Supplementary Material for McGan: Mean and Covariance Feature Matching GAN

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A. Subspace Matching Interpretation of Covariance Matching GAN

Let $\Delta_\omega = \Sigma_\omega(\mathbb{P}) - \Sigma_\omega(\mathbb{Q})$. Δ_ω is a symmetric matrix but not PSD, which has the property that its eigenvalues λ_j are related to its singular values as given by: $\sigma_j = |\lambda_j|$ and its left and right singular vectors coincides with its eigenvectors and satisfy the following equality $u_j = \text{sign}(\lambda_j)v_j$. One can ask here if we can avoid having both U, V in the definition of IPM_Σ since at the optimum $u_j = \pm v_j$. One could consider $\delta E_\omega(\mathbb{P}_r, \mathbb{P}_\theta)$ defined as follows:

$$\max_{\omega \in \Omega, U \in \mathcal{O}_{m,k}} \underbrace{\mathop{\mathbb{E}}_{x \sim \mathbb{P}_r} \left\| U \Phi_\omega(x) \right\|^2}_{\text{Energy in the subspace of real data}} - \underbrace{\mathop{\mathbb{E}}_{z \sim p_z} \left\| U \Phi_\omega(g_\theta(z)) \right\|^2}_{\text{Energy in the subspace of fake data}},$$

and then solve for $\min_{g_\theta} \delta E_\omega(\mathbb{P}_r, \mathbb{P}_\theta)$. Note that:

$$\begin{aligned}\delta E_\omega(\mathbb{P}_r, \mathbb{P}_\theta) &= \max_{\omega \in \Omega, U \in \mathcal{O}_{m,k}} Trace(U^\top(\Sigma_\omega(\mathbb{P}_r) - \Sigma_\omega(\mathbb{P}_\theta))U) \\ &= \max_{\omega \in \Omega} \sum_{i=1}^{k} \lambda_i(\Delta_\omega)\end{aligned}$$

δE_ω is not symmetric furthermore the sum of those eigenvalues is not guaranteed to be positive and hence δE_ω is not guaranteed to be non negative, and hence does not define an IPM. Noting that $\sigma_i(\Delta_\omega) = |\lambda_i(\Delta_\omega)|$,we have that:

$$\text{IPM}_\Sigma(\mathbb{P}_r, \mathbb{P}_\theta) = \sum_{i=1}^{k} \sigma_i(\Delta_\omega) \geq \sum_{i=1}^{k} \lambda_i(\Delta_\omega) = \delta E_\omega(\mathbb{P}_r, \mathbb{P}_\theta).$$

Hence δE is not an IPM but can be optimized as a lower bound of the IPM_Σ. This would have an energy interpretation as in the energy based GAN introduced recently (Zhao et al., 2017): the discriminator defines a subspace that has higher energy on real data than fake data, and the generator maximizes his energy in this subspace.

B. Mean and Covariance Matching Loss Combinations

We report below samples for McGan, with different $\text{IPM}_{\mu,q}$ and IPM_Σ combinations. All results are reported for the same architecture choice for generator and discriminator, which produced qualitatively good samples with IPM_Σ (Same one reported in Section 6 in the main paper). Note that in Figure 7 with the same hyper-parameters and architecture choice, WGAN failed to produce good sample. In other configurations training converged.

Figure 7. Cifar-10: Class-conditioned generated samples with $\text{IPM}_{\mu,1}$(WGAN). Within each column, the random noise z is shared, while within the rows the GAN is conditioned on the same class: from top to bottom *airplane, automobile, bird, cat, deer, dog, frog, horse, ship, truck*.

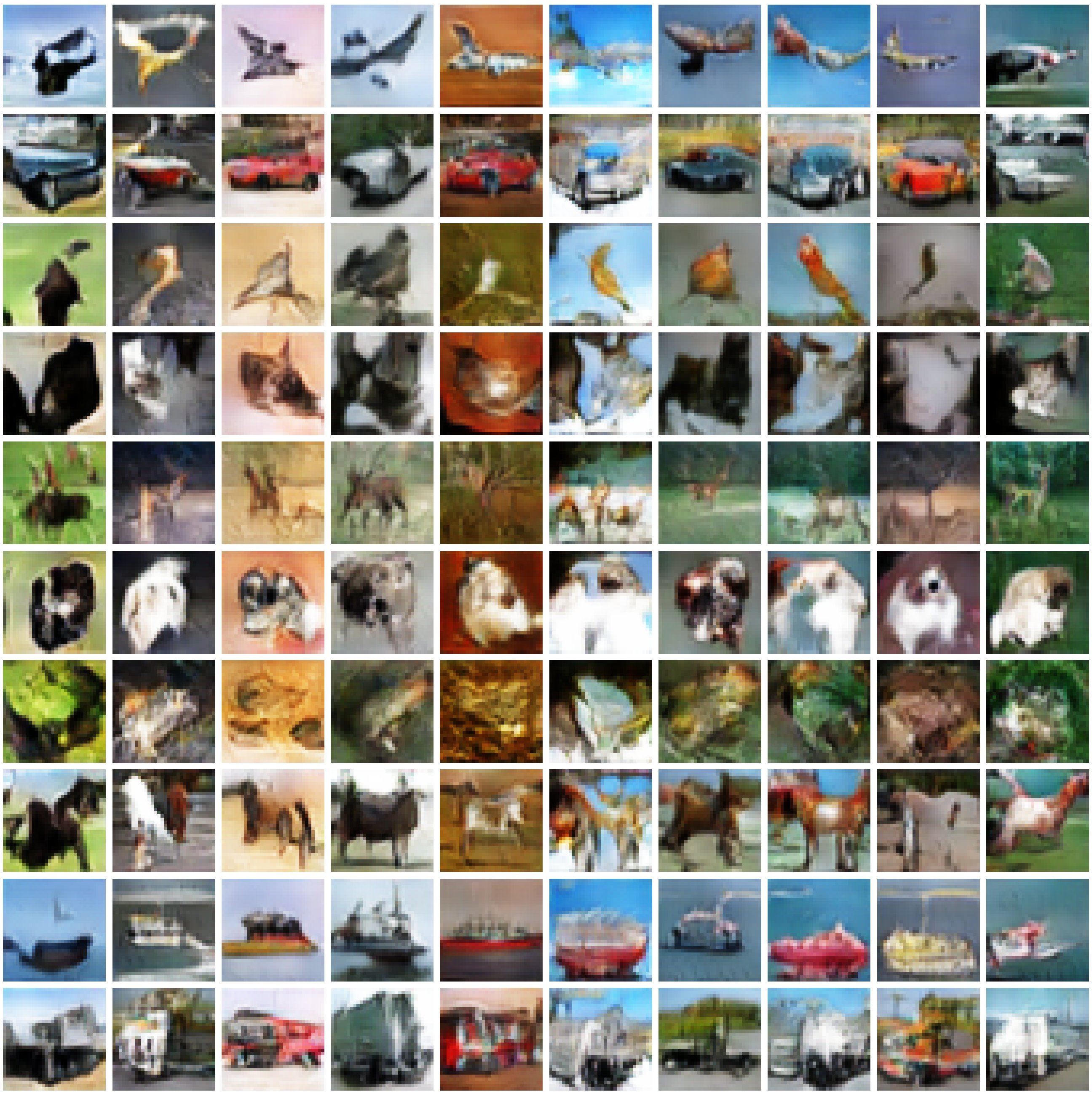

Figure 8. Cifar-10: Class-conditioned generated samples with $\text{IPM}_{\mu,2}$. Within each column, the random noise z is shared, while within the rows the GAN is conditioned on the same class: from top to bottom *airplane, automobile, bird, cat, deer, dog, frog, horse, ship, truck*.

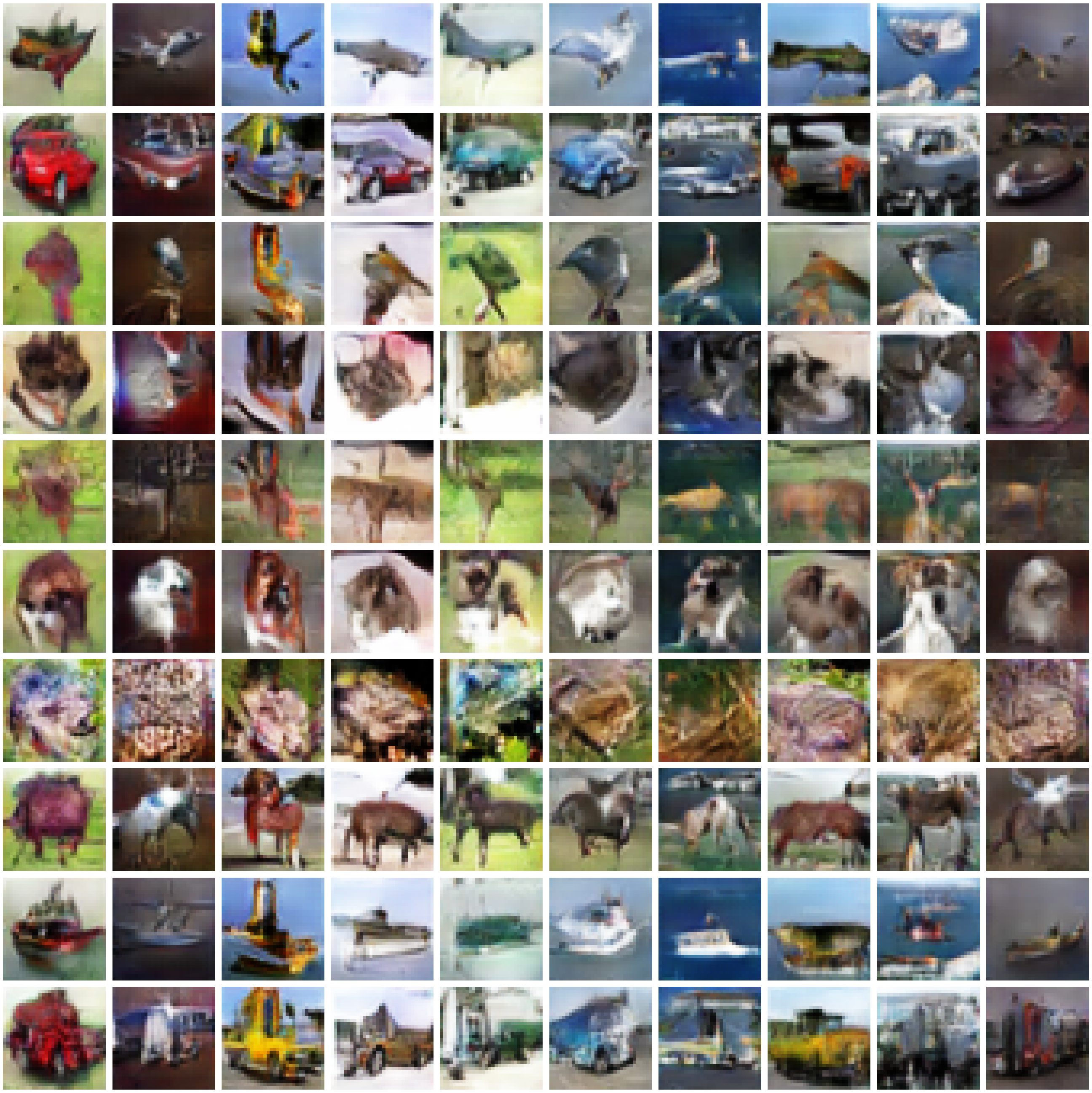

Figure 9. Cifar-10: Class-conditioned generated samples with IPM_{Σ}. Within each column, the random noise z is shared, while within the rows the GAN is conditioned on the same class: from top to bottom *airplane, automobile, bird, cat, deer, dog, frog, horse, ship, truck*.

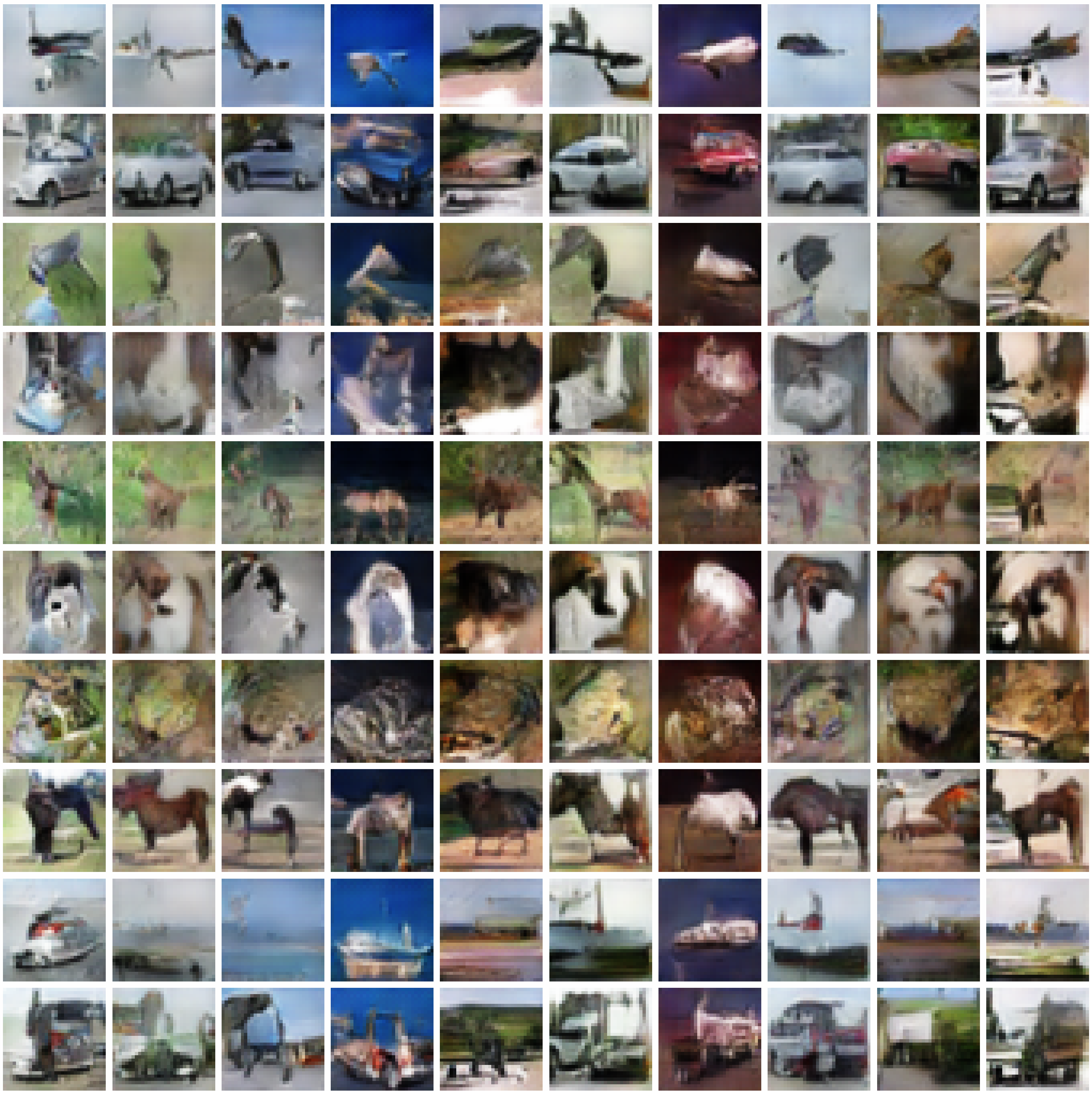

Figure 10. Cifar-10: Class-conditioned generated samples with $\mathrm{IPM}_{\mu,1}$+ IPM_{Σ}. Within each column, the random noise z is shared, while within the rows the GAN is conditioned on the same class: from top to bottom *airplane, automobile, bird, cat, deer, dog, frog, horse, ship, truck*.

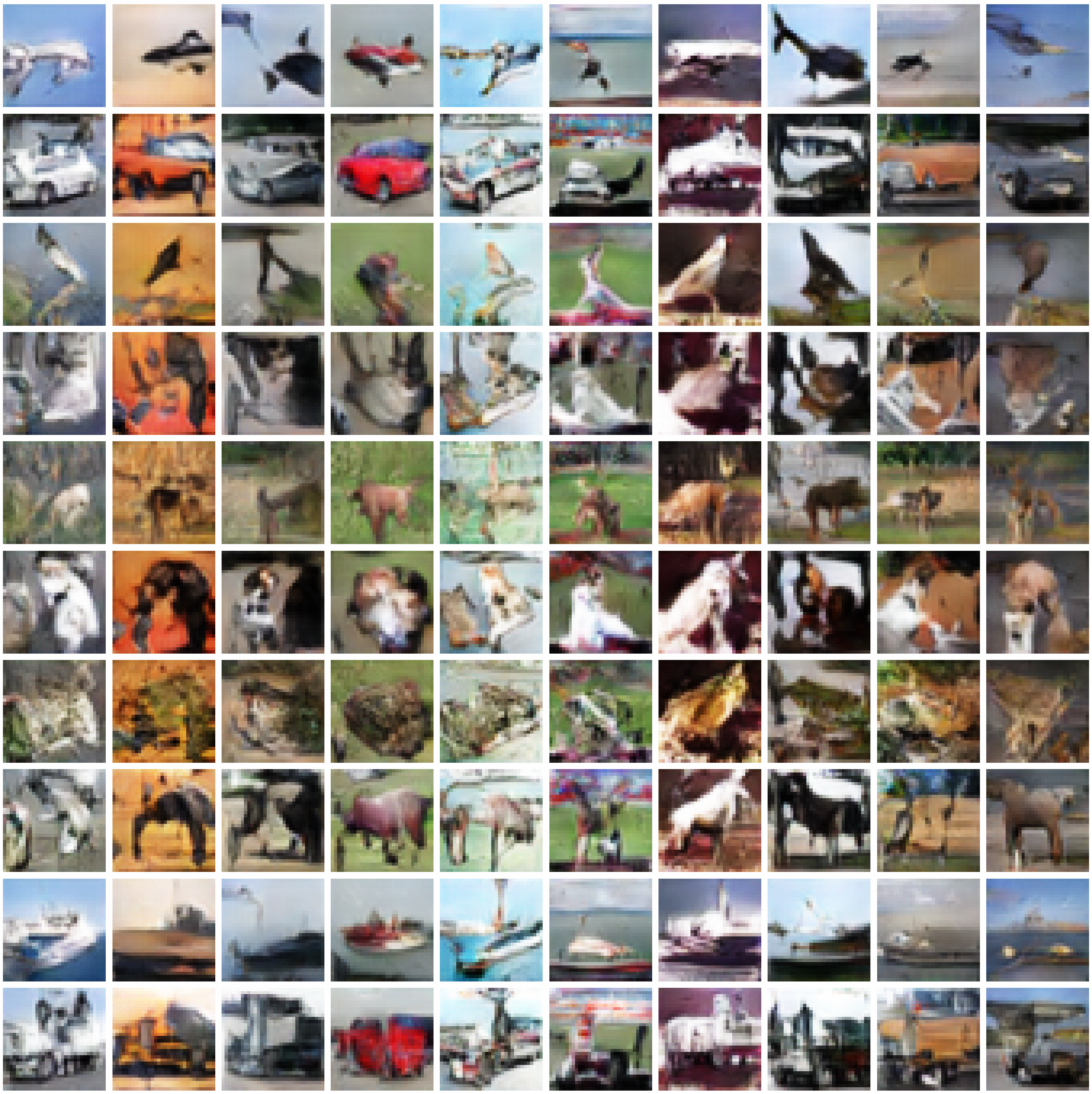

Figure 11. Cifar-10: Class-conditioned generated samples with $\text{IPM}_{\mu,2}$+ IPM_{Σ}. Within each column, the random noise z is shared, while within the rows the GAN is conditioned on the same class: from top to bottom *airplane, automobile, bird, cat, deer, dog, frog, horse, ship, truck*.