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Trans-Planckian wimpzillas

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Two previously proposed conjectures—gravitational trans-Planckian particle creation in the expanding universe, and the existence of ultra-heavy stable particles with masses up to the Planck scale (wimpzillas)—are combined in a proposal for trans-Planckian particle creation of wimpzillas. This new scenario leads to a huge enhancement in their production compared to mechanisms put forward earlier. As a result, it requires the trans-Planckian particle creation parameter to be rather small to avoid overproduction of such particles, much less than that is required for observable effects in the primordial perturbation spectrum. This ensures also that wimpzillas are mainly created at the end of primordial inflation. Conditions under which trans-Planckian wimpzillas can constitute the present dark matter are determined.

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I. INTRODUCTION

Creation of pairs of all types of particle and antiparticles by a strong external gravitational field is a direct analogue of electron–positron creation in a strong electromagnetic field–an unambiguous prediction of quantum electrodynamics. The former effect also has a solid field-theoretical basis. Its relevance in cosmology was recognized by Schrödinger as early as 1939 [1]. In his paper, "The proper vibrations of the expanding universe," Schrödinger discussed what he referred to as the "alarming phenomenon" of particle creation in an expanding universe. The use of the word 'alarming' suggests that emergence of particles from the quantum vacuum simply due to the expansion of the universe seemed to Schrödinger at the time to signify some internal inconsistency of quantum field theory. However, such concerns completely disappeared after the construction of a rigorous theory for gravitational creation of particles and the energy-momentum tensor of quantum fields in cosmological backgrounds, beginning with the pioneering papers in Refs. [2] (the de Sitter background), [3, 4] (a Friedmann-Robertson-Walker (FRW) background), [5] (an anisotropic homogeneous background and the energy-momentum tensor), [6] (gravitons in a FRW background), and others. For many years after that, this theory was considered as a purely theoretical exercise, far removed from any practical applications. Now, observations verifying the direct consequences of this effect have become one of the main topics in experimental and theoretical cosmology, since gravitational particle creation in the expanding universe during a primordial inflationary (de Sitter) stage serves as the physical mechanism for the generation of the observed inhomogeneities in the matter density and the cosmic microwave background (CMB) radiation temperature [7] (along with a predicted, but yet unobserved, primordial gravitational wave background [8]).

The usual calculations of density perturbations assuming the minimal possible choice for the initial quantum state for perturbations (namely, adiabatic vacuum initial conditions for each Fourier mode), have led to predictions that have been confirmed by observations. This is considered as one of the most remarkable successes of the whole inflationary scenario. Moreover, it should be noted that even alternatives like the Pre-Big-Bang [9] or ekpyrotic [10] scenarios, in which there is no inflationary stage, still use the phenomenon of particle creation in the expanding universe to produce the observed perturbations. So the relevance of particle creation is not tied to a specific cosmological scenario.

However, the phenomenon may have even more richness: Consider the hypothesis of the existence of supermassive

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particles with a rest mass $m > 10^{10}$ GeV and a lifetime exceeding the age of the universe. These hypothetical particles were dubbed wimpzillas in Ref. [11]. Here the question of how to produce them in the early universe arises once more. One possibility is the standard gravitational particle creation scenario first studied in Refs. [11, 12]. Then, production at the end of inflation appears to be the most efficient for this aim. In such a scenario, production of particles of mass in excess of H_I is strongly suppressed. Since we know that $H_I \lesssim 10^{14}$ GeV from the upper limit on the contribution of primordial gravitational waves to the measured CMB fluctuations, wimpzillas with masses exceeding 10^{14} GeV cannot be produced by this mechanism. An additional possibility for production of higher mass particles is provided by the preheating process [13], *i.e.,* the rapid creation of massive particles by inflaton oscillations after the end of inflation in the regime of a broad parametric resonance. Here, one may expect creation of particles with masses up to 10^{16} GeV [11], but not more.[1]

Now the question arises if it may be possible to create even more wimpzillas than the known mechanisms listed above permit. We put forward a new idea to use a (hypothetical, of course) mechanism of *trans-Planckian particle creation* (TPPC) for this aim. The TPPC effect during inflation was first proposed in Ref. [16]. Though the initial emphasis in TPPC investigations concentrated on the uniqueness of the inflationary predictions for perturbations in standard quantum field theory (QFT), it soon became clear that TPPC is a new physical effect that exists not only during inflation, but for all of cosmic history (if it exists at all), and it requires breaking some postulates of standard QFT for its existence (in particular, local Lorentz invariance). Ref. [17] contains an extensive list of further references on this topic, including various microscopic mechanisms that might lead to such an effect. We will not discuss possible mechanisms of the TPPC effect in this paper. Rather, we will restrict ourselves to a purely phenomenological description of its outcome in terms of 'out' parameters defined after the TPPC effect ceases, and we will use its observational consequences to place new and severe restrictions on the possible strength of the TPPC effect.

In Sec. II the TPPC description is introduced and several subtle points about the TPPC effect are discussed, in particular, why the TPPC effect generically *may not* be reduced to standard particle creation from a non-vacuum initial quantum state, and what restricts the possible amount of TPPC. These points are necessary both for the strict definition of what we mean by TPPC, and since some confusion still seems to exist in the literature regarding this topic. A reader interested in the direct application to observational effects may skip this section in the first reading. In Sec. III, TPPC creation of wimpzillas is calculated and new restrictions of the TPPC effect are obtained from it. Final conclusions are presented in Sec. IV.

II. TPPC VERSUS STANDARD PARTICLE CREATION FROM A NON-VACUUM INITIAL STATE

As has been pointed out above, standard QFT predictions for particle creation in curved space-time, including the inflationary predictions for perturbations, are based on an adiabatic in-vacuum for *all* modes with a sufficiently large momentum. Of course, this is crucial for the uniqueness of the final result. Let us emphasize that the choice of an initial quantum state is a physical, not a technical, problem. The whole discussion of TPPC historically began from the reconsideration of the well known fact that for any Fourier mode $\mathbf{k}$ ($k = |\mathbf{k}|$) of small inhomogeneous perturbations of a FRW background with a scale factor $a(t)$, the physical momentum $p = k/a(t)$ was once very large at sufficiently early times during the expansion of the universe and may have greatly exceeded the Planck mass $M_{Pl} = 1/\sqrt{G}$ ($\hbar = c = 1$ is assumed throughout the paper). Thus, observable modes of density and temperature fluctuations emerged from a 'trans-Planckian' region.

However, this does not result in anything dangerous for a Lorentz-invariant theory and does not preclude the unambiguous determination of the vacuum state so long as $\omega^2(p) - p^2$ remains much less than M_{Pl}^2, where ω is the physical frequency. Let us recall that exactly the same trans-Planckian problem arises formally in the calculation of electron-positron pair creation in a constant electric field $\mathbf{E}_0$ if the gauge $\mathbf{A} = -\mathbf{E}_0 t$ is used. However, it can be easily verified (see, *e.g.,* Ref. [18]) that by using the adiabatic vacuum as an initial condition for all Fourier modes at $t \to -\infty$ and integrating over all momenta p, in spite of the unlimited growth of particle energy in this limit, one correctly reproduces the Heisenberg–Euler action (more exactly, its analytical continuation to the electric field case) obtained by fully covariant methods. Thus, using the canonical approach to QFT we are even *obliged* to consider field modes with an arbitrarily high momentum p, but in the adiabatic vacuum state, in order to obtain correct results: no momentum cutoff is permitted.

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[1] Here we speak about boson production. Mechanisms of fermion production during preheating [14], or during inflation through direct coupling of a wimpzilla to an inflaton [15], are capable of producing fermions with a rest mass as large as the Planck mass. The vast difference between fermion and boson cases reflects the non-gravitational origin of these kind of mechanisms.

Still, in the cosmological case, as well in the toy model with an electric field, the initial state may well be a non-vacuum state and contain some particles (well defined up to exponentially small terms so long as their momenta or rest mass exceed the Hubble parameter $H \equiv \dot{a}/a$). So, the physical question concerns their origin. One possibility, which does not require a change of basic physical laws, is that these particles were produced either before the beginning of inflation or during some previous stage(s) of inflation with a higher curvature. Indeed, *e.g.,* in a model with two stages of inflation divided by a matter-dominated or radiation-dominated period, some modes enter the second inflationary stage in a non-vacuum state [19]. However, the average number of particles and their energy density in the initial state taken at some moment $t = t_0$ should be finite and not too large to avoid an excessive back-reaction incompatible with the assumed behavior of a FRW background:

$$\langle \rho_{part} \rangle = \frac{g_s}{(2\pi)^3 a^4} \int_{k=aH}^{\infty} d^3k \; \omega(k) \langle n(\mathbf{k}) \rangle \;\lesssim H^2 M_{Pl}^2 \tag{1}$$

at $t = t_0$ (g_s is the statistical weight).[2] This requires that $k^3 \langle n(\mathbf{k}) \rangle \to 0$ as $k \to \infty$ for any kind of quantum state. As a result, the effect of a physically admissible non-vacuum initial state on the perturbation power spectrum is very transient in k-space. Deviations of spectra from the vacuum result, proportional to $\sqrt{\langle n(\mathbf{k}) \rangle}$, must become negligible for large k (late Hubble radius crossing times $t - t_0 \gg H^{-1}$ during inflation). For this conclusion to be valid, it is not necessary that the initial state of each mode decay to vacuum (due to, *e.g.,* some particle interactions) and it may even remain as it is. This shows, in particular, that the slowness of relaxation pointed out recently in Ref. [20] does not present any obstacle for the unambiguity of predictions of the inflationary scenario. The self-consistency condition (1) is crucial for all discussions of which initial states are typical. It plays the same role as the fixing of the energy for a microcanonical ensemble or temperature for a canonical one in statistical physics. Without it, of course, particles of arbitrary high energy would be typical at all times including at present.

In addition, creation from a non-vacuum initial state results in a drastic deviation of the initial power spectrum of density perturbations from the approximately flat (Harrison-Zel'dovich) spectrum that is easily distinguishable. Within present observational bounds, such behavior is possible only for scales close to the present Hubble radius; see Refs. [17, 21] for the recent comparison of different local features in the power spectrum with the 3-year WMAP data.

Thus, significant (over a wide interval of scales) corrections to the standard inflationary predictions are possible only with a hypothesis of the existence of a new effect—*trans-Planckian particle creation*. Observational signatures of this effect give us tools to study and constrain physics in the trans-Planckian regime, which explains the recent attention devoted to this subject. The main feature distinguishing TPPC from standard gravitational particle creation is that the characteristic energy of created particles and antiparticles (the symmetry between matter and antimatter is still respected) at the moment of their creation is not $E \sim H$ (for a rest mass $m \ll H$) but $E \sim \Lambda$, where Λ is some new scale that should necessarily be connected with some kind of (possibly soft) Lorentz invariance breaking, otherwise such a process is impossible.[3] The natural candidate for Λ is the Planck mass; however, there are other possible scales: the superstring scale, the scale associated with duality (a minimum length scale), scales associated with extra dimensions, and so on. In these cases, one expects that $10^{-3} M_{Pl} \lesssim \Lambda \lesssim M_{Pl}$.

Soon after the initial proposal, it was emphasized in Ref. [22] that if the TPPC effect exists at all, it should be "everlasting:" *i.e.,* it should not be restricted to only the de Sitter background, but it should occur during all periods of the universe's expansion up to the present time. Indeed, the main reason for the possible existence of the TPPC effect—continuous drift of all Fourier modes from the trans-Planckian region of momenta to the sub-Planckian one—remains the same for any kind of expansion. Of course, the TPPC effect should become weaker for a smaller curvature (a curvature-independent effect is immediately excluded by its absence at present [22]).

Since the TPPC phenomenon produces particles of energy Λ, ultra-high energy particles should be created long after primordial inflation (and even now!). The dependence of TPPC on H is crucial for determining the moment of time when created particles have the most important effect on any observable. Furthermore, one could hardly expect that it is possible to break Lorentz invariance while keeping conformal invariance intact. Therefore, TPPC should be "democratic," and the creation of photons, neutrinos, electrons, *etc.*, should be possible, in addition to creation of scalar (density) perturbations and gravitons (tensor perturbations). This was used by two of us in Ref. [23] to obtain strong restrictions on the TPPC effect using observed limits on the flux of the diffuse X-ray background.

Note that the TPPC effect, which corresponds to a non-standard contribution to the imaginary part of the (off-shell) graviton propagator, should be distinguished from vacuum polarization effects (corrections to the real part of the propagator). The latter ones certainly exist and lead, in particular, to H_I^2/M_{Pl}^2 corrections to the standard

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[2] In generalizations of the general theory of relativity, such as scalar-tensor gravity, $F(R)$ theory where R is the Ricci scalar, brane gravity, *etc.*, G_{eff}^{-1} (depending on H, ρ, and other quantities) replaces M_{Pl}^2 on the right-hand side of Eq. (1).

[3] Unless explicitly stated, we will assume $H \ll \Lambda$.

inflationary predictions (here H_I is the value of H during inflation–more exactly, during the last 60 e-folds of it).[4] However, this only results in an effective renormalization of H_I and other parameters defining the primordial spectra. On the other hand, the TPPC effect is a much bolder hypothesis, and it may well not exist at all. But if it exists, it leads to unique consequences such as the generation of super-high-energy cosmic rays at the present time [23] and oscillations in the primordial power spectra of scalar and tensor perturbations with amplitudes not decreasing with the growth of k [25].

Following Refs. [22, 23, 25, 26], for a quantum field ϕ, the TPPC effect can be completely, though only phenomenologically, described in terms of the Bogoliubov coefficients α_k and β_k (the Bogoliubov coefficients satisfy $|\alpha_k|^2 - |\beta_k|^2 = 1$) in the expression for the time-dependent part $\phi_k(t)$ of the mode wave functions multiplying the Fock annihilation operator $\widehat{a}_k$ in the Heisenberg representation valid during the WKB regime $p = k/a(t) \gg H$, but after the mode has reached the sub-Planckian region $p \ll \Lambda$:

$$\phi_k = \frac{1}{\sqrt{2\omega a^3}} \left(\alpha_k e^{-i\int \omega dt} + \beta_k e^{i\int \omega dt} \right) \ , \tag{2}$$

where $\omega = \sqrt{p^2 + m^2}$ and the rest mass m are assumed to be much less than Λ. Though only the leading terms (the zeroth- and first-order ones) of the WKB solutions are explicitly written in this formula, the full WKB series is there.

The 'final' condition (2) (final in the sense that TPPC ceases) corresponds to the mode $\mathbf{k}$ being in a pure squeezed quantum state. This assumption is taken by analogy with the usual particle creation from vacuum where the final state is a squeezed one, too. However, since we need the average particle number $\langle n(\mathbf{k}) \rangle \equiv |\beta_k|^2$ only for the calculation of TPPC of wimpzillas (see Sec. III below), it is straightforward to generalize the condition (2) to an arbitrary pure quantum state having the same $\langle n(\mathbf{k}) \rangle$. In principle, further generalization to the density matrix description is also possible and does not present any problem. However, since usual particle creation both in cosmology and black hole physics does not result in the local loss of coherence, *i.e.,* in the transformation of a pure state into a mixed one, it is natural to suppose (at least, at the first step) that the TPPC effect does not lead to it either.

The surface $p^2 = \Lambda^2$ was dubbed the 'new-physics hypersurface' in Ref. [26] (see Ref. [27] for a comparison of this approach to other ones). Corrections to the perturbation spectra produced during inflation are proportional to β_k, while the number of created particles is $n_k = |\beta_k|^2$. As was pointed out above, there is no TPPC effect in flat space-time, so as expected, $\beta_k = 0$ for $H = 0$. The main hypothesis assumed in all studies of TPPC is that β_k is not exponentially suppressed, but only power-law suppressed for $H \ll \Lambda$, where H is taken at the moment $k = a\Lambda$ when the given mode crosses the new physics hypersurface (otherwise, the effect would not be of practical interest). Thus, $|\beta_k|$ may be modeled as

$$|\beta_k| = b \left(\frac{H_k}{\Lambda} \right)^{\gamma} \ll 1, \quad \text{with } \gamma > 0 \ , \tag{3}$$

where H_k is the value of the expansion rate when a mode with comoving momentum k crossed the trans-Planckian region: $H_k \equiv H(k = a\Lambda)$.

Of course, the use of only one time-dependent parameter H_k is an over-simplification. Really, from general covariance one should expect that for a FRW background β_k depends not only on H but also on $\dot{H}/H^2$ in such combinations that the actual dependence is on the generally covariant quantities $R \equiv R^{\mu}{}_{\mu}$ and $R_{\mu\nu}R^{\mu\nu}$. Moreover, this dependence is most probably non-local since even the usual particle creation effect is always non-local. Also, the answer should contain the step function $\theta(H)$ since the very TPPC effect may come into existence only due to the expansion of the universe. Thus, the formula (3) is a crude approximation which, however, becomes a rather good one during slow-roll inflation since $R_{\mu\nu}$ is approximately constant and uniquely related to R there.

In principle, the parameter γ need not even be an integer. In particular, just such a case is realized in the concrete model proposed in Ref. [28] where four-dimensional Lorentz invariance is softly violated due to brane-world effects (see Ref. [29] for earlier works in a similar direction). However, mostly integer values of γ were considered previously. The condition of the absence of an excessive back-reaction for TPPC requires

$$|\beta_k| \lesssim H_k M_{Pl} \Lambda^{-2} \ , \tag{4}$$

which is valid both during inflation [30] and after it [22] (see also [31, 34]). It is obtained under the assumption that created particles appear just at the new-physics hypersurface $p = \Lambda$, and do not exist before that. The comparison of Eq. (4) with the corresponding condition for the standard quantum field theory of Eq. (1) (with $\langle n(\mathbf{k}) \rangle = |\beta_k|^2$)

[4] Effects of the same order were considered in Refs. [24] and other papers.

clearly shows the profound difference between the two cases: First, integration over k from the Lorentz non-invariant scale $k = a\Lambda$ up to infinity is completely omitted and modeled as a boundary term at $k = a\Lambda$; second, this boundary term is estimated at different moments of time for different k, in contrast to Eq. (1) imposed at the same $t = t_0$ for all modes.

One may ask how the mode wave function (2) would look at earlier times (in particular, if one wishes to define initial conditions for all modes $\mathbf{k}$ at the same moment of time). The answer is that in standard QFT where TPPC is absent, it has essentially the same form as (2) with constant α_k and β_k since it remains in the WKB regime. For a non-standard QFT admitting TPPC, the answer is, of course, strongly model dependent and generically not known at all. Then, however, condition (1) ceases to be valid soon in the past due to the decrease of $a(t)$ (especially rapid during inflation). In particular, even the partial contribution to the integral over d^3k in (1) from modes having $k = a\Lambda$ at the end of inflaton increases by more than $e^{240} \approx 10^{104}$ times when shifted to an initial moment (presumably the same for all modes) taken slightly earlier than the moment when a comoving scale equal to the present Hubble radius first crossed the Hubble radius during inflation. So, in order not to violate (1), either γ should be very large or b should be exponentially small if we wish to obtain the result (3) using standard particle creation from a non-vacuum initial state. Moreover, not only the power-law behaviour (3) with an arbitary γ but any phenomenological outcome of TCCP not being *exactly* zero for a sufficiently small H *may not* result from any non-vacuum initial state in standard QFT if the de Sitter stage beginning in our Universe now is stable since $|\beta_k| \to const$ at $k \to \infty$ for that state.

This shows that the condition (1) yields a quantitative criterion to distinguish genuine TPPC from that which may be explained without it. Note that this conclusion is opposite to the main statement of Ref. [35] about the impossibility to discriminate between the usual and trans-Planckian particle creation using observational data. Though for a test QFT in an external gravitational field, any final outcome of TPPC like (2) may be formally expressed as the usual creation from a non-vacuum initial state taken at some earlier time, this construction becomes inconsistent when the energy-momentum tensor of particles is taken into account. In other words, we *may not* simply employ standard QFT and think that particles really exist if condition (1) is violated. Thus, particles created by TPPC, with their energy and momentum in the usual sense, should come to existence at some later time only, not at the initial hypersurface with equal time for all modes $\mathbf{k}$.

Inserting Eq. (3) into Eq. (4) we see that for $\gamma \geq 1$, Eq. (4) is satisfied for all $H \leq \Lambda$ if $b \leq M_{Pl}/\Lambda$. Thus, in the $\gamma \geq 1$ case, Eq. (4) is not very restrictive. However, if $\gamma < 1$, then the inequality in Eq. (4) is violated for sufficiently small H regardless of b.

The case $\gamma = 1$ is required to obtain noticeable corrections to the primordial spectra generated during inflation. The above mentioned results of Ref. [23] show that the $\gamma = 1$ TPPC effect must be strongly suppressed for usual elementary particles: $b < 10^{-6} M_{Pl}/\Lambda$. This result leads to the absence of any noticeable features in the CMB temperature anisotropy. On the other hand, ultra-high energy particles created in this way at the present time still can be seen through an excess of cosmic rays at energies above the Greizen-Zatsepin-Kuzmin limit (if such an excess will prove to exist).

In Eq. (3), it is assumed that Λ is the *minimal* energy scale connected to 'new physics' and that this expression is valid for all energy scales $\omega \ll \Lambda$. Inflation is supposed to occur below this scale also: $H_I \ll \Lambda$. To avoid the upper limit of Ref. [23] keeping the possibility to detect some TPPC features in CMB fluctuations, a more complicated model with *two* scales Λ and $\Lambda_1 \ll \Lambda$ was proposed in Ref. [32], where Eq. (3) with $\gamma = 1$ is valid for $\Lambda_1 \ll H \ll \Lambda$ and inflation occurs also in this energy interval. On the other hand, $|\beta_k|$ is strongly suppressed for $k \ll \Lambda_1$. Of course, assuming inflation to occur above any new-physics scale, especially that related to some kind of the Lorentz invariance breaking, strongly undermines the power and beauty of the standard inflationary calculations based just on the assumption of the *absence* of any radically new physics up to the scale of inflation. In particular, if Lorentz invariance is not valid during inflation, then even the necessity to invoke inflation to explain causal connections in the observed part of the universe may well be called into question. Fortunately, present observations do not require us to go so far: the direct search of superimposed oscillations of potentially trans-Planckian origin in the power spectrum of density perturbations using the 3-year WMAP CMB data [17] (see also Ref. [33]) does not give any statistically significant evidence for them. All this puts the existence of the TPPC effect with $\gamma \leq 1$ under serious question. However, larger values of γ are not excluded. In the next Section, we show that TPPC of wimpzillas presents a new possibility to probe that range of γ.

III. TPPC OF WIMPZILLAS

Let us combine the TPPC and wimpzilla ideas and study the cosmological implications of the creation of wimpzillas by the expansion of the universe due to trans-Planckian effects. Due to the universality of gravitational interactions, TPPC, if exists at all, should occur for all types of particles and for all times. Moreover, trans-Planckian production of wimpzillas is not suppressed as long as their rest mass satisfies $m_X < \Lambda$. Thus, the TPPC effect opens a possibility for a new and more effective way to produce super-heavy particles with masses up to M_{Pl} (if $\Lambda \sim M_{Pl}$). In turn, supermassive particles present a new possibility for TPPC to reveal its properties in a different regime: since wimpzillas are not thermalized, the present abundance of particles of mass in excess of H_I should reflect the TPPC properties, in particular, the value of γ.

The number density of created X (wimpzilla) particles is given by the expression

$$n_X = \frac{1}{2\pi^2 a^3}\int_0^{a\Lambda} dk\, k^2 \left|\beta_k\right|^2 \ , \tag{5}$$

where the upper limit just reflects the assumption that particles are created with the physical momentum Λ at the moment of time when $k/a(t) = \Lambda$, *i.e.,* at different moments of time for different modes. After that, their momentum simply is redshifted as the scale factor a increases.

The next step is to determine the moment when TPPC produces the main effect. Let us insert here $|\beta_k|$ from Eq. (3). The dependence of the expansion rate H on a depends on the cosmic epoch as

$$H \propto \begin{cases} a^0 & \text{inflation} \\ a^{-3/2} & \text{matter dominated} \\ a^{-2} & \text{radiation dominated} \ . \end{cases} \tag{6}$$

We see that for any positive value of γ, the contribution to the total number density from particles that emerged from the trans-Planckian region during inflation is dominated by those created toward the end of this epoch (those created earlier are redhifted away). During the matter-dominated era, late-time TPPC dominates if $\gamma \leq 1$, while if $\gamma > 1$, then the largest contribution comes from the particles created at the beginning of the matter-dominated phase. During the radiation-dominated era, late-time TPPC dominates for $\gamma \leq 3/4$, but for $\gamma > 3/4$ early-time created particles make the main contribution to n_X.

In view of the discussion above, let us turn to the case $\gamma > 1$ from here on. Then the integral in Eq. (5) is dominated by the contribution from the end of inflation. For the accuracy needed, it is sufficient to put $H = H_I = const.$ in this region. Then the X-number density after inflation would be

$$n_X = \frac{b^2}{6\pi^2}\left(\frac{H_I}{\Lambda}\right)^{2\gamma} \Lambda^3 \left(\frac{a_{EI}}{a}\right)^3 \ , \tag{7}$$

where the EI subscript denotes the end of inflation.

Now the development depends a bit on the evolution of the universe between the end of inflation and "reheating," the beginning of the radiation-dominated era. We will assume the maximum temperature of the universe in the radiation-dominated era was T_{RH}. If entropy was conserved between reheating and today, the ratio of the X-number density to the entropy density s is

$$\frac{n_X}{s} = \frac{b^2}{3\pi}\frac{H_I^{2\gamma-2}}{\Lambda^{2\gamma-3}}\frac{T_{RH}}{M_{Pl}^2} \ . \tag{8}$$

Since it is not very useful to have the result depend on the unknown parameter T_{RH}, we define a reheating efficiency factor r as

$$r = \frac{T_{RH}}{M_{Pl}^{1/2} H_I^{1/2}} \ . \tag{9}$$

If reheating is very efficient, then $r \sim 1$. If the extraction of the inflaton energy density is an inefficient, prolonged affair, then it is possible that $r \ll 1$. It is also convenient to define a dimensionless parameter λ relating the trans-Planckian scale Λ to the Planck scale:

$$\Lambda = \lambda M_{Pl} \ . \tag{10}$$

One expects $10^{-3} \lesssim \lambda \lesssim 1$.

In terms of these dimensionless parameters, the ratio of the number density to the entropy density becomes

$$\frac{n_X}{s} = \frac{1}{3\pi}\frac{r b^2}{\lambda^{2\gamma-3}}\left(\frac{H_I}{M_{Pl}}\right)^{2\gamma-3/2} \ . \tag{11}$$

It is straightforward to convert n_X/s into an expression for $\Omega_X h^2$, where as usual Ω_X is the present ratio of the X energy density to the critical density and h is Hubble's constant in units of 100 km s^{-1} Mpc^{-1}:

$$\Omega_X h^2 = 3 \times 10^{26} \frac{rb^2}{\lambda^{2\gamma-3}} \left(\frac{H_I}{M_{Pl}}\right)^{2\gamma-3/2} \frac{m_X}{M_{Pl}} \ . \tag{12}$$

In the case $\gamma = 2$, to avoid overproduction of wimpzillas, we need:

$$b^2 \times \frac{m_X}{M_{Pl}} \lesssim 5 \times 10^{-28} \ \frac{\lambda}{r} \left(\frac{H_I}{M_{Pl}}\right)^{-5/2} \ . \tag{13}$$

Thus, if wimpzillas exist, the strength of the TPPC effect (the coefficient b) should be small even for $\gamma = 2$. Moreover, we see that trans-Planckian effects can produce supermassive dark matter (wimpzillas) as massive as the Planck mass in an abundance to result in $\Omega_X h^2 \sim 0.15$, even for H_I as low as 10^8 GeV, corresponding to an energy density during inflation of about $(10^{13}\,\mathrm{GeV})^4$.

On the other hand, in the case of the two-scale trans-Planckian model of Ref. [32] discussed above, we can put $\gamma = 1$ in Eq. (13) if $H_I \simeq \Lambda_1$, so that there is no TPPC creation after inflation. Then

$$\Omega_X h^2 = 3 \times 10^{26}\, rb^2 \lambda^{3/2} \left(\frac{\Lambda_1}{\Lambda}\right)^{1/2} \frac{m_X}{M_{Pl}} \ . \tag{14}$$

So, even for this model, it is impossible to have noticeable trans-Planckian effects in CMB fluctuations (which requires $b = \mathcal{O}(1)$) and the existence of wimpzillas at the same time—trans-Planckian creation of the latter ones appears to be too strong.

IV. CONCLUSIONS

We have proposed to combine the hypothesizes of the wimpzillas and TPPC existence and calculate the amount of these superheavy particles created due to the expansion of the universe. It has been shown that a rather weak TPPC effect with $\gamma \simeq 2$ can produce the necessary amount of such particles to comprise the present dark matter in the universe, even if their mass is comparable to the Planck mass. These particles are mainly created at the end of inflation. Thus, a new possibility for the origin of dark matter opens.

On the other hand, existence of wimpzillas places new upper bounds on any more fundamental theory predicting TPPC, which are significantly stronger than those known previously. In particular, it is not possible to obtain both super-heavy dark matter particles and observable corrections to the primordial perturbation spectrum – TPPC cannot bear two fruits at once.

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