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FROM HADRON GAS TO QUARK MATTER II *)

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ABSTRACT

We describe a quark-gluon plasma in terms of an ideal quark-gluon gas supplemented with two essential corrections: the vacuum pressure B (from the bag model) and first order interaction in the running coupling constant α_s. Such a gas has a critical curve P = 0 which is similar to and can be made coincident with the bootstrap critical curve found in the first lecture. We therefore argue that these possibly coinciding critical curves separate two phases in which strongly interacting matter can exist: a hadronic phase and a quark-gluon plasma phase. There is a finite region of coexistence between these two phases, which is determined by the usual Maxwell construction. Having thus joined the two models along their possibly common critical curves, we try to confront our model with experiments on relativistic heavy ion collisions. In order to do so, plausible (but not very safe) assumptions are made about the history of a collision and how each state contributes to observable quantities like temperature or transverse momenta. Our results are compatible with experiments, but many questions remain open. A signature of the quark-gluon phase surviving hadronization is suggested.

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1. INTRODUCTION

Having computed the grand partition function Z for the hadronic gas in the first part of these lectures[1], we can now turn to the discussion of possible experiments involving collisions of heavy nuclei and calculate particular features of the inclusive particle spectra that are determined by our equations of state. The physical picture underlining this description is that of centrally colliding nuclei with a large number of nucleons participating in the formation of a hot and dense region of hadronic matter[2],[3],[4].

Our intuition demands that in such sufficiently dense and excited hadronic gas, the individual hadrons dissolve into a new phase of matter consisting of the hadronic constituents; the quark-gluon plasma may be formed[5]. The occurrence of this new phase is not in contradiction with the yet unsuccessful attempts to liberate quarks in high energy collisions. The currently accepted theory of hadronic structure and interactions, quantum chromodynamics supplemented with its phenomenological extension: the MIT Bag model[6], allows the formation of large space domains filled with a quark-gluon plasma. Such a state is expected to be unstable and to decay again into individual hadrons, following its free expansion. The mechanism of quark confinement requires that all quarks recombine to form hadrons again . Thus the quark-gluon plasma may be only a transitory form of hadronic matter formed under special conditions and therefore quite difficult to detect experimentally.

We will begin this lecture with a short summary of the relevant QCD facts and beliefs and give the equations of state for the quark-gluon plasma, which extend the domain of hadronic matter described by the statistical Bootstrap approach beyond the critical curve. We further note that when one identifies the 'elementary input particles' of the bootstrap as being the SU(2) isospin subsets of the baryonic decouplet and the mesonic octet, we naturally expect a smooth matching of both phases of hadronic matter and a quantitive description of the phase transition from the hadronic gas phase described in the first lecture to the quark-gluon plasma phase described here.

Having discussed the properties of the phase transition in some detail, both in a thermal bath and in hadronic collisions, we will turn our attention to the study of temperatures and mean transverse momenta of pions and nucleons produced in nuclear collisions. We will argue that most information about the hot and dense phase of the collision is lost, as most of the particles originate in the cooler and more dilute hadronic gas phase of matter. In order to obtain reliable information on quark matter, we must presumably perform more specific experiments. We briefly point out that the presence of numerous $\bar{s}$ quarks in the quark plasma suggests, as a characteristic experiment, the observation of $\bar{\Lambda}$ hyperons. We conclude these lectures with an outlook on likely future developments of the theory and experiment.

2. QCD AND THE QUARK-GLUON PLASMA

We begin with a summary of the relevant postulates and results that characterize the current understanding of strong interactions in quantum chromodynamics (QCD). The most important postulate is that the proper vacuum state in QCD is not the (trivial) perturbative state that we (naively) imagine to exist everywhere and which is little changed when the interactions are turned on/off. In QCD the true vacuum state is believed to have a complicated structure which originates in the glue ('photon') sector of the theory. The perturbative vacuum is an excited state with an energy density B above the true vacuum. It is to be found inside hadrons where perturbative quanta of the theory, in particular quarks, can therefore exist. The occurrence of the true vacuum state is intimately connected to the glue-glue interaction; unlike QED these massless photons of QCD also carry a charge - colour - that is responsible for the quark-quark interaction.

In the above discussion, the confinement of quarks is a natural feature of the hypothetical structure of the true vacuum. If it is, for example, a colour superconductor, then an isolated charge cannot occur. Another way to look at this is to realize that a single coloured object would, according to Gauss' theorem, have an electric field that can only end on other colour charges. In the region penetrated by this field, the true vacuum is displaced, thus effectively raising the mass of a quasi-isolated quark by the amount $B.V_{field}$.

Another feature of the true vacuum is that it exercises a pressure on the surface of the region of the perturbative vacuum to which quarks are confined. Indeed, this is just the idea of the original MIT bag model[6)]. The Fermi pressure of almost massless light quarks is in equilibrium with the vacuum pressure B. When many quarks are combined to form a giant quark bag, then their properties inside can be obtained using standard methods of many-body theory[7)]. In particular, this also allows the inclusion of the effect of internal excitation through a finite temperature and through a change in the chemical composition.

A further effect that must be taken into consideration is the quark-quark interaction. We shall use here the first order contribution in the QCD running coupling constant $\alpha_s(q^2) = g^2/4\pi$. However, as $\alpha_s(q^2)$ increases when the average momentum exchanged between quarks decreases, this approach will have only limited validity at relatively low densities and/or temperatures. The collective screening effects in the plasma are of comparable order of magnitude and should reduce the importance of perturbative contribution.

As u and d quarks are almost massless inside a bag, they can be produced in pairs and at finite temperatures some $q\bar{q}$ pairs will be present. In particular also $s\bar{s}$ pairs will be produced and we will return to this point below. Furthermore, real gluons can be present when $T \neq 0$ and will be included here in our considerations. However, until we properly understand the vacuum problem, this term must be distrusted. Fortunately it is of marginal importance in our considerations.

As was outlined in the first lecture, a complete description of the thermodynamical behaviour of a many-particle system can be derived from the grand partition function Z. For the case of the quark-gluon plasma in the perturbating vacuum, one finds an analytic expression through first order in α_s neglecting the quark masses. We obtain for the quark Fermi gas[5),8)] with $N = 3$ colours

$$\ln Z_q(\beta,\lambda) = \frac{gV}{6\pi^2}\beta^{-3}\left[\left(1-\frac{2\alpha_s}{\pi}\right)\left(\frac{1}{4}\ln^4\lambda_q + \frac{\pi^2}{2}\ln^2\lambda_q\right) + \left(1-\frac{50}{21}\frac{\alpha_s}{\pi}\right)\frac{7\pi^4}{60}\right] \tag{1}$$

where $g = (2\,s + 1)\,(2\,I + 1)\,N = 12$ counts the number of the components of the quark gas, and λ_q is the fugacity related to the quark number. As each quark has baryon number 1/3, we find

$$\lambda_q^3 = \lambda = e^{\mu/T} \tag{2}$$

where λ, as in the first lecture, allows the conservation of the baryon number. Consequently

$$3\mu_q = \mu \tag{3}$$

The glue contribution is[7)]

$$\ln Z_g(\beta,\lambda) = V\frac{8\pi^2}{45}\beta^{-3}\left(1-\frac{15\alpha_s}{4\pi}\right) . \tag{4}$$

We notice the two relevant differences with the photon gas: (i) the occurrence of the factor eight associated with the number of gluons; (ii) the glue-glue interaction as gluons carry the colour charge.

Finally, let us introduce the true vacuum term as

$$\ln Z_{vac} = -\beta B V \tag{5}$$

This leads to the required positive energy density B within the volume occupied by the coloured quarks and gluons and to a negative pressure on the surface of this region. At this stage, this term is entirely phenomenological as discussed above. The equations of state for the quark-gluon plasma are easily obtained by differentiating

$$\ln Z = \ln Z_q + \ln Z_g + \ln Z_{vac} \tag{6}$$

with respect to β, λ and V. The energy, baryon number density and pressure are respectively:

$$\varepsilon = -\frac{1}{V}\frac{\partial}{\partial\beta}\ln Z = \frac{6}{\pi^2}T^4\left[\left(1-\frac{2\alpha_s}{\pi}\right)\times\right.$$
$$\left.\times\left(\frac{1}{4\cdot 3^4}\ln^4\lambda+\frac{\pi^2}{2\cdot 3^2}\ln^2\lambda\right)+\left(1-\frac{50}{21}\frac{\alpha_s}{\pi}\right)\frac{7\pi^4}{60}\right]+$$
$$+\frac{8\pi^2}{15}T^4\left(1-\frac{15\alpha_s}{4\pi}\right)+B \tag{7}$$

$$\nu = \frac{1}{V}\lambda\frac{\partial \ln Z}{\partial\lambda} = \frac{2T^3}{\pi^2}\left[\left(1-\frac{2\alpha_s}{\pi}\right)\left(\frac{1}{3^4}\ln^3\lambda+\frac{\pi^2}{9}\ln\lambda\right)\right] \tag{8}$$

$$P = T\frac{\partial \ln Z}{\partial V} = \frac{2T^4}{\pi^2}\left[\left(1-\frac{2\alpha_s}{\pi}\right)\left(\frac{1}{4\cdot 3^4}\ln^4\lambda+\frac{\pi^2}{2\cdot 3^2}\ln^2\lambda\right)\right.$$
$$\left.+\left(1-\frac{50}{21}\frac{\alpha_s}{\pi}\right)\frac{7\pi^4}{60}\right]+\frac{8\pi^2}{45}T^4\left(1-\frac{15\alpha_s}{4\pi}\right)-B \tag{9}$$

We first notice the relativistic relation between the energy density and pressure: $\varepsilon - B = 3\,(P + B)$, that is

$$P = \frac{1}{3}\left(\varepsilon - 4B\right) \tag{10}$$

This simple equation of state of the quark-gluon plasma is slightly modified when the finite quark masses are considered, or when the QCD coupling constant α_s is dependent on the dimensional parameter Λ. From Eq. (10) it follows that when the pressure vanishes, the energy density is $4B$, independent of the values of μ and T which fix the line $P = 0$. We recall that this has been precisely the kind of behaviour found for the hadronic gas in the preceding lecture[1]. This coincidence of the physical observables strongly suggests that the different parameters of the theory should be chosen in such a way that both lines $P = 0$ coincide; we will return to this point again below. For $P > 0$ we have $\varepsilon > 4B$ (we recall that in the hadronic gas we always had $\varepsilon \leq 4B$). Thus, in this domain of $\mu - T$ we have the quark-gluon plasma exposed to an external force.

In order to obtain an idea of the form of the $P = 0$ critical curve in the $\mu - T$ plane for the quark-gluon plasma, we rewrite Eq. (9) using Eqs. (2) and (3) for $P = 0$:

$$B = \frac{\left(1 - \frac{2\alpha_s}{\pi}\right)}{162\,\pi^2}\left[\mu^2 + (3\pi T)^2\right]^2 - \frac{T^4\pi^2}{45}\left[\left(1 - \frac{5\alpha_s}{3\pi}\right)\cdot 12 - \left(1 - \frac{15\alpha_s}{4\pi}\right)\cdot 8\right] \tag{11}$$

Here, the last term is the glue pressure contribution. If the true vacuum structure is determined by the glue-glue interaction, then this term could be modified significantly. We find that the greatest lower bound on temperature T_q at $\mu = 0$ is about

$$T_q \approx B^{1/4} \tag{12}$$

This result can be considered to be correct to within 20%. However, its order of magnitude is as expected. Taking Eq. (11) as it is, we find for $\alpha_s = 1/2$ $T_q = 120$ MeV. Omitting the gluon contribution to the pressure we find $T_q = 135$ MeV. It is quite likely that with the proper treatment of the glue field, of the plasma corrections and perhaps with some larger $B^{1/4}$, the desired value of $T_q = 190$ MeV corresponding to the statistical bootstrap choice would follow. Furthermore, allowing some $T - \mu$ dependence of α_s we then can easily obtain an agreement between the critical curves.

Let us further note that for $T \ll \mu$ the baryon chemical potential tends to

$$\mu_B = 3\mu_q \Rightarrow 3B^{1/4}\left[\frac{2\pi^2}{(1-\frac{2\alpha_s}{\pi})}\right]^{1/4} = 1010 \text{ MeV} \quad \left[\alpha_s = 1/2\, ,\ B^{1/4} \doteq 145 \text{ MeV}\right] \tag{13}$$

In concluding this discussion of the $P = 0$ line of the quark-gluon plasma, let us note that the choice $\alpha_s \sim 1/2$ is motivated by the fits to the charmonium and ypsilonium spectra as well as to deep inelastic scattering. In both these cases spacelike domains of momentum transfer are explored. The much smaller value of $\alpha_s \sim 0.2$ is found in timelike regions of momentum transfer in $e^+e^- \to$ hadrons experiments. In the quark-gluon plasma, as described up to first order perturbation theory, positive and negative momenta transfers occur: the perturbative corrections to the radiative T^4 contribution is dominated by timelike momenta transfers, while the correction to the μ^4 term originates from spacelike quark-quark scattering.

3. PHASE TRANSITION FROM HADRONIC GAS TO THE QUARK-GLUON PLASMA

In the previous section we described the form of the $P = 0$ critical curve for the quark plasma. In order to have a consistent description of both phases, it is not necessary that both critical curves coincide, though this would be the preferable case. As the quark plasma is the phase into which individual hadrons dissolve, it is sufficient if the quark plasma pressure vanishes within the boundary set for non-vanishing positive pressure of the hadronic gas. It is quite satisfactory for the theoretical development that this is the case. In Fig. 1a) a qualitative picture of both $P = 0$ lines is shown in the $\mu - T$ plane. Along the dotted straight line at constant temperature we show in Fig. 1b) the pressure as a function of the volume (a P-V diagram). The volume is obtained by inverting the baryon density at a constant fixed baryon number

$$V = \frac{\langle b \rangle}{\nu} \tag{14}$$

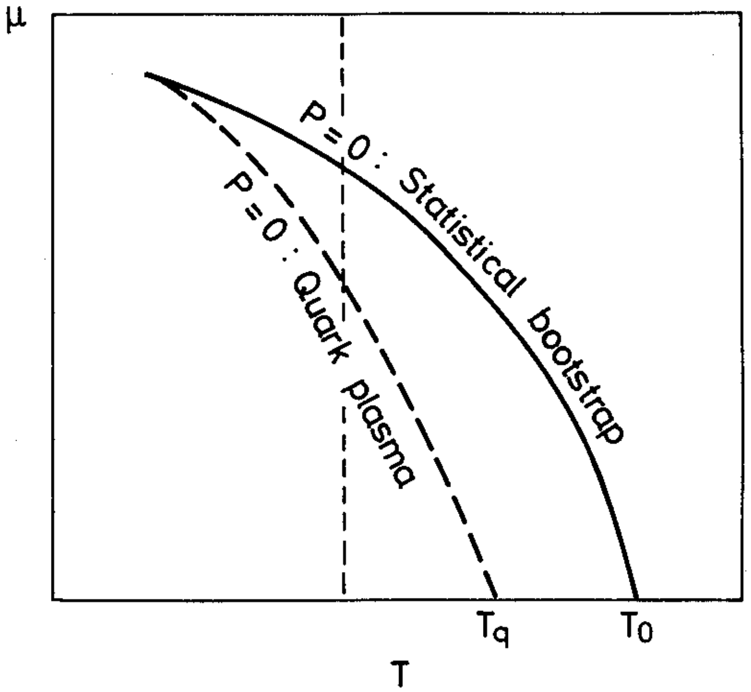

Fig. 1a : The critical curves (P = 0) of the two models in the T - μ plane (qualitiatively). The region below the full line is described by the statistical bootstrap model, the region above the broken line by the quark-gluon plasma. The critical curves can be made to coincide.

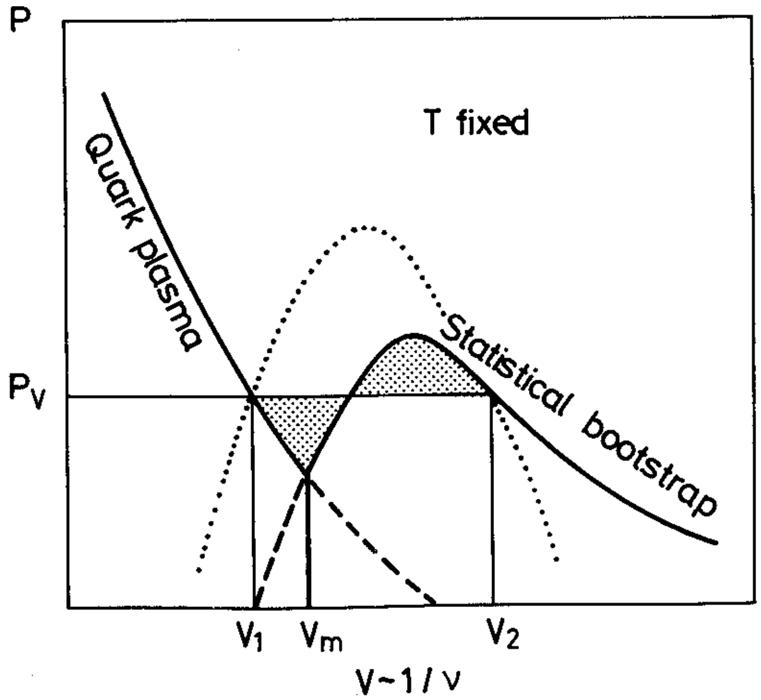

Fig. 1b : P - V diagram (qualitative) of the phase transition (hadron gas to quark-gluon plasma) along the broken line T = const. of Fig. 1a. The coexistence region is found from the usual Maxwell construction (the shaded areas are equal).

The behaviour of $P(V, T = \text{const.})$ for the hadronic gas phase is as described in the statistical bootstrap model. For large volumina, we see that P falls with rising V. However, when hadrons get close to each other so that they form larger and larger lumps, the pressure drops rapidly to zero; the hadronic gas becomes a state of few composite clusters (internally already consisting of the quark plasma). The second branch of the $P(V, T = \text{const.})$ line meets the first one at a certain volume $V = V_m$.

The phase transition occurs for $T = \text{const.}$ in Fig. 1b) at a vapour pressure P_v obtained from the conventional Maxwell construction - the shaded regions in Fig 1b) are equal. Between the volumina V_1 and V_2 matter coexists in the two phases with the relative fractions being determined by the magnitude of the actual volume. This leads to the occurrence in the $\mu - T$ plane (Fig. 1a) of a third region - the coexistence region of matter, in addition to the pure quark and hadron domains. For $V < V_1$ corresponding to $\nu > \nu_1 \sim 1/V_1$ all matter has gone into the quark plasma phase.

The dotted line in Fig. 1b) encloses (qualitatively) the domain in which the coexistence between both phases of hadronic matter seems possible. We further note that at low temperatures, $T \lesssim 50$ MeV, the plasma and hadronic gas critical curves meet each other in Fig. 1a). As this is just the domain where our description of the hadronic gas fails[*)] this should not be considered as a serious flaw of our theory.

The qualitative discussion presented above can be easily supplemented with quantitative results; we will not do so here, but turn our attention immediately to the modifications forced onto this simple picture by the experimental circumstances in high energy nuclear collisions.

4. NUCLEAR COLLISIONS

We assume that in relativistic collisions triggered to small impact parameters[2)] a single hot central fireball of hadronic matter can be produced. We are aware of the whole problematics connected with such an idealization. A proper treatment should include collective motions and distribution of collective velocities, local temperatures and so on[9)]; triggering for small impact parameters hopefully eliminates some of the complications. However, except for rare events, not all nucleons from projectile and target nuclei will participate in the formation of the fireballs[3)]. In particular, in nearly symmetric collisions (projectile

*) because we neglect Bose-Einstein and Fermi-Dirac statistics as well as some low energy features of the interaction.

and target nuclei are similar) we can argue that the numbers of participants in the centre of mass of the fireball originating in the projectile or target are the same. Therefore, it is irrelevant how many nucleons do form the fireball - the above symmetry argument leads, in a straightforward way, to a formula for the centre of mass energy per participating nucleon:

$$U := \frac{E_{c.m.}}{A} = m_N \sqrt{1 + (E_{K,lab}/A)/2m_N} \tag{15}$$

where $E_{k,lab}/A$ is the projectile kinetic energy per nucleon in the laboratory frame. While the fireball changes its baryon density and chemical composition ($\pi + p \leftrightarrow \Delta$ etc.) during its lifetime through a change in temperature and chemical potential, the conservation of energy and baryon number assures us that U in Eq. (15) remains constant, assuming that the influence on U of pre-equilibrium emission of hadrons from the fireball is negligible. As U is the total energy per baryon available, we can, supposing that kinetic and chemical equilibrium have been reached, set it equal to the ratio of thermodynamic expectation values of the total energy and baryon number

$$U = \frac{\langle E \rangle}{\langle b \rangle} = \frac{\mathcal{E}(\beta,\lambda)}{\nu(\beta,\lambda)} \tag{16}$$

Thus we see that the experimental value of U Eq. (15) fixes through Eq. (16) a relation between allowable values of (β, λ): the available excitation energy defines the temperature and the chemical composition of hadronic fireballs. In Fig. 2 a,b) these paths are shown for a choice of kinetic energies $E_{k,lab}/A$ in a) in the μ - T plane and in b) in the ν - T plane. In both cases, only the hadronic gas domain is shown. We wish to note several features of the curves shown in Fig. 2) relevant in later considerations:

1) beginning at the critical curve, the chemical potential first drops rapidly when T decreases and then rises slowly as T decreases further (Fig. 2a); this corresponds to a monotonically falling baryon density with decreasing temperature (Fig. 2b), but implies that in the initial expansion phase of the

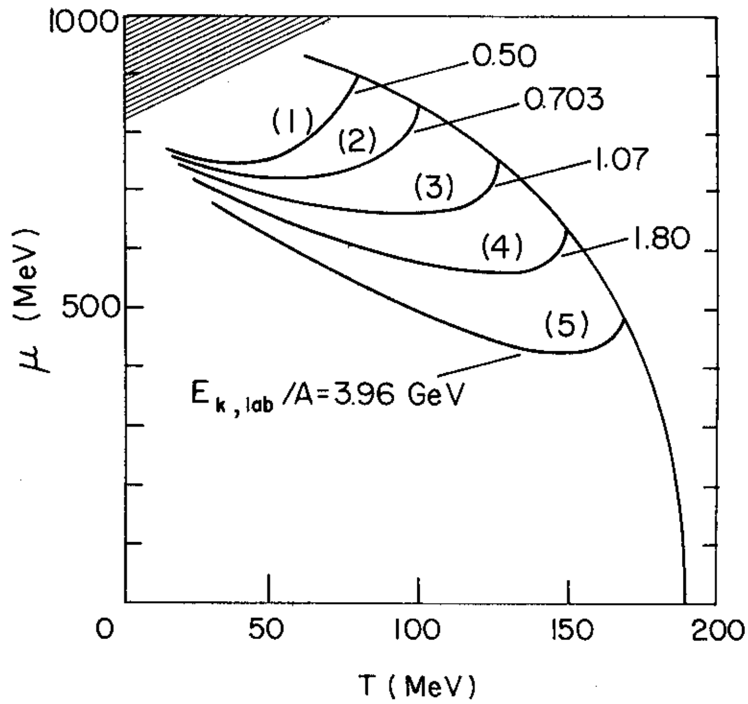

Fig. 2a : The critical curve of hadron matter (bootstrap) together with some "cooling curves" in the T - μ plane. While the system cools down along these lines it emits particles; when all particles have become free, it comes to rest on some point on these curves ("freeze out"). In the shaded region our approach may be invalid.

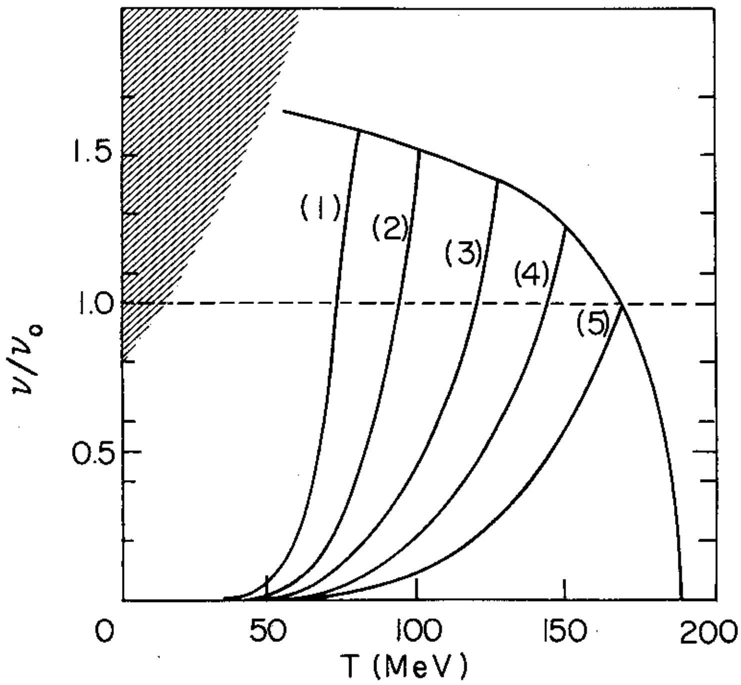

Fig. 2b : The critical curve of hadron matter (bootstrap) together with some "cooling curves" (same energies as in Fig. 2a) in the variables T and ν/ν_0 = (baryon number density)/(normal nuclear baryon number density). In the shaded region our approach may be invalid.

fireball, the chemical composition changes more rapidly than the temperature;

2) the baryon density in Fig. 2b) is of the order of 1-1.5 of normal nuclear density. This is a consequence of the choice of $B^{1/4} = 145$ MeV. Were B twice as large, i.e., $B^{1/4} = 172$ MeV - so far not excluded by any experiment - then the baryon densities in this figure would double to 2-3 ν_o. Furthermore, we observe that along the critical curve of the hadronic gas the baryon density falls with rising temperature. This is easily understood as, at higher temperature, more volume is taken up by the mesons;

3) inspecting Fig. 2b) we see that at given U the temperatures at the critical curve and those at about ~ 1/2 ν_o differ little (10%) for low U, but more significantly for large U. Thus, highly excited fireballs cool down more before dissociation ('freeze out'). As particles are emitted all the time while the fireball cools down along the lines of Fig. 2), they carry kinetic energies related to various different temperatures; the inclusive single particle momentum distribution will yield only averages along these cooling lines.

Calculations of this kind can also be carried out for the quark plasma. They are, at present, uncertain due to the unknown values of α_s and $B^{1/4}$. Fortunately, there is one particular property of the equations of state of the quark-gluon plasma that we can easily exploit. Combining Eq. (10) with Eq. (16) we obtain

$$P = \frac{1}{3}\left(U\nu - 4B\right) \tag{17}$$

Thus, for a given U (the available energy per baryon in a heavy ion collision) Eq. (17) describes the pressure-volume (~ $1/\nu$) relation. By choosing to measure P in units of B and ν in units of normal nuclear density, $\nu_o = 0.14/\mathrm{fm}^3$ we find:

$$P/B = \frac{4}{3}\left(\gamma U/m_N \cdot \nu/\nu_o - 1\right)$$

with (18)

$$\gamma := m_N \nu_o / 4B = .56 \quad \left[B^{1/4} = 145\,\mathrm{MeV};\ \nu_o = .14/\mathrm{fm}^3\right]$$

Here, γ is the ratio of the energy density of normal nuclei ($\varepsilon_N = m_N\nu_o$) and of quark matter or of a quark bag ($\varepsilon_q = 4B$). In Fig. 3) this relation is shown for three projectile energies: $E_{k,lab}/A = 1.80$ GeV, 3.965 GeV, 5.914 GeV corresponding to U = 1.313 GeV, 1.656 GeV and 1.913 GeV respectively. We observe that even at the lowest energy shown, the quark pressure is zero near the

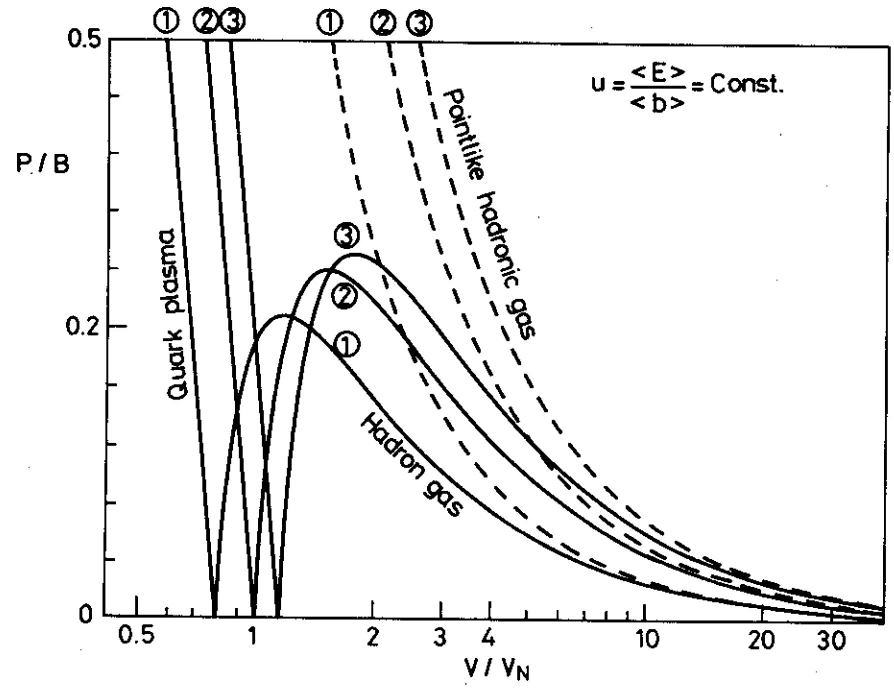

Fig. 3 : P - V diagram of "cooling curves" belonging to different kinetic lab. energies per nucleon: ① 1.8 GeV; ② 3.965 GeV; ③ 5.914 GeV. In the history of a collision the system comes down the quark lines and jumps somewhere over to the hadron curves (Maxwell). Broken lines show the diverging pressure of pointlike bootstrap hadrons.

baryon density corresponding to 1.3 normal nuclear density, given the current value of B.

Before discussing this point further, we note that the hadronic gas branches of the curves in Fig. 3) show a quite similar behaviour to that shown at constant temperature in Fig. 1b). Remarkably enough, both branches meet each other at $P = 0$ since both have the same energy density: $\varepsilon = 4B$ and therfore $V(P = 0) \sim \sim 1/\nu = U/\varepsilon = U/4B$. However, what we cannot see inspecting Fig. 3) is that there will be a discontinuity in the variables μ and T at this point except if parameters are chosen so that the critical curves of both phases coincide. Indeed, near to $P = 0$ the results shown in Fig. 3) should be replaced by points obtained from the Maxwell construction. The pressure in a nuclear collision will never fall to zero, it will correspond to the momentary vapour pressure of the order of 0.2B as the phase change occurs.

A further aspect of the equations of state for the hadronic has is also illustrated in Fig. 3). Had we ignored the finite size of hadrons (one of the Van der Waals effects) in the hadron has phase then, as shown by the dash-dotted lines, the phase change could never occur because the point particle pressure would diverge where the quark pressure vanishes. In our opinion, one cannot say it often enough: inclusion of the finite hadronic size and of the finite temperature when considering the phase transition to quark plasma, lowers the relevant baryon density [from 8-14 ν_0 for cold point-nucleon matter] to 1-3 ν_0 (depending on the choice of B) in 2-5 GeV/A nuclear collisions.

5. INCLUSIVE PARTICLE SPECTRA

It seems, inspecting Fig. 2) again, that a possible test of the equations of state for the hadronic gas consists of the measurement of the temperature in the hot fireball zone, and to do this as a function of the nuclear collision energy. The plausible assumption made is that the fireball follows the 'cooling' lines shown in Fig. 2) until final dissociation into hadrons. This presupposes that the surface emission of hadrons during the expansion of the fireball does not significantly alter the available energy per baryon. This is more likely true for sufficiently large fireballs; for small ones, pion emission by the surface may influence the energy balance. As the fireball expands, the temperature falls and the chemical composition changes. The hadronic clusters dissociate and more and more hadrons are to be found in the 'elementary' form of a nucleon or a pion; their kinetic energies are reminiscent of the temperature found at each phase of the expansion.

To compute the experimentally observable final temperature[10] we shall argue that a time average along the cooling curves must be performed. Not knowing the reaction mechanisms too well, we assume that the temperature decreases approximately linearly with the time in the significant expansion phase. We further have to allow that a fraction of particles emitted can be reabsorbed in the hadronic clusters. This is a geometric problem and, in first approximation the ratio of the available volume Δ to the external volume V_{ex} is the probability that an emitted particle be not reabsorbed, i.e., that it can escape:

$$R_{esc.} = \frac{\Delta}{V_{ex}} = \left(1 - \frac{\varepsilon(\beta,\lambda)}{4B}\right) \qquad (20)$$

The relative emission rate is just the integrated momentum spectrum

$$R_{emis.} = \int \frac{d^3p}{(2\pi)^3}\, e^{-\sqrt{p^2+m^2}/T + \mu/T} = \frac{m^2 T}{2\pi^2} K_2\left(\frac{m}{T}\right) e^{\mu/T} \qquad (21)$$

The chemical potential acts only for nucleons. In the case of pions it has to be dropped from the above expression. For the mean temperature we thus find:

$$\langle T \rangle := \frac{\int_c R_{esc.} \cdot R_{emis.} \cdot T\,dT}{\int_c R_{esc.} \cdot R_{emis.}\, dT} \tag{22}$$

c indicates here a line integral along that particular cooling curve in Fig. 2) which belongs to the energy per baryon fixed by the experimentalist.

The average temperature, as a function of the range of integration over T, reaches different limiting values for different particles. The limiting value obtained thus is the observable "average temperature" of the debris of the interaction, while the initial temperature T_{cr} at given $E_{k,lab}$ is difficult to observe. When integrating along the cooling line (Eq. 22) we can easily, at each point, determine the average hadronic cluster mass. The integration for protons is interrupted (protons are 'frozen out') when the average cluster mass is about half the nucleon isobar mass. We have also considered baryon density dependent

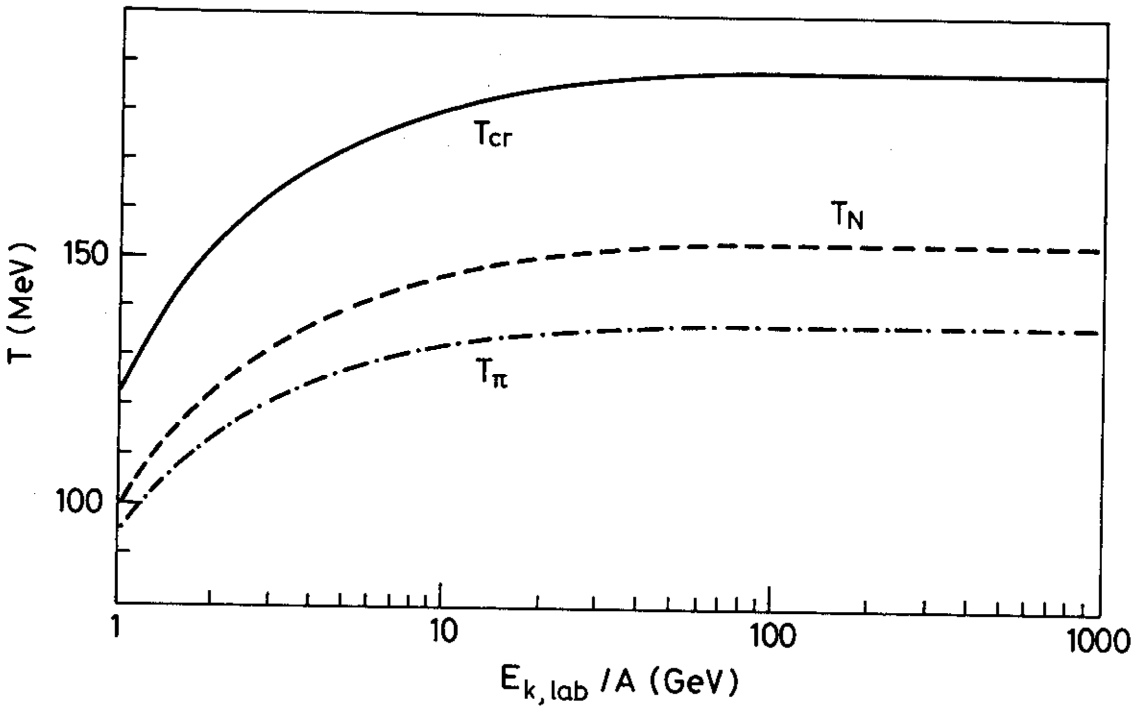

Fig. 4 : Mean temperatures for nucleons and pions together with the critical temperature belonging to the point where the "cooling curves" start off the critical curve (see Fig. 2a). The mean temperatures are obtained by integrating along the cooling curves. Note that T_N is always greater than T_π.

freeze-out, but such a procedure depends strongly on the unreliable value of B.

Our choice of the freeze-out condition was made in such a way that the nucleon temperature at $E_{k,lab}/A = 1.8$ GeV is 120 MeV as suggested by recent BEVELAC experiments[11]. The actual model dependence of our freeze-out introduces an uncertainty of ± 10 MeV on the average temperature. In Fig. 4) we show the expected pion and nucleon average temperatures as well as the initial temperature T_{cr} as a function of the heavy ion kinetic energy in the range 1-1000 GeV. Two effects contribute to this result:

1) the particular shape of the cooling curves (Fig. 2a): the chemical potential drops rapidly from the critical curve, thereby damping relative baryon emission at lower T. Pions, which do not feel the baryon chemical potential, continue being created also at lower temperatures.

2) the freeze-out of baryons occurs earlier than the freeze-out of pions. We have not included here a third effect - the isospin conservation[12]

This is also the place to comment on our choice of the bootstrap constant H so that $T_o \sim 190$ MeV: the observed[13] "experimental T_o" of the order of 160 MeV is an average in the above sense; our T_o leads, for very large E_k, to an average value of the order of 160 MeV.

In practice, the temperature is most reliably measured through the measurement of mean transverse momenta of the particles. It may be more practical therefore, to calculate the average transverse momentum of the emitted particles. In principle, to obtain this result we have to perform a similar averaging as above; for the average transverse momentum at given T,μ we find[14]

$$\langle p_\perp(m,T,\mu)\rangle_p = \frac{\int p_\perp e^{-(\sqrt{p^2+m^2}-\mu)/T} d^3p}{\int e^{-(\sqrt{p^2+m^2}-\mu)/T} d^3p} = \frac{\sqrt{\frac{\pi m T}{2}}\, K_{5/2}\left(\frac{m}{T}\right) e^{\mu/T}}{K_2\left(\frac{m}{T}\right) e^{\mu/T}} \tag{23}$$

The average over the cooling curve is then:

$$\langle\langle p_\perp(m,T,\mu)\rangle_p\rangle_c = \frac{\int_c \frac{\Delta}{V_{ex}} T^{3/2} \sqrt{\frac{\pi m}{2}}\, K_{5/2}\left(\frac{m}{T}\right) e^{\frac{\mu}{T}} dT}{\int_c \frac{\Delta}{V_{ex}} T K_2\left(\frac{m}{T}\right) e^{\frac{\mu}{T}} dT} . \tag{24}$$

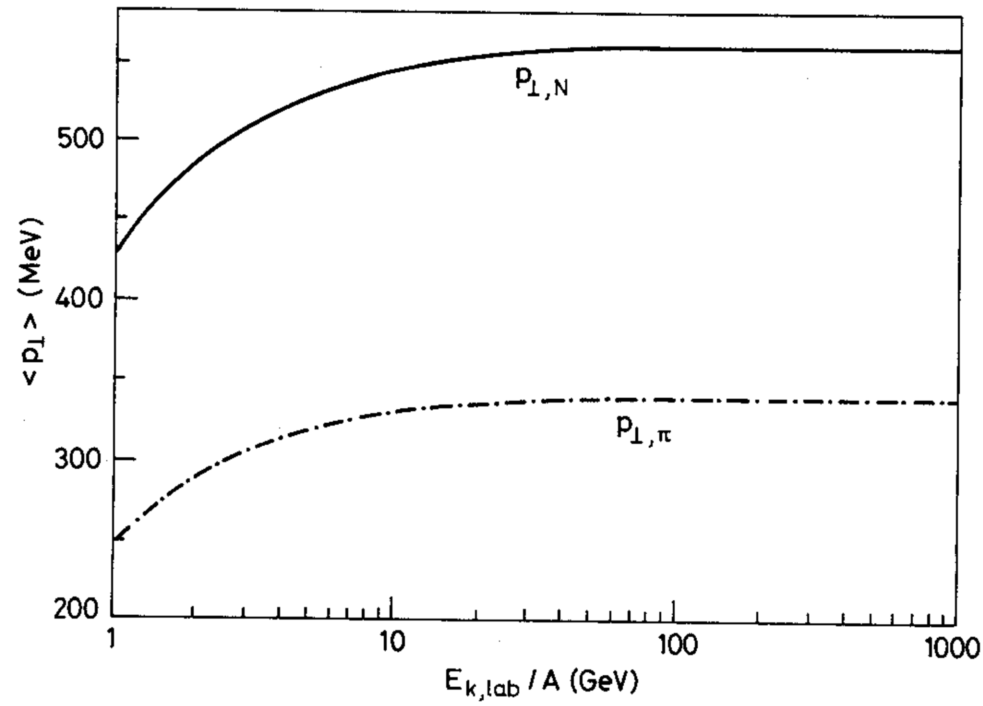

Fig. 5 : Mean transverse momenta of nucleons and pions found by integrating along the "cooling curves".

We did verify numerically that the order of averages does not matter;

$$\langle p_\perp(m, \langle T\rangle_c, \mu)\rangle_p \approx \langle\langle p_\perp(m, T, \mu)\rangle_p\rangle_c \tag{25}$$

which shows that the mean transverse momentum is also the simplest (and safest) method of determining the average temperature (indeed better than fitting ad hoc exponential type functions to $p_\perp$ distributions). In Fig. 5) we show the dependence of the average transverse momenta of pions and nucleons on the kinetic energy of the heavy ion projectiles.

6. EXPERIMENTS ON QUARK MATTER

From the averaging process described here, we have learned that the temperatures and transverse momenta of particles originating in the hot fireballs are more reminiscent of the entire history of the fireball expansion than of the initial hot compressed state, perhaps present in the form of quark matter. We may generalize this result and then claim that most properties of inclusive spectra are reminiscent of the equations of state of the hadronic gas phase and that the memory of the initial dense state is lost during the expansion of the fireball as the hadronic gas rescatters many times while it evolves into the final kinetic and chemical equilibrium state.

In order to observe properties of quark-gluon plasma we must design a thermometer, an isolated degree of freedom weakly coupled to the hadronic matter. Nature has, in principle (but not in praxis) provided several such thermometers: leptons and heavy flavours of quarks. We would like to point here to a particular phenomenon perhaps quite uniquely characteristic of quark matter; first we note that, at a given temperature, the quark-gluon plasma will contain an equal number of strange (s) quarks and antistrange ($\bar{s}$) quarks, naturally assuming that the hadronic collision time is much too short to allow for light flavour weak interaction conversion to strangeness. Thus, assuming equilibrium in the quark plasma, we find the density of the strange quarks to be (two spins and three colours):

$$\frac{s}{V} = \frac{\bar{s}}{V} = 6\int\frac{d^3p}{(2\pi)^3}e^{-\sqrt{p^2+m_s^2}/T} = 3\frac{Tm_s^2}{\pi^2}K_2\left(\frac{m_s}{T}\right) \tag{26}$$

(neglecting, for the time being, the perturbative corrections and, of course, ignoring weak decays). As the mass of the strange quarks, m_s, in the perturbative vacuum is believed to be of the order of 280-300 MeV, the assumption of equilibrium for $m_s/T \sim 2$ may indeed be correct. In Eq. (26) we were able to use the Boltzmann distribution again, as the density of strangeness is relatively low. Similarly, there is a certain light antiquark density ($\bar{q}$ stands for either $\bar{u}$ or $\bar{d}$):

$$\frac{\bar{q}}{V} \cong 6\int\frac{d^3p}{(2\pi)^3}e^{-|p|/T-\mu_q/T} = e^{-\mu_q/T}\cdot T^3\frac{6}{\pi^2} \tag{27}$$

where the quark chemical potential is, as given by Eq. (3), $\mu_q = \mu/3$. This exponent suppresses the $q\bar{q}$ pair production as only for energies higher than μ_q is there a large number of empty states available for the q.

What we intend to show is that there are many more $\bar{s}$ quarks than antiquarks of each light flavour. Indeed:

$$\frac{\bar{s}}{\bar{q}} = \frac{1}{2}\left(\frac{m_s}{T}\right)^2 K_2\left(\frac{m_s}{T}\right)e^{\mu/3T} \tag{28}$$

The function $x^2K^2(x)$ is, for example, tabulated in Ref. 15). For $x = m_s/T$ between 1.5 and 2, it varies between 1.3 and 1. Thus, we almost always have more $\bar{s}$ than $\bar{q}$ quarks and, in many cases of interest, $\bar{s}/\bar{q} \sim 5$. As $\mu \to 0$ there are about as many $\bar{u}$ and $\bar{q}$ quarks as there are $\bar{s}$ quarks.

When the quark matter dissociates into hadrons, some of the numerous $\bar{s}$ may, instead of being bound in a $q\bar{s}$ Kaon, enter into a $(\bar{q}\bar{q}\bar{s})$ antibaryon and, in particular, a $\bar{\Lambda}$ or $\bar{\Sigma}^0$. The probability for this process seems to be comparable to the similar one for the production of antinucleons by the antiquarks present in the plasma. What is particularly noteworthy about the $\bar{s}$ carrying antibaryons is that they can only be produced in direct pair production reactions. Up to about $E_{k,lab}/A = 3.5$ GeV this process is strongly suppressed by the energy-momentum conservation and because for free p-p collisions the threshold is at about 7 GeV. We thus would like to argue that a study of the $\bar{\Lambda}$, $\bar{\Sigma}^0$ in nuclear collisions for $2 < E_{k,lab}/A < 4$ GeV could shed light on the early stages of the nuclear collisions in which quark matter may be formed.

7. SUMMARY AND OUTLOOK

Our aim has been to obtain a description of hadronic matter valid for high internal excitations. By postulating the kinetic and chemical equilibrium we have been able to develop a thermodynamic description valid for high temperatures and different chemical compositions. In our work we have found two physically different domains; firstly the hadronic gas phase, in which individual hadrons can exist as separate entities, but are sometimes combined to larger hadronic clusters; and in the second domain, individual hadrons dissolve into one large cluster consisting of hadronic constituents - the quark-gluon plasma.

In order to obtain a theoretical description of both phases we have used some 'common' knowledge and plausible interpretations of the currently available experimental observation. In particular, in the case of hadronic gas we have completely abandoned a more conventional Lagrangian approach in favour of a semi-phenomenological statistical bootstrap model of hadronic matter that incorporates those properties of hadronic interaction that are, in our opinion, most important in nuclear collisions.

In particular, the attractive interactions are included through the rich, exponentially growing hadronic mass spectrum $\tau(m^2,b)$ while the introduction of the finite volume of each hadron is responsible for an effective short-range repulsion. Aside from these manifestations of strong interactions, we only satisfy the usual conservation laws of energy, momentum and baryon number. We neglect quantum statistics since a quantitative study has revealed that this is allowed above $T \approx 50$ MeV. But we allow particle production, which introduces a quantum physical aspect into the otherwise 'classical' theory of Boltzmann particles.

Our considerations lead us to the equations of state of hadronic matter which reflect what we have included in our considerations. It is the quantitative nature of our work that allows a detailed comparison with the experiment. This work has just begun and it is too early to say if the features of strong interactions that we have chosen to include in our considerations are the most relevant ones. It is important to observe that the currently predicted temperatures and mean transverse momenta of particles agree with the experimental results available at $E_{k,lab}/A =$ $= 2$ GeV [BEVELAC - see Ref. 11)] and at 1000 GeV [ISR - see Ref. 13)] as much as a comparison is permitted. For example, the pion and nucleon mean transverse momenta show the required substantial rise, see Fig. 5). Further comparisons involving, in particular, particle multiplicities and strangeness production are under consideration.

Finally, we mention the internal theoretical consistency of our two-fold approach. With the proper interpretation, the statistical bootstrap leads us, in a straightforward fashion, to the postulate of a phase transition to the quark-gluon plasma. This second phase is treated by a quite different method; in addition to the standard Lagrangian quantum field theory of weakly interacting particles at finite temperature and density, we also introduce the phenomenological vacuum pressure and energy density B. This term is required in a consistent theory of hadronic structure. It turns out that $B^{1/4} \sim 150$ MeV is just, to within 20%, the temperature of the quark phase before dissociation into hadrons. This is smaller than the maximal hadronic temperature $T_o \simeq 190$ MeV but consistent with it. T_o is fixed to this value in the statistical bootstrap through the choice of the constant H. This choice of T_o assures that the fall-off of transverse cross-sections at high energies is $e^{-6p_\perp}$ $(T = 1000/6 = 160$ MeV$)$.

Perhaps the most interesting aspect of our work is the realization that the transition to quark matter will occur at very much lower baryon density for highly excited hadronic matter than for matter in the ground state $(T = 0)$. Using the currently accepted value B and the fixed value of H we find that at $\nu \sim 1 -$ $- 3\,\nu_o$, $T = 150$ MeV, the quark phase may be formed. The detailed study of the different aspects of this phase transition, as well as of possible characteristic signatures of quark matter, must still be carried out. However, our initial results look very encouraging, as the required baryon density and temperatures are well within the range of the 2-10 GeV per nucleon fixed target heavy nucleon collisions. We look forward to the next generation of heavy ion accelerators which may provide us with the required experimental information.

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