KEK Preprint 99-61
August 1999
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Preprint accepted to Physics Letters A

High Energy Accelerator Research Organization (KEK), 1999

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Mechanical quality factor of a cryogenic sapphire test mass for gravitational wave detectors

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A sapphire cylinder, of 100 mm diameter and 60 mm length, was suspended on a pair of sapphire fibers to simulate a mirror for a cryogenic laser interferometric gravitational wave detector. A mechanical quality factor of 2.5×10^8 was obtained for the second axial symmetric mode of the cylinder at 4.2 K.

Key words: Cryogenics; Gravitational wave detector; Laser interferometer; Thermal noise; Sapphire; mechanical quality factor

1 Introduction

For the first direct detection of a gravitational waves, large scale laser interferometric gravitational wave detectors, such as LIGO[1], VIRGO[2], GEO[3] and TAMA[4], are now under construction. However, there is little chance of success in the next few years as target events like a binary neutron star coalescence are very rare at the required amplitude of detection. To improve the detection rate advanced techniques are necessary that enhance the sensitivity to expand our observable horizon. Note that one order of magnitude improvement of the amplitude sensitivity increases the chance of detection by three orders.

Some advanced experimental technics have been already developed. For example, an X-isolator[5] has been implemented to decrease seismic noise, which is the dominant noise source below a few tens of Hz. Also, power recycling[6] has

Preprint accepted to Physics Letters A

been developed to enhance the effective laser power within the interferometer to reduce the effect of laser Shot noise, which dominates over several hundred Hz. The sensitivity of the detector between the seismic and Shot noise regions is limited by the thermal noise[7] of the suspension system. Typically, a suspension system contains a low acoustic loss test-mass coated to form an optical mirror, suspended by a low loss pendulum wire. We are developing a cryogenic mirror suspension system in order to reduce the thermal noise. This development presented in this paper is part of the Japanese LCGT project (Large scale Cryogenic Gravitational wave Telescope)[8].

The thermal noise consists of both the pendulum motion of the suspended mirror and the elastic vibration of the mirror itself. The amplitude of the thermal vibration obeys the equation

$$\langle x^2 \rangle \propto \frac{T}{Q} \tag{1}$$

where T is the temperature and Q the mechanical quality factor (Q-factor). Our strategy to reduce the thermal noise includes the use of low acoustic loss materials for the mirror test mass and the suspension fiber, along with the cooling of the whole mirror suspension system to cryogenic temperature. We have chosen sapphire for both the mirror and the fiber, since it has high Q-factor and thermal conductivity[9] (which is important for cooling the mirror). Earlier studies have shown that a sapphire cylinder suspended carefully can achieve a Q-factor of 5.0×10^9 at 4 K[10].

Previously, we have reported on thermal conductivity measurements of a sapphire mirror suspension supported by sapphire fibers[11]. We showed that a sapphire fiber can transfer heat due to absorption of high laser power incident on the mirror to a 4.2 K thermal ground. The equilibrium temperature of the test mass when applied to the TAMA detector was shown to be 15 K. The TAMA detector is a room temperature interferometric gravitational wave detector with 300 m base line, which is now operating in Japan[4].

We have measured the temperature dependence of Q-factor of a sapphire cylinder on the cryogenic mirror suspension system as well as the mirror of the gravitational wave detectors, in order to prove that the cryogenic suspension system is low loss and does not spoil the high Q-factor of the sapphire mirror.

We show that the Q-factor of the sapphire cylinder in the cryogenic mirror suspension system experimentally increases as the suspension was cooled. From this result combined with the previous attained equilibrium temperature, the mirror thermal noise was reduced by more than one order of magnitude compared to a conventional room temperature laser interferometric gravitational wave detector.

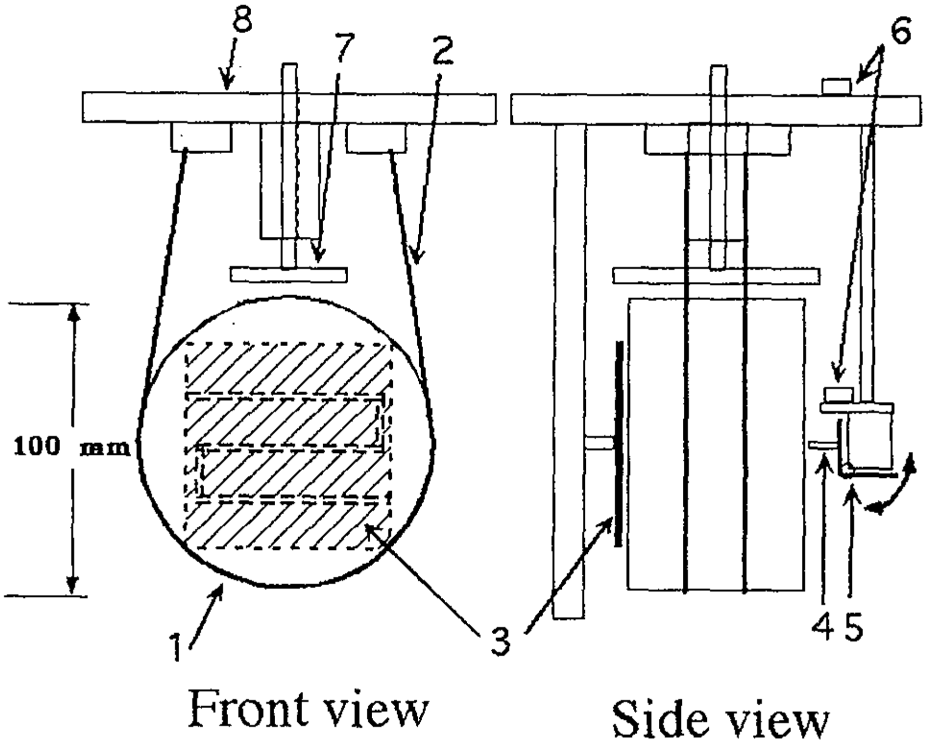

Fig. 1. Set up of the experiment. 1) a sapphire test mass of 100 mm in diameter and 60 mm in length; 2) sapphire fibers of 250 μm in diameter; 3) electrodes of the capacitance transducer; 4) a PZT vibrator; 5) a driver; 6) thermometers; 7) a mirror protection which consists of a shape memory spring; 8) 4.2 K thermal ground. Q-factor was calculated from decay time of free oscillation. The sapphire cylinder was suspended by a pair of sapphire fibers. The end of the fibers were firmly fixed to the 4.2 K cryogenic thermal ground. The excitation was controlled by a PZT vibrator and a driver. The PZT vibrator was pushed to be touched to the center of the sapphire cylinder and released during free oscillation measurement. Oscillation was detected by the capacitance transducer.

2 Experiment and Results

The ring down method was implemented to measure the Q-factor of two axial symmetric modes of the sapphire cylinder. After building up the vibration of a certain mode by an exciter, the damping time of free oscillation was measured by a capacitance transducer. The exciter consisted of a PZT vibrator and a flap for moving the PZT vibrator. The flap pushed the vibrator onto the end surface of the sapphire cylinder during excitation, then pulled the vibrator away when measuring the free oscillation. The heat production of the exciter was negligibly small so the sapphire cylinder stayed at cryogenic temperature. Thus, we regarded the thermometer attached on the exciter as the temperature.

Figure 1 shows the experimental setup. We used a sapphire cylinder[12] of

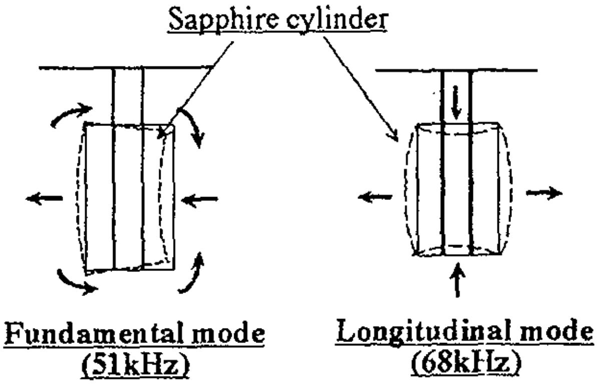

Fig. 2. Schematic view of measured modes. The transducer is only sensitive to axial symmetric modes. Each mode was identified by its resonant frequency.

100 mm in diameter and 60 mm in length. Orientation of crystal axis was perpendicular to the flat surface. Both sides of the sapphire cylinder were polished to be optically flat, however the cylindrical surface remained sandy. We coated thin films of aluminum on both flat surfaces to act as electrodes for the capacitance transducer. The sapphire cylinder was suspended by two loops of sapphire fibers of 250 μm in diameter[13] which were the same as used in the thermal conductivity experiment[11]. Upper ends of the sapphire fibers were fixed to a 4.2 K thermal ground. The whole assembly was installed in a vacuum chamber and evacuated down to 10^{-4}Pa. The vacuum chamber was dipped into liquid He in a cryostat.

Figure 2 shows schematic side views of the observed modes. Table 1 summarizes resonant frequencies and Q-factors measured at typical temperature points. Since the contribution of the modes with lower resonant frequency is more significant to the thermal noise of the interferometric gravitational wave detector, we measured the first (fundamental) and the second lowest (longitudinal) modes with axial symmetry. Other modes without axial symmetry do not contribute to the thermal noise in the interferometric gravitational wave detectors, because their contributions are smoothed out as long as the laser power distributes as Gaussian TEM_{00} mode.

Table 1
Resonant frequencies of the modes measured and of Q-factors at typical temperatures.

	fundamental mode		longitudinal mode	
temperature	resonant frequency	Q-factor	resonant frequency	Q-factor
4.2 K	50.977 kHz	9.6×10^7	67.759 kHz	2.5×10^8
15 K	50.977 kHz	1.0×10^8	67.759 kHz	2.0×10^8
78 K	50.971 kHz	2.9×10^7	67.756 kHz	3.5×10^7
300 K	50.633 kHz	3.1×10^6	67.548 kHz	4.6×10^6

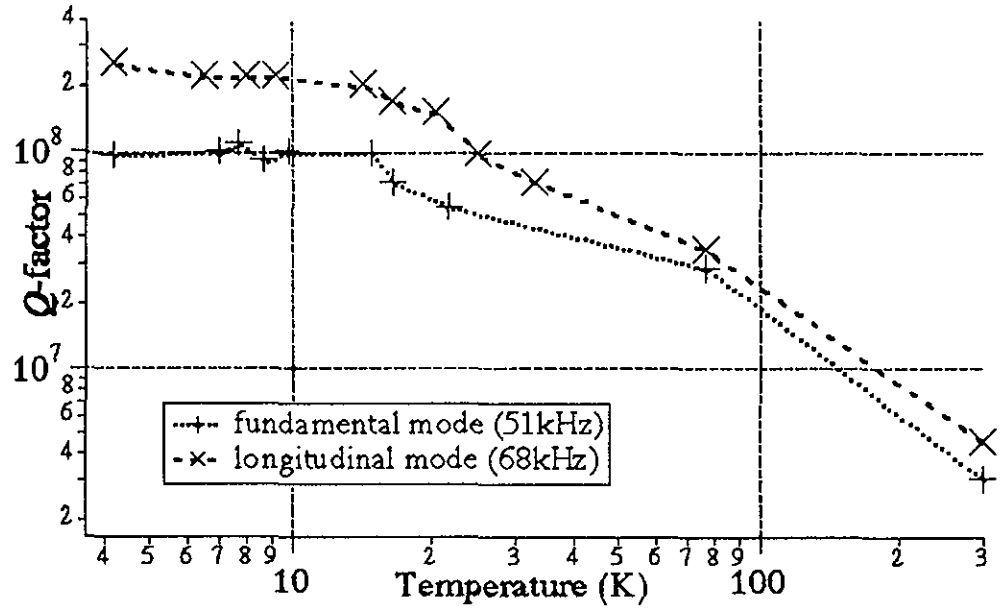

Fig. 3. Temperature dependence of the Q-factor of the sapphire test mass suspended by a pair of sapphire fibers.

Each acoustic mode was identified with the assistance of finite element program, ANSYS. *Calculations differed from the measured frequency by 11% for* the fundamental mode and 1% for the longitudinal mode. We guess that these discrepancies were due to the assumption of free vibrations, which ignored the effect of the wire suspension system.

Figure 3 shows the temperature dependence of the Q-factors. We obtained Q-factors of over 10^8 for both modes below 15 K. According to Eq. 1, with an estimated equilibrium temperature of 15 K[11], the root mean square amplitude of thermal oscillation is reduced by factor of 25 for the "fundamental mode" and by factor of 29 for the "longitudinal mode" in comparison with those at room temperature.

3 Discussion

In this section we discuss the limits of the measured Q-factors. A measured Q-factor is considered to be a superposition of individual loss mechanisms as

$$\frac{1}{Q_{meas}} = \sum \frac{1}{Q_i}. \tag{2}$$

We list possible loss mechanisms that may affect our experiment in the following.

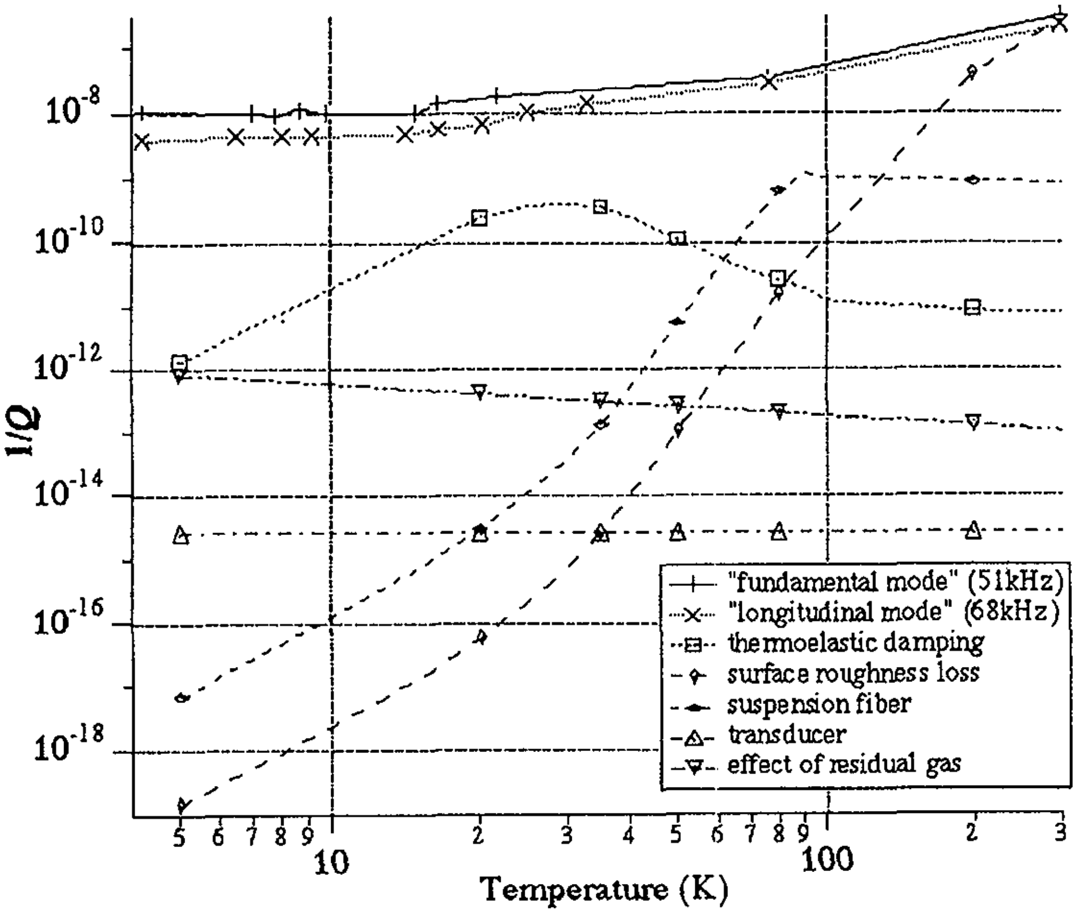

Fig. 4. Estimated contribution of five loss mechanisms in this experiment.

$1/Q_{te}$: effect of thermoelastic damping[10,14,15]
$1/Q_{surf}$: loss due to surface roughness of the sapphire cylinder[10]
$1/Q_{fiber}$: loss of the suspension fiber[10]
$1/Q_{trans}$: loss due to the transducer[16]
$1/Q_{gas}$: loss due to residual gas[17]
$1/Q_{fric}$: loss due to Coulomb friction between the sapphire fiber and the sapphire cylinder
$1/Q_{intrinsic}$: intrinsic loss of the sapphire cylinder

We evaluated Q_i's for each loss mechanism of the longitudinal mode except for the Coulomb friction and for the intrinsic loss. Figure 4 shows that these estimated Q_i's are much smaller than Q_{meas} at cryogenic temperature.

Although we were not successful in estimating the contribution of the Coulomb friction between the sapphire cylinder and the suspension fiber, we could experimentally conclude that the effect of the Coulomb friction was negligible within the experimental accuracy. This is because time dependence of the decay of the Coulomb damped oscillation is different from those of other loss mechanisms. If the Coulomb friction was dominant, the amplitude of a free os-

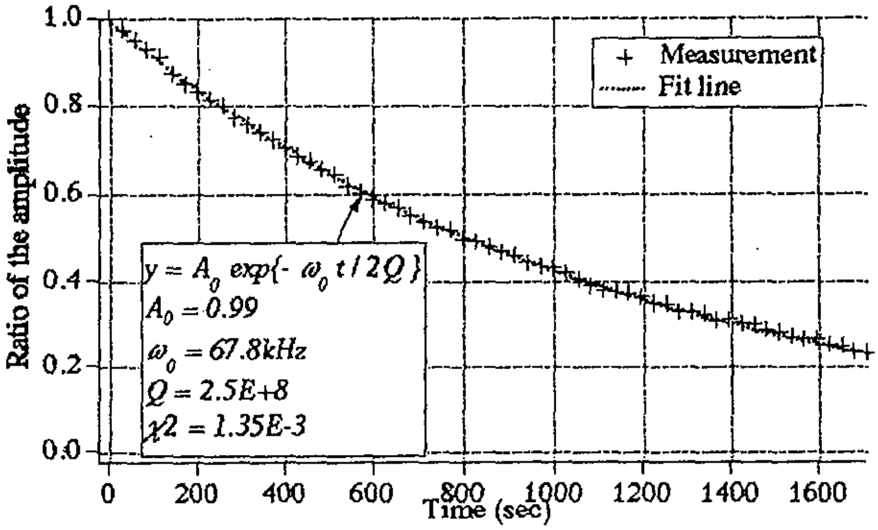

Fig. 5. Least square fit of the measured free decay of the longitudinal mode at 4.2 K. Also fit line of an exponential function is shown with $\chi^2 = 1.35 \times 10^{-3}$.

cillation decays linearly with time. Figure 5 shows a measured free decay with an exponential fitting function. This exponential dependence clearly shows that the Coulomb friction is not dominant in our experiment.

Finally, we can conclude that the dominant component of Q_{meas} was $Q_{intrinsic}$ at cryogenic temperature. From the obtained Q-factors, the cryogenic mirror suspension system exhibited low enough loss not to spoil the Q-factor of a well suspended cylindrical sapphire mirror of a gravitational wave detector.

4 Conclusion

The sapphire mirror/fiber suspension system was shown to function appropriately for a cryogenic interferometric gravitational wave detector. The Q-factor of the suspended cylindrical sapphire mirror was 2.5×10^8 at 4.2 K. Based on this result, the cryogenic sapphire mirror thermal noise was reduced by one order of magnitude compared to conventional gravitational wave detector operating at room temperature. Although many things have to be address to accomplish the cryogenic interferometer, for example contamination of a mirror surface due to cooling, we consider that the cryogenic mirror suspension system is promising for the LCGT project.

Acknowledgement

The authors acknowledge Prof. H. Sugawara, the director general of KEK and Prof. Y. Totsuka, the director general of ICRR for their encouragement and support. We wish to thank Daido Steel Co., Ltd for offering a shape-memory spring for a mirror protection system. This study was supported by the Joint Research and Development Program of KEK and by a grant-in-aid prepared by the Ministry of Education, Science, Sports and Culture.

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Appendix

A Thermoelastic damping

Thermoelastic damping is a kind of internal friction[14] in which the dissipation mechanism is caused by thermal conduction due to the temperature gradient in the oscillating cylinder[15]. Braginsky calculated the dissipation of a longitudinal mode of a bar[10] to be

$$\frac{1}{Q_{te}} = \frac{\kappa T \alpha^2 \rho \omega_0}{9c^2}, \tag{A.1}$$

where κ is thermal conductivity, T is temperature, α is the coefficient of thermal expansion, ρ is the density, ω_0 is resonant angular frequency and c is the heat capacity per unit volume. Applying this equation to our system, we have obtained a value ranging from 1.4×1.0^{-12} to 4.0×1.0^{-10}, which was lower than the result of our measurement in all temperature range.

B Loss due to surface roughness

The sapphire cylinder has possible damaged layer on its surface due to machining and polishing. Such a damaged layer consists of randomly oriented small crystallites which invade over a hundred microns deep[10]. Each of these crystallites have heat inhomogeneity during vibration and it will make heat loss similar to that of the thermoelastic damping. In general, dissipation of the thermoelastic damping is expressed as[7]

$$\frac{1}{Q} = \frac{E T \alpha^2}{c} \frac{\omega_0 \tau}{1 + (\omega_0 \tau)^2}, \tag{B.1}$$

$$\tau = a^2 \frac{c}{\kappa}, \tag{B.2}$$

where a is the typical size of heat distribution. The typical size for the surface loss is estimated to be the size of the small crystallite, which was, for example, given 10^{-6} m[10]. Additionally, we have to consider the ratio of the damaged volume to the total to evaluate the ratio of the dissipation energy to the total energy. Finally we obtain

$$\frac{1}{Q_{surf}} = 4h(\frac{1}{2L} + \frac{1}{D})\frac{ET\alpha^2}{c}\frac{\omega_0\tau}{1+(\omega_0\tau)^2}, \tag{B.3}$$

where h is the depth of the damaged layer, L is the length of the sapphire cylinder and D is the diameter of the sapphire cylinder. Using this equation and typical values[10], $1/Q_{surf}$ at the room temperature is 2.8×10^{-7} which is close to the our measurement of 2.2×10^{-7}. We recognized our result at the room temperature was limited by this surface loss. However the surface roughness loss is much smaller than the result of our measurement at cryogenic temperature.

C Loss of suspension fiber

The vibration of the sapphire cylinder causes the suspension fibers to oscillate with the same frequency. Since the suspension fiber has loss mechanisms as well, energy dissipation occurs. To estimate this dissipation we need to know two things. The first is how much energy is used to oscillate the fibers. The second is how much dissipation the sapphire fiber causes.

The former is represented by the ratio of oscillation energy of the fibers and the sapphire cylinder, and is given by

$$\frac{4m(x\omega_0)^2}{M(X\omega_0)^2}, \tag{C.1}$$

where m is the mass of the fiber, x is the amplitude of the oscillation of the fiber, M is the effective mass of the vibration mode and X is the amplitude of the oscillation of the sapphire cylinder. Although x is not equal to X precisely, we assume that $x = X$ for the order estimation and $4m/M = 8.3 \times 10^{-5}$ in our case.

In this case we consider that the thermoelastic or the structure damping of the sapphire fiber is the dominant source of dissipation. The case of the thermoelastic damping is plotted in Fig. 4. Since the dissipation due to the structure damping of the sapphire fiber is reduced by the factor of $4m/M$, we can regard it negligible compared with the dissipation due to the structure damping of the sapphire cylinder, which is less than the result of our measured Q-factor. From these consideration we conclude that the fiber loss is not limiting our measurement.

D Q degradation due to transducer

The capacitance transducer has an electrically couples to the sapphire cylinder, which can cause a dissipative loss. According to the equivalent circuit analysis[16], the capacitance transducer characterized by a capacitance C, a bias field strength E_0 and load resistance R affects the loss as

$$\frac{1}{Q_{trans}} = \beta \frac{\omega_0 CR}{1 + (\omega_0 CR)^2}, \tag{D.1}$$

where

$$\beta = \frac{E_0^2 C}{M\omega_0^2}, \tag{D.2}$$

is a coupling constant, ω_0 is the resonant frequency, and M is the effective mass of the vibration mode. Since $C = 10\,\mathrm{pF}$, $R = 200\,\mathrm{M\Omega}$, $E_0 = 2 \times 10^5\,\mathrm{V/m}$ and $M = 0.94\,\mathrm{kg}$, it gives $1/Q_{trans} = 2.6 \times 10^{-15}$ and $\beta = 2.2 \times 10^{-12}$ for $\omega_0/2\pi = 68\,\mathrm{kHz}$. Thus the effect of $1/Q_{trans}$ for the result of our measurement is negligible.

E Loss due to residual gas

The vacuum pressure was kept around 10^{-4} Pa in the vacuum chamber during the measurement. Applying the fact that mean free path of molecules are longer than characteristic length of the experimental apparatus, so we have to treat this dissipation as momentum transfer caused by collisions of gas molecules and the surface of the sapphire cylinder. This mechanism may cause a dissipation as

$$\frac{1}{Q_{gas}} \approx \frac{PA}{M\omega_0}\sqrt{\frac{\mu}{k_B T}}, \tag{E.1}$$

where P is the pressure, A is the surface area of the sapphire cylinder, μ is the mass of the molecule and k_B is the Boltzmann constant[17]. We assume that dominant residual gas is helium and the pressure is 10^{-3}Pa. $1/Q_{gas}$ given by Eq. E.1 is in a range from 10^{-13} to 10^{-12}.