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HEAVY MESONS, HEAVY BARYONS AND HEAVY MULTIQUARKS IN POTENTIAL MODELS *)

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A B S T R A C T

We review some recent work done on heavy hadron spectroscopy in the framework of potential models. We stress the difficulty of deducing the QQQ or $QQ\bar{Q}\bar{Q}$ potential energy from the $Q\bar{Q}$ potential if the latter is purely empirical. On the other hand, the bag model, although quite involved to handle, provides a unified derivation of the potential governing the quarks inside the mesons, the baryons, and the multiquarks.

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1. HEAVY MESONS

The ability of potential models to describe the spectrum of heavy mesons is now well established. Logarithmic or power-law potentials are especially attractive, because of their simplicity and their nice scaling properties[1]. For instance, A. Martin recently proposed a potential[2]

$$V^{I}(r) = A + Br^{\beta} , \qquad \beta = 0.100 . \tag{1}$$

He successfully fitted the Υ, J/ψ, $F(c\bar{s})$, and ϕ families. The ϕ family means the $\phi(1020)$, the DCI $\phi'(1650)$ [3], and the E(1.42), with $J^{PC} = 1^{++}$ interpreted here as an $s\bar{s}$ P-state rather than as a glueball[4]. Moreover, the quark masses used in the fit are compatible with the experimental value of M_D-M_K and M_B-M_K.

Simple potentials such as V^I can be understood as a parametrization of the intermediate-range part of more traditional potentials, which are of the type "Coulomb-plus-linear"[5]

$$V^{II}(r) = -\frac{4}{3}\frac{\alpha_s}{r} + \lambda r + b , \tag{2}$$

and provide also very good fits[6]. The form of V^{II} has clear theoretical motivations. The Coulomb term comes from one-gluon exchange, and the linear behaviour at large distances is common to strong coupling expansion, and to string or bag models.

The bag model, precisely, provides a continuous interpolation from the Coulomb régime to the vortex limit. This application of the bag model was first presented at Moriond in 1975 by Hasenfratz, Kuti and Szalay[7] and has recently been developed in more detail[8-10]. The bag model for heavy quarks is basically the same as the one for light quarks. It is, however, handled differently. In the latter case, the MIT group assumed that highly relativistic quarks oscillate freely inside a fixed spherical cavity. The mass of the hadrons was obtained by minimization on the radius[11]. The case of heavy quarks, on the other hand, is treated in an adiabatic approximation similar to the Born-Oppenheimer treatment of the molecular spectrum. For a given interquark separation, the bag energy is minimized with respect to the size and the shape of the bag. The minimum is then plugged into the Schrödinger equation as the $Q\bar{Q}$ potential. In other words, this picture means that the bag re-adjusts itself almost immediately to an optimal configuration when the quarks move. With only two parameters -- the QCD coupling constant α_s and the bag pressure B -- the $Q\bar{Q}$ potential obtained in the bag model describes the spectrum of the J/ψ and Υ families[8] very well.

2. HEAVY BARYONS

Starting from a phenomenological $Q\bar{Q}$ potential such as V^I, the simplest and most commonly adopted strategy[12] for computing the baryon spectrum consists in adding QQ potentials given by

$$V_{QQ}(r) = \frac{1}{2} V_{Q\bar{Q}}(r) . \tag{3}$$

This is a special case of the "additive" model, in which the potential energy of a colour singlet $(Q_1Q_2...Q_n)$ made of n quarks or antiquarks is

$$V(Q_1Q_2...Q_n) = \sum_{i<j} \lambda_i \cdot \lambda_j \; V_8(r_{ij}) . \tag{4}$$

If one uses the Martin potential V^I and the rule (3), one gets[13] for the Ω^-(sss) the very satisfactory value $M(\Omega^-) = 1.662$ GeV (after spin corrections), to be compared with the experimental one $M(\Omega^-) = 1.672$ GeV. Note that, in Ref. 13, the three-body problem is treated carefully by the method of the hyperspherical expansion[14], which turns out to be very well suited to the study of baryons in the quark model.

This success in reproducing the mass of the Ω^-, as well as other successes in describing the properties of light baryons[12], must not overshadow the theoretical questions concerning prescriptions (3) or (4)[15]. For two-body forces, dominance of colour-octet exchanges seem reasonable, unless there would be confining forces between hadrons. A small amount of non-confining colour-singlet exchange cannot, however, be excluded. Moreover, three-body forces can very well be present. In perturbation theory, the three-gluon vertex induces three-body forces at the order α_s^2. Considerations of lattice gauge theories or of string models also suggest that the linear potential $V^L_{Q\bar{Q}} = \lambda r_{Q\bar{Q}}$ has to be generalized into

$$V^L_{QQQ} = \lambda \text{ Min } (d_1 + d_2 + d_3) , \tag{5}$$

where d_i is the distance between the quark i and a "junction" point chosen such that the sum of the d_i's is minimal[16]. Such a behaviour is recovered in a bag model calculation of the QQQ potential[17]. Asymptotically, this generalized linear term V^L_{QQQ} corresponds to a "Y-shape" (three-arm star) for the bag. However, as for the $Q\bar{Q}$ case, we have the property of "precocious linearity", i.e. when the interquark separations increase, the asymptotic régime is obtained much earlier for the potential than for the shape of the bag. For instance, the linear potential V^L_{QQQ} plays an important role in binding the ccc ground state (charm = 3), but the corresponding bag remains always almost spherical.

The potential V^L_{QQQ} in Eq. (5) is manifestly of the three-body type. Unfortunately, it is rather well approximated numerically by $\sum_{i<j} 0.5\,\lambda\, r_{ij}$ [16], which is exactly what would give the application of rule (4) to $V^L_{Q\bar{Q}}$. So the study of baryon spectroscopy would hardly distinguish between simple additive models of type (4) and more complicated models involving three-body forces. On the other hand, the above remark implies that the extrapolation from the mesonic to the baryonic sector is quite safe for phenomenological applications, since different approaches lead to the same effective interaction.

3. MULTIQUARKS

Going from n = 3 to n = 4 quarks (or antiquarks) does not only mean more technical difficulties. In fact, this raises a fundamental question: Do we have narrow hadronic "molecules", i.e. in more precise terms, stable or metastable multiquark hadrons which do not split spontaneously into smaller colour singlets by simple quark rearrangement? For n = 4, the question is whether or not a $QQ\bar{Q}\bar{Q}$ composite lies below the threshold made of two quarkonia.

The literature on multiquarks already contains a great variety of papers[18]. Most of them concern light quarks, for which the chromomagnetic forces play the most crucial role. Recently we have studied the problem for heavy quarks[19], where the spin-independent potential is presumably dominant. Some of our preliminary results are presented below.

First we considered additive potentials of type (4). For n = 2 ($Q\bar{Q}$) or n = 3 (QQQ), the operators $\lambda_i\cdot\lambda_j$ have a well-defined value, $-^{16}/_3$ and $-^{8}/_3$, respectively, and the colour wave function is factorized. For $n \geq 4$, there are different ways of building a colour singlet. The wave function thus has several components and the potential (4) is a matrix in the colour space. We have assumed that the colour wave function is still factorized, i.e. that we have only one spatial wave function. This is largely justified in the case of identical quarks, where the constraints due to the Pauli principle reduce considerably the possibility of colour mixing. In our approximation, we are dealing with a single-channel potential

$$V(Q_1Q_2\ldots Q_n) = \sum_{i<j} a_{ij}\, V_8(r_{ij}) \,, \qquad (6)$$

with $a_{ij} = \langle\lambda_i\cdot\lambda_j\rangle$, $\sum_{i<j} a_{ij} = -^{8}/_{3}n = C^2_n\bar{a}$. Such an interaction has the following properties:

i) If we denote by $M_n^{(S)}$ the mass of the ground state for a symmetric potential ($a_{ij} = \bar{a} \ \forall\ i,j$), then

$$\frac{M_2^{(S)}}{2} \leq \frac{M_3^{(S)}}{3} \leq \ldots \leq \frac{M_n^{(S)}}{n} \ . \tag{7}$$

This means, for instance, that with an additive potential of type (4), we have $M(\Omega^-) > {}^3\!/_2\, M(\phi)$ [including the spin-spin corrections if we consider V_8 as being the spin-triplet potential].

ii) For n fixed, if we compare several (a_{ij}) distributions with a given total strength $\sum_{i<j} a_{ij}$, the symmetric case always gives the heaviest mass for the ground state:

$$M_n(a_{ij}) \leq M_n^{(S)} \ . \tag{8}$$

iii) For n = 4, the "true" diquonium with colour wave function $|QQ - \bar{Q}\bar{Q}\rangle = |\bar{3} - 3\rangle_1$ is always above the threshold:

$$2M(Q\bar{Q}) < M(\bar{3} - 3) \ . \tag{9}$$

The results (7), (8), and (9) are rigorous and independent of the confining potential V_8.

iv) For a "mock" baryonium of colour structure $|QQ - \bar{Q}\bar{Q}\rangle = |6 - \bar{6}\rangle_1$, there are compelling reasons for believing that it satisfies for most potentials V_8:

$$2M(Q\bar{Q}) < M(6 - \bar{6}) < M(\bar{3} - 3) \ . \tag{10}$$

This means that there is no narrow $QQ\bar{Q}\bar{Q}$ state within the additive model (6).

So far, however, we considered only quarks with equal masses. We also studied the effect of the potential (6) on some configurations involving two different masses m and m′. We assumed that the static potential V_8 is mass (or flavour) independent, as suggested by QCD and by the ability of $Q\bar{Q}$ potentials to describe different quarkonium families simultaneously ($s\bar{s}$, $c\bar{c}$, $b\bar{b}$, $c\bar{s}$ ...).

a) The crypto-exotic (in flavour) state $X = QQ'\bar{Q}\bar{Q}'$ has two thresholds: $T_1 = Q\bar{Q} + \bar{Q}'\bar{Q}'$ and $T_2 = Q\bar{Q}' + Q'\bar{Q}'$. Within our assumptions, we can prove that $T_1 \leq T_2$, and it seems almost sure that we have always $T_1 \leq X_1$, i.e. X is very broad and never shows up as a structure in the spectrum.

b) The genuine exotic $Y = QQ\bar{Q}'\bar{Q}'$ may become stable in additive potential models (6) if the ratio of masses m/m′ is large enough. This property and, eventually, the critical value of m/m′ depends upon the specific potential V_8 that we consider. With the model V^I [Eq. (1)] and the prescription (4), a state such as $tt\bar{s}\bar{s}$ would be bound provided $m(t) \gtrsim 10$ GeV. With the model V^{II} [Eq. (2)], the state $tt\bar{c}\bar{c}$ also has a chance if $m(t) \gtrsim 15$ GeV.

The estimate of the $QQ\bar{Q}\bar{Q}$ masses can also be done with the bag model. As already stressed, in this framework there is no extra parameter, nor is there any extra parameter when going from the $Q\bar{Q}$ to the $QQ\bar{Q}\bar{Q}$ case. We have estimated the potential within the approximation of a spherical bag centred at the centre of mass of the four quarks. The resulting $QQ\bar{Q}\bar{Q}$ potential has multibody components and is more attractive at short distances than one would expect from the additive rule (3). When this potential is plugged into the Schrödinger equation, narrow multiquarks do not, however, proliferate. In fact, we very often obtain states at the edge of the threshold, and so to arrive at conclusions regarding their stability would require some refinements in our calculation. This could concern the effect of surface tension, of higher order terms in α_s inside the bag, of non-spherical deformations, etc.

To conclude, we wish to underline the following points:

1) The potential models work very well for mesons.

2) They can safely be extended to the sector of baryons.

3) The existence of narrow heavy multiquarks could be a good test of the nature of interquark forces. For instance, a model with pairwise forces and colour-octet exchange dominance provides less attraction than the more collective interaction arising from the bag model.

4) In any case, from our preliminary calculations, there is no tendency to a proliferation of heavy multiquarks. If any, they would more likely appear in flavour-exotic configurations such as $tt\bar{s}\bar{s}$ involving very different masses rather than in crypto-exotic configurations with nearly equal masses such as $b\bar{b}c\bar{c}$.

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