TWO-PION EXCHANGE NUCLEAR POTENTIAL - CHIRAL CANCELLATIONS

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It is shown that

~~We show that~~ chiral symmetry is responsible for large cancellations in the two-pion exchange nucleon-nucleon interaction, which are similar to those occuring in free pion-nucleon scattering.

The two-pion exchange nucleon-nucleon potential ($\pi\pi$E-NNP) in the framework of chiral symmetry has recently attracted considerable attention, especially as far as the restricted pion-nucleon sector is concerned [1-5]. This process is closely related to the pion-nucleon scattering amplitude, as pointed out many years ago by Brown and Durso [6] and is described in detail in Ref. [7]. In the non-linear realization of chiral symmetry, one possibility is to write the pion-nucleon interaction Lagrangian as a sum of scalar and pseudoscalar terms, as follows:

$$\mathcal{L}_{PS+S} = \cdots - g\overline{N}\left[\sqrt{f_\pi^2 - \phi^2} + i\tau\cdot\phi\gamma_5\right]N + \cdots , \tag{1}$$

where g is the πN coupling constant, f_π is the pion decay constant and N and ϕ are the nucleon and pion fields, respectively.

In the case of pion-nucleon scattering, this Lagrangian yields a tree amplitude which contains poles in the s and u channels, as well as a scalar contact interaction, which is the signature of chiral symmetry. At low energies, this last term cancels a large part of the pole contributions, giving rise to a final amplitude which is much smaller than the individual contributions.

As far as nucleon-nucleon scattering is concerned, the two-pion exchange amplitude to order g^4 is given by five diagrams, usually named box, crossed box, triangle (twice) and bubble [5], given in Fig. 1.

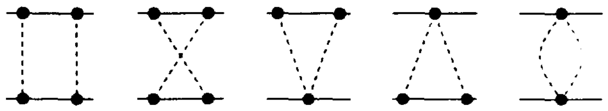

FIG. 1. Loop diagrams for the two pion exchange NN potential calculated in the minimal chiral model.

The first two diagrams contain only nucleon propagators and are independent of chiral symmetry, whereas the triangles and the bubble involve the scalar interaction and hence are due to the symmetry. When one considers the potential instead of the amplitude, the iterated OPEP has to be subtracted from the box diagram.

In this work we show that, as in pion-nucleon scattering, there are large cancellations among the various individual contributions to the interaction, that yield a relatively small net result, and thus prevent the perturbative explosion of the amplitude. Using the potential in coordinate space produced recently [5] and parametrized in Ref. [8], one finds two important cancellations within the scalar-isoscalar sector of the $\pi\pi$E-NNP. The first of them happens between the triangle and bubble contributions, as shown in Fig. 2.

The other one occurs when the remainder from the previous cancellation (S) is added to the sum of the box and crossed box diagrams (PS). In this last case, the direct inspection of the profile functions for the potential, given in Fig 3, provides just a rough estimate of the importance of the cancellation, since the iterated OPEP is not included there.

It is a well established fact [4,9] that the form of the potential depends on the procedure adopted for subtracting the iterated OPEP, and therefore it is important to quantify this contribution. In order to do that, we study the

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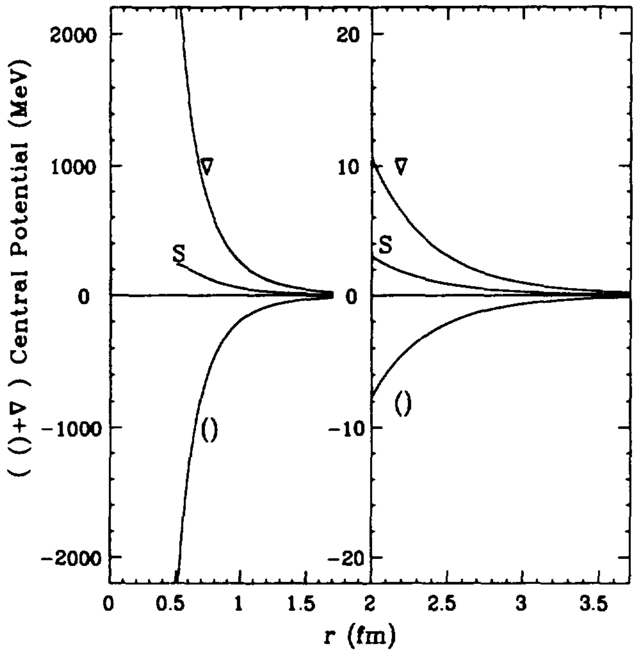

FIG. 2. Profile functions for the bubble (()) and triangle (∇) scalar-isoscalar potentials and for their sum (S), showing a strong cancellation between these two contributions. The graph at right is just an amplification.

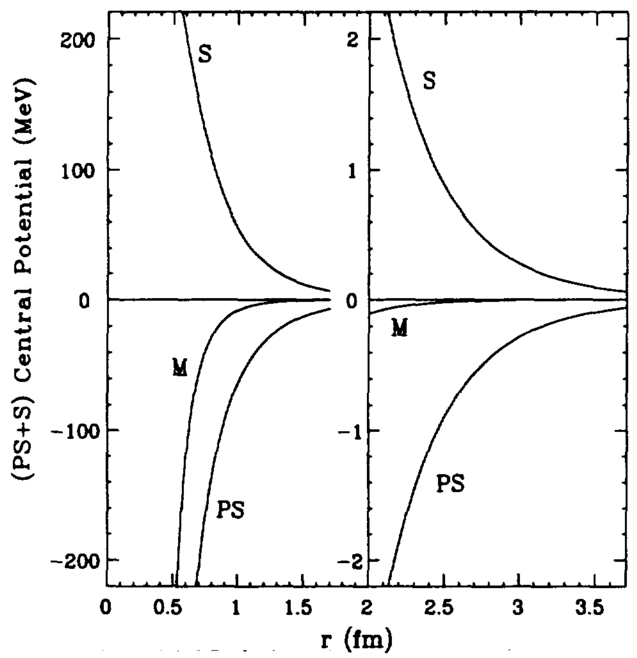

FIG. 3. Profile functions for the chiral (S) and pseudoscalar (PS) scalar-isoscalar potentials and for their sum (M), showing another strong cancellation between these two contributions. The graph at right is just an amplification.

chiral cancellations in the NN scattering problem, since the amplitudes obtained by solving a dynamical equation include automatically the iterated OPEP.

A nice feature of the NN interactions is that they occur in many different channels which emphasize different aspects of the interaction. In our case, we are interested in exhibiting the effects associated with chiral symmetry in the scalar-isoscalar two-pion exchange channel. Therefore we concentrate our study on singlet channels, where the strong effects associated with tensor OPEP interactions are not present. A suitable choice of observables also allow a separation of the dynamical effects according to their range. This is particularly useful in this problem, since it is well known that the use of the Chiral Perturbation Theory (CHPT) is associated with the inclusion of undetermined counterterms in the Lagrangian, involving higher orders of the relevant momenta [1,10]. However, in configuration space, these counterterms become delta functions which affect just the origin and hence are effective only for waves with low orbital angular momentum. In our derivation of the NN potential we used a Lagrangian which did not contain these contact terms and hence it is suited for medium and long distances. Thus, in order to avoid these undetermined short range effects, we consider only the $^1D_2$, $^1G_4$, $^1F_3$, and $^1H_5$ waves. For each channel, we decompose the full NN potential V as

$$V = U_\pi + U_{PS} + U_S + U_C \ , \tag{2}$$

where U_π is the OPEP, U_C represents the short ranged core contributions, U_{PS} is due to the box and crossed box diagrams whereas U_S is associated with the chiral triangle and bubble interactions. Using the variable phase method. it is possible to write the phase shift for angular momentum ℓ as [11,12]

$$\delta_\ell = -\frac{m}{k}\int_0^\infty dr\ V\ P_\ell^2 \ . \tag{3}$$

In this expression, the structure function P_ℓ is given by

$$P_\ell = \hat{j}_\ell \cos D_\ell - \hat{n}_\ell \sin D_\ell \ , \tag{4}$$

where $\hat{j}_\ell$ and $\hat{n}_\ell$ are the usual Bessel and Neumann functions multiplied by their arguments and D_ℓ is the variable phase. Using the decomposition of the potential given in Eq. 2, one writes the perturbative result

$$\delta_\ell = -\frac{m}{k}\int_0^\infty dr \left\{ U_\pi \hat{j}_\ell^2 \right.$$
$$\left. + \left[U_\pi \left(P_\ell^2 - \hat{j}_\ell^2\right) + U_{PS} P_\ell^2 + U_S P_\ell^2 \right] + U_C P_\ell^2 \right\}$$
$$\equiv \delta_\ell)_{\pi L} + [\delta_\ell)_{\pi I} + \delta_\ell)_{PS} + \delta_\ell)_S] + \delta_\ell)_C \ . \tag{5}$$

In this expression, the first term represents the perturbative long range OPEP (πL), the second the iterated OPEP (πI), the third the part due to the box and crossed-box diagrams (PS), the fourth the contribution from chiral symmetry (S). The last one is due to the core and

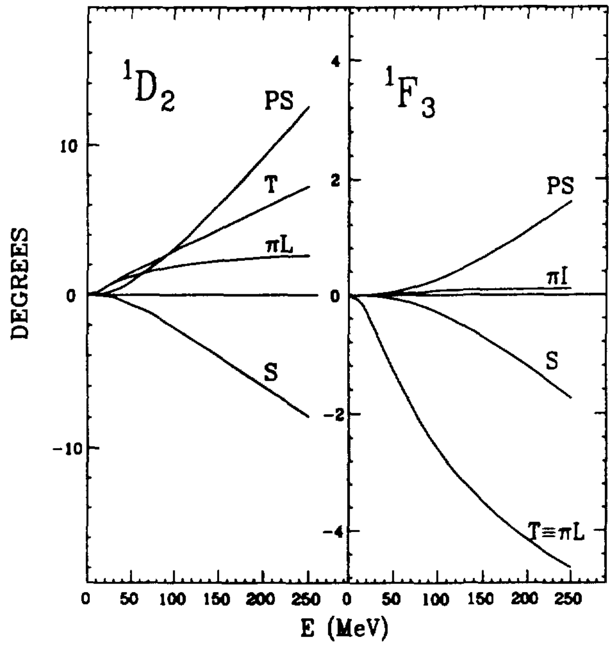

FIG. 4. Contributions for the long-OPEP (πL), iterated OPEP (πI), pseudoscalar (PS) and chiral (S) terms of the potential to the phase shifts for $^1D_2$ and $^1F_3$ waves. The total phase shifts are indicated by (T).

vanishes for waves with $L \neq 0$. In Fig. 4 we show the partial contributions to the $^1D_2$ and $^1F_3$ phase shifts as functions of energy.

There it is possible to see two important features of the two pion exchange interaction. One of them is that the iterated OPEP contribution is comparatively small, indicating that ambiguities in the definition of the potential do not have numerical significance. The second one concerns the large cancellations of the medium range contributions, represented by the terms in the square brackets in Eq. 5. In the case of the $^1F_3$ wave the cancellation is almost complete and the total phase shift is very close to that due to the long-OPEP term. The same pattern also holds for the $^1G_4$ and $^1H_5$ waves [13].

These results show that chiral symmetry, in the restricted pion-nucleon sector, is responsible for large cancellations in the two-pion exchange interaction. This process is therefore similar to threshold pion-nucleon or pion-deuteron [14] scattering amplitudes, where the main role of the symmetry is to set the scale to the problem to be small.

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