

TRN IL 8102070 ✓

WIS-81/8/Feb-ph

Estimate of the Gluon Condensate from Monte Carlo Calculations*

T. Banks

Tel Aviv University, Ramat Aviv, Israel

R. Horsley, H.R. Rubinstein and Ulli Wolff***

Physics Department, Weizmann Institute, Rehovot, Israel

ABSTRACT

Using existing Monte Carlo data we estimate the value of the gluon condensate $\phi = \langle 0 | \frac{\beta(g)}{g} F_{\mu\nu}^2 | 0 \rangle$. Given the limitations of the method and the available data we ~~find~~ reasonable agreement in both sign and magnitude with the value needed in QCD sum rule calculations.

*This work was partially supported by the United States-Israel Binational Science Foundation.

Partially supported by the Israel Commission for Basic Research

***On leave of absence from the Max-Planck-Institut, München, Germany

The QCD vacuum is undoubtedly very different from the state described by renormalized perturbation theory: strong infrared vacuum fluctuations are responsible for the forces that implement quark confinement and the chiral order parameter $\langle \bar{\psi}\psi \rangle$ has a finite vacuum expectation value even for a theory of massless quarks. Several years ago Shifman, Vainshtein and Zakharov [1] argued that another non-perturbative feature of the QCD vacuum was a non zero expectation value for the trace of the energy momentum tensor (in the massless quark theory). Incorporating this expectation value into a duality scheme based on the operator product expansion, they suggested that it was responsible for the large splitting between η_c and ψ , and other important non-perturbative effects. Their scheme and extensions by Reinders, Rubinstein and Yazaki [2] have been quite successful phenomenologically for systems of heavy quarks, bound states of heavy and light quarks [3], and even pure light quark systems [4], where its theoretical justification is most suspect. It is important to point out that the gluon condensate

$$\phi = \left\langle \frac{\beta(g)}{g} F_{\mu\nu}^2 \right\rangle \quad (1)$$

appears multiplied by different Wilson coefficients in the various applications of the method, and that the power corrections required by the phenomenology are those expected from the dimensions of this operator.

The theoretical basis of SVZ's use of ϕ is a matter of some debate [5]. Although connected matrix elements of the operator $\frac{\beta(g)}{g} F_{\mu\nu}^2$ are finite in perturbation theory, the vacuum expectation value is infinite. SVZ suggest that the appropriate definition of the operator is to "normal order it in the perturbative vacuum", i.e.

$$\phi = \left\langle \frac{\beta_B(g_0)}{g_0} \left(F_B^2 - \left\langle F_B^2 \right\rangle_p \right) \right\rangle \quad (B=\text{bare}) \quad (2)$$

where $\langle F_B^2 \rangle_p$ is the vacuum expectation computed in perturbation theory [6]. Non-perturbative contributions to ϕ are then supposed to be finite. Checks of this conjecture could presumably be made, for example by computing the higher loop corrections to ϕ around an instanton. To our knowledge no such calculations have been carried out. The present paper is an attempt to evaluate (2) by a fit to Monte Carlo simulations of lattice gauge theory. The success of such a fit is a partial confirmation of the conjecture of SVZ on the finiteness of (2). Given the crudity of our methods, our results are quite encouraging. We find that the non-perturbative contribution to ϕ has the same sign and order of magnitude as the phenomenological value used by SVZ and RRY. The appearance of the right order of magnitude is non-trivial because it requires that a certain numerical coefficient in our fit be of order 10^9 and this agrees with Monte Carlo data. Of course, much more theoretical work is needed before we can conclude that the SVZ conjecture is verified.

I. Connection between lattice theory and continuum

Although (2) is well defined in perturbation theory (including non-perturbative semiclassical expansions) it does not make sense when applied to an "exact", i.e. Monte Carlo evaluation of ϕ . The point is that the perturbation series is at best asymptotic and it is only defined once a summation method is prescribed. Clearly one possible summation method is just to say that the sum of the series equals the exact value for the function in which case $\phi = 0$. This is obviously not what was intended by

SVZ. We propose instead the following pragmatic definition of (2). Evaluate a certain number of terms in the series and find a range of values of $\beta = \frac{4}{g_0^2}$ for which the Monte Carlo data deviates from the series by more than the estimated errors in the series (as represented by the highest term calculated). Of course, we restrict ourselves to a range where successive terms in the series are not growing. The difference between the series and the data defines the non-perturbative part of ϕ . Clearly we are going to have to work at values of β that are not too large if we are to have any hope of separating out the non-perturbative piece. A priori we have no right to expect that the behaviour of ϕ at these values of β will be anything like its large β asymptote. However, previous Monte Carlo studies of lattice gauge theories appear to show that the functions of the theory take on their asymptotic form as soon as β is above the so-called weak-strong transition. We assume that this is also true for ϕ . SVZ's conjecture that (2) is finite in the continuum limit then implies that the lattice plaquette defined in (5) behaves like

$$W = 1 - \frac{d_0}{\beta} - \frac{d_1}{\beta^2} - \frac{d_2}{\beta^3} - A \beta \frac{2b_1}{b_0^2} e^{-\frac{\beta}{2b_0}} \quad (3)$$

even for values of β at which we can extract it. This is a renormalization group argument and b_0, b_1 are the first two perturbative coefficients of the β function. The constant A is related to the continuum value of ϕ

$$\frac{\alpha}{\pi} \langle F^2 \rangle = \frac{-\phi}{4\pi^2 b_0} = \Lambda_{\text{MOM}}^4 A_{\text{NS}} \begin{cases} 1.3 \times 10^{-8} & \text{for SU(2)} \\ 8.7 \times 10^{-9} & \text{for SU(3)} \end{cases} \quad (4)$$

where Λ_{MOM} is the QCD scale parameter in the momentum subtraction scheme. To derive (4) we have used the relation between the lattice scale parameter and Λ_{MOM} in the Feynman gauge calculated by Hasenfratz

and Hasenfratz[7]. Note that for values of Λ_{MOM} of a few hundred MeV the phenomenological value $\frac{\alpha}{\pi} \langle F^2 \rangle = .012 (\text{GeV})^4$ can be reproduced only if $\Lambda \sim 10^8$. Notice

that for SU(2) [8]
$$b_0 = \frac{11}{24\pi^2}, b_1 = \frac{17}{96\pi^4}, \Lambda_{\text{LATT}} = \frac{1}{57.5} \Lambda_{\text{MOM}}$$

while for SU(3)[8]
$$b_0 = \frac{11}{16\pi^2}, b_1 = \frac{51}{128\pi^4}, \Lambda_{\text{LATT}} = \frac{1}{83.5} \Lambda_{\text{MOM}}$$

The necessity of making our fit in a restricted range of β is troublesome but it is actually a general feature of Monte Carlo calculations in QCD. For large β (actually $\beta \sim 4-5$ for SU(2)) the Compton wavelengths of particles in the lattice gauge theory become larger than the finite lattice of the Monte Carlo calculation. Even if the Monte Carlo data were exact for the finite lattice they cannot be assumed to give the correct infinite volume limit at values of β much larger than this. On the other hand if β is too small we cannot expect to obtain a good approximation to the continuum ($\beta \rightarrow \infty$) theory. Thus all successful Monte Carlo calculations in lattice gauge theory depend on the existence of a range of intermediate β in which important quantities already take on their asymptotic ($\beta \rightarrow \infty$) forms. In our case the problem is slightly exacerbated by the fact that we are calculating a quantity which has a non-trivial perturbation expansion. We must go to small enough β to separate the perturbative and non-perturbative effects. Nonetheless we have been able to find a fit to ϕ in the same region in which other authors have fit the string tension.

II. Monte Carlo fit

We will now use existing Monte Carlo data to evaluate (2). The first step in the calculation is to find the relation between the plaquette average energy and ϕ . To that effect consider a hypercubic lattice with spacing a embedded in a continuum and with axes $\mu_\alpha = \delta_{\mu\sigma}$. For an arbitrary point x the $SU(N)$ Wilson plaquette, in the μ, ν plane adjacent to x is given by

$$W = \frac{1}{N} \operatorname{Re} \langle \operatorname{tr} U_{x, x+a\mu} U_{x+a\mu, x+a\mu+a\nu} U_{x+a\mu+a\nu, x+a\nu} U_{x+a\nu, x} \rangle \quad (5)$$

By this we mean the expectation value of the 1×1 Wilson loop. In (5)

$$U_{x, x+a\mu} = P \exp \left(ig_0 \int_x^{x+a\mu} A_{B, \alpha}(\xi) d\xi_\alpha \right) \quad (6)$$

and W has been normalized to belong to $[0, 1]$. By standard methods we obtain

$$W = 1 - \frac{a^4}{2} g_0^2 \frac{1}{N} \operatorname{tr} \langle F_{\mu\nu}^2 \rangle + O(a^5) \quad (7)$$

$$= 1 - \frac{a^4}{48N} g_0^2 \langle F_B^2 \rangle + O(a^5) \quad (8)$$

where in (8) we have used x independence, and we have averaged over planes and orientations (12 of them).

Combining (2) and (8) we obtain, in the continuum limit

$$\phi = \lim_{a, g_0 \rightarrow 0} \frac{\beta_B(g_0)}{g_0^3} \frac{48N}{a^4} [W_P - W] \quad (9)$$

where N is the order of the $SU(N)$ group. W_p is the perturbative part of W . For large β the perturbative part W_p is given by

$$W_p = 1 - \frac{d_0}{\beta} - \frac{d_1}{\beta^2} - \frac{d_2}{\beta^3} + \dots \quad (10)$$

In principle these coefficients should be calculated by weak coupling perturbation theory on a finite lattice. This has been done only for d_0^* . However, Lautrup and Nauenberg [9] have fit the coefficients d_1 and d_2 to their high accuracy large β Monte Carlo data for $SU(2)$. We have performed a similar fit on the (lower statistics) data of Creutz and Pietarinen for $SU(3)$ [10]. The resulting values are

	$SU(2)$	$SU(3)$
d_0	3/4	4/3
d_1	0.13	0.8
d_2	0.29	-

Our strategy is to attempt to match the Monte Carlo data for W to the formula

$$W = W_p - A \beta \frac{2b_1/b_0^2}{\beta} e^{-\beta/2b_0} \quad (11)$$

Then using (9) and the expression

$$\Lambda_{LATT} = \frac{1}{a} \left(\frac{4}{\beta} b_0 \right)^{-b_1/2b_0^2} e^{-\beta/8b_0} \quad (12)$$

* We are presently considering extending this calculation to higher orders.

We find

$$\phi = -b_0 48N(4b_0)^{2b_1/b_0^2} A \Lambda_{LATT}^4 \quad (13)$$

then (4) obtains.

For SU(2) the Monte Carlo is very accurate and we obtain a good fit (see fig. 1) down to $\beta = 2.1$. We can extract the non-perturbative part below $\beta = 2.4$, and our best estimate for A is 10^8 to 10^9 .

For comparison we note that Bahnot and Rebbi's fit to the string tension (SU(2)) [11] was performed in the interval $2 < \beta < 3$. It is generally accepted that the weak to strong transition for SU(2) occurs at about $\beta \sim 2$. Data below 2 cannot be expected to conform to the asymptotic form given by (11). It is interesting to note that the weak coupling perturbation series is ceasing to be asymptotic in this region. The $1/\beta^2$ and $1/\beta^3$ terms are of the same order of magnitude. Thus it does not make sense to add more terms to the series. Nonetheless we may attempt a polynomial fit in the region $2 < \beta < 2.5$. Of course such a fit succeeds (any continuous function can be fit by a polynomial on a compact interval), but the resulting coefficients are too large to be interpretable as the next terms in the perturbation series ($O(10)$). On the other hand the large value of A in our fit is expected in order to give the phenomenological order of magnitude of ϕ . For SU(3) the Monte Carlo (due to Creutz and Pietarinen) [10] is cruder but still sufficient. We find an adequate fit for $3.6 < \beta < 4.1$. See Fig. 2 with $A \approx (5 \pm 3) \cdot 10^9$. Note that all the points that do not lie on our curve lie below the conventional value for the weak to strong transition ($\beta \sim 4$). Thus it is reasonable to disregard them. With this value of A, $\langle \frac{\alpha}{\pi} F^2 \rangle = 43.5 (\Lambda_{MOM})^4$.

To summarize: we have found a fit to the Monte Carlo data for the action density in SU(2) and SU(3) lattice gauge theories which is consistent with the conjecture of SVZ that the perturbatively normal ordered operator $\frac{\beta(g)}{g} F_{\mu\nu}^2$ has a finite non zero value in continuum QCD. The deviation from weak coupling perturbation theory on which our fit is based is an order of magnitude above the statistical errors of the Monte Carlo calculation in a significant range of β above the weak strong transition. Higher order terms in a perturbation series cannot reproduce this deviation and the exponential behaviour required by the renormalization group is clearly discernible.

Our calculation is of course subject to all the usual criticisms of Monte Carlo studies in QCD. (Finite lattice effects, absence of fermions etc.). Nonetheless we believe that certain aspects of our fit are quite clearcut and will survive in more careful studies. These are:

1) The sign of ϕ which is obtainable simply from the fact that the data lies below the perturbative curve. This agrees with the sign of the phenomenologically motivated fit to ϕ .

2) The large value of A. This is necessary in order to obtain even order of magnitude agreement with the continuum result.

3) As for the functional form of $W(\beta)$ we do not claim to have proven that it is that given by the renormalization group. However it is significant that the data does not disagree with the renormalization group fit. Furthermore there is no doubt that some sort of exponential dependence can be discerned in the range of β studied. Polynomial fits are of course possible but highly artificial.

Acknowledgements

We thank B. Lautrup and E. Pietarinen for correspondence and for providing unpublished Monte Carlo data, and many colleagues for discussions. U.W. acknowledges a fellowship of the Minerva Committee for Israeli-German cooperation in research.

References

1. M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147, (1979) 385.
2. L.J. Reinders, H.R. Rubinstein and S. Yazaki, Phys. Lett. 95B (1980) 103.
3. L.J. Reinders, H.R. Rubinstein and S. Yazaki, Phys. Lett. 97B (1980) 257, and to be published.
4. M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B147 (1979) 448.
5. V.A. Novikov, M.A. Shifman, A.I. Vainshtein and V.I. Zakharov, Nucl. Phys. B174 (1980) 378.
6. H. Kluberg-Stern and J.B. Zuber, Phys. Rev. D12 (1975) 467;
J.C. Collins, A. Duncan and S.D. Joglekar, Phys. Rev. D16 (1977) 438.
R. Fukuda, Phys. Rev. D21 (1980) 485.
7. A. Hasenfratz and P. Hasenfratz, Phys. Lett. 93B (1980) 165.
8. W.E. Caswell, Phys. Rev. Lett. 33 (1974) 244;
D.R.T. Jones, Nucl. Phys. B75 (1974) 531.
9. B. Lautrup and M. Nauenberg, CERN preprint TH 2945 (September 1980), and private communication.
10. E. Pietarinen, Helsinki preprint HU-TFT-80-49 and private communication.
11. G. Bhanot and C. Rebbi, CERN preprint TH 2979 (November 1980).

Figure Captions

Figure 1: SU(2) plaquette energy versus $\beta = 4/g_0^2$.
----- Perturbative fit (see (10)) $d_0 = \frac{3}{4}$, $d_1 = 0.13$, $d_2 = 0.29$.
.... Monte Carlo data [9]. Typical errors are below 0.5%.
--- Fit according to (a) $A=10^8$; (b) $A=10^9$; (c) $A=10^{10}$.

Figure 2: SU(3) Plaquette energy versus $\beta=4/g_0^2$.
---- Perturbative fit (see(10)) $d_0 = \frac{4}{3}$, $d_1 = .8$, $d_2 = 0$
+ Creuz data for Monte Carlo and
.... Pietarinen data from [10].
Typical errors are below 1%.
--- Fit according to (a) $A=10^8$; (b) $A=3 \times 10^9$,
(c) $A=10^{10}$.

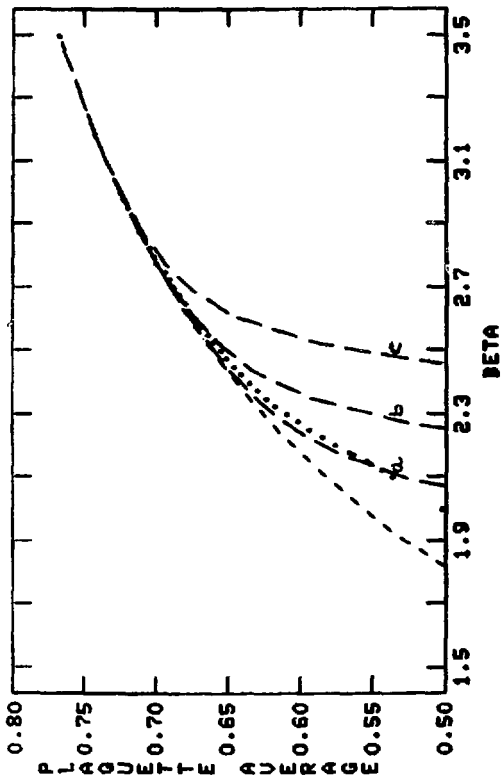


Fig. 1

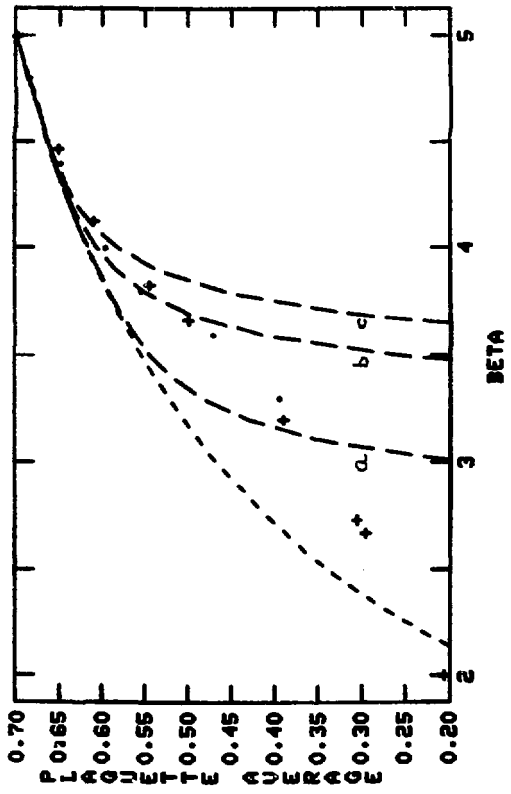


Fig. 2

