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Optimisation and Approximation in Abstract Argumentation: The Case of Admissibility

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Abstract

We propose two soft notions of the notion of admissibility in abstract argumentation. The first one weakens the defence notion by allowing, to a certain degree, undefended attacks, and the second one allows, to a certain degree, conflicts within sets of arguments. We analyse these new semantical notions based on the computational complexity of optimisation and approximation. Finally, we discuss and analyse soft notions for preferred semantics.

1 Introduction

Abstract argumentation frameworks (AFs) (Dung 1995) model rational decision-making based on the representation of arguments and their relationships as a directed graph. Here, *arguments* are identified by vertices, and an *attack* from one argument to another is represented as a directed edge. The reasoning process involves identifying sets of arguments (*extensions*) that can be considered *jointly* acceptable. One of the most basic notions here is *admissibility*, which states that an acceptable set is conflict-free and should defend itself against any threat.

In recent years, *approximate* notions of admissibility have gained some interest (Craandijk and Bex 2020; Delobelle, Mailly, and Rossit 2023; Kuhlmann and Thimm 2019; Malmqvist et al. 2020; Malmqvist 2022; Thimm 2021). In particular, the recent editions of the *International Competition of Computational Models of Argumentation* (ICCMA)[1] have had *approximation tracks*. However, despite the name, these tracks are actually about *heuristic algorithms* for solving decision problems and the term *approximation algorithm* (Vazirani 2001) actually refers to an algorithm that solves an *optimisation problem* and has a theoretically guaranteed *approximation quality*. In this work, we follow the latter interpretation as well. In a recent work, Thimm (2024) presented soft notions of stable semantics for abstract argumentation frameworks and introduced optimisation problems that model reasoning with these notions, together with an analysis of the approximation complexity of these problems. Here, we continue this work and analyse two soft notions of *admissibility*, denoted by k-*admissibility* and k-*admissibility**. The first notion weakens the constraint of full defence by allowing sets of arguments to be k-*admissible* if those sets can defend themselves against at least k arguments. The second notion weakens the constraint of *conflict-freeness* by saying that a set of arguments is k-*admissible** if the set defends itself against every attack from the outside and respects at least k attacks (we will provide a formal treatment of these concepts later). These weaker notions allow us, for example, to rank sets of arguments based on their "closeness" to being admissible, cf. (Skiba et al. 2021). Such a notion of closeness can be helpful in finding the closed set of arguments that satisfy conditions that are not satisfied by any acceptable set.

For both notions (k-admissibility and k-admissibility*), we consider the optimisation problem of finding the maximum value k such that a given argument is in a k-*admissible* (resp. k-*admissible**) set. We analyse the computational complexity of these *exact* problems and show that they are both $\mathbf{FP}^{\mathbf{NP[log]}}$-complete. Furthermore, we analyse the hardness of *approximating* the solutions to these optimisation problems and show that the problem is $\mathbf{Poly}$-$\mathbf{APX}$-complete for k-*admissibility* and $\mathbf{log}$-$\mathbf{APX}$-hard for k-*admissibility** under $\mathbf{PTAS}$-reductions. So both problems are generally hard to approximate. Based on the notions of k-admissibility/k-admissibility* we also briefly consider the k-preferred/k-preferred* semantics, which generalise the classical preferred semantics for abstract argumentation frameworks (recall that a preferred extension of an AF is the subset-maximal admissible set).

The rest of this paper is structured as follows. In Section 2 we recall necessary background information. Section 3 discusses our two variants for softening admissibility and analyses the corresponding optimisation problems. The softening of the preferred semantics is discussed in Section 4. Section 5 concludes the paper. All proofs can be found in the supplementary material[2]. The propositions and proofs mirror the results of (Thimm 2024) for stability and are adapted for admissibility.

2 Preliminaries

Abstract argumentation frameworks (Dung 1995) are a formalism that allows the representation of conflicts between pieces of information using arguments and attacks between arguments.

[1] http://argumentationcompetition.org/

[2] http://mthimm.de/misc/ksmt_kr24_appendix.pdf

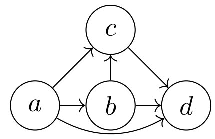

Figure 1: Abstract argumentation framework F_1 from Example 1.

Definition 1. An *abstract argumentation framework* (AF) is a directed graph $F = (A, R)$ where A is a finite set of *arguments* and R is an *attack relation* $R \subseteq A \times A$.

For an AF $F = (A, R)$, an argument a is said to *attack* an argument b if $(a, b) \in R$. We say that, a set $E \subseteq A$ *defends* an argument a if every argument $b \in A$ that attacks a is attacked by some $c \in E$. For $a \in A$, we define $a_F^- = \{b \mid (b, a) \in R\}$ and $a_F^+ = \{b \mid (a, b) \in R\}$. In other words, a_F^- is the set of attackers of a and a_F^+ is the set of arguments attacked by a. For a set of arguments $E \subseteq A$ we extend these definitions to E_F^+ and E_F^- via $E_F^+ = \bigcup_{a \in E} a_F^+$ and $E_F^- = \bigcup_{a \in E} a_F^-$, respectively. If the AF is clear in the context, we sometimes omit the index.

Semantics for AFs are mostly based on two basic concepts: *conflict-freeness* and *admissibility*.

Definition 2. Given $F = (A, R)$, a set $E \subseteq A$ is a

- *conflict-free* (cf) set iff $\forall a, b \in E$, $(a, b) \notin R$;
- *admissible* (ad) set iff it is *conflict-free* and it defends its elements;
- *preferred* extension (pr) iff it is a $\subseteq$-maximal admissible set;
- *stable* extension (st) iff E is conflict-free and $E \cup E_F^+ = A$.

We use $\mathsf{cf}(F)$ and $\mathsf{ad}(F)$ to denote the sets of conflict-free and admissible sets of an argumentation framework F, respectively, and $\mathsf{pr}(F)$ and $\mathsf{st}(F)$ to denote the preferred and stable extensions of F.

Example 1. Consider the AF F_1 depicted as a directed graph in Figure 1. F_1 has two admissible sets $E_1 = \{a\}$, $E_2 = \emptyset$. E_1 is a preferred and stable extension.

An argument a is *credulously accepted* in F with respect to the extension semantics σ if there exists a set $a \in E$ such that $E \in \sigma(F)$, argument a is *skeptically accepted* if $a \in E$ for all $E \in \sigma(F)$.

We continue work from (Thimm 2024), where two *soft* notions of stable semantics were introduced that weaken each aspect of stable semantics. For an AF $F = (A, R)$ and a set $S \subseteq A$ define

$$S_F^{\circledast} = \{(a, b) \in R \mid a \notin S \text{ or } b \notin S\}$$

In other words, $S_F^{\circledast}$ is the set of attacks *satisfied* by S, in particular, $S_F^{\circledast} = R$ iff S is conflict-free. An attack $(a, b) \in R \setminus S_F^{\circledast}$ is also called *violated attack* of F by S.

Definition 3. Let $F = (A, R)$ be an AF, $k \in \mathbb{N}$, $S \subseteq A$.

- S is a *k-stable extension* iff $S \in cf(F)$ and $|S \cup S_F^+| \geq k$.
- S is a *k-stable* extension* iff $S \cup S_F^+ = A$ and $|S_F^{\circledast}| \geq k$.

So S is a k-stable extension if S is conflict-free and contains and attacks at least k arguments. Observe that a $|A|$-stable extension is a stable extension. Furthermore, S is a k-stable* extension if all arguments are either contained in S or attacked by it, and S satisfies at least k attacks. Observe that a $|R|$-stable* extension is a stable extension. In (Thimm 2024), the reasoning problems of finding the maximal value for k such that an argument is contained in some k-stable/k-stable* extension, have been investigated in-depth, in particular showing that the *approximation complexity* for these problems are $\mathbf{Poly\text{-}APX}$-complete and $\mathbf{log\text{-}APX}$-hard, respectively (under $\mathbf{PTAS}$-reductions).

3 Soft Notion of Admissibility

In the remainder of this paper, we apply the methodology of (Thimm 2024) for the semantical concept of *admissibility* instead of *stability*. A number of papers have discussed weakening admissibility, such as the *weak admissibility* of (Baumann, Brewka, and Ulbricht 2022), where (in particular) attacks by self-contradicting arguments are ignored or the *graded semantics* of Grossi and Modgil (2019), where each attacker of an argument must be attacked by a certain number of arguments or the works by Vassiliades et al. (Vassiliades et al. 2023) and Dauphn (Dauphin 2020).

Admissibility is defined upon two clear concepts: in order for a set to be admissible it must be conflict-free and defend itself against all attacks. We consider now two soft notions of admissibility that each weakens one of these two constraints. In particular, we weaken the constraint that all attacks must be defended, which we call the "requirement of full defense" in the following, in Section 3.1 and that a set must be conflict-free in Section 3.2.

3.1 Softening the Requirement of Full Defense

We start by softening the requirement that all attacks onto a set must be defended, but we still require that the set must be conflict-free. For $F = (A, R)$ and $S \subseteq A$ let

$$S_F^{\diamondsuit} = \{a \in A | \exists b \in S : (a, b) \in R \land \neg\exists c \in S : (c, a) \in R\}$$

denote the set of arguments a that attack some argument in S but are not attacked by some argument from S. We call $S_F^{\diamondsuit}$ the set of *unattacked attackers*. Correspondingly, let

$$S_F^{\#} = A \setminus S_F^{\diamondsuit}$$

denote the complement set, i. e., the set of arguments that are either in S, do not attack S, or attack S and are attacked by S.

Definition 4. Let $F = (A, R)$ be an AF, $k \in \mathbb{N}$ and $S \subseteq A$. We say that S is a *k-admissible set* iff $S \in cf(F)$ and $|S_F^{\#}| \geq k$.

In other words, a conflict-free set S is k-admissible if at least k arguments are "compatible" with S being admissible (so are either in S, not attacking S, or properly defended against). We denote the set of *k-admissible* sets for an AF $F = (A, R)$ as $ad_k(F)$.

Proposition 1. Let $F = (A, R)$, $k \in \mathbb{N}$, and $S \subseteq A$.

1. S is an $|A|$-admissible set iff S is an admissible set.
2. If S is a k-admissible set then S is a k'- admissible set for all $k' < k$.
3. $S \in cf(F)$ iff S is a 0-admissible set.
4. If S is a k-stable extension, then S is a k-admissible set.

In particular, item 4 from the above proposition shows that our new soft notion of admissibility behaves to the k-stable semantics from (Thimm 2024), as classical admissibility behaves to stability (in the sense that every stable extension is admissible).

Example 2. Consider again F_1 from Example 1. Then since $ad(F_1) = ad_{|A|}(F_1)$ we have $ad_4(F_1) = \{\{a\}, \emptyset\}$. For the remaining $k \in \{1, 2, 3\}$ we get:

$$ad_3(F_1) = \{\{b\}\} \cup ad_4(F_1)$$
$$ad_2(F_1) = \{\{c\}\} \cup ad_3(F_1)$$
$$ad_1(F_1) = \{\{d\}\} \cup ad_2(F_1)$$

Reasoning with k-admissibility is as hard as reasoning with admissibility (Dvořák and Dunne 2017).

Proposition 2. Let $k \in \mathbb{N}$, $F = (A, F)$, $S \subseteq A$, and $a \in A$.

1. Deciding whether $S \in ad_k(F)$ is in $\mathbf{P}$.
2. Deciding whether $a \in \bigcup ad_k(F)$ is $\mathbf{NP}$-complete.
3. Deciding whether $a \in \bigcap ad_k(F)$ is trivial.
4. Deciding $ad_k(F) \neq \emptyset$ is trivial.
5. Deciding $ad_k(F) \neq \{\emptyset\}$ is $\mathbf{NP}$-complete.

The larger the value k, the "closer" a k-admissible set is to being admissible. Therefore, it is interesting to find the largest k' such that an argument a is part of a k'-admissible set. We therefore consider the optimisation problem[3]:

MAXADM
Input: An AF $F = (A, R)$, $a \in A$
Output: $mad(F, a) = max_k\{k | a \in \bigcup ad_k(F)\}$

Note that self-attacking arguments cannot be in any k-admissible set, so for any non-self attacking argument a we get $mad(F, a) \geq 1$.

Example 3. Let us continue with Example 2. Then the solutions of MAXADM for arguments a, b, c, d with respect to F_1 are:

$$mad(F_1, a) = 4 \quad mad(F_1, b) = 3$$
$$mad(F_1, c) = 2 \quad mad(F_1, d) = 1$$

For the (exact) optimisation of MAXADM we show that this problem is $\mathbf{FP}^{\mathbf{NP[log]}}$-complete.

Proposition 3. MAXADM is $\mathbf{FP}^{\mathbf{NP[log]}}$-complete.

Next we look at the problem of approximating the solution to MAXADM. So we want to maximise the *approximation ratio* AR for an algorithm, with $AR = \frac{APP}{OPT}$, where OPT is the optimal solution of MAXADM and $APP \leq OPT$ is the solution of an approximation algorithm. Then $AR = 1$ indicates an optimal solution and a lower value of AR indicates a worse approximation quality of an algorithm. We show that MAXADM is $\mathbf{Poly\text{-}APX}$-complete under a $\mathbf{PTAS}$-reductions. $\mathbf{Poly\text{-}APX}$ is a complexity class consisting of optimisation problems that can be approximated by polynomial-time algorithms with an approximation rate of polynomial size with respect to the input size, i. e., for MAXADM we can guarantee $AR \geq \frac{1}{f(|A|)}$ where f is a polynomial and A is the set of arguments of AF $F = (A, R)$.

Proposition 4. MAXADM is $\mathbf{Poly\text{-}APX}$-*complete* under $\mathbf{PTAS}$-reductions.

The above result (and also its proof technique) mirrors the result we have for k-stability in (Thimm 2024).

[3] We define $max\ \emptyset = -\infty$

3.2 Softening the Requirement of Conflict-Freeness

The relaxation of the conflict-free requirement is similar to the k-stable* semantics. While the k-stable* semantics requires a full range, we require defence against any attack from outside the set. On the other hand, we allow for conflicts within the set.

Definition 5. Let $F = (A, R)$ be an AF, $k \in \mathbb{N}$, and let $S \subseteq A$. We say that S is a *k-admissible* set* iff $S^-_F \setminus S \subseteq S^+_F$ and $|S^{\circledast}_F| \geq k$.

In other words, a set S which defends itself from every attack against outside is a k-admissible* set if the number of attacks satisfied by this set is at least k. We denote the set of k-admissible* sets for an AF $F = (A, R)$ by $ad^*_k(F)$.

Proposition 5. Let $F = (A, R)$, $k \in \mathbb{N}$, and $S \subseteq A$.

1. S is a $|R|$-admissible* set iff S is an admissible set.
2. If S is a k-admissible* set then S is a k'-admissible* set for all $k' < k$.
3. If S is a k-stable* extension then S is a k-admissible* set.

Example 4. Consider F_1 from Example 1. Then we have:

$$ad^*_6(F_1) = \{\{a\}, \emptyset\}$$
$$ad^*_5(F_1) = \{\{a, b\}, \{a, c\}, \{a, d\}\} \cup ad^*_6(F_1)$$
$$ad^*_4(F_1) = ad^*_5(F_1)$$
$$ad^*_3(F_1) = \{\{a, b, c\}, \{a, b, d\}, \{a, c, d\}\} \cup ad^*_4(F_1)$$
$$ad^*_2(F_1) = ad^*_3(F_1)$$
$$ad^*_1(F_1) = ad^*_2(F_1)$$
$$ad^*_0(F_1) = \{\{a, b, c, d\}\} \cup ad^*_1(F_1)$$

Similar to k-admissibility, reasoning with k-admissibility* is as hard as reasoning with admissibility (Dvořák and Dunne 2017).

Proposition 6. Let $k \in \mathbb{N}$, $F = (A, F)$, $S \subseteq A$, and $a \in A$.

1. Deciding whether $S \in ad^*_k(F)$ is in $\mathbf{P}$.
2. Deciding whether $a \in \bigcup ad^*_k(F)$ is $\mathbf{NP}$-complete.
3. Deciding whether $a \in \bigcap ad^*_k(F)$ is trivial.
4. Deciding $ad^*_k(F) \neq \emptyset$ is trivial.
5. Deciding $ad^*_k(F) \neq \{\emptyset\}$ is $\mathbf{NP}$-complete.

For k-admissible* sets, we see that the larger the value of k, the closer these sets are to being admissible. So defining the optimisation problem in a similar way to MAXADM is the obvious next step.

MAXADM*
Input: An AF $F = (A, R)$, $a \in A$
Output: $mad^*(F, a) = max_k\{k|a \in \bigcup ad^*_k(F)\}$

Note that $A \in ad^*_0(F)$ for each AF $F = (A, R)$, so $mad^*(F, a) \geq 0$ for each argument $a \in A$.

Example 5. Let us continue with Example 4. The solution of MAXADM* for arguments a, b, c, d for F_1 are:

$$mad^*(F_1, a) = 6$$
$$mad^*(F_1, b) = mad^*(F_1, c) = mad^*(F_1, d) = 5$$

Determining the exact solution of MAXADM* has the same computational complexity as the determining the exact solution of MAXADM.

Proposition 7. MAXADM* is $\mathbf{FP}^{\mathbf{NP[log]}}$-complete.

The approximation problem of MAXADM* is $\mathbf{log\text{-}APX}$-hard under $\mathbf{PTAS}$-reductions. This means that we can approximate the problem with an polynomial-time algorithm with an approximation ratio bounded by the logarithm of a polynomial depending on the size of the input, i.e. $AR \geq \frac{1}{log(f(|R|))}$, where f is a polynomial and R is the set of attacks of AF $F = (A, R)$. The membership of MAXADM* to $\mathbf{log\text{-}APX}$ will be shown in future work.

Proposition 8. MAXADM* is $\mathbf{log\text{-}APX}$-hard under $\mathbf{PTAS}$-reductions.

The above result (and also its proof technique) mirrors the result we have for k-stability* in (Thimm 2024).

4 Softening Preferred Semantics

Typically, the notion of admissibility lacks the granularity necessary for effective reasoning, thus the *preferred semantics* has been defined to provide greater expressiveness (Dung 1995). The preferred extensions are the subset-maximal admissible sets, so the preferred semantics is much more restrictive than the notion of admissibility. In this section we discuss a softening of the preferred semantics based on our two softenings of the admissibility notion.

Definition 6. Let $F = (A, R)$ be an AF and S be a k-admissible set and S' a k'-admissible set. The set S *dominates* S' iff $S \supsetneq S'$ and $k \geq k'$.

In other words, a set S dominates a set S' if S is a superset of S' and S defends itself against at least the same number of attacks than S'. Define a relation *dominates** analogously by using k-admissibility* instead of k-admissibility.

We use the notion of dominance to define k-preferred (k-preferred*) semantics.

Definition 7. Let $F = (A, R)$ be an AF and $S \subseteq A$ be a k-admissible (k-admissible*) set. S is a *k-preferred* (*k-preferred**) extension iff there is no $S' \subseteq A$ that dominates (dominates*) S.

Let $pr_k(F)$ and $pr^*_k(F)$ denote the sets of k-preferred and k-preferred* extensions for an AF $F = (A, R)$.

Proposition 9. Let $F = (A, R)$, $k \in \mathbb{N}$, and $S \subseteq A$.

1. S is an $|A|$-preferred extension iff S is a preferred extension.
2. S is a $|R|$-preferred* extension iff S is a preferred extension.
3. If S is a k-preferred extension then S is a k'-preferred extension for all $k' < k$.
4. If S is a k-preferred* extension then S is a k'-preferred* extension for all $k' < k$.
5. If S is a k-stable* extension then S is a k-preferred* extension.

Note that k-stable extensions are not necessarily k-preferred extensions[4]. Thus, k-stable semantics and k-preferred semantics do not perfectly mirror their classical counterparts, where every stable extension is also a preferred extension.

Example 6. Consider F_1 from Example 1. Then $\{a\}$ is the only preferred extension, hence $pr_4(F_1) = pr^*_6(F_1) = \{\{a\}\}$. For all other k the k-preferred and k-preferred* extensions coincide with their respective k-admissible and k-admissible* sets.

Reasoning with k-preferred and k-preferred* semantics is as hard as reasoning with preferred semantics.

Proposition 10. Let $k \in \mathbb{N}$, $F = (A, F)$, $S \subseteq A$, $a \in A$, and $\hat{pr}_k \in \{pr_k, pr^*_k\}$.

1. Deciding whether $S \in \hat{pr}_k(F)$ is $\mathbf{coNP}$-complete.
2. Deciding whether $a \in \bigcup \hat{pr}_k(F)$ is $\mathbf{NP}$-complete.
3. Deciding whether $a \in \bigcap \hat{pr}_k(F)$ is $\mathbf{\Pi_2^P}$-complete.
4. Deciding $\hat{pr}_k(F) \neq \emptyset$ is trivial.
5. Deciding $\hat{pr}_k(F) \neq \{\emptyset\}$ is $\mathbf{NP}$-complete.

Next, we define the optimisation problems for k-preferred and k-preferred* semantics.

MAXPR
Input: An AF $F = (A, R)$, $a \in A$
Output: $mpr(F, a) = max_k\{k|a \in \bigcup pr_k(F)\}$

MAXPR*
Input: An AF $F = (A, R)$, $a \in A$
Output: $mpr^*(F, a) = max_k\{k|a \in \bigcup pr^*_k(F)\}$

Note that the solutions of MAXPR are identical to those of MAXADM. Similarly, this equivalence holds for MAXPR* and MAXADM*.

Proposition 11. A solution of MAXADM/ MAXADM* is also a solution of MAXPR/ MAXPR*.

5 Conclusion

We introduced soft notions of admissibility and analysed their general properties, in particular in terms of complexity of optimisation and approximation. We also briefly discussed corresponding soft notions of preferred semantics.

[4]Consider AF $F = (\{a, b, c\}, \{(a, b)\})$, then $\{a\}$ is a 2-stable extension, however $\{a, c\}$ is 3-admissible set and thus dominates $\{a\}$. Hence, $\{a\}$ can not be a 2-preferred extension.

The soft notions of admissibility provide an intuitive way of modelling closeness in abstract argumentation. Skiba et al. (2021) have discussed the possibility of ranking sets of arguments based on their closeness to acceptable sets, such as preferred extensions. Using the soft notions to rank sets of arguments seems to be a promising approach. In future work, we will develop a semantics along the lines of Skiba et al.'s work. An addition future work is to investigate soft notions of completeness and define *k-complete* semantics.

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