

SUPPLEMENTARY MATERIAL

Results for the Algorithm 2 and Algorithm 3 experiments

In this Appendix, we present the full set of results for the experiments performed with Algorithms 2 and 3. In all the cases, we have consider values of R ranging from 5 to 70, number of particles 125, 216, 512 and 1000, and $\mu = 40, 80, 150$. The results are shown in Tables C.20 through C.43. In all the cases, we present the values of R and the number of times that not all particle pairs were recovered (“Fails”), together with minimum, maximum, average and standard deviation of the number of oracle queries.

R	Fails	Minimum	Maximum	Average	Standard deviation
5	718450	40	7418	2939.6492	1717.0896
10	41249	197	9648	5569.0876	852.5619
15	1505	654	9932	6009.2337	671.0525
20	58	1211	10080	6329.9406	669.5504
25	4	4001	10578	6648.2113	674.7998
30	0	4275	11155	6966.1094	679.7743
35	0	4324	11405	7283.0392	684.7065
40	0	4928	11613	7600.7908	690.3017
45	0	5189	12223	7918.7430	695.0099
50	0	5315	12237	8237.5179	699.4031
55	0	5641	12459	8553.4562	704.2803
60	0	5777	12843	8872.4479	708.3241
65	0	6195	12964	9189.5284	712.8600
70	0	6528	13612	9505.8860	718.6782

Table C.20. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 2 with 125 particles and $\mu = 40$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	2824	386	3712	2228.2688	217.8424
10	67	1165	4018	2548.1559	232.4877
15	1	1813	4630	2866.3376	246.8998
20	0	2027	4928	3183.3695	260.2819
25	0	2322	5271	3500.6617	273.3451
30	0	2597	5657	3818.5138	285.3814
35	0	2701	5959	4136.2548	297.1029
40	0	2964	6127	4453.5569	308.5054
45	0	3190	6810	4770.9495	319.0550
50	0	3463	6904	5088.6793	329.8067
55	0	3823	7400	5405.9980	339.7549
60	0	4015	7586	5723.9451	349.8596
65	0	4329	7833	6040.8812	359.2226
70	0	4647	8415	6358.7057	368.9050

Table C.21. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 3 with 125 particles and $\mu = 40$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	718628	37	15516	5901.4243	3448.7590
10	41357	503	18982	11181.1341	1714.4627
15	1556	1420	20896	12065.4905	1345.8739
20	54	2051	21332	12711.5539	1341.5124
25	2	6818	21172	13349.2033	1353.0338
30	0	8783	22207	13987.1952	1364.4461
35	0	8966	23816	14624.5805	1373.5282
40	0	9563	22670	15264.0114	1382.1603
45	0	10208	23718	15898.3035	1392.8924
50	0	11040	24741	16537.0394	1403.1162
55	0	11684	25428	17174.2995	1412.9613
60	0	12096	26344	17812.5433	1421.5418
65	0	12830	27214	18450.4361	1431.4450
70	0	13201	28141	19089.2051	1441.6392

Table C.22. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 2 with 216 particles and $\mu = 40$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	2377	898	7689	4825.1556	445.2583
10	59	2104	8534	5466.6574	473.8216
15	3	3780	9491	6103.8121	501.5960
20	0	4255	10959	6742.7040	528.9813
25	0	4824	11518	7379.4405	553.3524
30	0	5463	11685	8016.7392	577.5888
35	0	5995	12124	8654.2270	601.4419
40	0	6501	13166	9292.0317	623.7913
45	0	6944	13961	9930.4703	645.1531
50	0	7375	14465	10567.1632	666.4657
55	0	8018	15225	11204.3752	685.5598
60	0	8800	15461	11840.9150	705.6348
65	0	9069	16568	12478.6143	724.6778
70	0	9830	17082	13115.8680	743.3970

Table C.23. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 3 with 216 particles and $\mu = 40$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	718791	160	30868	11825.2023	6914.3259
10	41221	878	38631	22403.9427	3427.6403
15	1517	2051	40597	24178.9512	2702.7087
20	57	4604	39848	25475.3335	2691.4411
25	1	7201	43896	26750.3906	2707.0025
30	0	17789	43981	28031.4853	2729.6234
35	0	18180	44091	29304.0688	2750.3152
40	0	19496	46887	30585.1129	2770.6149
45	0	20388	48264	31859.0301	2788.2287
50	0	21896	49474	33138.2467	2807.0429
55	0	23460	49567	34416.2805	2826.3657
60	0	24140	50592	35691.7640	2850.4040
65	0	24539	51950	36970.7488	2867.3966
70	0	26362	54432	38242.6094	2887.9825

Table C.24. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 2 with 512 particles and $\mu = 40$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	1923	1910	15753	10145.7909	902.9648
10	44	4633	17451	11432.0083	960.5882
15	3	8536	20316	12710.2747	1015.7321
20	0	8806	22982	13986.8836	1067.2772
25	0	10351	22902	15264.6188	1117.9356
30	0	11131	23273	16540.8108	1166.4377
35	0	12455	25087	17819.0396	1212.3312
40	0	13582	26600	19095.3893	1255.3162
45	0	14479	28519	20374.3377	1298.8724
50	0	14841	28812	21648.2902	1342.0320
55	0	16772	30478	22927.2429	1380.5863
60	0	17651	32652	24205.4284	1419.4951
65	0	18730	34298	25486.3516	1457.7054
70	0	19923	34743	26760.8443	1494.1648

Table C.25. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 3 with 512 particles and $\mu = 40$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	718391	166	63409	23683.5436	13833.8447
10	40895	2324	75457	44862.0495	6874.5329
15	1492	5649	77415	48402.0877	5402.9775
20	72	9992	85992	50994.6734	5381.6955
25	2	20000	84930	53551.7566	5423.7290
30	0	34156	90053	56105.2792	5462.8954
35	0	36937	89677	58674.4402	5503.6964
40	0	38349	91133	61227.9910	5542.4209
45	0	40524	97136	63783.1961	5582.8703
50	0	43766	99471	66336.5767	5621.0076
55	0	46921	101023	68902.9617	5656.8642
60	0	46568	104222	71462.3188	5697.8280
65	0	48738	107966	74008.3424	5741.4485
70	0	54323	109460	76582.4883	5770.1841

Table C.26. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 2 with 1000 particles and $\mu = 40$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	1635	4265	31542	20964.4723	1823.0690
10	39	13441	35629	23536.2271	1936.5643
15	0	16590	37738	26093.3338	2046.9211
20	0	19203	43485	28652.9518	2151.9537
25	0	21504	46209	31208.2892	2250.5519
30	0	23149	48898	33770.6442	2342.3008
35	0	24765	49575	36319.7148	2434.8336
40	0	27083	56501	38880.8974	2525.8832
45	0	30029	56789	41439.2587	2609.3545
50	0	31775	58314	43996.7575	2693.1135
55	0	33189	61831	46553.6430	2770.0797
60	0	35264	65534	49114.7834	2850.8926
65	0	38689	67909	51672.2018	2926.7659
70	0	39434	71960	54225.1105	2999.7123

Table C.27. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 3 with 1000 particles and $\mu = 40$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	923296	25	13624	3635.9636	2856.8485
10	80365	279	15537	10336.4948	1842.9808
15	2888	624	16060	11098.5778	983.8430
20	98	1196	16695	11430.6496	942.7164
25	5	3176	17785	11750.7350	945.5938
30	0	8209	17179	12066.4249	948.4360
35	0	8317	17552	12385.7663	952.5189
40	0	8636	17778	12701.6750	956.0423
45	0	8748	17962	13021.0638	958.9156
50	0	9394	18442	13336.8252	963.1836
55	0	9767	19120	13654.1344	967.2429
60	0	9806	19565	13972.2885	970.1708
65	0	10307	19695	14291.1269	972.7903
70	0	10502	19788	14607.5681	976.9584

Table C.28. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 2 with 125 particles and $\mu = 80$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	2862	452	4268	2859.8728	231.2177
10	70	1063	4739	3179.8891	244.7497
15	4	2322	5091	3497.5590	258.2530
20	0	2572	5504	3815.2131	271.0638
25	0	2887	5860	4132.6062	283.2911
30	0	3173	6244	4449.8254	295.0987
35	0	3438	6949	4767.5163	306.8322
40	0	3646	6940	5085.1520	317.8033
45	0	3916	7512	5402.9935	327.4926
50	0	4177	7580	5720.4103	338.1065
55	0	4443	7898	6037.9105	348.5818
60	0	4525	8792	6355.0305	357.9535
65	0	5042	8803	6672.3907	367.2009
70	0	5055	9106	6990.2224	376.5325

Table C.29. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 3 with 125 particles and $\mu = 80$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	923535	53	26838	7278.0833	5732.9068
10	80164	466	30978	20764.6058	3686.2588
15	2893	1269	33464	22288.0032	1971.9172
20	98	3378	34485	22956.6472	1888.5995
25	4	6163	33952	23589.5022	1897.7596
30	0	16616	35536	24232.5031	1905.1978
35	0	16131	35181	24866.5963	1909.3065
40	0	18018	36800	25505.5612	1916.9934
45	0	17745	37909	26141.1385	1921.6054
50	0	18943	37610	26782.8205	1929.7497
55	0	19437	37705	27416.0201	1936.2964
60	0	20150	38316	28058.5007	1945.9851
65	0	20475	39788	28694.4024	1955.4758
70	0	20119	40084	29329.3894	1956.2711

Table C.30. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 2 with 216 particles and $\mu = 80$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	2392	1059	9052	6325.5507	474.2337
10	72	2573	10293	6967.4328	500.9015
15	0	5310	11499	7605.0946	527.7988
20	0	5832	11881	8242.9234	552.6728
25	0	6322	13802	8880.8130	577.3092
30	0	6792	13112	9517.5153	600.0381
35	0	7086	14341	10155.0858	621.8501
40	0	7719	14952	10793.2809	644.4091
45	0	8440	15185	11430.6318	664.4279
50	0	8552	15962	12067.8135	685.1698
55	0	9246	16926	12704.4590	703.8530
60	0	9872	17237	13343.4304	723.7423
65	0	10533	17705	13979.7554	743.0180
70	0	10967	18852	14618.0132	760.9026

Table C.31. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 3 with 216 particles and $\mu = 80$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	923253	187	52725	14611.2056	11490.1537
10	80134	1061	63761	41596.0359	7411.9602
15	2842	2385	67161	44658.7835	3943.5957
20	108	3890	67044	46001.3135	3790.5698
25	2	10629	68236	47280.9809	3798.1814
30	0	33531	69521	48549.5045	3805.9196
35	0	32346	70270	49839.0316	3822.6188
40	0	34265	71939	51111.6997	3840.7633
45	0	36299	73514	52389.4730	3858.2295
50	0	37650	74989	53670.5450	3874.8880
55	0	39547	76226	54943.0548	3881.4406
60	0	39633	76825	56222.5949	3896.9765
65	0	41269	79521	57496.3640	3908.8121
70	0	42185	80082	58775.9391	3928.2244

Table C.32. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 2 with 512 particles and $\mu = 80$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	1950	2115	19522	13470.1575	961.4043
10	54	4566	20804	14756.1635	1014.3651
15	4	11101	22472	16033.6005	1066.0397
20	0	12368	24762	17312.6776	1117.6257
25	0	12699	26817	18589.4395	1166.2992
30	0	14627	28121	19865.9294	1212.8327
35	0	15245	28624	21143.3044	1254.8767
40	0	16422	29459	22422.5002	1298.9550
45	0	17223	31994	23698.2628	1339.9998
50	0	18717	32834	24978.4924	1380.1082
55	0	19943	33593	26254.1776	1418.1368
60	0	20652	35096	27534.4299	1456.9480
65	0	21862	38463	28808.9218	1493.8088
70	0	23100	38009	30084.4850	1529.7581

Table C.33. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 3 with 512 particles and $\mu = 80$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	923323	255	104888	29222.7712	2856.8485
10	79935	2390	129834	83289.8695	1842.9808
15	2916	4740	134464	89407.7511	983.8430
20	88	7803	133875	92074.9924	942.7164
25	2	64634	135505	94648.8913	945.5938
30	0	66544	139891	97211.9290	948.4360
35	0	67639	142266	99766.2836	952.5189
40	0	71193	146028	102337.1896	956.0423
45	0	74491	149917	104881.7774	958.9156
50	0	73612	148087	107438.8263	963.1836
55	0	78205	158620	109989.6281	967.2429
60	0	80210	154721	112540.4592	970.1708
65	0	80626	158723	115106.1941	972.7903
70	0	82811	160788	117668.0933	976.9584

Table C.34. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 2 with 1000 particles and $\mu = 80$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	1738	4574	40682	28031.4885	1945.7548
10	56	11198	43337	30607.4803	2048.0667
15	1	19392	46624	33163.4896	2152.5102
20	0	25475	50528	35718.5258	2251.9494
25	0	27591	54305	38278.1468	2346.4738
30	0	30115	56163	40836.8827	2441.6654
35	0	31025	57841	43394.7480	2526.3774
40	0	33172	61129	45955.3550	2610.1783
45	0	36524	62045	48514.9762	2694.3705
50	0	37159	67122	51068.2165	2774.7045
55	0	39510	69679	53620.3119	2851.4596
60	0	43035	73106	56183.0602	2925.0693
65	0	44113	75208	58741.6703	3003.4321
70	0	46962	76631	61300.5532	3071.1791

Table C.35. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 3 with 1000 particles and $\mu = 80$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	991625	23	22017	3936.0119	3539.2533
10	141155	229	26817	18238.5071	4088.6913
15	5007	694	27105	19945.3744	1482.5957
20	178	1326	27614	20312.2466	1286.8417
25	7	3865	27146	20631.0459	1280.5572
30	1	15250	27272	20950.8806	1286.0155
35	0	15946	28345	21269.7710	1288.2153
40	0	15910	28047	21584.3052	1291.9420
45	0	16573	29365	21901.8913	1293.7654
50	0	16521	29823	22219.7388	1296.2946
55	0	17168	29288	22536.5065	1297.8384
60	0	17380	29745	22856.1961	1301.6468
65	0	17835	29883	23171.8584	1303.7534
70	0	17672	30728	23490.2205	1306.3670

Table C.36. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 2 with 125 particles and $\mu = 150$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	2954	342	5015	3567.7794	241.7000
10	77	1533	5699	3887.7511	254.1947
15	2	2880	5828	4205.6119	267.2645
20	0	3178	6341	4522.7434	280.1169
25	0	3603	6610	4840.4259	292.1643
30	0	3749	7134	5157.9179	303.3228
35	0	3966	7528	5475.1769	314.3134
40	0	4304	7636	5792.8299	324.8783
45	0	4590	8200	6110.0810	335.2959
50	0	4791	8346	6427.4647	345.2632
55	0	5012	8680	6745.1640	354.6144
60	0	5358	9359	7062.6664	364.8543
65	0	5557	9307	7380.1786	373.4239
70	0	5895	9825	7697.6120	382.8709

Table C.37. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 3 with 125 particles and $\mu = 150$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	991606	56	44866	7900.6697	7117.2576
10	140843	609	53792	36621.1976	8220.3127
15	5104	1370	54612	40048.2808	2980.3527
20	181	2722	54079	40784.4320	2579.0162
25	6	18818	54880	41431.0001	2574.3050
30	0	31161	55336	42063.5458	2579.5387
35	0	31338	56721	42704.7072	2586.1462
40	0	32303	57387	43336.8190	2587.6622
45	0	33082	57541	43977.8210	2591.9175
50	0	33625	58658	44618.2815	2599.5720
55	0	34127	59394	45255.0186	2604.8972
60	0	34587	60574	45886.6671	2610.6267
65	0	34515	59675	46530.9800	2616.4212
70	0	35837	61274	47168.8694	2618.2031

Table C.38. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 2 with 216 particles and $\mu = 150$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	2329	937	11030	8095.4643	497.3407
10	58	5365	12306	8737.4271	522.6686
15	1	6995	12657	9375.3278	548.0367
20	0	7495	13782	10012.7649	572.8338
25	0	7965	14121	10650.1500	596.2434
30	0	8176	15215	11287.6006	618.6946
35	0	9106	16204	11925.8409	640.3208
40	0	9565	16007	12562.8546	660.9694
45	0	10093	17563	13199.8430	681.4702
50	0	10632	17767	13837.7113	700.9023
55	0	11348	18398	14475.9172	720.4413
60	0	11748	19119	15114.0568	738.7913
65	0	12333	20315	15748.6789	757.0198
70	0	12957	20667	16386.4846	774.5531

Table C.39. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 3 with 216 particles and $\mu = 150$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	991722	171	87389	15787.1779	14214.6269
10	141218	1133	104564	73370.9227	16477.8624
15	5119	2584	106443	80267.4885	5956.5470
20	191	8396	110829	81734.6673	5176.2931
25	5	18438	112943	83029.7071	5154.9605
30	0	61565	112791	84295.9919	5163.6106
35	0	63555	112327	85583.6788	5176.9628
40	0	64111	113811	86853.8841	5194.6757
45	0	66188	115603	88128.6841	5194.7385
50	0	66729	117867	89402.4787	5205.8579
55	0	67905	117424	90693.6458	5219.0534
60	0	69315	119052	91965.4294	5231.4581
65	0	69658	124526	93241.4550	5240.3562
70	0	72515	124582	94512.2240	5245.4772

Table C.40. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 2 with 512 particles and $\mu = 150$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	1875	3230	23859	17501.6378	1012.9452
10	54	8240	25333	18787.8901	1061.4635
15	2	15497	26884	20064.1490	1112.5338
20	0	15898	28518	21342.7490	1160.4791
25	0	17546	30946	22622.6177	1206.2732
30	0	18106	32741	23897.8824	1249.8472
35	0	19230	32645	25176.4913	1293.0121
40	0	20251	34749	26454.0754	1335.0615
45	0	21517	35411	27730.7457	1376.7511
50	0	22288	37427	29007.4129	1415.1234
55	0	24033	38662	30284.7649	1452.5908
60	0	24633	39454	31562.9967	1489.6124
65	0	26049	42284	32839.0674	1528.9651
70	0	26944	42412	34118.9010	1563.3344

Table C.41. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 3 with 512 particles and $\mu = 150$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	991505	414	180395	31684.0844	28523.4480
10	140787	2180	206019	146916.3709	32947.0996
15	5019	5198	219209	160687.7883	11937.5891
20	147	12020	221281	163628.5918	10357.0073
25	6	36939	227243	166217.6099	10331.6578
30	0	117389	227737	168767.3977	10334.1867
35	0	127876	226940	171312.8985	10360.0652
40	0	129330	228330	173867.6444	10383.3829
45	0	132929	237269	176427.3668	10392.6679
50	0	135468	235256	178971.1799	10419.4715
55	0	139009	233572	181569.7748	10439.0855
60	0	141583	238208	184098.0041	10464.7435
65	0	142251	242487	186666.0778	10483.9050
70	0	142234	245575	189222.6928	10520.2681

Table C.42. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 2 with 1000 particles and $\mu = 150$

R	Fails	Minimum	Maximum	Average	Standard deviation
5	1673	6193	48431	36746.5779	2044.8347
10	42	30478	53071	39321.0275	2147.6401
15	3	31524	55663	41873.6086	2244.5502
20	0	32971	57502	44433.0812	2337.9566
25	0	35685	62610	46991.5814	2431.9973
30	0	37507	63685	49547.2117	2522.8804
35	0	40071	66778	52107.0978	2607.5169
40	0	40262	69429	54662.2551	2687.0371
45	0	44408	74472	57220.2541	2768.7009
50	0	46548	74999	59780.8423	2842.9850
55	0	48635	78220	62337.2892	2925.5149
60	0	48742	84650	64890.8063	2998.5694
65	0	53761	82784	67451.0666	3066.7362
70	0	55717	86997	70007.5573	3139.5400

Table C.43. # of fails and minimum, maximum, average and standard deviation of the number of oracle queries for 10^6 repetitions of Algorithm 3 with 1000 particles and $\mu = 150$