Linking component importance to optimisation of preventive maintenance policy[*]

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Abstract. In reliability engineering, time on performing preventive maintenance (PM) on a component in a system may affect system availability if system operation needs stopping for PM. To avoid such an availability reduction, one may adopt the following method: if a component fails, PM is carried out on a number of the other components while the failed component is being repaired. This ensures PM does not take system's operating time. However, this raises a question: Which components should be selected for PM? This paper introduces an importance measure, called *Component Maintenance Priority* (CMP), which is used to select components for PM. The paper then compares the CMP with other importance measures and studies the properties of the CMP. Numerical examples are given to show the validity of the CMP.

Keywords: cost-based component importance, preventive maintenance, Birnbaum importance, criticality importance

1 Introduction

1.1 Motivation

To improve the availability of engineered systems such as production lines and electricity transmission networks is the common pursuit of many firms. To achieve a high availability level, preventive maintenance may be used. However, performing preventive maintenance (PM) on a

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29   component in a system takes time and can therefore reduce the availability of the system if system

30   operation needs stopping for the PM. To avoid such a dilemma, one may adopt the following method: if a

31   component in the system fails, PM is carried out on a number of the other components while the failed

32   component is being repaired. However, this raises another question: Which components should be

33   selected for PM?

34   Reliability importance measures are developed to prioritise the components of a system in the light

35   of a given criterion and can offer guidance to improve system reliability/availability, reduce maintenance

36   cost, and improve system safety. For example, the Birnbaum importance is the partial derivative of the

37   system reliability with respect to the reliability of an individual component and measures the effect of the

38   reliability improvement of individual components on the improvement of the system reliability [1]. In the

39   reliability literature, many importance measures have been developed for various purposes, see [2] for an

40   excellent paper that reviews recent advances on importance measures. However, few importance

41   measures can be used to select such components for the above-mentioned purpose.

42   This paper extends the Birnbaum importance measure to an importance measure, called CMP

43   (Component Maintenance Priority), with which components can be selected for PM and the number of

44   components for PM may further be optimised.

1.2 Related work

46   Various component importance measures for binary coherent systems and state importance

47   measures for multi-state systems have been introduced in the literature. For example, the Fussell and

48   Vesely importance of a component is the probability that at least one minimal cut set containing the

49   components has failed, given that the system has failed; the criticality importance of a component is the

50   probability that the component has caused system failure, when the system is failed. The reader is

51   referred to the monograph by Kuo and Zhu [3] for detailed accounts of the theory and surveys of

52   commonly used importance measures.

53   The Birnbaum importance is probably the first importance measure introduced in the literature, for

54   the purpose of reliability improvement [1]. The Birnbaum importance measures the extent of the change

55   in the reliability of the system resulted from a change in the reliability of a component. It has been

56   extended to many variants considering different scenarios and applications, for example, cost-based

57   importance measures [4] that considers the lifecycle cost of maintaining each component in a system.

58   Other variants include performance based importance measures [5], joint component importance [6-8],

59   and joint component importance for multistate systems [9].

60   Recent development in importance measures also include: importance measure for systems with

61   degrading components [10], importance measure that estimates the effect of a component residing at

62   certain states on the performance of the entire multi-state systems [11], importance measure for

63   components when the system may be reconfigured [12], among many others ([13-15], for example).

64   Importance measures in the literature may be inter-related. Borgonovo shows that the Fussell-

65   Vesely importance, the criticality importance, the Birnbaum importance, the risk achievement worth and

66   the differential importance measure (DIM) are linked by simple relations [16]. Vaurio shows that DIM

67   and the criticality- importance yield the same ranking in realistic examples [17,18]. Furthermore,

68   according to Birnbaum [1], importance measures can be categorised into three classes according to the

69   knowledge needed for determining them: *structural importance measures*, *reliability importance*

70   *measures*, and *lifetime importance measures.* For example, Fig. 1 shows the process of available knowledge

71   and the corresponding importance measures that can be used.

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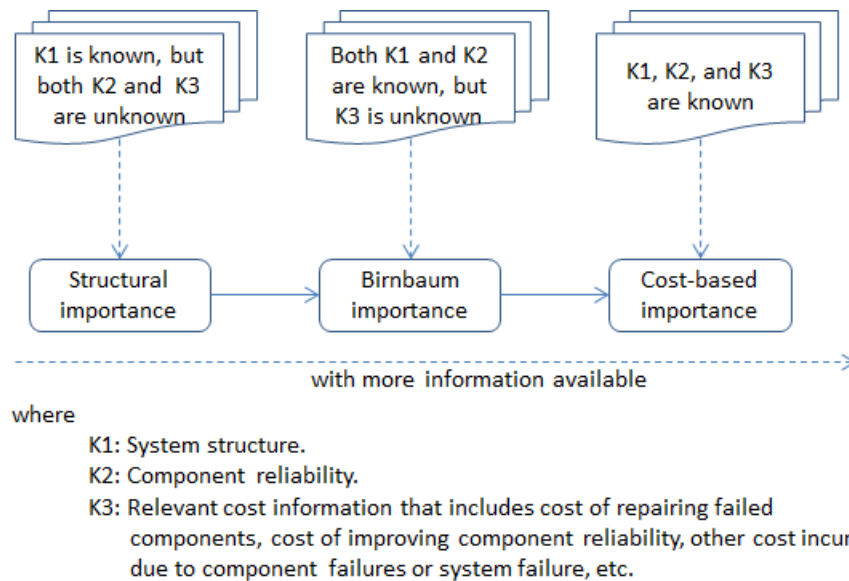
Fig. 1. Available knowledge and importance measures that can be applied.

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76   A common drawback of the Birnbaum importance measure and its variants is that they rank only

77   individual components and are not directly applicable to groups of components. The differential

78   importance measure (DIM), introduced by Borgonovo and Apostolakis in [19], overcomes this drawback

79   by defining the importance of a group of components using a first-order Taylor expansion, but it does not

80   account for the effects of interactions among components. Zio and Podofillini then extended the DIM

81   including both the first order and the second order Taylor expansion, which has a merit that is account of

82   the interactions of pairs of components [20].

83   The existing reliability literature, however, lacks an importance measure for solving the following

84   question: suppose that a component is failed and during the time when the component is being repaired,

85   other m components in the system can be selected for PM. This raises an interesting question: which m

86   components should be selected? This paper develops a new importance measure, the CMP, for answering

87   this question. The CMP can also be applied to schedule PM policy. Conventionally, optimisation of PM has

88   been centred on seeking the optimal intervals between consecutive PM activities. This paper, however,

89   optimises the number of maintenance personnel needed to minimise the expected cost in a given time

90 horizon. As such, in addition to its novelty of introducing a new importance measure, the paper also

91 creates the novelty of proposing a new method of optimising PM.

1.3 Summary

93   This rest of this paper is structured as following. Section 2 lists the notation and assumptions, and

94 discusses the justification of the assumptions. Section 3 introduces the CMP and discusses its relationship

95 with some existing importance measures. Section 4 gives upper and lower bounds of the expected

96 number of PM under two maintenance policies. Section 5 discusses issues relating to the CMP. Section 6

97 gives numerical examples. Section 7 concludes the findings of this paper.

2   Notation and Assumptions

99   The following notation and assumptions are used.

2.1 Notation

101   This paper uses the notation shown in Table 1.

102

103   Table 1. Notation.

$1_i\ (0_i)$	Component c_i is working (failed)
$\phi(\boldsymbol{p}(t))$	System reliability as a function of $\boldsymbol{p}(t)$
n	Number of components in a system
m	Number of components that can be preventively maintained simultaneously while a repair is being conducted on a failed component
$\boldsymbol{p}(t)$	$(p_1(t), \ldots, p_n(t))$
$p_i(t)$	Reliability of component c_i
x_i	Indicator: $x_i = 1$ if c_i is working, $x_i = 0$ otherwise
$\boldsymbol{X}_i$	$(x_1, x_2, \ldots, x_{i-1}, \cdot_i, x_{i+1} \ldots, x_n)$
$(x_i, \boldsymbol{p}_i(t))$	$(p_1(t), p_2(t), \ldots, p_{i-1}(t), x_i, p_{i+1}(t), \ldots, p_n(t))$
$[x]$	nearest integer number smaller than x
$\boldsymbol{1}_{ij}$	$(1_1, 1_2, \ldots, 1_{i-1}, 0_i, 1_{i+1}, \ldots, 1_{j-1}, 0_j, 1_{j+1}, \ldots 1_n)$
$F_j^{(1)}(t)$	$= P(\min\{X_{j_1}, X_{j_2}, \ldots, X_{j_m}\} < t)$, X_{j_k} is the lifetime of component j_k before the first PM
$f_j^{(1)}(t)$	$= \dfrac{\partial F_j^{(1)}(t)}{\partial t}$
T_j	A pre-specifed time length that is used in Policy B
$G_j^{(1)}(t)$	$= G_j^{(1)}(t) = P(\max\{\min\{X_{j_1}, X_{j_2}, \ldots, X_{j_m}\}, T_j\} < t)$
$g_j^{(1)}(t)$	$= \dfrac{\partial G_j^{(1)}(t)}{\partial t}$
$\mu_{j_k}(\cdot)$	hazard functions of component j_k before the first PM

2.2 Assumptions

105 A1. The system considered in this paper is a coherent system, which implies: each component is relevant,

106   and the structure function is increasing (non-decreasing) if the number of components increases.

107   A2. Once a component is failed, a certain symptom immediately appears and can be noticed. The failed

108   component can then be located.

109   A3. There are two types of PM: PM during system's downtime and PM during system's uptime, which are

110   denoted by PM_D and PM_U, respectively.

111   A4. Performing a PM (either PM_D or PM_U) on a component requires that the component stops working.

112   A5. A PM (either PM_D or PM_U) on a component can only be triggered when another component has failed.

113   A6. Component failures can have two different situations.

114   (A) When a critical component has failed, the system fails. Then the failed component is repaired. In

115   the meantime, PM_D is performed on other m selected components.

116   (B) When a non-critical component has failed, the system does not fail. Then the failed component is

117   repaired. In the meantime, PM_U is performed on other m selected components. The repair and the

118   PM_U will not affect the system operating.

119   A7. All of the components in the system are statistically independent.

120   **Remark 1**. From Assumption A3, the PM_U policy is: PM is performed when the system is still

121   working. This implies that a binary system (for example, a parallel system) under the PM_U policy may

122   never fail. Take the system in Fig. 2 as an example, under Assumptions A3, A4, and A5, the subsystem

123   constituted by components 5, 6, and 7 may never fail. This is because: once one of the three components

124   fails, the failed component will be immediately repaired while the subsystem is working. This will ensure

125   that the subsystem will never fail.

126   However, if we use the PM_U policy on multistate systems, performing PM_U can improve the

127   performance of the system. For example, for a water pumping station that is composed of three pumps, if

128   pump 1 degrades from a higher state to a lower one (or *is failed* as termed in this paper), its performance,

129   which is the amount of water a pump can pump, degrades and the pump may need repairing. The PM_U

130   that is performed on pump 2 can improve the performance of the pump, for example.

131   In essence, a jump from a higher state to a lower one in a multistate system (component) is the same

132   as the failure of a binary system (component). As such, in what follows, we simply focus our discussion on

133   binary system (component) cases.

3   A new importance measure: component maintenance priority

135   The main effort of this paper focuses on the development and analysis of a new importance measure

136   for binary systems. General cases are also discussed in this section.

3.1 Component maintenance priority

138   We first recall an importance measure with a similar definition as what we will define. This

139   importance measure is the conditional marginal reliability importance, defined in [8]. Its definition is

140   given below.

141   **Definition 1 (Conditional Marginal Reliability Importance (CMRI)**) [8]. The CMRI of component

142   c_j, given that component c_i is working or failed, is defined by

$$I_{j|i}^C(t) = \frac{\partial \phi(z_i, \boldsymbol{p}_i(t))}{\partial p_j(t)}, \tag{1}$$

143   where $z_i = 1$(or 0) means that the component c_i is working (or failed).

144   The authors of [8] claim that the CMRI can be used to decide to which components we should pay

145   more care in terms of maintenance.

146   **Remark 2.** Let's look at two typical systems: a series system and a parallel system.

147   • For a series system, we have the following two scenarios.

148   B1. If component c_i is working, or $z_i = 1$, then the system is working. In this case, no PM can be

149   performed on any component. This is because: according to Assumptions A3, A4 and A5,

150   neither PM$_D$ nor PM$_U$ can be performed. As PM$_D$ is only performed when the system is not

151   working, whereas PM$_U$ is performed when component c_i is failed. Hence, there is no need to

152   use $I_{j|i}^C(t)$ to rank the components.

153   B2. If $z_i = 0$, then $\phi(z_i, \boldsymbol{p}_i(t)) = 0$. $I_{j|i}^C(t) = 0$ for any $j \neq i$. That is, $I_{j|i}^C(t)$ cannot be used to rank

154   the components as they all are zeros.

155   • For a parallel system, similar to the series system, we have the following two scenarios.

156   B3. If $z_i = 1$, then $\phi(z_i, \boldsymbol{p}_i(t)) = 1$. $I_{j|i}^C(t) = 0$ for any $j \neq i$, that is, $I_{j|i}^C(t)$ cannot be used to rank

157   the components as they all are zeros.

158   B4. If $z_i = 0$, or component c_i is failed, according to Assumptions A3, A4 and A5, PM$_U$ can be

159   performed on unfailed components only when the number of the components is larger than 2.

160   This is because PM$_U$ is only performed on a component when the component stops working

161   and the system is working. When a 2-component parallel system includes one failed

162   component, the unfailed component must be working and cannot be stopped for PM$_U$.

163   From the above analysis, one can see that the measure $I_{j|i}^C(t)$ can only be used for the scenarios when

164   the number of components in a parallel system is larger than 2.

165   3.1.1   Component Maintenance Priority

166   The above analysis shows that the CMRI cannot be used to rank component importance under

167   Assumptions A3, A4, and A5. This necessitates introducing a new definition, which is given in the

168   following.

169   **Definition 2 (Component Maintenance Priority (CMP))**. If component i has failed, then under

170   Assumptions A1—A5, the CMP of component j is defined by

$$I_{j|i}^M(t) = H_{j|i} \frac{\partial \phi(\lambda_i, \boldsymbol{p}_i(t))}{\partial p_j(t)}, \tag{2}$$

171   where

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- $H_{j|i} = \begin{cases} 1 & \text{if } \phi(1_1, \ldots, 1_{i-1}, 0_i, 1_{i+1}, \ldots, 1_n) = 0 \\ \phi(0_i, 0_j, \mathbf{1}_{ij}) & \text{if } \phi(1_1, \ldots, 1_{i-1}, 0_i, 1_{i+1}, \ldots, 1_n) = 1 \end{cases}$, $(0_i, 0_{j,} \ldots, \mathbf{1}_{i,j})$ represents that

173
   components i, j stop working and all of the other components are working; and

174
- $\lambda_i = \chi\{\phi(1_1, 1_2, \ldots, 1_{i-1}, 0_i, 1_{i+1}, \ldots, 1_n) = 0\}$, $\chi\{.\}$ is an indicator function.

175 In Eq. (2), λ_i ensures that $\phi(\lambda_i, \boldsymbol{p}_i(t))$ is not constant, no matter whether component i is critical or

176 noncritical. This avoids the problems such as B2 and B3 listed in Remark 2. $H_{j|i}$ ensures that critical

177 components will not be selected for PM, given that component i is non-critical.

178 The CMP $I_{j|i}^M(t)$ can be used to suggest which components may be selected for PM so that the

179 reliability of the system can be maximally improved, given that component i has failed and repair needs

180 performing on it. This is a form of positive dependence that gives a downtime opportunity: component

181 failures can often be regarded as opportunities for PM of non-failed components. The positive

182 dependence has been discussed in maintenance optimisation for multi-component systems, see [21,22]

183 for example.

184 Below we give an example to show how the CMP works.

185 **Example 1.** Assume a system is structured as Fig. 2 and the components in the system have equal

186 reliability. That is, $p_1(t) = p_2(t) = \cdots = p_8(t) = R(t)$. Then the reliability of the system is

187 $\phi(\boldsymbol{p}(t)) = p_1(t)p_4(t)\left(1 - \prod_{i=2}^{3}(1 - p_i(t))\right)\left(1 - \prod_{i=5}^{8}(1 - p_i(t))\right).$

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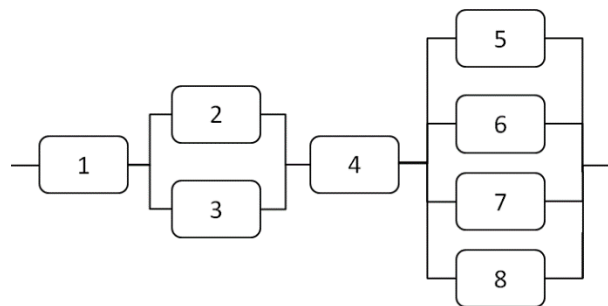
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190 Fig. 2. An example.

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192 Assume that a two-member maintenance team takes care of the system. This implies that the team

193 can repair a failed component, meanwhile carry out PM on another component while the failed

194 component is being repaired. We have the following straightforward analyses.

195 (a) if component 1 is failed, the system stops working. Then, based on the Birnbaum importance, PM

196   may be conducted on component 4.

197 (b) if component 2 fails while the system is working, then one of components 5, 6, 7, and 8 can be

198   selected for PM.

199 Based on Definition 2, we have $H_{j_1|1} = 1$ and $H_{j_2|2} = \phi(0_2, 0_{j_2}, \mathbf{1}_{2j_2})$, where $i, j \in \{1, 2, \ldots, 8\}$,

200 $j_1 \in \{2, 3, \ldots, 8\}$, and $j_2 \in \{1, 3, 4, \ldots, 8\}$.

7

(A) According to Definition 2, $I_{j_1|1}^M(t) = \frac{\partial \phi(1, p_2(t), \ldots, p_8(t))}{\partial p_{j_1}(t)}$. Then we have $I_{2|1}^M(t) = I_{3|1}^M(t) = R(t)(1 -$

$R(t))(1 - (1 - R(t))^4)$, $I_{4|1}^M(t) = (1 - (1 - R(t))^2)(1 - (1 - R(t))^4)$, and $I_{5|1}^M(t) = I_{6|1}^M(t) =$

$I_{7|1}^M(t) = I_{8|1}^M(t) = R(t)(1 - (1 - R(t))^3)(1 - R(t))^2$. It can easily be proved that $I_{4|1}^M(t) \geq I_{j|1}^M(t)$ for

$j = 2,3,5,6,7,8$. That is, based from Definition 2, if component 1 is failed, component 4 may be

selected for PM.

(B) According to Definition 2, $I_{j_2|2}^M(t) = H_{j_2|2} \frac{\partial h(p_1(t), \lambda_2, p_3(t), \ldots, p_8(t))}{\partial p_j(t)}$. Then we have $I_{1|2}^M(t) = I_{3|2}^M(t) =$

$I_{4|2}^M(t) = 0$, $I_{5|2}^M(t) = I_{6|2}^M(t) = I_{7|2}^M(t) = I_{8|2}^M(t) = (R(t))^3(1 - R(t))^3 > 0$. That is, if component 2

fails, the system is working. Then one of components 5, 6, 7 and 8 can be selected for PM.

The derived results (A) and (B) using Definition 2 agree with the analysed results (a) and (b).

Fig. 3. A four-component series system.

From the above example, it can also be found that $I_{j|i}^M$ may be zero, which differs from most existing

importance measures such as the Birnbaum importance measure, the joint importance measure, the

conditional importance measures, etc, which are always positive.

It can be found that $I_{j|i}^M(t) \neq I_{i|j}^M(t)$. The ordering ranked by $I_{j|i}^M(t)$ is apparently different from that

by $I_{i|j}^M(t)$. For example, assume a system is consisted of four components shown in Fig. 3 and their

reliabilities are $p_1(t), p_2(t), p_3(t), p_4(t)$ with $p_1(t) \geq p_2(t) \geq p_3(t) \geq p_4(t)$. Then, $I_{4|1}^M(t) = p_2(t)p_3(t)$

$I_{4|1}^M(t) \geq I_{3|1}^M(t) \geq I_{2|1}^M(t)$ and $I_{3|4}^M(t) \geq I_{2|4}^M(t) \geq I_{1|4}^M(t)$.

The relationships with other importance measures are discussed below.

- *Relationship with the Birnbaum importance*. The Birnbaum importance of a component is always
  positive. Since $I_{j|i}^M$, which is the CMP of component j given that component i has failed, may be zero,
  $I_{j|i}^M$ may be smaller than the Birnbaum importance of component j.

- *Relationship with the joint component importance.* The joint reliability importance, which is defined
  as the joint reliability importance as $I_{i,j}(t) = \frac{\partial^2 \phi(\boldsymbol{p}(t))}{\partial p_i(t) \partial p_j(t)}$ and is a measure of how 2 components in a
  system interact in contributing to the system reliability, as $I_{i,j}(t) = I_{j,i}(t)$. For special systems, we
  have the following results.

  (a) For series systems, we have $I_{j,i}(t) = I_{j|i}^M(t)$.

  (b) For parallel systems, If $n > 2$, we have $I_{j,i}(t) = -I_{j|i}^M(t)$; if $n = 2$, then $I_{j,i}(t) \neq I_{j|i}^M(t)$. This is
      because $I_{j|i}^M(t) = 0$ if $n = 2$.

232 • *Relationship with the conditional component importance.* For both series and parallel systems, we

233 have the relationship: $I_{j|i}^{M}(t) = H_{j|i}\big(I_{j|i}^{C}(t) + I_{j|i}^{C}(t)\big)$ for $n > 2$. If $n = 2$, $I_{j|i}^{M}(t) \neq I_{j|i}^{C}(t)$, and

234 $I_{j|i}^{M}(t) \neq I_{j|i}^{C}(t)$.

235 3.1.2 The expected number of PM and the number of components for PM

236 From Definition 2, an interesting concern is the expected number of PM of each component, based on

237 which one may design the system. For example, one may assume that the reliabilities of the components

238 in a system are equal, then calculate component's CMP. He can then allocate the real components with the

239 following rule: the component with the lowest reliability will be placed in the position with the largest

240 number of PM. Then the component will be preventively maintained more often than others.

241 $I_{j|i}^{M}(t)=0$ implies that component j is not selected for PM if component i fails. Hence, in a system, the

242 maximum number N_j of PM conducted on component j is given by

$$N_j = \sum_{i=1}^{n} \chi\{I_{j|i}^{M}(t) > 0\} \tag{3}$$

243 where we mean by the maximum number N_j, we have considered the fact that even if a PM is allowed, it is

244 not necessarily always done because of economic or manpower constraints.

245 Another interesting question is the number of components that can be preventively maintained

246 while a failed component is being repaired. There are two situations as following.

247 (A) If a critical component fails, then the system stops working. While the component is being repaired,

248 the rest $n - 1$ components can be maintained simultaneously.

249 (B) However, if a non-critical component fails, the system is still working. To keep the system working,

250 the number of other components that can be maintained is limited. The minimum number of

251 components to ensure the system working is n_c (where n_c is the number of components in the

252 shortest path set in the system), which implies that the rest $n - n_c - 1$ components can be

253 maintained simultaneously.

254 Hence, we may use the following remark, Remark 3, to summarise the above discussion.

255 **Remark 3.** For a n-component system, while the failed component is being repaired, the maximum

256 number m of components that can be preventively maintained simultaneously equals $n - 1$ or $n - n_c - 1$.

257 **Example 2**. In the system shown in Fig. 2, the shortest path set includes at least 4 components, i.e.,

258 component 1, component 4, one from components 2 and 3, and one from components 5, 6, 7, and 8. That

259 is, $n = 8$, and $n_c = 4$. Hence, the maximum number of components that can be maintained simultaneously

260 is $n - n_c - 1 = 3$ while a failed component is being repaired.

3.1.3 Importance for a group of components

Recall we mentioned that "A drawback of the Birnbaum importance measure and its variants is that it ranks only individual components but they are not directly applicable to groups of components" in Section 1.2. The differential importance measure (DIM), introduced by Borgonovo and Apostolakis in [19], overcome this drawback [19]. Below is the definition of the differential importance measure (DIM) of a given set of parameters, introduced in [19].

$$DIM\big(p_{j_1}(t), \cdots, p_{j_m}(t)\big) = \frac{\sum_{k=1}^{m} \frac{\partial \phi(\boldsymbol{p}(t))}{\partial p_{j_k}(t)} dp_{j_k}(t)}{d\phi(\boldsymbol{p}(t))}. \tag{4}$$

The DIM can be regarded as the fraction of the total change in system reliability that is due to a change in parameter components' reliabilities.

Similar to the other variants of the Birnbaum importance measure, the CMP ranks only individual components. The following quantity $I_{j_1,j_2,\cdots,j_m|i}^{M}(t)$ defines the improvement on the system if one improves the reliabilities of components $j_1, j_2, \cdots, j_m$ with amount $\Delta_{j_1}, \cdots, \Delta_{j_m}$, respectively. As the denominator in the right hand side of Eq (4) is constant, one can simply compare the enumerators of the DIM. As such, we can derive a similar result the following.

If component i is failed, then the component maintenance priority of a given set of reliability improvements $\big(\Delta_{j_1}, \cdots, \Delta_{j_m}\big)$ on m components $j_1, j_2, \cdots, j_m$ is given by

$$I_{j_1,j_2,\cdots,j_m|i}^{M}(t) = H_{j_1|i} \frac{\partial \phi(\lambda_i, \boldsymbol{p}_i(t))}{\partial p_{j_1}(t)} \Delta_{j_1} + \sum_{k=2}^{m} H_{j_k|i,j_1,j_2,\cdots,j_{k-1}} \frac{\partial \phi(\lambda_i, \boldsymbol{p}_i(t))}{\partial p_{j_k}(t)} \Delta_{j_k}, \tag{5}$$

where $H_{j_k|i,j_1,j_2,\cdots,j_{k-1}} = \begin{cases} 1 & \text{if } \phi(0_i, \mathbf{1}_i) = 0 \\ \phi(0_i, 0_{j_1}, \ldots, 0_{j_{k-1}}, \mathbf{1}_{i,j_1,j_2,\cdots,j_{k-1}}) & \text{if } \phi(0_i, \mathbf{1}_i) = 1 \end{cases}$, and

$\big(0_i, 0_{j_1}, \ldots, 0_{j_{k-1}}, \mathbf{1}_{i,j_1,j_2,\cdots,j_{k-1}}\big)$ represents that components $i, j_1, j_2, \cdots, j_{k-1}$ stop working and all of the other components are working.

3.2 Dynamic scenarios

The content in the preceding section, Section 3.1, does not consider the fact that reliability is a function of time.

The CMP is introduced for ranking maintenance priorities of the components of a system at a time when a component is failed. At a given time point, component reliabilities can be obviously regarded as constant. From a lifecycle perspective, however, as the components in the system can age and deteriorate, component reliabilities are time-dependent. From this regard, the rankings resulted from the CMP change over time. For example, in the system in Fig 2, at a time point, the reliability of component 5 may become larger than that of component 6, and consequently, their rankings by the CMP can change. This time-dependence property can cause a difficulty in estimating the expected number of PM needed in a period and further cause a difficulty in estimating the expected lifetime cost. However, estimating the upper and

290   lower bounds of the expected number of failures can be done, but it depends on maintenance policies

291   taken, as shown in Eqs (6) and (7) in Section 4.

4   Linking with maintenance policies

293   Once a component is failed, it is repaired. In the meantime, a given number of components are

294   selected for PM. A natural question posed here is: in case two components are failed within a short period

295   and both failures trigger PM on a component, will two PM be performed on the same component within a

296   short period? Such a scenario should be avoided because it is unnecessary to perform two PM within a

297   short period from a cost-effectiveness perspective. This leads to the following two possible maintenance

298   policies: one considers time since last PM and one does not.

299   Eq. (3) gives the expected number of PM for a given component at a time point. Below, we consider

300   the expected number of PM with a given time period $(0, T)$.

301   In this section, we make the following two assumptions.

302   A8. Repair on failed components are minimal repair, that is, the repair will bring the component back to

303   the status just before it failed. PM effect is imperfect, that is, a PM activity will bring the maintained

304   component to a status between as good as new and the time before the component was maintained.

305   A9. Time on PMU or PMU is negligible.

306   If a component, say, component i, fails, it will be repaired immediately. In the meantime, other m

307   components are selected for PM. The selection criterion differs between maintenance policies A and B.

308   **Maintenance Policy A**. The selection criterion is based on the component maintenance priority

309   $I_{j|i}^M(t)$, as defined in Definition 2. That is, m components, $j_1, \ldots, j_m$, with larger $I_{j|i}^M(t)$ are selected.

310   **Maintenance Policy B**. Components are selected with two steps: all components in the system are

311   ranked according to $I_{j|i}^M(t)$ (with $j = 1, \ldots, i-1, i+1, \ldots, n$); then m components, $j_1, \ldots, j_m$, with the largest

312   $I_{j|i}^M(t)$ values are selected. If the calendar age of a selected component since its last PM is older than a pre-

313   specified value, T_{j_k} say (for $k = 1, \ldots, m$), then a PM will be conducted on it. Otherwise, no PM will be

314   conducted on those with ages younger than the pre-specified values.

315   For a given period $(0, T)$, the lower and upper bounds of the expected number of PM on a set of

316   components $\{j_1, \ldots, j_m\}$ under Policy A and Policy B are given in Eqs. (6) and (7). The set can be all of the

317   components in a system or a subset of components in a system.

318   In the following, we give the expected number of PM within $(0, T)$ under the above two maintenance

319   policies.

4.1 Bounds of the expected number of PM under maintenance policy A

321   Denote N_j^A by the total expected number of PM on components $\{j_1, \ldots, j_m\}$ under maintenance policy

322   A within time period $(0, T)$. Suppose the failure of a component among a set of components $\{J_1, J_2, \ldots, J_M\}$

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323 may trigger PM on a subset components in components $\{j_1, \ldots, j_m\}$. Let $\mu_{J_k}(\cdot)$ be the hazard functions of

324 component J_k before the first PM is conducted on the component.

325 For two identical items in which one is preventively maintained and one is not, the item with PM

326 should have fewer failures than the one without PM. The expected number of failures of component J_k is

327 $\int_0^T \mu_{J_k}(\tau)d\tau$ if it is not preventively maintained and minimal repair is conducted upon failures during the

328 time interval $(0, T)$. If the failure of a component among components $\{J_1, J_2, \ldots, J_M\}$ triggers PM on a subset

329 components of the m components $\{j_1, \ldots, j_m\}$, the maximum total expected number of PM is

330 $m \sum_{k=1}^{M} \int_0^T \mu_{J_k}(\tau)d\tau$. Hence, we have $N_J^A \leq m \sum_{k=1}^{M} \int_0^T \mu_{J_k}(\tau)d\tau$.

331 The time to the first failure among the set of components $J_1, J_2, \cdots, J_m$ is $\min\{X_{J_1}, X_{J_2}, \ldots, X_{J_m}\}$. Let

332 $f_J^{(1)}(t) = \frac{\partial F_J^{(1)}(t)}{\partial t}$, $F_J^{(1)}(t) = P(\min\{X_{J_1}, X_{J_2}, \ldots, X_{J_M}\} < t)$, where X_{J_k} is the time-to-first-failure of

333 component J_k. Since PM on components $\{j_1, \ldots, j_m\}$ are conducted only if one of the components in

334 $\{X_{J_1}, X_{J_2}, \ldots, X_{J_m}\}$ fails and the probability that the first failure occurs is $F_J^{(1)}(t)$, the lower boundary of N_J^A

335 will be $m_j F_J^{(1)}(t)$. Hence, $m_j F_J^{(1)}(t) \leq N_J^A$, where $m_j (< m)$ is the minimum number of components that

336 can be simultaneously maintained.

337 Hence, if maintenance policy A is applied and PM takes effect, then the expected number N_J^A of PM of

338 a set of components $\{j_1, \ldots, j_m\}$ within time interval $(0, T)$, has bounds given in the following.

$$m_j F_J^{(1)}(t) \leq N_J^A \leq m \sum_{k=1}^{M} \int_0^T \mu_{J_k}(t)dt. \tag{6}$$

4.2 Bounds of the number of PM under maintenance policy B

340 Denote N_J^B as the expected number of PM on components $\{j_1, \ldots, j_m\}$ under maintenance policy B

341 within time period $(0, T)$.

342 If maintenance policy B is applied, then the maximum expected number of PM on component j_k is

343 $\left[\frac{T}{T_{j_k}}\right]$, where T_{j_k} is the pre-specified age for PM and $[t]$ as the nearest integer number larger than t. The

344 maximum expected number of PM on the set of components $\{j_1, \ldots, j_m\}$ is not greater than $\sum_{k=1}^{m}\left[\frac{T}{T_{j_k}}\right]$.

345 Following the discussion in Section 4.1, we denote $T_J = \min\{T_{J_1}, T_{J_2}, \ldots, T_{J_m}\}$. If $\min\{X_{J_1}, X_{J_2}, \ldots, X_{J_M}\} <$

346 T_J, no PM will be conducted. Hence, the probability that the first PM will be conducted within time period

347 $(0, T)$ is $P\big(T_j < \min\{X_{j_1}, X_{j_2}, \ldots, X_{j_m}\} < T\big) = P\big(\min\{X_{j_1}, X_{j_2}, \ldots, X_{j_m}\} < T\big) - P\big(\min\{X_{j_1}, X_{j_2}, \ldots, X_{j_m}\} <$

348 $T_j\big) = F_J^{(1)}(T) - F_J^{(1)}(T_j)$. The optimum scenario is that no failure to occur since the first PM. $\Big(F_J^{(1)}(T) -$

349 $F_J^{(1)}(T_j)\Big) m_j \leq N_J^B$, where $m_j (< m)$ is the minimum number of the components that can be conducted on

350 the set of components $\{j_1, \ldots, j_m\}$.

12

351   Based on the above discussion, if maintenance policy B is applied, then, N_j^B, the expected number of

352   PM of a set of components $\{j_1, \ldots, j_m\}$ within time interval $(0, T)$, has bounds given in the following.

$$\left(F_j^{(1)}(T) - F_j^{(1)}(T_j)\right)m_j \leq N_j^B \leq \sum_{k=1}^{m}\left[\frac{T}{T_{j_k}}\right]. \tag{7}$$

5 Discussion

354   **Optimisation of maintenance policies**. Conventionally, optimisation of PM has been centred on

355   seeking the optimal intervals between consecutive PM activities. From the above discussion, however, it

356   can be seen that the optimal number of components that may be preventively maintained can be sought

357   to minimise the expected cost in a given time horizon.

358   **Maintenance time**. If time of maintenance is considered, then $I_{j|i}^m(t) = \chi\{T_i^C \geq T_j^P\}H_{j|i}\frac{\partial\phi(\lambda_i, \boldsymbol{p}_i(t))}{\partial p_j(t)}$

359   may be used, where T_i^C is the repair time on the failed component i and T_j^P is the time of PM on

360   component j. As $T_i^C \geq T_j^P$ means that the time of PM is shorter than that of repairing the failed component

361   i, $\chi\{T_i^C \geq T_j^P\}$ ensures that only those components with shorter PM time will be selected. Normally, the

362   condition $T_i^C \geq T_j^P$ can be easily satisfied as repair (or corrective maintenance) involves more tasks such

363   as fault diagnosis, fault location and fault removal, whereas PM is pre-scheduled and it is conducted by

364   following a pre-specified procedures. Of course, in case other scenarios on repair time are considered,

365   one can easily amend $\chi\{T_i^C \geq T_j^P\}$ to fit for purpose.

366   **Reliability-based, cost-based, or geography-based importance measures.** This paper extends

367   the Birnbaum importance measure to a measure, maintenance priority measure, which is based on

368   system reliability. It is obvious that other criteria can also be applied, for example, system reliability may

369   be replaced with the lifecycle cost or a function associating with geography convenience. For geography-

370   based importance measures, if a component fails, one may choose some other components that are

371   geographically easy to approach to be maintained. For example, in case of the offshore wind mills, if a

372   component fails, then other components close to the failed one may also inspected and maintained.

373   **No symptom appears upon failures**. If no symptom appears upon failure, failure can be detected

374   only when a critical component has failed or a cut set has failed. In this case, maintenance including repair

375   and PM are conducted while the system is not being operated. As such, In this case, $H_{j|i}$ in Definition 2 can

376   be ignored as it is used to ensure that PM does not stop system working. One may therefore consider

377   using the following measure

$$I_{j|i}^{M'}(t) = \frac{\partial\phi(\lambda_i, \boldsymbol{p}_i(t))}{\partial p_j(t)}, \tag{8}$$

378   to rank the importance.

13

6 A numerical example

The above sections discuss maintenance policies A and B. In the following, for the sake of simplicity, we use maintenance policy A as an example.

We consider the system shown in Fig. 2. Assume $p_k(t) = \exp\left\{-\left(\frac{t}{\alpha_k}\right)^{\beta_k}\right\}$, where $\alpha_k = 18 - k$, $\beta_k = 1 + 0.03k$, and $k = 1, 2, \ldots, 8$. Suppose that a failed component is replaced with a new identical one. Suppose that the PM effect on component k follows a linear PM model [23], i.e., the failure rate of component k after the j-th PM is given by

$$h_{k,j}(t) = h_{k,j-1}(t - a_k t_j), \tag{9}$$

where $h_{k,0}(t) = \frac{\beta_k}{\alpha_k}\left(\frac{t}{\alpha_k}\right)^{\beta_k - 1}$, $t \in (t_j, +\infty)$, t_j is the calendar age of the system after the j-th PM is conducted on component k, $0 < a_k < 1$, $k = 1, 2, \ldots, 8$, and $j = 1, 2, \cdots$.

It can be seen that the shortest path sets should include components 1 and 4, one of components 2 and 3, and one of components 5, 6, 7, and 8. That is, the number of components in the shortest path set is $n_c = 4$.

We use Monte Carlo simulation to estimate the average numbers of component failures and the average number of system failures. Suppose that the total life is 5 years (or 60 months), which can be seen as a PM contract (see [24], for example). We repeat the simulation for 3,000 times. Column 1 includes the number, m, of components that are selected for PM and row 1 includes the settings of parameters a_k, where $a_k = k\eta$. N_c and N_s in column 2 are the average number of component failures and system failures, respectively. The results are shown in Table 2. When no PM is performed, the average number of component failures is $N_c = 48.633$ and the average number of system failures is $N_s = 9.022$, which are not shown in the Table. Values 46.521 and 8.958 in cells (2,3) and cell (3,3) in the table are the total numbers of component failures and system failures within 6 years if $m = 1$ (i.e., 1 component can be preventively maintained) and $a_k = 0.01k$ (for $k = 1, 2, \ldots, 8$). It can be observed from the table that

- If m increases and η keeps constant, both N_c and N_s show decreasing trends; and
- If η increases and m keeps constant, N_c and N_s show decreasing trends.
- All N_c in the table are smaller than 48.633 (i.e., the number of component failures when no PM is conducted) and all N_s are smaller than 9.022 (i.e., the number of system failures when no PM is conducted).
- One can also observe that N_c changes more drastically than N_s. This is because of the following reasons.
  - the system only fails if component 1 or component 4 fails;
  - both component 1 and component 4 are only preventively maintained when one of them fails; and
  - if component 1 (or component 4) fails, then component 4 (or component 1) usually has the top priority of being selected for PM.

14

413   Table 2 also indirectly illustrates the use of the importance for a group of components defined in Eq.

414   (5), as the effect of both reliability improvement (i.e., $a_k = k\eta$ in the table) and a group of components for

415   Pm (i.e., $m > 1$) on the system reliability is illustrated in the table.

416   One may also optimise the number of components on which PM can be conducted. For example,

417   suppose that conducting a PM costs £30, a system failure can incur £40, a component failure can incur

418   £10, and. Then on the maintenance effect in column 2 (i.e., $\eta = 0.01$) in Table 2, the cost analysis is shown

419   in Table 3. The total costs for $m = 1, 2, 3$ and 4 are shown in the last column in Table 3. For example, if

420    $m = 1$, then £30 × 1 + £10 × 46.521 + £40 × 8.958 = 853.53. The costs are 853.53, 840.70, 842.97, and

421   859.13 for $m = 1, 2, 3,$ and 4, respectively. As a result, one may select $m = 2$ as its corresponding cost

422   840.70 is the minimum.

423

424   Table 2. Comparison of the number of failure within 5 years over the number of components for PM.

		η =0.01	η =0.015	η =0.02	η =0.025	η =0.03
$m = 1$	N_c	46.521	45.627	44.922	43.920	43.186
	N_s	8.958	8.827	8.855	8.674	8.558
$m = 2$	N_c	42.334	40.518	39.389	38.651	37.817
	N_s	8.934	8.832	8.735	8.666	8.396
$m = 3$	N_c	39.693	36.568	34.208	32.536	31.463
	N_s	8.901	8.823	8.724	8.514	8.453
$m = 4$	N_c	38.629	35.117	32.934	31.307	30.271
	N_s	8.821	8.570	8.328	8.010	7.574

425

426   Table 3. Cost analysis over the number of components for PM

m	N_c for η =0.01	N_s for η =0.01	Cost on PM (= £30 × m)	Cost on component Failure (= £10 × N_c)	Cost on system Failure (= £40 × N_s)	Total cost
1	46.521	8.958	30	465.21	358.32	853.53
2	42.334	8.934	60	423.34	357.36	840.70
3	39.693	8.901	90	396.93	356.04	842.97
4	38.629	8.821	120	386.29	352.84	859.13

7   Conclusions

428   Based on the analysis of the conditional component importance proposed in [8], this paper extends

429   the Birnbaum importance measure to a measure called the component maintenance priority (CMP), with

430   which a pre-specified number of components may be selected for preventive maintenance (PM) while a

431   failed component is being repaired. The CMP differs from most of the existing component importance

432   measures as the CMP may be zero and the latter are usually positive. Here, a component with a zero CMP

433   implies that PM should not be conducted on it.

434   The CMP can be used to schedule PM policy, as illustrated in the example in Section 6. Different from

435   conventional PM optimisation methods that optimise the interval between PM activities, this paper

436 optimises the number of components on which PM can be conducted while a failed component is being

437 repaired.

438 Our future research is to investigate component maintenance priority when maintenance cost and

439 reliability improvement cost are considered. That is, the ideas of this paper and that from reference [3]

440 will be extended.

Acknowledgement

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