A. Supplementary Material

A.1. Additional Details for Proof of Theorem 1

We have assumed that all paths from v_i to v_ℓ go through v_j, and we wish to show that there exist functions $\tilde{F}$ and $\tilde{H}$ such that

$$v_j = F(v_i) := \tilde{F}(v_i, v_A)$$
$$v_\ell = H(v_j) := \tilde{H}(v_j, v_B)$$

and all nodes in v_A and v_B are either not reachable from v_i or have no path to v_ℓ. Note that if a variable v_k is not reachable from v_i, then the scalar value v_ℓ may still depend on v_k, but the derivatives $\frac{d^q v_\ell}{dv_i^q}$ do not depend on v_k, so it is safe to treat v_k as a fixed constant relative to the dual number $\langle v_\ell, dv_i \rangle_q$. If v_k has no path to v_ℓ, neither v_ℓ nor the derivatives $\frac{d^q v_\ell}{dv_i^q}$ depend on v_k.

The construction of $\tilde{F}$ is easy:

$$\tilde{F}(v_i, v_A) := f_{ij}(v_i, v_{0:i-1})$$

The nodes $v_{0:i-1}$ precede v_i in the topological ordering, and hence have no path from i.

To construct $\tilde{H}$, we reason about the partial computation $f_{j\ell}(v_{0:\ell})$ from v_j to v_ℓ. Recall that this is defined recursively starting with $\varphi_\ell(v_k)_{k \prec \ell}$, and terminating whenever a variable v_p is reached for $p \leq j$. Consider any such variable v_p that is reached by the calculation. Then v_p must satisfy $p \prec q$ for $q > j$ (otherwise the recursion would not reach v_p), and, furthermore, there must be a path from v_q to v_ℓ (otherwise the recursion does not reach v_q). In other words, there is a path $v_p, v_q, \ldots, v_\ell$ for $q > j$. However, this implies that v_p is *not* reachable from v_i, otherwise we would contradict the assumption that all paths from v_p to v_ℓ go through v_j. Therefore, if we consider the subset of variables $v_B \subseteq v_{0:j-1}$ on which $f_{j\ell}$ depends, none of these variables is reachable from v_i. Therefore we can write:

$$\tilde{H}(v_j, v_B) = f_{j\ell}(v_j, v_B)$$

where we omit from $f_{j\ell}$ the arguments on which it does not depend. This completes the argument.

A.2. Additional Parameter Estimation Experiments

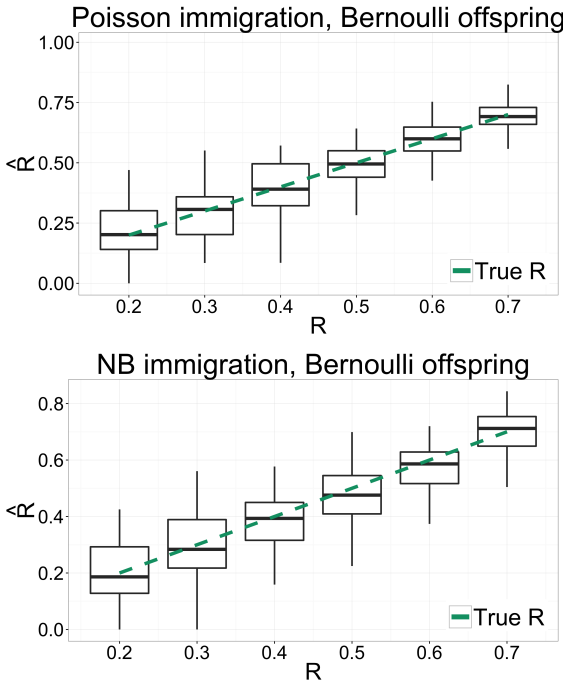

Figure 5. Estimates of R for two additional models. Titles indicate immigration and offspring distribution. 50 trials summarized as box plot for each model, parameter combination.