USE OF WATERSHEDS IN CONTOUR DETECTION

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ABSTRACT

A non-parametric method is developped for contour extraction in a grey image. This method relies in defining the contours as the watersheds of the variation function (gradient modulus) of the light function (considered as a relief surface). Two application examples are described : bubble detection in a radiographic plate, and facet detection in fractures in steel.

I - INTRODUCTION

Two traditional methods are generally used in contours detection : The first one consists of detecting the strong values of the gradient in an image. This method requires the choice of a threshold value of the gradient modulus. Depending upon this value, the contours are, either thin but not closed, or on the contrary, well closed but too thick, therefore lacking in precision. The second method lies in image segmentation starting from the grey levels histogram. It is based upon the idea that the phases of interest correspond to the most frequent grey values. Unfortunately, this method requires a more or less important smoothing of the histogram and is particularly inoperative when dealing with a great number of phases.

In this paper, we propose a non-parametric contour detection method (the advantage being that no threshold value is used). Giving to its principle, this method gives closed contours. Two examples will illustrate it : bubbles detection in a radiographic plate, and display of facets in a metallic fracture.

II - DESCRIPTION OF THE METHOD

II-1) The tools

II-1-1) Gradient modulus

Let f be the grey function of an image, supposed to be continuous.

We shall denote the variation of f at point x(u,v) of R^2 by the function f defined by :

$$g(x) = \lim_{\varepsilon \to 0} \frac{Sup_{B(x,\varepsilon)}[f] - Inf_{B(x,\varepsilon)}[f]}{2\varepsilon}$$

with

$Sup_{B(x,\varepsilon)}[f]$ maximum value of the function f in the ball of radius ε centered in x.

$Inf_{B(x,\varepsilon)}[f]$ minimum value of f in $B(x,\varepsilon)$

If f is continously differentiable, it is easy to show that the variation of f is nothing other than the gradient modulus :

$$g(x) = |\text{grad } f(x)| = [(\frac{\partial f}{\partial u})^2 + (\frac{\partial f}{\partial v})^2]^{1/2}$$

II-1-2) Thresholds

Thresholding f at level λ defines two sets :
the set, denoted X_λ, of all points x of R^2 such that f(x) is less than or equal to λ :

$$X_\lambda = \{x \in R^2 : f(x) \leq \lambda\}$$

and the set, denoted Y_λ, of all points x of R^2 such that f(x) is strictly less than λ.

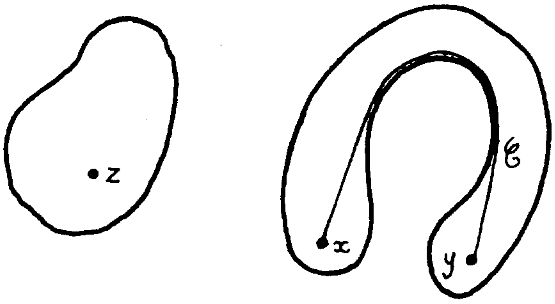
$$Y_\lambda = \{x \in R^2 : f(x) < \lambda\}$$

Notice that the family $\{X_\lambda\}$ for $0 \leq \lambda$ perfectly defines the function f; indeed, we have :

$$\forall\, x \in R^2, \quad f(x) = Inf(\lambda | x \in X_\lambda)$$

II-1-3) Zones of influence

Let X be a part of R^2, not necessarily connected. It is possible to define the distance between two points x and y of X as the smallest length of the arcs, if they exist, enclosed in X and joining x to y. If there exists no such arc, the distance is conventionally equal to infinity (Figure 1)



$d_X(y,x) = |\mathcal{C}|$

$d_X(x,z) = +\infty$

Figure 1 : Geodesic distance

This distance is called the geodesic distance. Given a point x of X and a subset Y of X, the geodesic distance between x and Y is :

$$d_X(x,Y) = \operatorname*{Inf}_{y \in Y} d_X(x,y)$$

Let Y be a subset of X consisting of n set $K_1,\ldots,K_n$, and disjoined pairwise:

$$Y = \bigcup_{p=1}^{n} K_p \quad \text{with} \quad \forall\, p \neq q,\ K_p \cap K_q = \emptyset$$

A zone of influence $I_p(Y;X)$, consisting of the set of all points of X at a finite distance from K_p and closer to K_p than to any other K_q (with respect to the geodesic distance), can be associated to every K_p (Figure 2).

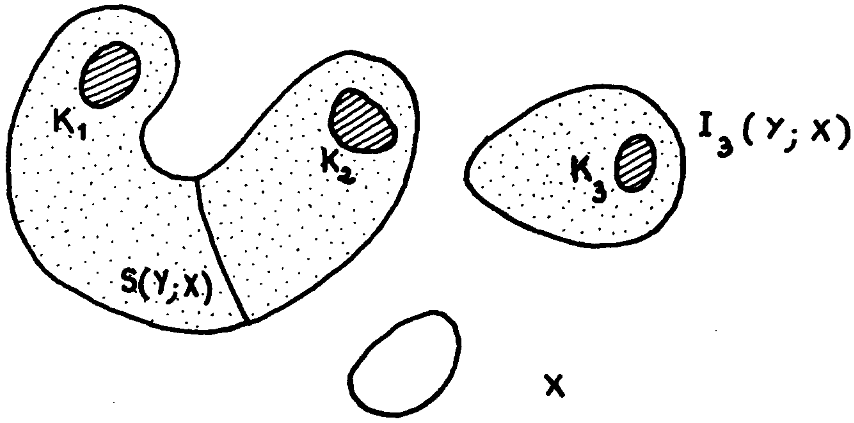


Figure 2 : Zones of influence

The points of X which do not belong to any zone of influence are either points of X at an infinite distance from Y or points equidistant from two different connected components of Y. The set of these latter points is called the "Skeleton by zone of influence" of Y with respect to X. It is denoted by S(Y ; X), and it is possible to prove that it is locally a finite union of simple arcs.

II-2) Use of the tools

II-2-1) Minima of a function

Let f be a function defined in R^2 and $\{X_\lambda\}$ be its corresponding family of sets. The function f is said to have at point x a minimum of height λ (with $\lambda = f(x)$) if :

$$d_{X_\lambda}(x, Y_\lambda) = +\infty$$

(see Figure 3)

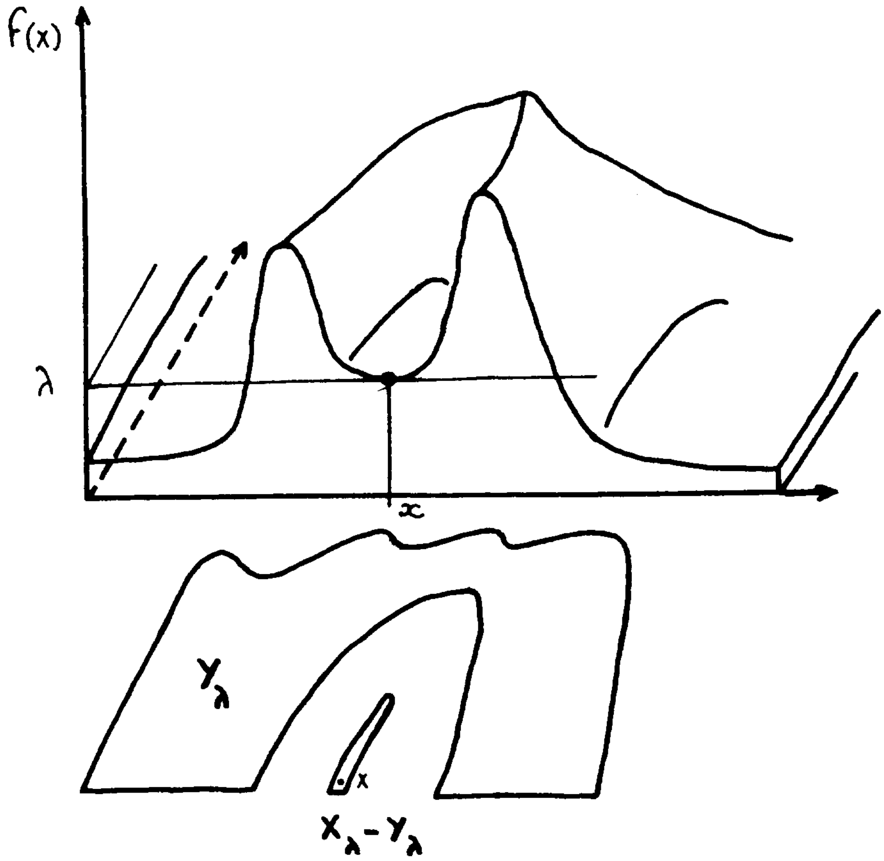


Figure 3 : Minima of a function

We denote by M(f) the set of the minima of the function f.

II-2-2) Notions of catchment basins and watersheds

These two terms are derived from geography. For better understanding, we shall use geographic vocabulary in this section.

The graph of a function f can be regarded as a topographic surface. f(x) is the height at point x.

Let us consider a drop of water on this topographic surface. The water streams down, reaches a minimum of height and stops there. The set of all points of the surface which the drops of water reaching this minimum can come from can be associated with each minimum. Such a set of points is a catchment basin of the surface. Notice that several catchment basins can overlap. Their common points form the watersheds.

In a more formal way, let Z be the set of the watersheds, and Z_λ the subset of Z of those points which are at height λ. Obviously, we have :

$$Z = \bigcup_{\lambda} Z_\lambda$$

Suppose that we know Z_μ, for every μ strictly less than λ. We shall try to determine Z_λ.

$(Y_\lambda - \bigcup_{\mu<\lambda} Z_\mu)$ is the set of points whose height is less than λ, and belonging to one, and only one, catchment basin. Let x be a point at height λ. If $d_{X_\lambda}(x, Y_\lambda - \bigcup_{\mu<\lambda} Z_\mu) < +\infty$, and if this distance is the same for two different connected components of $(Y_\lambda - \bigcup_{\mu<\lambda} Z_\mu)$, x appears to be equidistant from two different catchment basins, and consequently, must be considered as a point of Z. In other words, we define :

$$Z_\lambda = S(Y_\lambda - \bigcup_{\mu<\lambda} Z_\mu \; ; X_\lambda)$$

This definition gives a mode of operation for building catchment basins and watersheds.

III - APPLICATION TO CONTOUR DETECTION

In a picture, we define as contours the watersheds of the variation function g.

This definition seems to be somewhat arbitrary. It has, however, three major advantages :

- it gives a rigorous mathematical definition of the objects under study.
- it furnishes a mode of operation
- the result is visually satisfactory.

As a matter of fact, every object in a picture corresponds to a minimum of the grey variation function.

IV) EXAMPLES OF USE

IV-1) Contour detection in a micrography of fractures in steel

The fractures under study are cleavage fractures. The facets of cleavage (see Figure 4) do not present homogeneous grey values.

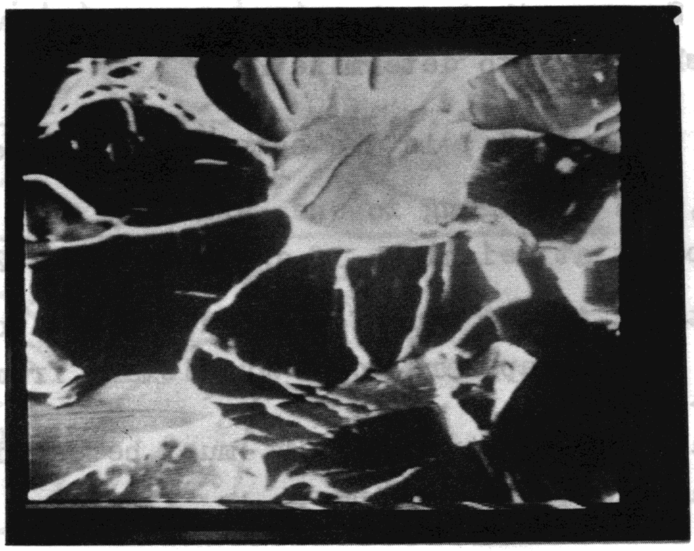


Figure 4 : Cleavage fractures in steel

Figure 5 shows the result of this procedure of contour detection applied to the previous picture.

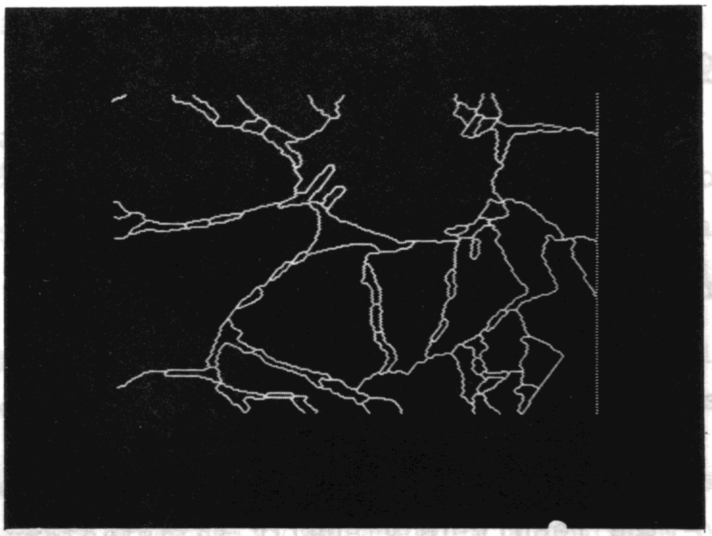


Figure 5 : Result of contour detection

IV-2) Bubbles detection in a radiography

The difficulty here lies in the fact that the bubbles are surrounded by diffraction halos which make them appear larger than they are, and therefore show very fuzzy contours (Figure 6)

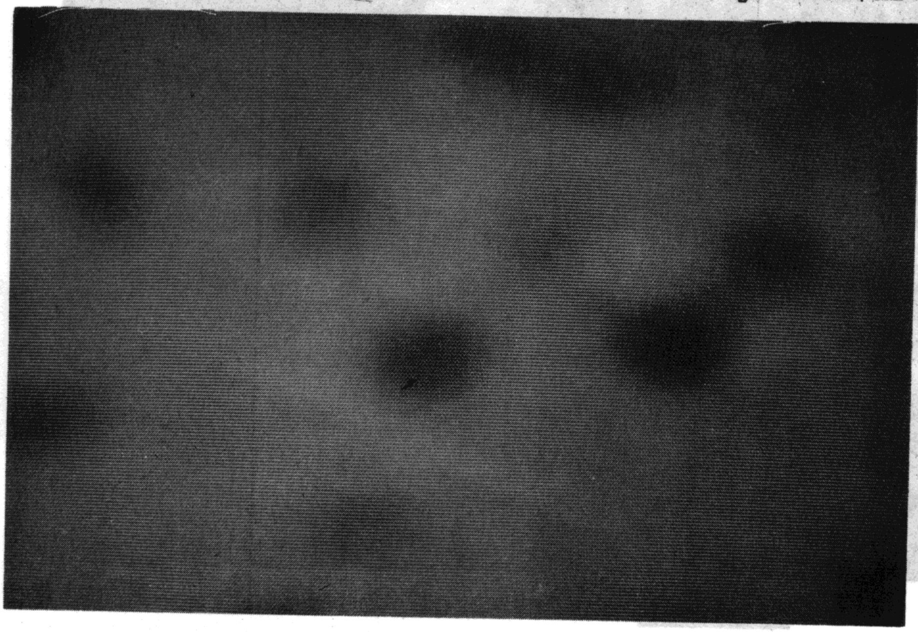


Figure 6 : Bubbles in a radiographic plate

The set Z of contours is constructed using the method previously described (Figure 7).

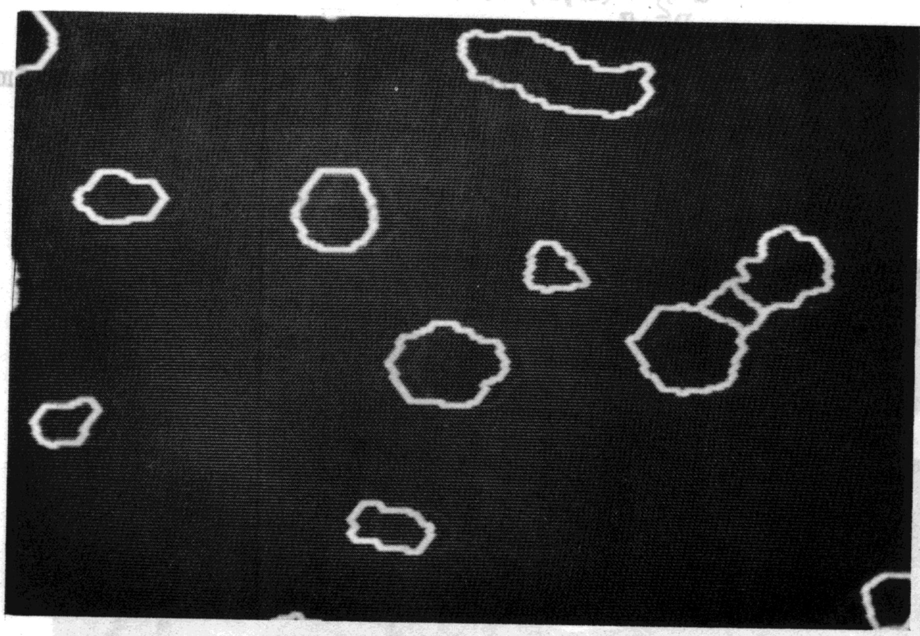


Figure 7 : Result of segmentation

As can be seen, the image is over-segmented. It is easy to suppress the unnecessary contours. For that, we call a bubble every connected component of the partition defined by the contours which contains a minimum of the light function f.

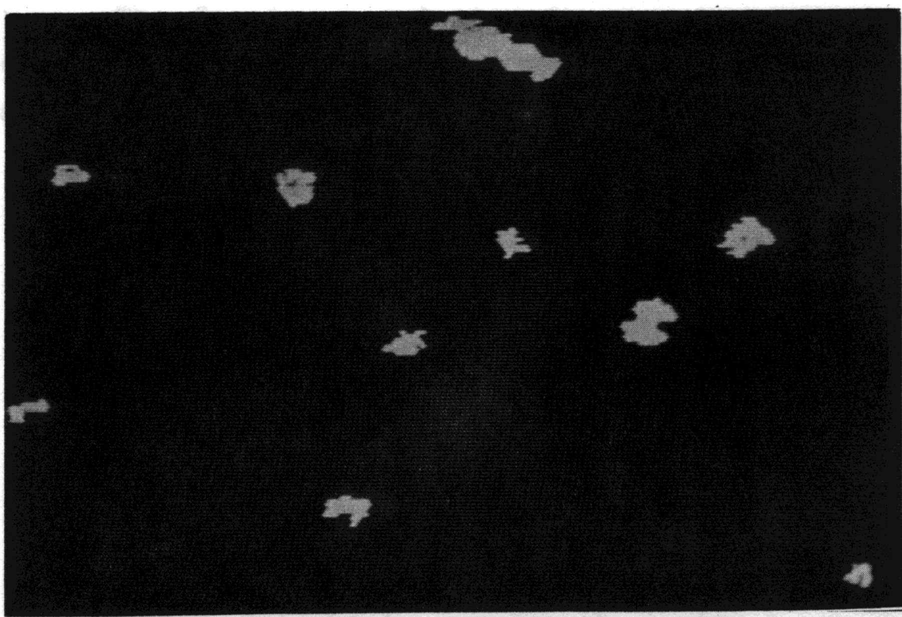


Figure 8 : Minima of the function f.

Hence, we keep only those points x of the contours picture for which :

$$d_{R^2-Z}(x,M) < +\infty$$

Where Z is the contours set, and M, the minima of the function f (Figure 8).

Figure 9 show the final result.

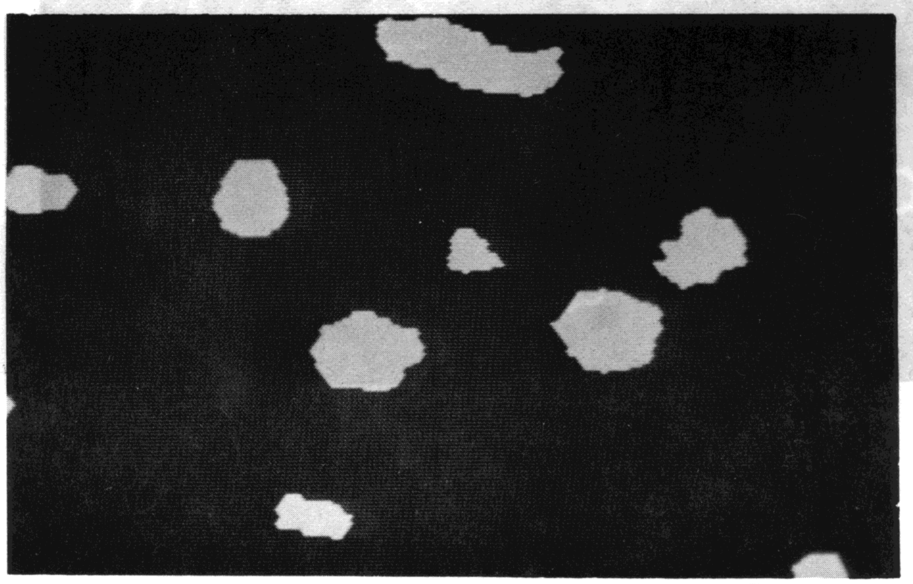


Figure 9 : Final result of the detection of bubbles

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BIBLIOGRAPHY

LANTUEJOUL C., BEUCHER S. : On the importance of the field in image analysis. Proceedings of the ICSS, Salzbourg 1979

BEUCHER S., LANTUEJOUL C. : On the change of space in image analysis. Proceeding of the ICSS, Salzbourg 1979

LANTUEJOUL C. : Détection de bulles sur un cliché micrographique par élimination des halos de diffraction qui les grossissent. Internal report, 1979.

BEUCHER S., HERSANT T. : Analyse quantitative de surfaces non planes. Application à la description de faciès de rupture fragile par clivage. Rapport final D.G.R.S.T. (Aide n° 76.7.1209,1210), 1979.