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Multipole Analysis for Absolute Magnetic Field Measured by Multi-Probe Pulsed-NMR Method

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***Abstract*—For accelerator dipole magnets, the study on multipole coefficients in a local region is useful for understanding magnet characteristics and checking a construction procedure. The multi-probe pulsed-NMR method is a candidate for measurement of the local multipole coefficients. For the absolute fields measured by the multi-probe pulsed-NMR method, a rigorous application of the Fourier expansion is found to yield multipole coefficients. It is made clear that the direction of the dipole field is unable to be determined by the Fourier expansion alone. For estimating alignment accuracy of pulsed-NMR probes, the relationship between errors of multipole coefficients and those of probe position is derived by the Fourier expansion method. For applying of multi-probe pulsed-NMR method to accelerator dipole magnets, we designed and fabricated a field measuring system using two probes. The probes had NMR coils wound around glass ampuls with a diameter of about 1.4 mm. The NMR signal was processed with a phase lock loop circuit to produce the NMR frequency. Dipole field uniformity and multipole coefficients were measured at 0.35 T for a 1.5 m long dipole magnet. It was confirmed that the field data by this NMR system had an accuracy of 10^{-6}.**

I. INTRODUCTION

Large collider accelerators such as LHC [1] are equipped with a great number of superconducting magnets. The dipole magnets are required to have a highly homogeneous magnetic field around the beam axis. The relative field deviation in the region about 1 cm in diameter should be less than $2\sim3\times10^{-4}$ without use of additional correcion coils. The magnetic field in the beam region is expressed by superposition of multipole components with normal and skew coefficients in a two-dimensional treatment [2]. The multipole components of the magnetic field need to be measured with a high accuracy. For accelerator operation, the magnetic field is tailored to be as uniform as possible: According to the measured data, the multipole components are decreased to values below desired tolerance by the use of the correction multipole coils.

The magnetic field in the accelerator magnets has often been measured by the rotating-coil method [3]. The rotating-coil measurement usually gives the magnetic field integrated typically in a few meters along the beam axis. The local multipole characteristics along the beam axis are of current interest. The local multipole measurement is useful for understanding magnet characteristics [4] and for improving the magnet fabrication procedure. However, the rotating-coil method is unsuited for supplying local field distribution. Hall probes are a candidate for measuring the local field distribution at individual positions. Well-calibrated probes have been produced an accuracy of 5×10^{-5} in field measurement [5]. Hall probes, however, are sensitive only to the magnetic field perpendicular to the Hall plane, so that troublesome problems may arise in setting small probes precisely in the magnet bore tube [6].

The alternative to the Hall probe method is the use of the multi-probe pulsed-NMR system [7], [8]. NMR methods have an excellent accuracy without calibration: It readily reaches seven-digit or more accuracy. In addition, the temperature of NMR probes has no influence on the measurement accuracy in principle. The NMR method is capable of making high-accuracy measurement in bore tubes at room or cryogenic temperatures without calibration. The pulsed-NMR probe can also be made small in a size of mm range, so that it is applicable to studying a local field homogeneity. The excellent accuracy in field measurement may lead to highly advanced understanding of the magnet characteristics. The pulsed-NMR method is suited to measurement in the straight section of the magnet, whereas it is not easy to use the NMR system in the magnet end region due to its limited dynamic range in field measurement.

The pulsed-NMR method has already been demonstrated to be usable for field homogeneity measurement at the SSC Laboratory [9]. This method supplies only the absolute value of the magnetic field $|B|$ due to the properties of NMR. Since $|B|$ is unsuited to the Fourier expansion in a sense of complete two-dimensional magnetic field analysis, the multipole analysis was made approximately. The measured $|B|$ was regarded as the field in the vertical direction B_y for the purpose of direct use of the Fourier expansion. It is not clear to what extent the substitution of $|B|$ by B_y disturbed the resultant multipole components. It is necessary to find a better multipole expansion procedure which makes complete use of pulsed-NMR results with excellent precision.

In this paper, we attempt to develop a method of obtaining multipole coefficients by means of rigorous treatment of the absolute field. In addition, details on the pulsed-NMR field measuring system with two probes are described. Finally, field measurements for a dipole magnet and the performance of the system are discussed.

II. MULTIPOLE ANALYSIS FOR PULSED-NMR METHOD

In the pulsed-NMR method, the absolute fields in dipole magnets are measured at many points on a circle with a certain radius, as shown in Fig. 1. The probes are sensitive to the absolute strength of the magnetic field in the x-y plane in this configuration. The fields at individual positions (r,θ)

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were analyzed [7] for giving the multipole components. Since the measured absolute fields at individual points were assumed to approximately equal to the vertical field B_y, the field in the vertical direction B_y at the point (r,θ) was expressed by

$$By(r,\theta) = |B_0| + |B_0| \sum_{n=1}^{\infty} r^n [b_n \cos n\theta - a_n \sin n\theta], \tag{1}$$

where B_0 is the fundamental dipole field in the y-direction, a_n and b_n are the skew and normal $2(n+1)$-pole coefficients, respectively. For the measurement with multi-probe (total number of probes is M), the coefficients a_n and b_n are written by Fourier inversion formulas :

$$a_n = [-2/(M\overline{|B|}r^n)] \sum_{m=0}^{M-1} |B(m)| \sin(2\pi nm/M), \tag{2.a}$$

$$b_n = [2/(M\overline{|B|}r^n)] \sum_{m=0}^{M-1} |B(m)| \cos(2\pi nm/M), \tag{2.b}$$

where $|B(m)|$ is the absolute field measured at point m, and $\overline{|B|}$ the average value of $|B(m)|$. Measurement at 36 points, for instance, gives multipole components up to 18-th pole.

Unlike Ref. [7], we take a rigorous expression without approximation. The two-dimensional field around the beam axis is written by a complex value B as $B = B_y + \mathrm{i} B_x$. The complex field B is expanded with $z = x + \mathrm{i}y = re^{\mathrm{i}\theta}$ and expressed as

$$B = B_0 \sum_{n=0}^{\infty} c_n z^n = B_0 \sum_{n=0}^{\infty} (b_n + \mathrm{i}\, a_n) r^n (\cos n\theta + \mathrm{i} \sin n\theta) . \tag{3}$$

The complex value $c_n = b_n + \mathrm{i}\, a_n$ shows the multipole components again. We concentrate our treatment on dipole magnets, where $b_0 = 1$ for the fundamental dipole component and $a_0 = 0$ in definition. It is noted that the total field B is composed of the fundamental field B_0 and the sum of other higher harmonic fields B_h as $B = B_0 + B_h$, where $B_h = B_0 \sum_{n=1}^{\infty} (b_n + \mathrm{i}\, a_n) r^n (\cos n\theta + \mathrm{i} \sin n\theta)$. One can see that $|B|^2$ is written by $|B|^2 = B_0^2 + 2B_0 \,\mathrm{Re}(B_h) + B_h B_h^*$, where B_h^* is the complex conjugate. In this paper, a scalar quantity D is introduced as $D = |B|^2 - |B_h|^2$. The value of D is completely Fourier-expanded without any approximation as

$$D = B_0^2 + 2B_0^2 \sum_{n=1}^{\infty} r^n (b_n \cos n\theta - a_n \sin n\theta). \tag{4}$$

As the first approximation, the initial value of D is taken to be $D(1) = |B|^2$ by the use of the measured absolute values. This $D(1)$ gives the quantities of $B_0(1)$, $b_n(1)$, $a_n(1)$ through Eq. (4). The procedure in Ref. [7] corresponds to the first approximation. Using the field $B_0(1)$ and the coefficients $b_n(1)$ and $a_n(1)$ leads to the second approximation to D as

$$D(2) = |B|^2 - |B_0(1) \sum_{n=1}^{\infty} c_n(1)\, z^{n-1}|^2. \tag{5}$$

The values of $B_0(2)$, $b_n(2)$ and $a_n(2)$ can be determined by applying the Fourier expansion of Eq. (4) again. The calculation is iterated m times until satisfying $B_0(m)c_n(m) \approx B_0(m-1)c_n(m-1)$ with acceptable degree. The convergence is reached in a few repetitions because $|B_0| >> |B_h|$ for the accelerator dipole magnets.

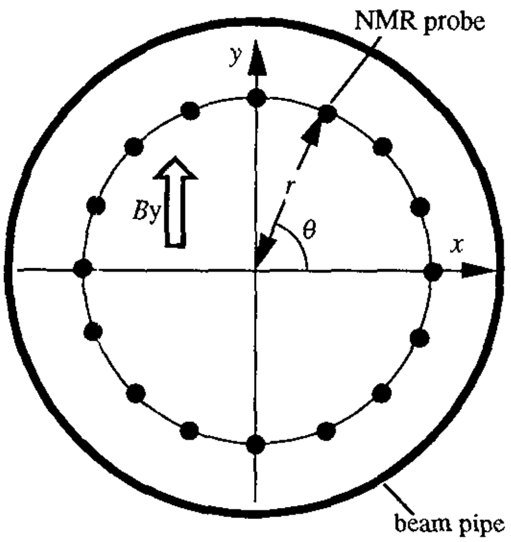

Fig. 1. Example of NMR probe configuration in measuring the magnetic field in accelerator dipole magnet. The z axis is taken along the beam axis. Each coil is placed on the point along the circumstance with radius r. In this case, the total number of measuring points M is 16.

Next we suppose that the vertical axis of the pulsed-NMR system tilts slightly by ϕ against the true direction of the dipole component. Then, c_n changes to $c_n' = c_n e^{\mathrm{i}\phi}$, so that ϕ in c_n' gives no contribution to D, as inferred from Eq. (5). Therefore, ϕ is unable to be determined by the Fourier expansion in the absolute field measurement. For determination of ϕ, for instance, we can use a probe coil to measure the field in the x-direction B_{0x}'. It is desired to carefully set the whole NMR probe system to realize $B_{0x}' = 0$ during a magnet current ramp.

In the practical pulsed-NMR measurement, there may be some geometrical problems arising from alignment errors on probe positions. A rough analytical estimation is made on the basis of the Fourier expansion on the condition that randomness of individual position errors is ignored. We assume that the probe location deviates from the original correct position (r,θ) by Δr and $\Delta\theta$. When $\Delta r/r << 1$, $\Delta\theta/\theta << 1$ and the products of $\Delta r \Delta\theta$ are negligibly small, the expression in Σ of Eq.(4) yields

$$\Delta b_n / b_n = n\Delta r / r - n\Delta\theta (a_n / b_n) , \tag{6.a}$$

$$\Delta a_n / a_n = n\Delta r / r + n\Delta\theta (b_n / a_n) . \tag{6.b}$$

These equations give the following indications: Δr and $\Delta\theta$ induce a greater increase in Δa_n and Δb_n at larger n, and a large value of a_n decreases Δb_n through the change in $\Delta\theta$. When $\Delta r/r = 0.01$ (for instance, r = 10 and Δr = 0.1 mm) and $b_n \approx a_n$, we obtain $\Delta b_n / b_n \approx \Delta a_n / a_n \approx 1\%$ for n = 1 and 10% for n = 10. If $\Delta\theta$ = 0.01 rad (0.6 degree), we have $\Delta b_n / b_n \approx \Delta a_n / a_n \approx 1\%$ for n = 1, and 10% for n = 10. It is suggested that $\Delta r/r < 0.01$ and $\Delta\theta < 0.01$ rad should be satisfied in the construction of NMR probe system.

III. PULSED-NMR FIELD MEASUREMENT SYSTEM

In the NMR method, radio frequency (rf) pulses are supplied through an NMR coil to the nuclear spin system which is placed in a static magnetic field. When the supplied rf pulses almost satisfy the NMR condition, a decaying rf signal is induced in the NMR coil. The observed rf pulses are called the free-induction-decay (FID) signal. The frequency of the FID signal is exactly equal to the NMR frequency. The relation between the NMR frequency ($f_{\rm NMR}$) and the static field (B_0) is expressed by $f_{\rm NMR} = \gamma_g B_0/(2\pi)$, where γ_g is the nuclear gyromagnetic ratio and equals 2.675147×10^8 radT^{-1}s^{-1} for hydrogen nuclei. Counting $f_{\rm NMR}$ gives the strength of the static field with an excellent accuracy. For the high accuracy field uniformity measurement in accelerator dipole magnets, we designed and fabricated the pulsed-NMR field measuring device equipped with two small NMR probes.

Figure 2(a) shows a block diagram of the measurement setup. The device consists of oscillator, transmitting (Tx) amplifier, pre-amplifier, main amplifier, frequency counter and the NMR probe. The ac wave generated from the oscillator is converted to rf pulses through a Tx gate. A Tx amplifier forms intense rf pulses and transmits them to the matching network in the pre-amplifier. A receiving (Rx) gate circuit prevents the pre- and main amplifiers from being saturated by the input rf pulses. The same coaxial cable is used for the transmission of the input rf pulses and the reception of the FID signal in this system. The FID signal is a decaying rf wave and is generated at regular intervals. After the FID signal is amplified by the main amplifier, the frequency of the FID signal is modulated into an intermediate frequency (IF) as shown in Fig. 2(b). The frequency of the FID signal is counted by a frequency counting module based on a phased locked loop (PLL) circuit. In this module, a wave shaper circuit slices off a part of the signal and produces rf pulses at certain regular intervals. The phase of the rf pulses is detected by a phase lock unit by comparison with the output of a voltage controlled oscillator (VCO). The deviation signal between two signals is fed back to the VCO. Then the output frequency of the VCO equals that of the IF of the FID signal. The absolute value of the magnetic field is obtained from the frequency. The coaxial cable length l is taken as $n\lambda_{\rm NMR}/2\,(n = 1, 2 \ldots)$, where $\lambda_{\rm NMR}$ is the wavelength of the NMR signal in the cable [10]. Tuning the inductance of the NMR coil L and the capacitance of the matching capacitor C enables us to optimize the propagation performance of the probe module.

IV. MEASUREMENT AND RESULTS

Measurement of the field homogeneity and multipole coefficients was carried out for a 1.5-m long dipole magnet by using the manufactured system. The measurement along the beam axis (z scan) was performed from 250 to 750 mm from the edge of the yoke. The scanning device with the NMR probes was installed between pole pieces. A probe holder was attached on a tip of a long aluminum rod. Rotation of the probe holder around the central axis enabled us to measure the field on the circumstance at 36 points (M=36) per 10 degrees. In this measurement, we utilized two probes for simplicity. One was set on a point of 30 mm away from the axis for signal detection through rotation. The other was fixed at the center of the holder on the axis. The latter one had a role of the reference probe for compensating a fluctuation of the field caused by time variation of the transport current. The NMR coils for mapping were wound around a glass ampul with a diameter of either 1.4 or 2.4 mm. The ampul for the reference coil had the diameter of 2.4 mm. All ampuls were filled with water solution of copper sulfate (4.8 wt %). Protons in these ampuls were used as the source of the signal. The all data were obtained at 0.35 T corresponding to a transport current of 350 A. The power supply had a long-term stability of 5×10^{-5}. The frequency of the device was optimized with $f_{\rm NMR}$ for 0.35 T (14.9 MHz).

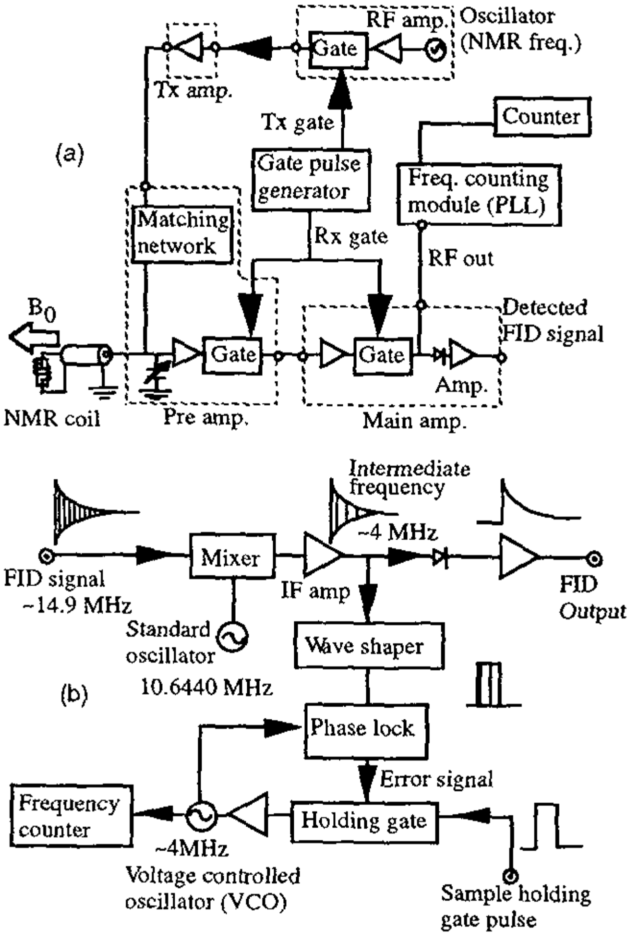

Fig. 2. Pulsed-NMR system for the magnetic field measurement: (a) Block diagram of the pulsed-NMR devices. (b) Block diagram of the frequency counting module using PLL circuit and the typical waveform at each point.

The frequency of the two NMR probes ratio was measured as a function of θ at the center of the magnet (z = 750 mm and r = 30 mm). According to Eq. (4), multipole components up to 18-th pole for the magnet were calculated from measurement results. Fig. 3 shows profiles of multipole coefficients $|c_n|$ at the center of the magnet. As seen in Fig. 3, the coil with 1.4 mm diameter yields better accuracy than that of 2.4 mm. Minimizing the size of the coil leads to mapping with higher accuracy. In the use of 1.4 mm probe, the normalized sextupole component ($|c_2|$), which is one of the most important components for dipole magnets, is predominant and the value is to 3.33×10^{-4} with a standard deviation of 1.80×10^{-6}. For all other components, the

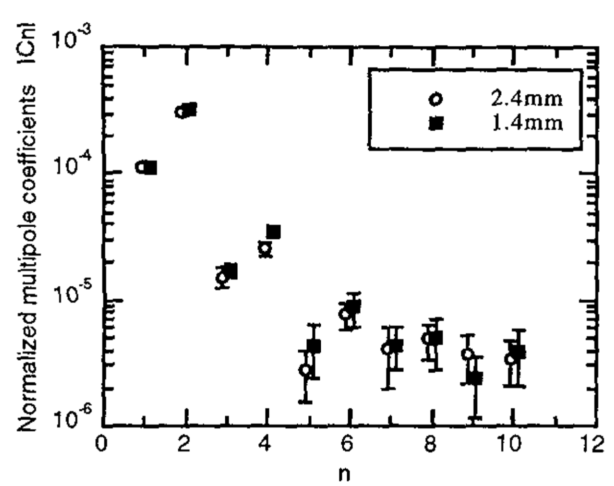

Fig. 3. Normalized multipole coefficients $|c_n|$ up to 18-th pole for harmonic number n. Sextupole component (n=2) is dominant.

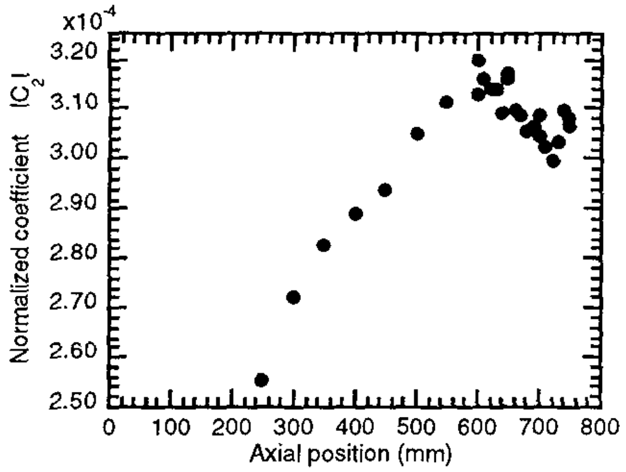

Fig. 4. Axial distribution of sextupole component at r=30 mm. The center of the magnet locates at z=750mm.

deviation within order of 10^{-6} was obtained. In our scanning device, the positioning error was estimated to be 50 μm. It corresponds to about 3×10^{-6} in $\Delta\theta$. The field homogeneity was also measured by a rotation-coil with a length of 4 cm. Results from the pulse-NMR coil were averaged. It was confirmed that these data agreed with data obtained from a rotating-coil method within a typical error of about 20 % except for the 14-th pole component. The measured values of sextupole are plotted in Fig.4 as a function of z. A clear axial variation can be seen in the end region of the pole piece in detail. The influences on multipole components caused by the manufacturing error of pole pieces was observed.

V. CONCLUSION

We developed the procedure to calculate the multipole coefficients of the field in accelerator dipole magnets by rigorous treatment for the pulsed-NMR method. The absolute value of the field were step-wisely Fourier-expanded. This procedure improves the quality of the analyzed multipole coefficients in the pulsed-NMR method. A high accuracy magnetic field homogeneity measurement apparatus was developed on the basis of the multi-probe pulsed-NMR method. The dipole field homogeneity and multipole strength were precisely measured at 0.35 T for the 1.5 m long dipole magnet with an accuracy of 10^{-6}.

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