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NEW LIGHT ON DIPOLE SUM RULES

H. Krasemann
CERN - Geneva

A B S T R A C T

We discuss the dipole sum rules in the context of spin independent first order perturbation theory to a Schrödinger potential model. When applied to charmonium, thirty per cent corrections to the bounds on $\psi' \to \gamma P_c/\chi$ emerge. New upper and lower bounds on charmonium photon transitions are given. The same amount of corrections is discovered in a Klein-Gordon potential model which explicitly contains relativistic kinematics. We then predict photon transition rates for upsilons.

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INTRODUCTION: THE STATUS OF $c\bar{c}$ E1 TRANSITIONS AND CORRECTIONS

The charmonium system and especially its spectrum can be understood reasonably well in terms of a non-relativistic potential model with the Schrödinger equation[1),2)]. One test for the degree of non-relativity (which, however, will turn out to be misleading except for the spectrum) is usually a calculation of the first order relativistic correction to the kinetic energy of a c quark in the J/ψ [2)], which turns out to be well below 10% [corresponding to $(v/c)^2 < 0.4$]. Nevertheless we observe big discrepancies between theory and experiment for decays, especially lepton pair decays and radiative transitions. Adopting the naïve model, we can fit the one or the other, but not both[2)]. The sensitive parameter here is the quark mass m_c. With low $m_c \approx 1$ GeV we can fit the lepton pair decay to lowest order

$$\Gamma(J/\Psi \longrightarrow \mu^+\mu^-) = 16\,\pi\,\alpha^2 e_c^2\, |J/\Psi(0)|^2\, M_{J/\Psi}^{-2} \tag{1}$$

and with high $m_c \approx 3.5$ GeV we can fit the electric dipole (E1) radiation

$$\Gamma_{E1}(\Psi' \longrightarrow \gamma\, P_c/\chi) = \frac{4}{3}\;\frac{2j+1}{9}\,\alpha\, e_c^2\;k^3 |\langle\Psi'|r|P_c/\chi\rangle|^2 \tag{2}$$

where j is the spin of the P_c/χ and e_c the quark charge, J/ψ, P_c/χ and ψ' are Schrödinger model wave functions.

A good value for the constituent c quarks mass lies between one and two GeV, implying that there are substantial corrections to both formulae (1) and (2). The corrections to (1) are widely discussed in the literature and we will not take this up here. Instead we will concentrate on the corrections to (2). What is the status ? Experimentally, all three transitions (2) are measured to be (16±5) keV [3)], while a model calculation with $m_c = 1.6$ GeV gives (36,50,58) keV for $j = (2,1,0)$. Can we tune the model so that it matches experiment ? The answer is "no", because the best quantity to calculate,

$$k\;|\langle\Psi'|r|P_c/\chi\rangle|^2\;, \tag{3}$$

is only sensitive to m_c [it is $\sim m_c^{-1}$, therefore a large m_c makes the rates (2) small] and very insensitive to any other parameter of the model. For example, varying the power α of a power potential $V(r) \sim r^{\alpha}$ between $\alpha = 0$ and $\alpha = \infty$ changes (3) by less than 10%[4)].

Unable to tune (3), we have to ask for corrections to (2) beyond the strictly non-relativistic model. A list of (relativistic) corrections contains

i) finite size effects (= corrections to the long photon wave length approximation),

ii) recoil corrections,

iii) higher multipoles beyond E1,

iv) extra terms in the transition Hamiltonian,

v) admixtures of different radial and orbital eigenstates, and of a $D\bar{D}$ (or $D\bar{D}^*$, etc.) continuum

vi) corrections to the size of the wave functions.

Finite size effects, which have been discussed by Novikov et al.[5], and recoil corrections both show up as an extra factor $\exp(-i\vec{k}\cdot\vec{r}/2) \neq 1$ in the transition matrix element. For processes (2), $(k\,r/2)^2$ lies between 0.04 and 0.25, but a simple calculation[5] shows that there is an extra suppression factor of 10 coming from the three dimensional nature of the problem. Thus corrections i) and ii) will be negligible. Higher multipoles iii) may be controlled by an expansion of the same $\exp(-i\vec{k}\cdot\vec{r}/2)$, again only the even powers are significant (this corresponds to an expansion of either the electric or the magnetic field of the classical photon). For $j = 0$, multipoles higher than E1 are excluded, for the other states they should be a few per cent at most. It is amusing to note that this order of magnitude estimate leads us to expect at most 10% M2 in $\chi_2(3.55) \to \gamma\, J/\psi$. Extra terms in the transition Hamiltonian which appear as higher terms in the non-relativistic reduction iv) were first discussed by Novikov et al.[5]. More recently, this item has been taken up again by Karl, Meshkov and Rosner[6], who essentially show that no big effects appear. An admixture of different radial eigenstates in either ψ' or P_c/χ can be mimicked by tuning the potential. We have seen that this has no big effects. An admixture of different orbital eigenstates generally occurs in a relativistic treatment. It will only be significantly large if the states which mix lie nearby in energy. Therefore a 1^3D_1 admixture to the 2^3S_1 wave in ψ' is much more likely than any $\ell \geq 3$ admixture in the $\ell = 1$ P_c/χ states. But even in this case, the $1^3D_1 - 2^3S_1$ mixture induced by the bound state dynamics is negligibly small[2),5]. The $\psi' - \psi''$ mixing occurs predominantly via the coupling of both states to $D\bar{D}$: the ψ'' decays Zweig allowed to $D\bar{D}$ and the ψ' are at least very close to threshold with a large virtual coupling to $D\bar{D}$. Radiative decays of the ψ' are mainly affected twofold: the $c\bar{c}$ content becomes renormalized and a strong (positive) interference between the transitions $2S \to 1P$ and $1D \to 1P$ occurs. In the model of Eichten et al.[7] the net effect, however, is only a 14 - 24% reduction of the rates (2). It is amusing to note that a change of the sign of the D wave admixture in ψ would make the S-D interference in the $\gamma\,\chi(0^+)$ decay negative and would thus help to make the rates more similar. This is because the ratio of the amplitudes,

$$\frac{A(1^3D_1 \longrightarrow \gamma\, 1^3P_j)}{A(2^3S_1 \longrightarrow \gamma\, 1^3P_j)} = \left(5, \frac{-5}{4}, \frac{1}{20}\right)^{1/2} \text{for } j = (0,1,2) \tag{4}$$

is not uniform for the three P waves. To summarize this point (correction v), mixing effects are very likely to play an important role among the corrections to naïve non-relativistic dipole rates. We will now turn to another very important correction, namely relativistic effects on the size of $c\bar{c}$ wave functions vi). As a first attempt we will concentrate on spin independent effects. Since these will turn out to be quite large we also expect sizeable spin dependent corrections. To some extend these have been discussed by Jackson[8]. The spin independent corrections we study are a consequence of the modification of the non-relativistic equation of motion. A relativistic equation of motion exhibits a different relation between energy level differences and the size of the wave functions as compared to the non-relativistic case: for fixed level differences the relativistic wave functions will occupy less space and therefore dipole moments will be smaller. This can be seen in two ways. First, we discuss the dipole sum rules in the context of a model in first order perturbation theory to the non-relativistic case. There we will find a 30% reduction of the upper bounds on process (2) as given by the dipole sum rules. We then turn to a relativistic model using the Klein-Gordon (KG) equation. For charmonium, the transition rates of this KG model are reduced by the same 30% ÷ 40% as compared to the non-relativistic model.

E1 SUM RULES IN FIRST ORDER PERTURBATION THEORY

The $c\bar{c}$ Hamiltonian is

$$H = H^{(0)} + H^{(1)} \tag{5}$$

where in the centre of mass system

$$H^{(0)} = \frac{p^2}{2\mu} + V(r) . \tag{6}$$

Here μ is the reduced mass and $V(r)$ the static part of the potential, while $H^{(1)}$ contains all non-static corrections. To first order perturbation theory all terms in $H^{(1)}$ are known[9),10)]. Even numerical calculations of $H^{(1)}$ for the $c\bar{c}$ system exist[11),12)]. To this first order all spin dependent terms leave the centres of gravity of multiplets invariant, so that it is consistent to consider these centres of gravity and the spin independent corrections alone. The energy difference k in (3) will change from zeroth to first order, while the matrix element of the dipole operator in (3) will not (wave functions are unaffected in first order). With the numerics of Ref. 11)

$$k^{(1)}/k^{(0)} \approx 0.7 \tag{7}$$

where m_c = 1.8 GeV [a smaller quark mass than 1.8 GeV should make the ratio (7) even smaller]. Expression (3) can now be rewritten as

$$\frac{k^{(1)}}{k^{(0)}} \, k^{(0)} \left|\langle\psi'|r|P_c/\chi\rangle\right|^2 . \tag{8}$$

The important observation is that the dipole sum rules only bound $k^{(0)}|\langle\psi'|r|P_c/\chi\rangle|^2$ but not the whole of (8). To see this, remember that the sum rules are derived from the uncertainty relation $[\vec{x},\vec{p}] \leq 3i$ ($\hbar \equiv 1$) by using the <u>lowest order</u> equation of motion

$$\vec{p} = i\mu \left[H^{(0)}, \vec{x} \right] \tag{9}$$

to replace $\vec{p}$. It follows that

$$\langle [\vec{x}, [H^{(0)}, \vec{x}]] \rangle \leq 3/\mu \tag{10}$$

and with the insertion of a complete set of final states one arrives at

$$\sum_f k^{(0)}_{fi} \left|\langle f|\vec{x}|i\rangle\right|^2 \leq 3/2\mu , \tag{11}$$

where $k^{(0)}_{fi} = \langle f|H^{(0)}|f\rangle - \langle i|H^{(0)}|i\rangle$. The consequence of this is a modification of all E1 sum rules by factors (7). With the numerics of Ref. 11) we have worked out the following Table of new dipole sum rule bounds.

transition	TRK SR	WK SR
$2^3S_1 \to \gamma 1^3P_2$	-	<25
$2^3S_1 \to \gamma 1^3P_1$	-	<35
$2^3S_1 \to \gamma 1^3P_0$	-	<40
$1^3P_2 \to \gamma 1^3S_1$	<410	>135 + 200 ± 80
$1^3P_1 \to \gamma 1^3S_1$	<310	>105 + 105 ± 40
$1^3P_0 \to 1\gamma\ {}^3S_1$	<150	>50 + 40 ± 15

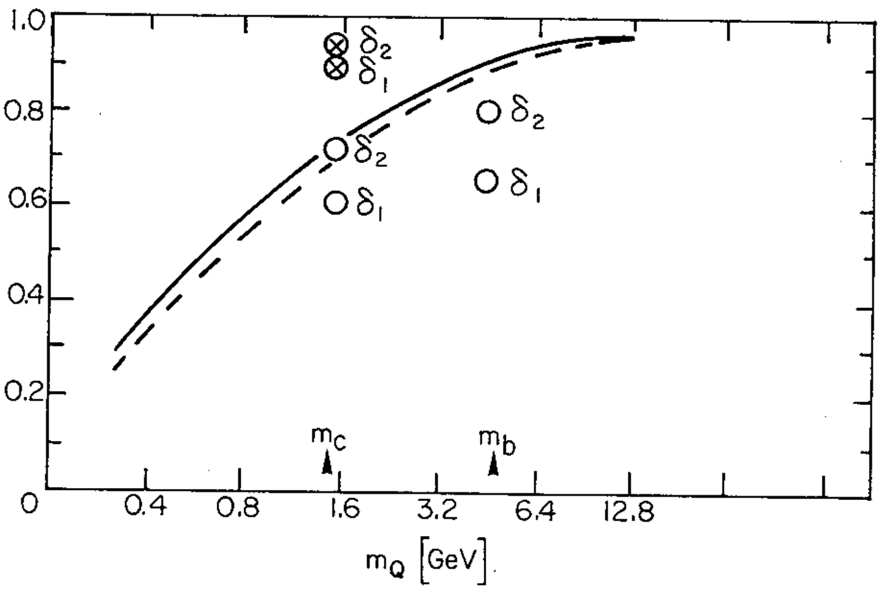
Upper and lower limits on $c\bar{c}$ E1 transitions (in keV) using the Thomas-Reiche-Kuhn and Wigner-Kirkwood sum rules to first order perturbation theory with $m_c =$ = 1.8 GeV. We further use $\Gamma(\psi' \to \gamma P_c/\chi) = (16 \pm 5)$ kev[3]. Where the bound is expressed as two numbers, the second number and the error arise from this experimental measurement.

We note much less discrepancy between the measured rates (2) and the bounds in the Table than in a lowest order treatment[2]. Especially the upper and lower bounds on $P_c/\chi \to \gamma\ J/\psi$ no longer leave a big gap.

THE KLEIN-GORDON MODEL

The author has studied a charmonium potential model using the Klein-Gordon (KG) equation[13]. A considerable reduction of the quantity (3) as compared to the non-relativistic model was found. These findings have been checked by a detailed study of the $c\bar{c}$ system using the Dirac equation. In the introduction we noted that the quantity (3) varies by less than 10% for all confining power potentials. In fact, it never deviates by more than 10% from the sum rule value $3/2\mu$ (11). Consequently, we study the quantities

$$\delta_1 = \mu\, k_{1P,2S}\, |\langle 1P | r | 2S \rangle|^2 \qquad (12)$$
$$\delta_2 = \frac{2}{3}\mu\, k_{1S,1P}\, |\langle 1S | r | 1P \rangle|^2$$



The quantities of (12) as a function of m_Q in a non-relativistic Schrödinger potential model (⊗) and in a KG potential model (○). Both models are of the Coulomb + linear type (standard model) and fit $c\bar{c}$ and $b\bar{b}$ respectively. The solid line shows δ_2 (the broken line δ_1) in a KG model with a pure (scalar) linear potential $V(r) = 0.8$ GeV/fm.

which are one in the non-relativistic harmonic oscillator and ≥ 0.9 in any non-relativistic monominal potential model which confines. The figure shows δ_1 and δ_2 in various models. One readily sees that the non-relativistic limit is not reached before $m_Q \approx 6 \div 10$ GeV. By comparison to the pure linear model it becomes evident that the Coulombic part of the potential plays a role for δ only in $b\bar{b}$, not in $c\bar{c}$. For $c\bar{c}$ the dipole sum rules are quite close to model rates, but for $b\bar{b}$ we need the model. We have done such a model calculation (using the KG equation) and find it most convenient to express the result as a modification of the formulae (6.11) through (6.14) of the second of Ref. 2). The rates are given by those formulae with

$$\bar{k}\,|r_{fi}|^2 = \frac{1}{m_b} \times \begin{array}{l} 0.62 \times 4 \\ 0.81 \times 2 \\ 0.66 \times 5 \\ 0.89 \times 5 \\ 0.53 \times 2 \\ 0.73 \times 3 \end{array} \quad \text{for} \quad \begin{array}{l} 3S \longrightarrow 2P \\ 2P \longrightarrow 1D \\ 2P \longrightarrow 2S \\ 1D \longrightarrow 1P \\ 2S \longrightarrow 1P \\ 1P \longrightarrow 1S \end{array} \tag{13}$$

where $\bar{k}$ is the average energy difference between the centres of gravity of the spin multiplets. Note that the dipole sum rules would give one for each decimal fraction shown in (13) except for $3S \to 2P$, where no bound exists.

CONCLUSION

We have shown that a good part of the discrepancy between the measured rates $\psi' \to \gamma P_c/\chi$ and their non-relativistic theory can be identified as relativistic size effects on the wave functions. We only discussed spin independent effects but it is conceivable that spin dependent effects may be of similar magnitude at least in charmonium. Such spin dependent effects would also affect ratios of these rates. Another source of this discrepancy is the neglect [with exception of Ref. 1)] of $D\bar{D}$ admixtures in S and P wave functions and of S - D wave mixing. The existing calculation shows relatively few net effects, but this topic certainly remains open. In principle the ratios of our rates are also strongly affected by S - D mixing.

In $b\bar{b}$ transitions relativistic effects are already much smaller, but the dipole sum rules are much less saturated than in $c\bar{c}$ because of the increasing importance of the Coulomb part of the potential. A calculation had to be done.

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