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CONSIDERATIONS ON THE MODULI SPACE OF CALABI-YAU MANIFOLDS

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ABSTRACT

We report on recent results on the geometry of the moduli space of Calabi-Yau compactifications and their implications for superstring four-dimensional effective Lagrangians.

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In this lecture I will review some of the general properties of the geometry of the moduli space[1] of a wide physically interesting class of superstring compactifications, namely Calabi-Yau vacua[2].

In a broader sense, in string theory, Calabi-Yau compactifications are referred to as (2,2) vacua[3], referring to the superconformal properties[4] of the string internal degrees of freedom which are used to define a four-dimensional superstring model[5].

In any string compactification to four dimensions we require space-time supersymmetry to be unbroken, in order to define a sensible string theory. This of course may also solve the hierarchy problem of the weak interaction scale[6], provided we tacitly assume that there is some, as yet unknown, mechanism which will generate both the weak scale and supersymmetry breaking at energies $E \leq 0(1\ TeV)$.

A general property of Calabi-Yau compactifications of superstrings down to four dimensions is that there is a general relation[3][2][7] between some massless neutral multiplets and the multiplets charged under the gauge symmetry group G which is, in general, at least at the compactification scale, $E_6 \times E_8$.

More specifically, a Calabi-Yau space is specified by some topological numbers, the Hodge numbers of the manifold, which count the number of independent harmonic (1,1) forms and (2,1) forms which exist on the manifold. The Euler number is simply given by $2(h_{(1,1)} - h_{(2,1)})$ and is related to the net chirality of fermion representations in a given Calabi-Yau space. Indeed for each (1,1) and (2,1) harmonic forms in the internal manifold there is a massless scalar field $M_a(x)(a = 1, \dots h_{(1,1)}), N_\alpha(x)(\alpha = 1, \dots h_{(2,1)})$ in Minkowski space and for each such form there is also a corresponding 27 (for (1,1) forms) and $\overline{27}$ (for (2,1) forms) left-handed family of the E_6 gauge group (singlet with respect to the residual E_8). A model is therefore chiral if the number of harmonic (1,1) forms is different from the number of harmonic (2,1) forms, i.e. if the Euler number does not vanish.

The neutral scalar fields M, N are usually called moduli fields[8)-12] in the sense that their vacuum expectation value is completely undetermined by the equations of motion, and therefore $< M >, < N >$ are just free parameters for the internal metric of the Calabi-Yau space. For Calabi-Yau threefolds the moduli have the geometrical meaning of deformation parameters of the Kähler structure ((1,1) moduli) and of the complex structure ((2,1) moduli) respectively[9][10].

The relation of the scalar moduli fields and the underlying two-dimensional conformal field theory is best seen from their interpretations as flat directions of the scalar potential $V(M, N, P)$ of the theory. Here by P we denote any other scalar field which may be charged or neutral under G but which comes from the gauge degrees of freedom.

The moduli fields have the property that

$$\frac{\partial V}{\partial M} = 0\ \forall M \tag{1}$$

in contrast with the P fields, for which the equation $\frac{\partial V}{\partial P} = 0$ fixes their value at some point P_0.

In the background field approach[13] massless space-time fields such as the graviton and scalar fields appear as "coupling constants" in an underlying two-dimensional σ-model.

The requirement of conformal invariance beyond the tree level, namely the statement that the β function associated to these couplings vanishes, is nothing but their effective space-time equation of motion[13], i.e.

$$\beta_{g_{\mu\nu}} = 0 \text{ yields } R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = T_{\mu\nu} \tag{2}$$

and for a generic scalar field ϕ

$$\beta_\phi = 0 \text{ yields } \frac{\partial V}{\partial \phi} = 0 \text{ for } \phi = < \phi > \tag{3}$$

since the other terms depend on $\partial_\mu \phi$.

From eq. (3) we see that a flat direction corresponds to a coupling constant $< \phi >$ of the underlying conformal field theory for which the theory is exactly conformal invariant. If we call V_ϕ the vertex operator which corresponds to the moduli massless excitation then ϕV_ϕ is a conformal invariant perturbation for all ϕ and V_ϕ is called an exactly marginal operator.

The motion in the space of conformal field theory is given by the geometry of the ϕ manifold, i.e. the "coupling constant" space.

Let us assume this space to be some differentiable manifold: Zamolodchikov has shown that this space can be regarded as a Riemannian space with metric given by[1]

$$< V_{\phi_I}(1) V_{\phi_J}(0) > = G_{IJ}(\phi) \tag{4}$$

Using the fact that V are truly marginal operators from their operator product expansion, it may then be shown that in the effective Lagrangian the $\phi_I(x)$ kinetic term is given by

$$G_{IJ}(\phi) \partial \mu \phi_I \partial_\mu \phi_J \tag{5}$$

and moreover

$$V(\phi) \equiv 0 \qquad 6)$$

when we set all other (non-moduli) fields to their v.e.v.

Considerable progress in the knowledge of the metric $G_{I\bar{J}}$ has been gained over the last year with a number of techniques which give quite general and remarkable results.

It is the aim of this lecture to discuss these properties and relate them to the dynamics of a given superstring compactification.

Let us consider the number of moduli fields of a given Calabi-Yau compactification.

If we were geometers and considered only deformation of the metric then the number of (real) moduli would be $h_{(1,1)} + 2h_{(2,1)}$. However, in superstrings, space-time supersymmetry gives us additional scalar degrees of freedom from the non-gauge sector, namely those coming from the (internal components of the) antisymmetric tensor $B_{\mu\nu}$ (they are exactly $h_{(1,1)}$) and two more coming from the dilaton ϕ and the space-time components $b_{\mu\nu}$ of the antisymmetric tensor[2]. Therefore in any superstring compactified on a Calabi-Yau threefold the non-gauge sector gives $2(h_{(1,1)} + h_{(2,1)} + 1)$ degrees of freedom which are exactly suitable to be used as coordinates of a complex (Kähler) manifold, as required by $N = 1$ space-time supersymmetry[14] present in heterotic string compactification.

Just from this fact we know that the (neutral) moduli of heterotic string compactifications are coordinates of a Kähler manifold of complex dimension $(h_{(1,1)} + h_{(2,1)} + 1)$. The one-dimensional manifold associated to the dilaton is readily seen to be (at string tree level) $\frac{SU(1,1)}{U(1)}$. There are several ways to find this result. One is to use the space-time Peccei-Quinn symmetry associated to $b_{\mu\nu}, S \to S + ic$..

By duality transformation ϕ and $b_{\mu\nu}$ can be put in a complex field S. Then the Kähler potential must be of the form $K(ReS)$. But (ReS) is the dilaton coupling whose power is fixed at tree level to give a kinetic[15] term

$$\frac{1}{(ReS)^2}\left((\partial_\mu ReS)^2 + (\partial_\mu ImS)^2\right) \tag{7}$$

which can be rewritten as

$$K_{S\overline{S}}\,\partial_\mu S\partial_\mu\overline{S} \quad \text{with} \quad K = -log(S+\overline{S}) \tag{8}$$

Dixon, Kaplunowsky and Louis have shown[12], using superconformal Ward identities, that the moduli manifold has the product structure

$$\mathcal{M} = \frac{SU(1,1)}{U(1)} \times \mathcal{M}_{h_{(1,1)}} \times \mathcal{M}_{h_{(2,1)}} \tag{9}$$

where $\mathcal{M}_{h_{(1,1)}}, \mathcal{M}_{h_{(2,1)}}$ are two Kähler manifolds of complex dimensions $h_{(1,1)}$ and $h_{(2,1)}$ respectively.

This result was first pointed out by Seiberg[8] and then proved in refs. 9) and 10) with different methods.

One of these proofs uses $N = 2$ space-time supersymmetry[8] which also gives additional insights on the structure of the moduli space[9][11].

The occurrence of $N = 2$ space-time supersymmetry comes about because Calabi-Yau spaces can be used to compactify type II rather than heterotic superstrings.

Since the moduli metric $G_{I\bar{J}}(\phi)$ does not know which specific superstring theory one is compactifying, the term given by eq. 5) in the effective Lagrangian is common to heterotic and type II theories, but in the second case, because the number of space-time supersymmetries is doubled, it has to satisfy the additional constraint coming from the second space-time supersymmetry.

Of course it is conceivable that this constraint is inherited from the Ward identities of the underlying (2,2) superconformal algebra. With no surprise this turns out to be precisely the case[12].

Much useful information on string dynamics comes from exploiting the symmetries of the effective Lagrangian, the most powerful being local supersymmetry. For example, the non-renormalization theorems on the heterotic superstring effective superpotential and the way they may be violated are easily seen in the effective Lagrangian approach[16].

General properties of superstring compactifications on (4,0) or (4,4) superconformal field theories and the extensive use of $N = 2$ and $N = 4$ space-time supersymmetry in those cases is another example[8][9].

We now focus our attention on the Calabi-Yau vacua in four dimensions.

For these compactifications we can see the degrees of freedom in a pure space-time picture assuming the compactification scale R is much larger than the string size $\alpha'^{1/2}$. In this regime we may use the point-field limit of 10-dimensional superstrings which is 10-dimensional $N = 1$ supergravity. For heterotic superstrings[17] we have 10D-supergravity coupled to a Yang-Mills $E_8 \times E_8$ (or $SO(32)$) multiplet[18]. For type II strings we have type II A (non-chiral) and type II B (chiral) supergravity[19].

The bosonic fields which give rise to scalars in four dimensions are

$$G_{\hat{\mu}\hat{\nu}}, B_{\hat{\mu}\hat{\nu}}, \phi, A^A_{\hat{\mu}} \tag{10}$$

for heterotic superstrings,

$$G_{\hat{\mu}\hat{\nu}}, B_{\hat{\mu}\hat{\nu}}, \phi, A_{\hat{\mu}}, A_{\hat{\mu}\hat{\nu}\hat{\rho}} \tag{11}$$

for type IIA superstrings, and

$$G_{\hat{\mu}\hat{\nu}}, B^c_{\hat{\mu}\hat{\nu}}, \phi^c, A_{\hat{\mu}\hat{\nu}\hat{\rho}\hat{\sigma}} \tag{12}$$

for type II B superstrings. Here $B^c_{\hat{\mu}\hat{\nu}}, \phi^c$, denote complex antisymmetric tensor and scalar fields in ten dimensions.

The (1,1) and (2,1) forms in Calabi-Yau compactifications come as follows: we split $\hat{\mu} = \mu, I$ ($\mu = 1 \ldots 4,\ I = 1 \ldots 6$) and the $I = (i, \bar{\imath})$ $(i = 1, 2, 3)$.

Then in heterotic strings the (1,1) and (2,1) forms come respectively from

$$G_{i\bar{\jmath}}, b_{i\bar{\jmath}}\ ;\ G_{ij} \tag{13}$$

In type II A strings they come from

$$G_{i\bar{\jmath}}, b_{i\bar{\jmath}}, A_{\mu i\bar{\jmath}}\ ;\ G_{ij}, A_{i\bar{\jmath}\bar{k}} \tag{14}$$

and in type II B strings from

$$G_{i\bar{\jmath}}, b^c_{i\bar{\jmath}}, A_{\mu\nu i\bar{\jmath}}\ ;\ G_{ij}, A_{\mu i\bar{\jmath}\bar{k}} \tag{15}$$

The reason we have as many $27, \overline{27}$ families as (1,1) and (2,1) forms is because we identify the $SU(3)$ holonomy connection with the $SU(3)$ gauge connection[2] in the decomposition of $E_8 \to E_6 \times SU(3)$.

$$A_J^{(i,27)} \to A_j^{(i,27)}, A_{\bar{\jmath}}^{(i,27)} \tag{16}$$

The full spectrum of the scalar fields in the three theories compactified on the same Calabi-Yau space is as follows:

$$\text{heterotic case:}\quad M_a, N_\alpha, \phi^A_a, \phi^{\overline{A}}_\alpha, S \tag{17}$$

$$[a = 1 \ldots h_{(1,1)}, \alpha = 1 \ldots h_{(2,1)}, A \in 27, \overline{A} \in \overline{27}]$$

where M_a correspond to $g_{i\bar{\jmath}}, b_{i\bar{\jmath}}, N_\alpha$ to g_{ij} and S to ϕ and $b_{\mu\nu}$.

$$\text{Type II A case:}\quad M_a, N_\alpha, C_\alpha, S, C \tag{18}$$

when C_α correspond to the $A_{i\bar{\jmath}\bar{k}}$ modes and C to the A_{ijk} mode.

$$\text{Type II B case:}\quad M_a, C_a, S_1, S_2, N_\alpha \tag{19}$$

when M_a, C_a correspond to $g_{i\bar{\jmath}}, b^c_{i\bar{\jmath}}, A_{\mu\nu i\bar{\jmath}}$ and S_1, S_2 correspond to $\phi^c, b^c_{\mu\nu}$.

Since in type II A theories there are 4 degrees of freedom for each (2,1) form and in type II B theories there are 4 degrees of freedom for each (1,1) form, we

conclude that the (2,1) and (1,1) moduli belong to $N = 2$ (space-time) hypermultiplets respectively in type II A and type II B theories[8),9)].

In the chirality reversed theory the same moduli belong to vector multiplets; indeed in type II A theories there are $h_{(2,1)} + 1$ gauge vectors coming from $A_{\mu i j \bar{k}}$ and $A_{\mu ijk}$. The additional vector is the graviphoton. From $N = 2$ space-time supersymmetry arguments[20)] we know that the interaction of vector multiplets and hypermultiplets consistent with $N = 2$ supergravity is a non-linear σ-model of the form

$$\mathcal{M}_{SK} \times Q \tag{20}$$

where $\mathcal{M}$ is a (special) Kähler manifold (to be defined later) for the vector multiplets[20)] and Q is a quaternionic manifold for the hypermultiplets[20)21)22)].

If we write in brackets the (complex) and (quaternionic) dimensions of these manifolds in type II A and II B theories we have[9)]

$$M^A = \mathcal{M}^A(h_{(1,1)}) \times Q^A(h_{(2,1)} + 1) \tag{21}$$

$$M^B = \mathcal{M}^B(h_{(2,1)}) \times Q^B(h_{(1,1)} + 1) \tag{22}$$

The additional hypermultiplet which raises the Q dimension from h to $h+1$ comes from the dilaton and antisymmetric tensor sectors.

It is worth mentioning at this point that while the $\mathcal{M}$ Kähler manifolds contain the same moduli fields which appear in heterotic strings, the Q manifolds are obtained by gluing together moduli scalars with non-moduli scalars which actually, in string theory, come from the Ramond-Ramond sector of the left-right superconformal algebra.

The first observation at this point is that the manifolds $\mathcal{M}^A$ and $\mathcal{M}^B$ must coincide with the submanifolds of heterotic strings when we freeze one of the two sets of the topologically distinct moduli. The fact that the full manifold is a product space as given by eq. (9) comes by setting to zero the R-R fields in type II theories. For example, setting $C_\alpha = C = 0$ in type II A we obtain that[9)]

$$Q_{(h_{2,1}+1)} \to \mathcal{M}_{(h_{(2,1)})} \times \frac{SU(1,1)}{U(1)} \tag{23}$$

and the same is true for the type II B theory.

We conclude that from pure space-time arguments we can indeed prove eq. (9).

We now come to the next question.

Which is the structure of the $\mathcal{M}^{A(B)}$ special Kähler manifolds?

The answer is given by $N = 2$ space-time supersymmetry[20)23)]. A special Kähler manifold is a Kähler manifold whose curvature $R_{a\bar{b}c\bar{d}}$ satisfies the additional constraint[23)]

$$\begin{aligned} R_{a\bar{b}c\bar{d}} &= G_{a\bar{b}}G_{c\bar{d}} + G_{a\bar{d}}G_{c\bar{b}} \\ &- e^{2K} C_{acp}\overline{C}_{\bar{b}\bar{d}\bar{q}} G^{p\bar{q}} \end{aligned} \tag{24}$$

where $G_{a\bar{b}}$ is the Kähler metric and $G^{a\bar{b}}$ its inverse.

Here C_{abc} is a holomorphic (totally symmetric) tensor which because of the Bianchi identity, satisfies the integrability condition[12)24)25)]

$$\mathcal{D}_{[d}e^{2K}C_{a]cp} = 0 \tag{25}$$

which in turns implies $C_{abc} = e^{-2K}\mathcal{D}_a\mathcal{D}_b\mathcal{D}_c(e^{2K}S)$, where S is a scalar function.

Eq. (24) has also been derived from superconformal Ward identities[12)] between scattering amplitudes of moduli fields and charged fields in which case the holomorphic tensor C_{abc} has the meaning of the Yukawa coupling for 27 (or $\overline{27}$) families[12)26)]

$$C_{abc}(27)^3 \quad , \quad C_{\alpha\beta\gamma}(\overline{27})^3 \tag{26}$$

Eq. (24) gives a further constraint on the Kähler potential K which defines the Kähler metric

$$G_{a\bar{b}} = \partial_a\partial_{\bar{b}}K \tag{27}$$

A metric which satisfies eq. (24) can be found in a special coordinate system which is the one actually used in $N = 2$ supergravity tensor calculus[20)23)].

If we define by Z^a the moduli coordinates and by $f(Z^a)$ an arbitrary holomorphic function of the moduli, then it is not difficult to show that the following ansatz [20)23)]

$$K = -\ell n Y \tag{28}$$

$$Y = 2f + 2f^* - (f_a - f_a^*)(Z^a - Z^{*a}) \left(f_a = \frac{\partial f}{\partial Z^a} \right) \tag{29}$$

$$C_{abc} = f_{abc} = \frac{\partial}{Z^a}\frac{\partial}{Z^b}\frac{\partial}{Z^c} f \tag{30}$$

solves eq. (24) for any $f(Z)$.

We are led to the conclusion that in a special coordinate system, called the special gauge, the entire geometry of the Calabi-Yau moduli space is encoded in two holomorphic functions of the moduli fields $f^A(M), f^B(N)$.

There are profound implications for superstring dynamics which come from this specific structure of the moduli space and its relation to the Yukawa couplings. The first one is that $(27)^3$ and $(\overline{27})^3$ couplings can only depend on their separate

moduli[12)26)27)], i.e. $(27)^3$ couplings can only depend on the M parameters and $(\overline{27})^3$ couplings on the N parameters. This results in an exact (string tree level) result[27)]. A result which is true (to any finite order) in σ-model perturbation theory, i.e., in a power expansion in α'/R^2, is the fact that Yukawa couplings for 27 families are just constants and cannot depend on the moduli parameters.

This is related to the Peccei-Quinn symmetry of the $b_{i\bar{j}}(x)$ fluctuations which in turn imply that the f function is strictly a cubic polynomial [9)]

$$f(Z) = d_{abc}Z^aZ^bZ^c \tag{31}$$

The coefficients d_{abc} are quantized and are topological objects, given by the intersection matrices of (1,1) forms[28)]

$$d_{abc} = \int_{C_3} B_a \wedge B_b \wedge B_c \tag{32}$$

over the Calabi-Yau space.

This result is however spoiled by world-sheet instanton effects, which give rise to an explicit Z-dependence on the Yukawa couplings[26)]. We will comment later on this effect.

In the case of (2,1) moduli, the $(\overline{27})^3$ Yukawa couplings depend on the moduli; however, there are no string corrections to these couplings (perturbative or non-perturbative) due to the fact that the σ-model coupling expansion parameter α'/R^2 is precisely one of the (1,1) moduli which is forbidden to mix with the (2,1) moduli from the previous considerations. Therefore the $(\overline{27})^3$ coupling can be evaluated exactly at the σ-model tree level or in the point-field theory limit[12)26)27)]. In this limit an exact formula of the f^B function is given by[10)11)]

$$f^B = -\frac{i}{2}\int_{C_3} \Omega \wedge (\alpha_0 + Z^i\alpha_i) \tag{33}$$

where Ω is a holomorphic three-form in projective coordinates for the moduli and $\alpha_0, \alpha_i (i = 1 \dots h_{2,1})$ (with β^o, β^i) is a cohomology basis in H^3 dual to the homology cycles A^a, B_a.

$$\begin{aligned} &\int_{C_3} \alpha_a \wedge \beta^b = \delta_a^b \\ &\int_{A^b} \alpha_a = \int_{C_3} \alpha_a \wedge \beta^b = \delta_a^b \\ &\int_{B_a} \beta^b = \int_{C_3} \beta^b \wedge \alpha_a = -\delta_a^b \end{aligned} \tag{34}$$

We want now to explore another consequence of eq. (24), namely the relation between the moduli metric and the matter metric. In heterotic strings we know that the full scalar self-couplings in the effective $N = 1$ supergravity action are determined by the function[29)]

$$G \;=\; K + \ell n|W|^2 \tag{35}$$

where W is the superpotential.

In our case

$$W(M^a, N^\alpha, \phi^a, \widetilde{\phi}^\alpha) = C_{abc}(M)\phi^a\phi^a\phi^c + C_{\alpha\beta\gamma}(N)\phi^\alpha\phi^\beta\phi^\gamma \tag{36}$$

(E_6 gauge indices and couplings being understood).

From eq. (24) we know then that under Kähler transformations of the moduli spaces we must have

$$\begin{aligned} K^A &\to K^A - \Lambda^A - \overline{\Lambda}^A \quad , \quad C_{abc} \to C_{abc}e^{2\Lambda^A} \\ K^B &\to K^B - \Lambda^B - \overline{\Lambda}^B \quad , \quad C_{\alpha\beta\gamma} \to C_{\alpha\beta\gamma}e^{2\Lambda^B} \end{aligned} \tag{37}$$

where $\Lambda^A = \Lambda^A(M)$ and $\Lambda^B = \Lambda^B(N)$ are holomorphic parameters of the moduli.

This is a consequence of the fact that the C tensors are holomorphic. The full Kähler potential of the moduli + matter field space is of the form[12)]

$$K \;=\; K^A + K^B + 0(\phi^2) \;+\; \text{higher order terms}$$

The crucial fact is that the matter-dependent part must be Kähler inert under the Kähler transformations of the moduli subspace. Under this requirement

$$K \to K - \Lambda_A - \overline{\Lambda}_A - \Lambda_B - \overline{\Lambda}_B \tag{38}$$

and in order for G to be invariant, both terms in W must scale as $We^{\Lambda_A+\Lambda_B}$. This is achieved by using the following Kähler transformations for the ϕ fields

$$\phi_a \to \phi_a\, e^{\frac{-\Lambda^A+\Lambda^B}{3}} \quad \widetilde{\phi}_\alpha \to \widetilde{\phi}_\alpha\, e^{\frac{\Lambda^A-\Lambda^B}{3}} \tag{39}$$

It is now easy to construct functions of the matter fields which are Kähler inert.

The simplest ones (quadratic in the ϕ's) are

$$e^{(K^B-K^A)/3}\phi^a G_{a\bar{b}}\overline{\phi}^{\bar{b}} \quad , \quad e^{(K^A-K^B)/3}\widetilde{\phi}^\alpha G_{\alpha\bar{\beta}}\overline{\widetilde{\phi}}^{\bar{\beta}} \quad , \quad m_{a\alpha}\phi^a\widetilde{\phi}^\alpha \tag{40}$$

From eq. (40) we easily extract the matter field metric (for $<\phi_a>=<\tilde{\phi}_\alpha>=0$) to be

$$G_{\phi_a\bar{\phi}_{\bar{b}}} \;=\; G_{a\bar{b}}e^{(K^B-K^A)/3} \quad , \quad G_{\phi_\alpha\bar{\phi}_{\bar{\beta}}} \;=\; G_{\alpha\bar{\beta}}e^{(K^A-K^B)/3} \tag{41}$$

a result derived from conformal field theory arguments in ref. 12).

If we go to higher order terms in the matter fields we can construct many Kähler-invariant functions. A definite form is probably obtained in the point-field theory limit, i.e. by compactifying 10D supergravity on a Calabi-Yau manifold, using the fact that the metric can only have a simple dependence on the charged fields since they come from the 10D gauge fields. However, contrary to the moduli case, we expect in this situation string corrections in σ-model perturbation theory[30)]. We also remark that we have further assumed that the moduli space has no isometries which may change eq. (41).

We would like to end this summary by discussing, in deeper detail, the non-perturbative effects which spoil the point-field limit result of eq. (31) for the 27 families Yukawa couplings.

A case which can be discussed in great detail is an orbifold limit[31)] of a Calabi-Yau space. At the orbifold points (in the case of the Z_3 orbifold) the moduli space has an enhanced gauge symmetry $SU(3)$ and for some values of the nine untwisted (1,1) moduli parameters an extra gauge symmetry $U(1)^6$.

The smooth Calabi-Yau space which corresponds to a blown-up Z_3 orbifold has 36 modular complex parameters[2)], 27 of them coming from the blowing up modes. In the orbifold limit we remain with the 9 untwisted modular parameters and locally the parameter space of the Z_3 orbifold is the symmetric space[32)33)] $SU(3,3)/SU(3) \times SU(3) \times U(1)$.

This is a homogeneous symmetric space with Kähler metric compatible with eqs. (29) and (31).

If we call $T_{i\bar{j}}$ the 9 moduli fields ($i, \bar{j} = 1,2,3$) the d coefficient is simply given by[28)9)]

$$d_{abc} \;=\; \varepsilon_{ijk}\varepsilon_{\bar{i}\bar{j}\bar{k}} \qquad \begin{aligned} a &= (i,\bar{\imath}) \\ b &= (j,\bar{\jmath}) \\ c &= (k,\bar{k}) \end{aligned} \tag{42}$$

In the field theory limit the Yukawa couplings for the $(27)^3$ families corresponding to these nine modes are just constant. This is also true in string theory. However if we take the 27 additional families corresponding to the blowing up modes, in the field theory limit they are also constant and with the following symmetries[28)]

$$d_{ijk} = 0 \text{ if } i \neq j \neq k \quad d_{iii} \text{ constant} \tag{43}$$

In string theory, due to world sheet instanton corrections, what happens is that the d coefficients become dependent on the untwisted moduli. The d's which were zero are exponentially suppressed, while the d's which were constant approach a constant only in the $R^2/\alpha' \to \infty$ limit[34].

The remarkable fact is that the T dependence of the Yukawa couplings seems to be controlled by a new symmetry, called space-time duality, which has to do with the fact that the moduli space is not really a smooth manifold but rather a conifold on which some points must be identified[35)-37)].

In the language of string theory this fact is ultimately related to the fact that a string theory compactified on a torus of radius R is equivalent to the same theory compactified on a torus with radius α'/R[35)-38)].

If we think of the moduli space as the space which classifies distinct conformal field theories, this space has to be modded out by the duality group (Z_2 in the simplest example $R \to \frac{\alpha'}{R}$) which connect equivalent couplings.

Space-time duality symmetry seems to be a powerful tool in order to control some non-perturbative world sheet effects in string theory and also in order to explain different gauge symmetry groups occurring in superstring compactifications.

Indeed, much progress has been recently made in understanding to what extent duality symmetry is a general phenomenon of generic four-dimensional superstring models[38)-45)].

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