Contents

1 INTRODUCTION 1

1.1 Probability Spaces 1
1.2 Time Series 3
1.3 Examples of Stochastic Processes 5
1.4 Properties of the Autocovariance and Autocorrelation Functions 7
1.5 Complex Valued Time Series 10
1.6 Periodic Functions and Periodic Time Series . . . 12
1.7 Vector Valued Time Series 14

2 MOVING AVERAGE AND AUTOREGRESSIVE PROCESSES . . . 18

2.1 Moving Average Processes 18
2.2 Absolutely Summable Sequences and Infinite Moving Averages 26
2.3 An Introduction to Autoregressive Time Series . . 36
2.4 Difference Equations 41
2.5 The Second Order Autoregressive Time Series . . 52
2.6 Alternative Representations of Autoregressive and Moving Average Processes 56
2.7 Autoregressive Moving Average Time Series . . . 66
2.8 Vector Processes 70
2.9 Prediction 75

3 INTRODUCTION TO FOURIER ANALYSIS 93

3.1 Systems of Orthogonal Functions—Fourier Coefficients 93
3.2 Complex Representation of Trigonometric Series 114

3.3 Fourier Transform—Functions Defined on the Real Line 115
3.4 Fourier Transform of a Convolution 120

4 SPECTRAL THEORY OF TIME SERIES 126

4.1 The Spectrum 126
4.2 Circulants—Diagonalization of the Covariance Matrix of Stationary Processes 133
4.3 The Spectral Density of Moving Average and Autoregressive Time Series 139
4.4 Vector Processes 153
4.5 Measurement Error—Signal Detection 166

5 SOME LARGE SAMPLE THEORY 179

5.1 Order in Probability 179
5.2 Convergence in Distribution 193
5.3 Central Limit Theorems 199
5.4 Approximating a Sequence of Expectations . . . 201
5.5 Gauss-Newton Estimation of Nonlinear Parameters 211
5.6 Instrumental Variables 220

6 ESTIMATION OF THE MEAN AND AUTOCORRELATIONS . . . 230

6.1 Estimation of the Mean 230
6.2 Estimators of the Autocovariance and Autocorrelation Functions 236
6.3 Some Central Limit Theorems for Stationary Time Series 244
6.4 An Example 257
6.5 Estimation of the Cross Covariances 262

7 THE PERIODOGRAM, ESTIMATED SPECTRUM 275

7.1 The Periodogram 275
7.2 Smoothing, Estimating the Spectrum 287
7.3 Examples 301
7.4 Multivariate Spectral Estimates 308

8 ESTIMATION FOR AUTOREGRESSIVE AND MOVING AVERAGE TIME SERIES 327

8.1 First Order Autoregressive Time Series 327
8.2 Higher Order Autoregressive Time Series 333
8.3 Moving Average Time Series 342
8.4 Autoregressive Moving Average Time Series . . . 358
8.5 Nonstationary Autoregressive Time Series 366
8.6 Prediction with Estimated Parameters 382

9 REGRESSION, TREND, AND SEASONALITY 387

9.1 Global Least Squares 388
9.2 Grafted Polynomials 393
9.3 Autocorrelations Estimated from the Least Squares Residuals 398
9.4 Moving Averages—Linear Filtering 402
9.5 Differences 413
9.6 Some Effects of Moving Average Operators . . . 414
9.7 Regression with Time Series Errors 419
9.8 Regression Equations with Lagged Dependent Variables and Time Series Errors 429

BIBLIOGRAPHY 452

INDEX 463