

Recent CKMfitter updates on global fits of the CKM matrix

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On behalf of the *CKMfitter* Collaboration

CKM 2021

University of Melbourne, Australia (Online)

- **Theorists + experimentalists performing a global analysis of measurements (inputs from HFLAV, FLAG) determining the CKM matrix parameters in the framework of the SM and some of extensions**

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CKM matrix

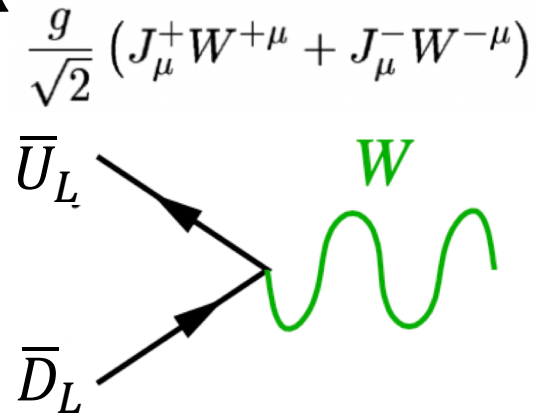
- Yukawa couplings not necessarily diagonalized in interaction eigenstates \Rightarrow mass eigenstates different from interaction eigenstates

- Unitary matrix needed to diagonalize mass matrix

$$J_\mu^+ = \bar{U}_L^I \gamma_\mu D_L^I + \bar{\nu}_L^I \gamma_\mu \ell_L^I,$$

$$J_\mu^+ = \bar{U}_L \gamma_\mu V_{CKM} D_L + \bar{\nu}_L \gamma_\mu \ell_L,$$

$$\begin{pmatrix} d^I \\ s^I \\ b^I \end{pmatrix} = \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \begin{pmatrix} d \\ s \\ b \end{pmatrix}$$

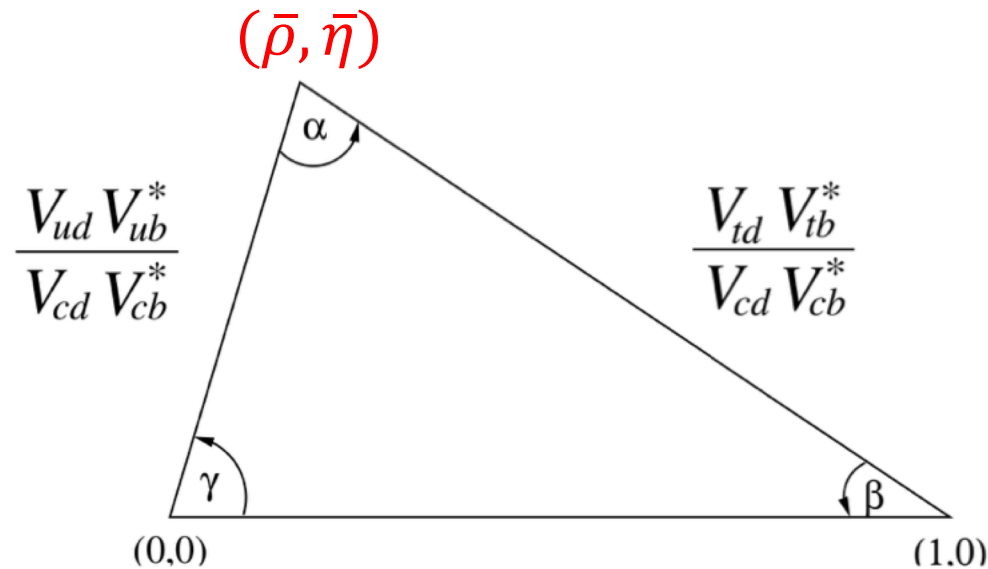


- One complex phase (3 generations) for CP violation in SM
- Constraints from unitary conditions:

$$\sum_{i \text{ or } j} V_{ij} V_{ij}^* = 1 \quad \sum_i V_{ij} V_{ik}^* = 0 \quad \sum_j V_{ij} V_{kj}^* = 0$$

Unitary triangle and parameter definition

$$V_{ud}V_{ub}^* + V_{cd}V_{cb}^* + V_{td}V_{tb}^* = 0$$



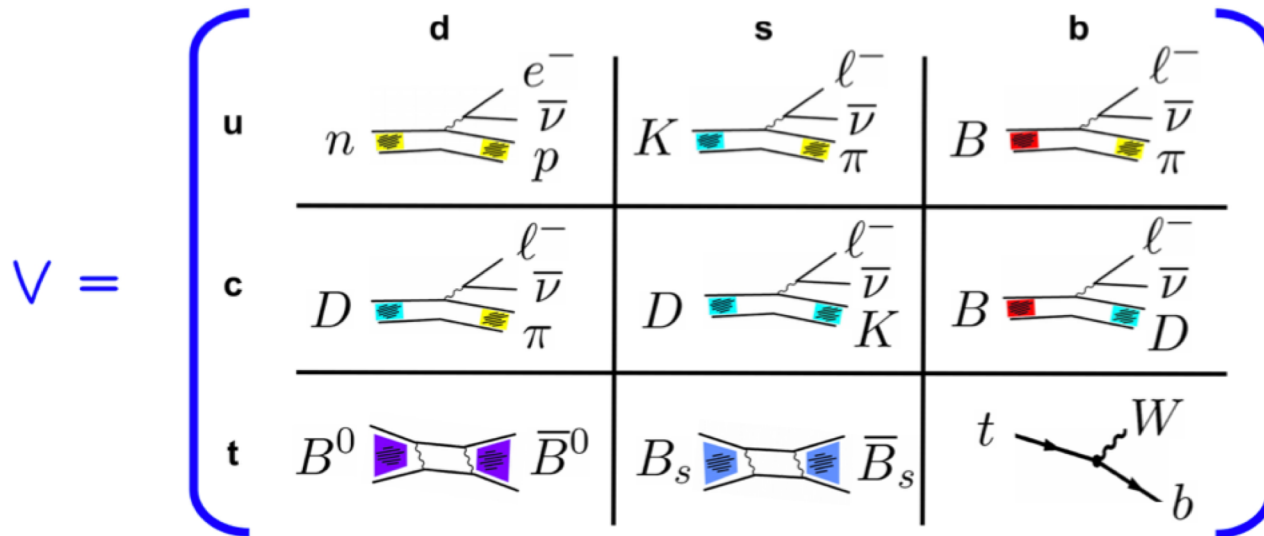
- **Most popular one: similar size of three angles**
- **Closely related to B decays**

Wolfenstein parameterization:

$$\lambda^2 = \frac{V_{us}V_{us}^*}{V_{ud}V_{ud}^* + V_{us}V_{us}^*} \quad A^2\lambda^4 = \frac{V_{cb}V_{cb}^*}{V_{ud}V_{ud}^* + V_{us}V_{us}^*} \quad \bar{\rho} + i\bar{\eta} = -\frac{V_{ud}V_{ub}^*}{V_{cd}V_{cb}^*}$$

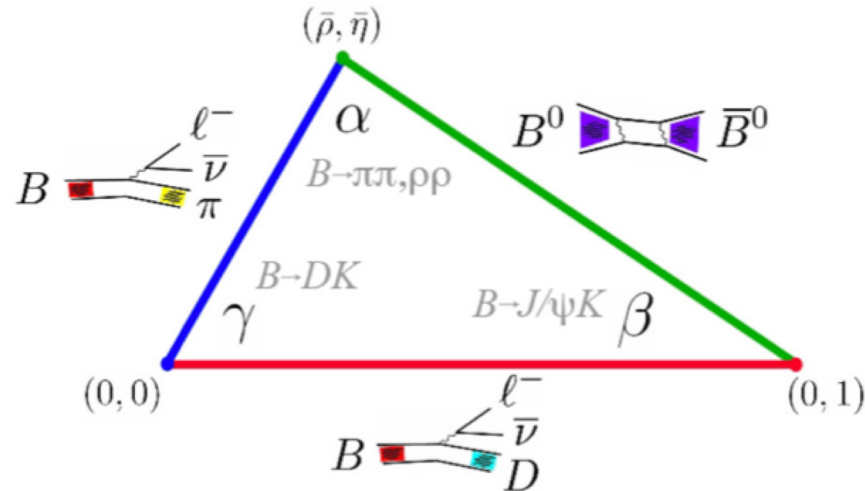
X and y axis

CKM observables (1)



- $|V_{ud}|$: superallowed nuclear β decays
- $|V_{us}|$: $K \rightarrow \pi l \nu$, $K \rightarrow l \nu$, $\tau \rightarrow K \nu$ etc. + form factors, decay constants
- $|V_{cs}|$, $|V_{cd}|$: (semi-)leptonic charm decays + Lattice inputs
- $|V_{ub}|$, $|V_{cb}|$: (semi-)leptonic B decays + Lattice inputs
- $|V_{td}|$, $|V_{ts}|$: Δm_d , Δm_s + bag parameters, decay constants

CKM observables (2)



- α : $B \rightarrow \pi\pi, B \rightarrow \rho\pi, B \rightarrow \rho\rho$, isospin analyses
- β : $B \rightarrow (\bar{c}c)K, B \rightarrow Dh^0$, time-dependent CP violation
- γ : $B \rightarrow DK$, ADS/GLW/GGSZ
- ϕ_s : $B_s^0 \rightarrow (c\bar{c})(KK, \pi\pi)$, time-dependent CP violation
- $-2\beta_s + \gamma$: $B_s \rightarrow D_s K$, not included yet
- $V_{td}^* V_{ts}$ and $V_{cd}^* V_{cs}$: ϵ_K + lattice inputs

Statistical approach

- Frequentist statistics based on a χ^2 analysis
- χ^2_{min} : indication of goodness-of-fit
- $\Delta\chi^2$: calculation of Confidence Level (CL) or p-values
- *Range* fit scheme (*Rfit*): special treatment of theoretical uncertainties

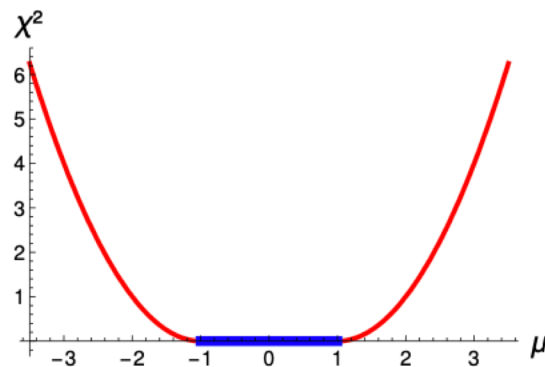
Theoretical inputs: mainly from Lattice papers (**with error budgets**); different systematic uncertainties **combined linearly**

Example in 1D, $0 \pm 1_{stat} \pm 1_{theo}$ ($N_{dof} = 1$)

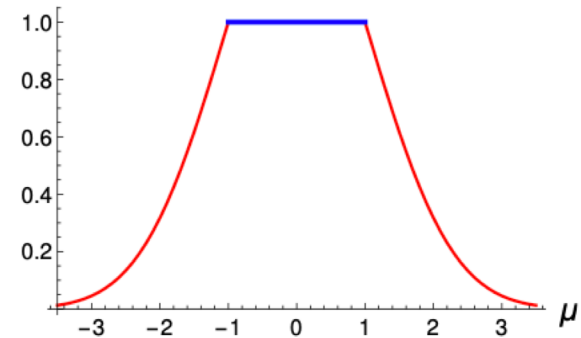
Treated as Gaussian

$$\mathcal{L} = \mathcal{L}_{exp} \times \mathcal{L}_{theo}$$

Treated as range



p-value



χ^2 : flat bottom, quadratic walls

Recent updates on V_{ud}

- Precision on $|V_{ud}|$ led by superallowed $0^+ \rightarrow 0^+$ nuclear β decays
- 2020 survey by Hardy and Towner, including recent calculations for radiative corrections and new improved measurements

NEW

Theoretical

Our 2019 update (also from Hardy and Towner)

$$V_{ud} = 0.97373 \pm 0.00031$$

$$V_{ud} = 0.97418 \pm 0.00021$$

- V_{ud} smaller by 0.00045, while uncertainties larger by 50%
- V_{ud} from our fits without direct measurements

INDIRECT

$$V_{ud} = 0.97440 \pm 0.00006$$

- Some tension seen if using new V_{ud} input directly without careful consideration
- Considering properly error budget from Hardy and Towner, in this update, we use

Our 2021 update

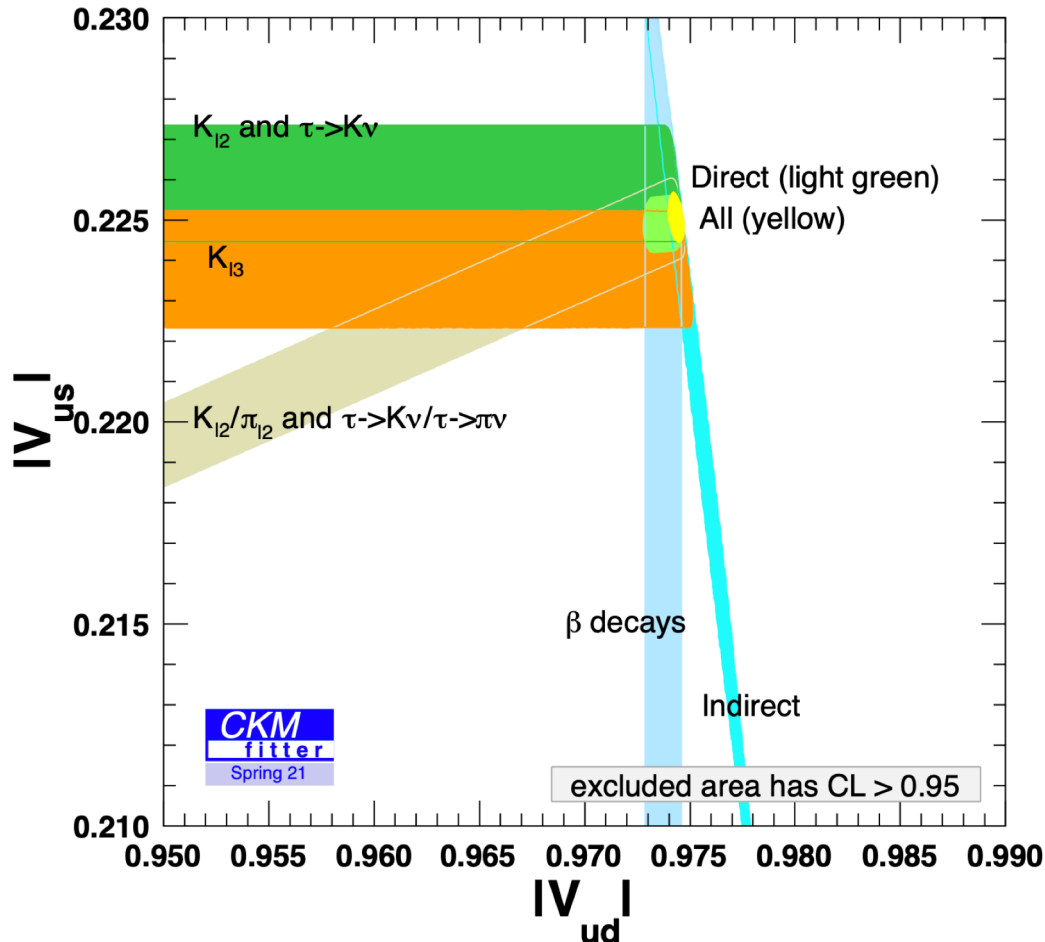
statistical

theoretical

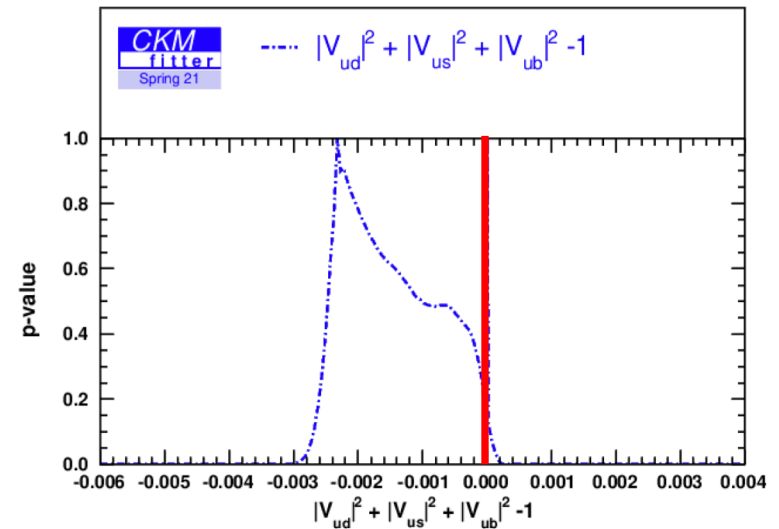
$$V_{ud} = 0.97373 \pm 0.00009 \pm 0.00053$$

Theoretical uncertainties summed up linearly

V_{ud} Vs V_{us}



- $|V_{us}|$ from $K \rightarrow l\nu, K \rightarrow \pi l\nu, \tau \rightarrow K\nu, \tau \rightarrow \pi\nu$ etc
- Still consistent with unitary $< 2\sigma$



$$V_{ud}V_{ud}^* + V_{us}V_{us}^* + V_{ub}V_{ub}^* - 1 = -0.00230^{+0.00218}_{-0.00023} (1\sigma)$$

$$-0.00230^{+0.00237}_{-0.00044} (2\sigma)$$

$$-0.00230^{+0.00242}_{-0.00065} (3\sigma)$$

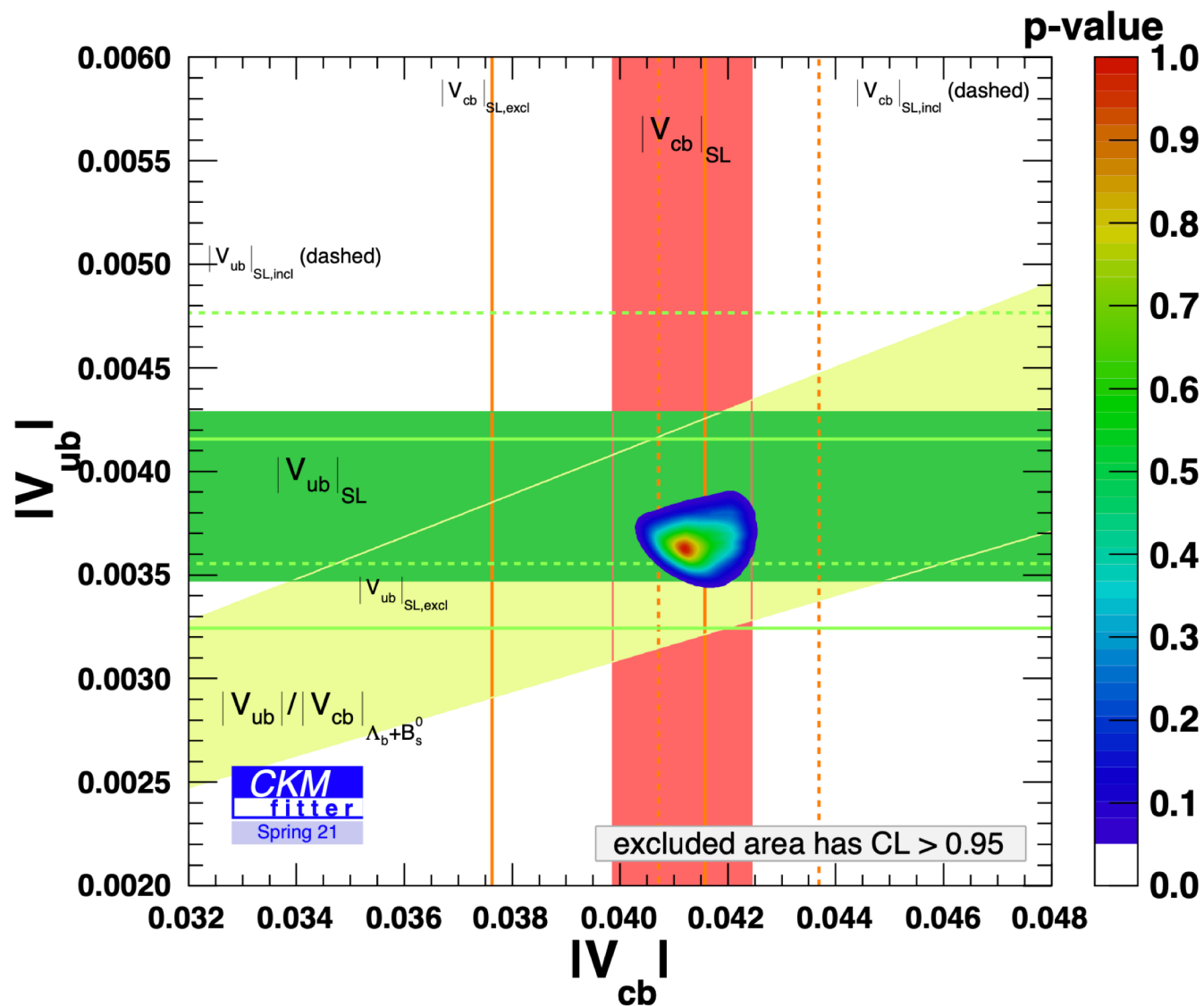
V_{ub} and V_{cb}

- Very small change on inclusive and exclusive V_{ub} and V_{cb} measurements

	Inclusive	Exclusive	Average
$V_{ub}(\times 10^{-3})$	$4.16 \pm 0.12 \pm 0.31$	$3.70 \pm 0.10 \pm 0.21$	$3.88 \pm 0.08 \pm 0.21$
$V_{cb}(\times 10^{-3})$	$42.2 \pm 0.4 \pm 0.5$	$39.6 \pm 0.6 \pm 0.5$	$41.15 \pm 0.34 \pm 0.45$

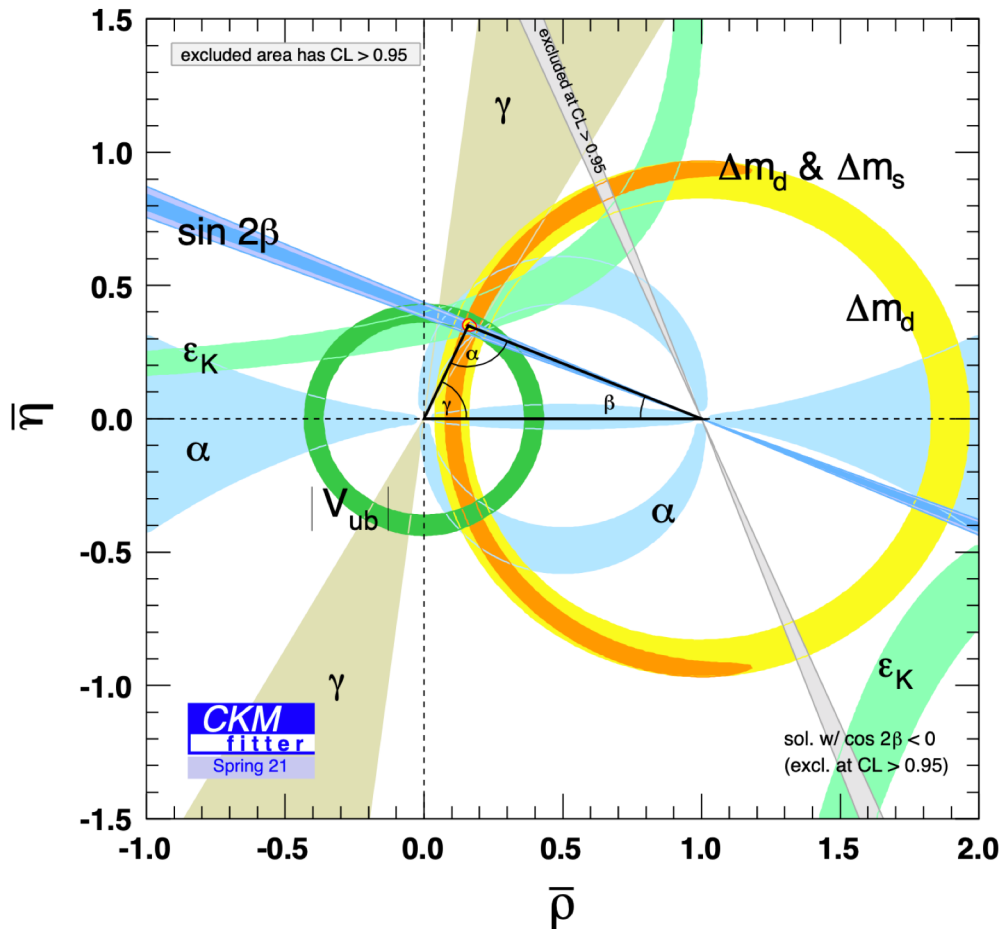
- New measurements of partial branching fractions of inclusive $B \rightarrow X_u lv$ decays with hadronic tagging (arXiv:2102.00020) included for V_{ub} inclusive
- V_{cb} exclusive: based on 2020 BGL refit with preliminary non-zero recoil FF ratio JLQCD inputs and new $D \rightarrow K\pi$ BF
- New ratio of $|V_{ub}|/|V_{cb}|$ from $B_s^0 \rightarrow K^+ \mu^- \nu_\mu$ and $B_s^0 \rightarrow D_s^+ \mu^- \nu_\mu$ (arXiv:1901.02561), only take high q^2 region which uses LQCD inputs (tension between high and low q^2)
- New $|V_{cb}|$ measurements from LHCb (arXiv:2001.03225) not used as knowledge of $B \rightarrow D^* lv$ required, care needed to consider larger correlations for $|V_{cb}|$ from these measurements

V_{ub} vs V_{cb}



Current status

- Inputs till spring 2021 (Moriond)
- χ^2 slightly increased compared to 2019 update, p-value $\sim 29\%$



Wolfenstein parameters:

$$A = 0.8132^{+0.0119}_{-0.0060}$$

$$\lambda = 0.25500^{+0.00024}_{-0.00022}$$

$$\bar{\rho} = 0.1566^{+0.0085}_{-0.0048}$$

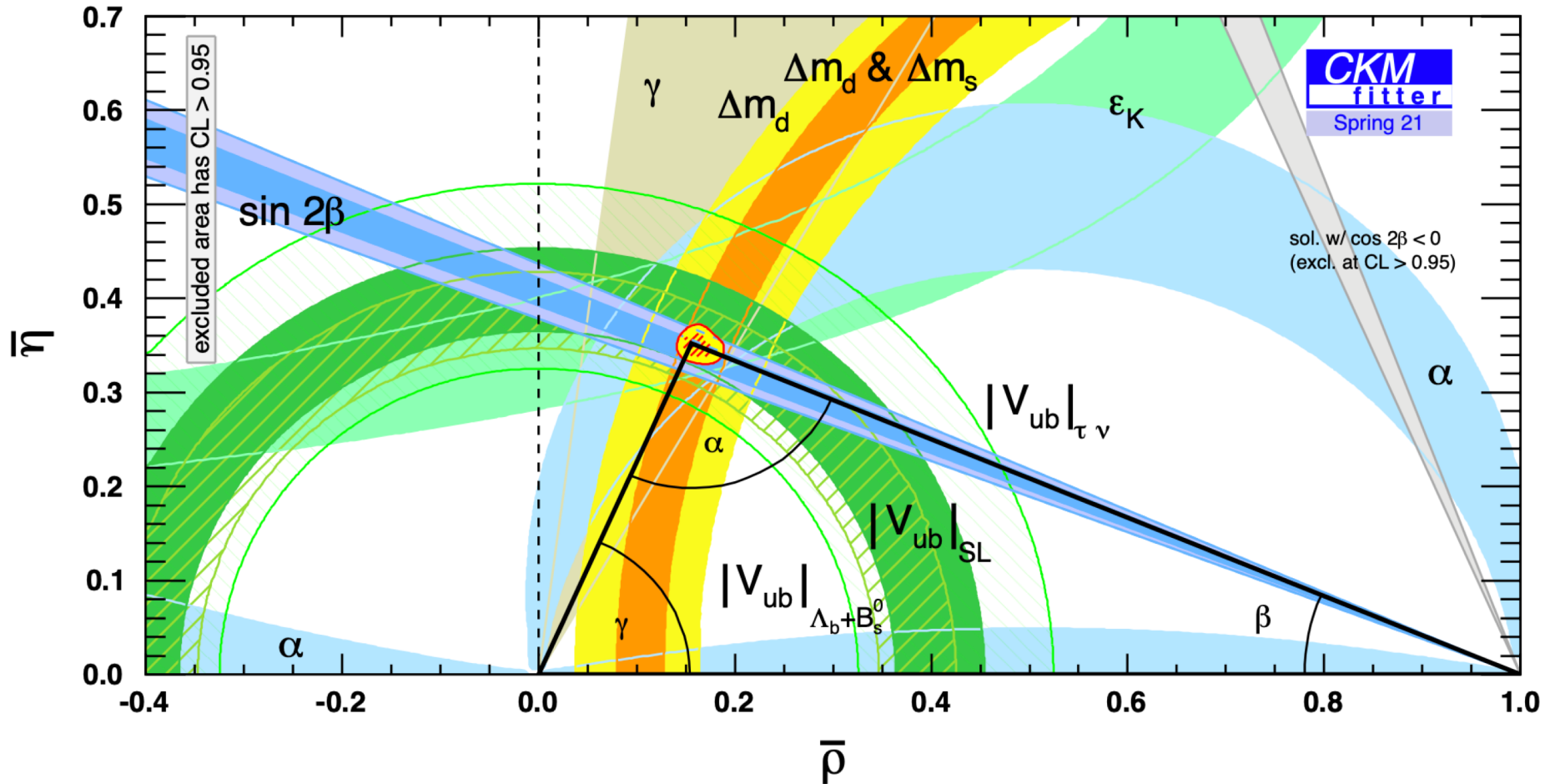
$$\bar{\eta} = 0.3475^{+0.0118}_{-0.0054}$$

Jarlskog invariant:

$$J = (3.044^{+0.068}_{-0.084}) \times 10^{-5}$$

@ 68% CL.

Zoomed version

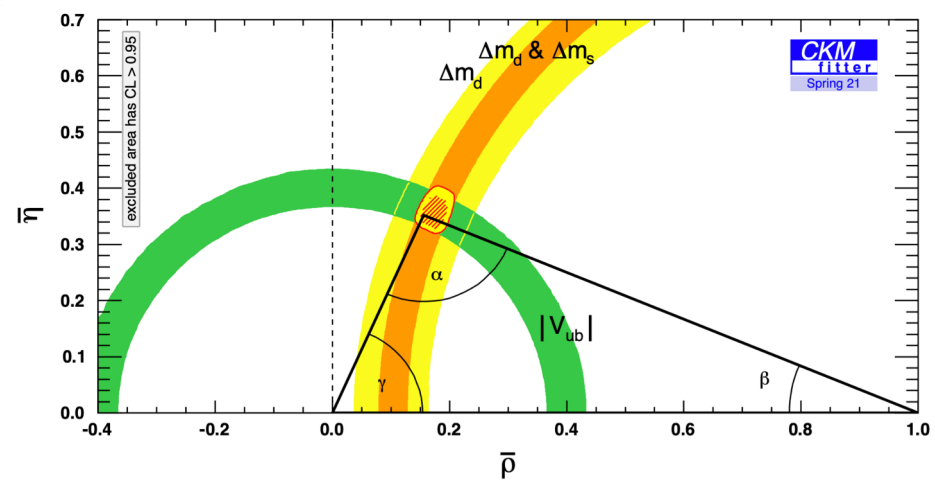
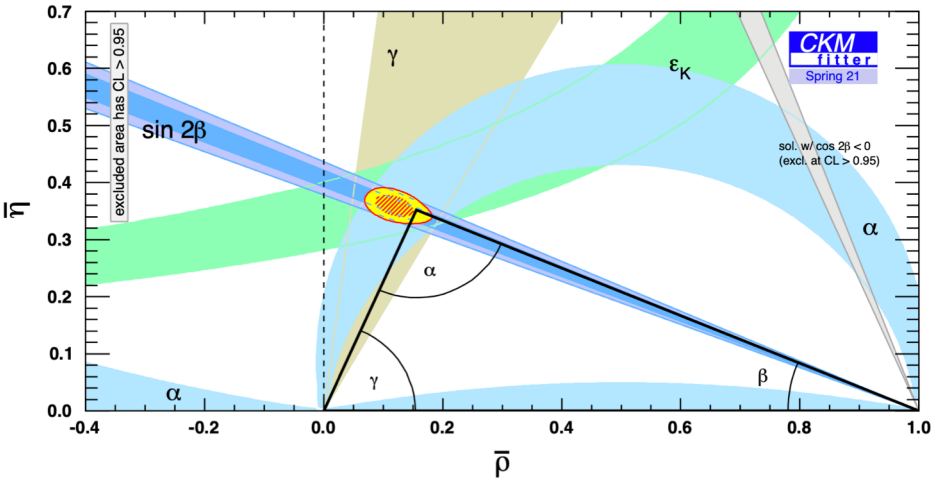


- Different contributions for $|V_{ub}|$ from $B \rightarrow \tau\nu$, inclusive + exclusive semi-leptonic measurements and $|V_{ub}|/|V_{cb}|$ ratio measurements are explicitly shown

Fits from different subsets

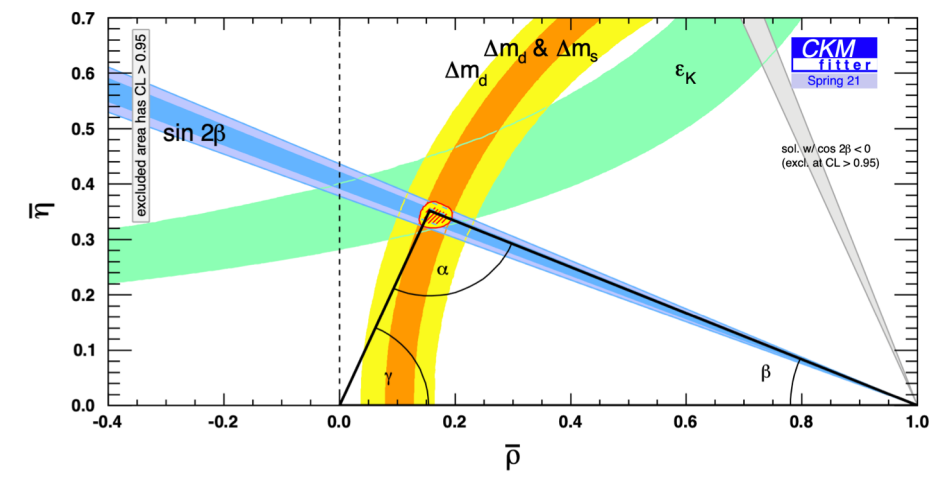
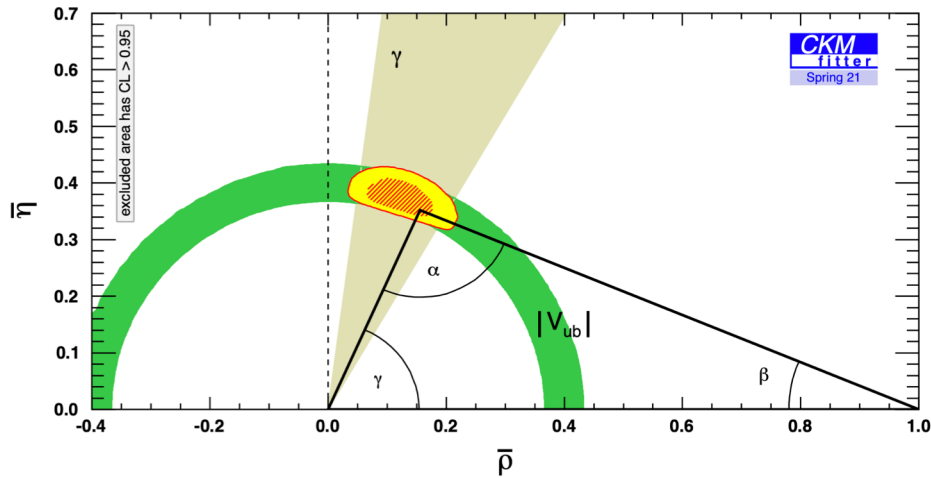
CP violating

CP conserving

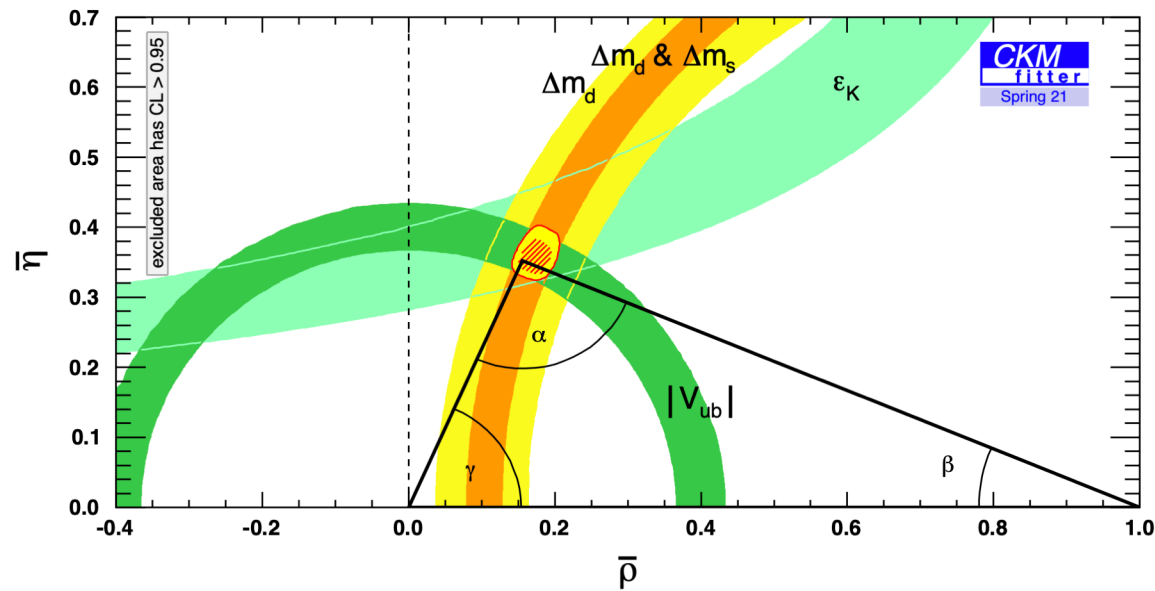
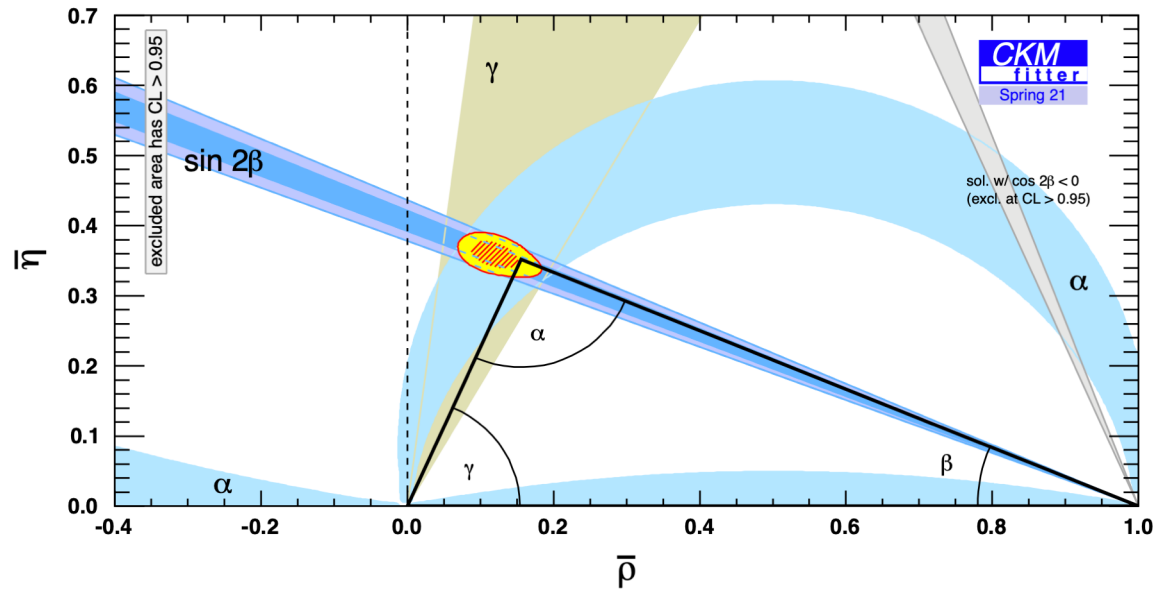


Tree

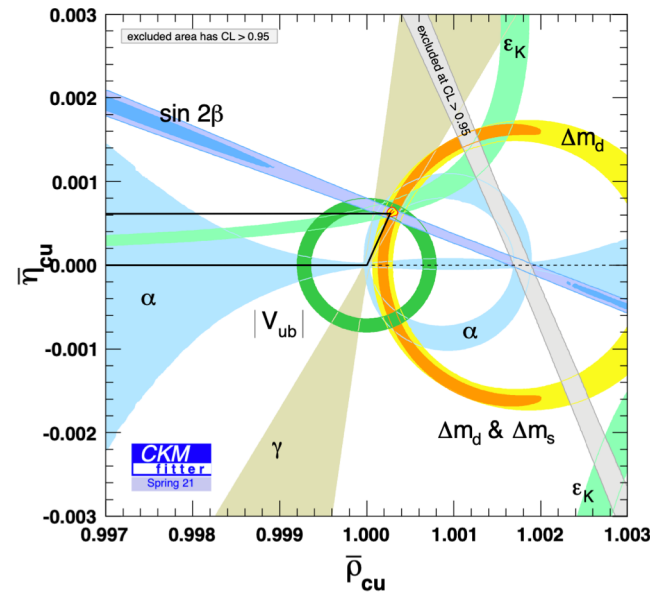
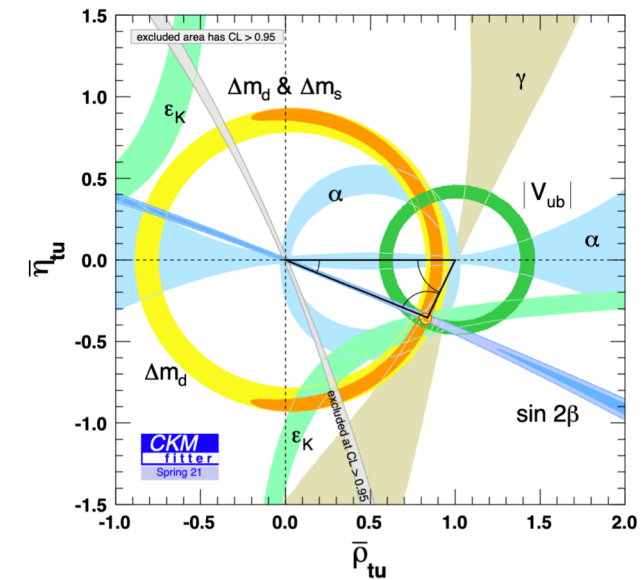
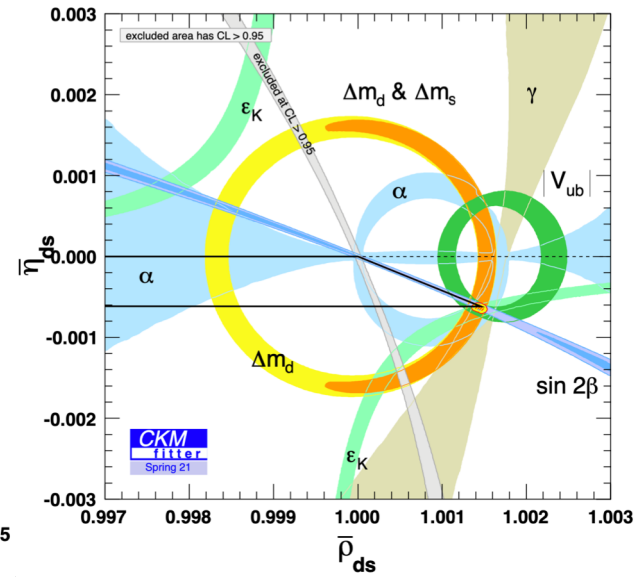
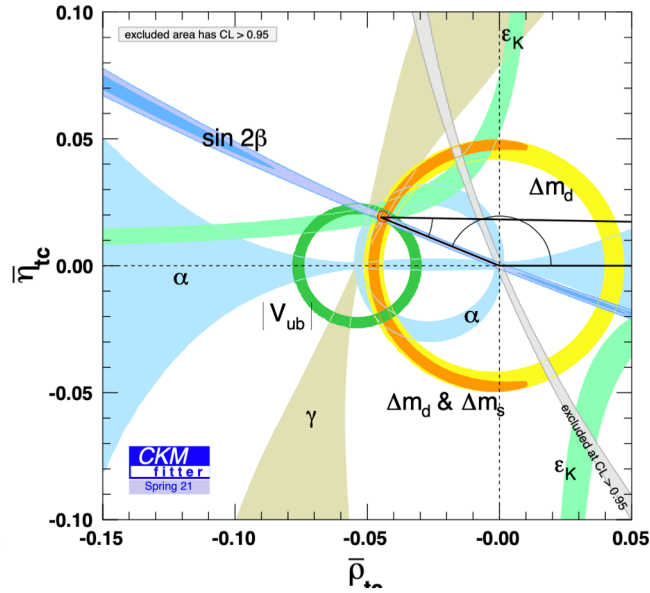
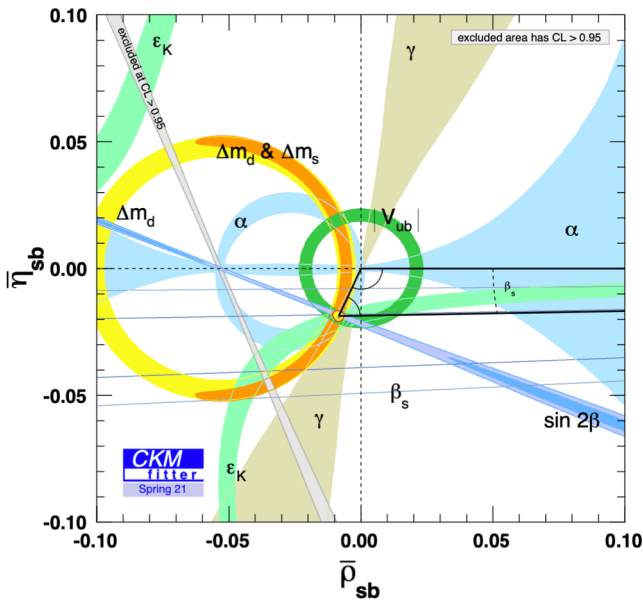
Loop



Angle vs magnitude

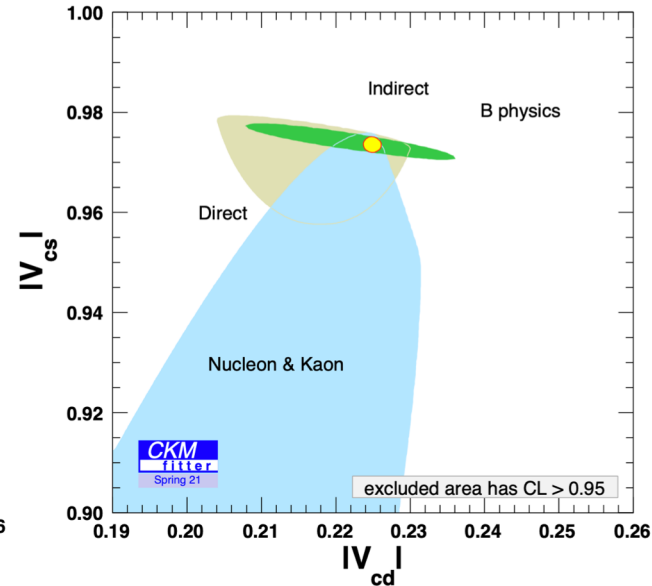
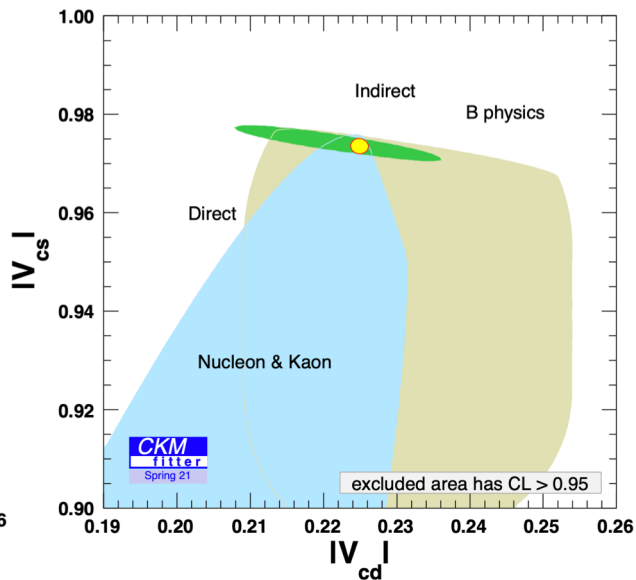
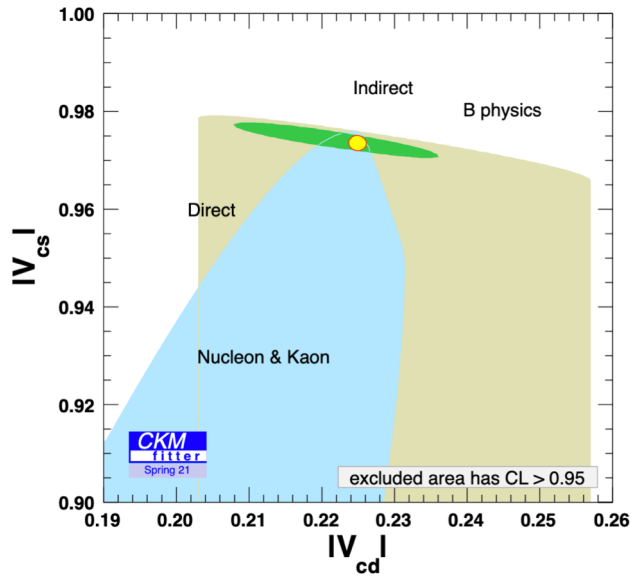


Other triangles



Consistency over
all triangles

V_{cd} vs V_{cs}



$|V_{cd}|$: $\nu - N$ scattering

$|V_{cs}|$: W decay

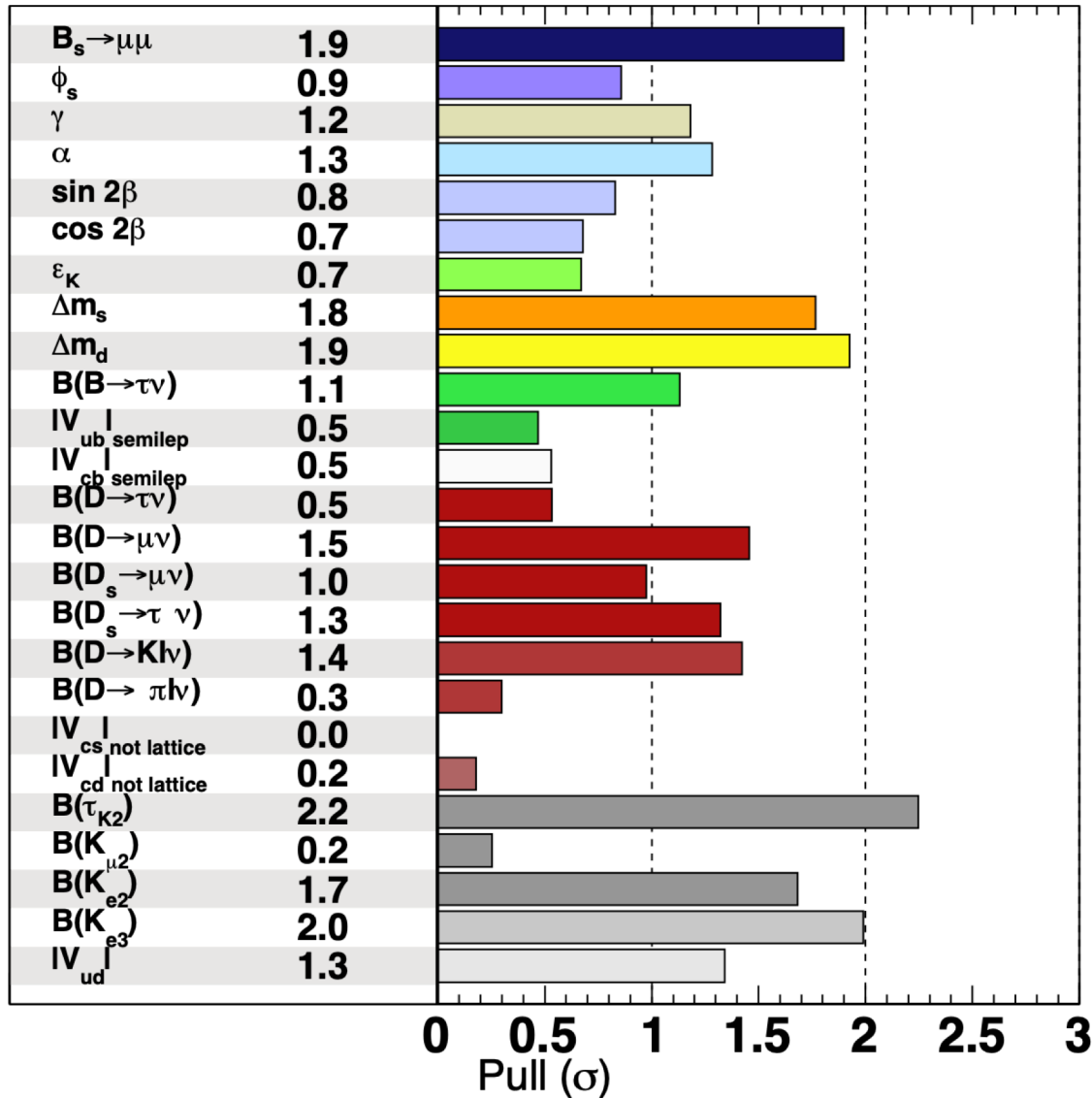
Semi-leptonic D decays

Leptonic D decays

Also other CKM elements + predictions for Br., ϕ_s etc.

	Direct	Indirect
$ V_{cs} $	$0.97508^{+0.00082}_{-0.00668}$	$0.97358^{+0.00015}_{-0.00026}$
$ V_{cd} $	0.2220 ± 0.0038	0.22483 ± 0.00030

Pull

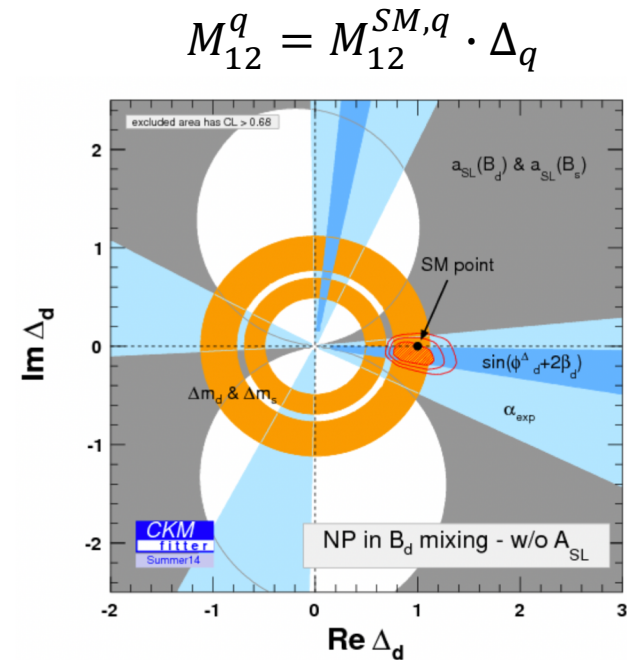
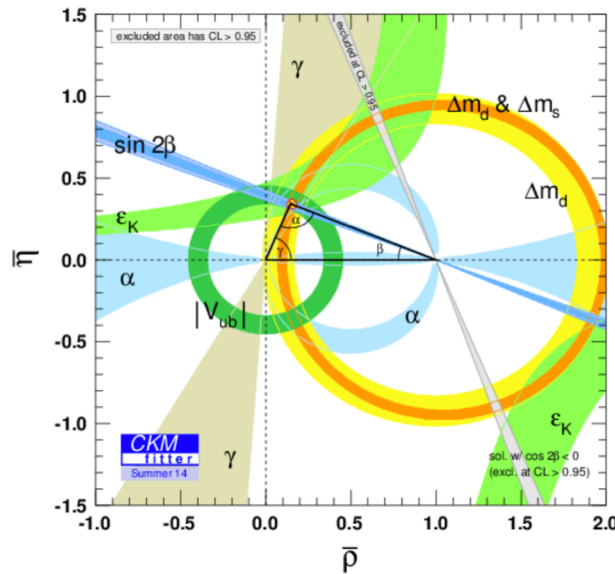


$$\text{Pull} = \sqrt{\chi_{\min}^2 - \chi_{\min, \text{!obser.}}^2}$$

No clear discrepancy seen

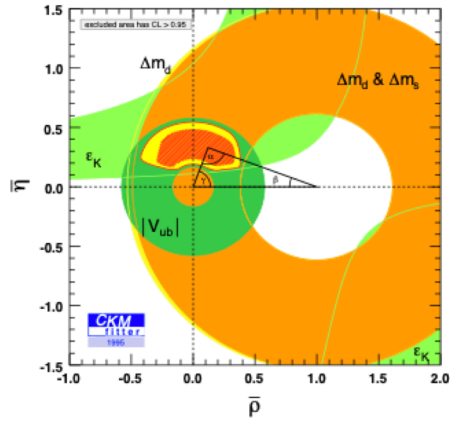
Some pull = 0 due to Rfit treatment of systematic uncertainties (theory)

- **CKMlive:** run dedicated analyses with the CKMfitter software
- **Your inputs:** set of related observables, theoretical and experimental inputs, fitting parameters and relations between them
- **Outputs:**

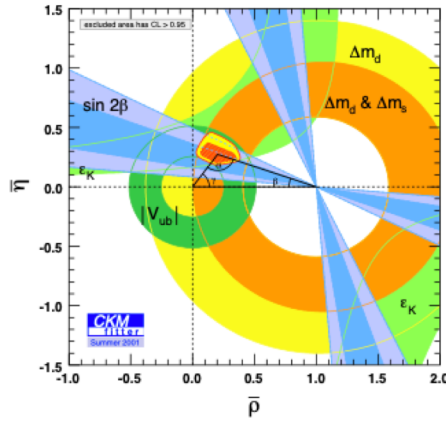


- **Supports:** ckmlive@clermont.in2p3.fr for questions; [tutorial](#) available

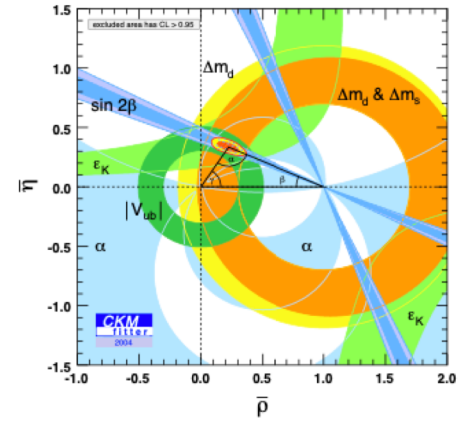
CKM status over years



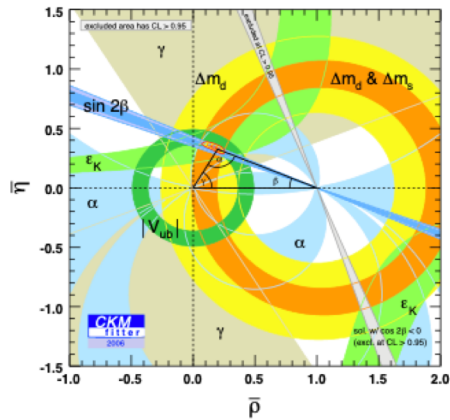
1995



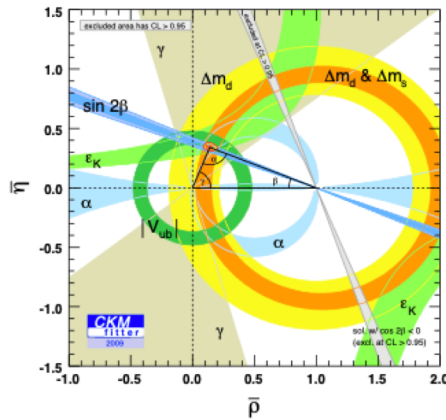
2001



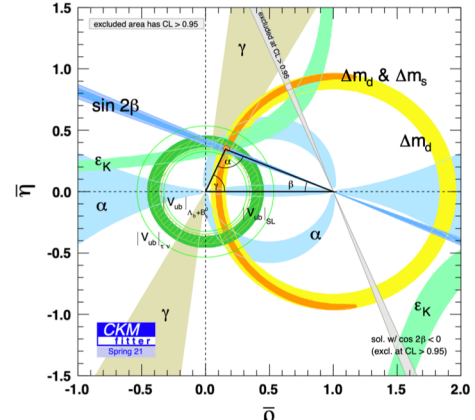
2004



2006



2009



2021

Thank you for your attention

Recent updates on V_{ud}

- Precision on $|V_{ud}|$ led by superallowed $0^+ \rightarrow 0^+$ nuclear β decays
- 2020 survey by Hardy and Towner, including recent calculations for radiative corrections:

$$\mathcal{F}t \equiv ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2G_V^2(1 + \Delta_R^V)}$$

Δ_R^V : process independent radiative correction

Reference	Δ_R^V (%)
Marciano and Sirlin [186] 2006	2.361 ± 0.038
Seng <i>et al.</i> [187,188] 2018/19	2.467 ± 0.022
Czarnecki, Marciano and Sirlin [189] 2019	2.426 ± 0.032
Adopted value	2.454 ± 0.019

OLD

Two new cal.

NEW (averaged)

δ_{NS} shifted G_V^2 to smaller values by 0.09%

Recent updates on V_{ud}

- Precision on $|V_{ud}|$ led by superallowed $0^+ \rightarrow 0^+$ nuclear β decays
- 2020 survey by Hardy and Towner, including recent calculations for radiative corrections:

$$\mathcal{F}t \equiv ft(1 + \delta'_R)(1 + \delta_{NS} - \delta_C) = \frac{K}{2G_V^2(1 + \Delta_R^V)}$$

$\mathcal{F}t$ merely changes

Uncertainties on $\mathcal{F}t$ larger (2.6 times) due to new theoretical terms in δ_{NS}