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RADIATIVE TRANSITIONS AND THE P WAVE LEVELS IN CHARMONIUM

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ABSTRACT

A value of $\frac{1}{2}$ or less for the ratio $[E(2^{++})-E(1^{++})]/[E(1^{++})-E(0^{++})]$ of the P level splittings in approximate agreement with the assignment of the states at 3.41, 3.50 and 3.55 to the 0^{++}, 1^{++} and 2^{++} P-wave levels, is obtained with a short-range Coulomb (Lorentz vector) potential together with a long-range linear (Lorentz scalar) confining potential. The radiative transition widths $\Gamma(\psi' \to 3.41+\gamma)$, $\Gamma(\psi' \to 3.50+\gamma)$, $\Gamma(\chi' \to 3.55+\gamma)$ are significantly smaller than those obtained in previous (one-channel) charmonium calculations. The best results were obtained by allowing the Coulomb coupling constant α_s to have a momentum dependence suggested by asymptotic freedom formulae.

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The $\chi(3.41)$ and two states at 3.50 and 3.55 GeV have been discovered[1-4] in radiative decays of the $\psi'(3.68)$ and have been interpreted as three of the four even-charge conjugation of charmonium predicted[5-7] to lie between $\psi(3.1)$ and $\psi'(3.68)$. Recently[1] a fourth state at 3.45 GeV has also been observed in radiative decay of the $\psi'(3.68)$, though unlike the other three this 3.45 state has no visible hadronic decay modes. The predicted states in question are 0^{++}, 1^{++}, and 2^{++}, being ^3P_J states of charmonium, and also 0^{-+}, a $^1S_0$ state, being the first radially excited state of paracharmonium commonly known as η_c'. Of the observed states the $\chi(3.41)$ is established from observed hadronic decays to be one of the series 0^{++}, 2^{++}, 4^{++}, ... The angular distribution in $\psi' \to \gamma + \chi(3.41)$ is in accord with that appropriate to the spin 0 assignment -- namely $(1 + \cos^2 \theta)$ -- and this spin 0 angular distribution is not observed in the 3.50 and 3.55 region[1]; these observations suggest that the 0^{++} state is at 3.41 and the 0^{-+} state at 3.45. In accord with these assignments, Chanowitz and Gilman[8] have given arguments for assigning $\chi(3.41)$, $\chi(3.50)$, and $\chi(3.55)$ to the 0^{++}, 1^{++}, and 2^{++} states, respectively, and the 3.45 to the 0^{-+} state. We adopt this orthodox point of view on the observed states and investigate the ^3P_J level splittings and the radiative widths $\Gamma(\psi' \to {}^3P_J)$ and their consequences for the dynamical parameters and properties of the charmonium system in general.

We use a potential method with relativistic kinematics to all orders of v/c, by using a Bethe-Salpeter equation with an instantaneous potential, as detailed below. For the short-range part of this potential we use, following standard practice, a form suggested by asymptotic freedom[9], taking one gluon exchange in the Feynman gauge

$$V \sim -\alpha_s \gamma^a_\mu \frac{1}{r} \gamma^{\mu b} \qquad (r \text{ small}), \tag{1}$$

where α_s is the (positive) asymptotic freedom coupling parameter and γ^a_μ, $\gamma^{\mu b}$ are the Dirac matrices acting on the particles a and b, respectively (being a quark anti-quark pair). For the longer range confining part we adopt the linear r dependence suggested by lattice gauge theories of infrared confinement and much used in charmonium calculations[6,7]:

$$V \sim \lambda\, 1^a\, r\, 1^b \qquad (r \text{ largish}), \tag{2}$$

where 1^a, 1^b are unit (Dirac) matrices. We have assumed in (2), in concordance with infrared confinement models[10], that the long-range potential is a multi-gluon effect and that the vector potential of one-gluon exchange is not appropriate. The Lorentz covaniant scalar (2) is the simplest form. Though it is overtly spin-independent, yet insertion into the Bethe-Salpeter equation reveals a spin-dependence of order 1/r, whereas the lattice gauge theories[10] suggest a much stronger spin-suppression as discussed by, for example, by Schnitzer[11,12]. It is instructive to expand the relativistic equation given below in powers of 1/m, m being the charmed quark mass (even though we solve our equations to all orders in 1/m or equivalently v/c) and to consider the resulting spin-orbit and tensor forces. For the spin-orbit coupling from (1) we get

$$\frac{3}{2m^2}\,\underline{L}\cdot\underline{S}\left(\frac{1}{r}\frac{\partial}{\partial r}\left(-\frac{\alpha_s}{r}\right)\right) = \frac{\alpha_s}{r^3}\,\frac{3}{4m^2}\left[J(J+1)-L(L+1)-S(S+1)\right] \tag{3}$$

and from (2) we also get a spin-orbit coupling

$$\frac{1}{2m^2}\,\underline{L}\cdot\underline{S}\left(-\frac{1}{r}\frac{\partial}{\partial r}(\lambda r)\right) = -\frac{\lambda}{r}\,\frac{1}{4m^2}\left[J(J+1)-L(L+1)-S(S+1)\right] \tag{4}$$

Both the potentials (1) and (2) are attractive (non-relativistically) but they give the opposite sign of spin-orbit potential, seen in (3) and (4), because of their different spin structure. Thus the Coulomb potential alone would order the P-levels, in ascending energy order, as $^3P_0$, $^3P_1$, $^3P_2$, while the scalar confining potential would order them oppositely as $^3P_2$, $^3P_1$, $^3P_0$. To the same order in 1/m the Coulomb potential (2), but <u>not</u> the confining potential (1), gives rise to a tensor force

$$\frac{3}{4m^2}\,\frac{\alpha_s}{r^3}\left[\frac{(\underline{\sigma}^{(a)}\cdot\underline{r})(\underline{\sigma}^{(b)}\cdot\underline{r})}{r^2} - \frac{1}{3}\,\underline{\sigma}^{(a)}\cdot\underline{\sigma}^{(b)}\right] \tag{5}$$

which gives a P-level splitting ordering the levels in ascending energy order, as $^3P_0$, $^3P_2$, $^3P_1$ thus co-operating with the Coulomb spin-orbit coupling in putting $^3P_0$ lowest. Thus in this perturbative treatment (in 1/m) discussion of this paragraph, the experimental quasi-fact[8] that $^3P_0$ is lowest, restricts λ to be not too large (in some measure related to the wave functions) relative to α_s; because m is fairly large this almost certainly holds also in the non-perturbative treatment of this paper.

EQUATION OF MOTION

We use an equation of motion which incorporates effects of relativistic kinematics. If we insert an instantaneous potential in the centre-of-mass system into the Bethe-Salpeter equation, we obtain the Salpeter equation[13,14] in momentum space:

$$\Phi(\underline{q}) = \int\left[\frac{\Lambda_+^a\Lambda_+^b\beta^a\beta^b V(\underline{q},\underline{q}')}{E-2W} - \frac{\Lambda_-^a\Lambda_-^b\beta^a\beta^b V(\underline{q},\underline{q}')}{(E+2W)}\right]\Phi(\underline{q}')\,d^3q' \tag{6}$$

where $\Phi(\underline{q})$ is the wave function in the relative momentum $\underline{q} = \underline{p}_a = -\underline{p}_b$; $W = \sqrt{m^2 + q^2}$, m being the quark mass; $\Lambda_{\pm}^{a,b} = \frac{1}{2}(1 \pm H_{a,b}/W)$ are the projection operaters for particles $\underline{a}$ and $\underline{b}$, where $H_a = \underline{\alpha}^a \cdot \underline{p}_a + \beta^a m$, $H_b = \underline{\alpha}^b \cdot \underline{p}_b + \beta^b m$. E is the energy of the two-particle state, and for bound states Eq. (6) is an eigenvalue equation in E.

For a non-relativistic system, or any system such that

$$\overline{E+2W} \gg \overline{E-2W} \tag{7}$$

it may be justified to drop the second term in (6) with the negative energy projection operators. [The precise justification in any particular case depends not only on (7) but also on the γ-matrix dependence of the potential, V.] Doing this, we obtain the simpler equation:

$$(E-2W)\,\Phi(\underline{q}) = \Lambda_+^a \Lambda_+^b \beta^a \beta^b \int V(\underline{q},\underline{q}')\,\Phi(\underline{q}')\,d^3q' \tag{8}$$

Alternatively we can regard (8), following Faustov[15], as a quasi-potential Logunov-Tavkheldize-Blankenbecler-Sugar equation[16].

In fact the equation (8), or its equivalent, is the basis for the derivation of the Fermi-Breit Hamiltonian[17], containing *inter alia* the spin-orbit interaction, or the treatment of positronium in the work of Schwinger[18] which has been widely followed in the charmonium literature. However, equation (8) is exact in the sense that no expansion in powers of 1/m (equivalently v/c), m being the charmed quark mass, is yet made.

We solve (8) to a high degree of accuracy using computer numerical methods. $V(\underline{q},\underline{q}') = V(\underline{q}-\underline{q}')$ is in our work just the Fourier transform of the potential (1), (2), or whatever instantaneous potential we wish to investigate. Our methods also allow us to solve the more complicated equation (6), and this we do, sometimes with results very close, in the cases presented here, to the solutions of (8).

WAVE FUNCTIONS FOR ψ AND χ MESONS

There is a well-known way of writing 2- (spin ½) fermion wave functions as a superposition of Dirac γ-matrices; this representation is particularly natural when the two fermions are particle and antiparticle, and the representation of spinor outer products, such as $u_a \bar{u}_b'$ or $u_b \bar{u}_a'$, as Dirac γ-matrices is almost immediate[19]. Using this representation the reduced Salpeter equation (8) may be written

$$(E - 2W)\,\Phi(\underline{q}) = \int d^3q'\ \Lambda_+(\underline{q})\beta\, V(\underline{q}-\underline{q}')\,\Phi(\underline{q}')\beta\,\Lambda_-(-\underline{q}) \tag{9}$$

where Φ is now a superposition of Dirac γ-matrices, $\Phi_{\alpha\beta}$ the first index α of the γ-matrices corresponding to particle a, and the second index β to particle b; $V(\underline{q}-\underline{q}')\Phi(q')$ in the integral is a symbolic notation involving γ-matrix multiplication on left (right) for γ-matrices of V corresponding to particles a (b), respectively; in the case of the potential (2), for example,

$$V(\underline{q}-\underline{q}')\ \ \Phi(\underline{q}')\ \sim\ \frac{1}{|\underline{q}-\underline{q}'|^2}\ \gamma_\mu\,\Phi(\underline{q}')\,\gamma^\mu$$

In (9), for a vector meson such as the $\psi(3.1)$ or $\psi'(3.68)$ we write $\Phi(\underline{q}) = \psi_i(\underline{q})$, where $i = 1, 2, 3$ is the spin-space suffix. The form of Eq. (9) immediately restricts the possible forms of ψ_i; this is a dynamical restriction[20] extra to those of parity rotational invariance and charge conjugation, and reduces the number of independent scalar functions in ψ_i from six to two so that ψ_i takes the form

$$\psi_i(\underline{q}) = \Lambda_+(\underline{q})\left\{-\beta\alpha_i\,\phi_s(q^2) + \beta\left(q_i\,\underline{\alpha}\cdot\underline{q} - \tfrac{1}{3}\alpha_i q^2\right)\phi_d(q^2)\right\}\Lambda_-(-\underline{q}) \tag{10}$$

In a non-relativistic situation with large m, $\Lambda_+(\underline{q}) \sim (1+\beta)/2$ and $\Lambda_-(\underline{q}) \sim (1-\beta)/2$, and a solution with ϕ_s alone would correspond to pure S-wave, since the quark spin α_i, is unmixed with orbital motion, and a solution with ϕ_d alone would correspond to pure D-wave. In the general case, substituting (10) into (9) gives two coupled integral equations in $\phi_s(q^2)$ and $\phi_d(q^2)$, and the solution is of the form (10) with both ϕ_s and ϕ_d present. In the charmonium case with $m \sim 1.3 - 2.0$ GeV/c^2 and forces of the form (1) and (2) with suitable λ and α_s to give $\psi(3.1)$ and $\psi'(3.68)$, an approximately non-relativistic situation results and the solutions of (9) fall into classes of

i) ϕ_s with a small admixture of ϕ_d ('S-wave solutions') and
ii) ϕ_d with a small admixture of ϕ_s ('D-wave solutions').

Non-relativistically both the 0^{++} and 1^{++} wave functions are pure P-waves, while the 2^{++} wave function is a mixture of P- and F-waves. Correspondingly there is only one independent function in either the 0^{++} or the 1^{++} wave functions, and substitution into (9) of either of these wave functions leads to an integral equation in one function; in the case of the 2^{++} a coupled integral equation in two functions results.

The use of the full Salpeter equation (6) rather than the reduced equation (8) or (9) still implies a restriction on the number of independent functions in the particle wave functions over and above those restrictions imposed by parity and charge conjugation, but the number of independent functions is doubled.

FORM OF THE POTENTIAL

The fundamental characteristics of our potential, except for the $r \to \infty$ behaviour, are conveyed by Eqs. (1) and (2). The precise form used is

$$V(r) = \left[\lambda r - \frac{\alpha_s}{r} \gamma^a_\mu \gamma^{\mu b} - C \right] e^{-\mu r} + C'(1 - e^{-\mu r}) \qquad (11)$$

where C, C′, and μ are constants. The non-relativistic aspect of the potential, obtained by putting $\gamma^a_\mu = \gamma^b_\mu = 1$, is illustrated in the figure. The factor $e^{-\mu r}$ damps the otherwise infinitely rising potential for large r; without such a damping, the integral equations (6), (8), or (9) would not be soluble, at any rate without special limiting procedures. Within the context of infrared confinement Kogut and Susskind[21)] have advocated such a potential as a screening effect of the original infinitely rising potential by the creation of quark-antiquark pairs. The outflow of probability to infinity associated with such a potential is due to the resulting charmed mesons, not bare quarks. We take $\mu = 0.1$ in GeV units corresponding to a range of about 2 fermis; our results are insensitive to making μ smaller -- corresponding to a larger range -- up to about 0.02, at which stage our numerical method begins to lose accuracy

The constant C sets the energy scale and includes all those spin-independent interactions not included in the r-dependent part of the potential. From an operational point of view, without this constant the quark mass might be restricted by the requirements of fitting $\psi(3.1)$ and $\psi'(3.68)$; in fact it turns out that C is rather small. The related constant $C' = V(\infty)$, whose effect is switched on for large r only, plays a formal role, enabling us to solve for all the charmonium energy levels, even the higher ones above 4 GeV/c^2, as discrete bound states.

Our important adjustable parameters are λ, α_s, m (the charmed quark mass), and C.

RESULTS FOR CONSTANT α_s

We solve the integral equations using matrix procedures, which will be described elsewhere. It is necessary either to regularize the potential or to modify the propagator $(E - 2W)^{-1} = (E - 2\sqrt{m^2 + p^2})^{-1}$ for very large momenta, and it is simplest to do the latter. We take as modified propagator $\left[(E - 2\sqrt{m^2 + p^2})(\Lambda^2 + p^2)\right]^{-1}$ with $\Lambda = 6.0$ GeV/c.

One can readily find values of λ, α_s, m, and C, with m in the region 1.5-2.0 GeV/c^2, such that 1^- mesons (predominantly S-wave) are at 3.1 and 3.68 GeV/c^2. The P-wave mesons then occur between 3.1 and 3.68, not in coincidence with the ψ'(3.68) as would be the case for a pure Coulomb potential ($\lambda = 0$). The difficult problem is to get the P-waves at the right positions, with the correct ordering and a specified level splitting, while at the same time maintaining the correct leptonic widths $\Gamma(\psi \to e^+e^-)$, $\Gamma(\psi' \to e^+e^-)$. The difficulty arises as follows. We know that the 0^{++} state is at 3.41 GeV and that the splitting between 0^{++} and next highest P-wave state, either 1^{++} or 2^{++}, is at least 0.09 GeV. As pointed out previously, the level ordering of itself, with 0^{++} lowest, demands a certain minimum ratio of $\alpha_s/\bar{r}_p^2$ to λ (and λ cannot be too small otherwise the P-wave states would be near to 3.68), and the relatively large level splitting increases the desirable magnitude of α_s. At the same time, larger α_s means that the ψ(3.61) and ψ(3.68) have larger wave functions at the origin, r = 0 [$\phi_s\big|_{r=0}$ in terms of Eq. (10) for the 1^- wave functions] leading to larger widths $\Gamma(\psi \to e^+e^-)$, $\Gamma(\psi' \to e^+e^-)$.

For evaluating these widths we use the Weisskopf-van Royen non-relativistic formula, with an extra factor of 3 for colour:

$$\Gamma_{e^+e^-} = \Gamma_{\mu^+\mu^-} = 3\left(\frac{1}{137}\right)^2 \frac{16\pi}{3} \; \frac{4}{9} \left(\psi(\underline{r}=0)/M\right)^2 \tag{12}$$

where $\psi(r)$ is the normalized wave function of the 1^- meson [in our case the Fourier transform of (10)] and M is the meson mass.

We have explored the features of the charmonium system in the space of the parameters (λ, α_s, m, C) and these features appear to be simple and smooth enough for us to build up a good picture by sampling. With our potential, and the charmed quark mass m in the region 2.0 > m > 1.5, it seems that the 0^{++} level is too high and, associatedly, the level splitting is too small. Our best results are in the neighbourhood of

$$(\lambda, \alpha_s, m, C) = (0.15, 0.46, 1.55, 0.53), \quad \text{GeV units.} \tag{13a}$$

These particular parameters give rise to the following mass spectrum:

$$1^- \text{ mesons } (\sim \text{s-wave}): \quad 3.1, \; 3.69, \; 4.07, \; 4.33 \tag{13b}$$

$$1^- \text{ mesons } (\sim \text{d-wave}): \quad 3.78, \; 4.12, \; 4.40 \tag{13c}$$

$$\text{P-wave mesons}: (0^{++}, 1^{++}, 2^{++}) \text{ at } (3.45, 3.505, 3.52) \tag{13d}$$

with leptonic and radiative decay widths of ψ, ψ' at reasonable values.

Since the P-wave splitting is too small, first-order formulae such as (3) - (5) strongly suggest that a lower charmed quark mass would be appropriate. For the charmed quark mass m in the region $1.5 > m > 1.0$ we find energy levels and leptonic and radiative decay widths for parameters in the neighbourhood of

$$(\lambda, \alpha_s, m, C) = (0.14, 0.49, 1.35, 0.1) \tag{14a}$$

These particular parameters (with $C' = 2$) give rise to the following mass spectrum:

$$1^- \text{ mesons } (\sim \text{s-wave}) : \quad 3.1, \; 3.7, \; 4.04, \; 4.26 \tag{14b}$$

$$1^- \text{ mesons } (\sim \text{d-wave}) : \quad 3.77, \; 4.08, \; 4.30 \tag{14c}$$

$$\text{P-wave mesons} : (0^{++}, 1^{++}, 2^{++}) \text{ at } (3.41, 3.50, 3.53) \tag{14d}$$

and the following widths:

$$\Gamma(\psi \to e^+e^-) = 4.3 \text{ keV} \qquad \Gamma(\psi' \to e^+e^-) = 1.2 \text{ keV} \tag{14e}$$

$$\Gamma[\psi' \to (0^{++}, 1^{++}, 2^{++}) + \gamma] = (26, 24, 16) \text{ keV} \tag{14f}$$

$$\Gamma[(0^{++}, 1^{++}, 2^{++}) \to \psi + \gamma] = (348, 747, 1064) \text{ keV} \tag{14g}$$

(The radiative widths in (14f) and (14g) are quoted using the correct experimental factor of k^3 in the dipole formula.)

We note a number of points:

i) That a large $0^{++} - 1^{++}$ splitting with these levels at 3.41 and 3.50, respectively, implies a relatively small $1^{++} - 2^{++}$ splitting -- indeed too small in the case above to agree with the experimental level at 2.55; this is a prediction of the potential used -- we do not have the freedom to vary totally independently the $1^{++} - 2^{++}$ splitting.

ii) The radiative widths of ψ' to the P-wave states are in marked disagreement with one experiment[22] which sets an upper limit of about 10 keV on the width of manochromatic γ-rays from ψ' decay, but are not in marked disagreement with some later experimental results on this width[1,23]. Eichten et al.[24] (who do not, of course, calculate the P-wave splitting) quote a rate $\Gamma(\psi' \to 0^{++} + \gamma) = 36$ keV in their calculation incorporating a charmed meson continuum which affects the ψ' wave function thus reducing the rate. To compare like with like, we should revert to their original calculations[5], which (with the correct γ-ray energy) would give $\Gamma(\psi' \to 0^{++} + \gamma) = 40$ keV, about 50% greater than our above calculation, (14f), of 26 keV.

iii) The value of $\Gamma^{\psi'}_{e^+e^-}$ is somewhat low.

iv) That the constant C has become very small so that the potential, in the region of the wave function, is almost purely $\lambda r - \alpha_s \gamma^a_\mu (1/r) \gamma^{\mu b}$.

v) The value of α_s being 0.49 is twice as much as that given by the use of the asymptotic freedom formula[25] for the ratio $\Gamma(\psi \to \text{leptons})/\Gamma(\psi \to \text{hadrons})$. However, in the calculations of Eqs. (16) we have not used the asymptotic freedom formula for α_s itself, and the use of this gives a different result, as reported below.

vi) The mass of the η_c and $\eta_{c'}$ (0^{-+}) on the above calculations are 3.04 and 3.2, respectively. We do not place any importance on these values because of the possibility of considerable mixing with η and η', and other effects associated with the two-gluon annihilation process of 0^{-+} mesons.

vii) The values of the $\sim$ S-wave and $\sim$ D-wave levels at 4.02 and 4.12 seem a little high to explain the marked dip in R at 4.0 as an interference effect of these resonances. However, we do not take this possible discrepancy too seriously because *inter alia* the effect of charmed meson channels[24] may be important in this region.

RESULTS WITH ASYMPTOTIC FREEDOM FORMULA FOR α_s

In the previous calculation, as in all the charmonium calculations reported so far, we have used a constant value of α_s which may be viewed phenomenologically or, from an asymptotic freedom point of view as some average α_s appropriate to meson in the 3-4 GeV region [which would be less than that appropriate to the $\phi(1016)$, for example][26]. However, the application to potential theory and the appropriate momentum at which α_s should be taken is not straightforward; this is implicitly acknowledged in the standard theory where for r large -- corresponding to small momenta being important -- a breakdown of asymptotic freedom and a linear confining potential λr, entirely different from the asymptotic freedom α_s/r, are envisaged. Presumably this could be reflected in at least some momentum dependence in the α_s of α_s/r.

The formalism of asymptotic freedom gives α_s as a function of the momentum, the formula appropriate to four flavours and three colours being

$$\alpha_s(p_2) = \left[1 - \frac{25}{12\pi}\alpha_s(p_1)\ln(p_1/p_2)^2\right]^{-1}\alpha_s(p_1) \tag{15a}$$

We have illustrated a different use of asymptotic freedom in charmonium calculations by using (15a) in our momentum space integral equation of motion (9) with

$$p_2 = \sqrt{q q'} \;; \quad p_1 = 1 \text{ GeV}/c \;; \alpha_s(p_2) = \alpha_s(p_1) = \tilde{\alpha}_s \text{ for } p_2 < p_1 \tag{15b}$$

This use corresponds to radiative corrections of the vertices only of the Bethe-Salpeter ladder diagram and thus is consistent with the standard Bethe-Salpeter 'ladder diagram only' usage. We regard our calculations as illustrative only because of the use of the particular four-flavour formula (15a) (though indeed it may well be the formula appropriate to the important momenta of the problem) and also because we have not generalized that formula to the one appropriate to two momenta ($\underline{q}$ and $\underline{q}'$) being involved but have instead approximated by putting $p^2 = \sqrt{qq'}$; however, on this latter point our experience shows that the integral equation is relatively insensitive to whether effects appear in $\underline{q}$ or in $\underline{q}'$.

Now our α_s for large momenta (which has a strong influence on wave functions at the origin $\underline{r} = 0$) is smaller than α_s for the moderate momenta which influenced the part of the wave functions responsible for the P-wave splitting. Consequently we hope for just that desirable enhancement of the P-wave splitting while preserving the correct lepton widths of ψ and ψ', which we were previously unable to find for $2.0 > m > 1.5$ GeV/c^2. This indeed happens, and for the following parameters

$$(\lambda, \tilde{\alpha}_s, m, C) = (0.13, 0.77, 1.6, 0.2) \tag{16a}$$

we find the results

$$1^- \text{ mesons } (\sim \text{s-wave}): \quad 3.1, \; 3.69, \; 3.98 \tag{16b}$$

$$1^- \text{ mesons } (\sim \text{d-wave}): \quad 3.77, \; 4.01 \tag{16c}$$

$$\text{P-wave mesons}: \; (0^{++}, 1^{++}, 2^{++}) \text{ at } (3.42, 3.50, 3.54) \tag{16d}$$

$$\Gamma(\psi \to e^+e^-) = 5.2 \text{ keV}, \quad \Gamma(\psi' \to e^+e^-) = 1.2 \text{ keV} \tag{16e}$$

$$\Gamma[\psi' \to (0^{++}, 1^{++}, 2^{++}) + \gamma] = 22, 21, 14 \text{ keV} \tag{16f}$$

$$\Gamma[(0^{++}, 1^{++}, 2^{++}) \to \psi + \gamma] = 120, 258, 367 \text{ keV} \tag{16g}$$

We note the following points which are different from the previous case (which had constant α_s and $m = 1.35$):

i) The ratio of the P-wave splittings $[E(2^{++}) - E(1^{++})]/[E(1^{++}) - E(0^{++})] = 1/2$, in agreement with experiment.

ii) The radiative transitions of $\psi'(3.684)$ are now less, being about the same as those found by Eichten et al.[24] using an explicit charmed meson channel for the 1^+ and 2^+, and considerably less for the 0^{++}.

iii) $\alpha_s(3.1) = 0.36$, a value which is still too large for the asymptotic freedom calculations on $\Gamma(\psi \to \text{hadrons})$, but considerably nearer than the previously quoted value of 0.49.

iv) The 0^{-+} occur at 3.0 and 3.67; the same remarks on the non-significance of the calculation of these levels apply.

Finally, we should remark that neither in the constant α_s nor the variable α_s case have me made an exhaustive search of the parameter space. An exhaustive search (which we could not carry out in any reasonable time) would perhaps reveal parameter values giving results even somewhat closer to the experimental indications. We think that in the present state of the theory of charmonium our present qualitative results on the effect of our new considerations are sufficient.

CONCLUSIONS

From our quoted and unquoted results we conclude the following:

I. i) A potential between quark and antiquark which is basically of the form $1^a \lambda r\, 1^b - \gamma^{\mu a} (\alpha_s/r)\, \gamma^b_\mu - C$ (where C turns out to be small, $\lesssim 0.2$ GeV), with constant α_s, can give the correct J/ψ and ψ' masses, approximately the correct leptonic widths, <u>and</u> the 0^{++} P-wave at 3.41 GeV with the 1^{++} P-wave at 3.50 GeV, in agreement with experiment.

ii) It then <u>predicts</u> the 2^{++} P-wave level to be in the region of 3.53 GeV, the experimental value being 3.55.

iii) The charmed quark mass m must be not more than 1.5 GeV/c^2; the radiative transitions $\psi' \to$ P-wave + γ are calculated to be somewhat wide in the region 15-26 keV.

II. i) With the same potential with a momentum-dependent α_s (corresponding to an asymptotic freedom formula with four flavours and three colours), the three P-wave levels can be found at approximately their correct energies improving on the situation in I. The relatively small $2^{++} - 1^{++}$ splitting is a successful <u>prediction</u>. This contrasts with the conclusions of previous authors[11,12,27] using a somewhat different method and potential, who find too large a ratio $[E(2^{++}) - E(1^{++})]/[E(1^{++}) - E(0^{++})]$.

ii) The charmed quark mass m can now be above 1.5 GeV/c^2; with m = 1.6 the radiative transitions $\psi' \to$ P-wave + γ are in the region 14-22 keV.

III. It is evident that the relative positions of the D-wave levels will provide an important clue to the nature of the potential. Our D-levels are higher than the corresponding S-levels. Linear confining potentials which are of the purely vector type $\gamma_\mu^a \gamma^{\mu b}$ will tend to give D-levels lower than S-levels[28].

It is evident that the experimental spectral information becoming available is capable, as it was hoped, of distinguishing sharply between different potentials, and perhaps of deciding whether any potential description be adequate. As an example of a potential awaiting detailed investigation along the lines of this paper, is notably a Coulomb short-range force [as given by (1), for example] together with a totally spin-independent linear confining force; this will probably give smaller values of α_s than we have found. The relative positions of the D-waves and S-waves can be vital due in these comparisons, and these might be discovered in the 3.9-4.5 GeV region, a though further problems of extra levels might await here.

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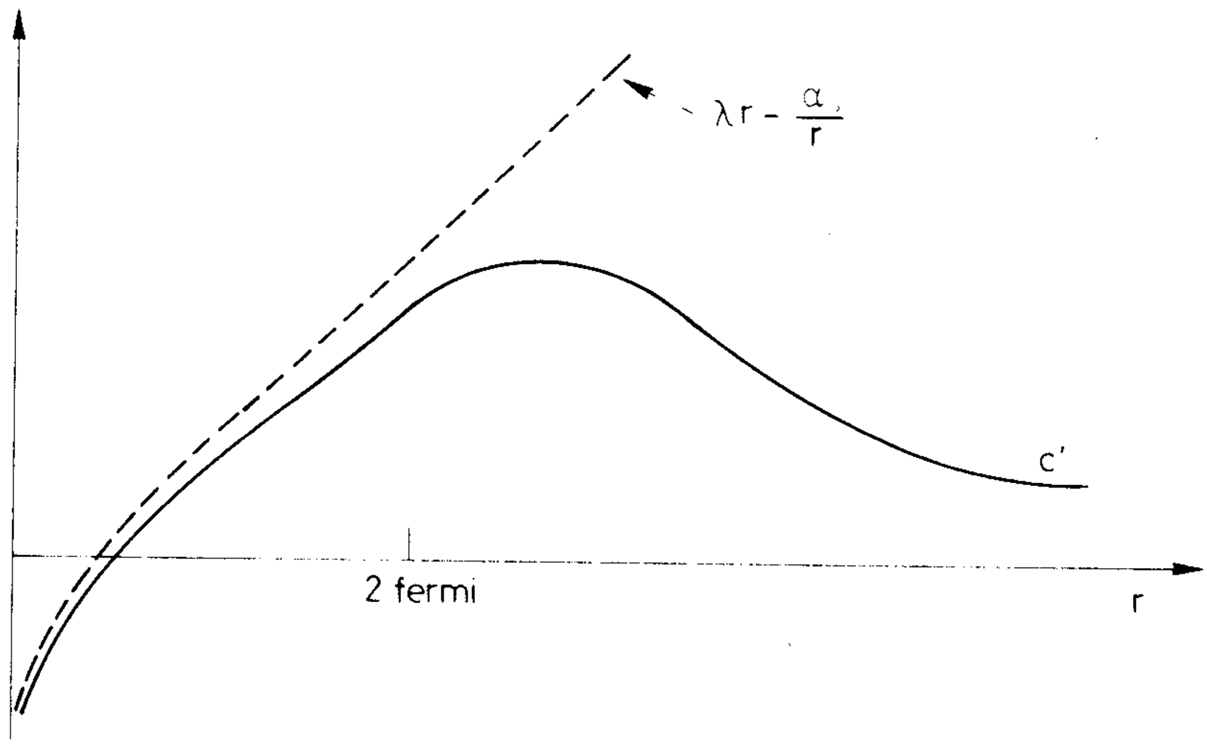

Fig. 1 Plot of the non-relativistic features of V(r), given in Eq. (11) of the text.