eliminate the spatial coordinate x, resulting in the transformed transverse displacement system:

$$\frac{d^2\bar{u}_i(t)}{dt^2} + 2v\sum_{j=1}^{\infty} A_{ij}\frac{d\bar{u}_j(t)}{dt} + (v^2 - 1)\sum_{j=1}^{\infty} B_{ij}\bar{u}_j(t) + \xi\mu_i^4\bar{u}_i(t) = 0, \quad i = 1, 2, 3, \ldots, \tag{13}$$

where the coefficients are defined as follows:

$$A_{ij} = \int_0^1 \widetilde{X}_i\widetilde{X}_j' \mathrm{d}x, \quad B_{ij} = \int_0^1 \widetilde{X}_i\widetilde{X}_j'' \mathrm{d}x. \tag{14a,b}$$

In the similar manner, initial conditions are also integral transformed to eliminate the spatial coordinate, yielding

$$\bar{u}_i(0) = 0, \quad \frac{d\bar{u}_i(0)}{dt} = v_0 \int_0^1 \widetilde{X}_i \sin(\pi x)\mathrm{d}x, \quad i = 1, 2, 3, \ldots \tag{15a, b}$$

For computational purposes, the expansion is truncated at sufficiently large order N. The coefficients A_{ij} and B_{ij} are obtained analytically through symbolic computation packages, such as the *Mathematica* [24], and automatically generated in Fortran form. The truncated system is then solved by a computer program developed in Fortran 90, based on the use of the subroutine DIVPAG from IMSL Library [25] with automatic control of the local relative error (10^{-8} is selected for this problem). For this purpose, the system composed by Eq. (13) is first rewritten as a first-order ODE system, i.e.

$$\mathbf{w}' = f(\mathbf{w}, t), \tag{16}$$

where the solution vector, $\mathbf{w}$, is defined as

$$\mathbf{w} = \left\{ \bar{u}_1, \bar{u}_2, \ldots, \bar{u}_N, \frac{\mathrm{d}\bar{u}_1}{\mathrm{d}t}, \frac{\mathrm{d}\bar{u}_2}{\mathrm{d}t}, \ldots, \frac{\mathrm{d}\bar{u}_N}{\mathrm{d}t} \right\}^{\mathrm{T}}. \tag{17}$$

Once the transformed potential, $\bar{u}_i$, has been numerically evaluated under controlled accuracy, the inversion formula (12b) is recalled to provide explicit analytical expressions for the original potentials, the dimensionless transverse displacement $u(x, t)$.

4. Results and discussion

We now present numerical results for the transverse displacement $u(x, t)$ of clamped axially moving beams by employing the GITT approach. For all the cases studied, $v_0 = 0.01$ is employed in the initial conditions (4a,b). The solution of the system (13) is obtained with $N \leqslant 50$ to analyze the convergence behavior.

The dimensionless transverse deflection $u(x, t)$, for three typical sets of values of axially moving velocity and flexural stiffness, *viz.* (i) $v = 0.15$, $\xi = 0.1$, (ii) $v = 1$, $\xi = 0.1$ and (iii) $v = 0.15$, $\xi = 1$, are presented in Tables 1–3. The convergence behavior of the integral transform solution is examined for increasing truncation terms N = 10, 20, 30, 40 and 50 at $t = 5$, $t = 20$, $t = 100$, respectively. For the dimensionless transverse deflection with $v = 0.15$ and $\xi = 0.1$ at $t = 5$, it can be observed that convergence is achieved essentially with a reasonably low truncation order ($N \leqslant 30$). For a full convergence to six significant digits, more terms (e.g., $N = 40$) are required. In addition, based on the numerical results, further investigation shows that all the solutions converge to the values with four significant figures at a truncation order of $N \leqslant 30$. The results at $t = 100$ indicate that the excellent convergence behavior of the integral transform solution does not change with time, verifying the good long-time numerical stability of the scheme. Through the comparisons between the results of Cases (i) and (ii), it can be observed that the increasing of v does not affect the convergence behavior, as shown in Tables 1 and 2, while the comparisons between the results of Cases (i) and (iii) demonstrate that the increasing of ξ can make the solution convergent at relatively low truncation orders (e.g., $N = 20$), as shown in Tables 1 and 3. For the same cases, the profiles of the transverse displacement at different time are illustrated in Figs. 1–3 with different truncation orders.

Next, the dynamic response is calculated at different axially moving velocities. Fig. 4 demonstrates the dimensionless midpoint dynamic deflection of the clamped beam without axial movement ($v = 0$), where the dimensionless flexural rigidity

Table 1
Convergence behavior of the dimensionless transverse displacement $u(x, t)$ of a clamped axially moving beam for $v = 0.15$ and $\xi = 0.1$

x	$N = 10$	$N = 20$	$N = 30$	$N = 40$	$N = 50$
$t = 5$					
0.1	0.00017961	0.00017987	0.00018008	0.00018008	0.00018007
0.3	0.00095469	0.00095545	0.00095541	0.00095540	0.00095539
0.5	0.00138727	0.00138818	0.00138819	0.00138825	0.00138824
0.7	0.00100125	0.00100210	0.00100207	0.00100207	0.00100206
0.9	0.00019338	0.00019366	0.00019381	0.00019381	0.00019381
$t = 20$					
0.1	−0.00000245	−0.00000040	−0.00000042	−0.00000046	-0.00000049
0.3	0.00013515	0.00013578	0.00013558	0.00013557	0.00013558
0.5	0.00030980	0.00030830	0.00030851	0.00030850	0.00030845
0.7	0.00022366	0.00022469	0.00022457	0.00022455	0.00022456
0.9	0.00002870	0.00003044	0.00003045	0.00003038	0.00003037
$t = 100$					
0.1	0.00015980	0.00015935	0.00015943	0.00015932	0.00015933
0.3	0.00082299	0.00082194	0.00082181	0.00082172	0.00082175
0.5	0.00117286	0.00117528	0.00117511	0.00117520	0.00117521
0.7	0.00085454	0.00085375	0.00085376	0.00085367	0.00085368
0.9	0.00016346	0.00016247	0.00016253	0.00016243	0.00016243

Table 2
Convergence behavior of the dimensionless transverse displacement $u(x,t)$ of a clamped axially moving beam for $v = 1$ and $\xi = 0.1$

x	$N = 10$	$N = 20$	$N = 30$	$N = 40$	$N = 50$
$t = 5$					
0.1	0.00010376	0.00010330	0.00010340	0.00010340	0.00010340
0.3	0.00082191	0.00082151	0.00082158	0.00082162	0.00082162
0.5	0.00140029	0.00140008	0.00140005	0.00140003	0.00140004
0.7	0.00098868	0.00098777	0.00098775	0.00098776	0.00098774
0.9	0.00015766	0.00015721	0.00015709	0.00015710	0.00015708
$t = 20$					
0.1	0.00001042	0.00000905	0.00000908	0.00000911	0.00000911
0.3	0.00042738	0.00042769	0.00042780	0.00042781	0.00042778
0.5	0.00111051	0.00111123	0.00111115	0.00111117	0.00111120
0.7	0.00102581	0.00102448	0.00102448	0.00102445	0.00102445
0.9	0.00021355	0.00021356	0.00021370	0.00021370	0.00021370
$t = 100$					
0.1	−0.00019196	−0.00019370	−0.00019369	−0.00019356	-0.00019357
0.3	−0.00101019	−0.00101004	−0.00101042	−0.00101053	-0.00101058
0.5	−0.00135663	−0.00135629	−0.00135640	−0.00135638	-0.00135640
0.7	−0.00095047	−0.00095246	−0.00095205	−0.00095197	-0.00095196
0.9	−0.00019424	−0.00018997	−0.00018984	−0.00018982	-0.00018985

Table 3
Convergence behavior of the dimensionless transverse displacement $u(x,t)$ of a clamped axially moving beam for $v = 0.15$ and $\xi = 1$

x	$N = 10$	$N = 20$	$N = 30$	$N = 40$	$N = 50$
$t = 5$					
0.1	0.00000164	0.00000199	0.00000199	0.00000199	0.00000200
0.3	0.00002353	0.00002375	0.00002380	0.00002381	0.00002380
0.5	0.00004377	0.00004369	0.00004361	0.00004363	0.00004363
0.7	0.00003385	0.00003406	0.00003410	0.00003412	0.00003411
0.9	0.00000503	0.00000539	0.00000539	0.00000539	0.00000540
$t = 20$					
0.1	0.00002067	0.00002009	0.00002007	0.00002009	0.00002009
0.3	0.00011580	0.00011567	0.00011565	0.00011565	0.00011565
0.5	0.00015621	0.00015611	0.00015607	0.00015605	0.00015605
0.7	0.00012854	0.00012837	0.00012836	0.00012836	0.00012836
0.9	0.00002519	0.00002459	0.00002457	0.00002458	0.00002458
$t = 100$					
0.1	0.00005094	0.00005080	0.00005077	0.00005078	0.00005078
0.3	0.00032736	0.00032688	0.00032674	0.00032670	0.00032669
0.5	0.00048935	0.00048986	0.00048971	0.00048977	0.00048976
0.7	0.00033050	0.00032998	0.00032984	0.00032980	0.00032979
0.9	0.00005189	0.00005187	0.00005185	0.00005185	0.00005185

$\xi = 0.1$. It can be observed that the amplitudes of the system do not change over time. The different behavior is found for the cases with translating velocities $v = 0.15$ and $v = 1$, as shown in Figs. 5 and 6, which present the effects of the dimensionless moving velocity on the transient amplitudes and vibration frequencies of the clamped axially moving beam. It can be seen that for both cases, $v = 0.15$ and $v = 1$, the amplitudes of the system increase with time. The comparisons between Figs. 5 and 6 show that as the translating velocity increases, the amplitudes increase and the vibration frequencies decrease.

Finally, we consider the influence of the dimensionless flexural rigidity ξ on dynamic responses of the midpoint of the clamped axially moving beam. The results at $\xi = 0.05$, 0.3 and 1 are shown respectively in Figs. 7–9. Note that the dimensionless moving velocity adopted is $v = 0.5$. It can be seen from the time-history of midpoint response computed that increasing the value of ξ leads to a decrease in amplitudes and an increase in vibration frequencies of the system.

5. Conclusions

The generalized integral transform technique (GITT) has been shown in this work to be an adequate approach for the analysis of the dynamic response of an clamped axially moving beam, providing a hybrid numerical-analytical solution for the transverse displacement. Excellent convergence behavior is shown for typical values of axially moving velocity and flexural stiffness. The investigation shows that all solutions converge to the values with four significant figures at a reasonable low truncation order of $N \leqslant 30$. Good long-time numerical stability is also verified. The comparisons between the numerical results show that the increasing of v does not affect the convergence behavior, while the increasing of ξ can make the solution converge faster. The parametric study indicates that the amplitudes of the system increase and the vibration

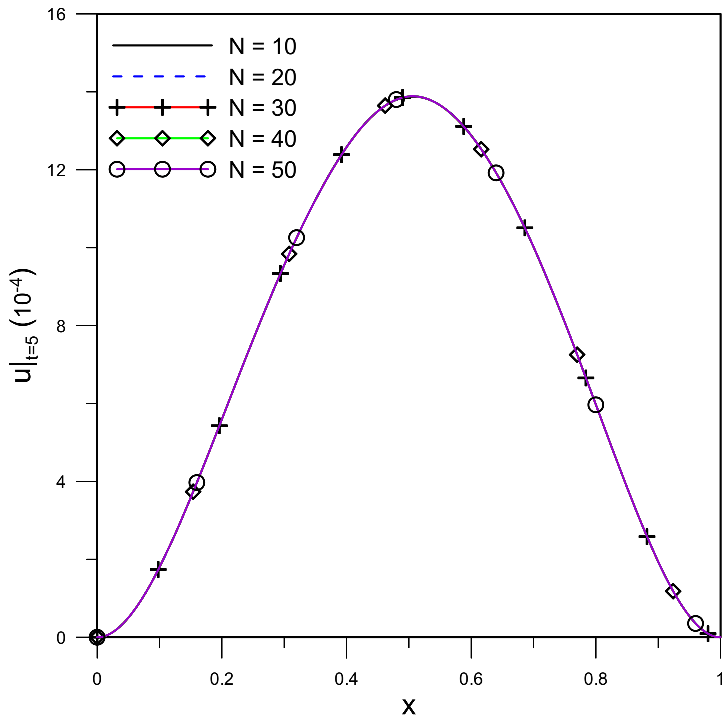

Fig. 1. GITT solutions with different truncation orders N for the dimensionless transverse displacement $u(x,t)$ of a clamped axially moving beam at $t = 5$ for $v = 0.15$ and $\xi = 0.1$.

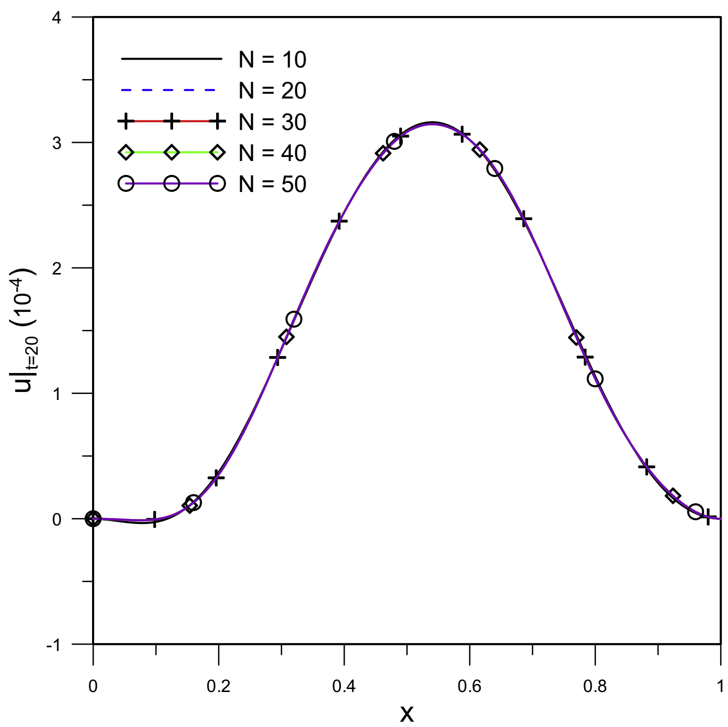

Fig. 2. GITT solutions with different truncation orders N for the dimensionless transverse displacement $u(x,t)$ of a clamped axially moving beam at $t = 20$ for $v = 0.15$ and $\xi = 0.1$.

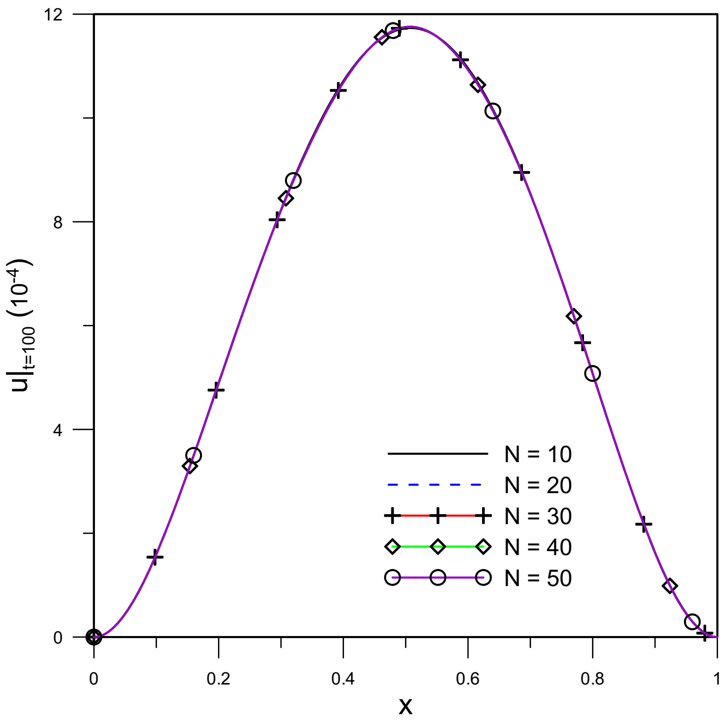

Fig. 3. GITT solutions with different truncation orders N for the dimensionless transverse displacement $u(x,t)$ of a clamped axially moving beam at $t = 100$ for $v = 0.15$ and $\xi = 0.1$.

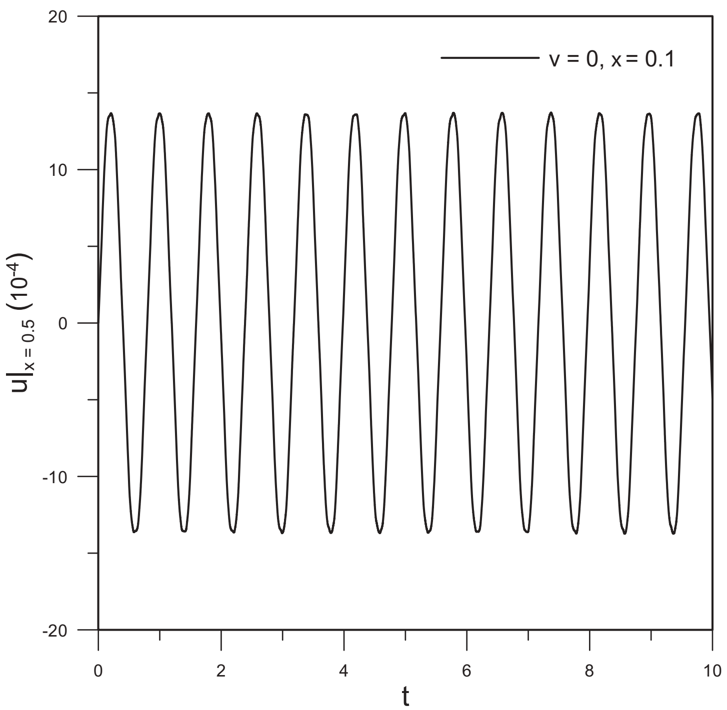

Fig. 4. Dimensionless transverse displacement $u(x,t)$ of a clamped axially moving beam at $x = 0.5$ for $v = 0$ and $\xi = 0.1$.

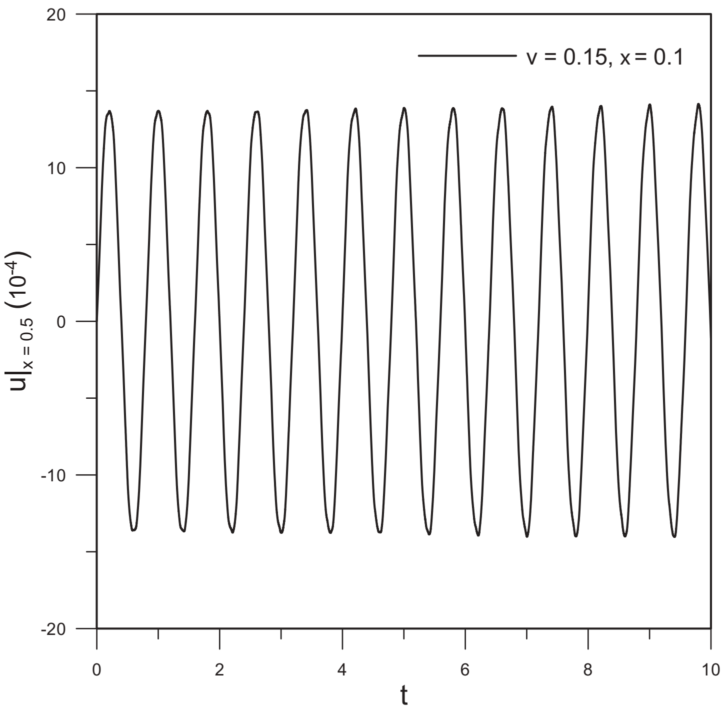

Fig. 5. Dimensionless transverse displacement $u(x,t)$ of a clamped axially moving beam at $x = 0.5$ for $v = 0.15$ and $\xi = 0.1$.

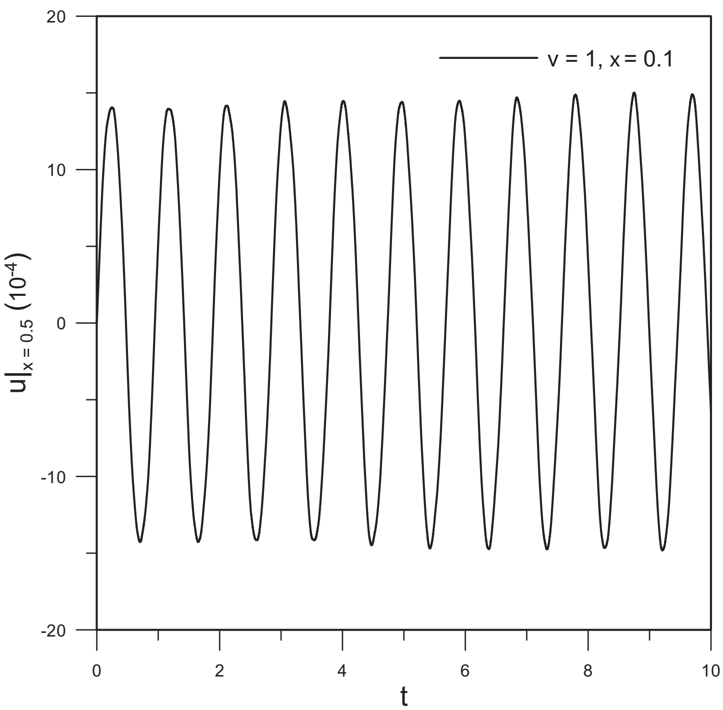

Fig. 6. Dimensionless transverse displacement $u(x,t)$ of a clamped axially moving beam at $x = 0.5$ for $v = 1$ and $\xi = 0.1$.

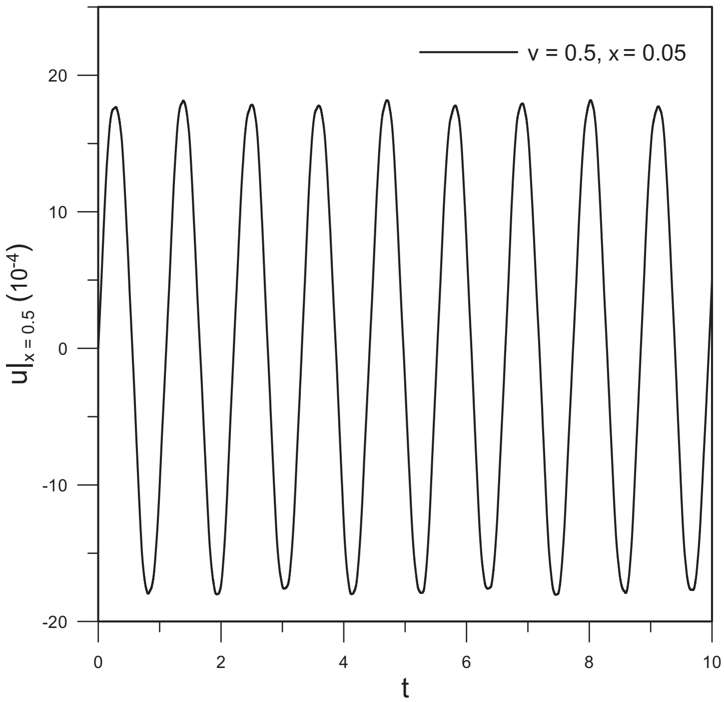

Fig. 7. Dimensionless transverse displacement $u(x,t)$ of a clamped axially moving beam at $x = 0.5$ for $v = 0.5$ and $\xi = 0.05$.

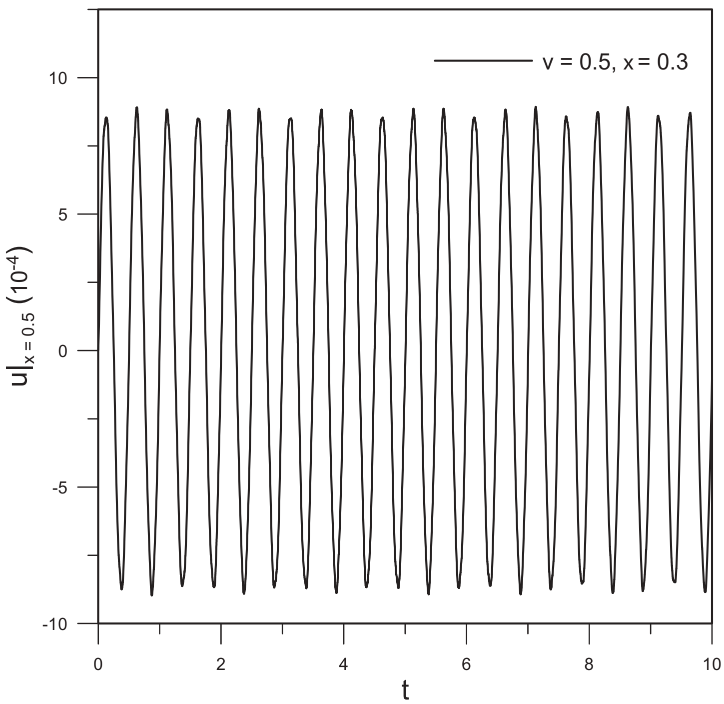

Fig. 8. Dimensionless transverse displacement $u(x,t)$ of a clamped axially moving beam at $x = 0.5$ for $v = 0.5$ and $\xi = 0.3$.

frequencies decrease with the translating velocity, and increasing the flexural rigidity leads to a decrease in amplitudes and an increase in vibration frequencies of the system. This approach can be either employed for benchmarking purposes, yielding sets of reference results with controlled accuracy, or alternatively, as an engineering simulation tool with lower truncation orders and exceptional computational performance.

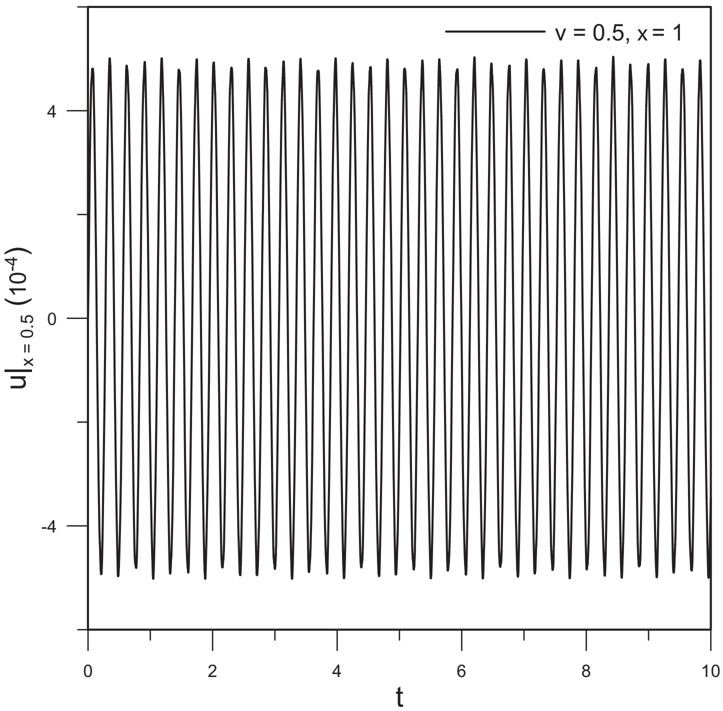

Fig. 9. Dimensionless transverse displacement $u(x,t)$ of a clamped axially moving beam at $x = 0.5$ for $v = 0.5$ and $\xi = 1$.

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