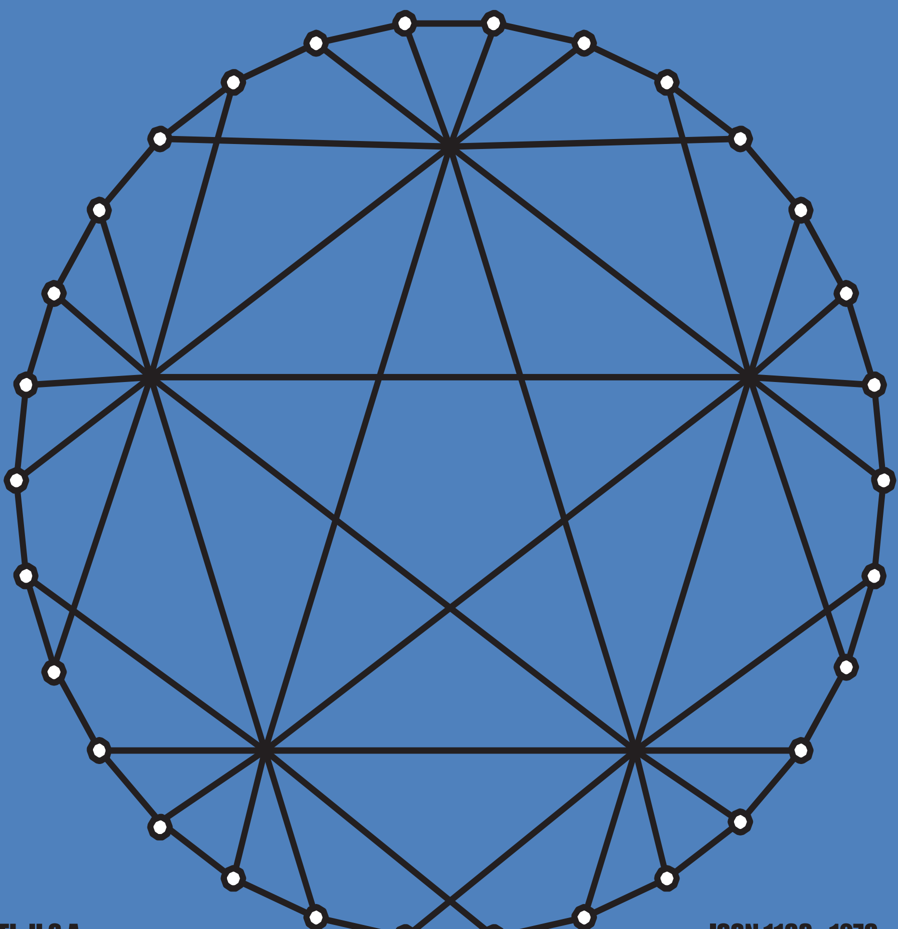
# BULLETIN of the INSTITUTE of COMBINATORICS and its APPLICATIONS

Volume 79
January 2017





Boca Raton, FL, U.S.A.

ISSN 1183 - 1278

BULLETIN OF THE ICA
Volume **79** (2017), Pages 108–115

# Classroom Note: An Application of Edge Clustering Centrality to Brain Connectivity

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**Abstract**

Edge clustering centrality measures the frequency an edge appears across the closed neighborhoods over all vertices. This problem is analogous to the structural graph theory problem of finding the maximum sized book graph ($K_2 \vee mK_1$, $m \geq 1$) in a given graph.

We apply this property to data from the Rochester Center for Brain Imaging and obtain results involving functional connectivity of the human brain. We give a series of exercises where students in an undergraduate discrete mathematics or graph theory course can experience an interdisciplinary real world application of mathematics.

# 1 Introduction

Graph theory can be used to model biological and social networks. In a social network, each person in the network is represented by a vertex, and for a pair of individuals who are connected in the network (e.g. who know each other), there is a corresponding edge in the graph. Certain individuals (and their relationships) will be more central to the network than others, and various metrics have been developed to quantify how well-connected each member of the network is (Borgatti [1] Pavlopoulos [2]) by measuring the centrality of *vertices*. In this paper, we instead consider the centrality of *edges*, with an interest in identifying connections that are more important to the network as a whole. The measure we use is called *edge clustering centrality*, with the edge clustering centrality of an edge $e$ in a graph $G$ defined as follows: $C_{cent}(e) = \sum_i f_i(e)$ where $f_i(e) = 1$ if $e \in E(G_i)$ and $f_i(e) = 0$ if $e \notin E(G_i)$ and where $G_i$ is the subgraph of $G$ induced by vertex $i$ and all of its neighbors. We note this problem is analogous to the structural graph theory problem of finding the maximum sized book graph ($K_2 \vee mK_1$, $m \geq 1$) in a given graph.

As a simple example, consider the below graph $G$ with 6 vertices and 8 edges, and its corresponding subgraphs $G_i$.

Here, $C_{cent}(BC) = 4$, $C_{cent}(AB) = C_{cent}(AC) = C_{cent}(BD)$

$= C_{cent}(CD) = 3$, and $C_{cent}(BE) = C_{cent}(CF) = C_{cent}(EF) = 2$. Since $BC$ has the highest clustering centrality, it can be considered to be the edge most important to the network's connectivity.

# 2 Edge clustering centrality in book graphs

The graph $G$ in the above example has as a subgraph a *book graph*. A book graph, denoted $B_m$, is a graph that consists either of $m$ triangles sharing a common edge or of $m$ quadrilaterals sharing a common edge. The former is called a *triangular book graph* and the latter a *quadrilateral book graph*.

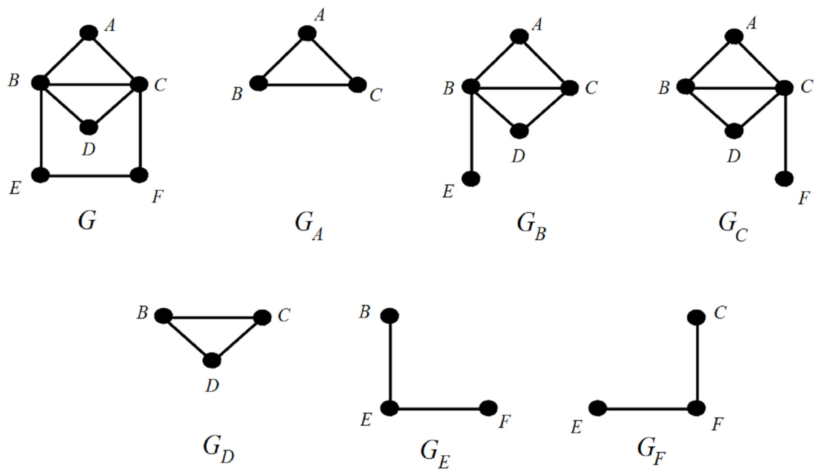



Figure 1: Closed neighborhoods of the vertices in $G$

These are shown in Figures 2 and 3 respectively. The common edge is called the *spine* of the book graph. Below are some of the smaller book graphs in each of the two categories.

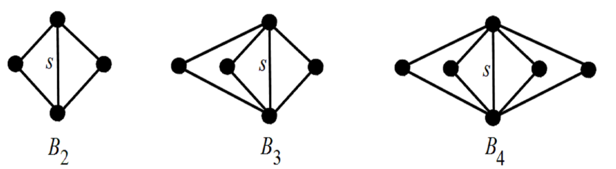



Figure. 2. Triangular book graphs (each with spine $s$)

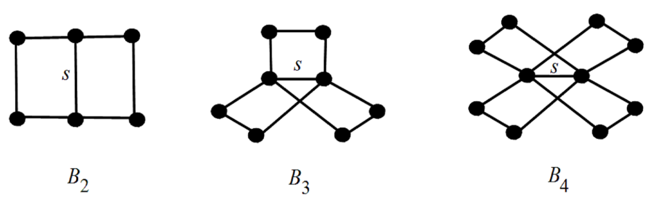
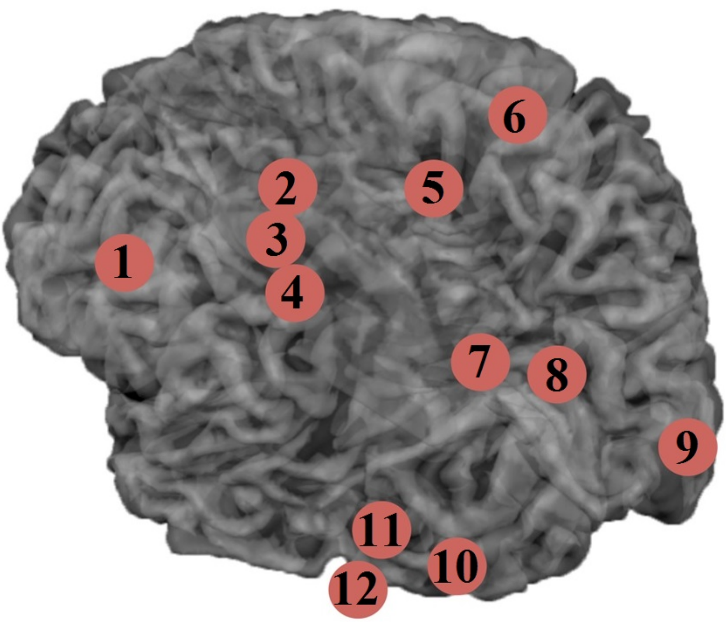



Figure 3. Quadrilateral book graphs (each with spine $s$)

For book graphs, and graphs that contain book graphs as subgraphs, there is an efficient way of identifying the edge(s) with greatest edge clustering centrality. In particular, the spine of a book graph will have the highest edge clustering centrality of any edge in the graph. Moreover, the spine of the largest book subgraph in a graph will have the highest edge clustering centrality of any edge in that graph. (The student exercises at the end explore these ideas further.) Note that the graph $G$ in Figure 1 has the triangular book graph $B_2$ as its (only) book subgraph; its spine (edge $BC$) was found to have the highest edge clustering centrality.

## 3 An application from neuroscience

Functional magnetic resonance imaging (fMRI) is a technique that measures brain activity and is used in many studies to understand how a healthy brain functions and/or how its normal function is disrupted due to disease [4]. We consider a graph model of the brain, where vertices correspond to physical regions of the brain (Regions of Interest, or ROIs), and edges correspond to correlations in the functional activation profiles associated with the two vertices [3]. In particular, an edge is present between two regions of the brain if there is a strong correlation between the time series of Blood Oxygenated Level Dependent (BOLD) signals between the respective regions. Figure 4 (which appeared in [3]) depicts a model of the brain with 12 regions selected across the temporal, parietal, and frontal lobes.



1. Left Ventral Premotor Cortex
2. Left Dorsal Premotor Cortex
3. Right Hand Motor Representation
4. Left Anterior Intraparietal Sulcus
5. Right Foot Motor Representation
6. Left Hand Motor Representation
7. Left Dorsal Occipital Cortex
8. Right Medial Fusiform Gyrus
9. Right Lateral Occipital Complex
10. Left Lateral Occipital Complex
11. Left Medial Fusiform Gyrus
12. Left Post. Middle Temporal Gyrus

Figure 4. A model with 12 regions of the brain

The fMRI data used in the model was collected at the Rochester Center for Brain Imaging at the University of Rochester, from 12 healthy right-handed individuals performing two different tasks, where they were shown images of various tools: hammer, scissors, screwdriver, knife, pliers, and corkscrew. [3]. In one task, participants were asked to pantomime the use of an object with their right hand during the fMRI, and during the second task, participants were asked to identify the images. Figure 5 (which appeared in [3]) shows the edges where regions had higher correlations for pantomiming than for identification.

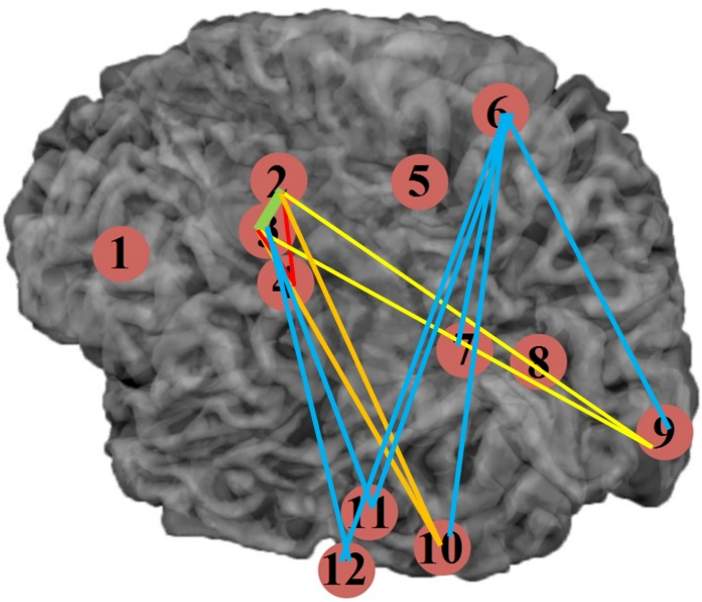



Figure 5. Edge clustering centrality: pantomiming greater than viewing

Note the triangular book graph $B_3$, with edges highlighted in red, orange, and yellow. The connections among these regions comprising the book graph are expected, since they are involved in accessing manipulation knowledge from the visual structure of objects and actually manipulating the tools [3]. The spine of this book graph is the connection between the Left Dorsal Premotor Cortex and Right Hand Motor Representation. In fact, this edge has the highest edge clustering centrality of any edge in the graph, indicating that it is the most prominent connection in the network. Figure 6 shows the edges where regions had higher correlations for identification than for pantomiming.

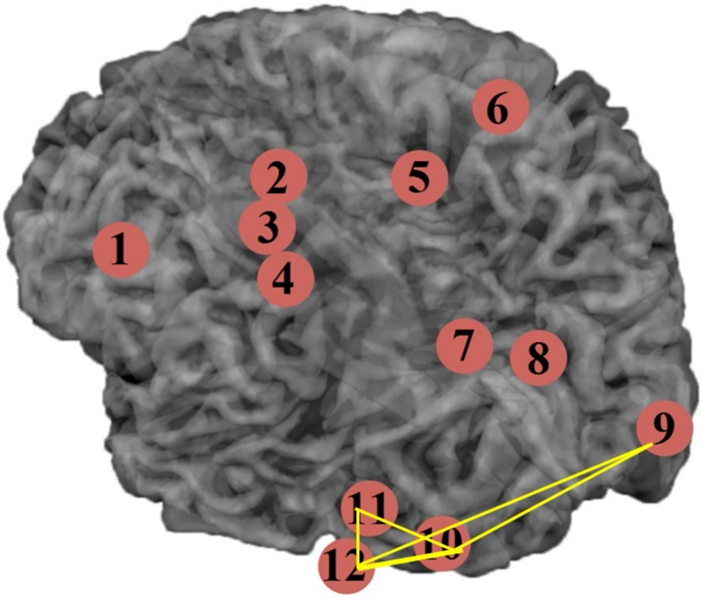



Figure 6. Edge clustering centrality: viewing greater than pantomiming

In this graph (also appearing in [3]), we observe that four regions form the triangular book graph $B_2$. These connections are expected, since the four regions support the visual processing and representation of tools [3]. The spine of this book graph is the connection between the Left Posterior Middle Temporal Gyrus and Left Lateral Occipital Complex. This edge has the highest edge clustering centrality of any edge in the graph, indicating that it is the most prominent connection in the network.

# 4 Student Exercises

In this section, we include a collection of exercises based on the concepts developed in this paper. These exercises would be appropriate for implementation in an undergraduate discrete mathematics or graph theory course.

**Exercise 1.** For the graph in Figure 5, calculate the edge clustering centrality for each edge $e$ by applying the formula for $C_{cent}(e)$ that was provided in the introduction to this paper; confirm that the spine of the embedded book graph in fact has the highest edge clustering centrality. Repeat this exercise for the graph in Figure 9.

**Exercise 2.** Repeat the above exercise for the graph in Figure 6.

**Exercise 3.** Repeat the above exercise for the quadrilateral book graphs in Figure 3.

**Exercise 4.** An alternative definition of the quadrilateral book graph $B_m$ is the graph Cartesian product $S_{m+1} \times P_2$, where $S_m$ is a star graph and $P_2$ is the path graph on two vertices [5]. Show that in fact this is equivalent to forming $B_m$ from a collection of $m$ quadrilaterals sharing a common edge.


# Acknowledgements:

This work was supported by NSF Grant #1019532, STEM Real World Applications of Mathematics. Roger Vargas's research was supported by a NSF Research Experiences for Undergraduates Grant at the Rochester Institute of Technology #1358583, under the direction of Darren Narayan. The acquisition and analysis of the MRI data reported herein was supported by NIH Grant NS089609 to Bradford Mahon. Preparation of this manuscript was supported by NIH Grant NSO89069 and NSF Grant 1349042 to Bradford Mahon. Frank Garcea was supported by a University of Rochester Center for Visual Science pre-doctoral training fellowship (NIH training Grant 5T32EY007125-24).

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[5] http://mathworld.wolfram.com/BookGraph.html