University Transportation Research Center - Region 2

Final Report

Disaster Relief Routing Under Uncertainty: A Robust Optimization Approach

Performing Organization: State University of New York (SUNY)

May 2017

Sponsor:
University Transportation Research Center - Region 2

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The Region 2 University Transportation Research Center (UTRC) is one of ten original University Transportation Centers established in 1987 by the U.S. Congress. These Centers were established with the recognition that transportation plays a key role in the nation's economy and the quality of life of its citizens. University faculty members provide a critical link in resolving our national and regional transportation problems while training the professionals who address our transportation systems and their customers on a daily basis.

The UTRC was established in order to support research, education and the transfer of technology in the field of transportation. The theme of the Center is "Planning and Managing Regional Transportation Systems in a Changing World." Presently, under the direction of Dr. Camille Kamga, the UTRC represents USDOT Region II, including New York, New Jersey, Puerto Rico and the U.S. Virgin Islands. Functioning as a consortium of twelve major Universities throughout the region, UTRC is located at the CUNY Institute for Transportation Systems at The City College of New York, the lead institution of the consortium. The Center, through its consortium, an Agency-Industry Council and its Director and Staff, supports research, education, and technology transfer under its theme. UTRC's three main goals are:

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16. Abstract

This report addresses the capacitated vehicle routing problem (CVRP) and the split delivery vehicle routing problem (SDVRP) with uncertain travel times and demands when planning vehicle routes for delivering critical supplies to the affected population in need after a disaster. A robust optimization approach is used to formulate the CVRP and the SDVRP with uncertain travel times and demands for five objective functions: minimization of the total number of vehicles deployed (minV), minimization of the total travel times/travel costs (minT), minimization of the summation of arrival times (minS), minimization of the summation of demand-weighted arrival times (minD), and minimization of the latest arrival time (minL). The minS, minD, and minL are critical for deliveries to be fast and fair in routing for relief efforts, while the minV and minT are common cost-based objective functions in the traditional VRP. A two-stage heuristic method that combines the insertion algorithm and tabu search is used to solve the VRP models for large-scale problems. The solutions of the CVRP and the SDVRP are compared for different examples. Keywords: Robust Optimization, Vehicle Routing Problem, Tabu Search, Insertion Algorithm

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Executive Abstract

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Contents

List of Figures

List of Tables

1 Introduction

There are significant devastating effects of natural and man-made disasters. For example, the Hurricane Katrina in August 2005 is a well-known disaster, resulting in damage estimates exceeding 200 billion U.S. dollars (Burby, 2006). More recently, two severe earthquakes occurred in Nepal on April 25th and May 12th of 2015, which caused at least 8,000 deaths, 25,000 injuries, and approximately two million homeless people (Binns and Low, 2015). Unfortunately, the scale of natural disasters is becoming larger. Therefore, the importance of effective management of disasters cannot be overemphasized as it is directly relevant to human life, health, and welfare.

Disaster management is typically divided into three phases: preparation, immediate response, and reconstruction (Kovács and Spens, 2007) or four phases: mitigation added after preparation (Pearce, 2003). In this project, we focus on the immediate response phase in the context of disaster relief operations and humanitarian logistics, which takes part in the aftermath of disasters. Specifically, we tackle the relief routing problems to effectively and equitably deliver critical supplies to the affected population.

The vehicle routing problems (VRPs) are one type of important problems considered in disaster relief operations, especially in the immediate response phase, as vehicles are an essential part of the supply chain for delivering supplies. The VRP aims to design optimal delivery or collection routes from one or several depots to a number of geographically scattered customers, subject to some constraints (Laporte, 1992). In disaster relief operations, vehicle routing problems are involved in detailed information collection, medical aid deliver, medical supply deliver, food supply deliver, etc (Luis et al., 2012).

It is not logical to assume that the vehicle capacity is always sufficient to carry all the demand from a customer locations and, therefore, the location may need to be visited multiple times (Yi and Kumar, 2007), which implies split delivery. Özdamar et al. (2004) point out that in emergencies the load to be transported is quite large and a vehicle's capacity is usually sufficient to carry only a small part of the load. Wang et al. (2014) also state that an affected area can be served more than one time when the demand of the disaster area is greater than the capacity of the vehicle, given the large demand of relief at the affected areas in the post-earthquake. Therefore, the split delivery vehicle routing problem (SDVRP) should play an important role in disaster relief operations to handle large demands.

The SDVRP, which was introduced in Dror and Trudeau (1989), is relatively new compared with the capacitated vehicle routing problem (CVRP). The SDVRP allows a demand node to be visited by more than one vehicle, while the CVRP requires that a demand node be visited exactly once. The SDVRP has attracted researchers' interest because of the potential cost savings (Dror and Trudeau, 1989; Archetti et al., 2006). The variants of the SDVRP and algorithms to solve the SDVRP and its variants have been extensively studied in recent years. In this project, the SDVRP with uncertain travel times and demands is addressed in the context of disaster relief operations, and is compared with the CVRP counterpart. Uncertainties in travel times and demands are critical factors in planning a vehicle route after a disaster because the optimal deterministic routes could be even infeasible due to a small perturbation in parameters caused by uncertainties. Therefore, it is essential to mitigate the impact of uncertainty in planning a vehicle route. We aim to enhance disaster relief vehicle routing operations by taking into uncertainties explicitly. To do so, robust-optimization based models of the SDVRP with uncertain travel times and demands are proposed to consider different objectives in disaster relief operations. To the best of our knowledge, there are no such robust models of the SDVRP in the context of humanitarian logistics in the literature.

Stochastic programming and robust optimization are two main modeling approaches that can handle uncertainty. Stochastic programming has some disadvantages in the VRP for disaster relief operations. It requires the known probability distribution function and generally needs heavy computations (Bertsimas et al., 2011; Sim, 2004). However, we may not know the exact information or even the probability distribution of the uncertainties in travel times and demands. When we can only estimate the range of uncertain parameter, we still need to find efficient and effective solution to VRPs for the immediate response operations. In that case, stochastic programming may not perform well. To address this issue, robust optimization can be a good option to formulate the VRP with uncertainty in context of disaster relief operations.

The rest of this report is organized as follows. In Section 2, the literature review of robust optimization in VRP is provided. In Section 3, the deterministic models of CVRP and SDVRP are described as the basis of the robust models, then the robust models of CVRP and SDVRP are presented. The proposed algorithms are illustrated in Section 4. The results are shown in Section 5. Conclusion follows in Section 6.

2 Literature Review

Uncertainty is an important and frequently encountered issue in the VRP for the humanitarian logistics. Among others, the uncertainty in demands and travel times are frequently considered in the literature (Allahviranloo et al., 2014; Braaten et al., 2017). Robust optimization is a modeling methodology of optimization problems in which part of (or all) data are uncertain and only known to belong to some uncertainty sets (Ben-Tal and Nemirovski, 2002). Without the probability distribution information regarding such uncertain data, a solution constructed by robust optimization can be feasible for any realization of the uncertainty in a given set (Bertsimas et al., 2011). Robust optimization has been applied in various areas such as emergency logistics planning (Ben-Tal et al., 2011; Najafi et al., 2013) and value-based performance and risk management in supply chains (Hahn and Kuhn, 2012).

In the CVRP research, Sungur et al. (2008), Erera et al. (2010), Ben-Tal et al. (2011), Gounaris et al. (2013), and Allahviranloo et al. (2014) use robust optimization to address the demand uncertainty. Regarding the uncertain travel times, Braaten et al. (2017) consider a robust version of the CVRP with time windows, in which travel times are uncertain. Han et al. (2013) consider the CVRP with uncertain travel times in which a penalty is incurred for each vehicle that exceeds a given time limit. Agra et al. (2013) address the CVRP with time windows and travel times that belong to an uncertainty polytope. We note that Lee et al. (2012) consider uncertain travel times and demands at the same time in the CVRP, while most other papers focus on only one of the two. Furthermore, Solano-Charris et al. (2014) apply robust optimization for the CVRP with uncertain travel costs. Chen et al. (2016) apply robust optimization for the road network daily maintenance routing problem with uncertain service times. We also note that most papers mentioned above consider the objective of minimizing the total travel time (or travel costs), which may not be relevant to the humanitarian logistics.

In the literature, only limited number of papers, e.g., Bouzaiene-Ayari et al. (1992), Yu et al. (2012), and Lei et al. (2012), that focus on the SDVRP with stochastic demands are found. Bouzaiene-Ayari et al. (1992) propose a heuristic algorithm for the SDVRP with stochastic demands. Yu et al. (2012) address the large scale stochastic inventory routing problem with split delivery and service level constraints. Lei et al. (2012) present a paired vehicle recourse policy for the SDVRP with stochastic demands. An adaptive large neighborhood search heuristic is applied for solving this problem. To the best of our knowledge, there are no robust models of the SDVRP with uncertain travel times and demands for different objective functions in the literature.

In this project, we explicitly consider travel time and demand uncertainty in the CVRP and the SDVRP, and explore several objectives that may better suit the humanitarian logistics such as minimizing the summation of arrival times and the latest arrival time.

3 Models

In this section, the deterministic models for the CVRP and the SDVRP with different objectives are presented as a basis for the robust counterparts. The robust models of the CVRP and the SDVRP are then proposed.

3.1 Deterministic Models of the CVRP

In the deterministic CVRP, let the depot be located at node 0 and the set of nodes except the depot is denoted by $N = \{1, ..., n\}$. The set of all nodes is then $N_0 = \{0\} \cup \{1, ..., n\}$. The set of all arcs is denoted by A such that $G = G(N_0, A)$ represents the road network. The travel time between nodes i and j in N_0 is denoted by $t_{ij}, \forall (i, j) \in A$. In our assumption, the travel cost between nodes i and j is equivalent to its travel time by setting the travel cost as one for the unit travel distance. The demand from node i is denoted by q_i, which needs to be served as soon as possible.

The set of available vehicles is denoted by $K = \{1, ..., |K|\}$, and k is the index of a vehicle. All vehicles are assumed to be homogeneous, and C denotes the capacity of a vehicle. The decision variables $x_{ij}, \forall (i, j) \in A$ are binary variables that indicate whether a vehicle travels from i to j.

In consistent with the well-known Miller-Tucker-Zelmin (MTZ) formulation of the VRP (Miller et al., 1960), a continuous variable $c_i, \forall i \in N$ is used, denoting the flow in the vehicle when it leaves the node i, to construct constraints that prevent sub-tours. A continuous variable $a_i, \forall i \in N$ denotes the arrival time of a vehicle at node i and an upper bound on the total travel times for each vehicle is denoted by T, which can be relaxed if needed in the problem context by assigning a sufficiently large number on it. We do not consider a time window for delivering critical supplies, as the objectives in the context of humanitarian logistics are on prompt deliveries and we assume that all the demand nodes can accept deliveries any time (e.g., shelters that are open 24 hours). However, time window constraints can be easily added when necessary. The deterministic model that minimizes the total number of vehicles deployed (minV) is formulated as follows:

$$
\begin{aligned}
&(\text{CVRP-minV}) \quad \min \sum_{i=1}^{n} x_{0i} && && (1)\\
&\text{s.t.} \quad \sum_{j \in N_0} x_{ij} = 1 && \forall i \in N && (2)\\
&\sum_{j \in N_0} x_{ij} - \sum_{j \in N_0} x_{ji} = 0 && \forall i \in N_0 && (3)\\
&t_{ij} \le a_j - a_i + T(1 - x_{ij}) && \forall i, j \in N && (4)\\
&t_{0i} x_{0i} \le a_i && \forall i \in N && (5)\\
&q_j \le c_j - c_i + C(1 - x_{ij}) && \forall i, j \in N && (6)\\
&q_i \le c_i \le C && \forall i \in N && (7)\\
&x_{ij} \in \{0, 1\} && \forall (i, j) \in A && (8)
\end{aligned}
$$

The objective (1) is to minimize the number of vehicles deployed. The constraints (2) require that each node should be visited once by exactly one vehicle and equations (3) are flow conservation constraints. The variables x_{ij} are associated with arrival times in inequalities (4), which also prevent subtours not including the depot. The appropriate minimum arrival time for each node is guaranteed in inequalities (5), and the inequalities (6) work in a similar fashion as inequalities (4). The capacity constraints are imposed in inequalities (7). To solve the CVRP-minV more effectively in the solvers, an additional constraint can be added in the model:

$$\sum_{i=1}^{n} x_{0i} \geq \frac{\sum_{i=1}^{n} q_i}{C} \tag{9}$$

The inequality (9) provides a tight lower bound of the objective function value, which can reduce the time for solving the model.

The model to minimize the total travel times (minT) is exactly as same as the CVRP-minV except the objective function. That is,

$$\begin{aligned} \text{(CVRP-minT)} \quad & \min \sum_{(i,j)\in\mathcal{A}} t_{ij}x_{ij} && (10) \\ \text{s.t.} \quad & (2)\text{–}(8) \end{aligned}$$

To minimize the summation of arrival times (minS), one more constraint to specify the number of vehicles available, $|K|$, needs to be added, because the optimal solution will be a trivial one that utilizes maximum vehicles, e.g., n vehicles, if there is no such constraint. The CVRP-minS can be formulated as:

$$\begin{aligned} \text{(CVRP-minS)} \quad & \min \sum_{i\in N} a_i && (11) \\ \text{s.t.} \quad & (2)\text{–}(8) \\ & \sum_{i\in N} x_{0i} = |K| && (12) \end{aligned}$$

To minimize the summation of demand-weighted arrival times (minD) can be formulated as:

$$\begin{aligned} \text{(CVRP-minD)} \quad & \min \sum_{i\in N} q_i a_i && (13) \\ \text{s.t.} \quad & (2)\text{–}(8), (12) \end{aligned}$$

At last, the model to minimize the latest arrival time (minL) is formulated as:

$$\begin{aligned} \text{(CVRP-minL)} \quad & \min a_l && (14) \\ \text{s.t.} \quad & (2)\text{–}(8), (12) \\ & a_i \leq a_l \quad \forall i \in N && (15) \end{aligned}$$

3.2 Deterministic Models of the SDVRP

The two-index formulation, e.g., x_{ij}, is used in the CVRP formulation. We now introduce the three-index formulation for the SDVRP while the notation for parameters remains the same to be consistent. The new decision variables $x_{ijk}, \forall(i,j) \in A, k \in K$ are binary, indicating whether vehicle k travels from i to j ($x_{ijk} = 1$) or not ($x_{ijk} = 0$). The amount of demand served by vehicle k to node i is denoted by $y_{ik}, \forall i \in N, k \in K$ and a continuous variable $a_{ik}, \forall i \in N, k \in K$ denotes the arrival time of vehicle k at node i. The minimum number of vehicles needed, K_{min}, for the SDVRP can be calculated by solving the following equation.

$$K_{min} = \left\lceil \frac{\sum_{i=1}^{n} q_i}{C} \right\rceil \tag{16}$$

In order to obtain feasible solution, $|K| \geq K_{min}$. Based on the deterministic model of the SDVRP in Berbotto et al. (2014), the subtour elimination constraints are modified to include a_{ik} in the deterministic models shown in this section. The model to minimize the total travel times of the SDVRP (SDVRP-minT) is formulated as follows.

$$\text{(SDVRP-minT)} \quad \min \sum_{(i,j)\in A} \sum_{k\in K} t_{ij} x_{ijk} \tag{17}$$

$$\text{s.t.} \quad \sum_{j\in N_0} \sum_{k\in K} x_{ijk} \geq 1 \qquad \forall i \in N \tag{18}$$

$$\sum_{j\in N_0} \sum_{k\in K} x_{0jk} \leq |K| \tag{19}$$

$$\sum_{j\in N_0} \sum_{k\in K} x_{0jk} \geq K_{min} \tag{20}$$

$$\sum_{j\in N_0} x_{ijk} - \sum_{j\in N_0} x_{jik} = 0 \qquad \forall i \in N_0, k \in K \tag{21}$$

$$t_{ij} \leq a_{jk} - a_{ik} + T(1 - x_{ijk}) \qquad \forall i, j \in N, k \in K \tag{22}$$

$$t_{0i} x_{0ik} \leq a_{ik} \qquad \forall i \in N, k \in K \tag{23}$$

$$y_{ik} \leq q_i \sum_{j\in N_0} x_{ijk} \qquad \forall i \in N, k \in K \tag{24}$$

$$\sum_{i\in N} y_{ik} \leq C \qquad k \in K \tag{25}$$

$$\sum_{k\in K} y_{ik} = q_i \qquad \forall i \in N \tag{26}$$

$$x_{ijk} \in \{0, 1\} \qquad \forall (i, j) \in A, k \in K \tag{27}$$

$$y_{ik} \geq 0 \qquad \forall i \in N, k \in K \tag{28}$$

The constraints (18) require that each node should be visited by at least one vehicle. The inequality (19) ensures that at most K vehicles depart from the depot. The inequality (20) ensures that at least K_{min} vehicles depart from the depot. The equations (21) are flow conservation constraints. The variables x_{ijk} are associated with arrival times in inequalities (22), which also prevent subtours not including the depot. The appropriate minimum arrival times for each node are guaranteed in inequalities (23). The inequalities (24) ensure that the node can only be served if the vehicle visits it. The inequalities (25) ensure that the maximum load of each vehicle does not exceed capacity C. The equations (26) require that the entire demand of each node is satisfied.

To minimize the summation of arrival times, the SDVRP-minS can be formulated as:

$$\text{(SDVRP-minS)} \quad \min \sum_{i\in N} \sum_{k\in K} a_{ik} \tag{29}$$

$$\text{s.t.} \quad (18)\text{–}(28)$$

To minimize the summation of demand-weighted arrival times, the SDVRP-minD can be formulated as:

$$\text{(SDVRP-minD)} \quad \min \sum_{i \in N} \sum_{k \in K} y_{ik} a_{ik} \tag{30}$$
$$\text{s.t.} \quad (18)\text{–}(28)$$

Note that the SDVRP-minD is a mixed integer nonlinear programming (MILP) model, as y_{ik} and a_{ik} are variables.

The objective to minimize the latest arrival time, a_l, is formulated as:

$$\text{(SDVRP-minL)} \quad \min a_l \tag{31}$$
$$\text{s.t.} \quad (18)\text{–}(28)$$
$$a_{ik} \leq a_l \quad \forall i \in N, k \in K \tag{32}$$

3.3 Robust Models of the CVRP

In robust optimization (RO), there are various ways to model uncertainty depending on how to define the sets to which the uncertain parameters belong. We assume that uncertainty sets in this project, which may be obtained by analyzing the historical data, are convex, closed, and bounded. Let us denote an uncertainty set by U, and the travel times and demands are subject to uncertainty. That is, $(t, q) \in U$ where t and q are the vectors such that $t = (t_{ij} : (i, j) \in A)$ and $q = (q_i : i \in N)$. Taking into account the uncertainty and considering that RO aims to find the best worst-case solutions, the robust CVRP-minV (RCVRP-minV) can be formulated as:

$$\text{(RCVRP-minV)} \quad \min \sum_{i=1}^{n} x_{0i} \tag{33}$$
$$\text{s.t.} \quad (2)\text{–}(3), (8)$$
$$\max_{(t,q) \in U} t_{ij} \leq a_j - a_i + T(1 - x_{ij}) \qquad \forall i, j \in N \tag{34}$$
$$\max_{(t,q) \in U} t_{0i} x_{0i} \leq a_i \qquad \forall i \in N \tag{35}$$
$$\max_{(t,q) \in U} q_j \leq c_j - c_i + C(1 - x_{ij}) \qquad \forall i, j \in N \tag{36}$$
$$\max_{(t,q) \in U} q_i \leq c_i \leq C \qquad \forall i \in N \tag{37}$$

Likewise, the robust CVRP-minT (RCVRP-minT) can be formulated as:

$$\text{(RCVRP-minT)} \quad \min_{x} \max_{(t,q) \in U} \sum_{(i,j) \in A} t_{ij} x_{ij} \tag{38}$$
$$\text{s.t.} \quad (2)\text{–}(3), (8), (34)\text{–}(37)$$

The robust counterparts for the CVRP-minS, CVRP-minD, and CVRP-minL can be formulated in a similar fashion as follows:

$$
\begin{aligned}
\text{(RCVRP-minS)} \quad & \min \sum_{i \in N} a_i && (39)\\
\text{s.t.} \quad & (2)\text{–}(3), (8), (12), (34)\text{–}(37)
\end{aligned}
$$

$$
\begin{aligned}
\text{(RCVRP-minD)} \quad & \min_{x} \max_{(t,q) \in U} \sum_{i \in N} q_i a_i && (40)\\
\text{s.t.} \quad & (2)\text{–}(3), (8), (12), (34)\text{–}(37)
\end{aligned}
$$

$$
\begin{aligned}
\text{(RCVRP-minL)} \quad & \min a_l && (41)\\
\text{s.t.} \quad & (2)\text{–}(3), (8), (12), (15), (34)\text{–}(37)
\end{aligned}
$$

Note that the objective functions of the RCVRP-minT and the RCVRP-minD are subject to uncertainty while the ones of other models are not.

Let us assume that there is no correlation between t and q, then $U = U_T \times U_Q$ where $t \in U_T$ and $q \in U_Q$. Indeed, this assumption can be made without loss of generality in our robust formulations because inequalities (34)–(37), and functions (38) and (40) consider only one type of uncertainty.

In RO, all uncertainty sets are assumed to be bounded. Accordingly, let $U_T = \{t \mid \bar{t} \le t \le \bar{t} + \hat{t}\}$ and $U_Q = \{q \mid \bar{q} \le q \le \bar{q} + \hat{q}\}$ where $\bar{t}$ and $\bar{q}$ are the nominal travel time and demand vectors, respectively, and $\hat{t}$ and $\hat{q}$ are vectors for the maximum travel delay and increased demand caused by the destabilized infrastructure after a disaster. Such uncertainty sets employed in this report are called box sets and we refer readers interested in a more general notion of uncertainty sets, e.g., ellipsoidal set and convex hull, to Ordóñez (2010) and Ben-Tal and Nemirovski (2002). Because there is only one uncertain factor per constraint, inequalities (34)–(37) can be rewritten as follows:

$$
\begin{aligned}
& \bar{t}_{ij} + \hat{t}_{ij} x_{ij} \le a_j - a_i + T(1 - x_{ij}) && \forall i, j \in N && (42)\\
& \bar{t}_{0i} + \hat{t}_{0i} x_{0i} \le a_i && \forall i \in N && (43)\\
& \bar{q}_j + \hat{q}_j x_{ij} \le c_j - c_i + C(1 - x_{ij}) && \forall i, j \in N && (44)\\
& \bar{q}_i + \hat{q}_i x_{0i} \le c_i \le C && \forall i \in N && (45)
\end{aligned}
$$

These new constraints are deterministic with given $\bar{t}_{ij}$, $\hat{t}_{ij}$, $\bar{q}_i$, and $\hat{q}_i$.

For the objective function of RCVRP-minT (38), it has uncertain travel times up to the number of arcs in the road network. By employing the concept of the *budget of uncertainty* (Bertsimas and Sim, 2004), its uncertainty set can be reformulated as:

$$
U_T = \left\{ t \mid \bar{t}_{ij} \le t_{ij} \le \bar{t}_{ij} + \hat{t}_{ij} x'_{ij}, (i,j) \in A, \sum_{(i,j) \in A} x'_{ij} \le \Gamma_T, x'_{ij} \in \{0, 1\} \right\} \tag{46}
$$

The parameter Γ_T is called the *budget of uncertainty* and it controls the degree of conservatism or robustness of the solution. The RCVRP-minT is then:

$$\text{(RCVRP-minT)} \quad \min_{x \in X} \sum_{(i,j) \in A} \bar{t}_{ij} x_{ij} + \max_{t \in U_T} \sum_{(i,j) \in A} \hat{t}_{ij} x'_{ij} \tag{47}$$

where X is the feasible set for x. We may relabel $\hat{t}_{ij}, (i,j) \in A$ in a decreasing order, i.e., $\hat{t}_{e_1} \geq \hat{t}_{e_2} \geq \cdots \geq \hat{t}_{e_m} \geq \hat{t}_{e_{m+1}} (= 0)$. Therefore, $\hat{t}_{e_i}$ is the ith greatest $\hat{t}_{ij}, (i,j) \in A$. For the sake of notational convenience, we also employ x_{e_i} that corresponds $\hat{t}_{e_i}$. The following Theorem 1 shows that the solution of RCVRP-minT can be found by solving multiple deterministic CVRP-minT problems.

Theorem 1. The solution of RCVRP-minT (47) can be computed as the minimum of $m+1$ deterministic VRP problems, for $l = 1, 2, ..., m+1$:

$$Z^l = \Gamma_T \hat{t}_{e_l} + \min_{x \in X} \left(\sum_{(i,j) \in A} \bar{t}_{ij} x_{ij} + \sum_{k=1}^{l} \left(\hat{t}_{e_k} - \hat{t}_{e_l} \right) x_{e_k} \right) \tag{48}$$

where m is the number of arcs in the road network. Let $l^* = \arg\min_l Z^l$, then $Z^* = Z^{l^*}$ and $x^* = x^{l^*}$ where x^l is the optimal solution of Z^l.

Proof. See Bertsimas and Sim (2003). □

The objective function of RCVRP-minD (40) can have uncertain demand nodes up to the number of nodes in the road network. A set $S_Q \subseteq U_Q, |S_Q| = \Gamma_Q$ is introduced, where Γ_Q is the budget of uncertainty, to the degree which the system is protected deterministically. Then the objective function of RCVRP-minD can be written as follows:

$$\text{(RCVRP-minD)} \quad \min \left(\sum_{i \in N} \bar{q}_i a_i + \max_{\{S_Q | S_Q \subseteq U_Q, |S_Q| \leq \Gamma_Q\}} \sum_{i \in S_Q} \hat{q}_i a_i \right) \tag{49}$$

This objective function is protected by:

$$\beta(\mathbf{a}, \Gamma_Q) = \max_{\{S_Q | S_Q \subseteq U_Q, |S_Q| \leq \Gamma_Q\}} \sum_{i \in S_Q} \hat{q}_i a_i \tag{50}$$

where $\mathbf{a}$ is vector of $a_i, \forall i \in N$.

Proposition 1. Equation (50) is equivalent to the following linear optimization problem:

$$\beta(\mathbf{a}, \Gamma_Q) = \max \sum_{i \in N} \hat{q}_i a_i z'_i \tag{51}$$

$$\text{s.t.} \quad \sum_{i \in N} z'_i \leq \Gamma_Q \tag{52}$$

$$0 \leq z'_i \leq 1 \qquad \forall i \in N \tag{53}$$

Proof. It is clear that the optimal solution value of function (51) consists of $\lfloor \Gamma_Q \rfloor$ variables z'_i at 1. This is equivalent to the selection of subset $\{S_Q \mid S_Q \subseteq U_Q, \mid S_Q \mid \leq \Gamma_Q\}$ with corresponding function $\sum_{i \in S_Q} \hat{q}_i a_i$. □

Theorem 2. The RCVRP-minD has the equivalent formulation as follows.

$$
\begin{aligned}
\text{(RCVRP-minD)} \min \sum_{i\in N} \bar{q}_i a_i + \Gamma_Q g' + \sum_{i\in N} p'_i & & (54)\\
\text{s.t.}\ \ (2)\text{–}(3), (8), (12), (42)\text{–}(45) & & \\
g' + p'_i \geq \hat{q}_i a_i & \quad \forall i \in N & (55)\\
p'_i \geq 0 & \quad \forall i \in N & (56)\\
g' \geq 0 & & (57)
\end{aligned}
$$

Proof. Consider the dual of function (51):

$$
\begin{aligned}
\min\ \Gamma_Q g' + \sum_{i\in N} p'_i & & (58)\\
\text{s.t.}\ g' + p'_i \geq \hat{q}_i a_i & \quad \forall i \in N & (59)\\
p'_i \geq 0 & \quad \forall i \in N & (60)\\
g' \geq 0 & & (61)
\end{aligned}
$$

By strong duality, since function (51) is feasible and bounded for $\Gamma_Q \in [0, |\mathcal{S}_\mathcal{Q}|]$, then the dual problem (58) is also feasible and bounded and their objective values coincide. Using Proposition 1, we have that function (50) is equal to the objective function value of function (58). Substituting (58)–(61), we obtain that function (49) is equivalent to function (54). □

3.4 Robust Models of the SDVRP

The robust models of the SDVRP use the same fashion of the robust models of the CVRP.

$$
\begin{aligned}
\text{(RSDVRP-minT)} \min_x \max_{(t,d)\in U} \sum_{(i,j)\in A} \sum_{k\in K} t_{ij} x_{ijk} & & (62)\\
\text{s.t.} \sum_{j\in N_0} \sum_{k\in K} x_{ijk} \geq 1 & \quad \forall i \in N & (63)\\
\sum_{j\in N_0} \sum_{k\in K} x_{0jk} \leq |K| & & (64)\\
\sum_{j\in N_0} x_{ijk} - \sum_{j\in N_0} x_{jik} = 0 & \quad \forall i \in N_0, k \in K & (65)\\
\max_{(t,d)\in U} t_{ij} \leq a_{jk} - a_{ik} + T(1 - x_{ijk}) & \quad \forall i, j \in N, k \in K & (66)\\
\max_{(t,d)\in U} t_{0i} x_{0ik} \leq a_{ik} & \quad \forall i \in N, k \in K & (67)\\
y_{ik} - \max_{(t,d)\in U} q_i \sum_{j\in N_0} x_{ijk} \leq 0 & \quad \forall i \in N, k \in K & (68)\\
\sum_{i\in N} y_{ik} \leq C & \quad k \in K & (69)\\
\sum_{k\in K} y_{ik} - \max_{(t,d)\in U} q_i = 0 & \quad \forall i \in N & (70)\\
x_{ijk} \in \{0, 1\} & \quad \forall (i, j) \in A, k \in K & (71)
\end{aligned}
$$

$$y_{ik} \geq 0 \qquad \forall i \in N, k \in K \tag{72}$$

The robust model to minimize the summation of arrival times can be formulated as:

$$\text{(RSDVRP-minS)} \min \sum_{i \in N} \sum_{k \in K} a_{ik} \tag{73}$$
$$\text{s.t.}(63)\text{–}(72)$$

The robust model to minimize the summation of demand weighted arrival times can be formulated as:

$$\text{(RSDVRP-minD)} \min \sum_{i \in N} \sum_{k \in K} y_{ik} a_{ik} \tag{74}$$
$$\text{s.t.}(63)\text{–}(72)$$

The robust model to minimize the latest arrival time is formulated as:

$$\text{(RSDVRP-minL)} \min a_l \tag{75}$$
$$\text{s.t.}(63)\text{–}(72), (32)$$

Inequalities (66) – (68), (70) can be written as:

$$\bar{t}_{ij} + \hat{t}_{ij} x_{ijk} \leq a_{jk} - a_{ik} + T(1 - x_{ijk}) \qquad \forall i, j \in N, k \in K \tag{76}$$
$$\left(\bar{t}_{0i} + \hat{t}_{0i}\right) x_{0ik} \leq a_{ik} \qquad \forall i \in N, k \in K \tag{77}$$
$$y_{ik} - (\bar{q}_i + \hat{q}_i) \sum_{j \in N_0} x_{ijk} \leq 0 \qquad \forall i \in N, k \in K \tag{78}$$
$$\sum_{k \in K} y_{ik} - (\bar{q}_i + \hat{q}_i) = 0 \qquad \forall i \in N \tag{79}$$

The main difference between the CVRP models and the SDVRP models is whether an arc (i, j) can be used by multiple vehicles or not. In the CVRP, an arc (i, j) can be used at most once, therefore, each t_{ij} is related to one variable x_{ij}. In contrast, an arc (i, j) in the SDVRP can be used by multiple vehicles. Therefore, t_{ij} is related to $x_{ijk}, k \in K$. A set $S \subseteq U_T, |S| = \Gamma_T$ is introduced, where Γ_T is the budget of uncertainty. Then the RSDVRP-minT can be written as follows:

$$\text{(RSDVRP-minT)} \quad \min \left(\sum_{(i,j) \in A} \sum_{k \in K} \bar{t}_{ij} x_{ijk} + \max_{\{S | S \subseteq U_T, |S| \leq \Gamma_T\}} \sum_{(i,j) \in S} \sum_{k \in K} \hat{t}_{ij} x_{ijk} \right) \tag{80}$$

As in the CVRP case, the objective function is protected by:

$$\beta(\mathbf{x}, \Gamma_T) = \max_{\{S | S \subseteq U_T, |S| \leq \Gamma_T\}} \sum_{(i,j) \in S} \sum_{k \in K} \hat{t}_{ij} x_{ijk} \tag{81}$$

where $\mathbf{x}$ is vector of decision variables.

Proposition 2. Equation (81) can be written as

$$\beta(\mathbf{x}, \Gamma_T) = \max_{\{S|S\subseteq U_T, |S|\leq\Gamma_T\}} \sum_{(i,j)\in S} \hat{t}_{ij} \sum_{k\in K} x_{ijk} \tag{82}$$

A new type of variable w_{ij} is introduced, which denotes the number of vehicles using arc (i, j). Therefore,

$$\beta(\mathbf{x}, \Gamma_T) = \max_{\{S|S\subseteq U_T, |S|\leq\Gamma_T\}} \sum_{(i,j)\in S} \hat{t}_{ij} w_{ij} \tag{83}$$

$$\text{s.t. } w_{ij} = \sum_{k\in K} x_{ijk} \qquad \forall (i,j) \in A \tag{84}$$

Equations (83) and (84) are equivalent to the following linear optimization problem:

$$\beta(\mathbf{x}, \Gamma_T) = \max \sum_{(i,j)\in A} \hat{t}_{ij} w_{ij} z_{ij} \tag{85}$$

$$\text{s.t. } (84)$$

$$\sum_{(i,j)\in A} z_{ij} \leq \Gamma_T \tag{86}$$

$$0 \leq z_{ij} \leq 1 \qquad \forall (i,j) \in A \tag{87}$$

Proof. Clearly the optimal solution value of function (85) consists of $\lfloor \Gamma_T \rfloor$ variables z_{ij} at 1. This is equivalent to the selection of subset $\{S \mid S \subseteq U_T, |S| \leq \Gamma_T\}$ with corresponding function $\sum_{(i,j)\in S} \hat{t}_{ij} w_{ij}$. □

Theorem 3. The RSDVRP-minT has the equivalent formulation as follows.

$$\text{(RSDVRP-minT)} \min \sum_{(i,j)\in A} \sum_{k\in K} \bar{t}_{ij} x_{ijk} + \Gamma_T g + \sum_{(i,j)\in A} p_{ij} \tag{88}$$

$$\text{s.t. } (63)\text{–}(65), (69)\text{–}(72), (76)\text{–}(79), (84)$$

$$g + p_{ij} \geq \hat{t}_{ij} w_{ij} \qquad \forall (i,j) \in A \tag{89}$$

$$p_{ij} \geq 0 \qquad \forall (i,j) \in A \tag{90}$$

$$g \geq 0 \tag{91}$$

$$0 \leq w_{ij} \leq |K| \qquad \forall (i,j) \in A \tag{92}$$

Proof. Consider the dual of Problem (85):

$$\min \ \Gamma_T g + \sum_{(i,j)\in A} p_{ij} \tag{93}$$

$$\text{s.t. } g + p_{ij} \geq \hat{t}_{ij} w_{ij} \qquad \forall (i,j) \in A \tag{94}$$

$$p_{ij} \geq 0 \qquad \forall (i,j) \in A \tag{95}$$

$$g \geq 0 \tag{96}$$

□

By strong duality, since function (85) is feasible and bounded for $\Gamma_T \in [0, |S|]$, then the dual function (93) is also feasible and bounded and their objective values coincide. Using Proposition 2, we have that function (81) is equal to the objective function value of function (93). Substituting (93)–(96), we obtain that function (80) is equivalent to function (88).

4 Heuristic Algorithms

For small-sized problems, the problems can be solved by using the solvers such as Gurobi. However, for large-scale problems, it is by no means practical to utilize the solvers, as the VRPs are NP-hard. Considering that the routing decisions need to be made quickly in the immediate response phase of disaster management, it is desirable to obtain the near-optimal solutions within a short time period. In light of this, we propose a heuristic algorithm for which the well-known insertion algorithm is modified and used in conjunction with tabu search method. The overall, high-level main framework is shown in Algorithm 1. In particular, the insertion algorithm is used to find the good feasible solution as an initial solution for tabu search. The maximum CPU time allowed is set by the users. Then, a tabu search method is implemented iteratively until the elapsed CPU time is greater than the maximum CPU time allowed. The best-so-far solution during the whole procedure is returned.

Main Framework for Solving the VRP Problems
Implement the insertion algorithm to construct a good feasible solution s^I;
Set s^I as the initial solution for the tabu search;
while *elapsed CPU time* $<$ *Max CPU time* **do**
  Implement one iteration of the tabu search method;
  Update best-so-far solution s^B;
end
Return best-so-far solution s^B

Algorithm 1: Main Framework

4.1 Extended Insertion Algorithms

We employ the insertion algorithm to find a good initial solution. It has been used for solving various vehicle routing problems, see, e.g., Campbell and Savelsbergh (2004); Campbell et al. (2008a). It constructs a reasonably good feasible solution by repeatedly and greedily inserting an unrouted customer node into a partially constructed feasible solution. The constructed solution from insertion algorithm is not guaranteed to be an optimal solution or a near-optimal solution. Therefore, it is used as the initial solution for a tabu search method in this project.

The insertion algorithm, presented in Campbell and Savelsbergh (2004) and Campbell et al. (2008a), constructs the routes that do not take demands into account. In this project, we extend the insertion algorithms in Campbell and Savelsbergh (2004) to consider the capacity constraints of the CVRP and the SDVRP with different objective functions. For the SDVRP, we further extend the insertion algorithm to consider the split delivery.

The notation used in the extended insertion algorithms proposed are as follows. Let N' denote the set of unassigned nodes, R' denote the set of assigned routes, and $|R'|$ denote the number of assigned routed (not including empty route). The index of a route is denoted by $r \in R$. The flow for route r is denoted by c^r, which means the total amount of demands of the nodes in current route r. A variable E is used to record the number of vehicles that can not take demand of node j because $C - c^r \leq q_j$. The end of route r is denoted by L_r. The objective value of R' is denoted by

f'. The objective value of $R_{i,j,r}$ is denoted by $f_{i,j,r}$. The objective value of R_j is denoted by f_j. The difference between two objective values is denoted by δ. The amount of demand served by route r to node j is denoted by y_{jr}. The remaining demand of node j is denoted by q'_j. The variables $i^\star$, $j^\star$, $r^\star$ and $\delta^\star$ are used to record the best choices of i, j, r, and δ, respectively.

The extended insertion algorithm for the CVRP-minV is shown in Algorithm 2. The objective of Algorithm 2 is to find a solution to minimize the number of vehicles needed in the problem. The algorithm starts with one route, and keeps adding the nodes into the set of routes. If a node cannot be added into any routes, then add a new route in the set of routes. Algorithm 2 stops when all nodes have been assigned into routes. When the CVRP-minV is solved, the number of vehicles is fixed and used for the CVRP-minT, CVRP-minS, CVRP-minD, and CVRP-minL. The insertion algorithms for the CVRP-minT, CVRP-minS, CVRP-minD, and CVRP-minL share the same structures, as shown in Algorithm 3. The algorithm starts with $|K|$ routes that only contain the depot. During the initialization, the algorithm searches $j^\star$ with the smallest $\delta^\star$ if add $j^\star$ at the end of route r for R'. After $j^\star$ is added into R', $j^\star$ is removed from N'. When $N' \neq \emptyset$, the algorithm searches $i^\star$, $j^\star$, $r^\star$ and $\delta^\star$ iteratively. By doing so, the algorithm can insert node $j^\star$ into R' with the smallest $\delta^\star$ in each iteration. In addition, during searching $i^\star$, $j^\star$, $r^\star$ and $\delta^\star$, the constraint $C - c^r \geq q_j$ is checked for all j and r. Therefore, the algorithm can guarantee R' is feasible for the capacity constraint.

The insertion algorithms for the SDVRP-minT, SDVRP-minS, SDVRP-minD, and SDVRP-minL share the same structures, as shown in Algorithm 5. In Algorithm 5, split delivery is allowed. The procedure to find $i^\star$, $j^\star$, $r^\star$, and $\delta^\star$ is as same as the one in Algorithm 3. The capacity constraint is replaced by $C - c^r \geq 0$ in Algorithm 5 because a vehicle can serve partial demand of a node. Once $j^\star$ is added into R', $y_{j^\star r^\star}$, $q'_{j^\star}$, and $c^{r^\star}$ are updated based on the condition $(C - c^r) < q'_{j^\star}$. If the full demand of a node has been served by the routes, then this node is removed from N'.

4.2 Tabu Search

The tabu search (TS) approach is a single solution based and deterministic method to search optimal solution. For tutorials, we refer readers to (Glover, 1990). The tabu search algorithm used in this project to solve the VRPs is shown in Algorithm 6.

In TS, the initial solution is given. In this reserch, the initial solution can be found by implementing the proposed extended insertion algorithm. The current solution during TS is denoted by R' and the tabu list is denoted by σ. The best-so-far solution during TS is denoted by R^{best}, and the objective value of R^{best} is denoted by f^{best}. In addition, the neighbor solution of R' is denoted by R'_h, and f'_h is the objective value of R'_h. The set of R'_h that satisfies all constraints is denoted by M, and R'_h are rearranged from the smallest f'_h to largest f'_h. The maximum CPU time that allows program to run is denoted by B_{max} and the elapsed CPU time is denoted by B. While $B < B_{max}$, TS is implemented iteratively. At each iteration, all R'_h of R' are found according the move operators shown in Algorithms 7–9. In Algorithms 7–9, the nodes are denoted by i and i' and the routes are denoted by r and r'.

Two types of neighbor solutions for the CVRP are defined in this report: exchange-node neighbor solutions and relocate-node neighbor solutions. The exchange-node neighbor solutions can be found by choosing two different nodes in the routes and switching the two nodes. The set of exchange-node neighbor solutions is denoted as M_1 in Algorithm 7. The relocate-node neighbor solutions can be found by removing one node from a route and relocate this node in another route. The set of relocate-node neighbor solutions is denoted as M_2 in Algorithm 8. The move operators for searching exchange-node neighbor solutions and relocate-node neighbor solutions are shown in Algorithms 7 and 8, respectively. For the SDVRP, besides exchange-node neighbor solutions and relocate-node

Extended Insertion Algorithm for CVRP-minV

Start one route $r \in R'$, each route only contains depot;

while $N' \neq \emptyset$ **do**

  for $j \in N'$ **do**

    initialize $E = 0$;

    for $r \in R'$ **do**

      if $C - c^r \geq q_j$ **then**

        $r^\star = r$

      else

        $E = E + 1$

      end

    end

    if $E = |R'|$ **then**

      add a new route r' in R';

      add j in r' ;

      $c^{r'} = q_j$;

    else

      add j in $r^\star$;

      $c^{r^\star} = c^{r^\star} + q_j$

    end

    Remove j from N';

  end

end

Return R', $|R'|$.

Algorithm 2: Insertion Algorithm Extended for CVRP-minV

Extended Insertion Algorithm for CVRP

start with K route $r \in R'$, each route only contains depot, $f' = 0$;
$c^r = 0, \forall r \in R'$;
for $r \in R'$ **do**
  $\delta^\star = \infty$;
  for $j \in N'$ **do**
    $R_j = R'$, add j at the end of route r for R_j;
    evaluate f_j, $\delta = f_j - f'$;
    if $\delta < \delta^\star$ **then**
      $\delta^\star = \delta, j^\star = j$
    end
  end
  add $j^\star$ at the end of route r for R', $c^r = q_{j^\star}$, $f' = f' + \delta^\star$;
  remove $j^\star$ from N';
end
while $N' \neq \emptyset$ **do**
  $\delta^\star = \infty$;
  for $j \in N'$ **do**
    for $r \in R'$ **do**
      if $C - c^r \geq q_j$ **then**
        for $i \in r$ **do**
          $R_{i,j,r} = R'$, insert j in front of i in route r for $R_{i,j,r}$;
          evaluate $f_{i,j,r}$, $\delta = f_{i,j,r} - f'$;
          if $\delta < \delta^\star$ **then**
            $\delta^\star = \delta, j^\star = j, i^\star = i, r^\star = r$
          end
        end
        $R_{i,j,r} = R'$, add j at the end of route r for $R_{i,j,r}$;
        evaluate $f_{i,j,r}$, $\delta = f_{i,j,r} - f'$;
        if $\delta < \delta^\star$ **then**
          $\delta^\star = \delta, j^\star = j, i^\star = L_r, r^\star = r$
        end
      end
    end
  end
  if $i^\star = L_{r^\star}$ **then**
    add $j^\star$ at the end of route $r^\star$ for R'
  else
    insert $j^\star$ in front of $i^\star$ in route $r^\star$ for R'
  end
  $f' = f' + \delta^\star$;
  $c^{r^\star} = c^{r^\star} + q_{j^\star}$;
  remove $j^\star$ from N';
end
return R', f'.

Algorithm 3: Extended Insertion Algorithm for CVRP

```
Function to find i⋆, j⋆, r⋆ and δ⋆ for Insertion Algorithm of SDVRP
δ⋆ = ∞;
for j ∈ N' do
    for r ∈ R' do
        if C − c^r ≥ 0 then
            for i ∈ r do
                R_{i,j,r} = R', insert j in front of i in route r for R_{i,j,r};
                evaluate f_{i,j,r}, δ = f_{i,j,r} − f';
                if δ < δ⋆ then
                    δ⋆ = δ, j⋆ = j, i⋆ = i, r⋆ = r
                end
            end
            R_{i,j,r} = R', add j at the end of route r for R_{i,j,r};
            evaluate f_{i,j,r}, δ = f_{i,j,r} − f';
            if δ < δ⋆ then
                δ⋆ = δ, j⋆ = j, i⋆ = L_r, r⋆ = r
            end
        end
    end
end
```

Algorithm 4: Function to Find $i^\star$, $j^\star$, $r^\star$ and $\delta^\star$ for Insertion Algorithm of SDVRP

neighbor solutions, two more types of neighbor solutions are defined: add-split-node neighbor solutions and delete-split-node neighbor solutions (Berbotto et al., 2014). The add-split-node neighbor solutions can be found by adding node i of route r into route r' if $i \notin r', i \neq i', r \neq r'$. The demand of node i is split and served by route r and route r'. The set of add-split-node neighbor solutions is denoted as M_3. The delete-split-node neighbor solutions can be found by choosing a node that is served by more than one route, and the node from one of the routes is removed. The set of delete-split-node neighbor solutions is denoted as M_4. The move operators for searching add-split-node neighbor solutions and delete-split-node neighbor solutions are shown in Algorithms 9 and 10.

Subsequently, all the neighbor solutions are evaluated and ranked according to their objective values. The best neighbor solution which is not in the tabu list is used as current solution for the next iteration, and added in the tabu list. At the end of each iteration, the tabu list is updated based on frequency. For example, when frequency is 50, a solution is stored in the tabu list for 50 iterations. After 50 iterations, this solution is removed from the tabu list. The best-so-far solution is saved during the whole procedure.

5 Results

5.1 Simple Examples

Two simple examples from Campbell et al. (2008b) are used to illustrate how different objective functions can influence the solutions, and one simple example from Huang et al. (2012) is used to show the difference between the CVRP and the SDVRP. These examples are shown in Figures 1, 2, and 3, respectively.

The main observation from the results, shown Table 2, is that the minS and minL objectives

Extended Insertion Algorithm for the SDVRP

start with K route $r \in R'$, each route only contains depot, $f' = 0$, $q'_j = q_j, \forall j \in N'$;

$y_{jr} = 0, \forall j \in N', \forall r \in R'$;

$c^r = 0, \forall r \in R'$;

for $r \in R'$ **do**

  $\delta^\star = \infty$;

  for $j \in N'$ **do**

    $R_j = R'$, add j at the end of route r for R_j;

    evaluate f_j, $\delta = f_j - f'$;

    if $\delta < \delta^\star$ **then**

      $\delta^\star = \delta, j^\star = j$

    end

  end

  add $j^\star$ at the end of route r for R', $f' = f' + \delta^\star$;

  if $C < q'_{j^\star}$ **then**

    $y_{j^\star r} = C$, $q'_{j^\star} = q'_{j^\star} - C$, $c^r = C$

  else

    $y_{j^\star r} = q'_{j^\star}$, $q'_{j^\star} = 0$, $c^r = q_{j^\star}$

  end

  if $q'_{j^\star} = 0$ **then**

    remove $j^\star$ from N'

  end

end

while $N \neq \emptyset$ **do**

  implement Algorithm 4 to find $i^\star$, $j^\star$, $r^\star$ and $\delta^\star$;

  if $i^\star = L_{r^\star}$ **then**

    add $j^\star$ at the end of route $r^\star$ for R'

  else

    insert $j^\star$ in front of $i^\star$ in route $r^\star$ for R'

  end

  $f' = f' + \delta^\star$;

  if $(C - c^r) < q'_{j^\star}$ **then**

    $y_{j^\star r^\star} = y_{j^\star r^\star} + (C - c^{r^\star})$, $q'_{j^\star} = q'_{j^\star} - (C - c^{r^\star})$, $c^{r^\star} = C$

  else

    $y_{j^\star r^\star} = y_{j^\star r^\star} + q'_{j^\star}$, $q'_{j^\star} = 0$, $c^{r^\star} = c^{r^\star} + q'_{j^\star}$

  end

  if $q'_{j^\star} = 0$ **then**

    remove $j^\star$ from N'

  end

end

Algorithm 5: Extended Insertion Algorithm for the SDVRP

```
Tabu Search Algorithm
initialize R', R^best = R', σ = ∅ ;
while B < B_max do
    M = ∅;
    find all R'_h of R' according to the move operators, and add them into M;
    for R'_h ∈ M do
        evaluate f'_h of R'_h;
    end
    rank R'_h ∈ M from the smallest f'_h to the largest f'_h;
    for R'_h ∈ M' do
        if R'_h ∉ σ then
            R' = R'_h;
            add R'_h to σ;
            if f'_h < f^best then
                R^best = R'_h;
                f^best = f'_h;
            end
        else
            if f'_h < f^best then
                R' = R'_h;
                add R'_h to σ;
                R^best = R'_h;
                f^best = f'_h;
            end
        end
        break the for-loop when R' is updated;
    end
    update σ based on frequency.
end
return R^best and f^best
```

Algorithm 6: Tabu Search Algorithm

```
Exchange-nodes Move Operator
for r ∈ R' do
    for r' ∈ R' do
        for i ∈ r do
            for i' ∈ r' do
                if i ≠ i' then
                    R'_h = R';
                    exchange i and i' in R'_h;
                    if R'_h satisfies all constraints then
                        add R'_h in M_1
                    end
                end
            end
        end
    end
end
return M_1
```

Algorithm 7: Exchange-nodes Move Operator

```
Relocate-node Move Operator
for r ∈ R' do
    for r' ∈ R' do
        for i ∈ r do
            for i' ∈ r' do
                if i ≠ i' and r ≠ r' then
                    R'_h = R';
                    take i out of r for R'_h, insert i after i' in r' for R'_h;
                    if R'_h satisfies all constraints then
                        add R'_h in M_2
                    end
                end
            end
        end
    end
end
return M_2
```

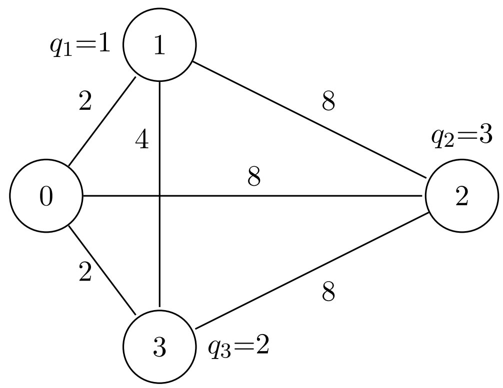
Algorithm 8: Relocate-node Move Operator

```
Add-splite-node Move Operator
for r ∈ R' do
    for r' ∈ R' do
        for i ∈ r do
            for i' ∈ r' do
                if i ∉ r', i ≠ i' and r ≠ r' then
                    R'_h = R', Y'_h = Y';
                    randomly generate a number α in range [1, y_ir];
                    insert i after i' in r' for R'_h;
                    for Y'_h, y_ir = y_ir − α, y_ir' = y_ir' + α;
                    if R'_h and Y'_h satisfy all constraints then
                        add R'_h in M_3
                    end
                end
            end
        end
    end
end
return M_3
```

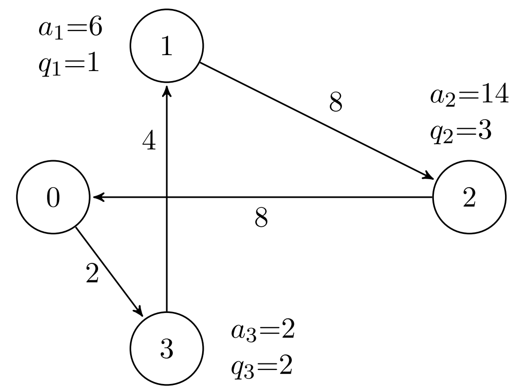
Algorithm 9: Add-split-node Move Operator

```
Delete-splite-node Move Operator
for r ∈ R' do
    for r' ∈ R' do
        for i ∈ r do
            if i ∈ r' and r ≠ r' then
                R'_h = R', Y'_h = Y';
                remove i from r;
                for Y', y_ir' = y_ir' + y_ir, y_ir = 0 ;
                if R'_h and Y'_h satisfy all constraints then
                    add R'_h in M_4
                end
            end
        end
    end
end
```

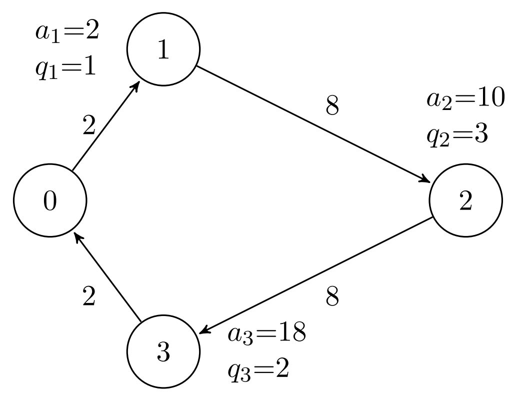
Algorithm 10: Delete-split-node Move Operator



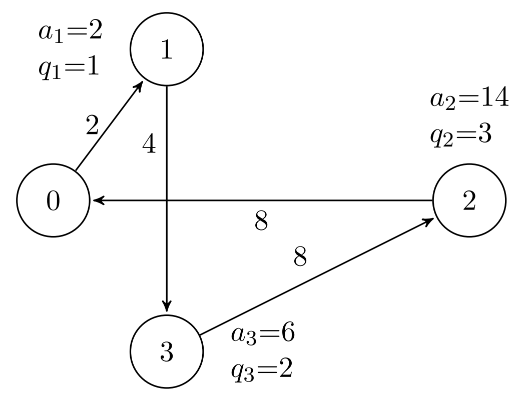
(a) Network of example 1



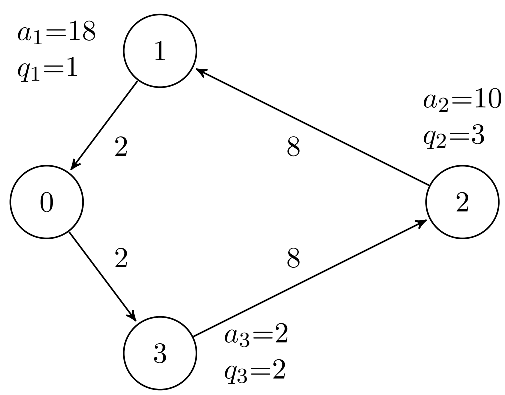
(b) Solution for CVRP-minV, SDVRP-minL



(c) Solutions for CVRP-minT, SDVRP-minT

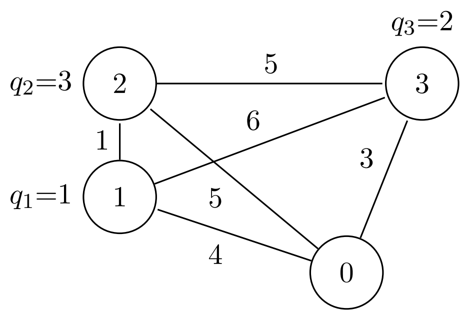


(d) Solutions for CVRP-minS, CVRP-minL, SDVRP-minS

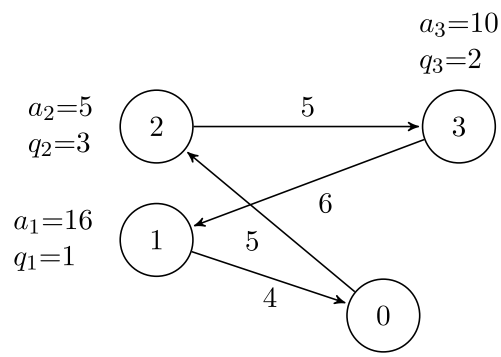


(e) Solutions for CVRP-minD, SDVRP-minD

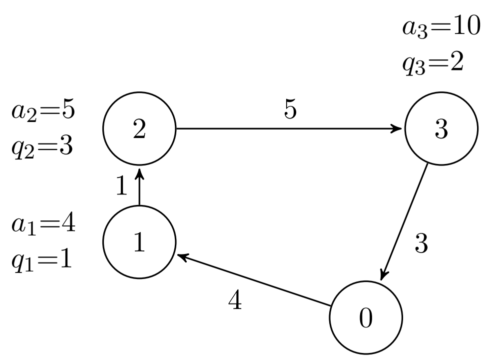
Figure 1: Network and Solutions of Example 1



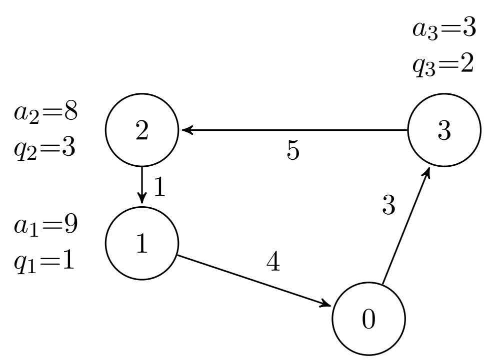
(a) Network of example 2



(b) Solution for CVRP-minTV

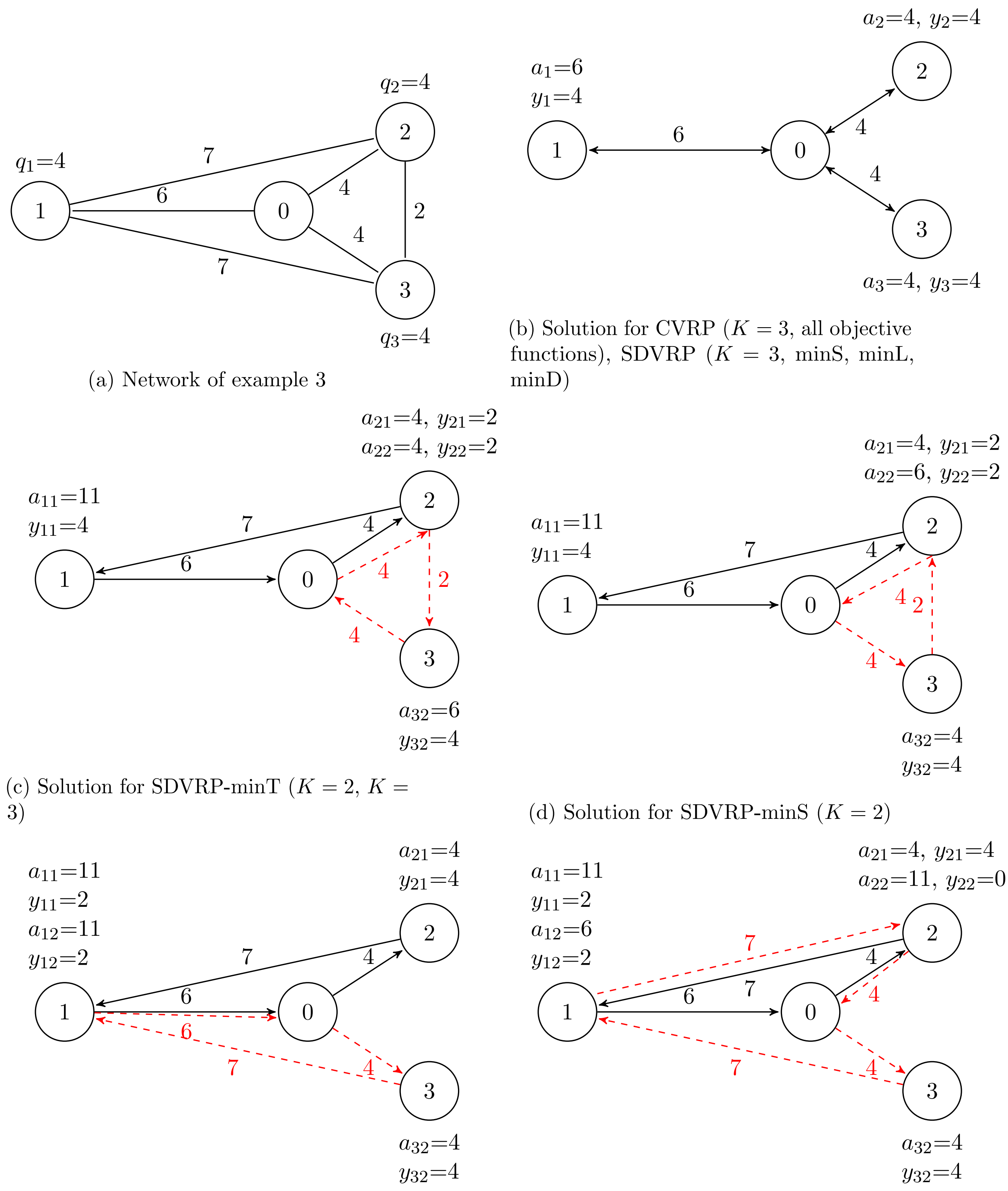


(c) Solutions for CVRP-minT, CVRP-minS, SDVRP-minS



(d) Solution for CVRP-minL, CVRP-minD, SDVRP-minT, SDVRP-minL, SDVRP-minD

Figure 2: Network and Solutions of Example 2



(a) Network of example 3

(b) Solution for CVRP ($K = 3$, all objective functions), SDVRP ($K = 3$, minS, minL, minD)

(c) Solution for SDVRP-minT ($K = 2$, $K = 3$)

(d) Solution for SDVRP-minS ($K = 2$)

(e) Solution for SDVRP-minL ($K = 2$)

(f) Solution for SDVRP-minD ($K = 2$)

Figure 3: Network and Solutions of Example 3

ensure the better response times for serving the demands of nodes at the possible expense of reduced efficiency (the increase total travel time). In disaster management, this implies that the minS and minL objectives allow delivering the critical supplies to the demand node faster, which is important for the sake of humanitarian logistics. The minD considers the demand-weighted arrival times at the nodes, which ensures the better response times for serving unit demand regardless of the node locations. In example 1, the minimum number of vehicles required is one for the CVRP and the SDVRP. The objective function values provided by the CVRP and the SDVRP are the same corresponding to the minT, minS, and minL.

The main observation, summarized from Table 3, is that the minS and minL objectives may provide different solutions depending on the network. In example 2, we can see that the optimal solution of minimizing the summation of arrival times does not guarantee the minimum latest arrival time. The optimal solution of minimizing the latest arrival time does not guarantee the minimum summation of arrival times either.

From Table 4, we may point out several observations. In this example, the CVRP nominal model requires at least three vehicles to produce feasible solutions, while the SDVRP nominal model requires at least two vehicles. Considering the minV, the SDVRP model allows better utilization of vehicles. In this example, the vehicles have not enough capacity to meet the entire demand of two nodes in the CVRP. Therefore, each vehicle can only serve one node in the CVRP. In the SDVRP, each node is allowed to be served by more than one vehicle. Therefore, vehicles can co-operate with each other to serve the demand. In the SDVRP, the solutions with minimum number of vehicles restrict the solution space. When three vehicles are allowed to be used in the SDVRP, we can see that the optimal solutions vary regarding minT, minS, minL, and minD. To minimize the total travel time, only two vehicles are used to serve the demand in the SDVRP nominal model even though three vehicles are available. To minimize the summation of arrival times, summation of demand-weighted arrival times, and latest arrival time, three vehicles are used to serve the nodes in order to reduce the response times. From these observations, we can see that the SDVRP provides more flexibility.

To test the robust models of the CVRP and the SDVRP, $\hat{t}_{ij}, \forall (i, j) \in A$ for examples 1–3 are generated as shown in Table 1. We use $\hat{q}_i = 1, \forall i \in N$ for examples 1–3. In Table 1, we can see that $\hat{t}_{01}$ and $\hat{t}_{10}$ are relatively very large, which implies that arcs $(0, 1)$ and $(1, 0)$ no longer function in the aftermath of a disaster. From Tables 2 and 3, we can see that arcs $(0, 1)$ and $(1, 0)$ are not selected in the solutions of the RCVRP and the RSDVRP for minT. For minS, minL, and minD, arc $(0, 1)$ is not selected in the solutions, and arc $(1, 0)$ can be selected because arc $(1, 0)$ does not influence the arrival time of the nodes (except depot).

For example 3 where we set $\bar{q}_i + \hat{q}_i = 5$, at least three vehicles are needed. From Table 4, the solutions of SDVRP models can avoid using arc $(0, 1)$ by visiting nodes 2 and 3 more than once. Based on the constraint of CVRP, there is only one feasible solution.

Table 1: Increased Travel Times of Examples 1–3

		Node			
	$\hat{t}_{ij}$	0	1	2	3
	0	0	100	3	5
Node	1	100	0	4	11
	2	3	4	0	8
	3	5	11	8	0

For the RCVRP-minT and the RSDVRP-minT, we can control the level of uncertainty of travel

Table 2: Results of Example 1

Nominal Model						
Model	Route	TV	TT	SUM	MAX	DA
CVRP-minV	0,3,1,2,0	1	22	22	14	52
CVRP-minT	0,1,2,3,0	1	20	30	18	68
CVRP-minS	0,1,3,2,0	1	22	22	14	56
CVRP-minL	0,1,3,2,0	1	22	22	14	56
CVRP-minD	0,3,2,1,0	1	20	30	18	52
SDVRP-minT	0,1,2,3,0	1	20	30	18	68
SDVRP-minS	0,1,3,2,0	1	22	22	14	56
SDVRP-minL	0,3,1,2,0	1	22	22	14	52
SDVRP-minD	0,3,2,1,0	1	20	30	18	52
Robust Model, Full Uncertainty						
Model	Route	TV	TT	SUM	MAX	DA
RCVRP-minV	0,2,1,3,0	1	45	72	38	204
RCVRP-minT	0,3,1,2,0	1	45	63	34	201
RCVRP-minS	0,3,1,2,0	1	45	63	34	201
RCVRP-minL	0,3,1,2,0	1	45	63	34	201
RCVRP-minD	0,3,2,1,0	1	137	65	35	183
RSDVRP-minT	0,2,1,3,0	1	45	72	38	204
RSDVRP-minS	0,3,1,2,0	1	45	63	34	201
RSDVRP-minL	0,3,1,2,0	1	45	63	34	201
RSDVRP-minD	0,3,2,1,0	1	137	65	35	183

time considered in the solutions by adjusting the value of Γ_T. When $\Gamma_T = 1$, one $\hat{t}_{ij}$ that increases the objective value with greatest amount is considered in the solution and the objective value. When $\Gamma_T = 2$, two $\hat{t}_{ij}$ that increase the objective value with greatest amount is considered in the solution and the objective value. When the value of Γ_T increases, more $\hat{t}_{ij}$ are considered in the solution and the objective value. As shown in Table 5, the optimal solutions may be the same when Γ_T increases, but the objective value increases due to more $\hat{t}_{ij}$ considered in the solution. For examples 1 and 2, the optimal solutions of RCVRP and RSDVRP are the same because one vehicle is used. For example 3, the only feasible solution of RCVRP is $[0, 1, 0], [0, 2, 0], [0, 3, 0]$ because of the constraint. For the RSDVRP of example 3, the optimal solutions avoid arcs $(0, 1)$ and $(1, 0)$.

For RCVRP-minD, we can control the level of uncertainty of demand considered in the solutions by controlling the value of Γ_Q. When the value of Γ_Q increases, more $\hat{q}_i$ are considered in the solution and the objective value. As shown in Table 6, the optimal solutions may be the same when Γ_Q increases, but the objective value increases due to more $\hat{q}_i$ considered in the solution.

5.2 Results from Heuristic Algorithms

For the large-scale examples, heuristic algorithms are used to obtain near-optimal solutions within the time limit. We use examples from http://neo.lcc.uma.es/vrp/vrp-instances/capacitated-vrp-instances/. Examples named $A - n32 - k5$, $A - n44 - k7$, and $E - n101 - k8$ are used. The number of nodes in these examples are 32, 44, and 101, respectively. The capacity of each vehicle in $A - n32 - k5$ and $A - n44 - k7$ is 100. The capacity of each vehicle in $E - n101 - k8$ is 200. For examples $A - n32 - k5$ and $A - n44 - k7$, we set CPU time = 300 seconds for one run of model

Table 3: Results of Example 2

Nominal Model						
Model	Route	TV	TT	SUM	MAX	DA
CVRP-minV	0,2,3,1,0	1	20	31	16	51
CVRP-minT	0,1,2,3,0	1	13	19	10	39
CVRP-minS	0,1,2,3,0	1	13	19	10	39
CVRP-minL	0,3,2,1,0	1	13	20	9	39
CVRP-minD	0,3,2,1,0	1	13	20	9	39
SDVRP-minT	0,3,2,1,0	1	13	20	9	39
SDVRP-minS	0,1,2,3,0	1	13	19	10	39
SDVRP-minL	0,3,2,1,0	1	13	20	9	39
SDVRP-minD	0,3,2,1,0	1	13	20	9	39
Robust Model, Full Uncertainty						
Model	Route	TV	TT	SUM	MAX	DA
RCVRP-minV	0,2,1,3,0	1	38	51	30	148
RCVRP-minT	0,2,1,3,0	1	38	51	30	148
RCVRP-minS	0,2,1,3,0	1	38	51	30	148
RCVRP-minL	0,3,2,1,0	1	130	55	26	160
RCVRP-minD	0,2,1,3,0	1	38	51	30	148
RSDVRP-minT	0,3,1,2,0	1	38	63	30	194
RSDVRP-minS	0,2,1,3,0	1	38	51	30	148
RSDVRP-minL	0,3,2,1,0	1	130	55	26	160
RSDVRP-minD	0,2,1,3,0	1	38	51	30	148

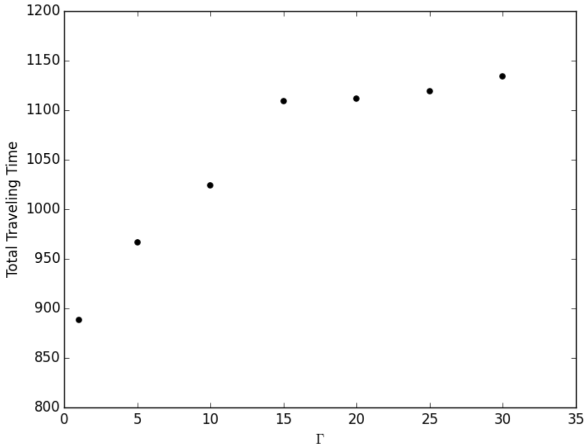
with minV, and CPU time = 3000 seconds for one run of model with other objectives. For example $E-n101-k8$, we set CPU time = 300 seconds for one run of model with minV, and CPU time = 9000 seconds for one run of model with other objectives.

For testing the robust models, we randomly generate $\hat{t}_{ij}$ in range $[0, 20]$, and $\hat{q}_i = 1$. From Tables 7–9, we can see that the performances of CVRP and SDVRP are at similar level in terms of the objective values within the limited CPU time. From Figure 4, we can see that the total travel time increases when Γ_T increases, because more $\hat{t}_{ij}$ in the solution are considered. As the near-optimal solutions are obtained from heuristic algorithms, we can see that the best-so-far solution for the smaller Γ_T may not always provide smaller total travel time than the best-so-far solution for the larger Γ_T.

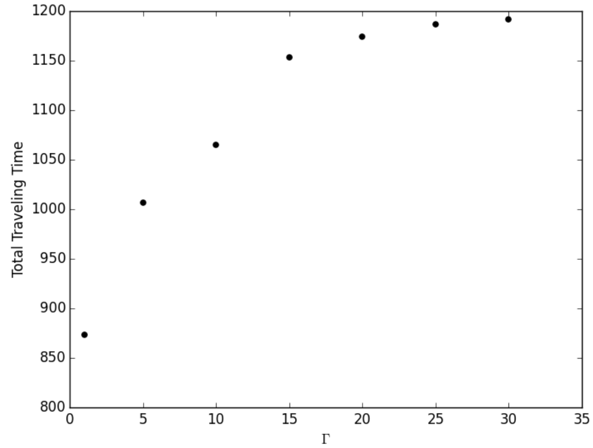
To test the RCVRP-minD with Γ_Q, $\hat{q}_i$ are randomly generated in range $[0, 3]$. From Figure 5, we can see that the summation of demand-weighted arrival times increases when Γ_Q increases, because more $\hat{q}_i$ in the solution are considered. As the near-optimal solutions are obtained from heuristic algorithms, we can see that the best-so-far solution for the smaller Γ_Q may not always provide smaller objective value than the best-so-far solution for the larger Γ_Q.

6 Conclusion

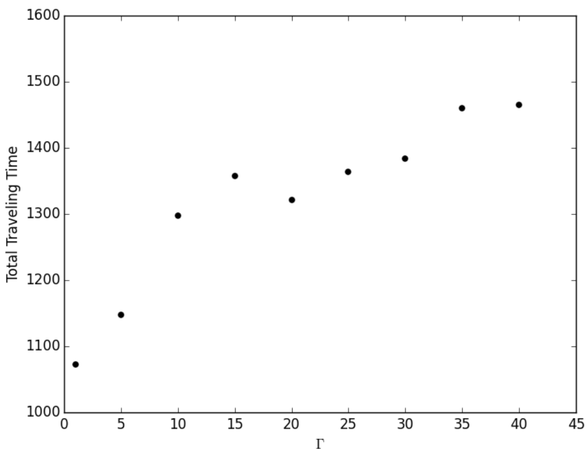
In this project, we explicitly considered the uncertainty in travel times and demands when planning vehicle routes for delivering critical supplies to the affected population in need after a disaster. To consider different scenarios, we proposed robust optimization approaches for the capacitated vehicle routing problems and the split delivery vehicle routing problems under uncertainty, respectively.



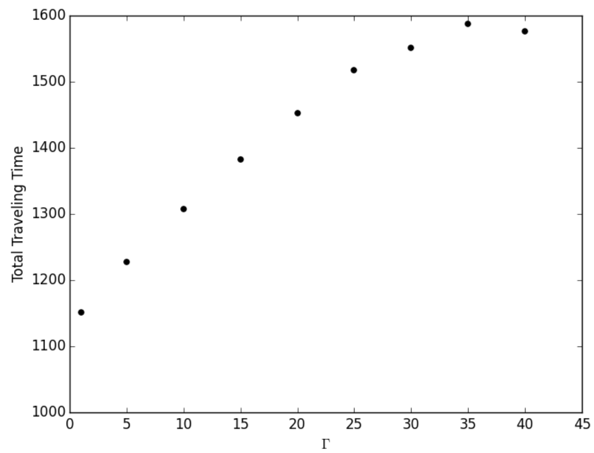
(a) RCVRP, $A-n32-k5$



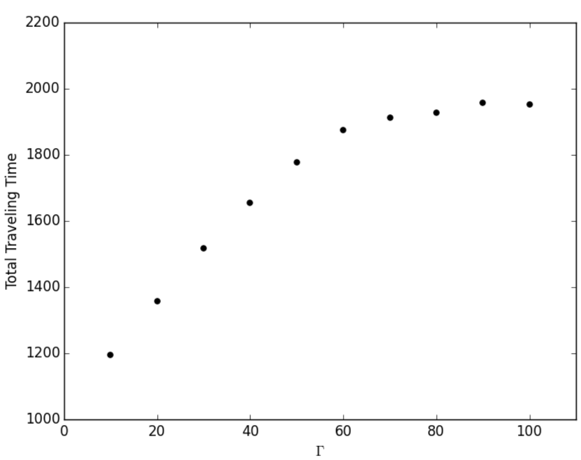
(b) RSDVRP, $A-n32-k5$



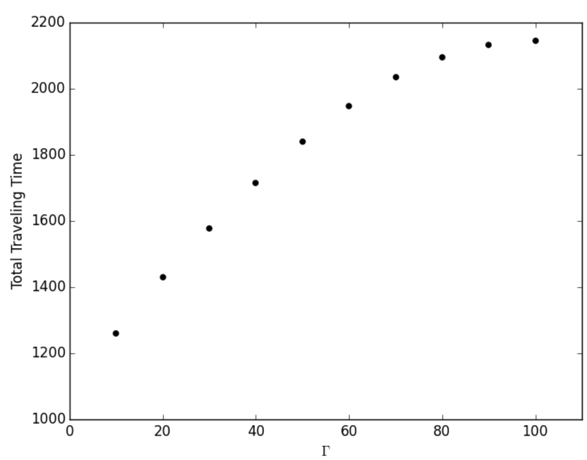
(c) RCVRP, $A-n44-k7$



(d) RSDVRP, $A-n44-k7$

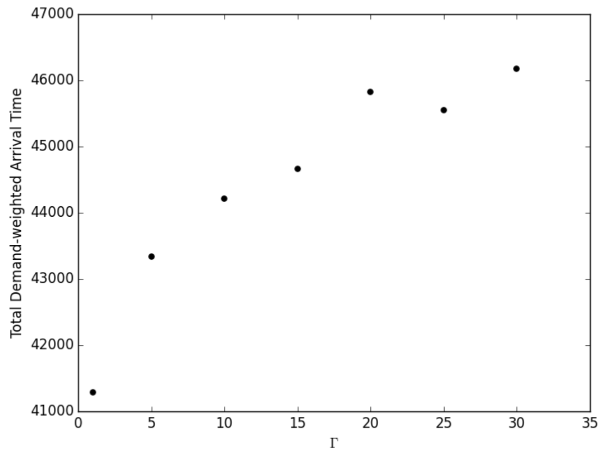


(e) RCVRP, $E-n101-k8$

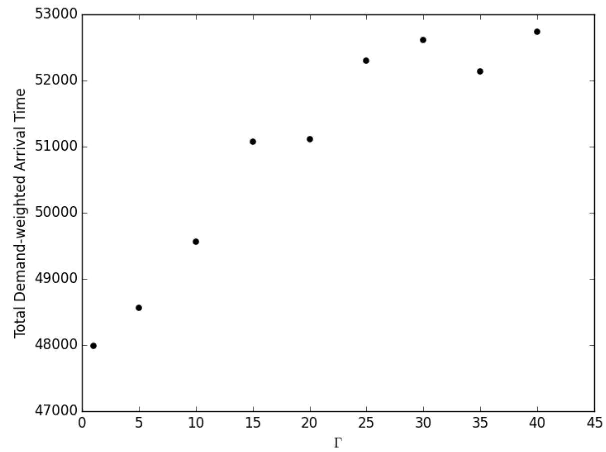


(f) RSDVRP, $E-n101-k8$

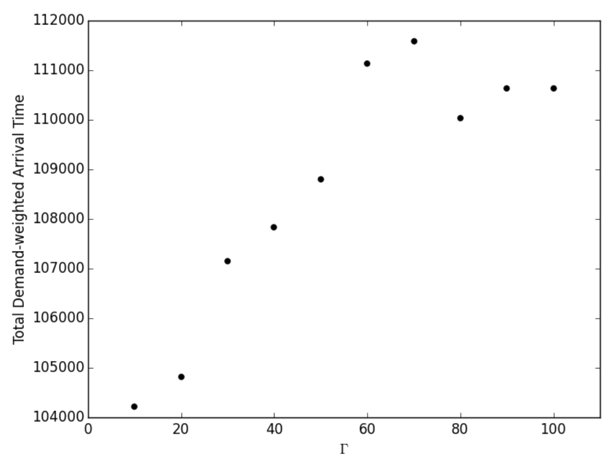
Figure 4: Robust Models of minT with Γ_T



(a) RCVRP, $A - n32 - k5$



(b) RCVRP, $A - n44 - k7$



(c) RCVRP, $E - n101 - k8$

Figure 5: Robust Models of minD with Γ_Q

Table 4: Results of Example 3

	Model	Routes	TV	TT	SUM	MAX	DA
		Nominal Model					
$K = 3$	CVRP-minV	[0,1,0], [0,2,0], [0,3,0]	3	28	14	6	56
	CVRP-minT	[0,1,0], [0,2,0], [0,3,0]	3	28	14	6	56
	CVRP-minS	[0,1,0], [0,2,0], [0,3,0]	3	28	14	6	56
	CVRP-minL	[0,1,0], [0,2,0], [0,3,0]	3	28	14	6	56
	CVRP-minD	[0,1,0], [0,2,0], [0,3,0]	3	28	14	6	56
$K = 2$	SDVRP-minT	[0,2,1,0], [0,2,3,0]	2	27	25	11	84
	SDVRP-minS	[0,3,2,0], [0,2,1,0]	2	27	25	11	80
	SDVRP-minL	[0,2,1,0], [0,3,1,0]	2	34	30	11	76
	SDVRP-minD	[0,2,1,0], [0,3,1,0]	2	34	30	11	76
$K = 3$	SDVRP-minT	[0,2,1,0], [0,2,3,0]	2	27	25	11	84
	SDVRP-minS	[0,1,0], [0,2,0], [0,3,0]	3	28	14	6	56
	SDVRP-minL	[0,1,0], [0,2,0], [0,3,0]	3	28	14	6	56
	SDVRP-minD	[0,1,0], [0,2,0], [0,3,0]	3	28	14	6	56
		Robust Model, Full Uncertainty					
	Model	Routes	TV	TT	SUM	MAX	DA
$K = 3$	RCVRP-minV	[0,1,0], [0,2,0], [0,3,0]	3	244	122	106	610
	RCVRP-minT	[0,1,0], [0,2,0], [0,3,0]	3	244	122	106	610
	RCVRP-minS	[0,1,0], [0,2,0], [0,3,0]	3	244	122	106	610
	RCVRP-minL	[0,1,0], [0,2,0], [0,3,0]	3	244	122	106	610
	RCVRP-minD	[0,1,0], [0,2,0], [0,3,0]	3	244	122	106	610
$K = 3$	RSDVRP-minT	[0,2,0], [0,2,1,3,0], [0,3,1,2,0]	3	104	142	38	237
	RSDVRP-minS	[0,3,0], [0,2,0], [0,2,1,0]	3	156	41	18	170
	RSDVRP-minL	[0,2,1,0], [0,2,3,0], [0,2,0]	3	164	56	18	210
	RSDVRP-minD	[0,3,0], [0,2,0], [0,2,1,0]	3	156	41	18	170

Table 5: Results of Robust Models of MinT with Γ_T, Simple Examples

	Example 1 CVRP, SDVRP		Example 2 CVRP, SDVRP		Example 3 SDVRP	
Γ_T	Route	TT	Route	TT	Route	TT
1	[0,2,1,3,0]	33	[0,3,1,2,0]	26	[0,2,0], [0,3,1,2,0], [0,2,1,3,0]	63
2	[0,3,1,2,0]	38	[0,2,1,3,0]	31	[0,3,0], [0,3,1,2,0], [0,2,1,3,0]	74
3	[0,3,1,2,0]	42	[0,2,1,3,0]	35	[0,2,0], [0,3,1,2,0], [0,2,1,3,0]	80
4	[0,3,1,2,0]	45	[0,2,1,3,0]	38	[0,2,0], [0,3,1,2,0], [0,2,1,3,0]	86

Table 6: Results of Robust CVRP Models of MinD with Γ_Q, Simple Examples

	Example 1		Example 2		Example 3	
Γ_Q	Route	DA	Route	DA	Route	DA
1	[0,3,2,1,0]	153	[0,2,1,3,0]	127	[0,1,0], [0,2,0], [0,3,0]	594
2	[0,3,2,1,0]	176	[0,2,1,3,0]	140	[0,1,0], [0,2,0], [0,3,0]	603
3	[0,3,2,1,0]	183	[0,2,1,3,0]	148	[0,1,0], [0,2,0], [0,3,0]	610

Table 7: Results of Example $A - n32 - k5$

Nominal Model					
Model	TV	TT	SUM	MAX	DA
CVRP-minV	5	2113	7254	542	87513
CVRP-minT	5	795	3085	207	35562
CVRP-minS	5	973	2312	142	29408
CVRP-minL	5	942	2536	144	32153
CVRP-minD	5	950	2410	138	28463
SDVRP-minT	5	886	3735	263	44024
SDVRP-minS	5	959	2325	145	29622
SDVRP-minL	5	1040	2478	144	30895
SDVRP-minD	5	1040	2478	144	30895
Robust Model, Full Uncertainty					
Model	TV	TT	SUM	MAX	DA
RCVRP-minV	5	2522	7743	566	104271
RCVRP-minT	5	1154	3969	239	51720
RCVRP-minS	5	1295	3237	211	44558
RCVRP-minL	5	1290	3468	192	45149
RCVRP-minD	5	1311	3623	213	42902
RSDVRP-minT	5	1217	3707	226	48005
RSDVRP-minS	5	1330	3210	191	44351
RSDVRP-minL	5	1358	3679	204	47985
RSDVRP-minD	5	1354	3544	205	43189

Table 8: Results of Example $A - n44 - k7$

Nominal Model					
Model	TV	TT	SUM	MAX	DA
CVRP-minV	6	2614	9131	512	107424
CVRP-minT	6	970	3862	208	53915
CVRP-minS	6	1272	2814	155	38074
CVRP-minL	6	1119	3037	123	40376
CVRP-minD	6	1107	2822	148	34198
SDVRP-minT	6	1009	3764	199	49064
SDVRP-minS	6	1118	2944	180	39282
SDVRP-minL	6	1118	2944	180	39282
SDVRP-minD	6	1121	2924	144	36959
Robust Model, Full Uncertainty					
Model	TV	TT	SUM	MAX	DA
RCVRP-minV	7	3065	10144	580	137494
RCVRP-minT	7	1451	4968	253	66387
RCVRP-minS	7	1643	3793	187	54005
RCVRP-minL	7	1838	4184	204	59997
RCVRP-minD	7	1769	3944	189	51895
RSDVRP-minT	7	1482	5571	306	73005
RSDVRP-minS	7	1735	3934	206	52893
RSDVRP-minL	7	1917	4783	205	56648
RSDVRP-minD	7	1727	4112	198	51121

Table 9: Results of Example $E-n101-k8$

Nominal Model					
Model	TV	TT	SUM	MAX	DA
CVRP-minV	8	2260	14751	360	204559
CVRP-minT	8	1031	6604	186	102722
CVRP-minS	8	1166	4436	134	66359
CVRP-minL	8	1154	5333	121	80143
CVRP-minD	8	1166	4436	134	66359
SDVRP-minT	8	1014	7294	223	101344
SDVRP-minS	8	1159	4970	153	68897
SDVRP-minL	8	1138	5843	133	79541
SDVRP-minD	8	1178	5112	146	67931
Robust Model, Full Uncertainty					
Model	TV	TT	SUM	MAX	DA
RCVRP-minV	8	3278	20644	476	324160
RCVRP-minT	8	1717	11090	258	177772
RCVRP-minS	8	1830	8378	233	131630
RCVRP-minL	8	1897	10296	199	164603
RCVRP-minD	8	1914	8623	229	128447
RSDVRP-minT	8	1766	11826	356	181360
RSDVRP-minS	8	1792	9249	254	132370
RSDVRP-minL	8	1886	9998	236	144022
RSDVRP-minD	8	1882	9783	275	130069

We presented different robust CVRP and SDVRP models depending on different objectives and the source of uncertain factors (e.g., travel time and demand) to mitigate the impact of the uncertainty and eventually to achieve enhanced resilience in the aftermath of disasters. From the examples presented in this report, we can see that the SDVRP can provide more flexible solutions when the demands from nodes are relatively large. The SDVRP model also avoids the arcs with large uncertain travel time selected in the optimal solution by allowing the visitation of nodes multiple times from different arcs. The small-sized problems can be solved by the solvers such as Gurobi in conjunction with Julia and Jump. For large-scale problems, we presented two-stages heuristic methods combining the insertion algorithm and tabu search to solve the VRP models.

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