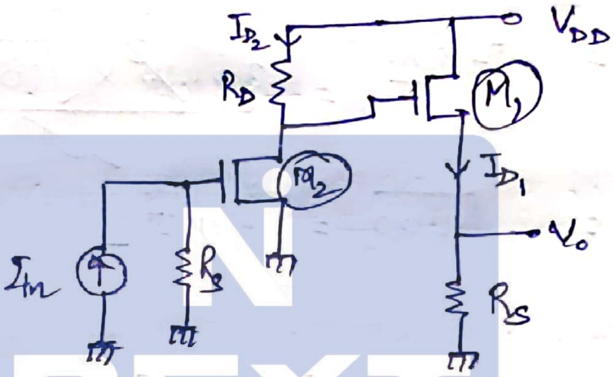
Q.1 (a) Given circuit shows a shunt - shunt feedback or voltage-shunt feedback.

Decoupled circuit:-

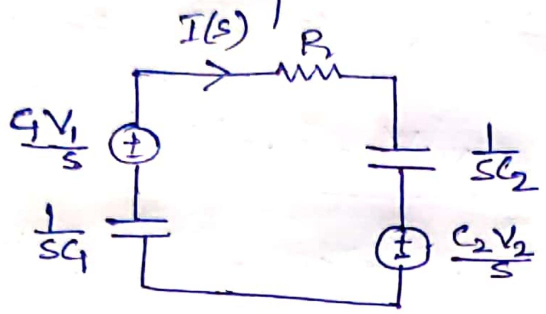


Now For M_2: & $V_{GS_2} = I_{in} R_s$

& For M_1: $V_{GS_1} = (V_{DD} - I_{D_2} R_D) - V_o$

& $V_o = I_{D_1} R_s$

Q.1(b) Turning given circuit into Laplace domain:–



Now Applying KVL:–

$$\frac{I(s)}{sC_1} - \frac{C_1V_1}{s} + RI(s) + \frac{I(s)}{sC_2} + \frac{C_2V_2}{s} = 0$$

$$\Rightarrow I(s)\left[\frac{1}{sC_1} + \frac{1}{sC_2} + R\right] = \frac{C_1V_1 - C_2V_2}{s}$$

$$\Rightarrow I(s)\left[\frac{sC_1C_2R + C_2 + C_1}{\cancel{s}C_1C_2}\right] = \frac{C_1V_1 - C_2V_2}{\cancel{s}}$$

$$\Rightarrow I(s) = \frac{[C_1V_1 - C_2V_2]C_1C_2}{C_1 + C_2 + sC_1C_2R}$$

$$= \frac{[C_1V_1 - C_2V_2]}{R\left[s + \frac{C_1+C_2}{RC_1C_2}\right]}$$

Taking inverse laplace transform:–

$$\boxed{i(t) = \frac{C_1V_1 - C_2V_2}{R} \cdot e^{-\left(\frac{C_1+C_2}{RC_1C_2}\right)t} \cdot u(t)}$$

Now total energy absorbed by resistor

$$= \int_0^\infty \left[\frac{K}{R} \cdot e^{-\frac{C_{eq}}{R}t}\right]^2 \cdot R\, dt$$

$$= \frac{K^2}{R} \cdot \int_0^\infty e^{-2(C_{eq}/R)t}\, dt$$

$$= \frac{K^2}{\cancel{R}} \cdot \frac{\cancel{R}}{-2C_{eq}} \cdot \left[e^{-2C_{eq}/R\, t}\right]\Big|_0^\infty$$

$$\boxed{\text{Energy} = \frac{K^2}{2C_{eq}}}$$ where $K = C_1V_1 - C_2V_2$

& $C_{eq} = \frac{C_1+C_2}{C_1C_2}$

Thus energy absorbed by R is $\boxed{\text{independent}}$ of the value of R.

$\boxed{\text{Q.1(c)}}$ $\vec{H} = 2\cos(\omega t - 3y)\hat{a}_z$

Given $\sigma = 0$; $\mu_r = 2$; $\epsilon_r = 5$

We know that for a medium

speed $(v) = \frac{\omega}{\beta} = \frac{1}{\sqrt{\mu\epsilon}}$

or $\frac{\omega}{3} = \frac{1}{\sqrt{\mu_r\epsilon_r}\cdot\sqrt{\mu_0\epsilon_0}} = \frac{3\times10^8}{\sqrt{2\times5}}$

$\Rightarrow \omega = \frac{9\times10^8}{\sqrt{10}}$ rad/s

$\Rightarrow \boxed{\omega = 2.85\times10^8 \text{ rad/s}}$

Now $\vec{E} = E_0 \cdot \cos(\omega t - 3y)\cdot\hat{a}_E$

deduced from the form of $\vec{H}$.

where $\hat{a}_E = \hat{a}_k \times \hat{a}_H = \hat{a}_y \times \hat{a}_z = \hat{a}_x$

& $\boxed{\frac{E_0}{H_0} = \eta}$ where $\eta = \sqrt{\frac{\mu}{\epsilon}} = \sqrt{\frac{\mu_r}{\epsilon_r}} \times 120\pi\ \Omega$

$\eta = 238.43\ \Omega$

$\Rightarrow E_0 = 238.43 \times 2\ V/m$

$$\boxed{E_0 = 476.86\ V/m}$$

Thus $\boxed{\vec{E} = 476.86 \cos(2.85 \times 10^8 t - 3y)\hat{a}_x\ V/m}$

Q.1 (d) Since we need causal sequence $\Rightarrow$ ROC is of the form $|z| > z_0$

Now $X(z) = \frac{1+2z^{-1}}{1-2z^{-1}+4z^{-2}} \times \frac{z^2}{z^2}$

$$= \frac{z(z+2)}{z^2-2z+4}$$

roots $= \frac{2 \pm \sqrt{4-16}}{2}$
$= \frac{2}{2} \pm \frac{2\sqrt{3}j}{2}$
$= 1 \pm \sqrt{3}j$

or $\frac{X(z)}{z} = \frac{z+2}{\cancel{z^2-2z+4}}$

$$\frac{X(z)}{z} = \frac{z+2}{(z-1-\sqrt{3}j)(z-1+\sqrt{3}j)}$$

Now $\frac{X(z)}{z} = \frac{A}{z-1-\sqrt{3}j} + \frac{B}{z-1+\sqrt{3}j}$

where A & B are constants.

$$A = (z-1-\sqrt{3}j)\cdot\frac{X(z)}{z}\Bigg|_{z=1+\sqrt{3}j} = \frac{3+\sqrt{3}j}{2\sqrt{3}j} = \frac{\sqrt{3}+j}{2j}$$

$$B = (z-1+\sqrt{3}j)\,\frac{X(z)}{z}\Bigg|_{z=1-\sqrt{3}j} = \frac{3-\sqrt{3}j}{-2\sqrt{3}j} = \frac{\sqrt{3}-j}{-2j}$$

$$\Rightarrow \frac{X(z)}{z} = \frac{\sqrt{3}+j}{2j}\cdot\frac{1}{z-1-\sqrt{3}j} - \frac{\sqrt{3}-j}{2j}\cdot\frac{1}{z-1+\sqrt{3}j}$$

$$\text{or } X(z) = \frac{\sqrt{3}+j}{2j}\cdot\frac{1}{1-(1+\sqrt{3}j)z^{-1}} - \frac{\sqrt{3}-j}{2j}\cdot\frac{1}{1-(1-\sqrt{3}j)z^{-1}}$$

Taking inverse Z transform; knowing that $\left\{\frac{1}{1-az^{-1}}\right\} \xrightarrow{ZT^{-1}} a^n u(n)$

$$\Rightarrow \cancel{X(z)}\; x(n) = \frac{\sqrt{3}+j}{2j}(1+\sqrt{3}j)^n u(n) - \frac{\sqrt{3}-j}{2j}(1-\sqrt{3}j)^n u(n)$$

Now $\frac{\sqrt{3}}{2}+\frac{1}{2}j = e^{j\pi/6}$; $\frac{\sqrt{3}}{2}-\frac{1}{2}j = e^{-j\pi/6}$

$\frac{1}{2}+\frac{\sqrt{3}}{2}j = e^{j\pi/3}$; $\frac{1}{2}-\frac{\sqrt{3}}{2}j = e^{-j\pi/3}$

$$\text{so; } x(n) = u(n)\left[\frac{e^{j\pi/6}}{j}\cdot 2^n\cdot e^{jn\pi/3} - \frac{e^{-j\pi/6}}{j}\cdot 2^n\cdot e^{-jn\pi/3}\right]$$

$$= 2^n u(n)\cdot\left[\frac{e^{j\left(\frac{n\pi}{3}+\frac{\pi}{6}\right)} - e^{-\left(\frac{n\pi}{3}+\frac{\pi}{6}\right)}}{2j}\right]\times 2$$

$$\boxed{x(n) = 2^{n+1}\sin\left(\frac{n\pi}{3}+\frac{\pi}{6}\right)u(n)}$$

Q(1)(e)

For right half:-

$$V_2 = I_2 \times 20\Omega$$

Now $V_1 = -\frac{I_2 \times 20}{2}$

$V_1 = -I_2 \times 10$

using turns ratio & dot convention

Now $120 = (4-6j) I_1 - I_2 \times 10$

But by turns ratio $I_2 = -\frac{I_1}{2}$

$\Rightarrow 120 = (4-6j) I_1 + \frac{I_1}{2} \times 10$

$120 = I_1 (4-6j+5)$

$120 = +I_1 (9+6j)$

or $I_1 = \frac{120}{+(9+6j)} = \cancel{19.73 \angle 99.5^\circ \text{ A}}$

~~Now $I_2 = \frac{I_1}{2} =$~~

$\boxed{I_1 = 11.1 \angle -33.7^\circ \text{ A}}$

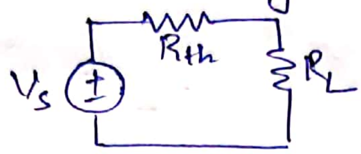
& $I_2 = -\frac{I_1}{2} = \boxed{5.55 \angle 146.3 \text{ A}}$

$\therefore V_0 = 20 \times I_2 \Rightarrow \boxed{111 \angle 146.3^\circ \text{ A}}$

Now complex power supplied by source $= V_{rms} \times I^*_{rms} = 120 \times 11.1 \angle 33.7^\circ$

$\boxed{S = 1332 \angle 33.7^\circ \text{ VA}}$

Q.5(a) Considering the thevenin equivalent



Now Case 1 :- $V_s = (R_{th} + 5) \times 4A$

Case 2 :- $V_s = (R_{th} + 20) \times 2A$

$$\Rightarrow \frac{R_{th} + 5}{R_{th} + 20} = \frac{1}{2}$$

$$2R_{th} + 10 = R_{th} + 20 \Rightarrow \boxed{R_{th} = 10\,\Omega}$$

Now $V_s = (10+5) \times 4 = \boxed{60V}$

(i) For max power transfer ; using theorem $R_L = R_{th}$

$$\Rightarrow \boxed{R_L = 10\,\Omega}$$

(ii) Max power delivered to load

$$= \frac{V_{th}^2}{4 \cdot R_{th}} = \frac{(60)^2}{4 \times 10} = \boxed{90W}$$

(iii) When delivering 50 W to load

$\Rightarrow$ Let load is $R\,\Omega$

$\Rightarrow$ Voltage $= V \times \dfrac{R}{R+10}$

& power $= \left(\dfrac{V \times R}{R+10}\right)^2 \times \dfrac{1}{R} = \dfrac{3600R}{(R+10)^2}$

$\Rightarrow \dfrac{\cancel{3600}^{72}R}{(R+10)^2} = \cancel{50} \Rightarrow R^2 + 20R + 100 = R \times 72$

$\cancel{R^2 + 19R + 100 = 0 \Rightarrow R =}$

$R^2 - 52R + 100 = 0 \Rightarrow R = \frac{52 \pm 48}{2} = 50\,\Omega$ or $2\,\Omega$

Now total power from source :—

$\frac{(60)^2}{10+50} = 60\,W$ or $\frac{(60)^2}{10+2} = 300\,W$

Powers delivered to load $= 50\,W$

$\Rightarrow$ Power transfer efficiency

$\frac{50}{60} \times 100 = \boxed{83.33\%}$ $\quad \frac{50}{300} \times 100 = \boxed{16.67\%}$

Q-5(b) Now we know that

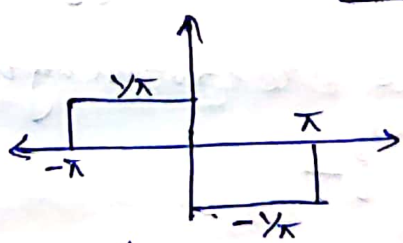
$$e^{-j\omega_0 t} \cdot x(t) \xleftrightarrow{FT} X(\omega + \omega_0)$$

$$\Rightarrow \left[\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right] x(t) \xleftrightarrow{FT} \frac{1}{2}\left[X(\omega+\omega_0) + X(\omega-\omega_0)\right]$$

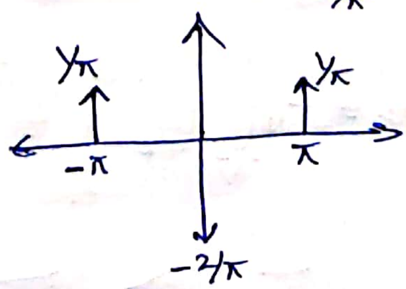
So; If $y(t) \xleftrightarrow{FT} Y(\omega)$

then $y(t) \cos 10(t) \xleftrightarrow{FT} \frac{1}{2}\left[Y(\omega+10) + Y(\omega-10)\right]$ — 1

Now $y'(t) =$



$y''(t) =$



Now $y''(t) = \frac{1}{\pi}\delta(t+\pi) - \frac{2}{\pi}\delta(t) + \frac{1}{\pi}\delta(t-\pi)$

$\Rightarrow (j\omega)^2 \cdot Y(\omega) = \frac{1}{\pi}\cdot\left[e^{j\omega\pi} + e^{-j\omega\pi}\right] - \frac{2}{\pi}$

or $Y(\omega) = \frac{1}{\pi\omega^2(-1)}\left[2\cos\pi\omega - 2\right]$

$Y(\omega) = \frac{2}{\pi\omega^2}\left[1 - \cos\pi\omega\right]$

$\boxed{\text{From (1)}}$

$\Rightarrow X(\omega) = \frac{1}{\cancel{2}}\left[\frac{\cancel{2}}{\pi(\omega+10)^2}\left[1-\cos(\pi\omega+10\pi)\right] + \frac{\cancel{2}}{\pi(\omega-10)^2}\left[1-\cos(\pi\omega-10\pi)\right]\right]$

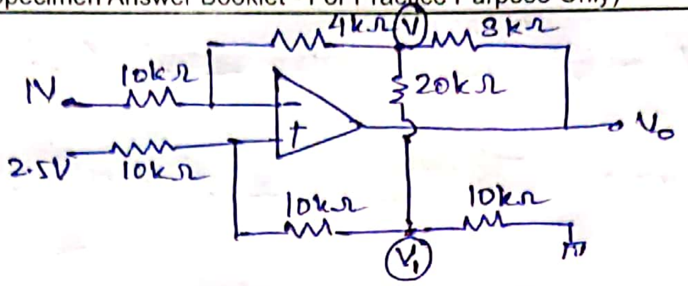
But $\cos(2\pi+\theta) = \cos(2\pi-\theta) = \cos(\theta-2\pi) = \cos\theta$

$\Rightarrow \boxed{X(\omega) = \frac{1}{\pi}\left[\frac{1-\cos\omega\pi}{(\omega+10)^2} + \frac{1-\cos\omega\pi}{(\omega-10)^2}\right]}$

Q.5(c) Poynting theorem states that the flow of energy in a flowing/travelling electromagnetic wave is given by :-

$$\oint\!\!\oint_S (\vec{E}\times\vec{H})\cdot d\vec{S} = -\frac{d}{dt}\iiint_V \left(\frac{\epsilon E^2}{2} + \frac{\mu H^2}{2}\right)dv - \int_V \sigma E^2 dv$$

Q.5(d)



Now using virtual ground & other op-amp properties, we know that current doesnt enter i/p terminals.

Now ~~$(2.5 - V_1) \times \frac{10}{10+10} = (1-V) \times$~~

At node with voltage V; using KCL;—

$$\frac{V-1}{14k\Omega} + \frac{V-V_1}{20k\Omega} + \frac{V-V_o}{8k\Omega} = 0 \quad \text{—— (I)}$$

$$\frac{V_1 - 2.5}{20k\Omega} + \frac{V_1 - V}{20k\Omega} + \frac{V_1}{10k\Omega} = 0 \quad \text{—— (II)}$$

& $2.5 - \frac{2.5 - V_1}{20k\Omega} \times 10k\Omega$

$$= 1 - \frac{1-V}{14k\Omega} \times 10k\Omega$$

$$1.5 = \frac{2.5 - V_1}{2} - \frac{(1-V)\times 5}{7}$$

$$\Rightarrow \frac{V_1}{2} = \left(\frac{2.5}{2} - 1.5\right) - \frac{5}{7} + \frac{5V}{7}$$

$$\frac{V_1}{2} = \frac{5V}{7} - \frac{27}{28} \Rightarrow \boxed{V_1 = \frac{10V}{7} - \frac{27}{14}}$$

Now from (II)

$$V_1\left(\frac{1}{20k\Omega} + \frac{1}{20k\Omega} + \frac{1}{10k\Omega}\right) = \frac{2.5}{20k\Omega} + \frac{V}{20k\Omega}$$

$$\Rightarrow \frac{V_1}{5} = \frac{1}{8} + \frac{V}{20} \Rightarrow \frac{1}{5}\left(\frac{10V}{7} - \frac{27}{14}\right) = \frac{1}{8} + \frac{V}{20}$$

$$\frac{2V}{7} - \frac{27}{70} = \frac{1}{8} + \frac{V}{20} \Rightarrow V\left(\frac{2}{7} - \frac{1}{20}\right) = \frac{27}{70} + \frac{1}{8}$$

$$V \times \frac{33}{140} = \frac{143}{280}$$

or $\boxed{V = \frac{13}{6} V} \Rightarrow \boxed{V_1 = \frac{7}{6} V}$

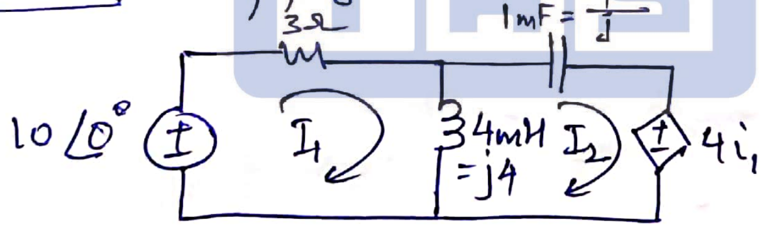
$\therefore$ Putting into (1):—

$$\frac{13/6 - 1}{14} + \frac{13/6 - 7/6}{20} + \frac{13/6 - V_o}{8} = 0$$

$$\frac{1}{12} + \frac{1}{20} + \frac{13}{48} = \frac{V_o}{8}$$

$\Rightarrow \boxed{V_o = \frac{97}{30} V}$ or $\boxed{V_o = 3.23 V}$

Q.1(e) Apply mesh analysis:—



Now $10 = 3I_1 + j4(I_1 - I_2)$ — (I)

$\frac{1}{j} I_2 + 4I_1 + j4(I_2 - I_1) = 0$ — (II)

From (I) we get $(3+j4)I_1 - j4I_2 = 10$

From (II) we get $3j\, I_2 + (4-4j)I_1 = 0$

$\Rightarrow I_2 = -\frac{4-4j}{3j} I_1$

$I_2 = \frac{4}{3}(1+j) I_1$ — (III)

$$\Rightarrow (3+4j) I_1 - j4\left(\frac{4}{3}+\frac{4j}{3}\right) I_1 = 10$$

$$\Rightarrow I_1\left[3+4j + \frac{16}{3} - \frac{16j}{3}\right] = 10$$

$$I_1 = \frac{10}{\frac{25}{3} - \frac{4j}{3}} = (1.17 + 0.19j)\ A$$

or $\boxed{i_1(t) = 1.185 \cos(10^3 t + 9.1^\circ)\ A}$

& $I_2 = \frac{4}{3}(1+j) I_1 = 2.234 \angle 54.1^\circ\ A$

or $\boxed{i_2(t) = 2.234 \cos(10^3 t + 54.1^\circ)\ A}$

Q.4(a) Given $y(n) - y(n-1) = x(n) + x(n-1)$

Taking z-transform:–

$$Y(z).\left[1 - z^{-1}\right] = X(z)\left[1+z^{-1}\right]$$

$$\Rightarrow \frac{Y(z)}{X(z)} = \frac{1+z^{-1}}{1-z^{-1}} = \boxed{\frac{z+1}{z-1}}$$

Now for the causal system:–

$$H(z) = \frac{1}{1-z^{-1}} + \frac{z^{-1}}{1-z^{-1}}$$

Now if $\frac{1}{1-z^{-1}} \xleftrightarrow{ZT^{-1}} u(n)$

~~the~~ we know by property that

$$u(n-1) \xleftrightarrow{ZT} z^{-1}\{ZT(u(n))\}$$

$$= \frac{z^{-1}}{1-z^{-1}}$$

$$\Rightarrow ZT^{-1}\{H(z)\} = u(n) + u(n-1)$$

or $\boxed{h(n) = u(n) + u(n-1)}$

(ii) Now $H(z) = \frac{1+z^{-1}}{(1-z^{-1})}$ & $X(z) = \frac{1}{1-z^{-1}}$

$$\Rightarrow Y(z) = \frac{(1+z^{-1})}{(1-z^{-1})^2}$$

or $$Y(z) = \frac{z(z+1)}{(z-1)^2}$$

Now $\cancel{\frac{Y(z)}{z}} = \cancel{\frac{z+1}{(z-1)^2}} =$

$$Y(z) = \frac{1}{(1-z^{-1})^2} + \frac{z^{-1}}{(1-z^{-1})^2}$$

Now we know that $\cancel{z}\; n u(n) \xrightarrow{ZT} \frac{z^{-1}}{(1-z^{-1})^2}$

& $x(n+1) \longrightarrow z \cdot X(z)$

$$\Rightarrow z \cdot \frac{z^{-1}}{(1-z^{-1})^2} \xrightarrow{ZT^{-1}} (n+1)\,u(n+1)$$

$$\Rightarrow \boxed{y(n) = (n+1)u(n+1) + n u(n)}$$

Now for $x(n) = 2^{-n}u(n)$; $X(z) = \frac{1}{1-\frac{1}{2}z^{-1}}$

or $\left(\frac{1}{2}\right)^n u(n)$

$$\Rightarrow Y(z) = \frac{1+z^{-1}}{(1-z^{-1})(1-\frac{1}{2}z^{-1})} = \frac{z(z+1)}{(z-1)(z-\frac{1}{2})}$$

or $\frac{Y(z)}{z} = \frac{z+1}{(z-1)(z-\frac{1}{2})} = \frac{A}{z-1} + \frac{B}{z-\frac{1}{2}}$

Now $A = (z-1)\frac{Y(z)}{z}\Big|_{z=1} = \frac{2}{1/2} = 4$

$B = (z-\frac{1}{2})\frac{Y(z)}{z}\Big|_{z=\frac{1}{2}} = \frac{3/2}{-1/2} = -3$

$$\Rightarrow \frac{Y(z)}{z} = \frac{4}{z-1} - \frac{3}{z-\frac{1}{2}} \Rightarrow Y(z) = \frac{4}{1-z^{-1}} - \frac{3}{1-\frac{1}{2}z^{-1}}$$

Taking IVT ; $\boxed{y(n) = 4u(n) - 3\left(\frac{1}{2}\right)^n u(n)}$

Q.4(b)

For the circuit in DC mode:-

$I_C = 10mA$; $V_{CC} = 15V$; $V_{BE} = 0.7V$

$\beta = 100V$ & $V_E = V_{CE} = 5V$

$\Rightarrow I_E = \frac{5V}{R_E} = \frac{(100+1)}{100} \times I_C = \frac{101}{100} \times 10mA$

$\Rightarrow R_E = \frac{5 \times 100}{101 \times 10mA} = \boxed{495\Omega}$

Now $V_{CE} = V_E = 5V \Rightarrow$ voltage across $R_C = 5V$

$\Rightarrow 10mA \times R_C = 5V$

or $\boxed{R_C = 500\Omega}$

Now $V_{th} = 15 \times \frac{R_2}{R_1+R_2}$

& $R_{th} = \frac{R_1 R_2}{R_1+R_2}$.

Now $V_{Th} - I_B R_{th} - 0.7 = V_E$

$\frac{15 R_2}{R_1+R_2} - 1\mu A \times \frac{R_1 R_2}{R_1+R_2} = 5 + 0.7$

or $\frac{R_2}{R_1+R_2}\left[15 - R_1 \times 10^{-6}\right] = 5.7$

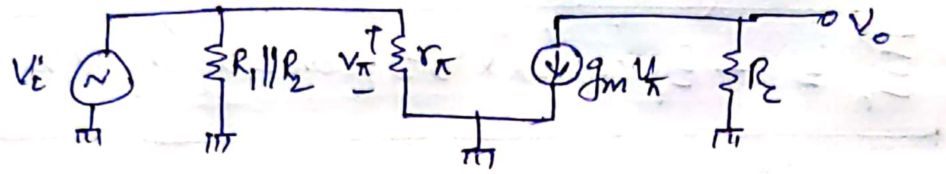
Lets choose $\boxed{R_1 = \cancel{10M\Omega} \quad 5M\Omega}$

$\Rightarrow \frac{R_2}{5\times10^6 + R_2} \times \left(15 - \frac{5}{\cancel{10}}\right) = 5.7$

or $1 + \frac{5\times10^6}{R_2} = \frac{10}{5.7}$

$\Rightarrow \dfrac{5\times10^6}{R_2} = \dfrac{4.3}{5.7} \Rightarrow \boxed{R_2 = 6.63\,M\Omega}$

Now for small signal gain :-



Now $V_o = -g_m V_\pi R_C$ where $V_\pi = V_i$

$\dfrac{V_o}{V_i} = -g_m R_C$

Now $g_m = \dfrac{I_C}{V_T} = \dfrac{10\,mA}{26\,mV} = \dfrac{5}{13}$

$\Rightarrow \dfrac{V_o}{V_i} = -\dfrac{5}{13}\times 500 = \boxed{-192.31}$

Q.4 (c) $\omega = 2\pi\times10^6$ rad/s ; $\sigma >> \omega\epsilon$

$\Rightarrow \eta = \sqrt{\dfrac{j\times\omega\mu}{\sigma}} = \sqrt{\dfrac{2\pi\times10^6\times4\pi\times10^{-7}}{5.8\times10^7}} \angle 45^\circ\,\Omega$

$\boxed{\eta = 3.7\times10^{-4} \angle 45^\circ\,\Omega}$

Now incident wave $\Rightarrow$

$\vec{E} = E_i \cos(\omega t - \beta x)\ \hat{a}_y$ V/m

Here $\omega = 2\pi\times10^6$ rad/s & $\beta = \dfrac{\omega}{3\times10^8}$ rad/m

$\beta = 0.021$ rad/m

so; $\boxed{\vec{E_i} = E_i \cos(2\pi\times10^6 t - 0.021x)\ \hat{a}_y \text{ V/m}}$

Now given $\eta_2 = 3.7\times10^{-4} \angle 45^\circ$ & $\eta_1 = 120\pi$

$\Gamma = (\eta_2 - \eta_1)/\eta_2 + \eta_1$

But $\eta_2 << \eta_1 \Rightarrow \Gamma \approx -1$

So; $E_r = -1 \times E_i$

& travelling wave in $-x$ direction.

$$\Rightarrow \boxed{\vec{E}_r = -E_i \cos(2\pi \times 10^6 t + 0.021x)\hat{a}_y \text{ V/m}}$$

For transmitted wave

$$\tau = \frac{2\eta_2}{\eta_1 + \eta_2} = \frac{2 \times 3.7 \times 10^{-4} \angle 45^\circ}{120\pi}$$

$$\tau = 2 \times 10^{-6} \angle 45^\circ$$

$$\Rightarrow E_t = 2 \times 10^{-6} E_i \quad \& \quad \beta_2 = \sqrt{\pi f \mu \sigma}$$

$$= \sqrt{\pi \times 10^6 \times \frac{4\pi}{10^7} \times 5.8 \times 10^7}$$

$$= 15132 \text{ m}^{-1}$$

$$\Rightarrow \cancel{\vec{E}_t = 2 \times 10^{-6} E_i \cos(2\pi \times 10^6 t - 15132x)\hat{a}_y \text{ V/m}}$$

However there is also $\alpha = 15132 \text{ m}^{-1}$

$$\text{So; } \boxed{\vec{E}_t = 2 \times 10^{-6} E_i e^{-15132x} \cos(2\pi \times 10^6 t - 15132x)\hat{a}_y \text{ V/m}}$$

Q.7(a) Now in steady state :—

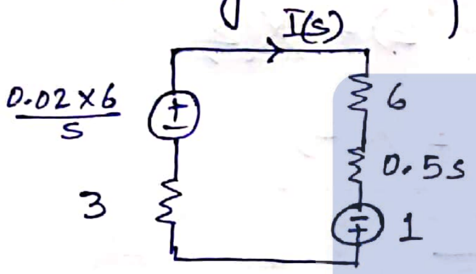
capacitors are open & inductors are short

$\Rightarrow i = \frac{10}{10} = 1A.$

$\Rightarrow Li(0^{-}) = \cancel{1A}\ 1V$

& $v(0^-) = 6i = 6V$

Moving to Laplace domain :—



Using KVL :—

$$\frac{0.12}{s} - 6I(s) - 0.5sI(s) + 1 - 3I(s) = 0$$

$$I(s)[-6-0.5s-3] + 1 + \frac{0.12}{s} = 0$$

$$\Rightarrow I(s) = \frac{1+\frac{0.12}{s}}{9+0.5s} = \frac{0.12+s}{s(0.5s+9)}$$

or $I(s) = \frac{2(s+0.12)}{s(s+18)} = \frac{A}{s} + \frac{B}{s+18}$

Now $A = s \cdot I(s)\Big|_{s=0} = \frac{2\times 0.12}{18} = \frac{1}{75}$

$B = (s+18)I(s)\Big|_{s=-18} = \frac{2\times(-17.88)}{-18} = \frac{149}{75}$

Now $I(s) = \frac{1}{75}\cdot\frac{1}{s} + \frac{149}{75}\cdot\frac{1}{s+18}$

or $i(t) = \frac{1}{75}\cdot u(t) + \frac{149}{75}\cdot e^{-18t}u(t)$

$$\boxed{i(t) = \frac{1}{75}[1+149e^{-18t}]u(t)}$$

Now $V(s) = \frac{0.12}{s} - 3I(s) \cancel{= \frac{8.1}{75 s}}$

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$$V(s) = \frac{2}{25}\cdot\frac{1}{s} - \frac{149}{25}\cdot\frac{1}{s+18}$$

$$\Rightarrow v(t) = \frac{2}{25}u(t) - \frac{149}{25}\cdot e^{-18t}u(t)$$

$$\boxed{v(t) = \frac{1}{25}\left[2 - 149e^{-18t}\right]u(t)}$$

Q.7(b) Now taking z transform:–

$$Y(z) = 0.95.z^{-1}Y(z) + X(z)$$

$$\Rightarrow Y(z)\cdot[1-0.95z^{-1}] = X(z)$$

or $$\boxed{\frac{Y(z)}{X(z)} = \frac{1}{1-0.95z^{-1}}} \longrightarrow 1$$

For $x(n) = \cos\left(\frac{\pi n}{20} - \frac{\pi}{2}\right)u(n)$

$$= \sin\left(\frac{\pi n}{20}\right)u(n)$$

$$= \frac{1}{2j}\left[e^{j\pi n/20} - e^{-j\pi n/20}\right]u(n)$$

Now $$X(z) = \frac{1}{2j}\left[\frac{1}{1-(e^{j\pi/20}).z^{-1}} - \frac{1}{1-(e^{-j\pi/20})z^{-1}}\right]$$

Now using 1 :–

$$Y(z) = \frac{1}{(1-0.95z^{-1})}\cdot\frac{1}{2j}\left[\frac{1}{1-e^{j\pi/20}z^{-1}} - \frac{1}{1-e^{-j\pi/20}z^{-1}}\right]$$

Now by final value theorem:–

$$y_{ss}(n) = \lim_{z\to 1}(z-1)\cdot Y(z)$$

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$\lim_{z \to 1} (z-1)\, Y(z) = 0$ or $\boxed{y_{ss}(n) = 0}$

$$Y(z) = \frac{1}{2j} \cdot \frac{z}{z-0.95} \cdot \left[\frac{z}{z-e^{j\pi/20}} - \frac{z}{z-e^{-j\pi/20}} \right]$$

$$= \frac{1}{2j} \cdot \frac{z}{z-0.95} \cdot \left[\frac{\cancel{z^2} - z \cdot e^{-j\pi/20} - \cancel{z^2} + z \cdot e^{j\pi/20}}{(z-e^{j\pi/20})(z-e^{-j\pi/20})} \right]$$

$$= \frac{1}{2j} \cdot \frac{z^2}{z-0.95} \cdot \left[\frac{e^{j\pi/20} - e^{-j\pi/20}}{z^2+1-z(e^{j\pi/20}+e^{-j\pi/20})} \right]$$

Let $\frac{\pi}{20} = \omega_0$ for calculation

$$\Rightarrow Y(z) = \frac{z^2}{z-0.95} \left[\frac{\sin \omega_0}{z^2 - 2z\cos\omega_0 + 1} \right]$$

$$\Rightarrow \frac{Y(z)}{z} = \frac{z \sin\omega_0}{(z-0.95)(z^2 - 2z\cos\omega_0 + 1)}$$

Let $$\frac{Y(z)}{z} = \frac{A}{z-0.95} + \frac{Bz + C}{z^2 - 2z\cos\omega_0 + 1}$$

$$= \frac{A(z^2 - 2z\cos\omega_0 + 1) + Bz^2 - Bz0.95 + Cz - C\times 0.95}{(z-0.95)(z - 2z\cos\omega_0 + 1)}$$

Now $A + B = 0$

$-2\cos\omega_0 A - B\times 0.95 + C = \sin\omega_0$

& $A - C\times 0.95 = 0$

$\Rightarrow B = -A$ & $C = \frac{A}{0.95}$

$\Rightarrow -2\cos\omega_0 A + 0.95A + \frac{A}{0.95} = \sin\omega_0$

$\Rightarrow A\left[2 - 2\cos\omega_0\right] = \sin\omega_0$

or $A = \frac{\sin\omega_0}{2 - 2\cos\omega_0}$; $B = \frac{\sin\omega_0}{2\cos\omega_0 - 2}$

& $C = \frac{1}{0.95}\left[\frac{\sin\omega_0}{2-2\cos\omega_0}\right]$

$$\Rightarrow \frac{Y(z)}{z} = \frac{\sin\omega_0}{2-2\cos\omega_0}\left[\frac{1}{z-0.95} + \frac{-z+\frac{1}{0.95}}{z^2-2z\cos\omega_0+1}\right]$$

$$\frac{Y(z)}{z} = \frac{\sin\omega_0}{2(1-\cos\omega_0)}\left[\frac{1}{z-0.95} + \frac{-(z-\cos\omega_0)}{(z-\cos\omega_0)^2+(\sin\omega_0)^2}\right]$$

$$+ \frac{\sin\omega_0}{2(1-\cos\omega_0)} \cdot \frac{\frac{1}{0.95}-\cos\omega_0}{(z-\cos\omega_0)^2+(\sin\omega_0)^2}$$

But we know that

$$ZT\{\cos\omega_0 n\, u(n)\} = \frac{z(z-\cos\omega_0)}{z^2-2z\cos\omega_0+1}$$

$$\& \quad ZT\{\sin\omega_0 n\, u(n)\} = \frac{z\sin\omega_0}{z^2-2z\cos\omega_0+1}$$

$$\therefore \frac{Y(z)}{z} = \frac{\sin\omega_0}{2(1-\cos\omega_0)} \cdot \left[\frac{1}{z-0.95} - \frac{z-\cos\omega_0}{z^2-2z\cos\omega_0+1}\right]$$

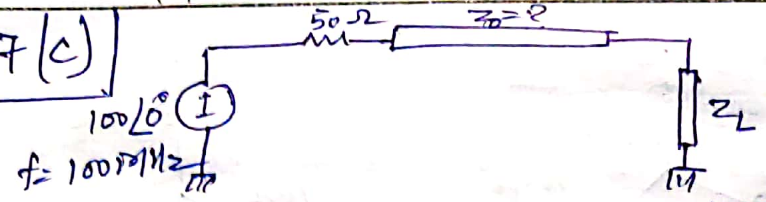
$$+ \frac{\sin\omega_0}{2\cancel{(1-\cos\omega_0)}} \cdot \frac{\cancel{(1-\cos\omega_0)}}{z^2-2z\cos\omega_0+1}$$

$$\Rightarrow y(n) = \frac{\sin\omega_0}{2(1-\cos\omega_0)}\left[(0.95)^n - \cos n\omega_0\right] + \frac{1}{2}\cdot\sin\omega_0 n$$

$$\boxed{y(n) = \frac{\sin\omega_0}{2(1-\cos\omega_0)}(0.95)^n + \frac{1}{2}\left[\sin\omega_0 n - \frac{\sin\omega_0 \cos\omega_0 n}{(1-\cos\omega_0)}\right]}$$

where $\omega_0 = \pi/20$

Q.7(c)



Now for the line ; $Z_0 = \sqrt{\frac{L}{C}} = \sqrt{\frac{0.25\times10^{-6}}{100\times10^{-12}}}$

$$\boxed{Z_0 = 50\,\Omega}$$

& $\beta = \omega\sqrt{LC} = 2\pi\times10^6\times100\sqrt{2.5\times10^{-17}}$

$= 200\pi\times10^6\times5\times10^{-9}\ m^{-1}$

$$\boxed{\beta = \pi \text{ rad/m}}$$

I) For $Z_L = 50\,\Omega$

; $\Gamma = \frac{Z_L - Z_0}{Z_L + Z_0} = 0 \Rightarrow \boxed{\Gamma = 0}$

$S = \frac{1+|\Gamma|}{1-|\Gamma|} = 1 \Rightarrow \boxed{S = 1}$

$Z_{in} = Z_0\left[\frac{Z_L + jZ_0\tan\beta l}{Z_0 + jZ_L\tan\beta l}\right] = \frac{Z_0\times\cancel{Z_0}}{\cancel{Z_0}}\left[\frac{\cancel{1+j\tan\beta l}}{\cancel{1+j\tan\beta l}}\right]$

$$\boxed{Z_{in} = 50\,\Omega}$$

II) For $Z_L = 100\,\Omega$

$\Gamma = \frac{100-50}{100+50} = \frac{1}{3} \Rightarrow \boxed{\Gamma = \frac{1}{3}}$

$S = \frac{1+\frac{1}{3}}{1-\frac{1}{3}} = \frac{4/3}{2/3} \Rightarrow \boxed{S = 2}$

$Z_{in} = 50\left[\frac{100 + j50\cancel{\tan\pi\times10}^{0}}{50 + j100\cancel{\tan\pi\times10}^{0}}\right] = 100\,\Omega$

or $\boxed{Z_{in} = 100\,\Omega}$

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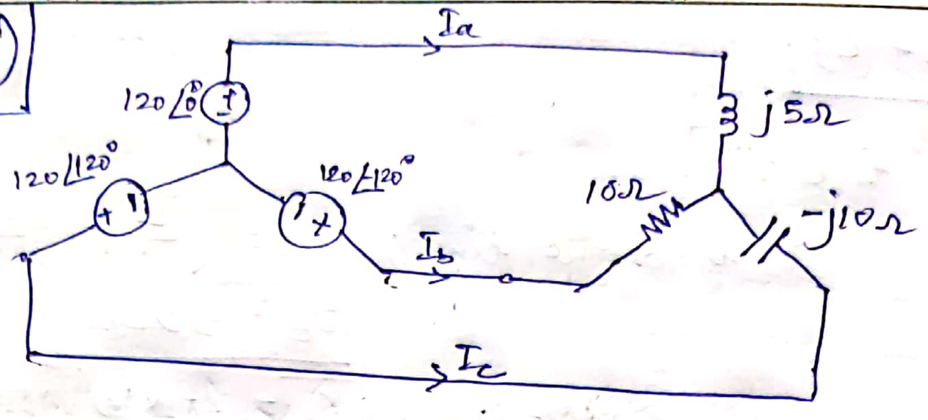
Q.3(a)



Now using KVL :—

$$120 - j5I_a + 10I_b - 120\angle -120^\circ = 0$$

$$\Rightarrow -jI_a + 10I_b = 207.85\angle -150^\circ \quad \text{(I)}$$

$$120 - j5I_a + (-j10)I_c - 120\angle 120^\circ = 0$$

$$\Rightarrow -j5I_a - j10I_c = 207.85\angle 150^\circ$$

Also $120\angle -120^\circ - 10I_b + (-j10)I_c - 120\angle 120^\circ = 0$

$$\Rightarrow -10I_b - j10I_c = 207.85\angle 90^\circ$$

or
$$\begin{bmatrix} -j & 10 & 0 \\ -j5 & 0 & -j10 \\ 0 & -10 & -j10 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 207.85\angle -150^\circ \\ 207.85\angle 150^\circ \\ 207.85\angle 90^\circ \end{bmatrix}$$

$$\Rightarrow (-j)^2 \begin{bmatrix} 1 & 10 & 0 \\ 5 & 0 & 10 \\ 0 & -10 & 10 \end{bmatrix} \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} 207.85\angle -150^\circ \\ 207.85\angle 150^\circ \\ 207.85\angle 90^\circ \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} -0.25 & 0.25 & -0.25 \\ 0.125 & -0.25 & 0.025 \\ 0.125 & -0.025 & 0.125 \end{bmatrix} \begin{bmatrix} 1\angle -150^\circ \\ 1\angle 150^\circ \\ 1\angle 90^\circ \end{bmatrix} \times \frac{207.85}{-1}$$

$$\Rightarrow \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \frac{207.85}{-1} \begin{bmatrix} -0.25\angle{-150} + 0.25\angle{150} - 0.25\angle{90^\circ} \\ 0.125\angle{-150} - 0.25\angle{150} + 0.25\angle{90^\circ} \\ 0.125\angle{-150} - 0.025\angle{150} + 0.125\angle{90^\circ} \end{bmatrix}$$

$$= \frac{207.85}{-1} \begin{bmatrix} 0 \\ 0.125\angle{30^\circ} \\ 0.1\angle{150^\circ} \end{bmatrix}$$

$$\Rightarrow \boxed{I_a = 0 \; ; \; I_b = -25.98\angle{30^\circ} \; ; \; I_c = -20.79\angle{150^\circ}}$$

For total complex power :-

$$S = V_A I_A^* + V_B I_B^* + V_C I_C^*$$

$$= 120 \times 0 + 120\angle{-120} \cdot (-25.98\angle{-30^\circ}) + (120\angle{120} \cdot (-20.79\angle{-150^\circ})$$

$$= 3117.6\angle{30^\circ} + 2494.8\angle{150^\circ} \text{ VA}$$

$$\boxed{S = 2857.56\angle{79.12^\circ} \text{ VA}}$$

Q3(b)(i) $\eta = 200 \angle 30^\circ$

Now $\vec{H} = 10 e^{-\alpha x} \cdot \cos(\omega t - \frac{1}{2}x)\hat{a}_y$ A/m

We know $\frac{E_0}{H_0} = |\eta| \Rightarrow \boxed{E_0 = 2000 \text{ V/m}}$

& $\hat{a}_E = -\hat{a}_k \times \hat{a}_H = -\hat{a}_x \times \hat{a}_y = -\hat{a}_z$

$\Rightarrow \vec{E} = -2000 e^{-\alpha x} \cos(\omega t - \frac{1}{2}x)\hat{a}_z$ V/m

Now $\beta = 0.5$ & we know $\frac{\sigma}{\omega\epsilon} = \tan\theta$
$= \tan(2\times 30^\circ)$
$= \sqrt{3}$

so, $\gamma = \sqrt{j\omega\mu(\sigma + j\omega\epsilon)} = \sqrt{j\omega\mu j\omega\epsilon\left(1 - j\frac{\sigma}{\omega\epsilon}\right)}$

$= j\omega\sqrt{\mu\epsilon} \cdot \sqrt{1 - j\sqrt{3}}$

$= j\omega\sqrt{\mu\epsilon} \cdot \sqrt{2} \cdot \angle -30^\circ$ ($\omega\sqrt{\mu\epsilon} \rightarrow \beta$)

$\gamma = \frac{1}{2} \times \sqrt{2} \times \angle -30+90^\circ = \frac{1}{\sqrt{2}} \angle 60^\circ$

$\Rightarrow \alpha = \frac{1}{\sqrt{2}} \cos 60$; $\beta = \frac{1}{\sqrt{2}} \sin 60$

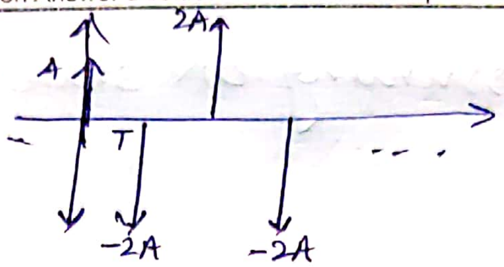
or $\boxed{\alpha = 0.354 \text{ m}^{-1}}$

skin depth $= \frac{1}{\alpha} = \boxed{2.825 \text{ m}}$

direction of wave polarization $= \hat{a}_z$

$$\boxed{\vec{E} = -2000 e^{-0.354x} \cos\left(\omega t - \frac{x}{2}\right)\hat{a}_z \text{ V/m}}$$

(ii) $f'(t) =$



or $f'(t) = A\left[\delta(t) - 2\delta(t-T) + 2\delta(t-2T) - 2\delta(t-3T) \ldots\right]$

$$= A\delta(t) - 2A\left[\delta(t-T) + \delta(t-3T) + \delta(t-5T) + \cdots\right]$$

$$+ 2A\left[\delta(t-2T) + \delta(t-4T) + \cdots\right]$$

Now taking Laplace transform :—

$$s.F(s) = A - 2A\left[e^{-sT} + e^{-3sT} + e^{-5sT} + \cdots\right]$$

$$+ 2A\left[e^{-2sT} + e^{-4sT} + \cdots\right]$$

$$= A - 2A \cdot \frac{e^{-sT}}{1-e^{-2sT}} + 2A \frac{e^{-2sT}}{1-e^{-2sT}}$$

$$= \frac{1-e^{-2sT} - 2e^{-sT} + 2e^{-2sT}}{1-e^{-2sT}} = \frac{1-2e^{-sT}+e^{-2sT}}{1-e^{-2sT}}$$

$$= \frac{(1-e^{-sT})^2}{(1-e^{-sT})(1+e^{-sT})} = \frac{1-e^{-sT}}{1+e^{-sT}}$$

or $\boxed{F(s) = \frac{1}{s} \cdot \frac{1-e^{-sT}}{1+e^{-sT}}}$