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# Multipole Analysis for Absolute Magnetic Field Measured by Pulsed-NMR Method

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**Abstract** - For accelerator dipole magnets, the study on multipole coefficients in a local region is useful for understanding magnet characteristics and checking a construction method. The multi-probe pulsed-NMR method is a candidate for obtaining the local multipole coefficients. For absolute fields measured by the multi-probe pulsed-NMR method, a rigorous application of The Fourier expansion is found to yield multipole coefficients. It is made clear that the direction of the dipole field is unable to be determined by the Fourier expansion alone. For estimating alignment accuracy of pulsed-NMR probes, the relationship between errors of multipole coefficients and those of probe position is derived by the Fourier expansion method.

## 1. Introduction

Large collider accelerators such as LHC[1] are equipped with a great number of superconducting magnets. The dipole magnets are required to have a highly homogeneous magnetic field around the beam axis. The relative field deviation in the region of a diameter of about 1 cm should be less than $2\sim3\times10^{-4}$ on the condition when there is no use of additional correction coils. The magnetic field in the beam region is expressed by superposition of multipole components with normal and skew coefficients $b_n$ and $a_n$[2] in a two-dimensional treatment. The multipole components of the magnetic field should be measured with high accuracy. For accelerator operation, the magnetic field is tailored to be as uniform as possible: According to the measured data, the multipole components are decreased to values below desired tolerance by the use of the correction multipole coils.

The magnetic field in the accelerator magnets has often been measured by the rotating coil method[3,4]. Since voltage is induced across a coil as a function of the rotation angle, field homogeneity is derived by multipole field analysis based on the Fourier expansion method[3,4]. The rotating coil measurement usually gives the magnetic field integrated typically



in a few meters along the beam axis. It is well known that multipole characteristics for poles of lower orders are changed in the magnet end region in comparison to those of the straight section: For instance, the magnet end part produces a considerably larger sextupole component. Since the rotating coil measurement can cover both the straight and the end regions simultaneously, the integrated signal is useful from the practical point of view. We are interested in local multipole characteristics along the beam axis. They show some particular changes being ascribed to inhomogeneity arising in magnet construction[5]. Some periodical tendency may appear reflecting such effects as a transposition pitch of superconducting cables[6] and eddy currents during current ramping[7]. In addition, there is a slow variation of magnetic field in a time scale of hours to days due to a flux creep in superconductors[8]. Hence, the local multipole measurement is useful for understanding magnet characteristics and for improving the magnet fabrication method. The rotating coil method is unsuited for supplying local field distribution.

Hall probes are small devices with a size of about 1 mm. They are a candidate for measuring the local field distribution at individual positions. Hall probes are sensitive only to the magnetic field perpendicular to the Hall plane, so that troublesome problems may arise in setting small probes precisely in the magnet bore tube[9]. Since they give a relative value of magnetic field, the Hall voltage should be calibrated in a known strength of magnetic field. Well-calibrated probes have produced an accuracy of $5 \times 10^{-5}$ in field measurement[10]. When a warm bore system is prepared for measurement of superconducting dipole magnets, however, a spatial temperature distribution appears in the bore tube along the beam axis, and the minimum temperature shows a value below 200 K in some cases. For use in actual measurement, therefore, the temperature may be different from that in the Hall probe calibration.

The alternative to the Hall probe method is the use of the multi-probe pulsed-NMR system[11]. NMR methods have an excellent accuracy in measuring field without calibration: It readily reaches seven-digit or more accurate. In addition, the temperature of NMR probes has no effect on the measurement accuracy in principle. The NMR methods are capable of making high-accuracy measurement in the bore tubes at room or cryogenic temperatures without the calibration. The pulsed-NMR probe can also be made small in size of mm range, so that it is applicable to studying a local field homogeneity. The excellent accuracy in field measurement may lead to highly advanced understanding of the magnet characteristics. The pulsed-NMR method is suited to measurement in the straight section of the magnet, whereas it is not easy to use the NMR system in the magnet end region due to its limited dynamic range in field measurement.

The pulsed-NMR method has already been demonstrated[12] to be usable for field homogeneity measurement at the SSC Laboratory. The pulsed-NMR method supplies only the absolute value of the magnetic field, $|B|$, due to the properties of NMR. Since $|B|$ is unsuited to the Fourier expansion, the multipole analysis was made approximately[12]: The measured absolute field $|B|$ was regarded as the field in the vertical direction $B_y$ for the purpose of direct use of the Fourier expansion. It is not clear to what extent the substitution of $|B|$ by $B_y$ disturbed the resultant multipole components. It is necessary to find a better multipole expansion procedure which makes complete use of the pulsed-NMR results with excellent precision.

In this paper, we attempt to develop a method of obtaining multipole coefficients by means of rigorous treatment of the absolute field. In addition, we will study the feasibility of finding the angle of the dipole field component with $y$-axis, and obtain the relationship between errors of the NMR probe positions and the multipole coefficients.

## 2. Fourier Expansion for Absolute Field Measurement

We consider that radio frequency (rf) pulses are supplied to the nuclear spin system which is in a static magnetic field, through an NMR coil. When the supplied rf pulses almost satisfy the NMR condition, decaying rf signals, shown in Fig. 1, are induced in the NMR coil. The observed rf pulses are called as the free-induction-decay (FID) signals. The frequency of the FID signal is exactly equal to the NMR frequency. The relation between NMR frequency ($f_{NMR}$) and static field ($B_0$) is expressed by

$$f_{NMR} = \frac{\gamma_g B_0}{2\pi}, \tag{1}$$

where $\gamma_g$ is the nuclear gyromagnetic ratio and equals $2.67522128 \times 10^8 \mathrm{radT^{-1}s^{-1}}$ for hydrogen nucleus. By counting $f_{NMR}$, the strength of the static field can be obtained with an excellent accuracy.

In the pulsed-NMR method, the absolute fields are measured at many points on a circle with a certain radius, as shown in fig.2. The probes are sensitive to the absolute strength of the magnetic field in the x-y plane in this configuration: The field orientation in the x-y plane makes no effect on the measured results, and this diminishes difficulties in setting the probes to some extent. The fields at individual positions $(r, \theta_k)$ are analyzed for giving the multipole components. Measurement at 36 points, for instance, gives multipole components up to 18th pole.

The two-dimensional field around the beam axis is given by a complex value $B$ as $B = B_x + iB_y$. The complex conjugate $B^*$ is expanded[13] with $z = x + iy = re^{i\theta}$. According to the usual convention, $B^*$ is expressed by



$$B^* = B_0 \sum_{n=0}^{\infty} c_n^* z^n, \tag{2}$$

that is

$$iB^* = B_y + iB_x = B_0 \sum_{n=0}^{\infty} (b_n + ia_n) r^n \{\cos n\theta + i\sin n\theta\}, \tag{3}$$

where $B_0$ is the fundamental dipole field in the $y$-direction. The complex value $c_n = a_n + ib_n$ which shows the multipole components: $b_n$ is the normal coefficient and $a_n$ the skew one.

We concentrate our treatment on dipole magnets, where $b_0 = 1$ for the fundamental dipole component and $a_0 = 0$ in definition. One can see that $|B|$ derived from Eq.(3) is not suited for the direct use of teh Fourier expansion. Therefore, account is taken of the fact that the total field $B$ is composed of the fundamental and other higher harmonic fields as

$$B = iB_0 + B_h. \tag{4}$$

It is known that $|B_0| >> |B_h|$ for actual dipole magnets, and

$$B_h^* = B_0 \sum_{n=1}^{\infty} c_n^* z^n. \tag{5}$$

The value of $|B|^2$ is written by

$$BB^* = (iB_0 + B_h)(-iB_0 + B_h^*) = B_0^2 + [-2\,\mathrm{Im}(B_h^*)] + B_h B_h^*. \tag{6}$$

In this paper, a scalar quantity $D$ is introduced as

$$D = |B|^2 - |B_h|^2, \tag{7}$$

although $B_h$ is unknown at first. The precise expression of $D$ is given through Eqs.(5) to (6) as

$$D = B_0^2 - 2B_0 \,\mathrm{Im}\left(B_h^*\right) = B_0^2 + 2B_0^2 \sum_{n=1}^{\infty} r^n \{b_n \cos n\theta - a_n \sin n\theta\}. \tag{8}$$

Meanwhile, we apply the Fourier expansion to the scalar value of Eq.(7) as

$$D = \sum_{n=1}^{\infty} r^n \{\tilde{b}_n \cos n\theta + \tilde{a}_n \sin n\theta\}, \tag{9}$$

where $\tilde{a}_0$ is meaningless in nature of the Fourier expansion. The coefficients $\tilde{b}_n$ and $\tilde{a}_n$ are evaluated by the values of $D_k$ at individual location $k$:

$$\tilde{b}_0 = \frac{1}{K} \sum_{k=1}^{K} D_k, \tag{10-a}$$

$$\tilde{b}_n = \frac{2}{K r^n} \sum_{k=1}^{K} D_k \cos n\theta_k , \tag{10-b}$$

and

$$\tilde{a}_n = \frac{2}{K r^n} \sum_{k=1}^{K} D_k \sin n\theta_k , \tag{10-c}$$

where n ≥ 1, $\theta_k = 2\pi(k-1)/K$ and $k = 1,2, ...,K$. The comparison between Eqs.(8) and (9) makes

$$|B_0| = \sqrt{\tilde{b}_0}, \tag{11-a}$$

$$b_n = \frac{\tilde{b}_n}{2 r^n B_0^2}, \tag{11-b}$$

and

$$a_n = -\frac{\tilde{a}_n}{2 r^n B_0^2}. \tag{11-c}$$

It is repeated that $B_h$ in Eq.(7) is unknown at the moment.

## 2.1 *First step*

We take the first approximation to $D$. The initial value $D(1)$ is set at the measured absolute value of $|B|$:

$$D(1) = |B|^2. \tag{12}$$

The values of $D(1)$ at individual positions give $\tilde{b}_n(1)$ and $\tilde{a}_n(1)$ for Eq.(9). The quantities of $B_0(1)$, $b_n(1)$ and $a_n(1)$ are determined with Eqs.(11), where the sign of $B_0(1)$ is found by the direction of transport current in the magnet coils. In reference[11], $|B|$ was assumed to equal $B_y$ and directly Fourier-expanded. If we consider a quantity of $D/B_0$, it is close to both $|B|$ and $B_y$



and can be Fourier expanded as deduced from Eq.(8). One can see that the method[11] corresponds to the present first approximation.

## 2.2 *Second and higher steps*

Using the central field $B_0(1)$ and multipole coefficients $b_n(1)$ and $a_n(1)$, the second approximation to $D$ is expressed by

$$D(2) = |B|^2 - \left| B_0(1) \sum_{n=1}^{\infty} c_n^*(1)\, z^{n-1} \right|^2 . \tag{13}$$

We apply the Fourier expansion Eq.(9) again to Eq.(13), and determine $B_0(2)$, $b_n(2)$ and $a_n(2)$ by Eqs.(11). These values obtained are more precise than those of the first approximation. This way of calculation is iterated $m$ times until $B_0(m)c_n(m) \approx B_0(m-1)c_n(m-1)$ with an acceptable degree. The convergence may be reached in a few repetitions because of $|B_0| >> |B_h|$ in the dipole magnets.

# 3. Problem in Practical Measurement

We suppose that the vertical axis of the pulsed-NMR measurement system may tilt slightly by $\phi$ against the true direction of the dipole component. The location $z' = re^{i\theta'}$ in the rotated system is related to the position $z$ before rotation: $z = re^{i\theta} = re^{i(\theta'+\phi)} = z'e^{i\phi}$. The field $B$ in Eq.(2) is modified into $B' = B'_x + iB'_y$ for the rotated system as

$$B'^* = B_0\, e^{i\phi} \sum_{n=0}^{\infty} c_n^*\, e^{in\phi}\, z'^n , \tag{14}$$

so that the fundamental dipole field $iB_0 e^{-i\phi}$ is given by

$$B'_{0x} + iB'_{0y} = B_0 \sin\phi + iB_0 \cos\phi . \tag{15}$$

The total field $B'$ is superposed by the dipole and higher harmonics fields as

$$B' = iB'_{0y} + (B'_{0x} + B'_h), \tag{16}$$

where $B_h'^* = B_0 \sum_{n=1}^{\infty} c_n'^* z'^n$, and $c_n'^* = c_n^*\, e^{i(n+1)\phi}$.

These equations lead to

$$B'B'^* = \{ iB'_{0y} + (B'_{0x} + B'_h)\}\{ -iB'_{0y} + (B'_{0x} + B'^*_h)\}$$
$$= (B'^2_{0y} + B'^2_{0x}) - 2B'_{0y}\,\mathrm{Im}(B'^*_h) + 2B'_{0x}\,\mathrm{Re}(B'^*_h) + B'_h B'^*_h . \qquad (17)$$

The scalar quantity $D = |B'|^2 + |B'_h|^2$ is expressed by

$$D = (B'^2_{0x} + B'^2_{0y}) - 2B'_{0y}\,\mathrm{Im}(B'^*_h) + 2B'_{0x}\,\mathrm{Re}(B'^*_h) . \qquad (18)$$

The definition of $B'^*_h$ transforms this formula in terms of $c'_n = a'_n + ib'_n$ into

$$D = (B'^2_{0y} + B'^2_{0x})$$
$$+ 2B_0 \sum_{n=1}^{\infty} r^n \{ (B'_{0y}b'_n + B'_{0x}a'_n)\cos n\theta' - (B'_{0y}a'_n - B'_{0x}b'_n)\sin n\theta' \} . \qquad (19)$$

When $D$ is expanded by Eq.(9) and compared with Eq.(19), we obtain

$$\tilde{b}_0 = B'^2_{0x} + B'^2_{0y} = |B_0|^2, \qquad (20\text{-a})$$

$$\tilde{b}_n = 2B_0(B'_{0y}b'_n + B'_{0x}a'_n), \qquad (20\text{-b})$$

and

$$\tilde{a}_n = -2B_0(B'_{0y}a'_n - B'_{0x}b'_n), \qquad (20\text{-c})$$

where $n \geq 1$. The relation $\tilde{c}_n = \tilde{a}_n + i\tilde{b}_n$ makes

$$\tilde{c}_n = 2i\, c'^*_n (B'_{0x} + iB'_{0y}) B_0 = -2\, c'^*_n |B_0|^2 e^{-i\phi},$$

that is

$$c'^*_n = -\frac{\tilde{c}_n e^{i\phi}}{2|B_0|^2} . \qquad (21)$$

If $\phi$ is determined, Eq.(21) gives the value of $c_n'^*$.



### 3.1. *First step*

We make the approximation of $D(1) = |B'|^2$, where $|B'|$ is the absolute value measured in the rotated system. The use of Eqs.(20) gives $|B_0(1)|$ and $c_n'^*(1) = -\tilde{c}_n(1)/2|B_0(1)|^2$ with assumption of $\phi = 0$ in Eq.(21).

### 3.2. *Second and higher steps*

At the second approximation, we use

$$D(2) = |B'|^2 - \left| B'_0(1) \sum_{n=1}^{\infty} c'^*_n(1)\, z'^n \right|^2 . \qquad (22)$$

Even if any value of $\phi$ in Eq.(21) is considered in Eq.(22), $\phi$ has no influence on $D(2)$ due to $|e^{i\phi}|=1$. For this reason, $\phi$ is set at 0 in this procedure. The expansion of Eq.(22) will give $|B_0(2)|$ and $c_n'^*(2)$. After $m$ iterations, we obtain $c_n'^*(m) = -(1/2|B_0(m)|^2)\tilde{c}_n(m)$ through the assumption of $\phi = 0$.

From the above description, one can see that $\phi$ is unable to be determined by the Fourier expansion in the absolute field measurement. The uncertainty of $\phi$ comes from the fact that there is no information about the field direction in the present NMR method. For determination of $\phi$ in Eq.(21), we need a direction of the field at one point, for example, $B'_x$ at the origin. In this case, the angle $\phi$ is evaluated from

$$B'_x = B_0 \sin\phi , \qquad (23)$$

and accordingly both $c_n'^* = e^{i\phi} c_n'^*(m)$ and $c_n^* = c_n'^* e^{-i(n+1)\phi}$ are obtained. A better approach is to set the probe system to realize $B'_{0x} = 0$ before the beginning of the pulsed-NMR measurement. For instance, a geometrically well defined coil can be installed and utilized as shown in fig. 2, where $B'_x$ is monitored while the magnet current ramps.

## 4. Relationship between Errors of Probe Positions and Multipole Coefficients

In the practical pulsed-NMR measurement, there may be some geometrical problems arising from alignment errors on probe positions or axis-tilting uncertainties. As a simple example, we are concerned with a deviation of probe position. A rough analytical estimation is made on the basis of the Fourier expansion on the condition that randomness of individual position errors is ignored. For consideration of the randomness, other methods are required such as numerical simulation or analytical estimation based on root-mean-squared sum. We assume that the probe location deviates from the original correct position $(r,\theta)$ by $\Delta r$ and $\Delta\theta$. We can obtain the following equation by rearranging Eq.(8):

$$D = B_0^2 + 2 B_0^2 \sum_{n=1}^{\infty} (r + \Delta r)^n \{ b_n \cos(n(\theta + \Delta\theta)) - a_n \sin(n(\theta + \Delta\theta)) \} \qquad (24)$$

Use of $\Delta r/r \ll 1$ and $\Delta\theta \ll 1$ gives

$$(r + \Delta r)^n \approx r^n + n r^{n-1} \Delta r, \qquad (25)$$

$$\cos n(\theta + \Delta\theta) \approx \cos n\theta - n\Delta\theta \sin n\theta, \qquad (26\text{-a})$$

and

$$\sin n(\theta + \Delta\theta) \approx \sin n\theta + n\Delta\theta \cos n\theta. \qquad (26\text{-b})$$

When products of $\Delta r\Delta\theta$ are negligibly small, the expression in $\Sigma$ of Eq.(24) turns into

$$\begin{aligned} & r^n \{ b_n - n a_n \Delta\theta + n b_n \Delta r/r \} \cos n\theta - r^n \{ a_n + n b_n \Delta\theta + n a_n \Delta r/r \} \sin n\theta \\ & = r^n [ (b_n + \Delta b_n) \cos n\theta - (a_n + \Delta a_n) \sin n\theta ], \end{aligned} \qquad (27)$$

where

$$\Delta b_n = -n a_n \Delta\theta + n b_n \Delta r/r, \qquad (28\text{-a})$$

and

$$\Delta a_n = n b_n \Delta\theta + n a_n \Delta r/r. \qquad (28\text{-b})$$

Then, we obtain

$$\Delta b_n/b_n = n(\Delta r/r) - n(a_n/b_n)\Delta\theta, \qquad (29\text{-a})$$

and

$$\Delta a_n/a_n = n(\Delta r/r) + n(b_n/a_n)\Delta\theta. \qquad (29\text{-b})$$

These equations give the following indications: $\Delta r$ and $\Delta\theta$ greatly increase $\Delta a_n$ and $\Delta b_n$ at larger $n$, and a large value of $a_n$ enhances $\Delta b_n$ through the change in $\Delta\theta$. If $\Delta r/r = 0.01$ for instance, $r = 10$ and $\Delta r = 0.1$ mm, and $b_n \approx a_n$, we obtain

$$\Delta b_n/b_n \approx \Delta a_n/a_n \approx 1\% \qquad \text{for } n = 1,$$



and

$$\Delta b_n/b_n \approx \Delta a_n/a_n \approx 10\% \qquad \text{for } n = 10.$$

When $\Delta\theta = 0.01$ rad, i.e. ~0.6 degree, we have

$$\Delta b_n/b_n \approx \Delta a_n/a_n \approx 1\% \qquad \text{for } n = 1,$$

and

$$\Delta b_n/b_n \approx \Delta a_n/a_n \approx 10\% \qquad \text{for } n = 10.$$

It is suggested that $\Delta r/r < 0.01$ and $\Delta\theta < 0.01$ rad should be satisfied in the construction of an NMR probe system.

## 5. Conclusion

We developed the procedure to calculate the multipole coefficients of the field in an accelerator dipole magnets by means of rigorous treatment. In this procedure, the absolute values of the field measured by the pulsed-NMR method were step-wisely Fourier-expanded. This procedure improves the quality of analyzed multipole coefficients in the pulsed-NMR method. We proved that the angle $\phi$ between the direction of the dipole field and the $y$-axis in measurement cannot be determined by the Fourier expansion method alone. It was made cleat that care should be taken on the field orientation in the use of the pulsed-NMR method. The errors of multipole coefficients $\Delta b_n$ and $\Delta a_n$ were simply estimated for the position deviations of the NMR probes $\Delta r$ and $\Delta\theta$ by the use of the Fourier expansion method.

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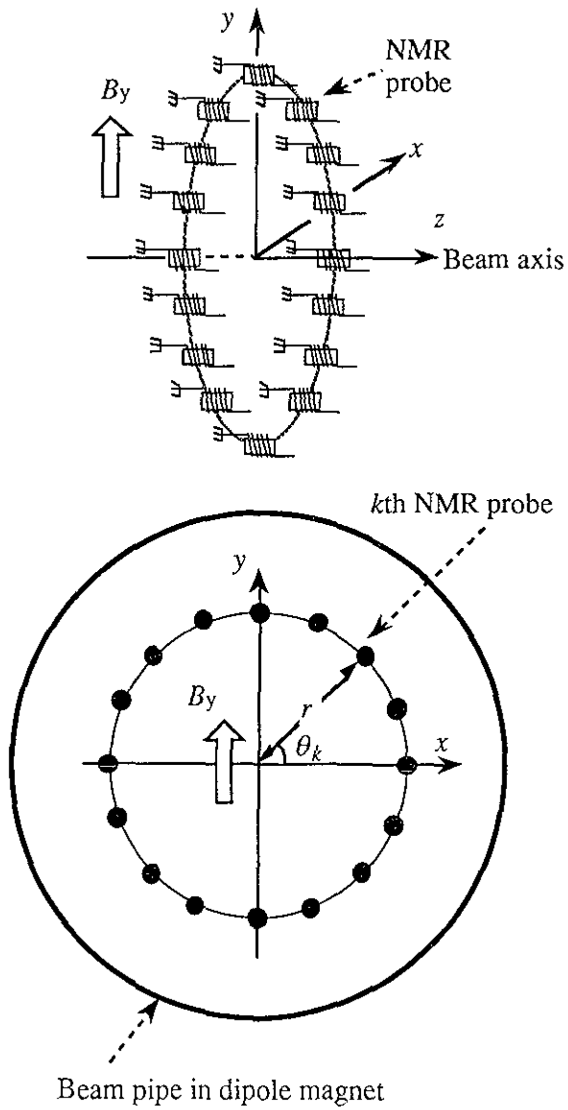


Fig. 1 Example of NMR probes in measuring the absolute values of the magnetic field. The angle is given by $\theta_k = 2\pi(k-1)/K$, where $K$ is the number of measuring points.

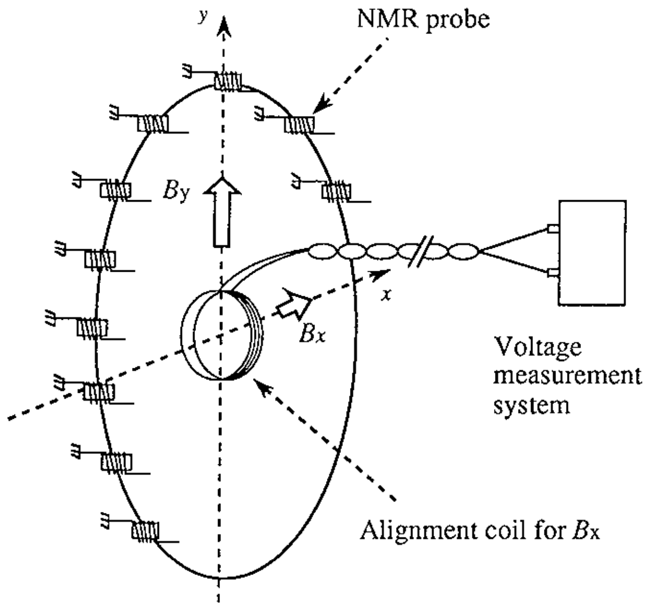


Fig.2 Arrangement of an alignment coil for finding the *y*-direction.
The axis of the alignment coil is placed in the *x*-direction.